Nucleon strange electromagnetic form factors from lattice QCD

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Biographical information

Undergraduate: 2003–2007, University of Toronto

Hon. B. Sc., Math and Physics

Summer research: knot theory, particle physics (ATLAS)

Graduate: 2007–2013, Massachusetts Institute of Technology

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Lattice QCD studies: spatial diquark correlations, nucleon structure

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Lattice QCD group (under Hartmut Wittig and Harvey Meyer)

- hadronic light-by-light scattering
- H dibaryon
- continued work on nucleon structure

Outline

- Introduction — electromagnetic form factors
- Nucleon matrix elements from lattice QCD
- Disconnected diagrams
- Results
- Conclusions and outlook

Electromagnetic form factors

We can probe the structure of a proton using virtual photons, which couple to quarks via the current

$$J_\mu = \tfrac{2}{3}\bar{u}\gamma_\mu u - \tfrac{1}{3}\bar{d}\gamma_\mu d - \tfrac{1}{3}\bar{s}\gamma_\mu s + \dots$$

Symmetries constrain matrix elements between proton states with momenta p and p':

$$\langle p'|J_\mu|p\rangle = \bar{u}(p')\left[\gamma_\mu F_1(Q^2) + \frac{i\sigma_{\mu\nu}(p'-p)^\nu}{2m_p}F_2(Q^2)\right]u(p),$$

where $Q^2 = -(p'-p)^2$ is the four-momentum transfer and $F_{1,2}$ are the Dirac and Pauli form factors.
Electric and magnetic form factors:

$$G_E(Q^2) = F_1(Q^2) - \frac{Q^2}{(2m_p)^2}F_2(Q^2), \qquad G_M(Q^2) = F_1(Q^2) + F_2(Q^2)$$

Electromagnetic form factors

In the *nonrelativistic limit*, $G_E(Q^2)$ and $G_M(Q^2)$ are Fourier transforms of the charge and magnetization densities in a proton.

- $G_E(0) = 1$, the charge of a proton
- $G_M(0) = \mu$, the magnetic moment of a proton, in units of the nuclear magneton $\mu_N = \frac{e}{2m_p}$

Even though this interpretation doesn't hold relativistically, it is still used to *define* the charge and magnetic radii using the derivatives at $Q^2 = 0$:

- $r_E^2 = -6G_E{}'(0)$
- $r_M^2 = -6G_M{}'(0)/\mu$

Relativistically, there is a rigorous interpretation of $F_1(Q^2)$ as the 2-D Fourier transform of the transverse charge density in the infinite-momentum frame.

Flavour separation

Define single-flavour form factors using unit charge:

$$J_\mu \to J_\mu^q \equiv \bar{q}\gamma_\mu q, \quad G_{E,M}(Q^2) \to G_{E,M}^q(Q^2).$$

Proton form factors from *ep* scattering involve all flavours:

$$G_{E,M}^{\gamma(p)} = \tfrac{2}{3}G_{E,M}^u - \tfrac{1}{3}G_{E,M}^d - \tfrac{1}{3}G_{E,M}^s + \dots$$

Form factors of a neutron are measured in experiments using scattering off ^{2}H or ^{3}He targets. Assuming isospin symmetry, we get

$$G_{E,M}^{\gamma(n)} = \tfrac{2}{3}G_{E,M}^d - \tfrac{1}{3}G_{E,M}^u - \tfrac{1}{3}G_{E,M}^s + \dots$$

By measuring the parity-violating asymmetry in elastic $\vec{e}p$ scattering, the contribution from Z exchange can be isolated. This gives

$$G_{E,M}^{Z(p)} = (1 - \tfrac{8}{3}\sin^2\theta_W)G_{E,M}^u - (1 - \tfrac{4}{3}\sin^2\theta_W)(G_{E,M}^d + G_{E,M}^s) + \dots$$

Neglecting heavier quarks, combining these three measurements yields the strange form factors $G_{E,M}^s$.

Lattice QCD

…is a regularization of Euclidean-space QCD such that the path integral can be done fully non-perturbatively

- Euclidean spacetime becomes a periodic hypercubic lattice, with spacing a and box size $L_s^3 \times L_t$.
- Path integral over fermion degrees of freedom is done analytically, for each gauge configuration. Solving the Dirac equation with a fixed source yields a source-to-all quark propagator.
- Path integral over gauge degrees of freedom is done numerically using Monte Carlo methods to generate an *ensemble* of *gauge configurations*.
- An ensemble with degenerate u and d quarks is called $N_f = 2$; adding a heavier s quark gives $N_f = 2 + 1$, etc.

The $a \to 0$ and $L_s, L_t \to \infty$ extrapolations need to be taken by using multiple ensembles.

Nucleon matrix elements using lattice QCD

To find matrix elements, compute

$$C_{2\text{pt}}(t,\vec{p}) = \sum_{\vec{x}} e^{-i\vec{p}\cdot\vec{x}} \langle N(\vec{x},t)\bar{N}(\vec{0},0)\rangle$$

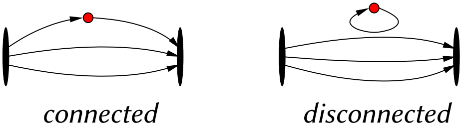
$$\xrightarrow{t\to\infty} e^{-E(\vec{p})t}|\langle p|\bar{N}|\Omega\rangle|^2$$

$$C_{3\text{pt}}(T,\tau;\vec{p},\vec{p}') = \sum_{\vec{x},\vec{y}} e^{-i\vec{p}'\cdot\vec{x}} e^{i(\vec{p}'-\vec{p})\cdot\vec{y}} \langle N(\vec{x},T)\mathcal{O}(\vec{y},\tau)\bar{N}(\vec{0},0)\rangle$$

$$\xrightarrow[T-\tau\to\infty]{\tau\to\infty} e^{-E(\vec{p}')(T-\tau)} e^{-E(\vec{p})\tau} \langle\Omega|N|p'\rangle\langle p'|\mathcal{O}|p\rangle\langle p|\bar{N}|\Omega\rangle$$

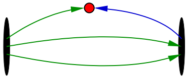
Then form ratios to isolate $\langle p'|\mathcal{O}|p\rangle$.

For $\mathcal{O}$ a quark bilinear, there are two kinds of quark contractions for $C_{3\text{pt}}$:



Connected contractions

We have efficient solvers for source-to-all quark propagators. Connected contractions can be computed using these via the *sequential propagator* technique.

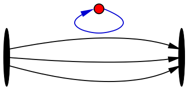


1. Fix the source and compute the forward propagator.
2. Fix the sink and T; compute the backward (sequential) propagator.
3. Combine the two to compute arbitrary $\mathcal{O} = \bar{q} \ldots q$, for all $\tau \in [0, T]$.

For the proton, these contribute for $q \in \{u, d\}$. If we take isovector $(u - d)$ observables, then these are the only contributing contractions.

Disconnected contractions

For strange quarks in the proton, these are the only contribution.
Disconnected light quarks are also needed for, e.g., the proton radius.



Using, e.g., $\mathcal{O} = \bar{q}\Gamma q$, these involve the disconnected loop,

$$T(\vec{q}, t, \Gamma) = -\sum_{\vec{x}} e^{i\vec{q}\cdot\vec{x}} \operatorname{Tr}[\Gamma D^{-1}(x,x)],$$

which involves the quark propagator $D^{-1}(x,y)$ from every point on a timeslice back to itself.

We can estimate the all-to-all propagator stochastically using noise sources η that satisfy $E(\eta\eta^\dagger) = I$. By solving $\psi = D^{-1}\eta$, we get

$$D^{-1}(x,y) = E(\psi(x)\eta^\dagger(y)).$$

Dilution

For a random vector η with components of magnitude $|\eta_i| = 1$, the diagonal of $\eta\eta^\dagger$ is exact and the variance comes from the off-diagonal parts.

$$\text{e.g.} \quad \eta = \begin{pmatrix} \eta_1 \\ \eta_2 \\ \eta_3 \\ \eta_4 \end{pmatrix}, \quad \eta\eta^\dagger = \begin{pmatrix} 1 & \eta_1\eta_2^* & \eta_1\eta_3^* & \eta_1\eta_4^* \\ \eta_2\eta_1^* & 1 & \eta_2\eta_3^* & \eta_2\eta_4^* \\ \eta_3\eta_1^* & \eta_3\eta_2^* & 1 & \eta_3\eta_4^* \\ \eta_4\eta_1^* & \eta_4\eta_2^* & \eta_4\eta_3^* & 1 \end{pmatrix}, \quad E(\eta\eta^\dagger) = I$$

Dilution: use a complete set of projectors $\{P_b | P_b^2 = P_b, \sum_b P_b = I\}$ to partition the components of η and eliminate parts of the variance:

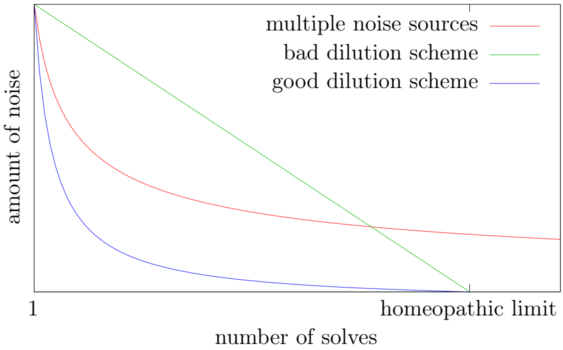
$$\eta^{(b)} \equiv P_b \eta; \qquad E\left(\sum_b \eta^{(b)}\eta^{(b)\dagger}\right) = I$$

$$\text{e.g.} \quad \eta^{(1)} = \begin{pmatrix} \eta_1 \\ \eta_2 \\ 0 \\ 0 \end{pmatrix}, \eta^{(2)} = \begin{pmatrix} 0 \\ 0 \\ \eta_3 \\ \eta_4 \end{pmatrix}, \quad \sum_b \eta^{(b)}\eta^{(b)\dagger} = \begin{pmatrix} 1 & \eta_1\eta_2^* & 0 & 0 \\ \eta_2\eta_1^* & 1 & 0 & 0 \\ 0 & 0 & 1 & \eta_3\eta_4^* \\ 0 & 0 & \eta_4\eta_3^* & 1 \end{pmatrix}$$

Dilution

In many cases, using N dilution projectors to target the most important parts of the noise yields a better than $1/\sqrt{N}$ reduction. Commonly used:

- Spin dilution
- Colour dilution
- Spatial dilution

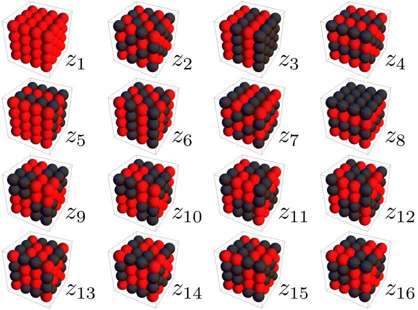


In the *homeopathic limit*, complete dilution is equivalent to fully computing a disconnected loop without stochastic estimation.

Hierarchical probing

(A. Stathopoulos, J. Laeuchli, K. Orginos, SIAM J. Sci. Comput. **35(5)** (2013) S299–S322 [1302.4018])

Use a sequence of specially-constructed spatial Hadamard vectors in order to progressively increase the level of spatial dilution.



red: $+1$
black: -1

Take the component-wise product $\eta^{[b]} \equiv z_b \odot \eta$ and average over b.

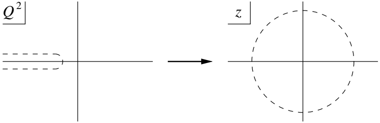
For each fixed number 2^n of Hadamard vectors, this is equivalent to a different spatial dilution scheme.

We use 128 three-dimensional Hadamard vectors to eliminate the variance from neighboring sites up to distance 7.

Fitting Q^2-dependence

We want to fit $G_{E,M}(Q^2)$ with curves to determine the radii and magnetic moment from the slope and intercept at $Q^2 = 0$.

- Common approach: use simple fit forms such as a dipole.
- Better: use z-expansion. Conformally map domain where $G(Q^2)$ is analytic in complex Q^2 to $|z| < 1$, then use a Taylor series:



[R. J. Hill and G. Paz, Phys. Rev. D **82** (2010) 113005]

$$z(Q^2) = \frac{\sqrt{t_{\text{cut}} + Q^2} - \sqrt{t_{\text{cut}}}}{\sqrt{t_{\text{cut}} + Q^2} + \sqrt{t_{\text{cut}}}},$$

$$G(Q^2) = \sum_k a_k z(Q^2)^k,$$

with Gaussian priors imposed on the coefficients a_k. Specifically,

- For G_E, set $a_0 = 0$ (charge conservation) and leave a_1 unconstrained.
- For G_M, leave a_0 and a_1 unconstrained.

Thus $r_{E,M}$ and μ are not directly constrained.

For higher coefficients, impose $|a_{k>1}| < 5 \max\{|a_0|, |a_1|\}$, and vary the bound to estimate systematic uncertainty.

Lattice calculation

RAPID COMMUNICATIONS

PHYSICAL REVIEW D **92,** 031501(R) (2015)

High-precision calculation of the strange nucleon electromagnetic form factors

Jeremy Green,[1,*] Stefan Meinel,[2,3,†] Michael Engelhardt,[4] Stefan Krieg,[5,6] Jesse Laeuchli,[7] John Negele,[8] Kostas Orginos,[9,10] Andrew Pochinsky,[8] and Sergey Syritsyn[3]

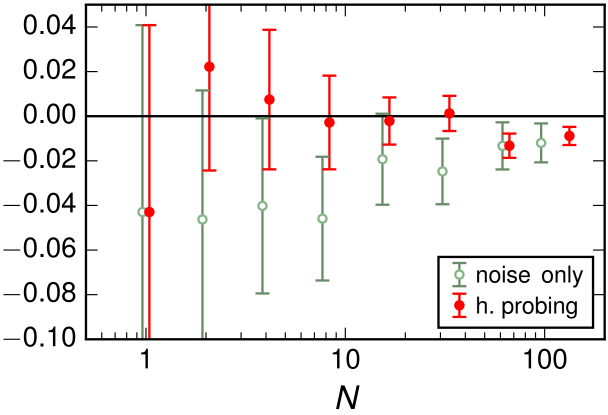
[arXiv: 1505.01803]

- $N_f = 2 + 1$ Wilson-clover fermions
- $a = 0.114$ fm, $32^3 \times 96$
- $m_u = m_d > m_{ud}^{\text{phys}}$, corresponding to pion mass 317 MeV
- $m_s \approx m_s^{\text{phys}}$
- 1028 gauge configurations
- disconnected loops for six source timeslices (128 Hadamard vectors, plus color+spin dilution)
- two-point correlators from 96 source positions

Hierarchical probing vs. many noise sources

Study using 1/3 of gauge configurations.

$G_M^{(\frac{2}{3}u-\frac{1}{3}d)}$ $(Q^2 \approx 0.11\ \mathrm{GeV}^2)$ (disconnected)

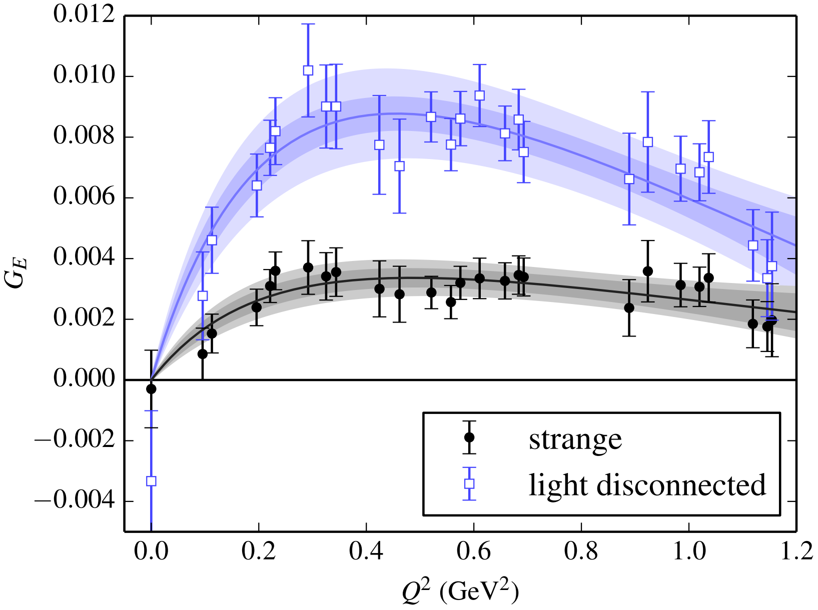


Equal cost at same N ($= N_{\mathrm{Hadamard}}$ or N_{noise}).

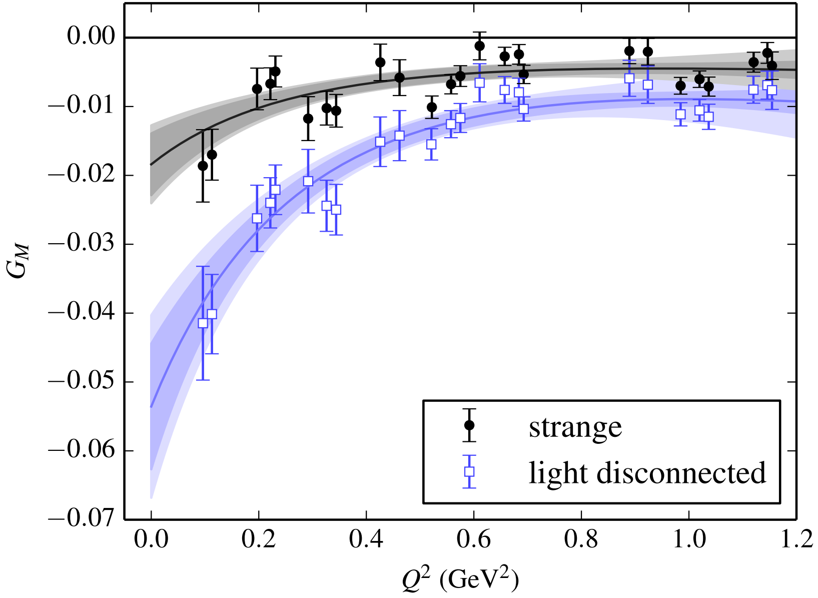
Points offset horizontally.

(S. Meinel, Lattice 2014)

Disconnected $G_E(Q^2)$



Disconnected $G_M(Q^2)$



Extrapolation to physical quark masses

By themselves, the disconnected light-quark form factors are unphysical, but they can be understood in partially quenched QCD and partially quenched chiral perturbation theory (ChPT): $q \in \{u, d, s\} \to q \in \{u, d, s, l, \tilde{l}\}$.

At leading one-loop order in partially quenched ChPT, the radii and magnetic moment depend on one meson mass:

$$m_{\text{loop}} = \begin{cases} m_K & \text{for strange quarks} \\ m_\pi & \text{for disconnected light quarks} \end{cases}$$

The ChPT expressions poorly describe our data, but we use them to motivate a crude physical-point extrapolation: interpolate in m^2_{loop} to the physical m^2_K.

Strange magnetic moment and radii at physical point

$$G_E^s(Q^2) = -\frac{1}{6}(r_E^2)^s Q^2 + O(Q^4), \qquad G_M^s(Q^2) = \mu^s - \frac{1}{6}(r_M^2)^s Q^2 + O(Q^4)$$

Best estimate at physical quark masses: use linear interpolation in m_{loop}^2:

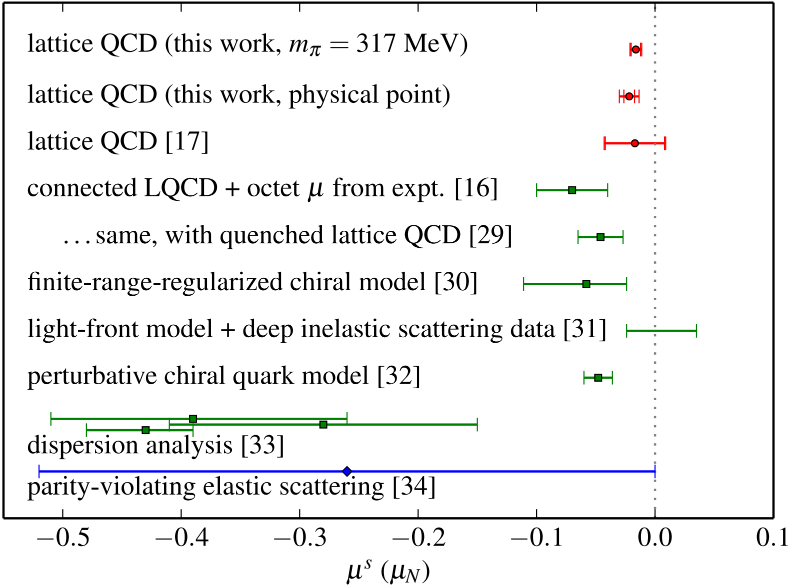
$$\begin{aligned}(r_E^2)^s &= -0.0067(10)(17)(15)\ \text{fm}^2,\\ (r_M^2)^s &= -0.018(6)(5)(5)\ \text{fm}^2,\\ \mu^s &= -0.022(4)(4)(6)\ \mu_N,\end{aligned}$$

where the uncertainties are

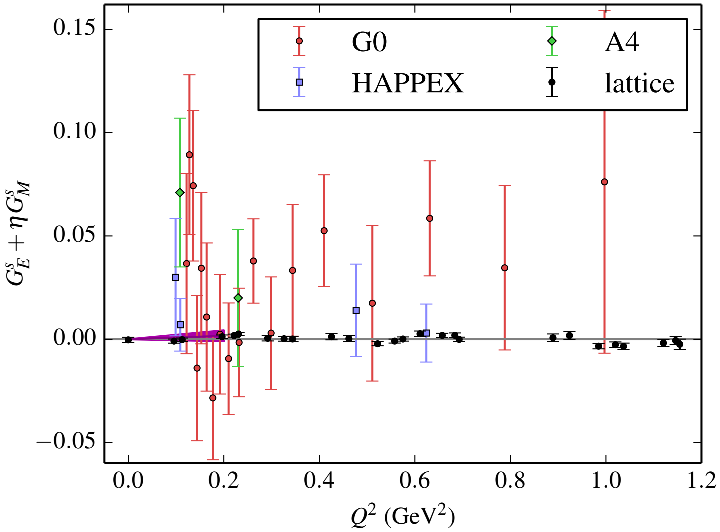
1. statistical
2. systematics at $m_\pi = 317$ MeV
3. physical-point extrapolation (= magnitude of shift from result at $m_\pi = 317$ MeV)

Including the charge factor of $-1/3$ yields a ~ 0.3–0.4% contribution to the proton r_E^2, μr_M^2, and μ, respectively.

Strange magnetic moment



Forward-angle scattering experiments



$\eta \approx AQ^2$, $A = 0.94\ \mathrm{GeV}^{-2}$

Conclusions and outlook

- High statistics and hierarchical probing methods are effective at producing a signal for the disconnected electromagnetic form factors.
- Strange quarks contribute a very small amount to the proton radii and magnetic moment ($\sim 0.3\%$).
- Obtaining a clear nonzero strange-quark signal will be a significant challenge for future parity-violating elastic scattering experiments, especially at forward scattering angles.
- Additional calculations, especially closer to physical quark masses, are needed to confirm the physical-point estimates.
- The same techniques can be applied for other operators, but renormalization is required. This will lead to calculations of the axial form factors as well as the decomposition of the proton's longitudinal and angular momentum into quark and gluon contributions.

Hadamard vectors

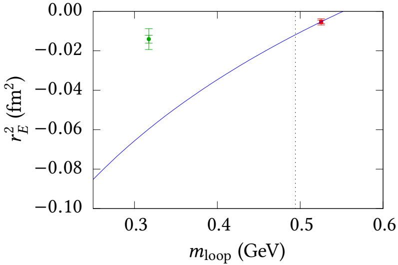
Hadamard vectors h_b can be used to obtain the same results as dilution, by taking the component-wise product $\eta^{[b]} \equiv h_b \odot \eta$ and averaging over b.
e.g. Hadamard vectors: $h_1 = (1,1,1,1)$, $h_2 = (1,1,-1,-1)$

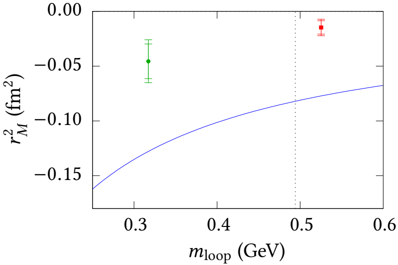
$$\eta^{[1]} = \begin{pmatrix} \eta_1 \\ \eta_2 \\ \eta_3 \\ \eta_4 \end{pmatrix}, \eta^{[2]} = \begin{pmatrix} \eta_1 \\ \eta_2 \\ -\eta_3 \\ -\eta_4 \end{pmatrix}, \quad \frac{1}{2} \sum_b \eta^{[b]} \eta^{[b]\dagger} = \begin{pmatrix} 1 & \eta_1\eta_2^* & 0 & 0 \\ \eta_2\eta_1^* & 1 & 0 & 0 \\ 0 & 0 & 1 & \eta_3\eta_4^* \\ 0 & 0 & \eta_4\eta_3^* & 1 \end{pmatrix}$$

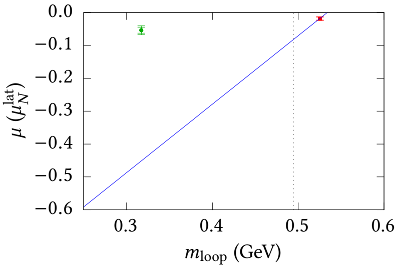
If we had used only $\eta^{[1]}$, we would also get the correct expectation value (only with more noise).
$\longrightarrow$ Hadamard vectors allow for progressively increasing the level of dilution, while making use of previous effort.

Partially quenched ChPT at leading one-loop order

Inputs: pseudoscalar decay constant and baryon axial couplings.







At this order, PQChPT poorly describes the data.
$\longrightarrow$ use simple linear interpolation in m_{loop}^2.