Non-perturbative methods for low-energy hadron physics

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JLab postdoc position interview, January 11th 2016

Biographical presentation

Biography

- Born in Madrid, Spain on March 6th, 1984
- 2002-2007: 5 year degree in Physics
  Universidad Complutense de Madrid (UCM)

  2006-2007 Erasmus student in Lund University
- 2007-2008: MSc in Fundamental Physics at UCM
- 2008-2012: PhD in Theoretical Physics, UCM

  Title: “Study of the **properties** and **nature** of the lightest **scalar mesons** and their relation to the spontaneous **chiral symmetry breaking**”

  Supervisor: J. R. Peláez

  Awarded the 2013 Extraordinary Doctorate Prize
- 2013: postdoc at IFIC (Valencia)

  with Prof. J. Nieves and A. Pich
- 2013-2016: postdoc at HISKP (Bonn)

  with Prof. B. Kubis and U.-G. Meißner
- 2015: awarded the Dr. Klaus Erkelenz Prize

Teaching Assistant

- 2010-2011: Degree in Physics, UCM

  Calculus I

  Calculus II

- 2013-2016: Master in Physics, Bonn University

  Advanced Quantum Physics

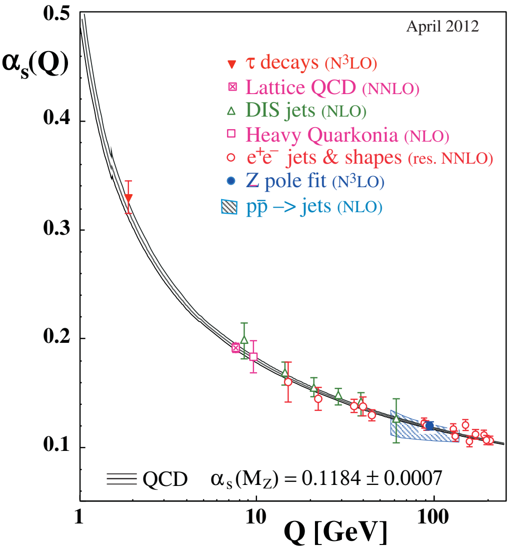
  General Relativity

  Advanced Quantum Field Theory

  Seminars on Advanced Topics in Quantum Field Theory

Research

Running coupling of QCD



Bethke et al. 2012

- Asymptotic freedom

$$\beta_{QCD} = \mu \frac{\partial}{\partial \mu} g$$

$$= -\left(\frac{11 N_c}{3} - \frac{2 N_f}{3}\right) \frac{g^3}{16\pi^2} + \mathcal{O}(g^5)$$

Gross, Politzer, Wilczek 1973 (Nobel prize 2004)

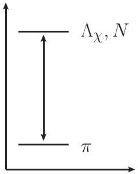
- QCD strongly coupled at low energies

  ⇒ Perturbation theory fails

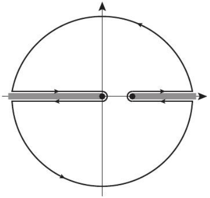
  ⇒ **Need non-perturbative methods**

Non-perturbative methods for low-energy Hadron Physics

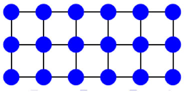
1. **Effective field theories**: symmetries, separation of scales
   $\hookrightarrow$ ChPT, UChPT, ChEFT, $\not\pi$EFT, HπEFT, NREFT $\cdots$



2. **Dispersion relations**: analyticity ($\simeq$ causality), unitarity ($\simeq$ probability conservation), crossing symmetry
   $\hookrightarrow$ Cauchy's theorem, analytic structure



3. **Lattice**: Monte-Carlo simulation
   $\hookrightarrow$ solve discretized version of QCD numerically



Research

- **Dispersive** methods for low-energy $\pi\pi$ and πN scattering
- **Non-ordinary** nature of the lightest **scalar mesons**
- Chiral symmetry restoration
- **Large-N_c** QCD for **mesons** and **baryons**

Dispersive methods for low-energy $\pi\pi$ and πN scattering

Motivation

- More observables than unknown constants

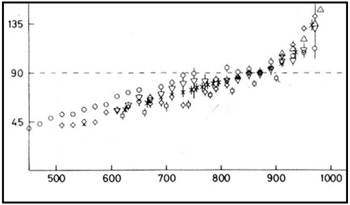
  $\hookrightarrow$ test of **ChPT** and the spontaneous **chiral symmetry** breaking
- Spectroscopy of **light scalars**
- Many hadronic processes of interest end with two or three pions

  $\hookrightarrow$ $\pi\pi$ **rescattering** thus relevant for final state interactions

Difficulties

- Data sets are incompatible in many regions

  $\hookrightarrow$ dominated by **systematics**
- Many analysis suffer from strong **model dependencies**



CERN-Munich $\pi\pi$ analyses of the same experiment

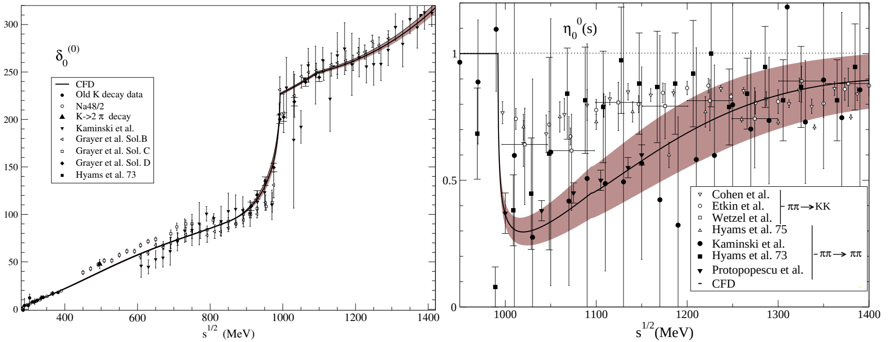
Grayer et al. (1974)

Model independent dispersive description of $\pi\pi$ scattering

Experimental fit up to $\sqrt{s} = 1.42$ GeV imposing **dispersive** constraints
$\hookrightarrow$ once- and twice-subtracted **Roy** equations

Roy (1971)

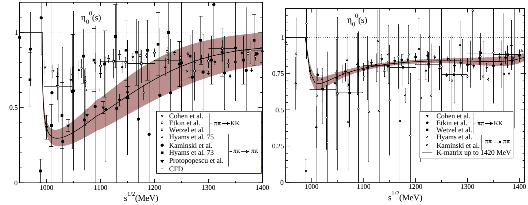
- Example: the S0-wave



García Martin, Kaminski, Peláez, JRE, Yndurain [PRD83 (2011)]

Dispersive analysis of low-energy $\pi\pi$: applications

- Long-standing dip vs non-dip **controversy** for the S0-wave inelasticity Pennington, Bugg, Zou, Achasov,...



$\hookrightarrow$ **non-dip** scenario **rejected** by dispersion relations

- **Dispersive** determination of the $\mathbf{f_0(500)}$ and $\mathbf{f_0(980)}$ pole parameters

  $\hookrightarrow$ triggered the PDG 2012 revision García-Martin, Kaminski, Peláez, JRE [PRL 107 (2011)]

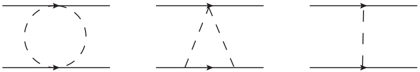
- **Analytic continuation** through **Padé approximants** Masjuan, JRE, Sanz-Cillero [PRD90 (2014)]

- **Normality** requirements for the **data** selection

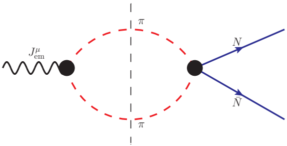
  $\hookrightarrow$ required for the χ^2-fit method Navarro, Ruiz-Arriola, JRE [PRD91 (2015)]

Motivation

- **Test** ChPT in the **baryon** sector
- Input for **NN interactions**
  ↪ extraction of the **LECs**



- Determination of the $\sigma_{\pi N}$
  ↪ fixes the scalar coupling of the nucleon → **dark matter** searches
- Nucleon **form factor**
  ↪ proton radius puzzle

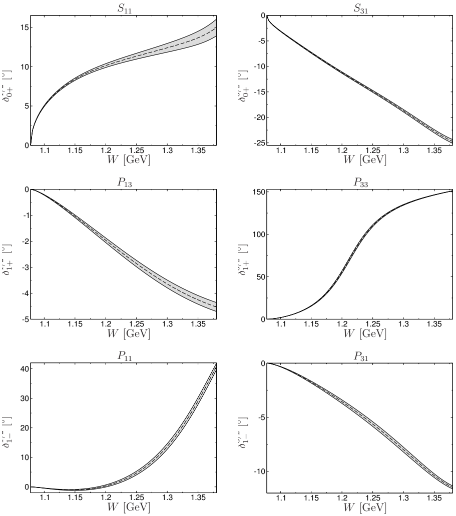


Aim

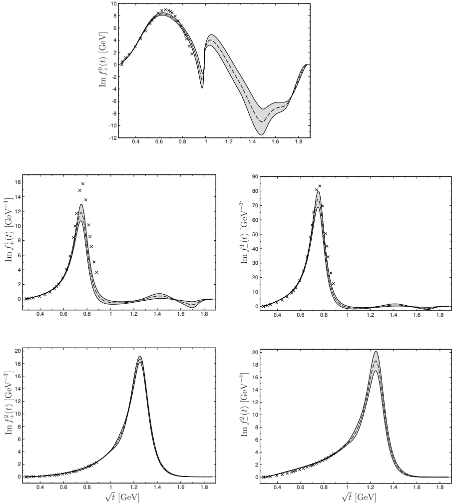
- Precision extraction of πN **scattering lengths** from **hadronic atoms**
- **Roy-equation constraints**: analyticity, unitarity, crossing symmetry

Dispersive analysis of low-energy πN scattering: results

- πN **s-channel** phase shifts



- πN **t-channel** partial waves



Hoferichter, JRE, Kubis, Meißner, accepted in Phys. Rept.

Dispersive analysis of low-energy πN scattering: applications

- Determination of the **sigma term**:

$$\sigma_{\pi N} = (59.1 \pm 3.5)\,\text{MeV}$$

$\hookrightarrow$ $\sigma_{\pi N}$ depends **linearly** on the scattering lengths

$$\sigma_{\pi N} = 59.1 + \sum_{I_s} c_{I_s} \Delta a_{0+}^{I_s}$$

Hoferichter, JRE, Kubis, Meißner, [PRL 115 (2015)]

- **Matching** to **ChPT**

	NLO	N^2LO	N^3LO
c_1 [GeV^{-1}]	-0.74 ± 0.02	-1.07 ± 0.02	-1.11 ± 0.03
c_2 [GeV^{-1}]	1.81 ± 0.03	3.20 ± 0.03	3.13 ± 0.03
c_3 [GeV^{-1}]	-3.61 ± 0.05	-5.32 ± 0.05	-5.61 ± 0.06
c_4 [GeV^{-1}]	2.17 ± 0.03	3.56 ± 0.03	4.26 ± 0.04

- Study of the **chiral convergence**

$\hookrightarrow$ loop diagrams with **Δ degrees of freedom** at N^3LO **spoil** the convergence

Hoferichter, JRE, Kubis, Meißner, [PRL 115 (2015)]

Ongoing results

- **Dispersive** determination of **Δ(1232)** pole parameters
- **Matching to ChPT** with explicit Δ's

  $\hookrightarrow$ **Large-N_c** constraints on Δ LECs
- **$\pi\pi$-continuum** contribution to the **nucleon** form factors

  $\hookrightarrow$ update of Höhler spectral functions, including also isospin breaking

criticism by Lee et al. 2015

Non-ordinary nature of the lightest scalar mesons

Motivation

- Responsible for the **attractive** part of the **nucleon-nucleon** interaction
- They have vacuum quantum numbers

  ↪ relevant for the **spontaneous chiral symmetry breaking**
- The **lightest glueball** expected to be a scalar meson

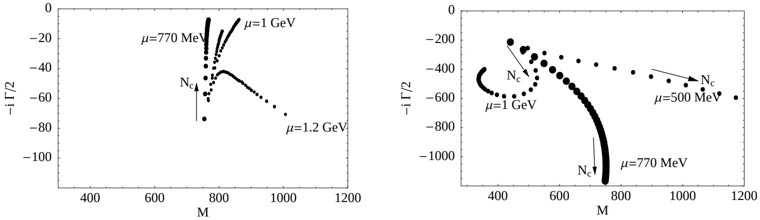
Still subject to debate

- Many resonances, some very wide and difficult to observe
- Not clear how to fit them in **SU(3) multiplets**
- Spectroscopic **classification**: $\bar{q}q$, glueballs, meson molecules, $\cdots$

UChPT + non-perturbative methods to clarify their nature

Main results

- **UChPT** at large N_c predicts a **non-$\bar{q}q$** nature for the lightest scalars — Oset, Oller, Peláez, Nieves, Arriola



- **Local duality** constraints the **nature** of the scalars mesons

  ↪ subdominant $\bar{q}q$ **component** for the $f_0(500)$ required to ensure **local duality**

  Peláez, Pennington, JRE, Wilson [PRD 84]

- Extension to $U(3)$ with resonance saturation

  ↪ including spectral-function sum rules — Guo , Oller, JRE [PLB712 (2012), PRD86 (2012)]

- **Bayesian** interpretation of the large N_c **short distance constraints**

  ↪ fixes the **scale** where the N_c **scaling** is applied — Ledwig, Nieves, Pich, Ruiz-Arriola, JRE [PRD90 (2014)]

Ongoing results

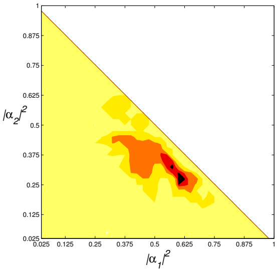
- Full **meson-meson** analysis in **U(3)** with **LECs** — Guo, JRE in progress
- **Composition** of the $f_0(500)$ in terms of **quarks and gluons**

$$|\sigma\rangle = \alpha_1 |(\bar{q}q)^2\rangle + \alpha_2 |\bar{q}q\rangle + \alpha_3 |gg\rangle + \cdots$$

▷ toy model: Fock expansion for the $f_0(500)$ meson in terms of three QCD states

$$H_{ii} = M_i + \frac{\Gamma_i}{2} = \mu \left(a_{ii} + i N_C^{\beta_i} b_{ii} \right) , \quad H_{ij} = H_{ji} = \mu N_C^{\beta_{ij}} \left(a_{ij} + i b_{ij} \right) , \quad P = \log \frac{\max |h_{ij}|}{\min |h_{ij}|}$$

↪ contour plot of P



↪ **natural** solutions dominated by the $|(\bar{q}q)^2\rangle$ **component** — Llanes-Estrada, Peláez, JRE in progress

Chiral symmetry restoration

- Contribution of the $f_0(500)$ to the **chiral symmetry restoration**

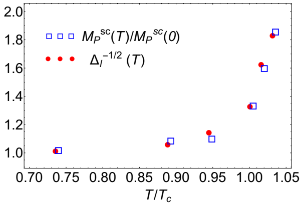
  $\hookrightarrow$ the $f_0(500)$ and $I = 2$ channels contribution **cancel** Nicola, Peláez, JRE [PRD82 (2010), PRD87 (2013)]

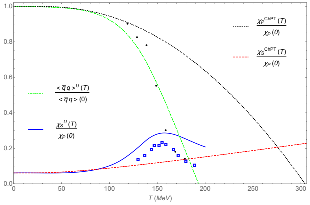
- Chiral symmetry restoration by **degeneration of chiral partners**

  $\triangleright$ pseudo-scalar susceptibility **proportional** to the quark condensate $\chi_P^{ChPT}(T) = \frac{\langle \bar{q}q \rangle_T}{m_q}$

  $\hookrightarrow$ relation satisfied in the **lattice**

  $\triangleright$ **scalar susceptibility** and the **quark condensate** in ChPT, UChPT and the **lattice**





Nicola, JRE [PRD88 (2013)]

- **Extension** to SU(3) in all channels and in the presence of the $U(1)_A$ anomaly Nicola, JRE in progress

Large-N_c QCD for Mesons and Baryons

Large-N_c QCD for **mesons**

Motivation: nature of non-ordinary meson

- Scaling of **masses**, **decay widths** and **couplings** of different **QCD configurations**
- **Broad mesons** in the large N_c limit? Cohen, Llanes-Estrada, Peláez, JRE [PRD90 (2014)]

Large-N_c QCD for **baryons**

- Large N_c relations between πN and $\pi\Delta$ LECs in preparation

And that's it, so far

Spare slides

Roy-equations for $\pi\pi$

- $\pi\pi \to \pi\pi \Rightarrow$ fully crossing symmetric in Mandelstam variables s, t, and $u = 4M_\pi - s - t$
- Start from twice-subtracted **fixed-t** DRs of the generic form

$$T^I(s,t) = c(t) + \frac{1}{\pi}\int_{4m_\pi^2}^{\infty} \frac{ds'}{s'^2}\left[\frac{s^2}{(s'-s)} - \frac{u^2}{(s'-u)}\right] \mathrm{Im}\, T^I(s',t)$$

- Subtraction functions c(t) are determined via crossing symmetry

  $\hookrightarrow$ functions of the I=0,2 scattering lengths: a_0^0 and a_0^2
- PW-expansion of these DRs yields the **Roy-equations** [Roy (1971)]

$$t_J^I(s) = ST_J^I(s) + \sum_{J'=0}^{\infty}(2J'+1)\sum_{I'=0,1,2}\int_{4m_\pi^2}^{\infty} ds' K_{JJ'}^{II'}(s',s)\mathrm{Im}\, t_{J'}^{I'}(s')$$

- $K_{JJ'}^{II'}(s',s) \equiv$ kernels $\Rightarrow$ analytically known

Roy-equations: flow information

- **Roy-equations** rigorously valid for a finite energy range
  $\Rightarrow$ introduce a matching point s_m
- only partial waves with $J \leq J_{\max}$ are solved
- assume isospin limit
- **Input**
  - High-energy region: $\mathrm{Im}\, t_{IJ}(s)$ for $s \geq s_m$ and for all J
  - Higher partial waves: $\mathrm{Im}\, t_{IJ}(s)$ for $J > J_{\max}$ and for all s
- **Output**
  - Self-consistent solution for $\delta_{IJ}(s)$ for $J \leq J_{\max}$ and $s_{\mathrm{th}} \leq s \leq s_m$
  - Constraints on subtraction constants

- $\pi H/\pi D$: bound state of π^- and p/d, spectrum sensitive to threshold πN amplitude
- Combined analysis of πH and πD:

  $a_0^+ \equiv a^+ = (7.5 \pm 3.1) \cdot 10^{-3} M_\pi^{-1}$

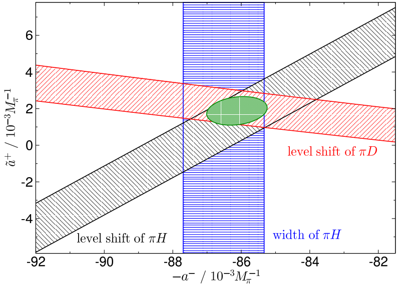
  $a_0^- \equiv a^- = (86.0 \pm 0.9) \cdot 10^{-3} M_\pi^{-1}$

  $\hookrightarrow$ Large a^+ suggests a large $\sigma_{\pi N}$,
- But: a^+ very sensitive to isospin breaking, PWA based on $\pi^\pm p$ channels

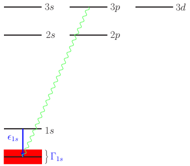
  $\hookrightarrow$ use instead

$$\frac{a_{\pi^- p} + a_{\pi^+ p}}{2} = (-0.9 \pm 1.4) \cdot 10^{-3} M_\pi^{-1}$$

- Isospin breaking in $\sigma_{\pi N}$ could be important
- We revisit the Cheng-Dashen low-energy theorem



[Baru et al. 2011]



$$\tilde{a}^+ = a^+ + \frac{1}{1+M_\pi/m_p}\left\{\frac{M_\pi^2 - m_{\pi^0}^2}{\pi F_\pi^2} c_1 - 2\alpha f_1\right\}$$

Results for the sigma-term

$$\sigma_{\pi N} = F_\pi^2 \left(d_{00}^+ + 2M_\pi^2 d_{01}^+ \right) + \Delta_D - \Delta_\sigma - \Delta_R$$

- subthreshold parameters output of the Roy–Steiner equations

$$d_{00}^+ = -1.36(3) M_\pi^{-1} \qquad [\text{KH: } -1.46(10) M_\pi^{-1}]$$

$$d_{01}^+ = 1.16(2) M_\pi^{-3} \qquad [\text{KH: } 1.14(2) M_\pi^{-3}]$$

- $\Delta_D - \Delta_\sigma = -(1.8 \pm 0.2)$ MeV [MH at al. 2012], $|\Delta_R| \lesssim 2$ MeV [Bernard, Kaiser, Meißner 1996]
- Isospin breaking in the CD theorem shifts $\sigma_{\pi N}$ by $+3.0$ MeV
- Final results:

$$\sigma_{\pi N} = (59.1 \pm 1.9_{\text{RS}} \pm 3.0_{\text{LET}}) \text{ MeV} = (59.1 \pm 3.5) \text{ MeV}$$

[MH, JRE, Kubis, Meißner]

- $\sigma_{\pi N}$ depends linearly on the scattering lengths

$$\sigma_{\pi N} = 59.1 + \sum_{I_s} c_{I_s} \Delta a_{0+}^{I_s}$$

- KH input $\Rightarrow \sigma_{\pi N} = 46$ MeV

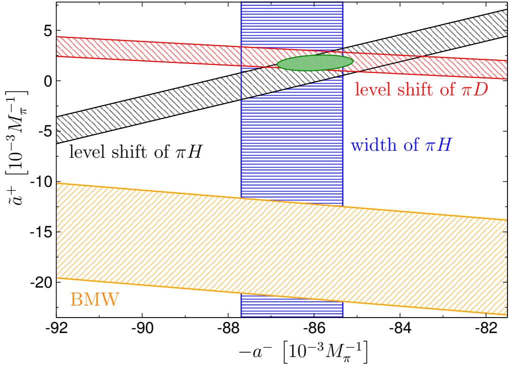
  $\hookrightarrow$ to be compared with $\sigma_{\pi N} = 45$ MeV [Gasser, Leutwyler, Socher, Sainio 1988]
- compare also $\sigma_{\pi N} \sim (64 \pm 8)$ MeV [Pavan et al. 2002]

Comparison with lattice $\sigma_{\pi N}$ results

- Recent lattice determination of $\sigma_{\pi N}$ from the BMW collaboration among others

  $\sigma_{\pi N} = 38(3)(3)\text{MeV}$ [Durr. et al. 2015]

- The linear dependence of $\sigma_{\pi N}$ on the scattering lengths introduces an additional constraint



- Fully inconsistent with the hadronic atom phenomenology

Nucleon strangeness

- Relate $\sigma_{\pi N}$ to strangeness content of the nucleon:

$$\sigma_{\pi N} = \frac{\hat{m}}{2m_N} \frac{\langle N|\bar{u}u + \bar{d}d - 2\bar{s}s|N\rangle}{1-y} = \frac{\sigma_0}{1-y}, \quad y \equiv \frac{2\langle N|\bar{s}s|N\rangle}{\langle N|\bar{u}u + \bar{d}d|N\rangle}$$

$(m_s - m)\,(\bar{u}u + \bar{d}d - 2\bar{s}s) \subset \mathcal{L}_{QCD}$ produces SU(3) mass splittings:

$$\sigma_{\pi N} = \frac{\sigma_0}{1-y}, \quad \sigma_0 = \frac{\hat{m}}{m_s - \hat{m}}\,(m_\Xi + m_\Sigma - 2m_N) \sim 26\text{ MeV}$$

- Higher-order corrections: $\sigma_0 \to (36 \pm 7)$ MeV — Borasoy, Meißner 1997
- Potentially large effects
  - ▷ from the decuplet
  - ▷ from relativistic corrections (EOMS vs. heavy-baryon)
  - ↪ may increase to $\sigma_0 = (58 \pm 8)$ MeV
- **Conclusion**:
  - ▷ $\sigma_{\pi N} = (59.1 \pm 3.5)$ MeV not incompatible with small y
  - ▷ chiral convergence of σ_0 (hence $\langle N|\bar{s}s|N\rangle$) very doubtful

Chiral low-energy constants

	NLO	N^2LO	N^3LO
c_1 [GeV^{-1}]	-0.74 ± 0.02	-1.07 ± 0.02	-1.11 ± 0.03
c_2 [GeV^{-1}]	1.81 ± 0.03	3.20 ± 0.03	3.13 ± 0.03
c_3 [GeV^{-1}]	-3.61 ± 0.05	-5.32 ± 0.05	-5.61 ± 0.06
c_4 [GeV^{-1}]	2.17 ± 0.03	3.56 ± 0.03	4.26 ± 0.04
$\bar{d}_1 + \bar{d}_2$ [GeV^{-2}]	—	1.04 ± 0.06	7.42 ± 0.08
$\bar{d}_3$ [GeV^{-2}]	—	-0.48 ± 0.02	-10.46 ± 0.10
$\bar{d}_5$ [GeV^{-2}]	—	0.14 ± 0.05	0.59 ± 0.05
$\bar{d}_{14} - \bar{d}_{15}$ [GeV^{-2}]	—	-1.90 ± 0.06	-12.18 ± 0.12
$\bar{e}_{14}$ [GeV^{-3}]	—	—	0.89 ± 0.04
$\bar{e}_{15}$ [GeV^{-3}]	—	—	-0.97 ± 0.06
$\bar{e}_{16}$ [GeV^{-3}]	—	—	-2.61 ± 0.03
$\bar{e}_{17}$ [GeV^{-3}]	—	—	0.01 ± 0.06
$\bar{e}_{18}$ [GeV^{-3}]	—	—	-4.20 ± 0.05

- Subthreshold errors tiny, chiral expansion dominates uncertainty
- $\bar{d}_i$ at N^3LO increase by an order of magnitude
  - ↪ due to terms proportional to $g_A^2(c_3 - c_4) = -16$ GeV^{-1}
  - ↪ mimic loop diagrams with Δ degrees of freedom
- What's going on with chiral convergence?
  - ↪ look at convergence of threshold parameters with LECs fixed at subthreshold point

Convergence of the chiral series

	NLO	N^2LO	N^3LO	RS
a^+_{0+} $[10^{-3} M_\pi^{-1}]$	-23.8	0.2	-7.9	-0.9 ± 1.4
a^-_{0+} $[10^{-3} M_\pi^{-1}]$	79.4	92.9	59.4	85.4 ± 0.9
a^+_{1+} $[10^{-3} M_\pi^{-3}]$	102.6	121.2	131.8	131.2 ± 1.7
a^-_{1+} $[10^{-3} M_\pi^{-3}]$	-65.2	-75.3	-89.0	-80.3 ± 1.1
a^+_{1-} $[10^{-3} M_\pi^{-3}]$	-45.0	-47.0	-72.7	-50.9 ± 1.9
a^-_{1-} $[10^{-3} M_\pi^{-3}]$	-11.2	-2.8	-22.6	-9.9 ± 1.2
b^+_{0+} $[10^{-3} M_\pi^{-3}]$	-70.4	-23.3	-44.9	-45.0 ± 1.0
b^-_{0+} $[10^{-3} M_\pi^{-3}]$	20.6	23.3	-64.7	4.9 ± 0.8

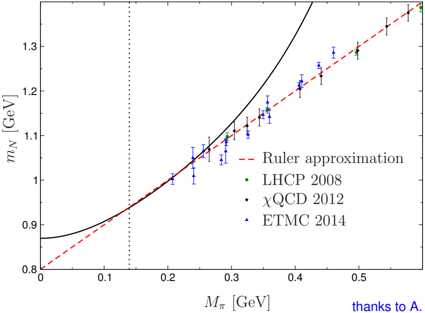
- N^3LO results bad due to large Delta loops
- Conclusion: lessons for few-nucleon applications
  1. either include the Δ to reduce the size of the loop corrections or use LECs from subthreshold kinematics — work in progress
  2. error estimates: consider chiral convergence of a given observable, difficult to assign a global chiral error to LECs

The “ruler plot” vs. ChPT

Lattice QCD simulations can be performed at different quark/pion masses

Pion mass dependence of m_N up to NNNLO in ChPT, using

- Input from Roy–Steiner solution



thanks to A. Walker-Loud for providing the lattice data

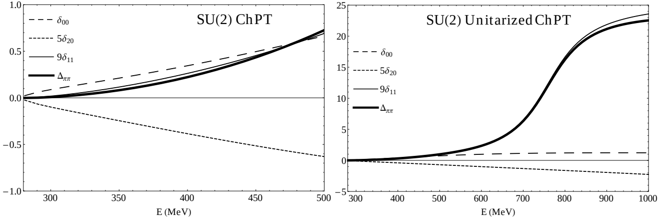
↪ range of convergence of the chiral expansion is very limited

↪ huge cancellation amongst terms to produce a linear behavior

Contribution of the $f_0(500)$ to the **chiral symmetry restoration**

- Influence in the **quark condensate** and in the **scalar susceptibility**

  $\hookrightarrow$ the $f_0(500)$ and scalar $I = 2$ channels contribution **cancel**



Nicola, Peláez, JRE [PRD82 (2010), PRD87 (2013)]

CAVEAT for Hadron Resonance Gas models which include the $f_0(500)$ and not the $I = 2$ channel

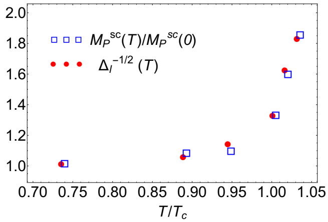
Chiral symmetry restoration by degeneration of chiral partners

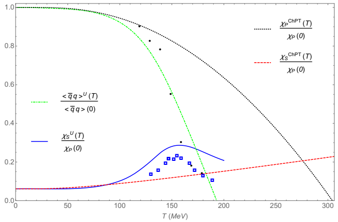
- Pseudo-scalar susceptibility **proportional** to the quark condensate in SU(2) ChPT

$$\chi_P^{ChPT}(T) = \frac{\langle \bar{q}q \rangle_T}{m_q}$$

$\hookrightarrow$ relation satisfied in the **lattice**

- **Scalar susceptibility** and the **quark condensate** in ChPT, UChPT and the **lattice**





Nicola, JRE [PRD88 (2013)]

- **Extension** to SU(3) in all channels and in the presence of the $U(1)_A$ anomaly

Nicola, JRE in progress

Large-N_c QCD for mesons

Motivation: nature of non-ordinary meson

- Scaling of **masses**, **decay widths** and **couplings** of different **QCD configurations**
  - ▷ states with fixed number of constituents

	$q\bar{q}$	$\pi\pi$	gg	$T_0(qq\bar{q}\bar{q})$
$q\bar{q}$	$O(1)$	$O\left(\frac{1}{\sqrt{N_c}}\right)$	$O\left(\frac{1}{\sqrt{N_c}}\right)$	$O(1)$
$\pi\pi$		$O(1)$	$O\left(\frac{1}{N_c}\right)$	$O\left(\frac{1}{\sqrt{N_c}}\right)$
gg			$O(1)$	$O\left(\frac{1}{\sqrt{N_c}}\right)$
$T_0(qq\bar{q}\bar{q})$				$O(1)$

  - ▷ the **polyquark** $(N_c - 1)q - (N_c - 1)\bar{q}$

	$q\bar{q}$	gg	$\pi\pi$	$T(qq\bar{q}\bar{q})$	$(N_c - 1)\pi$
$N_f = 1$	$(N_c - 1)!!\left(\frac{c}{N_c}\right)^{(N_c-4)/2}$	$N_c!!\left(\frac{c}{N_c}\right)^{(N_c-2)/2}$	$N_c!!\left(\frac{c}{N_c}\right)^{(N_c-4)/2}$	$N_c!!\left(\frac{c}{N_c}\right)^{(N_c-3)/2}$	c^{N_c-1}

- **Broad mesons** in the large N_c limit?

Cohen, Llanes-Estrada, Peláez, JRE [PRD90 (2014)]