



University
of Glasgow



Precise tests of the Standard Model from Lattice QCD

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Jefferson Lab, 8th Jan, 2016

Background

University of Calcutta
Degree : B.Sc (Hons); M.Sc
Subject : Physics
Specialisation: Particle physics



University of Kentucky
Degree: MS
Research : Proton Spin puzzle
using lattice QCD
Supervisor : Prof. Keh-Fei Liu
Collaboration : χ QCD
Phys.Rev. D91 (2015) 1, 014505

Topics

- Muon $g-2$
- Semileptonic decay
- Meson spectroscopy
- Quark mass tuning
- Non-perturbative renormalisation of lattice currents.

Invited talks

- **University of Cambridge**
Nov, 2015
- **Swansea University**
May, 2015
- **Excited QCD 2015**
Slovakia, March 2015
- **Durham University (plenary)**
(DiRAC Day –Sept, 2014)

Publications

1. **Phys. Rev. D** 91 (2015) 5, 054508 (Editor's suggestion)
2. **Phys. Rev. D** 89, 114501 (2014)
3. arXiv:1512.03270 (submitted to **Phys. Rev. D**)
4. 2 in preparation
5. 5 proceedings

Funding

- **Lord Kelvin Scholarship.**
- **Gilmour Scholarship**
- European Commission's Seventh Framework Programme (**FP7**) under **STRONGnet**, a **Marie Curie Initial Training Network (ITN)**

Communication with young physicists:

- Lab, recitation, demonstration for undergraduates
- Graduate level QFT grader
- Co-supervision - master's and junior PhD research students

Communication with public:

- University open days
- Royal Society master classes for high schoolers
- Science Slam 2015, Glasgow
- Explorathon'14 and Explorathon'15, European Researchers' Night, Scotland, funded by the European commission MARIE CURIE ACTIONS programme
- Research presentation organised by the Scottish Federation of University Women



Organisation: UK national conferences -

- Young Experimentalist And Theorist Institute - YETI'16, IPPP, Durham
- Young Theorists' Forum, Annual High Energy Physics Conference, Durham

Precision tests of the Standard Model using lattice QCD

Muon $g-2$:
Hadronic vacuum
polarisation from
lattice QCD

Phys. Rev. D 89,
114501 (2014);
arXiv:1512.03270
(submitted to
Phys. Rev. D)

Semileptonic
decay of
mesons:
D to K l v ;
scalar and
vector form
factors, V_{CS}

Meson mass and
decay constants

Vector mesons: $\phi, \rho,$
 $j/\psi, K^*$
Pseudoscalar : $\eta_S, \pi, K,$
 η_C, D, K

Proton Spin
Puzzle

Phys.Rev. D 91
(2015)
1, 014505

MS

Code writing
and data analysis

PhD

Testing, planning resources, writing scripts and modifying MILC codes,
data generation (2-pt, 3-pt), data analysis : risk analysis, co-ordination
and back up plans

Important ingredients :

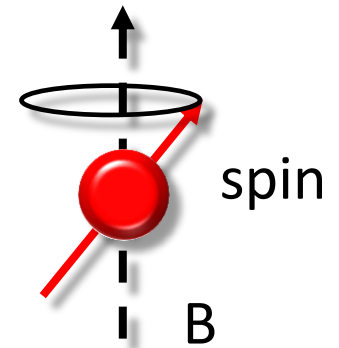
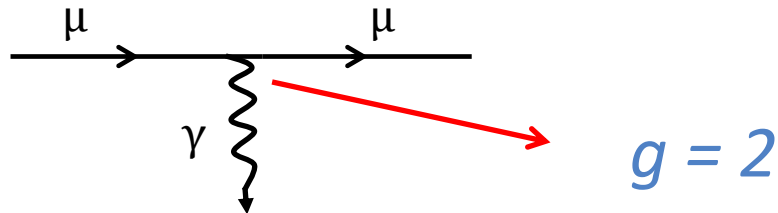
- Quark mass tuning [**Phys. Rev. D**91 (2015) 5, 054508 (Editor's suggestion)]
- Renormalisation of currents (non-perturbative)

**Hadronic vacuum polarisation contribution to the
muon anomalous magnetic moment
from lattice QCD**

Relevance and Timeliness

$$\vec{\mu} = g \frac{e}{2m} \vec{S}$$

$$a_{\mu} = \frac{g - 2}{2}$$



$$a_{\mu}^{exp} = 11\,659\,208.9(63) \times 10^{-10} \text{ (0.54 ppm)}$$

$$a_{\mu}^{SM} = 11\,659\,182.8(49) \times 10^{-10} \text{ (0.42 ppm)}$$

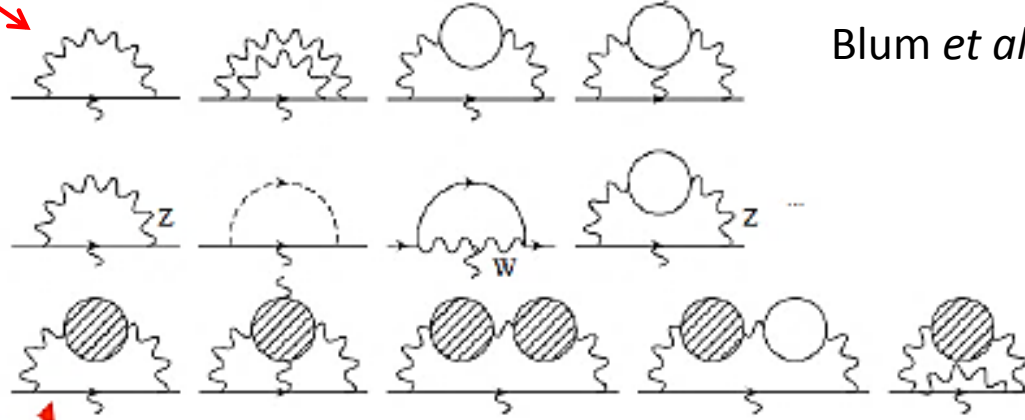
$$a_{\mu}^{exp} - a_{\mu}^{SM} = 26.1(8) \times 10^{-10} \quad \mathbf{3+\sigma !}$$

Expt. at FNAL & J-PARC will reduce uncertainty to **0.16 ppm** starting 2017



Current theoretical status of muon g-2

$$\frac{\alpha_{QED}}{2\pi} = 0.00116$$



Blum *et al.* 1301.2607

QED, EW & QCD
Contributes;
QED dominates

LO Hadronic vacuum polarisation (HVP)

Contribution	Result ($\times 10^{-10}$)	Error
QED (leptons)	11658471.8	0.00 ppm
HVP(lo) [1]	692.3	0.36 ppm
HVP(ho)	-9.8	0.01 ppm
HLbL [2]	10.5	0.22 ppm
EW	15.4	0.02 ppm
Total SM	11659180.2	0.42 ppm

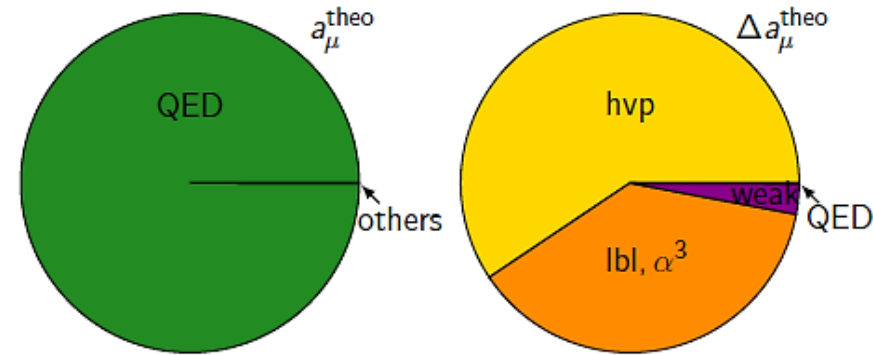


Fig. a: Result

Fig. b: Error

[1] Davier *et al.* Eur. Phys. J. C71 (2011) 1515

[2] Prades, de Rafael, Vainshtein, 0901.0306

HVP from lattice QCD

Target : better than 1% in HVP

In Euclidean space,

connected contribution for flavour i

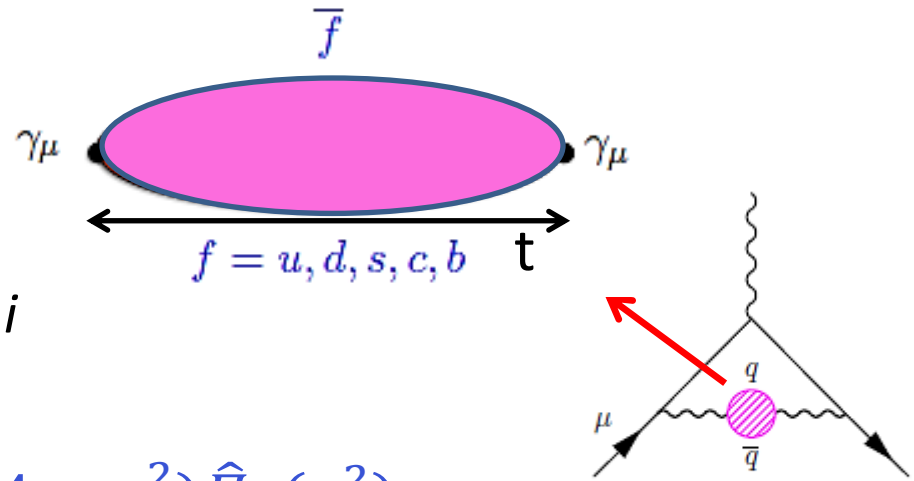
$$a_\mu^{HVP,i} = \frac{\alpha}{\pi} \int_0^\infty dq^2 f(q^2) (4\pi\alpha e_i^2) \hat{\Pi}_i(q^2)$$

On lattice

$f(q^2)$ known, divergent function of q^2

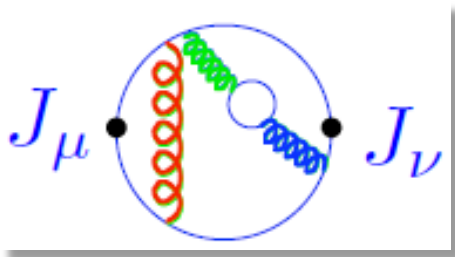
Renormalised Vacuum Polarisation function: $\hat{\Pi}(q^2) = \Pi(q^2) - \Pi(0)$

Integrand strongly peaked at $q^2 \sim \frac{m_\mu^2}{4} \sim 0.003 \text{ GeV}^2$



Blum, hep-lat/
0212018

HPQCD method for HVP calculation



Calculate correlation function on lattice

$$\langle j^j(\vec{x}, t) j^j(0) \rangle$$

Time moments of lattice current-current correlators

$$\begin{aligned} G_{2n} &\equiv a^4 \sum_t \sum_{\vec{x}} t^{2n} Z_V^2 \langle j^j(\vec{x}, t) j^j(0) \rangle \\ &= (-1)^n \frac{\partial^{2n}}{\partial q^{2n}} q^2 \hat{\Pi}(q^2) \Big|_{q^2=0} \end{aligned}$$

$$\hat{\Pi}(q^2) = \sum_{j=1}^{\infty} q^{2j} \Pi_j \quad \text{with} \quad \Pi_j = (-1)^{j+1} \frac{G_{2j+2}}{(2j+2)!}$$

Use Pade approximant (ratio of m/n polynomials) for better q^2 behaviour

Allows us to reconstruct $\hat{\Pi}(q^2)$ and integrate

Analysis Ingredients

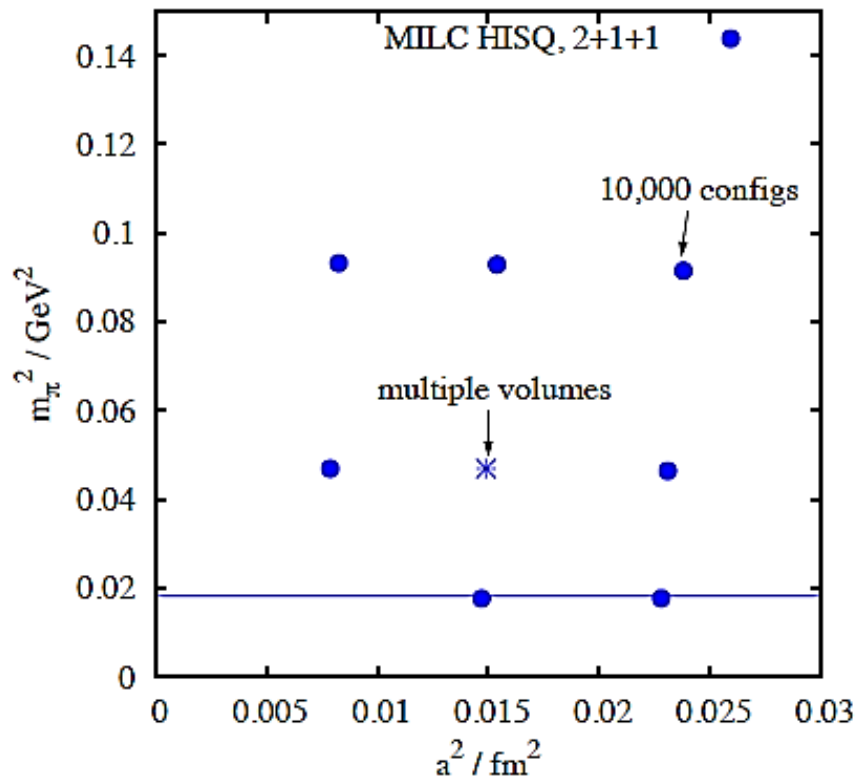
MILC configurations : up/down, strange, charm quarks in the sea

$$m_u = m_d$$

Multiple $m_{u/d}$
Including physical

Valence quark
Masses tuned
accurately

High statistics:
10,000 configurations
on coarsest lattice

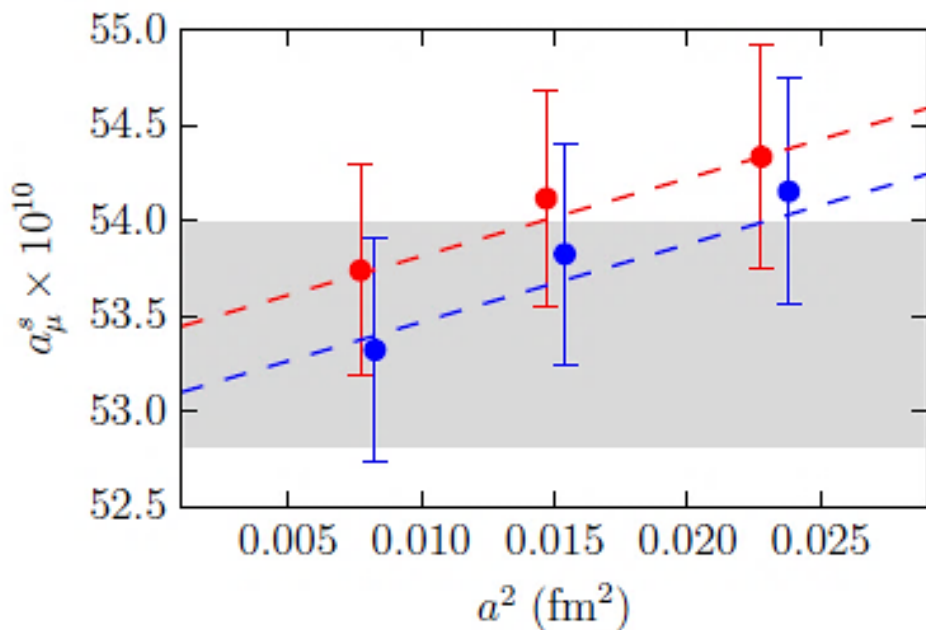


Lattice spacing
 $a \sim 0.09 - 0.15 \text{ fm}$

Multiple volumes
 $\sim (5-6 \text{ fm})^3$

Vector current renormalisation Z_V
calculated nonperturbatively
with high precision

Strange quark contribution to HVP



	a_μ^s
Uncertainty in lattice spacing (w_0, r_1):	1.0%
Uncertainty in Z_V :	0.4%
Monte Carlo statistics:	0.1%
$a^2 \rightarrow 0$ extrapolation:	0.1%
QED corrections:	0.1%
Quark mass tuning:	0.0%
Finite lattice volume:	< 0.1%
Padé approximants:	< 0.1%
Total:	1.1%

$$a_{\mu,lat}^s = a_\mu^s \times \left(1 + c_{a^2} \left(\frac{a\Lambda_{QCD}}{\pi} \right)^2 + c_{sea} \delta x_{sea} + c_{val} \delta x_{val} \right)$$

With $m_{u/d}^{lat} = m_{u/d}^{phys}$, after extrapolation to $a = 0$

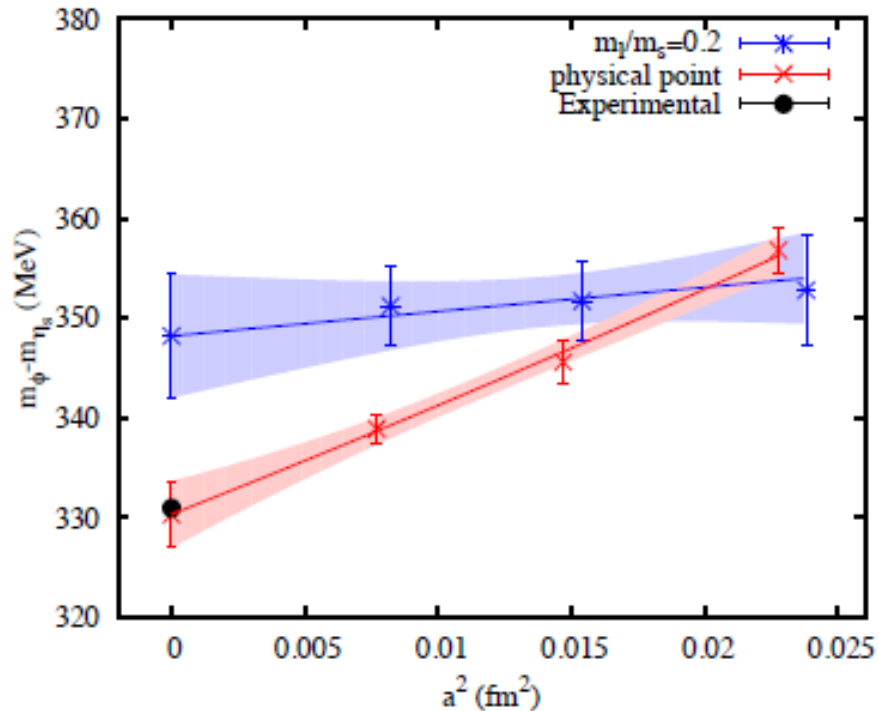
$$a_\mu^s = 53.41(59) \times 10^{-10}$$

Check strange-strange vector correlators

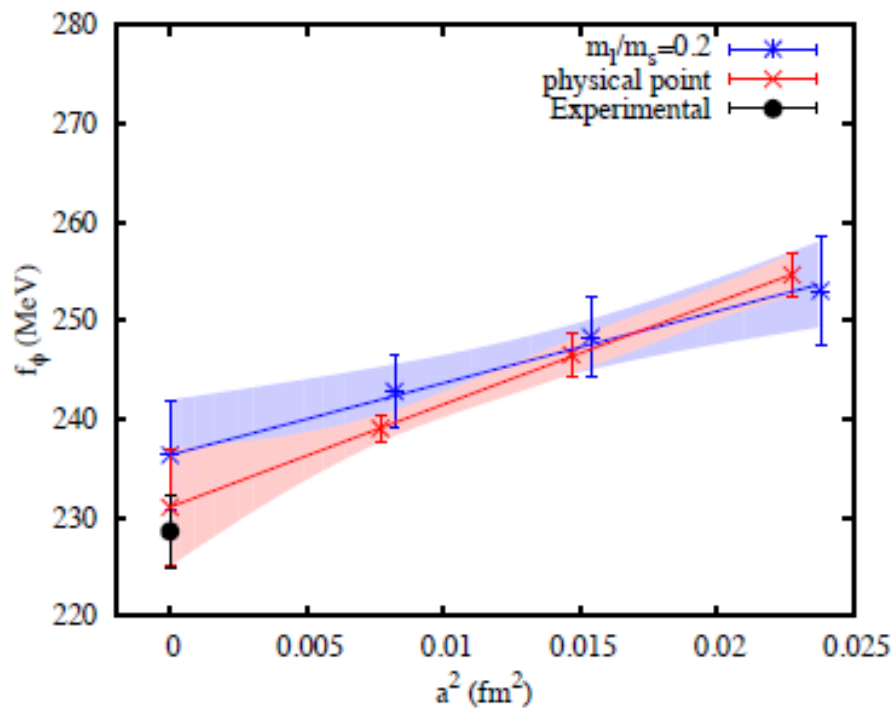
At large time : Correlator $\sim Ae^{-mt}$

ϕ -meson mass

Related to decay constant
(annihilation amplitude)



Experimental $\Gamma(\phi \rightarrow e^+ + e^-)$
used for f_ϕ



Light quark (connected) contribution to HVP

Additional challenges:

1. Noisy data
2. Finite volume error
3. $\pi\pi$ contribution

New ingredient to handle noisy data:

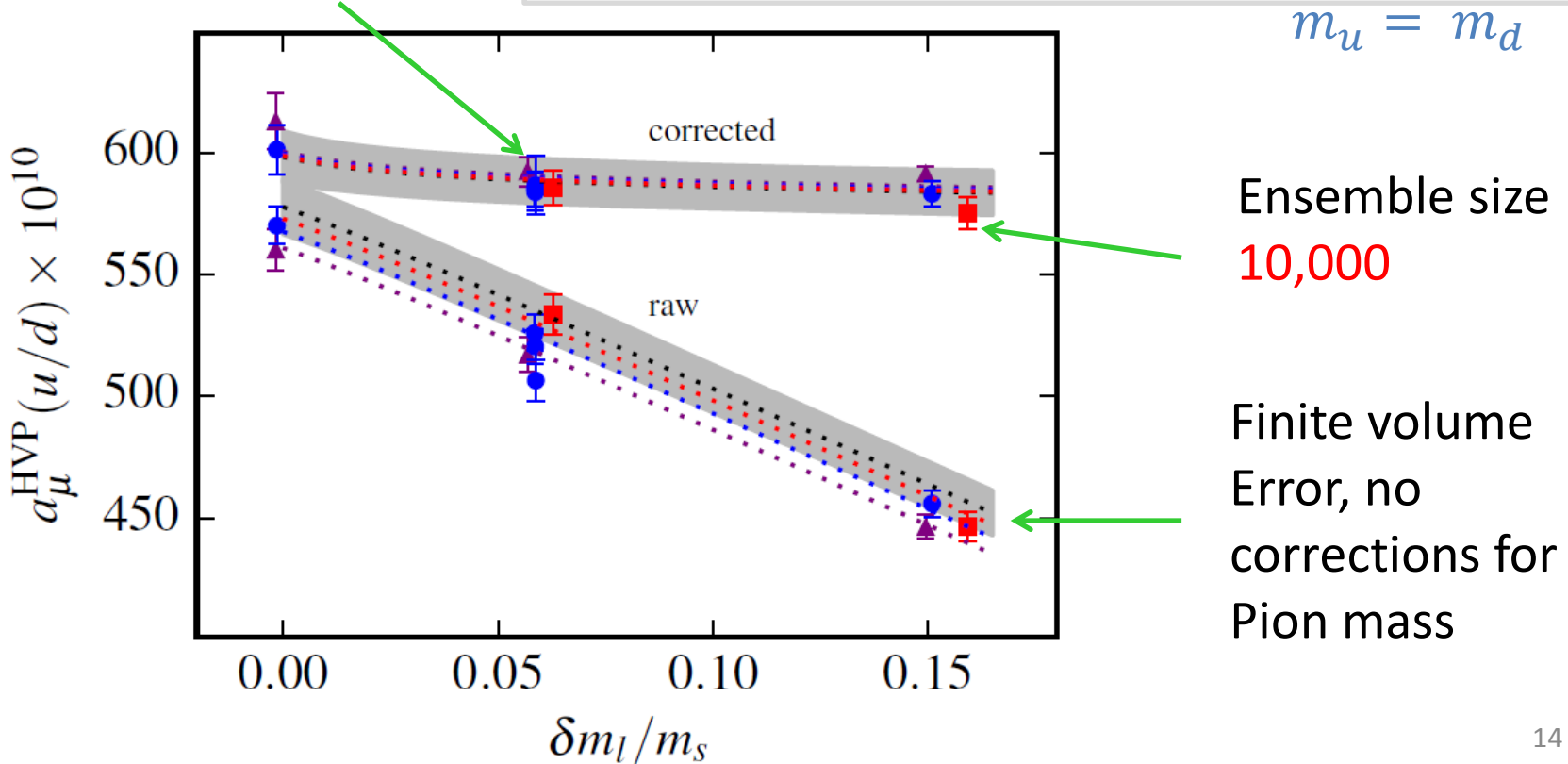
$$G(t) = \begin{cases} G_{data}(t), & t \leq t^* \\ G_{fit}(t), & t > t^* \end{cases}$$

← From Monte Carlo
← From multi-exponential fit

$$t^* = 1.5 fm = 6/\rho$$

+ Smearings at source-sink

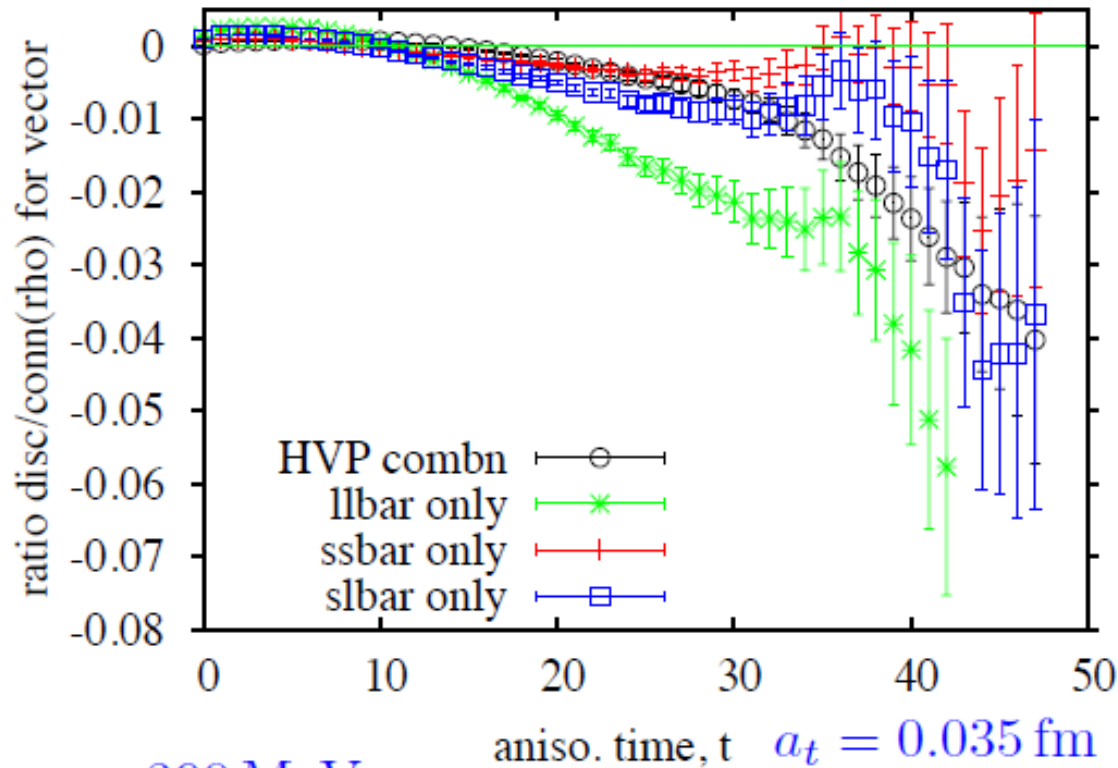
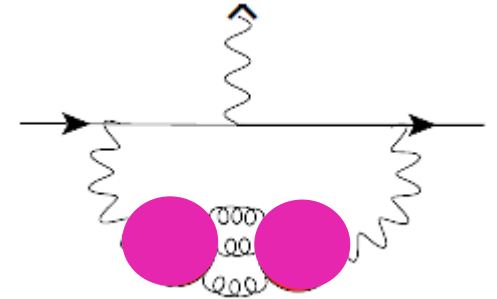
Multiple volumes



Quark-line disconnected contribution

Vanishes if $m_u = m_d = m_s$

since, $\sum_i e_i = 0$



$m_\pi = 390 \text{ MeV}$

aniso. time, t $a_t = 0.035 \text{ fm}$

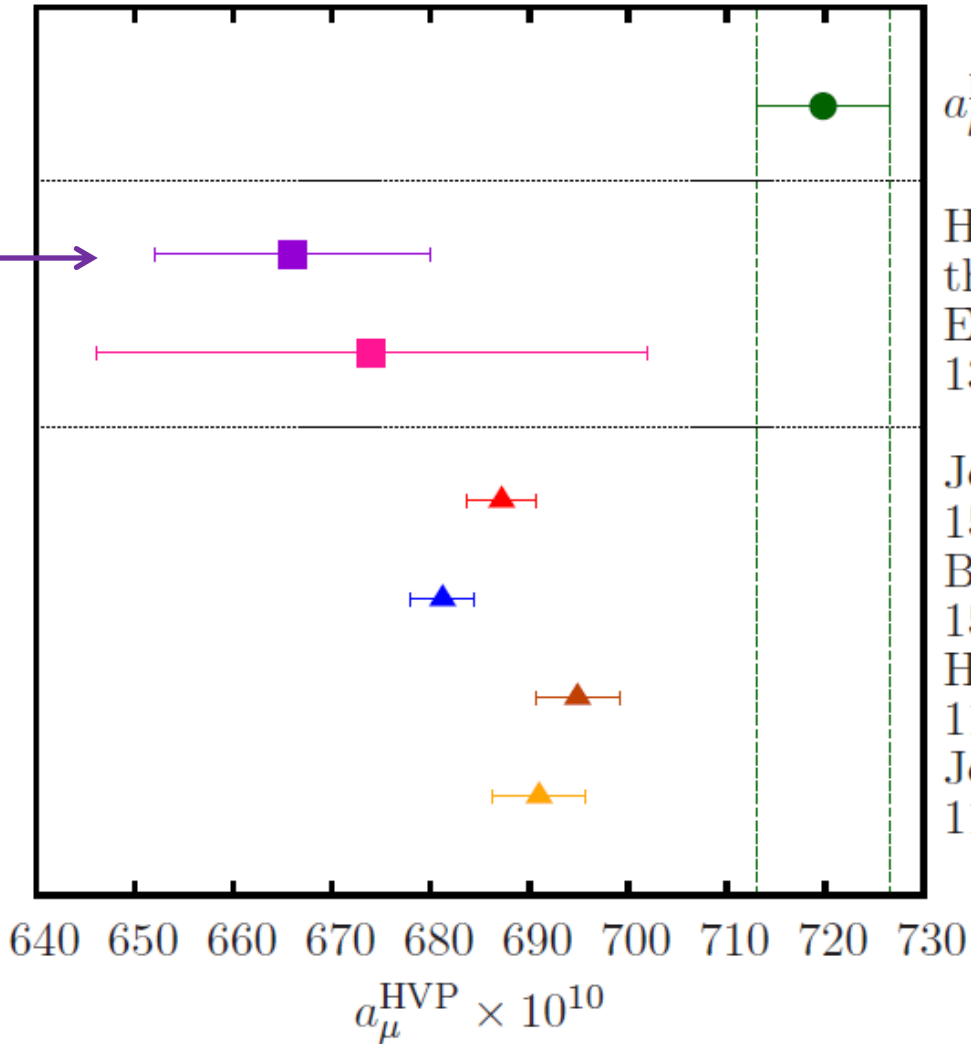
HVP disconnected contribution :

- Negative
- 0.2(1.5)% of the connected

Hadron Spectrum + HPQCD
arXiv: 1512.03270 (submitted to Phys. Rev. D)

Lattice – Continuum comparison

3.5 σ from
no new
physics \rightarrow



$a_\mu^{\text{HVP, nonewphysics}}$

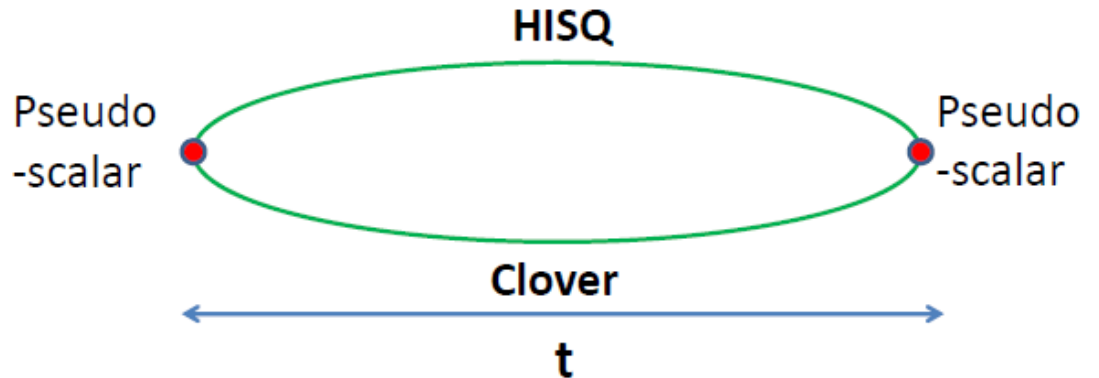
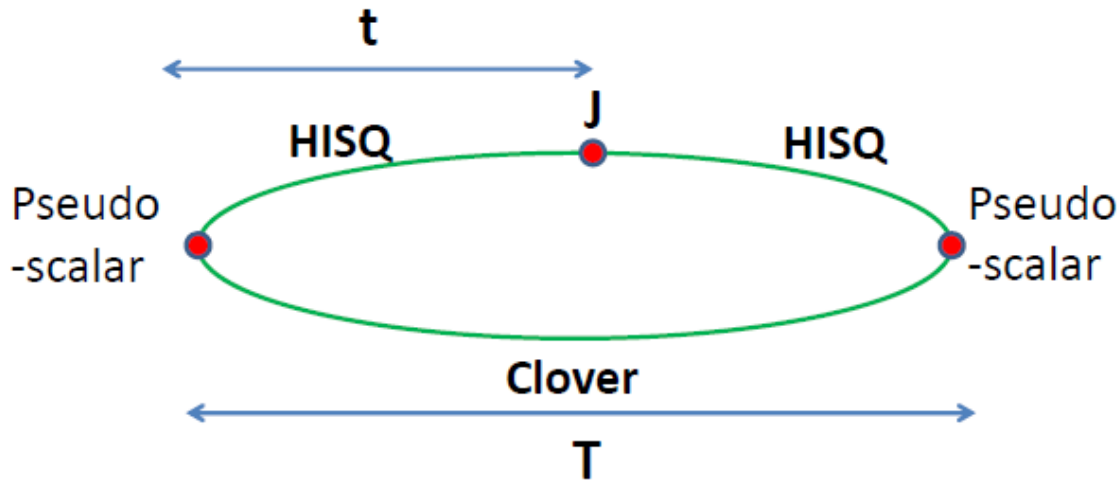
HPQCD
this paper
ETMC
1308.4327

Jegerlehner
1511.04473
Benayoun et al
1507.02943
Hagiwara et al
1105.3149
Jegerlehner et al
1101.2872

Lattice
calculation
using
u,d,s,c sea
quarks
+ physical
valence
quarks

In future : Increase statistics, QED and isospin effects, disconnected piece
More Calculations underway (Mainz, BMW, RBC/UKQCD, ...)

Nonperturbative renormalisation of vector currents

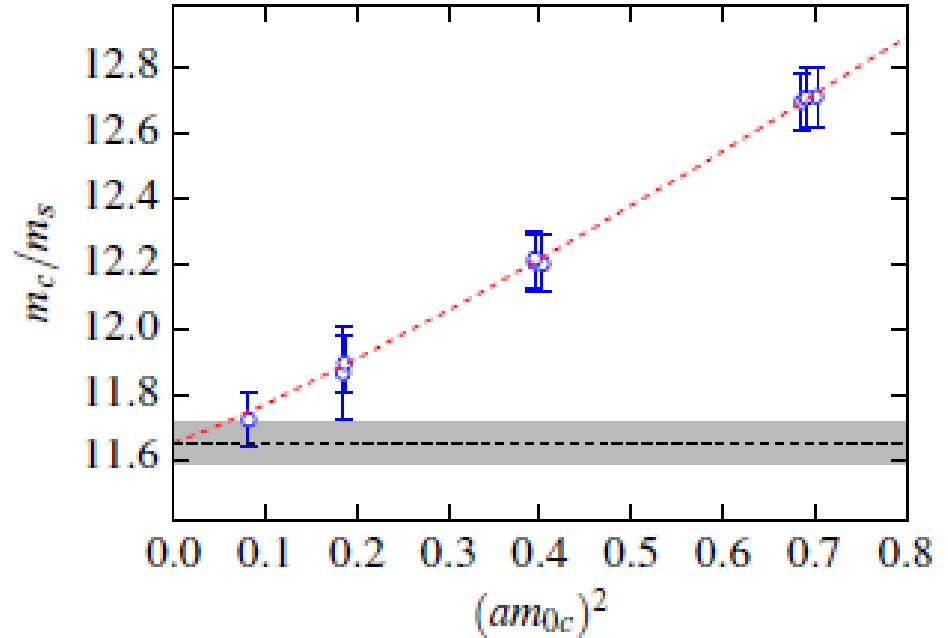
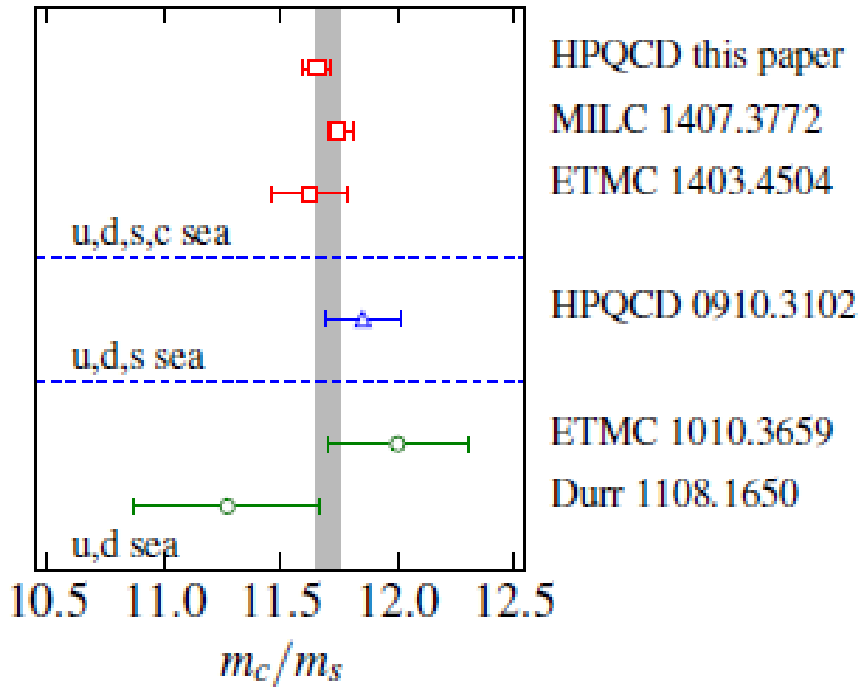


$$1 = Z_{V_{\bar{q}q}^4} \langle H_q | V_{\bar{q}q}^4 | H_q \rangle$$

B. Chakraborty et. al, PoSLATTICE 2013, 309

Quark mass tuning

$$m_{\overline{MS}}(\mu) = Z(a\mu)m_{latt}$$



$$\frac{(m_{q1,latt})}{(m_{q2,latt})_{a=0}} = \frac{m_{q1,\overline{MS}}(\mu)}{m_{q2,\overline{MS}}(\mu)}$$

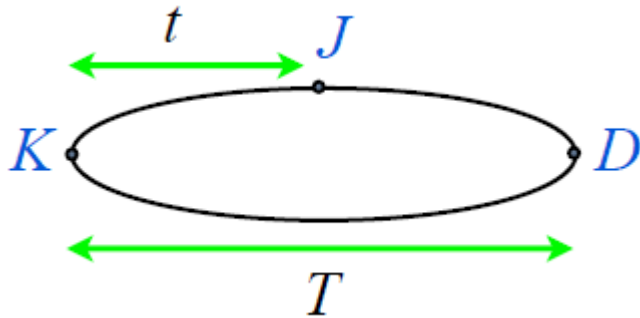
$$m_s^{\overline{MS}}(2 \text{ GeV}) = 92.2(13)\text{MeV}$$

Phys. Rev. D 91 (2015) 5, 054508

Flavour physics

Semileptonic decay : form factors; V_{CS}

$$\frac{d\Gamma^{D \rightarrow K}}{dq^2} = \frac{G_F^2 p^3}{24\pi^3} |V_{CS}|^2 |f_+^{D \rightarrow K}(q^2)|^2$$

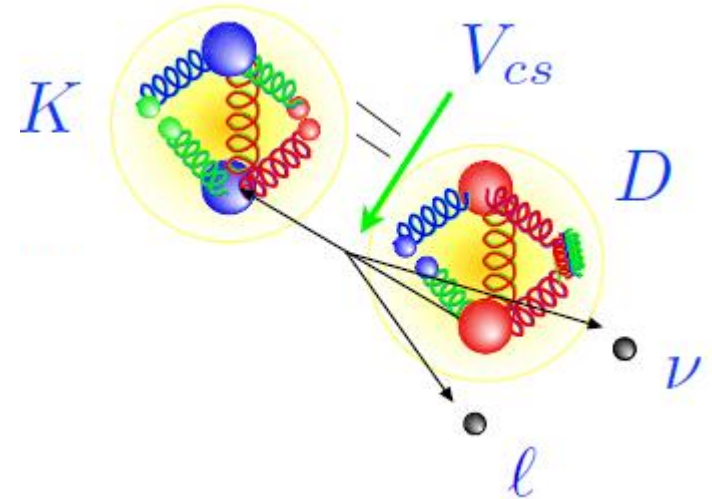


$$\langle K | S | D \rangle = f_0^{D \rightarrow K}(q^2) \frac{M_D^2 - M_K^2}{m_{0C} - m_{0S}}$$

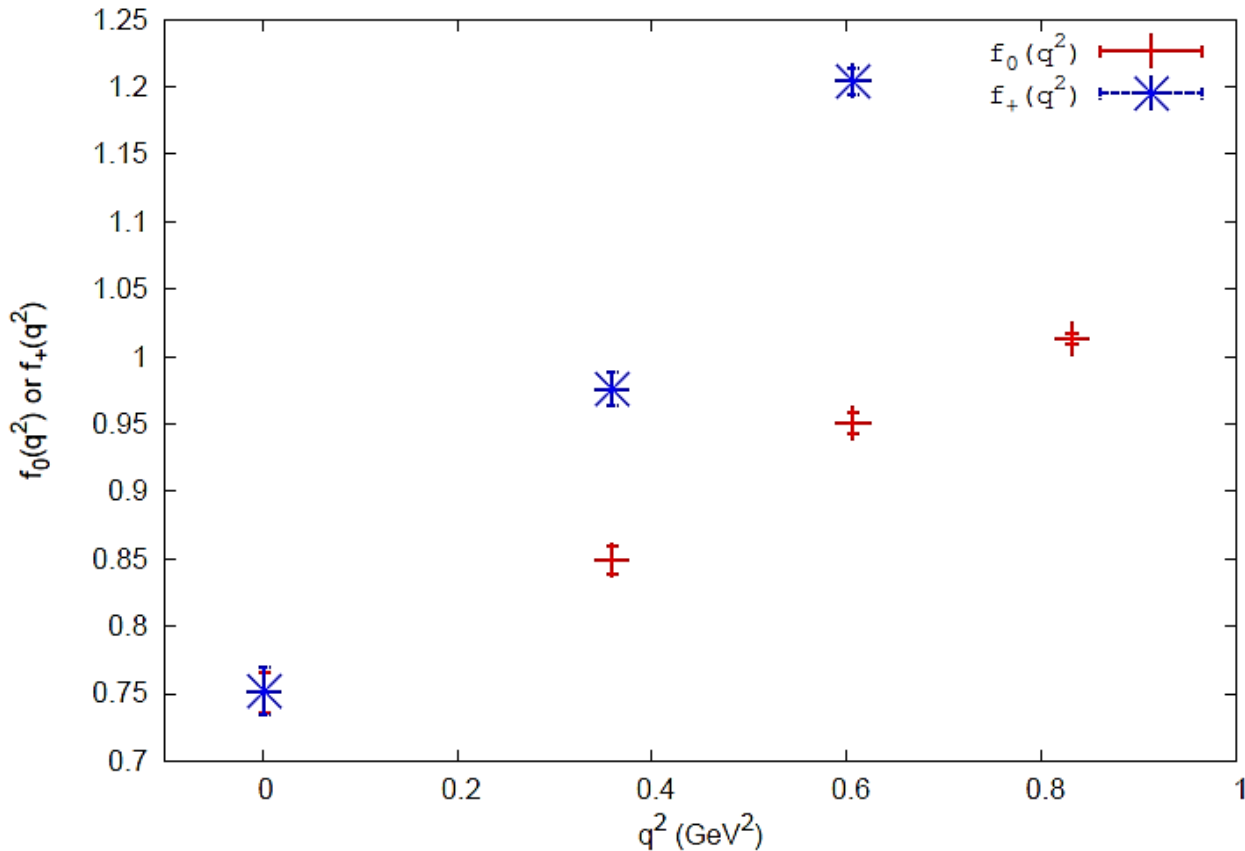
$$q^\mu = p_D^\mu - p_K^\mu$$

Twisted B.C.

$$\begin{aligned} \langle K | V^\mu | D \rangle &= f_+^{D \rightarrow K}(q^2) \left[p_D^\mu + p_K^\mu - \frac{M_D^2 - M_K^2}{q^2} q^\mu \right] \\ &+ f_0^{D \rightarrow K}(q^2) \frac{M_D^2 - M_K^2}{q^2} q^\mu \rightarrow f_0(0) = f_+(0) \end{aligned}$$



Shape of the form factors : $D \rightarrow K1\nu$



- Chiral – continuum extrapolation
- Include bin-bin correlation

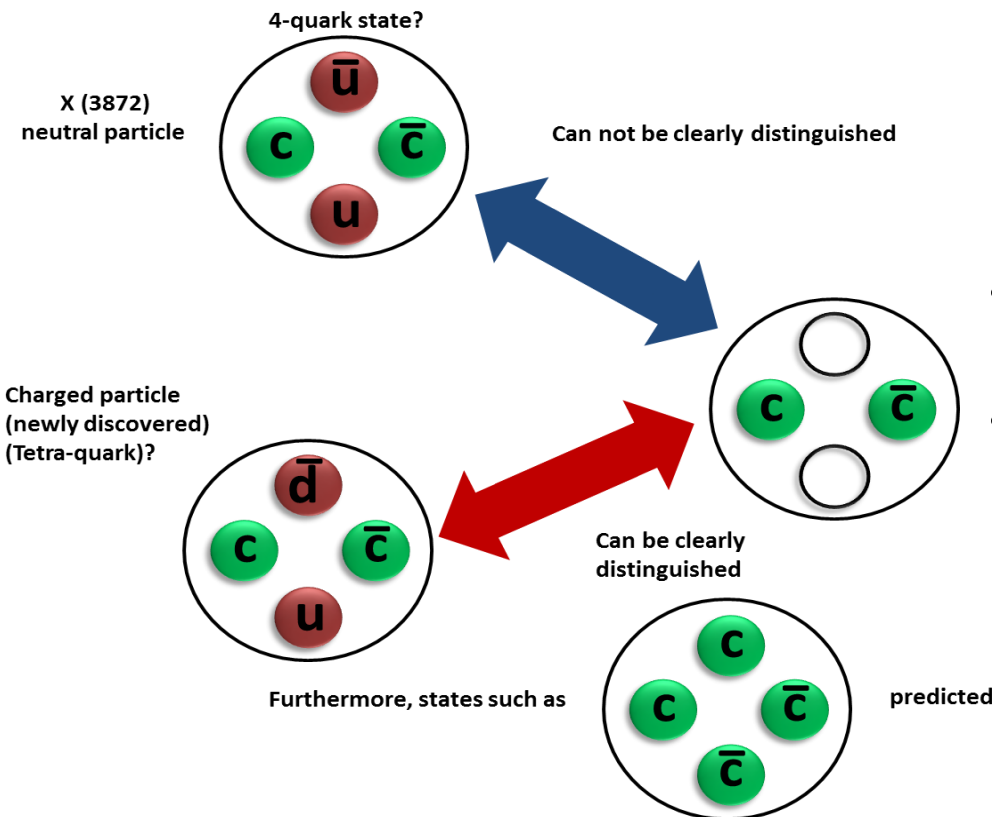
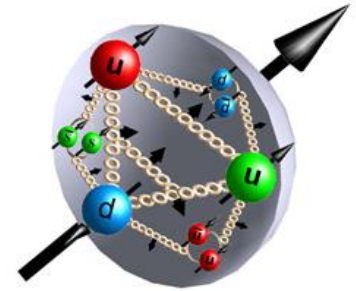
Transferable skills learned

- C, C++, Python, Bash.
- Handling MILC codes, Corrfitter, Isqfit.
- Written data generation and analysis scripts.
- Data generation: propagators, 2-pt correlators, 3-pt correlators, sequential technique, local and non-local currents, HISQ and clover quarks, Gaussian smearings, disconnected correlators, all-to-all propagators method using random noise vector.
- Data analysis : Bayesian multi-exponential fit, generalised eigenvalue method, jackknife, bootstrap, z-expansion, chiral-continuum extrapolation, finite volume corrections.
- Testing, planning resources, risk analysis, co-ordination and back up plans.
- Integrating individual effort with collaborative effort.
- Handling multiple projects – time management and organisation.

Future Plans: relevant to 12 GeV upgrade at JLAB

Nucleons

- The helicity distribution of the constituents within the nucleon
- Calculating GPDs and TMDs – physical quark, chiral symmetry, finer lattices, larger volume, disconnected contribution



Mesons

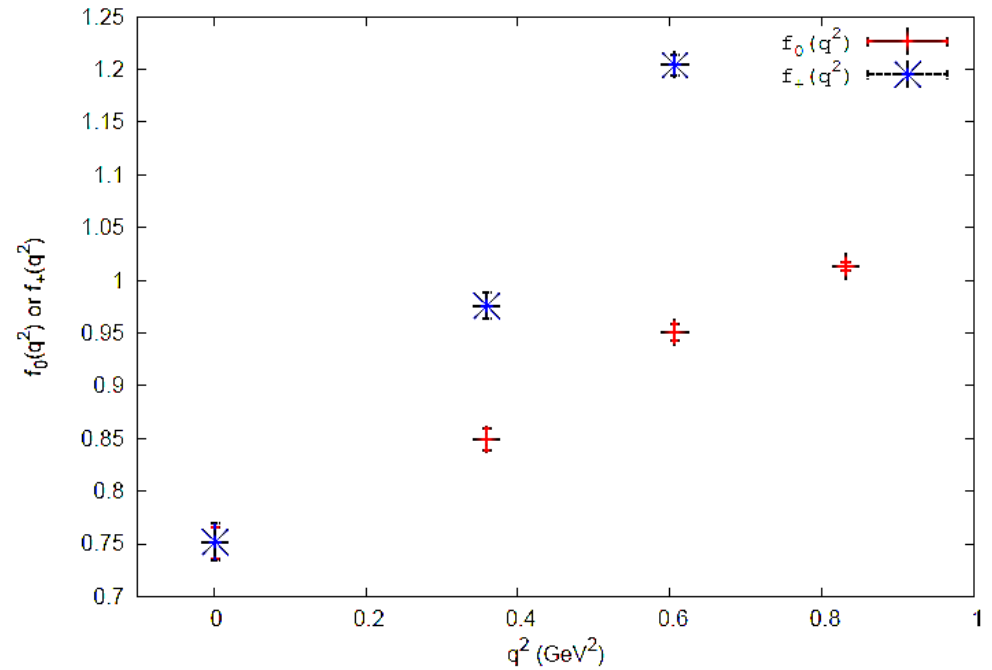
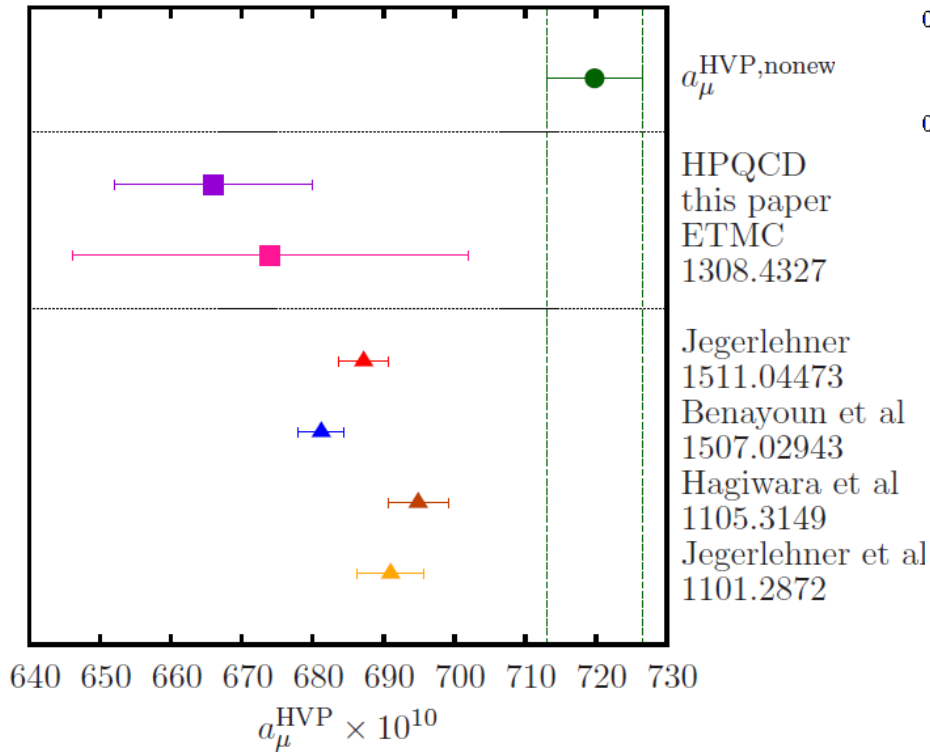
- Spectroscopy of exotic mesons; radiative transitions.
- Application of scattering theory methods to the phenomenology of multi-hadron final states.

Acknowledgement

PhD Advisor: Prof. Christine Davies (Glasgow)

Collaborators (during PhD): Dr Gordon Donald (Regensburg)
Dr Rachel Dowdall (Cambridge)
Pedro Gonçalves de Oliveira (Glasgow)
Dr Jonna Kopponen (Glasgow)
Prof. G. Peter Lepage (Cornell)
Prof. Mike Peardon (Dublin)
Prof. Sinead Ryan (Dublin)
Dr Thomas Teubner (Liverpool)

HVP contribution to the anomalous magnetic moment of muon



Form factors in D to K semileptonic decay