

EFFECTIVE NUCLEAR THEORIES
AND
LATTICE QCD

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A derivation of nuclear physics from the Standard Model.

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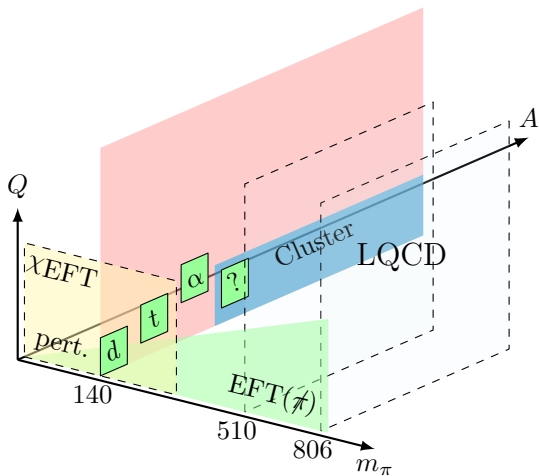
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A derivation of nuclear physics from the Standard Model.

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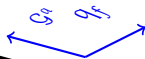
- **Consistent** understanding of nature;
- Analysis/exploration/discovery of **new** phenomena;
- **Stability** of the universe with respect to variations in fundamental constants.

NUCLEAR THEORY AS A COMBINATION OF EFTs.



THE THEORY OF STRONG INTERACTIONS, QCD.

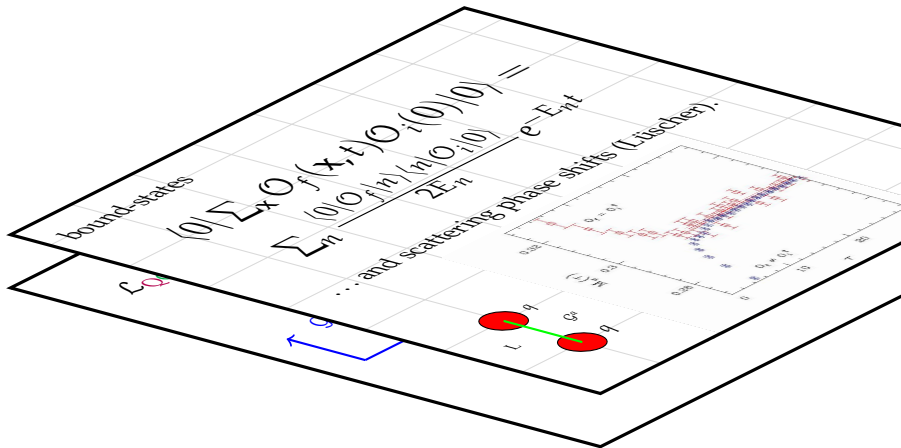
$$\mathcal{L}_{\text{QCD}} = \bar{q}(i\not{\partial} + g_s \not{G})q - \frac{1}{2} G_{\mu\nu}^a G^{\mu\nu a} + \bar{m} \cdot \bar{q}(1 - \not{0} \equiv \tau_3)q + \dots$$



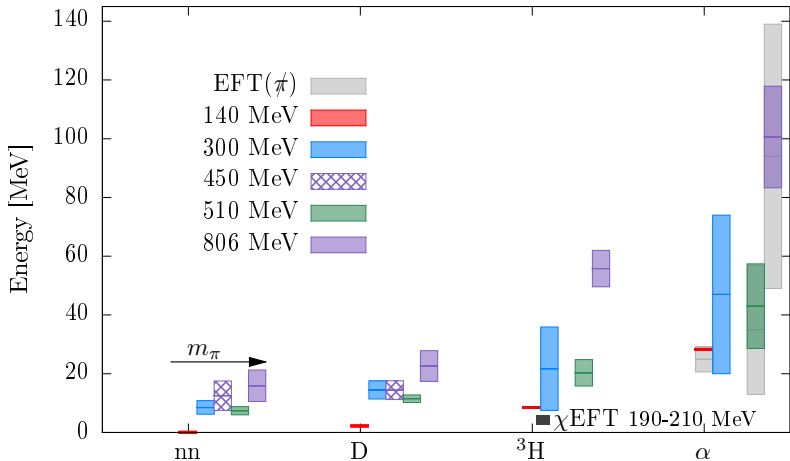
global SO flavor and
local SU color gauge symmetries

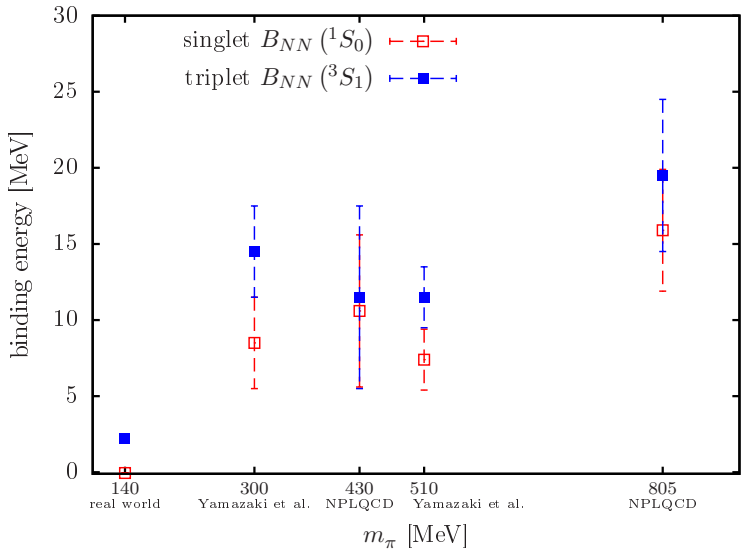
basic scales: $\Lambda_{\text{QCD}} \sim 1 \text{ GeV}$ and
 $m_{\pi} \sim 140 \text{ MeV}$

QCD SOLUTION IN DISCRETIZED SPACE TIME.

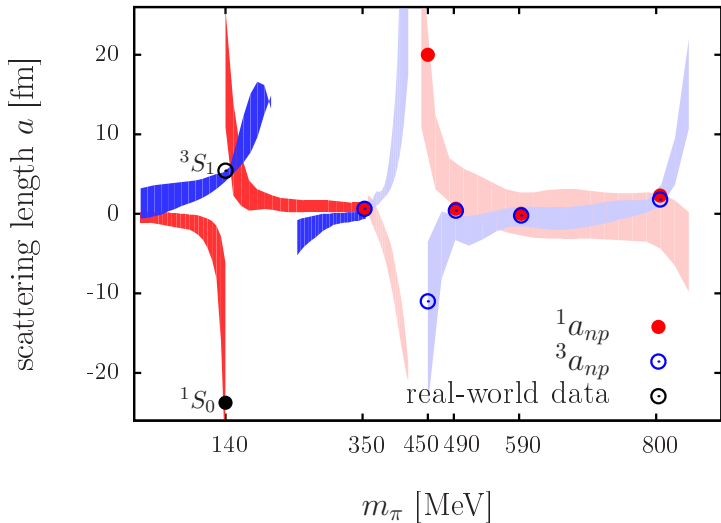


LATTICE NUCLEI FOR VARIOUS PION MASSES.



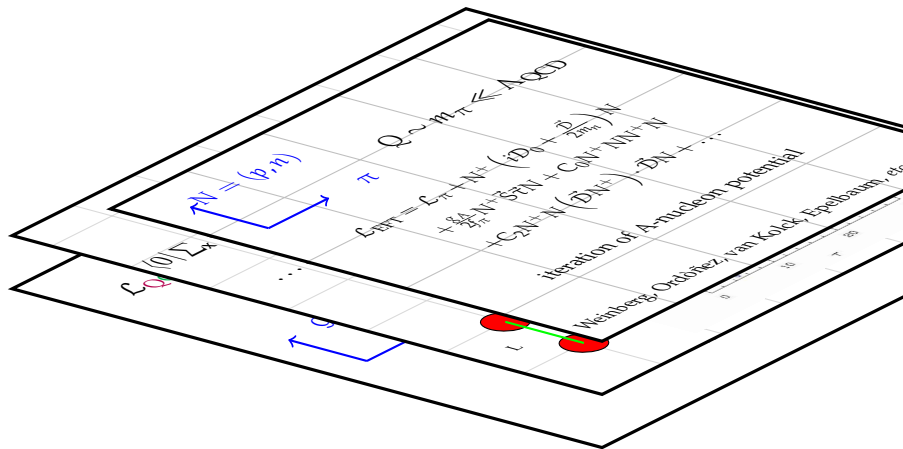
m_π DEPENDENCE OF TWO NUCLEONS (I).

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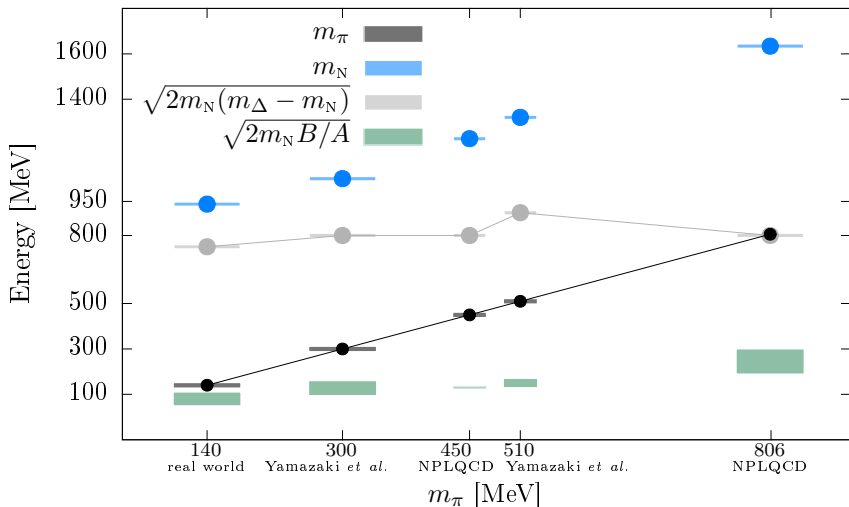


NPLQCD “data” and EFT extrapolation;

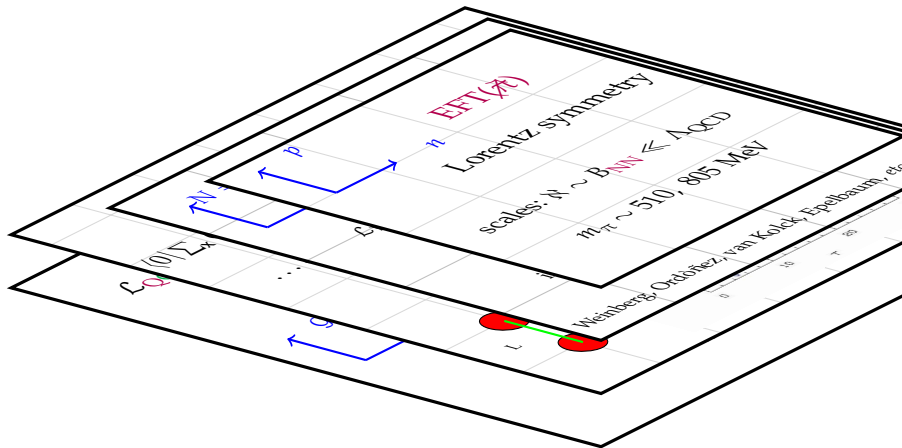
χ EFT AS AN EFFECTIVE THEORY OF QCD FOR MESONS AND NUCLEI.



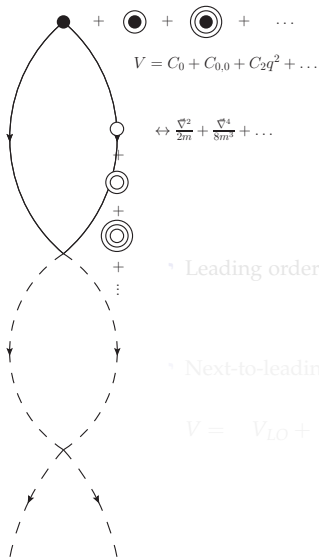
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EFT = LOOP AND VERTEX EXPANSION.



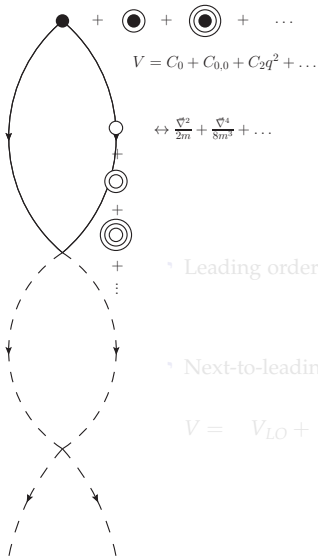
• Leading order:

$$V = \overset{\circ\circ}{\overset{\circ\circ}{C_{0,s}}} \hat{P}^{(1S_0)} + \overset{\circ\circ}{\overset{\circ\circ}{C_{0,t}}} \hat{P}^{(3S_1)} + \overset{\circ\circ}{\overset{\circ\circ}{D_{(*)}}} \hat{P}^{(S)}$$

• Next-to-leading order:

$$\begin{aligned}
 V = & V_{LO} + \left(\overset{\circ}{\overset{\circ}{C_{2,s}}} + \overset{\circ}{\overset{\circ}{C_{2,s}^2}} \right) \hat{P}^{(1S_0)} + \left(\overset{\circ}{\overset{\circ}{C_{2,t}}} + \overset{\circ}{\overset{\circ}{C_{2,t}^2}} \right) \hat{P}^{(3S_1)} + \overset{\circ}{\overset{\circ}{D_{(*)}}} \hat{P}^{(S)} \\
 & + \overset{\circ}{\overset{\circ}{C_{pp}}} \hat{P}_{pp}^{(1S_0)} + \frac{c^2}{4|r|}
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“Natural”, renormalized LECs:

$$C_{2n} = \frac{4\pi \mathcal{O}(1)}{m \mathbb{N} (M \mathbb{N})^n} \quad C'_{2n} = \frac{4\pi \mathcal{O}(1)}{m M^{2n+1}}$$

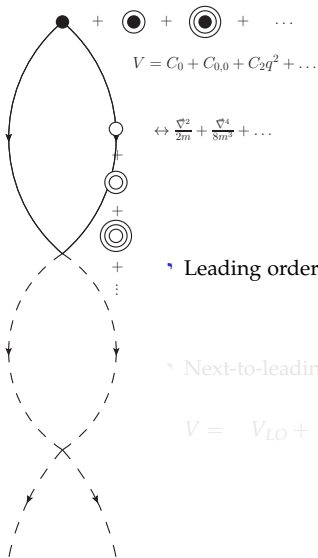
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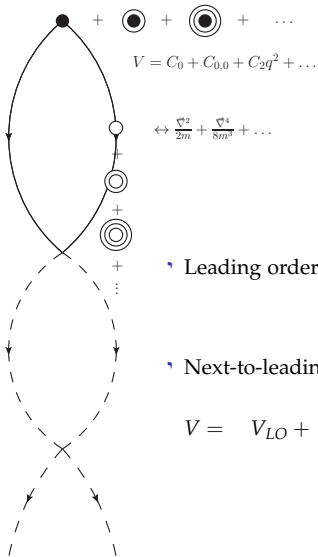
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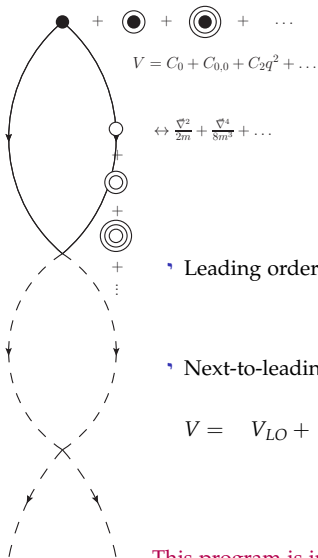
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$$V = C_0 + C_{0,0} + C_2 q^2 + \dots$$

$$\leftrightarrow \frac{\nabla^2}{2m} + \frac{\nabla^4}{8m^3} + \dots$$

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$$C_{2n} = \frac{4\pi\mathcal{O}(1)}{mN(MN)^n} \quad C'_{2n} = \frac{4\pi\mathcal{O}(1)}{mM^{2n+1}}$$

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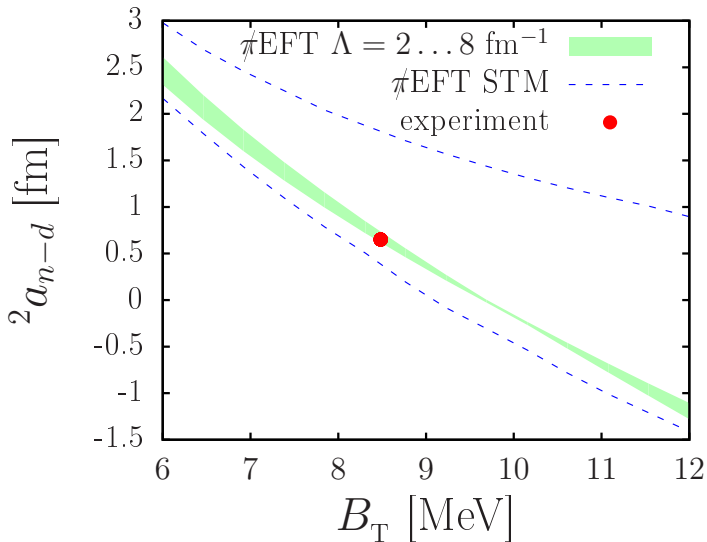
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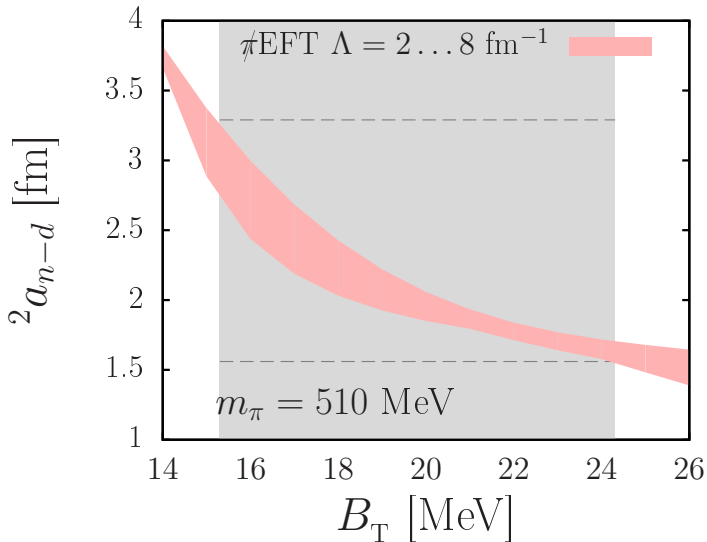
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This program is incomplete for $A > 2$ (even physical m_π)!

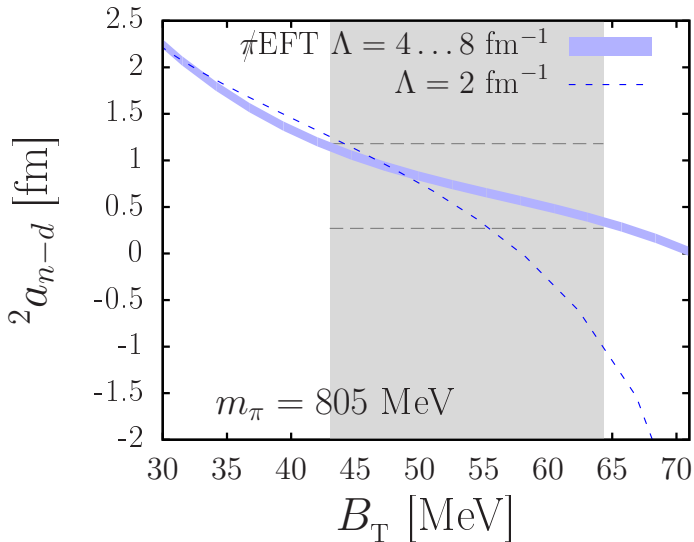
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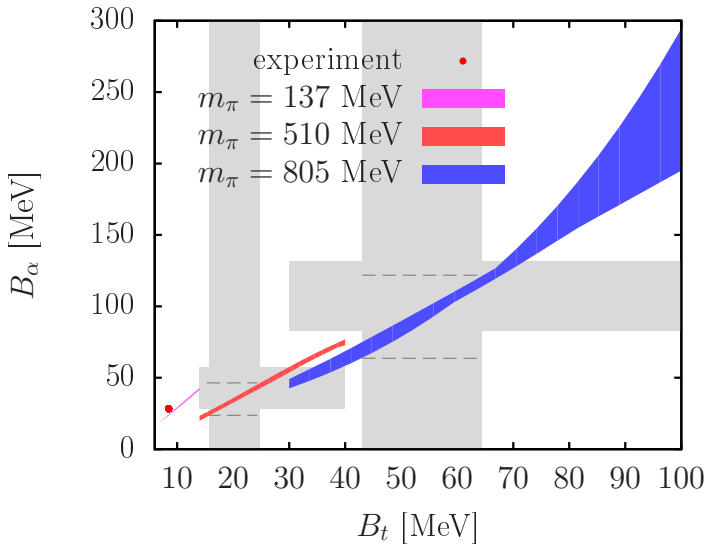
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THE T_{JON} 4-NUCLEON CHARACTERISTICUM.



THE FUTURE.

- m_π dependence of other nuclear characteristics:
 - 5 and 6 nucleons and reactions;
 - Electro-weak interactions (moments and decays);
 - Emergence of *strange* nuclei.
- Nuclei under **extreme** conditions
(magnetic and gravitational fields close to neutron stars).
- Refinement of **multi-nucleon EFTs** for physical and large pion masses.

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