The Angular Momentum Controversy: Resolution of a Conflict Between Laser and Particle Physics

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- **WHAT'S IT ALL ABOUT ?**

- **THE CONCEPTS INVOLVED**

- **THE PARTICLE PHYSICS CONTROVERSY** [Leader, Lorcé: Phys. Reps. 541 (2014) 163]

- **THE LASER PHYSICS POINT OF VIEW** [Bliokh, Nori:Phys. Reps. 592 (2015) 1]

- **WHAT LASER MEASUREMENTS TEACH US**

WHAT'S IT ALL ABOUT?

Normal massive particle

Total Angular Momentum = Orbital Part + Spin Part

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Normal massive particle

$$Total\ Angular\ Momentum = Orbital\ Part + Spin\ Part$$

Massless Particle : PHOTON

All textbooks on QED state:

The angular momentum of a photon (and gluon) **cannot** be split in a gauge-invariant way into an orbital and spin term

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See Physics Reports 541 (2014) 163

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A further cause of upset: the gluon polarization in a nucleon, a supposedly physically meaningful quantity, corresponds only to the **gauge-variant** gluon spin derived from Noether's theorem, evaluated **in a particular gauge**.

On the contrary, **Laser Physicists** have, for decades, been happily measuring physical quantities which correspond to orbital and spin angular momentum **evaluated in a particular gauge**.

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How can you reconcile the two points of view???

Is this just a matter of taste?

Or is there a genuine physical principle involved?

My point of view:

My point of view: Should be able to **test** what momentum and angular momentum is carried by an EM field!

THE CONCEPTS INVOLVED

REMINDER: Undergrad Physics

Kinetic momentum

.

Defined as mass times velocity

$$\boldsymbol{p}_{\text{kin}} = m\boldsymbol{v} = m\dot{\boldsymbol{x}}$$

Follows motion of particle.

Non-relativistic expression for the particle kinetic energy

$$E_{\text{kin}} = \boldsymbol{p}_{\text{kin}}^2/2m$$

Quantum Mechanics

Heisenberg uncertainty relations between position and momentum

$$[x_i, p_j] = i\hbar\,\delta_{ij}$$

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$$[x_i, p_j] = i\hbar\,\delta_{ij}$$

What is this $\boldsymbol{p}$? It is NOT the kinetic momentum

It is the **canonical momentum**, defined as

$$\boldsymbol{p}_{\mathsf{can}} = \partial L / \partial \dot{\boldsymbol{x}}$$

where L is the Lagrangian of the system

Comparison of $\boldsymbol{p}_{\text{can}}$ with $\boldsymbol{p}_{\text{kin}}$

For a particle moving in a potential $V(\boldsymbol{x})$

$$L = E_{\text{kin}} - V = \tfrac{1}{2} m\dot{\boldsymbol{x}}^2 - V(\boldsymbol{x})$$

so that

$$\boldsymbol{p}_{\text{can}} = m\dot{\boldsymbol{x}} = \boldsymbol{p}_{\text{kin}},$$

and there is no distinction between kinetic and canonical momentum.

What happens if an electromagnetic field is present?

Classical problem: charged particle, say an electron with charge e, moving in a fixed homogeneous *external* magnetic field $\boldsymbol{B} = (0, 0, B)$.

Particle follows a helical trajectory, so that at each instant, the particle kinetic momentum $\boldsymbol{p}_{\mathsf{kin}}$ points toward a different direction.

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The Lagrangian is given by

$$L = \tfrac{1}{2} m \dot{\boldsymbol{x}}^2 - e \dot{\boldsymbol{x}} \cdot \boldsymbol{A}$$

where $\boldsymbol{A}$ is the **vector potential** responsible for the magnetic field $\boldsymbol{B} = \boldsymbol{\nabla} \times \boldsymbol{A}$. It leads to

$$\boldsymbol{p}_{\text{can}} = \boldsymbol{p}_{\text{kin}}[\boldsymbol{x}(t)] - e\boldsymbol{A}[\boldsymbol{x}(t)]$$

A suitable vector potential

$$\boldsymbol{A} = \tfrac{1}{2}(-yB, xB, 0)$$

But, exactly the same magnetic field is obtained from the vector potential

$$\tilde{\boldsymbol{A}} = \boldsymbol{A} - \boldsymbol{\nabla}\alpha$$

$\alpha(\boldsymbol{x})$ is any smooth function.

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This change in $\boldsymbol{A}$ is called a gauge transformation.
It does not affect the physical motion of the particle.
But, it clearly changes $\boldsymbol{p}_{\text{can}}$.
$\boldsymbol{p}_{\text{can}}$ is a gauge **non-invariant** quantity.

key issue in the controversy: is such a quantity meaningful or measurable?

Completely analogous for **angular momentum** in Classical and Quantum Mechanics

Kinetic : $\boldsymbol{J}_{\text{kin}}$

Canonical : $\boldsymbol{J}_{\text{can}}$

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Kinetic : $\boldsymbol{J}_{\mathsf{kin}}$

Canonical : $\boldsymbol{J}_{\mathsf{can}}$

Completely analogous in Field Theory: Classical or Quantum

Kinetic : $\boldsymbol{p}_{\mathsf{kin}} \quad \boldsymbol{J}_{\mathsf{kin}}$ (usually called **Belinfante**)

Canonical : $\boldsymbol{p}_{\mathsf{can}} \quad \boldsymbol{J}_{\mathsf{can}}$

The Belinfante versions of momentum and angular momentum

As given in textbooks on Classical E and M.

$\boldsymbol{E} \times \boldsymbol{B} =$ **kinetic** momentum density; **Poynting vector**

$\boldsymbol{x} \times (\boldsymbol{E} \times \boldsymbol{B}) =$ **Belinfante** angular momentum density

Belinfante angular momentum density is gauge invariant BUT not split into **ORBITAL** and **SPIN** parts.

The opposite for **canonical** $\boldsymbol{J}_{\text{can}}$

Obtain expression from Lagrangian

$$\boldsymbol{J}_{\text{can}} = \underbrace{\boldsymbol{S}_{\text{can}}}_{\text{spin term}} + \underbrace{\boldsymbol{L}_{\text{can}}}_{\text{orbital angular momentum}}$$

BUT $\boldsymbol{S}_{\text{can}}$ and $\boldsymbol{L}_{\text{can}}$ NOT gauge invariant

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Under gauge transformation

$$\begin{aligned} \boldsymbol{J}_{\text{can}} \to \boldsymbol{J}_{\text{can}} &= \boldsymbol{S}'_{\text{can}} + \boldsymbol{L}'_{\text{can}} \\ &= [\boldsymbol{S}_{\text{can}} + \boldsymbol{\Delta}] + [\boldsymbol{L}_{\text{can}} - \boldsymbol{\Delta}] \end{aligned}$$

Example from Classical Electromagnetism: compare $\boldsymbol{J}_{\text{Belinfante}}$ and $\boldsymbol{J}_{\text{can}}$

For a free classical electromagnetic field, one has

$$\boldsymbol{J}_{\text{can}} = \underbrace{\int \mathrm{d}^3x \, (\boldsymbol{E} \times \boldsymbol{A})}_{\text{spin term}} + \underbrace{\int \mathrm{d}^3x \, E^i (\boldsymbol{x} \times \boldsymbol{\nabla} A^i)}_{\text{orbital term}}$$

and

$$\boldsymbol{J}_{\text{Belinfante}} = \int \mathrm{d}^3x \, [\boldsymbol{x} \times (\boldsymbol{E} \times \boldsymbol{B})]$$

Consider a left-circularly polarized (= positive helicity) beam, with angular frequency ω, and amplitude proportional to E_0, propagating along OZ, *i.e.* along the unit vector $\boldsymbol{e}_z$. Then

$$A^\mu = \left(0, \frac{E_0}{\omega} \cos(kz - \omega t), \frac{E_0}{\omega} \sin(kz - \omega t), 0\right)$$

gives the correct electric and magnetic fields. $\boldsymbol{E}$, $\boldsymbol{B}$ and $\boldsymbol{A}$ all rotate in the XY plane.

Look at J_z in the two cases. First the Canonical case

$$\boldsymbol{J}_{\text{can}} = \underbrace{\int \mathrm{d}^3x\,(\boldsymbol{E}\times\boldsymbol{A})}_{\text{spin term}} + \underbrace{\int \mathrm{d}^3x\,E^i(\boldsymbol{x}\times\boldsymbol{\nabla}A^i)}_{\text{orbital term}}$$

Note that

$$\boldsymbol{\nabla}\,A_{x,y} \propto \boldsymbol{e}_{(z)} \quad \text{so that} \quad (\boldsymbol{x}\times\boldsymbol{\nabla}\,A_{x,y})_z = 0$$

so only the spin term contributes to $J_{\text{can}},\, z$.

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Find

$$J_{\text{can},\,z,\text{ per unit volume}} = \frac{E_0^2}{\omega}$$

For one photon per unit volume $E_0^2 = \hbar\omega$ so that

$$J_{\text{can},\,z,\text{ per photon}} = \hbar \qquad \checkmark$$

Next the kinetic (Belinfante) case

$$\boldsymbol{J}_{\text{Belinfante}} = \int \mathrm{d}^3x \, [\boldsymbol{x} \times (\boldsymbol{E} \times \boldsymbol{B})]$$

$$(\boldsymbol{E} \times \boldsymbol{B}) \propto \, \boldsymbol{e}_z$$

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Suggests canonical is more reliable, closer to physical intuition

But what about the non-gauge invariance?????

THE PARTICLE PHYSICS CONTROVERSY

Chen, Lu, Sun, Wang and Goldman (Chen *et al*):

CLAIM::IT IS POSSIBLE to split photon or gluon angular momentum into a spin part and an orbital part in a GAUGE INVARIANT way !!!

THE PARTICLE PHYSICS CONTROVERSY

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CLAIM::IT IS POSSIBLE to split photon angular momentum into a spin part and an orbital part in a GAUGE INVARIANT way !!!

Put $\boldsymbol{A} = \boldsymbol{A}_{phys} + \boldsymbol{A}_{pure}$ with

$$\boldsymbol{\nabla}.\boldsymbol{A}_{phys} = 0 \qquad \boldsymbol{\nabla} \times \boldsymbol{A}_{pure} = 0$$

Corresponds exactly to the Helmholz decomposition into transverse $\boldsymbol{A}_{\perp}$ and longitudinal $\boldsymbol{A}_{\parallel}$ parts respectively

Adding a spatial divergence to $\boldsymbol{J}_{\text{kin}}$ they get, for photon part:

$$\begin{aligned}\boldsymbol{J}_{chen} &= \int d^3x\,(\boldsymbol{E}\times\boldsymbol{A}_\perp) + \int d^3x\, E^i[\boldsymbol{x}\times\boldsymbol{\nabla} A^i_\perp] \\ &= \boldsymbol{S}_{ch}(\text{photon}) + \boldsymbol{L}_{ch}(\text{photon})\end{aligned}$$

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Can show that under gauge transformation:

$$\boldsymbol{A}_\perp \to \boldsymbol{A}_\perp$$

so each term in $\boldsymbol{J}_{chen}$ is indeed gauge invariant.

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$\boldsymbol{A}_\perp$ is not, **in general**, a **local** field:

$$\boldsymbol{A}_\perp = \boldsymbol{A} - \frac{1}{\nabla^2}\boldsymbol{\nabla}(\boldsymbol{\nabla}\cdot\boldsymbol{A})$$

Recall

$$\frac{1}{\nabla^2}f(x) \equiv \frac{1}{4\pi}\int d^3x' \, \frac{f(\boldsymbol{x}')}{|\boldsymbol{x}-\boldsymbol{x}'|}$$

Summary of Particle Physics point of view

Don't like non-local fields

Worried that gluon polarization corresponds to canonical spin only in a particular gauge

THE LASER PHYSICS POINT OF VIEW

- Long ago van-Enk and Nienhuis studied exactly the orbital and spin operators proposed by Chen et al! (Actually introduced in Cohen-Tannoudj, Dupont-Roc and Grynberg: *Photons and Atoms 1989*, but they missed a key point)

- Pointed out that these **gauge-invariant** spin and orbital operators are just the Canonical ones evaluated in the Coulomb Gauge.

- Hence call them GAUGE INVARIANT CANONICAL: gic operators

- Showed that $\boldsymbol{S}_{gic}$ is NOT a genuine spin operator:

$$[S^i_{gic}, S^j_{gic}] = 0 \neq \epsilon_{ijk} S^k_{gic}$$

- Showed that only the HELICITY ——usually same as the Z-component——is a genuine AM

- Referred to "spin" and "orbital" in inverted commas

- Often used in Laser papers without inverted commas

Papers on Laser AM sometimes use the Belinfante version, sometimes the gic version.

My argument: Experiments on effect of laser beams on very small particles suggest the gic version is the physically correct one.

WHAT LASER MEASUREMENTS TEACH US

- Monochromatic beams: $e^{-i\omega t}$: superposition of plane waves

- Maxwell's Equations: free fields:

$$\boldsymbol{E} = \boldsymbol{E}_\perp = -\dot{\boldsymbol{A}}_\perp$$

so

$$\boldsymbol{A}_\perp = -\frac{i}{\omega}\boldsymbol{E} \quad \text{is local}$$

Real physical EM fields $(\boldsymbol{\mathcal{E}}, \boldsymbol{\mathcal{B}})$ expressed in terms of complex fields $(\boldsymbol{E}, \boldsymbol{B})$

$$\boldsymbol{\mathcal{E}} = Re(\boldsymbol{E}) \qquad \boldsymbol{E}(\boldsymbol{r}, t) = \boldsymbol{E}_0(\boldsymbol{r})\, e^{-i\omega t}$$

$$\boldsymbol{\mathcal{B}} = Re(\boldsymbol{B}) \qquad \boldsymbol{B}(\boldsymbol{r}, t) = \boldsymbol{B}_0(\boldsymbol{r})\, e^{-i\omega t}.$$

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The force on, and the torque (about the centre of mass of a **small** neutral object), in dipole approximation, are given by

$$\boldsymbol{F} = (\boldsymbol{\mathcal{P}}.\boldsymbol{\nabla})\,\boldsymbol{\mathcal{E}} + \dot{\boldsymbol{\mathcal{P}}} \times \boldsymbol{\mathcal{B}} \qquad \boldsymbol{\tau} = \boldsymbol{\mathcal{P}} \times \boldsymbol{\mathcal{E}}$$

where the induced electric dipole moment is given by

$$\boldsymbol{\mathcal{P}} = Re[\alpha \boldsymbol{E}(\boldsymbol{r}, t)]$$

and the complex polarizability is

$$\alpha = \alpha_R + i\,\alpha_I.$$

Study effect of force acting on the neutral dipole.

The total force splits into two terms

$$\boldsymbol{F} = \boldsymbol{F}_{\text{reactive}} + \boldsymbol{F}_{\text{dissipative}}$$

For oscillating fields must use **cycle average**: indicate by $<>$,

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For a classical electric dipole with momentum $\boldsymbol{P}_{\text{dipole}}$ it is $\boldsymbol{F}_{\text{dissapative}}$ that controls its rate of change of momentum

$$\left\langle \frac{d\boldsymbol{P}_{\text{dipole}}}{dt} \right\rangle = \langle \boldsymbol{F}_{\text{dissapative}} \rangle.$$

For linear momentum, gic version:

$$\boldsymbol{P}_{\text{gic}} = \int \mathrm{d}^3x \, \boldsymbol{p}_{\text{gic}} \quad \text{with} \quad \boldsymbol{p}_{\text{gic}} = \epsilon_0 \, \mathcal{E}^i \boldsymbol{\nabla} \mathcal{A}_\perp^i$$

Find

$$\langle \boldsymbol{F}_{\text{dissipative}} \rangle = \frac{\alpha_I \omega}{\epsilon_0} \langle \boldsymbol{p}_{\text{gic}} \rangle .$$

Hence

$$\left\langle \frac{d\boldsymbol{P}_{\text{dipole}}}{dt} \right\rangle = \frac{\alpha_I \omega}{\epsilon_0} \langle \boldsymbol{p}_{\text{gic}} \rangle$$

so that it is the gauge-invariant canonical version that seems to be physically relevant.

Next consider the torque about the centre of mass of the dipole. Find for the cycle average:

$$\langle \boldsymbol{\tau} \rangle = \frac{\alpha_I \omega}{\epsilon_0} \langle \boldsymbol{s}_{\text{gic}} \rangle.$$

So again it is the gauge invariant canonical version that seems to be physically relevant.

The foundation experiment on laser AM: Allen, Beijersbergen, Spreeuw and Woerdman 1992

Paraxial Approximation: beam propagating in the Z-direction

$$\boldsymbol{E} = i\omega\left(u(\boldsymbol{r}), v(\boldsymbol{r}), \frac{-i}{k}\Big(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\Big)\right)e^{i(kz-\omega t)}$$

where

$$\left|\frac{\partial u}{\partial z}\right| \ll k|u| \qquad \left|\frac{\partial v}{\partial z}\right| \ll k|v|$$

and all second derivatives and products of first derivatives are ignored.

For the case of circularly polarization

$$v = i\sigma_z u$$

where $\sigma_z = \pm 1$ for left/right circular polarization, and in cylindrical coordinates $(\rho,\ \phi,\ z)$

$$u(\rho, \phi, z) = f(\rho, z)e^{il\phi}.$$

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Using this calculate Belinfante and gic angular momentum.

Find for z-component of the Belinfante AM density

$$\langle j_{\mathrm{bel}} \rangle_z \simeq \epsilon_0 \omega \left[l|u|^2 - \frac{\sigma_z}{2} \rho \frac{\partial |u|^2}{\partial \rho} \right]$$

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Implies *per photon*

$$\langle j_{\mathrm{bel}} \rangle_z^{\mathrm{photon}} \simeq l\hbar - \frac{\sigma_z \hbar}{2|u|^2} \rho \frac{\partial |u|^2}{\partial \rho}.$$

For the gauge invariant canonical version one finds

$$\langle l_{\mathrm{gic}} \rangle \simeq \epsilon_0 \omega l |u|^2 \qquad \langle s_{\mathrm{gic}} \rangle \simeq \epsilon_0 \omega \sigma_z |u|^2$$

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implying the beautiful result *per photon*

$$\langle l_{\mathrm{gic}}\rangle_z^{\mathrm{photon}} \simeq l\hbar \qquad \langle s_{\mathrm{gic}}\rangle_z^{\mathrm{photon}} \simeq \sigma_z \hbar.$$

Which agrees with experiment??????

For small enough dipoles the angular momentum absorbed depends on the *local* AM density, which is quite different for the Belinfante and gic cases, even differing in sign between the beam axis and the beam periphery.

The first **semi-quantitative** test of the above was made by Garcés-Chávez, Mc Gloin, Padgett, Dulz, Schmitzer and Dholakia in 2003 who succeeded in studying the motion of a tiny particle trapped at various radial distances ρ from the axis of a so-called Bessel beam.

The transfer of orbital AM causes the particle to circle about the beam axis with a rotation rate Ω_{orbit} whereas the transfer of spin AM causes the particle to spin about its centre of mass with rotation rate Ω_{spin}.

For a Bessel beam, $|u|^2 \propto 1/\rho$ so, for the Belinfante case,

$$\Omega_{\text{orbit}} \propto 1/\rho^3 \qquad \text{and} \qquad \Omega_{\text{spin}} \propto 1/\rho,$$

which is precisely the behaviour found experimentally.

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NO!

Exactly the same functional dependence on ρ follows from the gic expressions. Since the **absolute** rotation rates depend upon detailed parameters which, according to the authors, were beyond experimental control, it is incorrect to interpret these results as evidence in favour of the Belinfante expressions.

Other experimental evidence?

Ghai, Senthilkumaran and Sirohi study the shift of the diffraction fringes in single slit diffraction of optical beams with a phase singularity:

They find: depends on l and not on σ_z.

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Recent review Bliokh and Nori: Canonical AM in the Coulomb gauge i.e the gic AM agrees with a wide range of experiments. [**Physics Report 592 (2015) 1**]

Two more arguments in favour of gic momentum and AM

Linear momentum:

1) Assume rate of change of momentum of the dipole is due to the momentum of the photons absorbed from the beam per second.

2) Take number of photons totally absorbed by the dipole per second $= 1/\hbar\omega$ times rate of increase of dipole's internal energy.

For paraxial beam can satisfy

$$\left\langle \frac{d\boldsymbol{P}_{\text{dipole}}}{dt} \right\rangle = \frac{\alpha_I \omega}{\epsilon_0} \langle \boldsymbol{p}_{\text{gic}} \rangle$$

ONLY if the average photon momentum is taken as

$$\langle \boldsymbol{p} \rangle^{\text{photon}}\Big|_{ave} \simeq \frac{1}{N} \langle \boldsymbol{p}_{\text{gic}} \rangle$$

where N is the number of photons per unit volume.

Angular momentum:

Similar argument: Can satisfy

$$\langle \boldsymbol{\tau} \rangle = \frac{\alpha_I \omega}{\epsilon_0} \langle \boldsymbol{s}_{\mathsf{gic}} \rangle.$$

ONLY if

$$\langle \boldsymbol{s} \rangle^{\mathsf{photon}} \Big|_{ave} \simeq \frac{1}{N} \langle \boldsymbol{s}_{\mathsf{gic}} \rangle.$$

SUMMARY

- Particle Physicists don't like van Enk-Nienhuisen=Chen et al= Gauge Invariant Canonical AM because, in general, $\boldsymbol{A}_{\perp}$ is non-local

- But it is local for monochromatic fields

- $\boldsymbol{j}_{gic}$ is split into $\boldsymbol{l}_{gic}$ and $\boldsymbol{s}_{gic}$

- But $\boldsymbol{S}_{gic}$ is not really an angular momentum vector. $[S^i_{gic}, S^j_{gic}] = 0$.

- Experiment shows that the gic versions play a central role in Laser Physics

- All components can, in principle, be measured, but only one component, the helicity, is a genuine AM.

- For a paraxial beam propagating in the Z-direction $\langle S_{\text{gic}} \rangle_z \simeq \langle \text{gic helicity} \rangle$ so this component is effectively a genuine AM.

- Finally, recognizing that the fundamental expressions are the gic ones, allows to avoid the disturbing claim that what is physically measured corresponds to a gauge-variant quantity evaluated in a particular gauge, i.e. the Coulomb one.