From quarks and gluons to the structure of hadrons

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March 2, 2016

Outline

Part I:

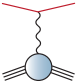
- **Introduction**
- **Bethe-Salpeter & Faddeev equations**
- **Form factors**
- **Tetraquarks**
- **Compton scattering**

Part II: Ongoing and future directions

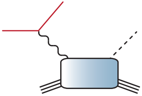
- **Meson & baryon spectroscopy**
- **Transition form factors**
- **Compton scattering, PDFs, GPDs**
- **From quarks and gluons to nuclei**

Introduction

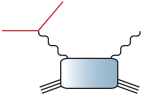
Ambitious experimental program underway with **JLab @ 12 GeV:**



Elastic scattering



Meson electro-production



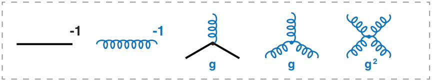
Compton scattering

- Search for **exotic mesons**
- Nucleon **form factors**, flavor separation, proton radius
- Nucleon **resonances** and transition form factors
- **Valence quark distributions** and flavor structure
- **Spatial and momentum tomography** of nucleons (GPDs, TMDs)
- Short-range structure of **nuclei,** quarks and gluons in the nucleus, EMC effect

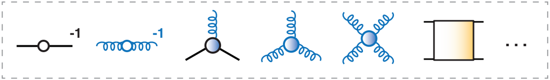
Introduction

QCD Lagrangian: $\mathcal{L} = \bar{\psi}\,(\partial\!\!\!/ + ig A\!\!\!/ + m)\,\psi + \frac{1}{4}\,F^a_{\mu\nu}\,F^{\mu\nu}_a$

- contains propagators and interactions:



- fully dressed n-point **Green functions** contain all **quantum effects:**

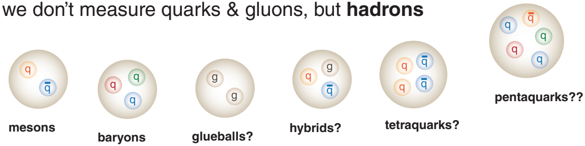


But!

- $\alpha(Q^2)$ becomes large at low momenta $\Longrightarrow$ need **nonperturbative methods!**

  Origin of **mass generation** and **confinement?**

- Quarks and gluons are **confined:** we don't measure quarks & gluons, but **hadrons**

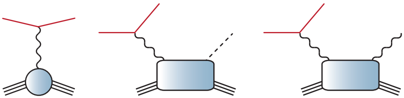


$\Longrightarrow$ need to understand **spectrum and interactions!**

Introduction

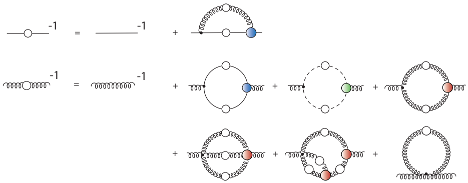
Requires **combined efforts** of experiment, theory and phenomenology:

- Amplitude analyses
- Hadronic reaction models
- Chiral effective field theory
- Microscopic approaches and models
- **Lattice QCD**



$$\langle 0 \,|\, T\, \psi(x)\, \bar{\psi}(y) \ldots |\, 0 \rangle = \int \mathcal{D}A\, \mathcal{D}\psi\, \mathcal{D}\bar{\psi}\, e^{-S}\, \psi(x)\, \bar{\psi}(y) \ldots$$

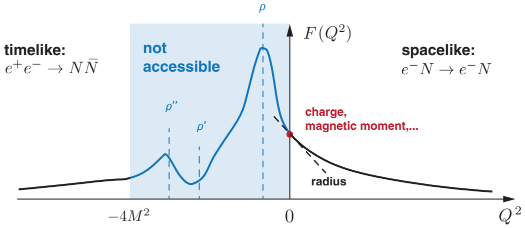
- **Dyson-Schwinger, Bethe-Salpeter, Faddeev equations**



- DSEs: quantum eqs. of motion
- nonperturbative, covariant
- all momentum scales, light and heavy quarks
- chiral symmetry
- truncations: model / neglect higher n-point functions to obtain **closed system**

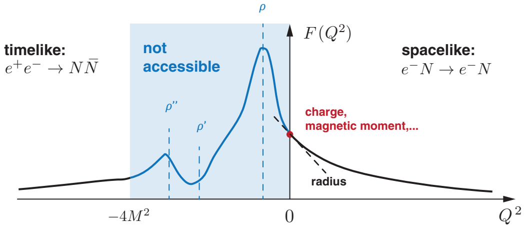
Introduction

Sketch of a generic electromagnetic form factor:

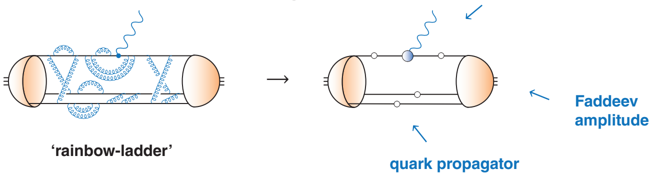


Introduction

Sketch of a generic electromagnetic form factor:



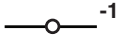
How can we calculate this from the **quark level?**

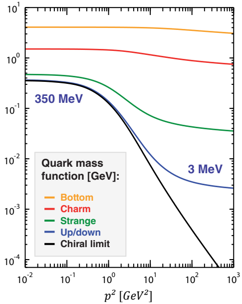


QCD’s Green functions

- **Quark propagator**

$$A(p^2)\,\big(i\not p + M(p^2)\big)$$

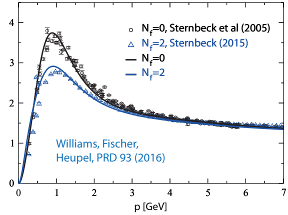




Dynamical chiral symmetry breaking generates ‘constituent-quark masses’

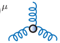
- **Gluon propagator**

$$\frac{D(p^2)}{p^2}\left(\delta^{\mu\nu} - \frac{p^\mu p^\nu}{p^2}\right)$$



- **Three-gluon vertex**

$$F_1\,[\,\delta^{\mu\nu}(p_1-p_2)^\rho + \delta^{\nu\rho}(p_2-p_3)^\mu + \delta^{\rho\mu}(p_3-p_1)^\nu\,] + \dots$$

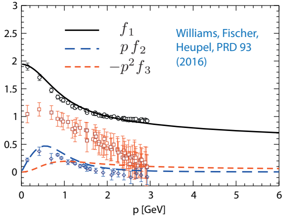


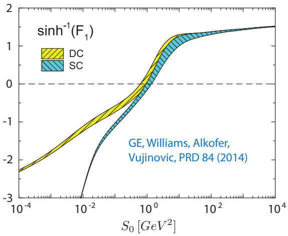
see also Huber & von Smekal, JHEP 1304 (2013)

Agreement between lattice, DSE & FRG within reach

- **Quark-gluon vertex**

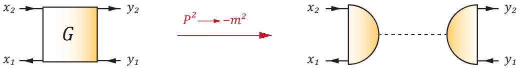
$$f_1\,\gamma^\mu + f_2\,ip^\mu + f_3\,p^\mu \not p + \dots$$





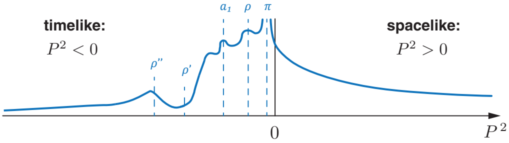
Hadrons?

- Hadron properties are encoded in **higher n-point functions**.
  For example, quark **four-point function** contains all possible **meson** poles:



$$\langle 0 \,|\, T\, \psi(x_1)\, \bar{\psi}(x_2)\, \psi(y_1)\, \bar{\psi}(y_2) \,|\, 0 \rangle \quad \sim \quad \sum \frac{\chi(q,P)\, \bar{\chi}(q',P)}{P^2+m^2}$$

"Bethe-Salpeter wave function"



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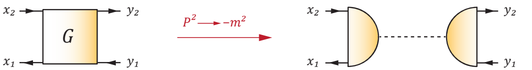

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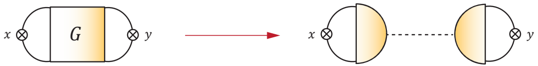
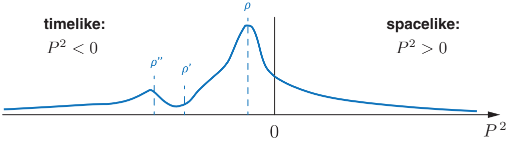
"Bethe-Salpeter wave function"

- **Lattice QCD:** construct gauge-invariant current correlators



$$\langle 0 \,|\, T\, \underbrace{\bar\psi(x)\, \mathcal{O}\, \psi(x)}_{J(x)}\, \underbrace{\bar\psi(y)\, \mathcal{O}\, \psi(y)}_{J(y)} \,|\, 0 \rangle \;=\; \int \mathcal{D}A\, \mathcal{D}\psi\, \mathcal{D}\bar\psi\; e^{-S}\, J(x)\, J(y) \quad \to \quad e^{-mt}$$

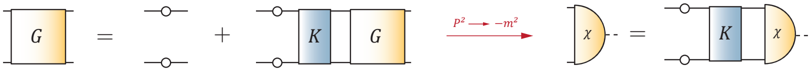


filter hadrons with definite J^{PC} and flavor quantum numbers,
pole in momentum space ⟹ exp. decay in Euclidean time

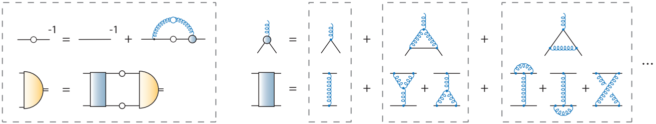
Hadrons?

- **Bethe-Salpeter approach:** use scattering equation to obtain G in the first place: $G = G_0 + G_0\,K\,G$

Homogeneous BSE for **BS wave function:**



- Kernel is connected to **quark Dyson-Schwinger equation** via **chiral symmetry** (can be derived from nPI effective action):



→ no **constant** quark mass unless NJL contact interaction
→ no **crossed-ladder** unless consistent quark-gluon vertex
→ cannot add **confinement** potential, drop **spin-orbit terms,** etc.

- In turn: em. gauge invariance, chiral symmetry, massless pion in chiral limit ... **for free**

Hadrons?

- **Bethe-Salpeter approach:** use scattering equation to obtain G in the first place: $G = G_0 + G_0\, K\, G$

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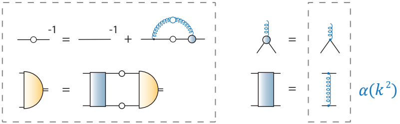
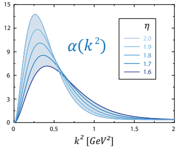




Maris, Roberts, Tandy, PRC 56 (1997), PRC 60 (1999)

Rainbow-ladder: gluon exchange with effective interaction

$$\alpha(k^2) = \alpha_{\text{IR}}\left(k^2/\Lambda^2, \eta\right) + \alpha_{\text{UV}}(k^2)$$

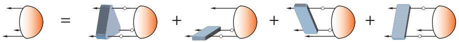
adjust scale Λ to observable, keep width η as parameter

see also: Qin, Chang et al., PRC 84 (1011), Binosi et al., PLB 742 (2015)

- In turn: em. gauge invariance, chiral symmetry, massless pion in chiral limit ... **for free**

Baryons

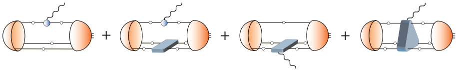
- Covariant Faddeev equation for **baryons:**
  keep 2-body interactions & rainbow-ladder,
  but no further approximations: $M_N = 0.94\,\text{GeV}$
  GE, Alkofer, Krassnigg, Nicmorus, PRL 104 (2010), GE, PRD 84 (2011)

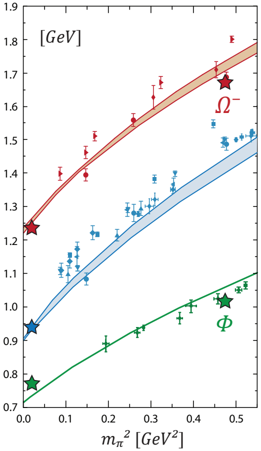


Relativistic bound states:
64 / 128 tensor structures for nucleon / Δ

- Octet & decuplet baryons, pion cloud effects,
  beyond rainbow-ladder
  Sanchis-Alepuz, Fischer, PRD 90 (2014), Sanchis-Alepuz, Fischer, Kubrak, PLB 733 (2014),
  Sanchis-Alepuz, Williams PLB 749 (2015)

- **Baryon form factors:**
  nucleon and Δ FFs, $N\to\Delta\gamma$ transition
  GE, PRD 84 (2011), Sanchis-Alepuz, Williams, Alkofer, PRD 87 (2013),
  Alkofer, GE, Sanchis-Alepuz, Williams, Hyp. Int. 234 (2015)



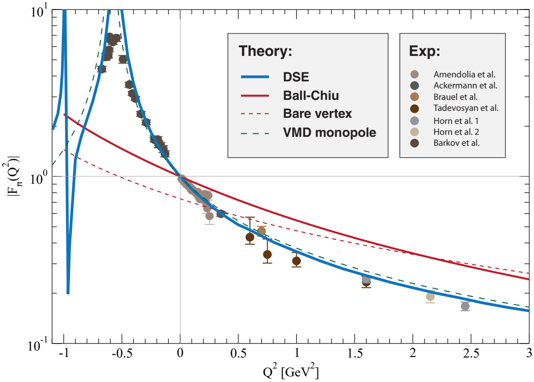


Delta:
Sanchis-Alepuz et al., PRD 84 (2011)

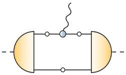
Nucleon:
GE, Alkofer, Krassnigg, Nicmorus, PRL 104 (2010);
GE, PRD 84 (2011)

ρ–meson:
Maris & Tandy, PRC 60 (1999)

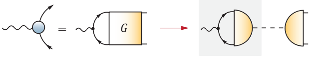
Pion form factor



A. Krassnigg (Schladming 2010),
Maris & Tandy, Nucl. Phys. Proc. Suppl. 161 (2006)

- Form factor from 

- **Timelike vector meson poles** automatically generated by quark-photon vertex BSE!



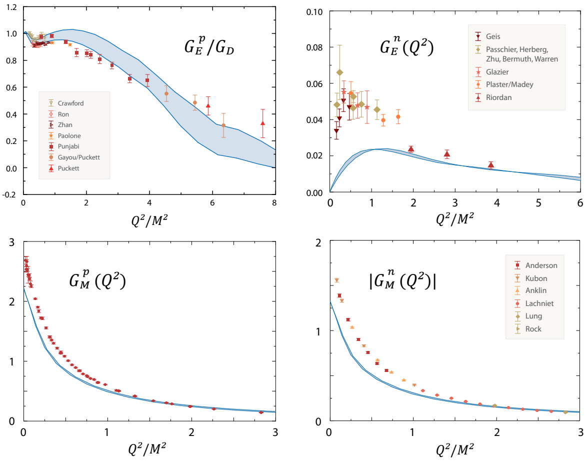
$\Rightarrow \Gamma^\mu =$ **Ball-Chiu** (em. gauge invariance)

$+$ **Transverse part** (vm. poles & dominance)

- Form factor at large Q^2
  Chang, Cloet, Roberts, Schmidt, Tandy, PRL 111 (2013)

- Include **pion cloud** effects:
  GE, Fischer, Kubrak, Williams, in preparation

Nucleon em. form factors



Three-body results:
all ingredients calculated,
model dependence shown
by bands GE, PRD 84 (2011)

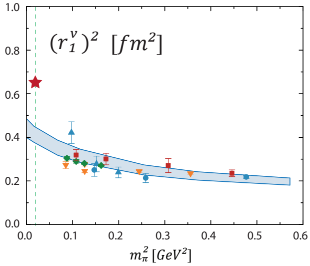
- **electric proton form factor:** consistent with data, possible zero crossing
- **magnetic form factors:** missing pion effects at low Q^2
- Similar for axial & ps. FFs, Δ elastic and $N\to\Delta\gamma$ transition
  GE, Fischer, EPJ A 48 (2012),
  Sanchis-Alepuz et al., PRD 87 (2013),
  Alkofer et al., Hyp. Int. 234 (2015)

$\Rightarrow$ **"quark core without pion-cloud effects"**

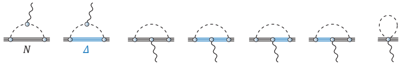
Nucleon em. form factors

Nucleon charge radii:

isovector (p-n) Dirac (F1) radius

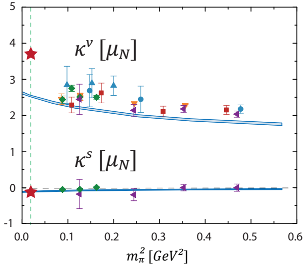


- **Pion-cloud effects** missing (⇒ divergence!), agreement with lattice at larger quark masses.



Nucleon magnetic moments:

isovector (p-n), isoscalar (p+n)



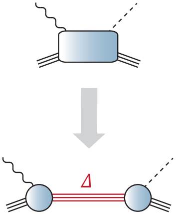
- **But:** pion-cloud **cancels** in κ^s ⇔ **quark core**

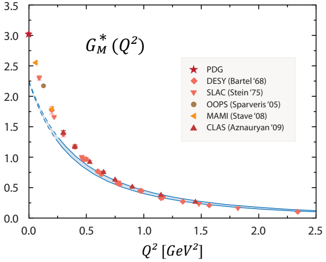
Exp: $\kappa^s = -0.12$

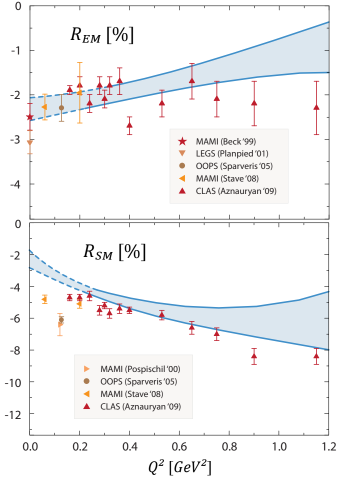
Calc: $\kappa^s = -0.12(1)$ **!!**

GE, PRD 84 (2011)

Nucleon-Δ-γ transition



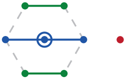


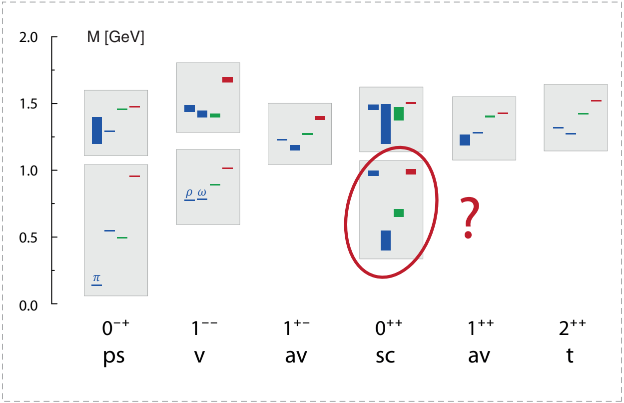


- **Magnetic dipole transition (G_M^*) dominant:** quark spin flip (s wave). "Core + 25% pion cloud"
- **Electric & Coulomb quadrupole ratios** small & negative, encode deformation. Reproduced without pion cloud: **OAM** from **p waves!**
  GE, Nicmorus, PRD 85 (2012)
- First three-body results similar
  Alkofer, GE, Sanchis-Alepuz, Williams, Hyp. Int. 234 (2015)

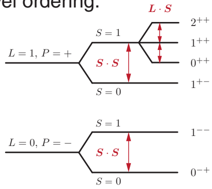
Meson spectrum

Light meson spectrum (PDG):
grouped with J^{PC} and flavor content

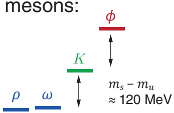




- Nonrelativistic level ordering:



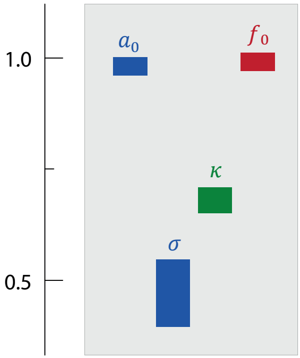
- Vector mesons:

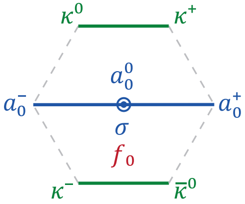


- **Pseudoscalar mesons?** spontaneous chiral symmetry breaking & axial anomaly
- **Scalar mesons?!**

Tetraquarks?

Light scalar (0^{++}) **mesons** don't fit into the conventional meson spectrum:





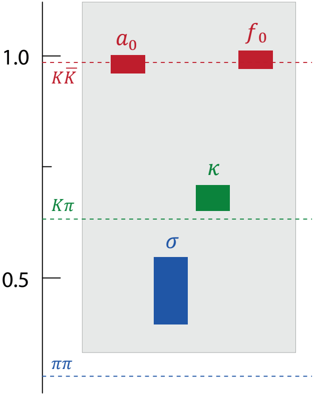
f_0 (980 MeV) $s\bar{s}$
κ (680 MeV) $u\bar{s}, d\bar{s}$
a_0 (980 MeV), σ (500 MeV) } $u\bar{u}, d\bar{d}, u\bar{d}$

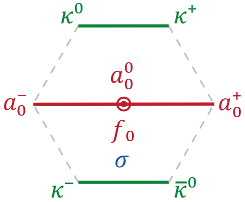
- Why are a_0, f_0 mass-degenerate?
- Why are their **decay widths** so different?

  $\Gamma(\sigma, \kappa) \approx 550$ MeV
  $\Gamma(a_0, f_0) \approx 50$–$100$ MeV
- Why are they so **light**?
  Scalar mesons ~ **p-waves,** should have masses similar to axialvector & tensor mesons ~ 1.3 GeV

Tetraquarks?

What if they were **tetraquarks** (diquark-antidiquark)? Jaffe 1977, Close, Tornqvist 2002, Maiani, Polosa, Riquer 2004



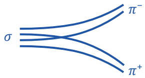


f_0 (980 MeV), a_0 (980 MeV) } $us\overline{us}$, ...

κ (800 MeV) $us\overline{ud}$, ...

σ (500 MeV) $ud\overline{ud}$

- Explains **mass ordering** & **decay widths**:
  f_0 and a_0 couple to K$\bar{\text{K}}$, large widths for σ, κ

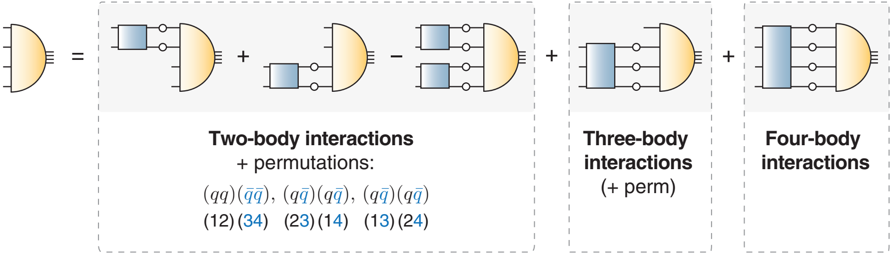


- Alternative: **meson molecules?**
  Weinstein, Isgur 1982, 1990; Close, Isgur, Kumano 1993

- Support for **non-q$\bar{\text{q}}$ nature** of σ from dispersive analyses, unitarized ChPT, large Nc, extended linear σ model, quark models
  Pelaez 2004, Weinberg 2013, Cohen, Llanes-Estrada, Pelaez, Ruiz de Elvira 2014, Londergan, Nebreda, Pelaez, Szczepaniak 2014, Parganlija, Giacosa, Rischke 2010, . . .

Tetraquarks

Four-quark bound-state equation:



Keep **two-body interactions** with **rainbow-ladder kernel:**
well motivated by many other studies, tetraquark is **s-wave**

Structure of the amplitude

General structure of **Bethe-Salpeter amplitude** $\Gamma(p,q,k,P)$ complicated:

$$\Gamma(p,q,k,P) = \sum_i f_i\left(p^2, q^2, k^2, \quad \ldots \quad\right) \quad \tau_i(p,q,k,P) \quad \otimes \quad \text{2 Color tensors} \quad \otimes \quad \text{Flavor}$$

9 Lorentz invariants

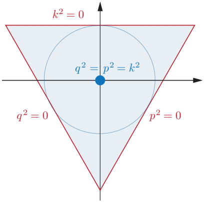
256 Dirac-Lorentz tensors

2 Color tensors: $3 \otimes \bar{3}$, $6 \otimes \bar{6}$ or $1 \otimes 1$, $8 \otimes 8$ (Fierz-equivalent)

Arrange Lorentz invariants into
multiplets of permutation group S4:
GE, Fischer, Heupel, PRD 92 (2015)

$$\Rightarrow f_i\left(\mathcal{S}_0, \triangledown, \triangle, \bigcirc\right)$$

- **Singlet:** $\mathcal{S}_0 = \frac{1}{4}(p^2+q^2+k^2)$
- **Doublet:** $\mathcal{D}_0 = \frac{1}{4\mathcal{S}_0}\begin{bmatrix} \sqrt{3}\,(q^2-p^2) \\ p^2+q^2-2k^2 \end{bmatrix}$
- **2 Triplets:** $\triangle$, $\bigcirc$



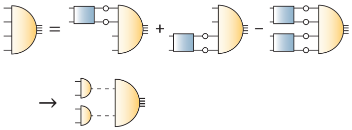
Keep **s waves** only:
Fierz-complete, **16** tensors:

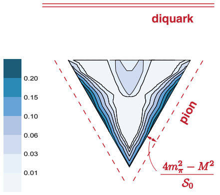
e.g. $\left\{ \begin{matrix} C^T\gamma_5 \otimes \gamma_5 C \\ C^T\gamma^\mu \otimes \gamma^\mu C \\ \ldots \end{matrix} \right\}$ in (12)(34)

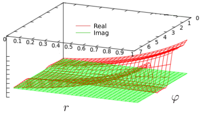
automatically includes also
$\gamma_5 \otimes \gamma_5$ in (23)(14), (31)(24)

Four quarks $\Rightarrow$ meson molecule

- Four-quark equation dynamically generates **pion poles** outside the integration domain







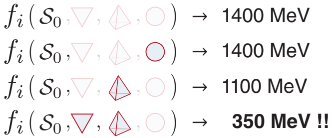
GE, Fischer, Heupel, PLB 753 (2016)

- Dynamical formation of a **'meson molecule'**, diquarks almost irrelevant!

- Pion poles drive tetraquark mass from 1.4 GeV to **~350 MeV**

- Poles enter integration domain above threshold $M > 2m_\pi$: the tetraquark becomes a **resonance**

- Four-body equation generates **bound state** together with its **decay channels!**

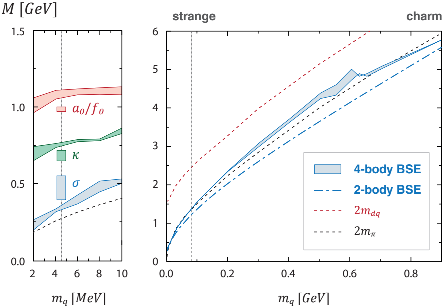


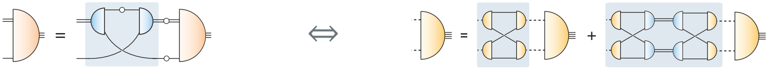
Four quarks ⇒ meson molecule

Same pattern for multiplet partners:
GE, Fischer, Heupel, PLB 753 (2016)

$\sigma,\ \kappa,\ a_0/f_0$ ~ **350**, **750**, **1080 MeV**

- Light scalar 'mesons' are light because they 'feel' **Goldstone nature of** π, η, K
- σ is resonance close to $\pi\pi$ threshold, becomes bound state in charm region
- Similar results from **meson-meson / diquark-antidiquark** two-body equations: analogous to quark-diquark model for baryons



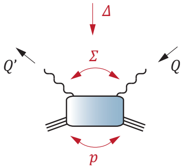


Oettel, Hellstern, Alkofer, Reinhardt, PRC 58 (1998)
GE, Cloet, Alkofer, Krassnigg, Roberts, PRC 79 (2009)

Heupel, GE, Fischer, PLB 718 (2012)

Same mechanism: **baryons** dominated by **diquarks, tetraquarks** by **pseudoscalar mesons**.
Resolves problem with dq-$\overline{\text{dq}}$ interpretation: ' 2 x 800 MeV - binding energy ' ~ 500 MeV?!

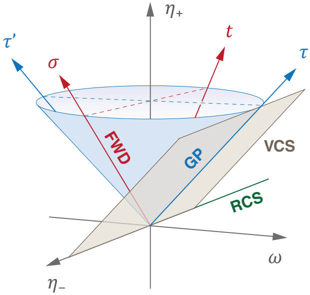
Compton scattering



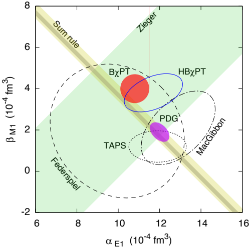
Four independent variables:

$$\eta_+ = \frac{Q^2 + Q'^2}{2m^2}, \quad \eta_- = \frac{Q\cdot Q'}{m^2},$$

$$\omega = \frac{Q^2 - Q'^2}{2m^2}, \quad \lambda = \frac{p\cdot\Sigma}{m^2}$$



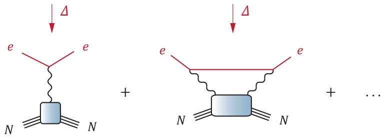
- **RCS:** nucleon polarizabilities
  Krupina & Pascalutsa, PRL 110 (2013)

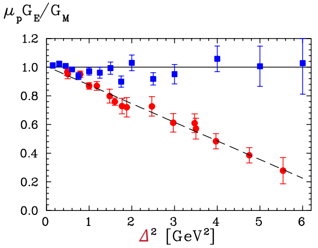


- **VCS:** generalized polarizabilities
- **DVCS:** handbag dominance, GPDs
- **Forward limit:** structure functions in DIS
- **Timelike region:** $p\bar{p}$ annihilation at PANDA
- **Spacelike region:** two-photon corrections to nucleon form factors, proton radius puzzle?

Compton scattering

- **Two-photon corrections to form factors:**
  can explain difference between Rosenbluth and polarization transfer measurements
  Guichon, Vanderhaeghen, PRL 91 (2003)



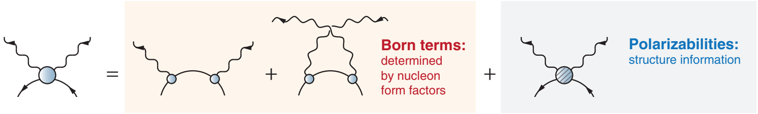


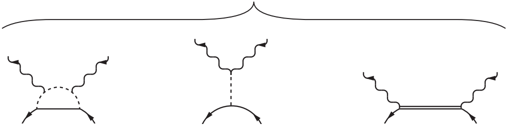
Arrington, Blunden, Melnitchouk
Prog. Part. Nucl. Phys. 66 (2011)

- **Proton radius puzzle:**
  can 2γ corrections explain difference between electron and muon measurements?
  So far: probably not, but . . .
  Carlson, Vanderhaeghen, 2011
  Birse, McGovern, EPJ A 48 (2012)

Compton scattering ...

Compton amplitude = sum of **Born terms + 1PI structure part:**

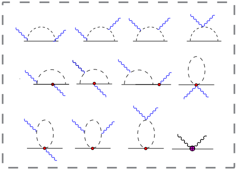




„Pion cloud“
(ChPT)

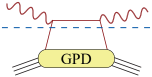
t-channel
meson exchange
$(\pi, \sigma, a_1, \ldots)$

s/u-channel
nucleon resonances
$(\Delta, N^*, \ldots)$



Griesshammer, McGovern, Phillips, Feldman,
Prog. Part. Nucl. Phys. 67 (2012)

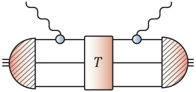
but also:



⇒ is there a common underlying
quark-level description?

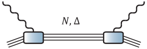
... at the quark level

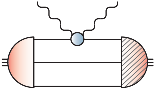
Closed expression for Compton amplitude at quark level
(here: rainbow-ladder, modulo crossing & permutation) GE, Fischer, PRD 85 (2012) & PRD 87 (2013)



s- and u-channel
nucleon resonances:

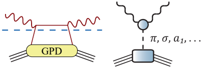
approximate this
(include all resonances)

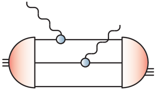




"Impulse approximation:"

calculate this: reproduces
handbag diagrams +
t-channel **meson poles**





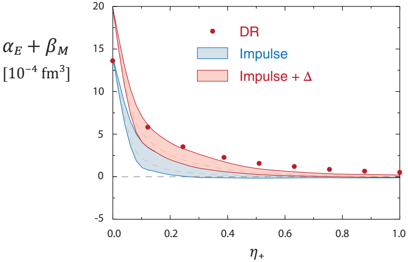
cat's ears diagrams

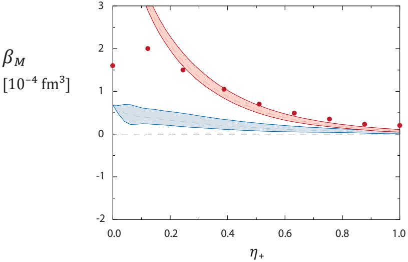
neglect this
(for the moment)

- √ crossing symmetry
- √ em. gauge invariance
- √ perturbative processes included
- √ s, t, u channel poles generated in QCD

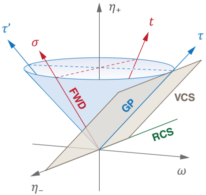
But only **sum** is **gauge invariant**, not individual diagrams ⇒ problem!
Solved by projecting onto full tensor basis (transverse + gauge)

Proton polarizabilities





First results: GE, 1601.04154



- bands = results inside cone (70% of radius)
- compared to GPs from dispersion relation
  Pasquini et al., EPJ A11 (2001),
  Downie & Fonvieille, EPJ ST 198 (2011)
- Impulse approximation:
  α_E dominated by handbag,
  β_M small due to cancellation

  $\Rightarrow$ **cf. meson electroproduction: "QCD background"!**
- Large $\varDelta$ contribution to β_M, expect large pion effects! Pascalutsa, Phillips, PRC 68 (2003), . . .

In total: polarizabilities ≈

Impulse app. (handbag + t-channel poles)
+ nucleon resonances (mostly $\varDelta$)
+ pion cloud (at low η_+)?

Summary Part I

Nucleon and Delta masses & form factors

- microscopic description works reasonably well, improvements underway
- need to include meson cloud effects
- nucleon resonances? → see Part II

Light scalar mesons as **tetraquarks**

- transition from four quarks to “meson molecule”
- resonances!
- many future applications → see Part II

Compton scattering

- hadronic vs. quark-level decomposition (general!)
- polarizabilities
- meson electroproduction from quark level? → see Part II

Outline

Part I:

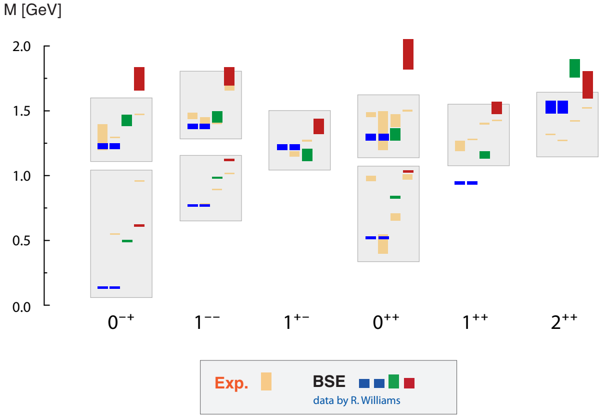
- **Introduction**
- **Bethe-Salpeter & Faddeev equations**
- **Form factors**
- **Tetraquarks**
- **Compton scattering**

Part II: Ongoing and future directions

- **Meson & baryon spectroscopy**
- **Transition form factors**
- **Compton scattering, PDFs, GPDs**
- **From quarks and gluons to nuclei**

Meson spectroscopy

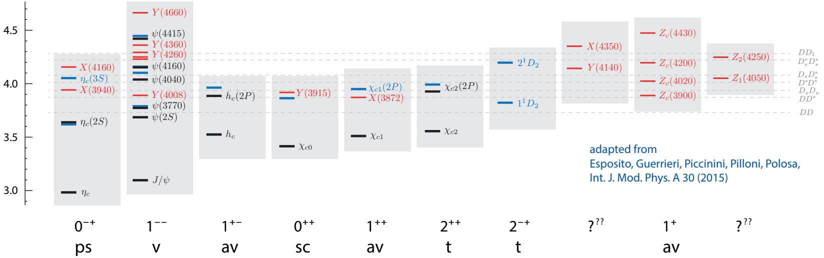
- **Beyond rainbow-ladder** is becoming state-of-the-art for **mesons**
  Fischer, Williams, PRL 103 (2009); Chang, Roberts, PRL 103 (2009); Williams, Fischer, Heupel, PRD 93 (2016)
- **Light meson spectrum** beyond rainbow-ladder:
  Sanchis-Alepuz, Williams, PLB 749 (2015)



- Gluon propagator, qg and ggg vertex solved in the process, no need for model interaction!
- **Radial excitations** and **exotics** now in right ballpark
- Recent results: **scalar mesons** above 1 GeV
  Williams, Fischer, Heupel, PRD 93 (2016)

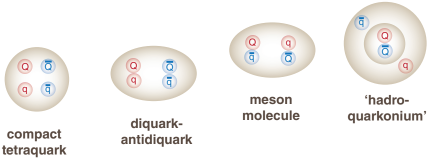
Meson spectroscopy

- **Tetraquarks** in **charmonium & bottomonium** spectrum: X(3872), Y(4260), charged Z states?



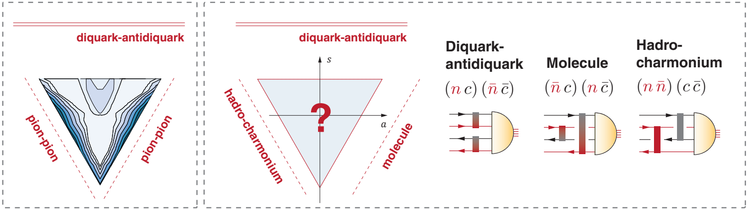
adapted from
Esposito, Guerrieri, Piccinini, Pilloni, Polosa,
Int. J. Mod. Phys. A 30 (2015)

- Can we **distinguish** different tetraquark configurations?

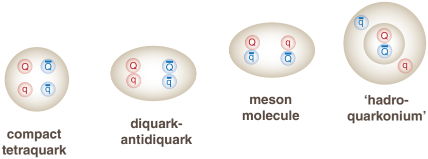


Meson spectroscopy

- Four-body BSE dynamically determines **strengths** of these components: four quarks rearrange themselves into dq-$\overline{\text{dq}}$, molecule, hadroquarkonium



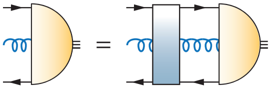
- Can we **distinguish** different tetraquark configurations?



Meson spectroscopy

- **Glueball** calculations underway
  Meyers, Swanson, PRD 87 (2013), Sanchis-Alepuz, Fischer, Kellermann, von Smekal, PRD 92 (2015)

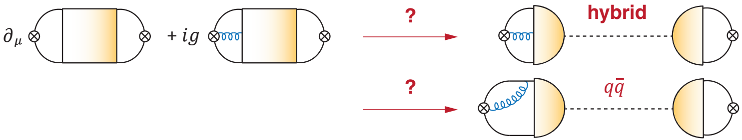
- **Hybrid mesons** with DSEs and BSEs:
  Collab. with C. Fischer, J. Segovia, R. Williams



  Study both within **three-body** and **"quark-diquark"** inspired approach:
  $q\bar{q}$ octet repulsive, qg triplet attractive

  Application to **hybrid baryons?**

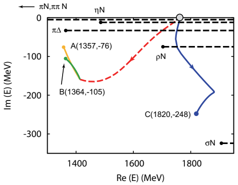
- **Exotic quantum numbers:** can be obtained with BSEs, but not on lattice?



Baryon spectroscopy

I \ J^P	$1/2^+$	$3/2^+$	$1/2^-$	$3/2^-$	...
$1/2$	P_{11} N(940) N(1440) N(1710) N(1880)	P_{13} N(1720) N(1900)	S_{11} N(1535) N(1650) N(1895)	D_{13} N(1520) N(1700) N(1875)	

- **"Missing resonances":** three-quark vs. quark-diquark composition?
- **Roper:** level ordering? 1st radial excitation of nucleon? Gluonic excitation? Molecule?
- "Quark core" vs. meson-baryon **coupled channel effects?**
- **Hybrid baryons?**
- Connection to quark-gluon substructure in QCD?



Suzuki et al., PRL 104 (2010)

Baryon spectroscopy

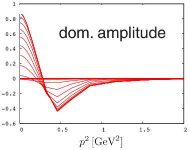
Nucleon resonances:
efforts with quark-diquark and three-quark approaches underway

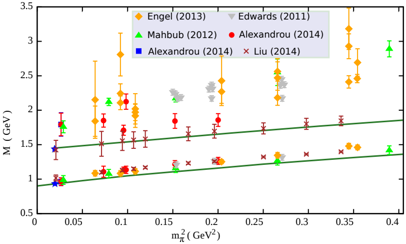
- Roper: 1st **radial excitation of nucleon**
  Segovia, El-Bennich, Rojas, Cloet, Roberts, Xu, Zong, PRL 115 (2015)

- First results from three-quark equation
  Sanchis-Alepuz, GE, Fischer, in preparation

$M_{N^*} = 1.45\,\text{GeV}$

	Nucleon	Roper
s-wave	66%	15%
p-wave	33%	61%
d-wave	1%	24%



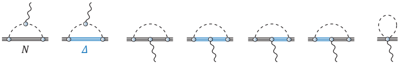


- **Same trend** for $N(1535)$, $\Delta(1620)$, $\Delta(1700)$, $\Delta(1910)$
  (dominated by negative-parity diquarks) GE, 1602.03462

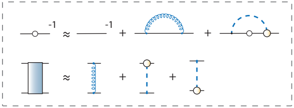
Baryon spectroscopy

Meson-baryon interactions?

- Calculate **“quark core”**, assume that chiral interactions will provide the rest. (But what is the core? Where do we stop?)

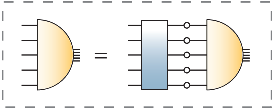


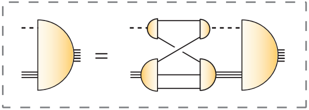
- Implement **quark-gluon topologies** that produce pion cloud effects... difficult! 4PI?



- Implement effective **pion cloud** at quark level
  Fischer, Nickel, Wambach, PRD 76 (2007), Cloet, Bentz, Thomas, PRC 90 (2014), Sanchis-Alepuz, Fischer, Kubrak, PLB 733 (2014)

- New avenue? Solve **five-quark equation** ⇒ system will dynamically rearrange itself into $N\pi, \dots$ if this is dominant





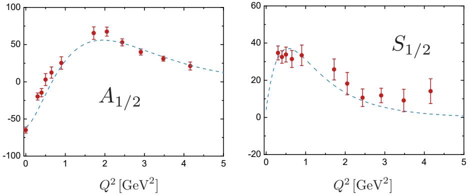
- Analogous to $\Lambda(1405)$: three-quark state or molecule?
  Hall et al., PRL 114 (2015)

- LHCb pentaquark?
  Aaij et al., PRL 115 (2015)

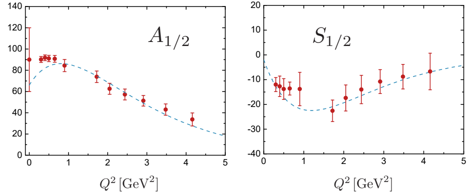
Technical challenge: **resonances!** Luescher method, finite volume?

Transition form factors

$N^*(1440)$



$N^*(1535)$



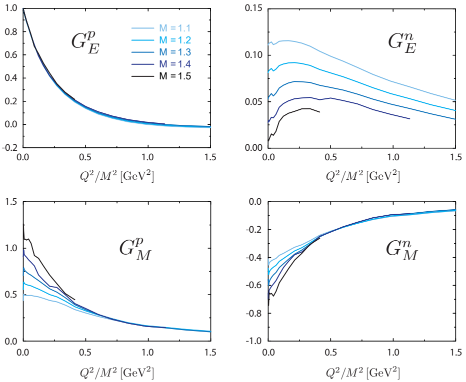
in $[10^{-3} GeV^{-1/2}]$

CLAS Aznauryan et al., PRC 80 (2009)
MAID Tiator et al., EPJ ST 198 (2011)

High-precision data on $N \to \gamma N^*$ transition form factors available. Theory predictions?

- **Roper:** in ballpark of data, missing pion cloud Segovia et al., PRL 115 (2015)
- $N(1535)$, etc.?

Transition form factors



High-precision data on $N \to \gamma N^*$ transition form factors available.
Theory predictions?

- **Roper:** in ballpark of data, missing pion cloud Segovia et al., PRL 115 (2015)
- N(1535), etc.? Example: N(1535) **elastic FF** (preliminary)

  Q^2 range limited by quark + diquark "threshold":

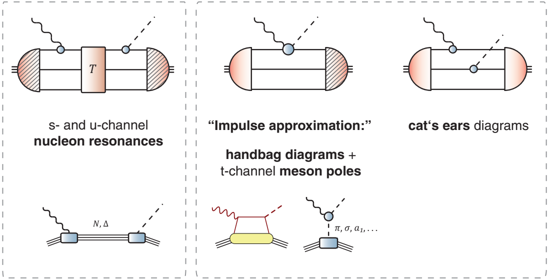
  $$Q^2 < 4\left[(m_q + m_{\text{ps}})^2 - M^2\right]$$

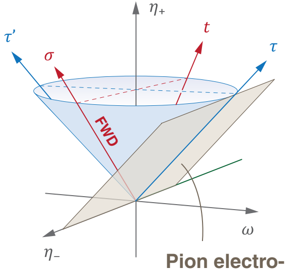
  Trick: calculate FFs with lower M, approach calculated mass from below.
  "Neutron" charge → 0: gauge invariance ok!
- Implement **pion cloud effects:** start with pion electromagnetic FF
  GE, Fischer, Kubrak, Williams, in preparation

Pion electroproduction

Extraction of nucleon resonances from electroproduction amplitudes
depends on knowledge of non-resonant **“QCD background”**

- decomposition analogous to Compton scattering
  (there: large contribution to $\alpha + \beta$!)





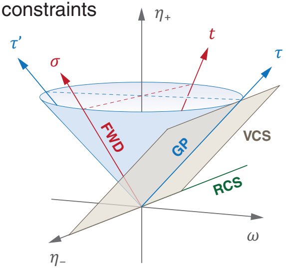
Pion electro-production ≙ VCS plane

- same phase space, but relevant kinematic region difficult to access
  ⇒ effective **scaling behavior,** similar to Compton scattering?
  ⇒ need amplitude decomposition with correct implementation of gauge invariance & analyticity

Compton scattering

Formalism for Compton scattering at quark level established:

- **Amplitude decomposition** with correct implementation of gauge invariance & analyticity
- **t-channel meson poles** reproduced GE, Fischer, PRD 87 (2013)
- general form of $N \to \gamma N^*$ **transition form factors** w/o kinematic constraints **and** offshell extension (→ for hadronic reaction models)
  GE, Ramalho, in preparation
- **scalar polarizabilities** GE, 1601.04154



Next steps:

- **spin polarizabilities**
- **two-photon corrections** to nucleon electromagnetic FFs
- **proton radius puzzle?**

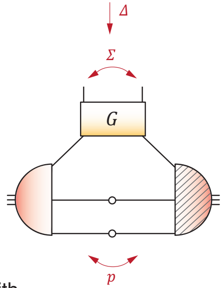
- **RCS/WACS:** handbag vs. pQCD? Fanelli et al., PRL 115 (2015)
- **DVCS,** generalized parton distributions
- Forward limit and nucleon structure functions
- Timelike Compton scattering?

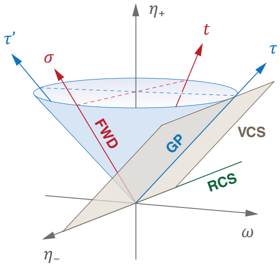
Nucleon structure

- Non-local four-point correlator:
  Wigner distribution, "mother" of TMDs, GPDs, PDFs, FFs

$$W(p,\Delta,z) = \langle\, N \,|\, \psi(\tfrac{z}{2})\, \bar{\psi}(-\tfrac{z}{2}) \,|\, N' \,\rangle$$

$$W(p,\Delta,\Sigma) =$$





- With **gauge link:** sum of diagrams with arbitrarily many gluons coming out of G

- At same light-cone time $z^+ = 0$:
  Lorcé, Pasquini, Vanderhaeghen, JHEP 1105 (2011)

TMDs $(\Sigma_+, \Sigma_\perp)$ → **PDFs** (Σ_+)

GPDs (Σ_+, Δ) → **PDFs** (Σ_+), **FFs** (Δ)

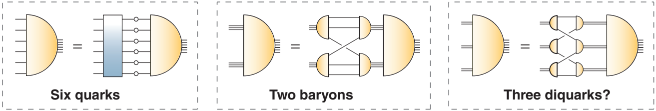
- Same kinematics as in **Compton amplitude:**
  4 Lorentz invariants, 32 "form factors"

- Establish basis free of kinematic constraints
  ⇒ only singularities in "FFs" are dynamical
  ⇒ facilitates extrapolating DSE results & modeling

- DSE studies for **pion GPDs** underway:
  Mellin moments → double distributions → GPDs
  Mezrag, Moutarde, Rodriguez-Quintero, 1602.07722

From quarks and gluons to nuclei

Transition from **quark-gluon to nuclear degrees of freedom**
⇒ generalize tetraquark studies to pentaquarks and hexaquarks



- only input are quarks and gluons
- dynamical generation of hadron poles, system is dominated by lowest-lying poles
- resonances!

Numerous open questions:

- Six ground states, one of them **deuteron** Dyson, Xuong, PRL 13 (1964)
- Dibaryon vs. **hidden-color** configurations? Bashkanov, Brodsky, Clement, PLB 727 (2013)
- d*(2380), H-Dibaryon?
- **Deuteron form factors** from quark level?
- **Microscopic origins of nuclear binding?**

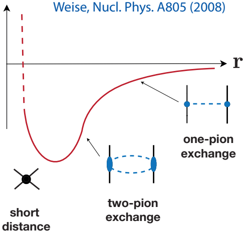
⇒ complementary to ongoing **lattice calculations!**
NPLQCD collab.: Beane et al., PRD 87 (2013), . . .

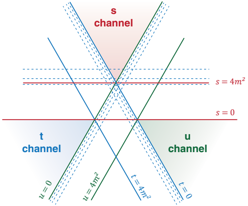
From quarks and gluons to nuclei

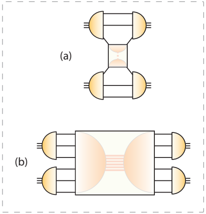
NN interaction?

- **NN potential:** long-range pion exchange vs. short-range repulsive core
- **Nuclear ab-initio calculations**
  Bedaque, van Kolck 2002, Bogner, Furnstahl, Schwenk 2010, Machleidt & Entem 2011, Epelbaum, Meissner 2012, Carlson et al. 2015, Vary et al. 2015
- **Lattice** (HAL-QCD): calculate scattering matrix, retroactively extract NN potential Aoki et al. 2012
- **Microscopic decomposition** analogous to FFs and other scattering amplitudes:

NN potential
Weise, Nucl. Phys. A805 (2008)







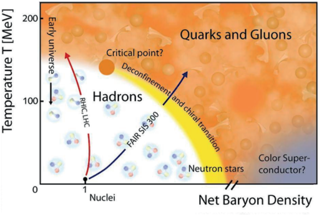
- only input are quarks and gluons
- **quark interchange** and **pion exchange** automatically included
- **dibaryon** exchanges

From quarks and gluons to nuclei

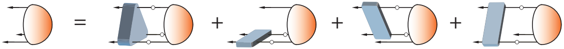
Nucleons in a nuclear environment:

- **Baryon effects** important at high nuclear densities: phase structure of QCD, critical endpoint?
- **Nucleon form factors** and **structure functions:** medium effects vs. short-range structure? Underlying QCD mechanism?
- Finite T and μ: DSE, FRG, model studies
  Qin et al., PRL 106 (2011), Fischer, Luecker, PLB 718 (2013), Wang et al., PRD 87 (2013), Cloet, Bentz, Thomas, PLB 642 (2006), ...

  Baryon back-reaction on phase diagram
  GE, Fischer, Welzbacher, PRD 93 (2016)



Need to solve **meson BSE** and **baryon Faddeev equation** at T and μ, analyze phase structure, calculate form factors and structure functions



Summary Part II: near future

Meson spectroscopy:

- beyond rainbow-ladder
- tetraquarks, glueballs, hybrids

Baryon spectroscopy:

- nucleon resonances & transition form factors
- microscopic background in electroproduction

Hadron structure:

- Compton scattering and its (many) applications
- Longitudinal, transverse and spin structure: GPDs, TMDs, PDFs

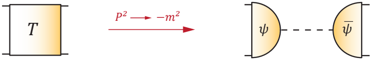
QCD and nuclei:

- Dibaryons and NN interaction from quark level
- Medium modifications of form factors and structure functions

Backup slides

Bethe-Salpeter equations

- Extract hadron properties from **poles** in $q\bar{q}$, qqq, $qq\overline{qq}$ **scattering matrices:**



- defines onshell **Bethe-Salpeter amplitude.** Simplest example: **pion**

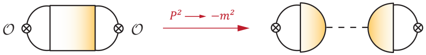
$$\psi(q,P) \;=\; \gamma_5 \;\big(\, f_1 + f_2\, \not{P} + f_3\, \not{q} + f_4\, [\not{q}, \not{P}] \,\big) \;\otimes\; \text{Color} \;\otimes\; \text{Flavor}$$

most general Dirac-Lorentz structure,
Lorentz-invariant dressing functions:

$$f_i = f_i(q^2, q\cdot P, P^2 = -m^2)$$

$\Longrightarrow$ pion is made of **s waves** and **p waves!**
(relative momentum ~ orbital angular momentum)

- Same in **lattice QCD:** construct gauge-invariant current correlators

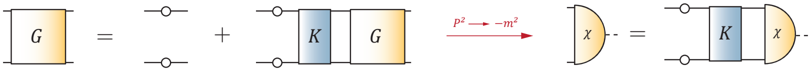


$$\langle (\bar{\psi}\,\mathcal{O}\,\psi)_x\, (\bar{\psi}\,\mathcal{O}\,\psi)_y \rangle = \int \mathcal{D}[\psi,\bar{\psi},A]\, e^{-iS}\, (\bar{\psi}\,\mathcal{O}\,\psi)_x\, (\bar{\psi}\,\mathcal{O}\,\psi)_y \quad \longrightarrow \quad e^{-mt}$$

Hadrons?

- **Bethe-Salpeter approach:** use scattering equation to obtain G in the first place: $G = G_0 + G_0\, K\, G$

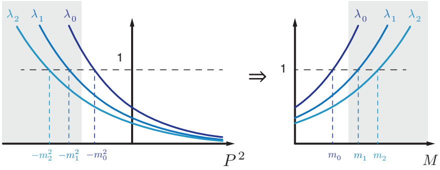
Homogeneous BSE for **BS wave function:**

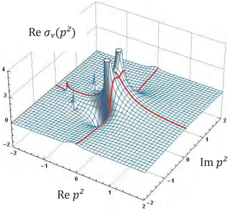


- BS wave function only makes sense **onshell**, but homogeneous BSE = **eigenvalue equation**, can be solved for offshell momenta:

$$K\,\psi_i = \lambda_i(P^2)\,\psi_i\,, \qquad \lambda_i \xrightarrow{P^2 \to -m_i^2} 1$$

Largest eigenvalue ⇔ ground state,
smaller ones ⇔ excitations





Restricted by singularities in **quark propagator** (no **physical threshold!**):

mesons: $M < 2m_p$
baryons: $M < 3m_p$

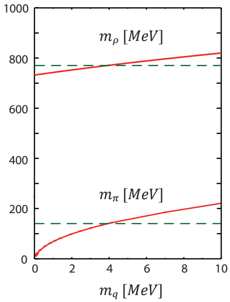
$m_p \sim 500 MeV$

Mesons

- **Dynamical chiral symmetry breaking** generates "constituent- quark masses"

$$S_0(p) = \frac{-i\not{p}+m}{p^2+m^2} \longrightarrow S(p) = \frac{1}{A(p^2)}\,\frac{-i\not{p}+M(p^2)}{p^2+M^2(p^2)}$$

- Pion is **Goldstone boson:** $m_\pi{}^2 \sim m_q$

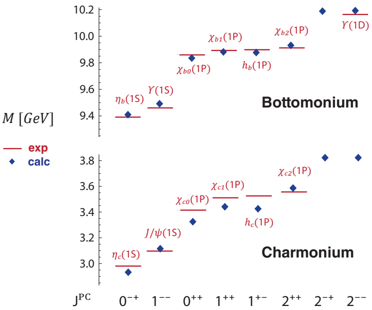


- Rainbow-ladder works well for **pseudoscalar & vector mesons:** masses, form factors, decays, ...

Maris, Roberts, Tandy, PRC 56 (1997), PRC 60 (1999);
Bashir et al., Commun. Theor. Phys. 58 (2012)

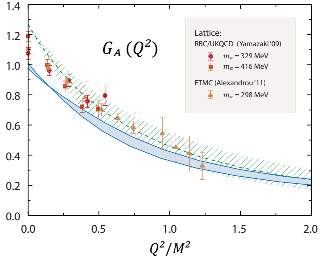
- Also **heavy mesons**

Fischer, Kubrak, Williams, EPJ A 51 (2015), Hilger et al., PRD 91 (2015)



Blank, Krassnigg, PRD 84 (2011)

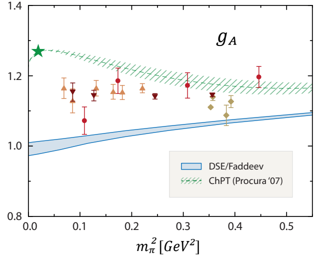
Axial form factors

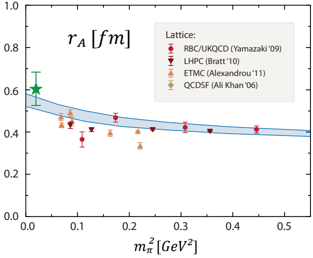


- looks like magnetic form factors:
  missing structure at low $Q^2 \Rightarrow g_A$ **too small**
- **Timelike meson poles:**
  a_1 in G_A, π & $\pi(1300)$ in G_P , $G_{\pi NN}$
- **Goldberger-Treiman relation**
  reproduced for **all** quark masses:

$$G_A(0) = \frac{f_\pi}{M_N}\,G_{\pi NN}(0)$$

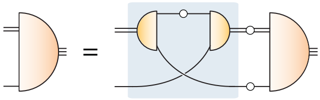
GE & Fischer, EPJ A 48 (2012)





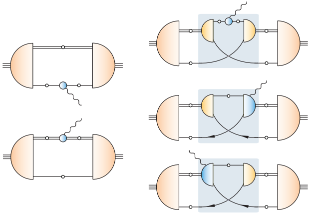
Quark-diquark model

- Assumption: separable qq scattering matrix ⇒ Faddeev equation simplifies to **quark-diquark BSE**



Oettel, Hellstern, Alkofer, Reinhardt, PRC 58 (1998),
Cloet, GE, El-Bennich, Klahn, Roberts, FBS 46 (2009)
Segovia, Cloet, Roberts, Schmidt, FBS 55 (2014)

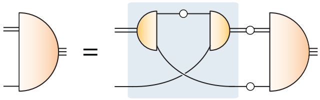
- **Quark exchange** between quark & diquark binds nucleon
- Calculate all quark and diquark ingredients from quark level ⇒ direct link to quark-gluon interaction! Rainbow-ladder: **scalar diquark ~ 800 MeV, axialvector diquark ~ 1 GeV**
- N and Δ masses & form factors very similar: **quark-diquark model is good approximation for three-body equation**
- Nucleon and Δ electromagnetic FFs, $N \to \Delta\gamma$ and $N \to \Delta\pi$ transition
  GE, Cloet, Alkofer, Krassnigg, Roberts, PRC 79 (2009), Nicmorus, GE, Alkofer, PRD 82 (2010), Mader, GE, Blank, Krassnigg, PRD 84 (2011), GE, Nicmorus, PRD 85 (2012)



Oettel, Pichowsky, von Smekal, EPJ A 8 (2000)

Quark-diquark model

- Assumption: separable qq scattering matrix ⇒ Faddeev equation simplifies to **quark-diquark BSE**



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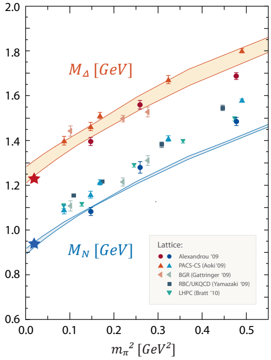
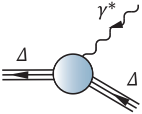
GE, Nicmorus, PRD 85 (2012)



Δ electromagnetic FFs

Almost no experimental information since Δ unstable: $\Delta \to N\pi$

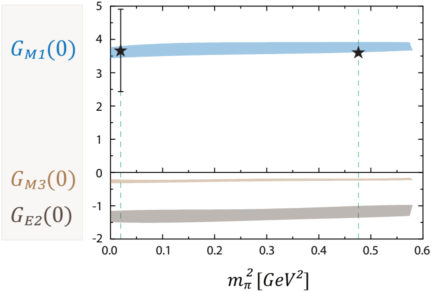


Magnetic moment $\mu_\Delta \sim 3.5$ with large errors (Δ^+).
But Ω^- (spin 3/2, *sss*) is stable w.r.t strong interaction,
magnetic moment $|\mu_\Omega| = 3.6(1)$. Accidental?

$$J^{\mu,\rho\sigma}(P,Q) = i\,\mathbb{P}^{\rho\alpha}(P_f)\left[\left(F_1^\star\,\gamma^\mu - F_2^\star\,\frac{\sigma^{\mu\nu}Q^\nu}{2M_\Delta}\right)\delta^{\alpha\beta} - \left(F_3^\star\,\gamma^\mu - F_4^\star\,\frac{\sigma^{\mu\nu}Q^\nu}{2M_\Delta}\right)\frac{Q^\alpha Q^\beta}{4M_\Delta^2}\right]\mathbb{P}^{\beta\sigma}(P_i)$$

Form factors at Q^2=0:

$G_{E_0}(0) = e_\Delta$	charge
$G_{E_2}(0) = \mathcal{Q}$	electric quadrupole moment
$G_{M_1}(0) = \mu_\Delta$	magnetic dipole moment
$G_{M_3}(0) = \mathcal{O}$	magnetic octupole moment

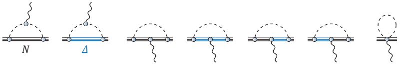


almost quark-mass independent,
match Ω^- magnetic moment
Nicmorus, GE, Alkofer, PRD 82 (2010)

Three-body results similar (except G_{M_3})
Sanchis-Alepuz, Alkofer, Williams, PRD 87 (2013)

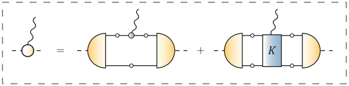
Pion cloud effects

- **Hadron level:**
  $N\pi$ contributions to nucleon self-energy;
  charge radii diverge in chiral limit,
  $\Delta \to N\pi$ decay cusps, etc.

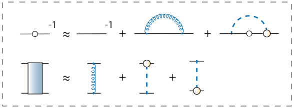


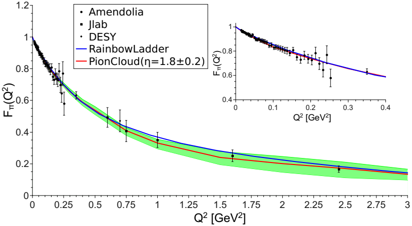
- **Baryons:** pion effects reduce N, Δ masses but also f_π (sets the scale) by similar amount: net effect small Sanchis-Alepuz, Fischer, Kubrak, PLB 733 (2014)

- **Pion form factor:** photon also couples to pion (necessary for gauge invariance), π exchange in quark-photon vertex



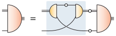
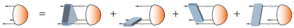
- **Quark level:**
  π contributions to quark self-energy,
  effective π exchange between quarks;
  pion not elementary field!
  Fischer, Nickel, Wambach, PRD 76 (2007)



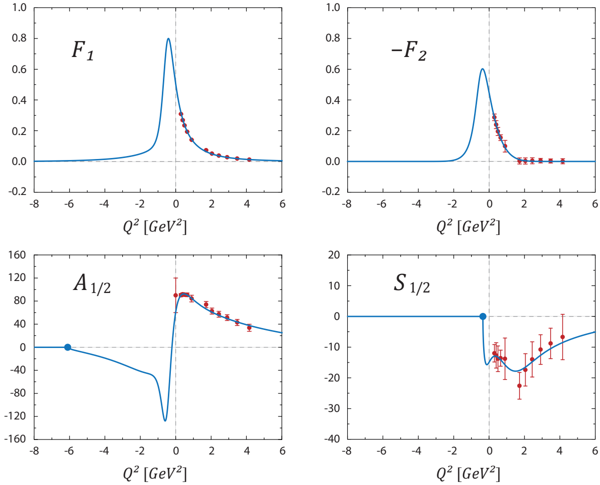


GE, Fischer, Kubrak, Williams, in preparation

DSE / Faddeev landscape $N \to N^*\gamma$

	Quark-diquark 			Three-quark 		
	Contact interaction	QCD-based model	DSE (RL)	RL	bRL	bRL + 3q
N, Δ masses	√	√	√	√	√	$\dots$
N, Δ em. FFs	√	√	√	√		
$N \to \Delta\gamma$	√	√	√	$\dots$		
Roper	√	√		$\dots$		
$N \to N^*\gamma$	√	√		$\dots$		
$N^*(1535), \dots$	$\dots$	$\dots$		$\dots$	$\dots$	
$N \to N^*\gamma$	$\dots$	$\dots$				

N*(1535)?



Form factors:
no kinematic constraints

CLAS data &
toy parametrization
with "ρ bump"

...vs. **helicity amplitudes**
in $[10^{-3} GeV^{-1/2}]$

kinematic zeros at
$Q^2 = -(m_R \pm m)^2$

see also
Ramalho & Tsushima, PRD 84 (2011)

Negative parity resonances?

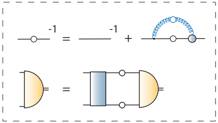
I \ J^P	$\frac{1}{2}^+$	$\frac{3}{2}^+$	$\frac{1}{2}^-$	$\frac{3}{2}^-$	…
$\frac{1}{2}$	P_{11} N(940) N(1440) N(1710) N(1880) sc, av, ps, v	P_{13} N(1720) N(1900) sc, av, ps, v	S_{11} N(1535) N(1650) N(1895) sc, av, ps, v	D_{13} N(1520) N(1700) N(1875) sc, av, ps, v	
$\frac{3}{2}$	P_{31} Δ(1910) av, v	P_{33} Δ(1232) Δ(1600) Δ(1920) av, v	S_{31} Δ(1620) Δ(1900) av, v	D_{33} Δ(1700) Δ(1940) av, v	

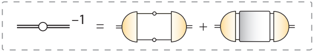
Diquarks: $I\,(J^P)$

- sc: $0\,(0^+)$
- av: $1\,(1^+)$
- ps: $0\,(0^-)$
- v: $1\,(1^-)$

N*(1535): the recipe

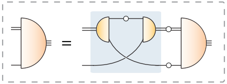
- Calculate quark DSE and **(pseudoscalar, vector)** diquark BSEs & propagators in complex plane



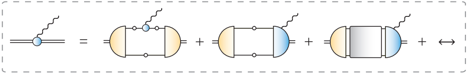


pseudoscalar diquark ~ 1 GeV
vector diquark ~ 1.1 GeV

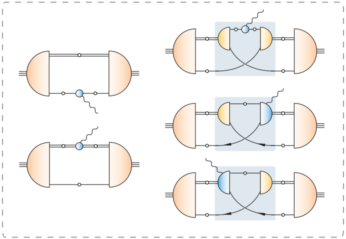
- Solve Faddeev equation, obtain N*(1535) mass and wave function



- Calculate quark-photon and **(pseudoscalar, vector scalar, axialvector)** diquark-photon vertices



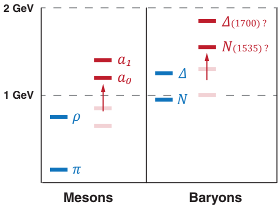
- Insert everything here and calculate **transition form factor:**



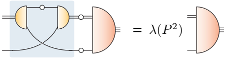
N*(1535): expectations vs. results

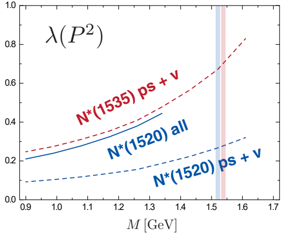
- **Mesons** and (opposite-parity) **diquarks** intrinsically linked in rainbow-ladder
- Ps & v mesons ok, but sc & av mesons too light, repulsive shifts beyond RL
  Fischer, Williams, PRL 103 (2009), Chang, Roberts, PRL 103 (2009)

  ⇒ also **ps & v diquarks** should be "too light"
  ⇒ same for $N^*(1535)$, etc. Chen et al., FBS 53 (2012)



- Three-body Faddeev equation: $M(1535) \sim 1.1\,\mathrm{GeV}$
  Sanchis-Alepuz, Williams, PLB 749 (2015)
- But **quark-diquark BSE** with ps & v diquarks only: $M(1535) \sim 1.65 \dots 1.70\,\mathrm{GeV}$!
- Including **sc & av diquarks** for $N^*(1535)$, $N^*(1520)$: not yet fully consistent (nor stable), but important for $N^*(1520)$. Near mass degeneracy?
- Transition form factor (with ps & v diquarks only) extremely small ⇒ **ps & v diquarks not enough?**





Nucleon resonances: $J^P = \frac{1}{2}^{\pm}$

- **Offshell nucleon-photon vertex** depends on 12 tensor structures (4 gauge + 8 transverse)

gauge	transverse	
γ^μ	$t^{\mu\nu}_{QQ}\,\gamma^\nu$	$t^{\mu\nu}_{QQ}\,ik^\nu$
ik^μ	$t^{\mu\nu}_{QQ}\,k\cdot Q\,\frac{i}{2}[\gamma^\nu, \not k]$	$t^{\mu\nu}_{QQ}\,k^\nu \not k$
$k^\mu \not k$	$\frac{i}{2}[\gamma^\mu, \not Q]$	$t^{\mu\nu}_{Qk}\,k\cdot Q\,\gamma^\nu$
$k\cdot Q\,\frac{i}{2}[\gamma^\mu, \not k]$	$\frac{1}{6}[\gamma^\mu, \not k, \not Q]$	$t^{\mu\nu}_{Qk}\,\frac{i}{2}[\gamma^\nu, \not k]$

- **onshell** everything collapses into 2 form factors:

$$J^\mu = \bar{u}_f \left[F_1 \gamma^\mu + \frac{F_2}{2m}\frac{i}{2}[\gamma^\mu, \not Q] \right] u_i$$

- No gauge part for $N \to N^*(\frac{1}{2}^{\pm})$ transition form factors (no conserved charge), hence

$$J^\mu = \bar{u}_f \begin{bmatrix} 1 \\ i\gamma_5 \end{bmatrix} \Big[\underbrace{\frac{F_1}{m^2}\, t^{\mu\nu}_{QQ}\,\gamma^\nu}_{Q^2\,\gamma^\mu_T} + \frac{F_2}{2m}\,\frac{i}{2}[\gamma^\mu, \not Q] \Big] u_i$$

$\Rightarrow$ F_1 and F_2 free of kinematic constraints

+ crossed term

$$t^{\mu\nu}_{AB} = A\cdot B\,\delta^{\mu\nu} - B^\mu A^\nu$$

- What about **offshell form factors** in nucleon Born term?

  γ^μ clashes with gauge invariance, only $t^{\mu\nu}_{QQ}\,\gamma^\nu$, $\frac{i}{2}[\gamma^\mu, \not Q]$ allowed: must use **Dirac current**

  Gauge invariance restored by 1PI part
  GE, Fischer, PRD 87 (2013)

  $\Rightarrow$ careful with **offshell form factors!**

Nucleon resonances: $J^P = \frac{3}{2}^{\pm}$

- **Offshell** $N \to N^*(\frac{3}{2}^{\pm})$ **transition vertices:**
  Rarita-Schwinger rep. contains spin-3/2 & 1/2;
  must get rid of offshell spin-1/2 background

+ crossed term

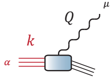
  Vertices must satisfy electromagnetic & spin-3/2 gauge invariance
  and invariance under point transformations:

$$Q^\mu \, \Gamma_{\mathrm{R}}^{\alpha\mu} = 0, \qquad k^\alpha \, \Gamma_{\mathrm{R}}^{\alpha\mu} = 0, \qquad \gamma^\alpha \, \Gamma_{\mathrm{R}}^{\alpha\mu} = 0$$

Pascalutsa, Timmermanns, PRC 60 (1999)
Shklyar, Lenske, PRC 80 (2009)
GE, Ramalho, in preparation

- General form of offshell $N \to \frac{3}{2}^{\pm}$ transition currents: 12 form factors

$$\begin{aligned} \varepsilon_{kQ}^{\alpha\mu} &= \gamma_5 \, \varepsilon^{\alpha\mu\rho\sigma} k^\rho Q^\sigma \\ t_{kQ}^{\alpha\mu} &= k \cdot Q \, \delta^{\alpha\mu} - Q^\alpha k^\mu \\ &\vdots \end{aligned} \quad \Leftrightarrow \quad \begin{aligned} & g_M \, (\partial^\mu \bar{\psi}^\alpha) \, \widetilde{F}^{\alpha\mu} \psi && \sim g_M \, \bar{\psi}^\alpha \, \gamma_5 \, \varepsilon_{kQ}^{\alpha\mu} \, A^\mu \psi \\ & g_E \, (\partial^\mu \bar{\psi}^\alpha) \, \gamma_5 \, F^{\alpha\mu} \psi && \sim g_E \, \bar{\psi}^\alpha \, \gamma_5 \, t_{kQ}^{\alpha\mu} \, A^\mu \psi \\ & && \vdots \end{aligned}$$



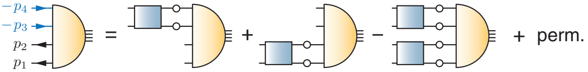
- **onshell** everything collapses into 3 form factors:

$$J^\mu = \frac{1}{m^2} \, \bar{u}_f^\alpha \begin{bmatrix} i\gamma_5 \\ 1 \end{bmatrix} \left[F_1 \, \varepsilon_{kQ}^{\alpha\mu} - F_2 \, t_{kQ}^{\alpha\mu} - \frac{F_3}{m} \, i t_{k\gamma}^{\alpha\beta} \, t_{QQ}^{\beta\mu} \right] u_i$$

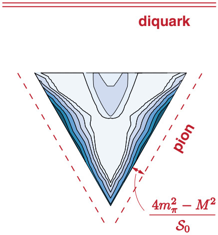
⇒ no kinematic constraints.
Jones-Scadron decomposition not good offshell!

Tetraquarks

- Solution of four-body equation (same input) reproduces mass pattern for **light scalar mesons:** σ, κ, a_0, f_0
  GE, Fischer, Heupel, 1508.07178 [hep-ph]

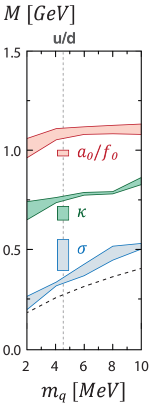


- BSE dynamically generates **pion poles** in wave function, drive σ mass from 1.5 GeV to ~350 MeV



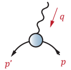
Four quarks rearrange to **"meson molecule", diquarks irrelevant**

Tetraquark is at the same time dynamically generated resonance!



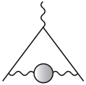
Muon g-2

- **Muon anomalous magnetic moment:**
  total SM prediction deviates from exp. by ~3σ

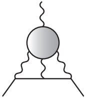


$$= ie\,\bar{u}(p')\left[F_1(q^2)\,\gamma^\mu - F_2(q^2)\,\frac{\sigma^{\mu\nu}q_\nu}{2m}\right]u(p)$$

- Theory uncertainty dominated by **QCD:**
  Is QCD contribution under control?



Hadronic vacuum polarization



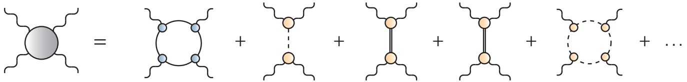
Hadronic light-by-light scattering

a_μ $[10^{-10}]$

Jegerlehner, Nyffeler, Phys. Rept. 477 (2009)

Exp:	11 659 208.9	(6.3)	
QED:	11 658 471.9	(0.0)	
EW:	15.3	(0.2)	
Hadronic:			
• VP (LO+HO)	685.1	(4.3)	
• **LBL**	**10.5**	**(2.6)**	**?**
SM:	11 659 182.8	(4.9)	
Diff:	26.1	(8.0)	

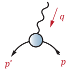
- **LbL amplitude:** ENJL & MD model results
  Bijnens 1995, Hakayawa 1995, Knecht 2002, Melnikov 2004, Prades 2009, Jegerlehner 2009, Pauk 2014



Quark loop	pseudoscalar exchange	scalar exchange	axialvector exchange	π, K loop	
2	8 ... 11	−1	2	−2	$(\times 10^{-10})$

Muon g-2

- **Muon anomalous magnetic moment:**
  total SM prediction deviates from exp. by ~3σ



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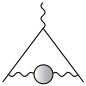
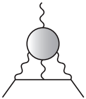


Hadronic vacuum polarization



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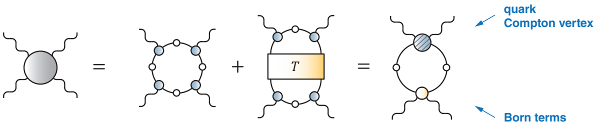
- **LbL amplitude** at quark level, derived from **gauge invariance:**
  GE, Fischer, PRD 85 (2012), Goecke, Fischer, Williams, PRD 87 (2013)



- **no double-counting, gauge invariant!**
- need to understand **structure of amplitude**
  GE, Fischer, Heupel, PRD 92 (2015)