Towards a first principles picture of the proton

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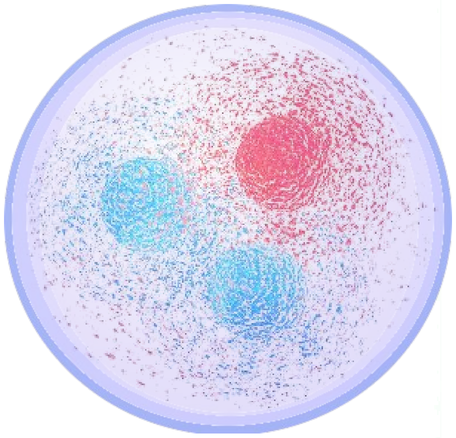
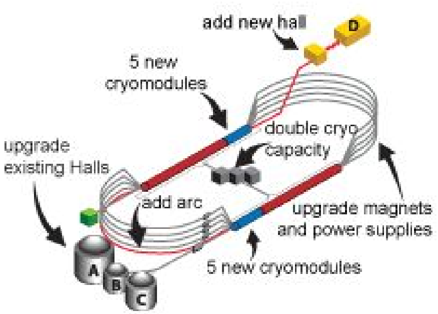
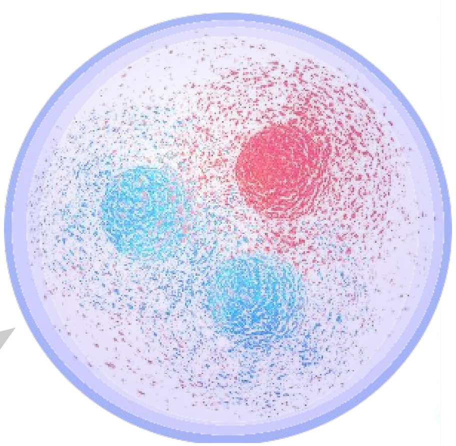
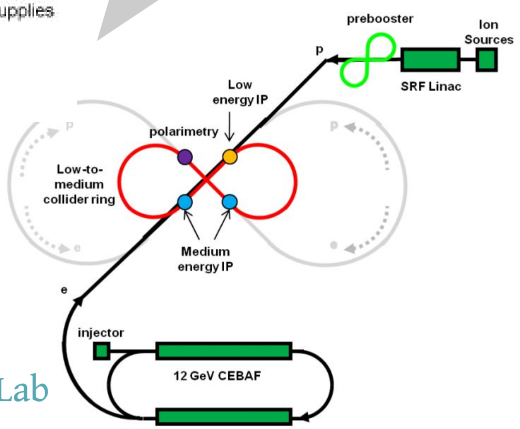
Jefferson Lab



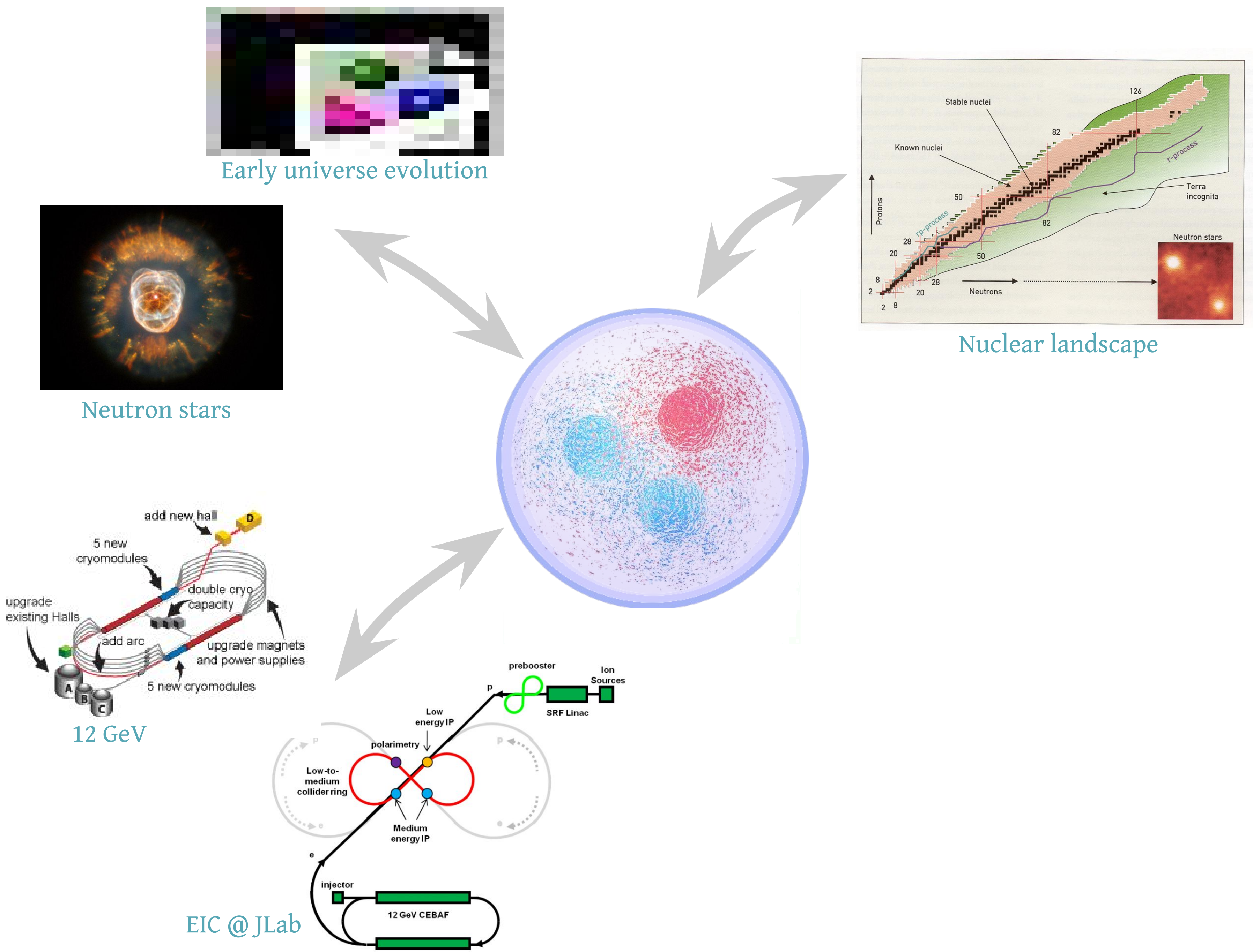


12 GeV

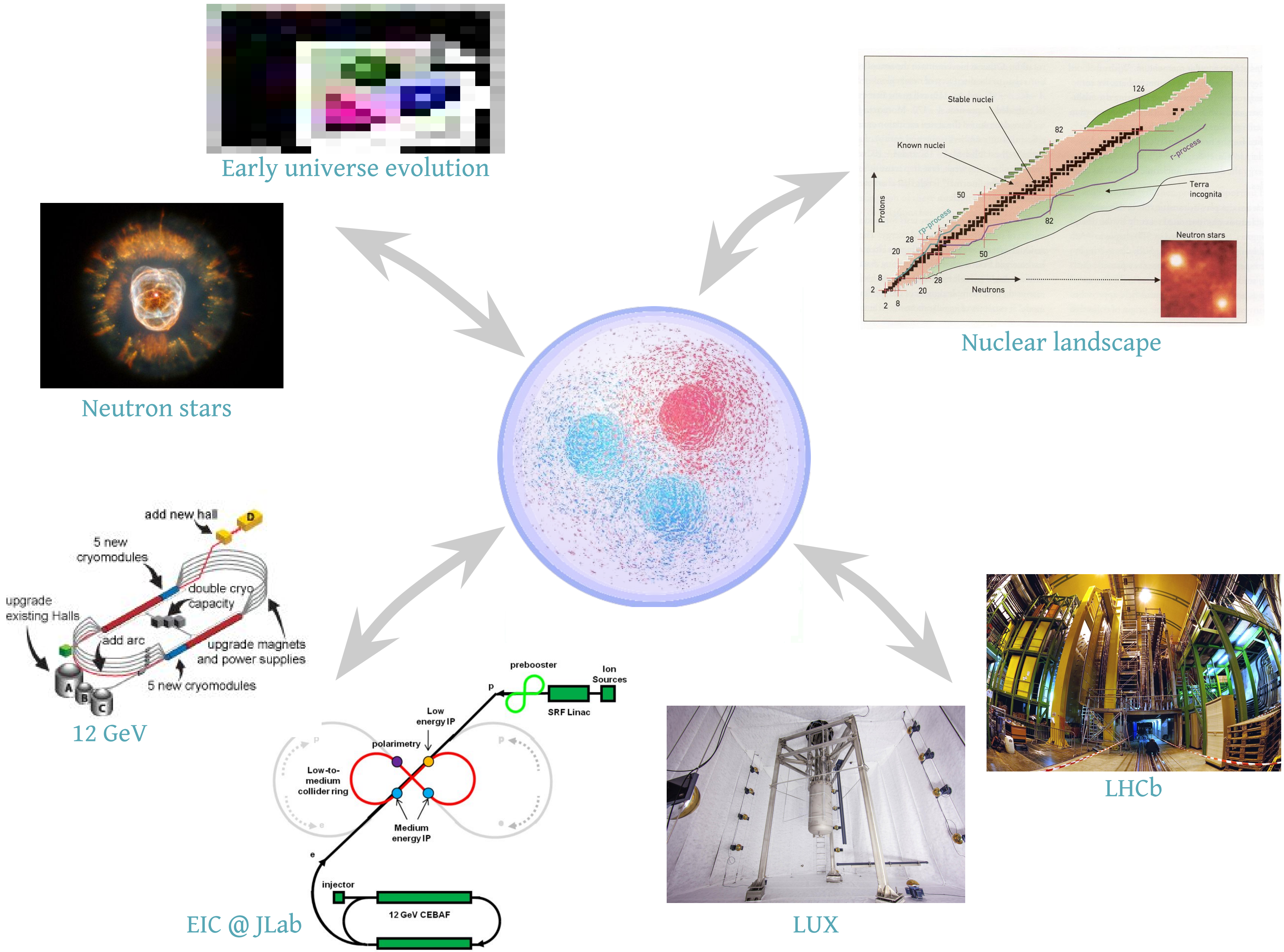




EIC @ JLab



Early universe evolution
Neutron stars
Nuclear landscape
Stable nuclei
Known nuclei
Terra incognita
Neutron stars
Protons
Neutrons
rp-process
r-process
126
82
50
28
20
8
2
12 GeV
add new hall
5 new cryomodules
double cryo capacity
upgrade existing Halls
add arc
upgrade magnets and power supplies
5 new cryomodules
EIC @ JLab
prebooster
Ion Sources
SRF Linac
Low energy IP
polarimetry
Low-to-medium collider ring
Medium energy IP
injector
12 GeV CEBAF



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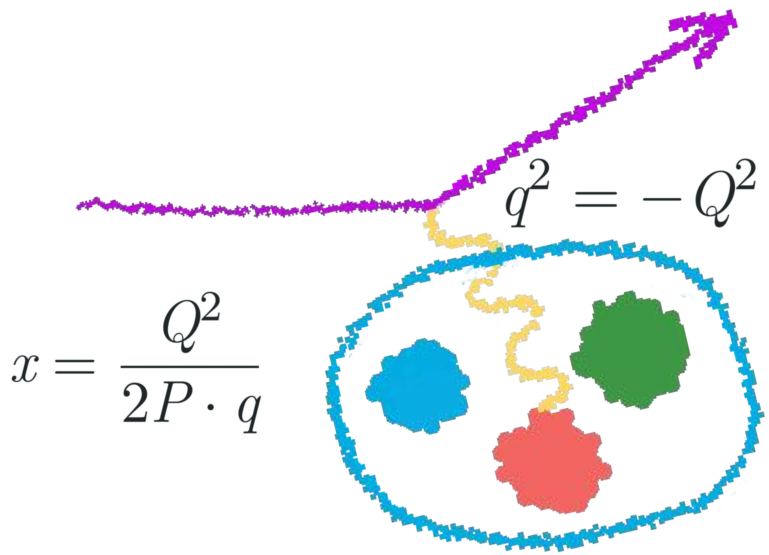
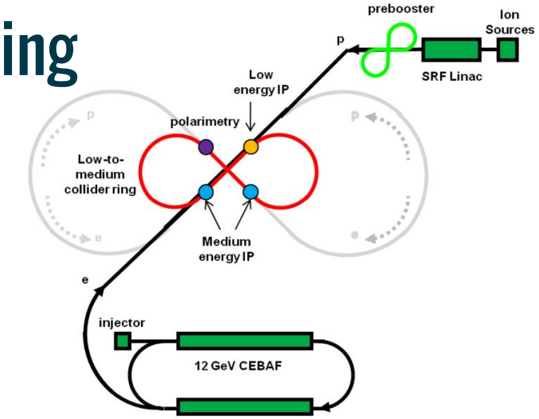
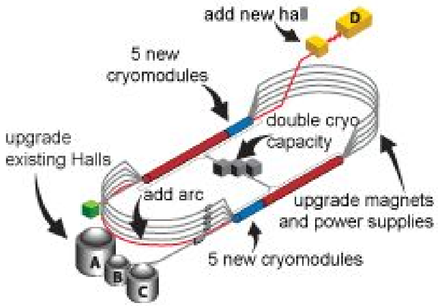
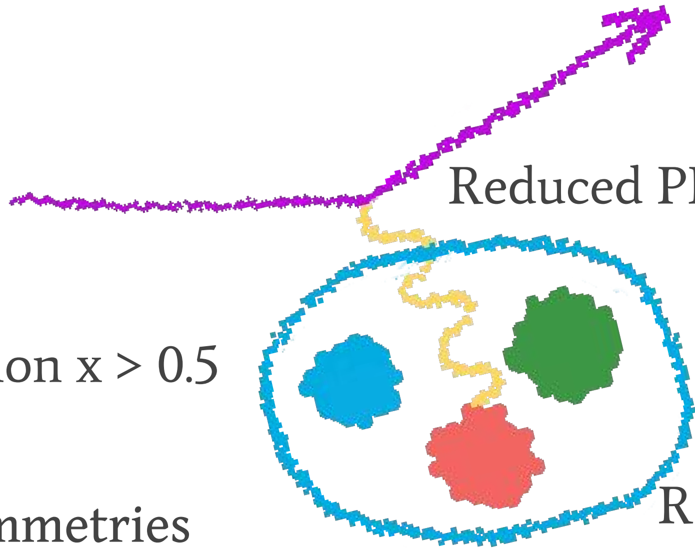
EIC @ JLab
LUX
LHCb


$q^2 = -Q^2$
$x = \frac{Q^2}{2P \cdot q}$

Inclusive and semi-inclusive deep inelastic scattering







Reduced PDF uncertainties $10^{-3} < x < 1$

PDFs in deep valence region $x > 0.5$

Kinematic reach to $x \sim 10^{-4}$

Nucleon polarisation asymmetries

Resolve origin of proton spin via gluon and sea quark helicities

Spin structure function g_2 and higher twists

Explore gluon and sea quark TMDs

Pion structure functions at high x

3D imaging via GPDs

Transversity distributions and GPDs

First principles picture:

Derived from QCD action

Independent of phenomenological modelling

Uncertainties systematically improvable

First principles picture:

Derived from QCD action

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Uncertainties systematically improvable

Lattice QCD

Direct determination of parton distributions

1. Nucleon structure on the lattice
2. PDFs from lattice QCD: an unsolved challenge
3. A solution: the gradient flow
4. No free lunch: the gradient flow scheme
5. Looking forward

Nucleon structure:

Nucleon mass

Axial charge

Form factors

Parton distributions

Transverse momentum and generalised parton distributions

Wigner functions

Nucleon mass

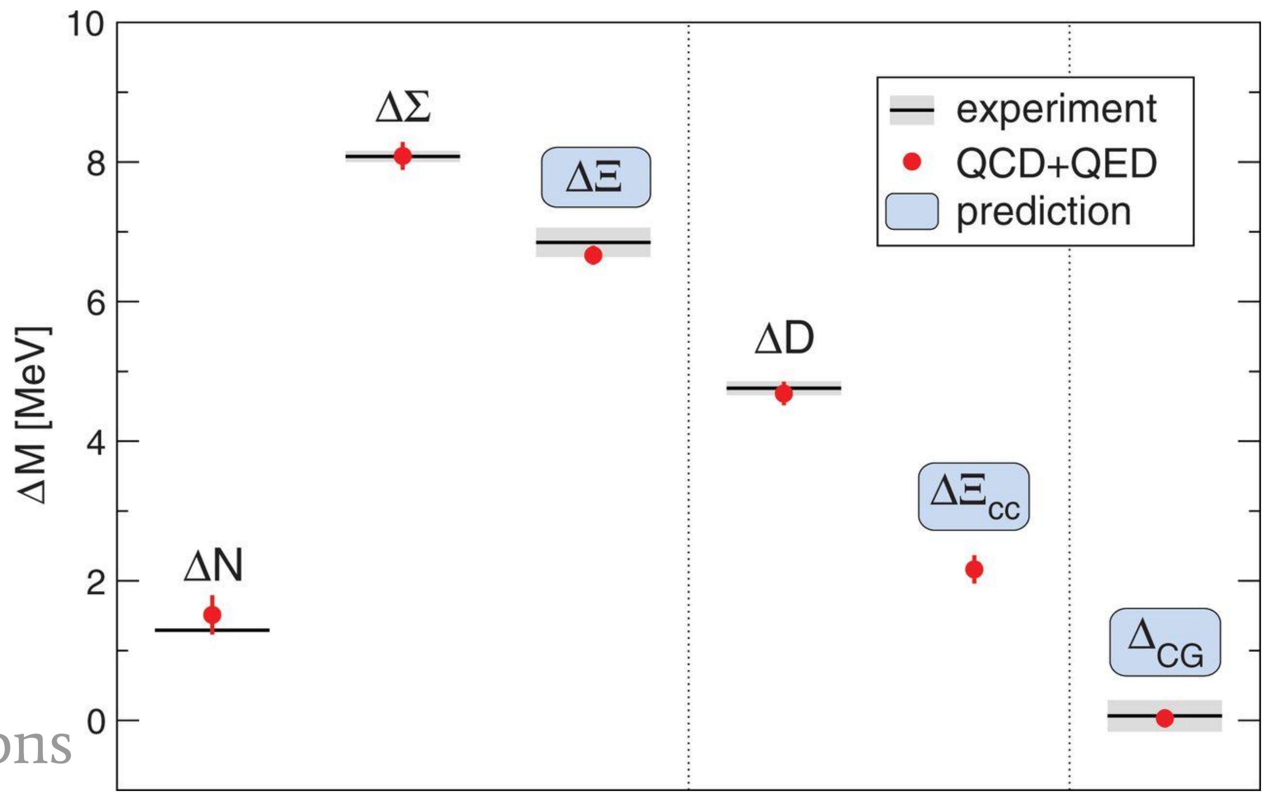
Axial charge

Form factors

Parton distributions

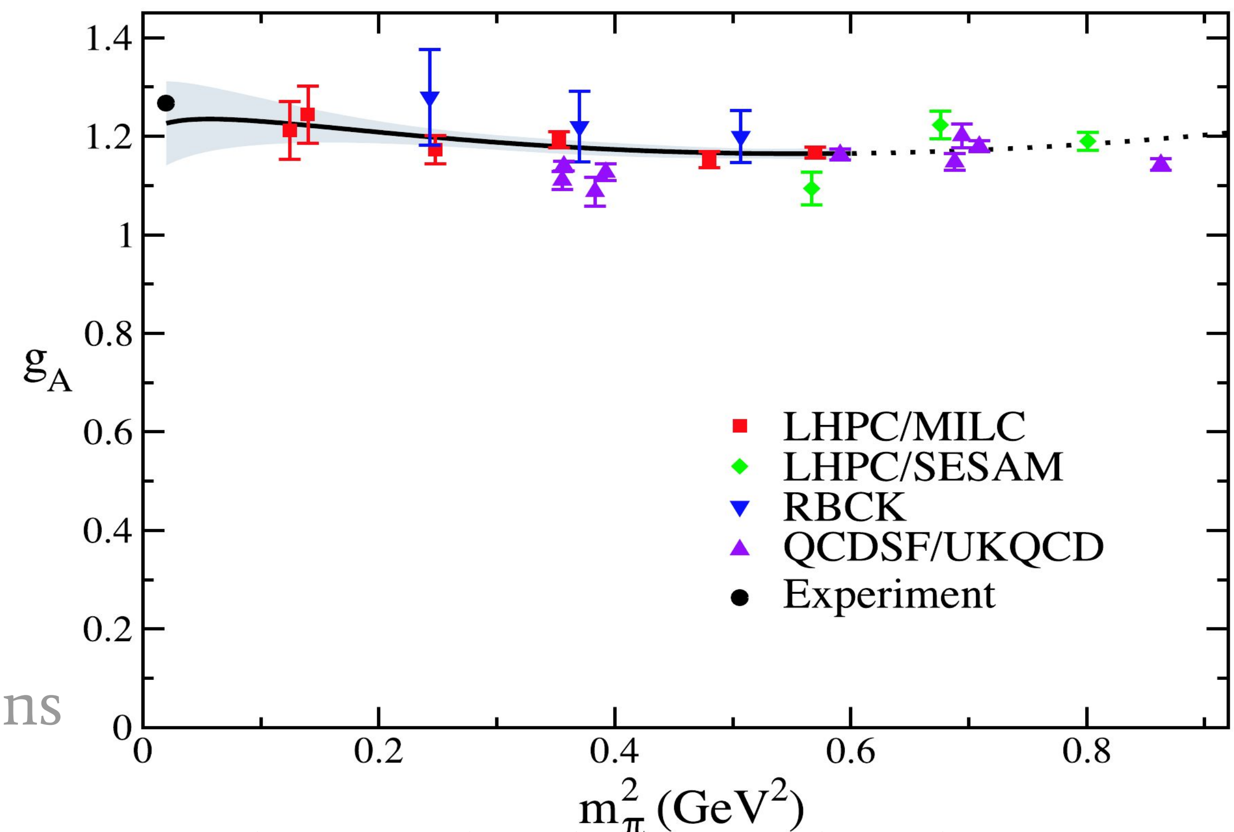
Transverse momentum and generalised parton distributions

Wigner functions



Borsanyi et al., Science 347 (2015) 1452

Edwards *et al*, Phys. Rev. Lett. 96 (2006) 052001



Nucleon mass

Axial charge

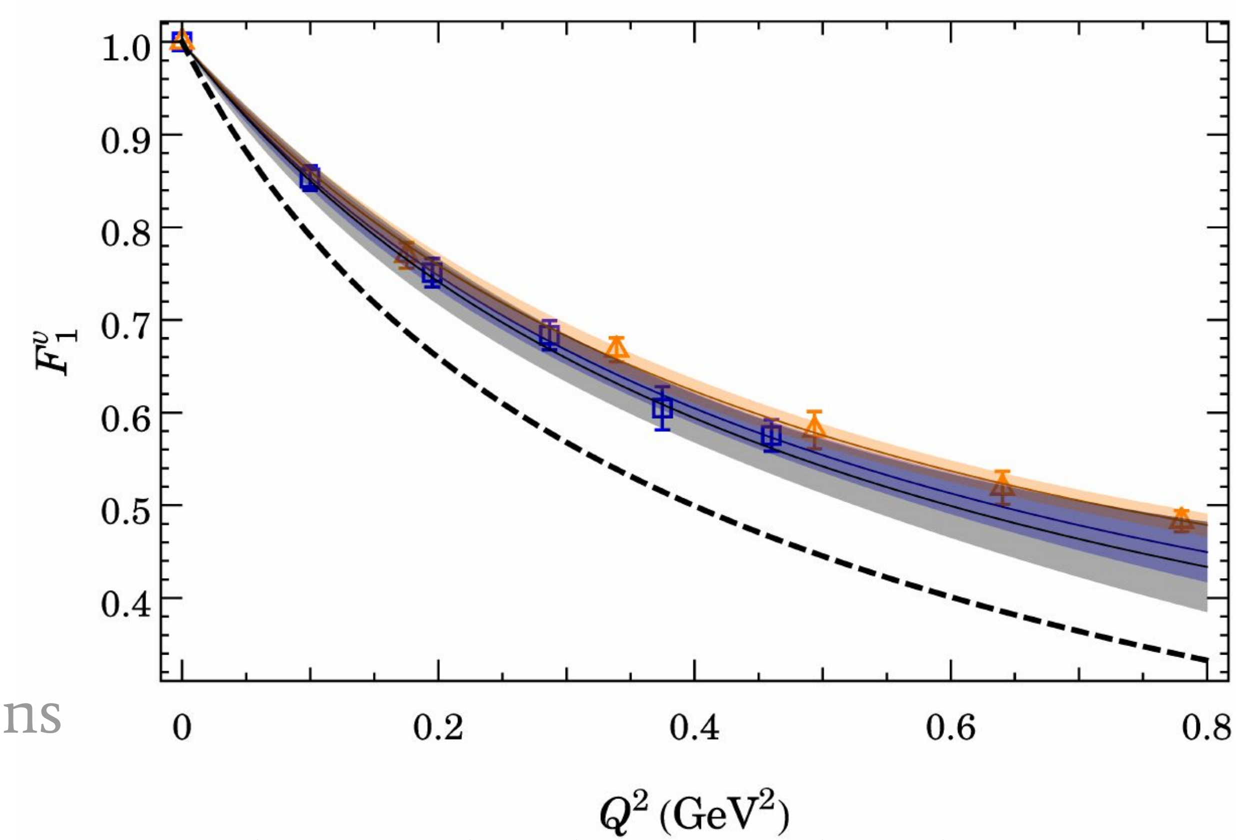
Form factors

Parton distributions

Transverse momentum and generalised parton distributions

Wigner functions

Bhattacharya et al, Phys. Rev. D 89 (2014) 094502



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Capture longitudinal momentum structure of constituents of fast-moving nucleons

Nucleon mass

Axial charge

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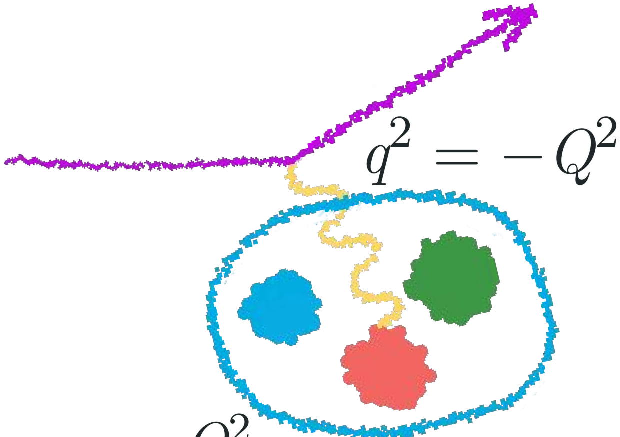
Capture longitudinal momentum structure of constituents of fast-moving nucleons

2. PDFs from lattice QCD:
An unsolved challenge

Deep inelastic scattering



$$x = \frac{Q^2}{2P \cdot q}$$

Decompose cross-section

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega \mathrm{d}E'} = \frac{e^4}{16\pi^2 Q^4} \ell^{\mu\nu} W_{\mu\nu}$$

Hadronic tensor

$$W_{\mu\nu}(p,q) = \frac{1}{4\pi} \int \mathrm{d}^4 x \, e^{iq\cdot x} \langle p, \lambda' \,|\, [j_\mu(x), j_\nu(x)] \,|\, p, \lambda \rangle$$

Express in terms of structure functions F_1, F_2, g_1, g_2

$$F(x, Q^2) = \int \mathrm{d}y \, C\left(\frac{x}{y}, \frac{Q^2}{\mu^2}\right) f_{q/N}(x, \mu^2)$$

(Light front) parton distributions universal

$$f_{q/N}(x, Q^2) = \frac{1}{4\pi} \int_{-\infty}^{\infty} \mathrm{d}y^- \, e^{-iy^- p^+} \langle N \,|\, \overline{\psi}(0^+, y^-, 0_\mathrm{T}) \gamma_+ U(y^-, 0) \, \psi(0) \,|\, N \rangle$$

Relate hadronic tensor to forward Compton amplitude

$$W_{\mu\nu} = \frac{1}{2\pi} \mathrm{Im}\{T_{\mu\nu}\}$$

Operator product expansion generates “twist” (dimension - spin) expansion

Twist-2 operators dominate in Bjorken limit

$$\overline{\psi}\gamma_{\{\mu_1} \overleftrightarrow{D}_{\mu_2} \ldots \overleftrightarrow{D}_{\mu_n\}} \psi - \text{traces}$$

Mellin moments

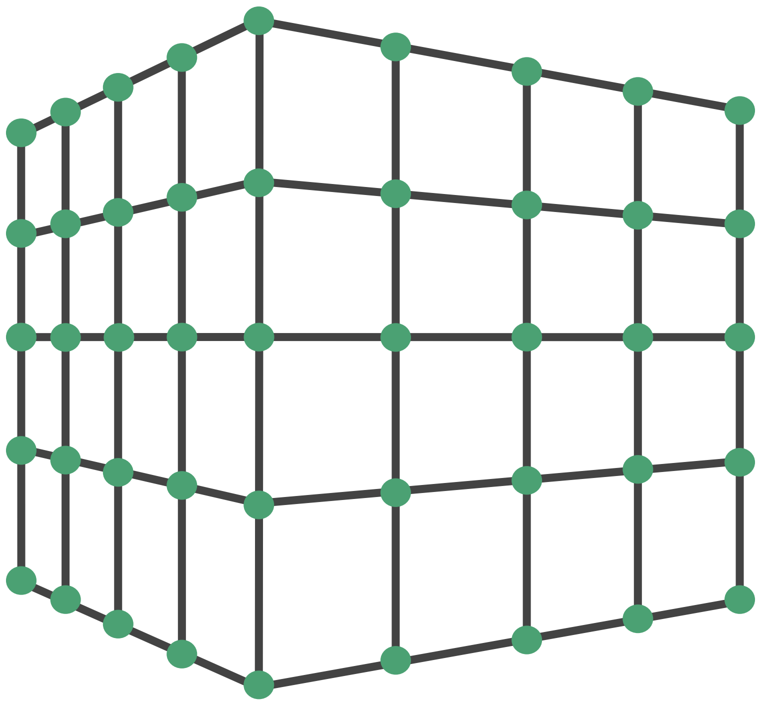
$$\langle x^n \rangle_{f_{q/N}} = \int_{-1}^{1} \mathrm{d}x \, x^n f_{q/N}(x)$$

$$2\langle x^n \rangle_{f_{q/N}} P_{\mu_1} \cdots P_{\mu_n} = \frac{1}{2} \langle N(P)|\overline{\psi}\gamma_{\{\mu_1} \overleftrightarrow{D}_{\mu_2} \cdots \overleftrightarrow{D}_{\mu_n\}} \psi|N(P)\rangle$$

Wick rotation of moments is trivial

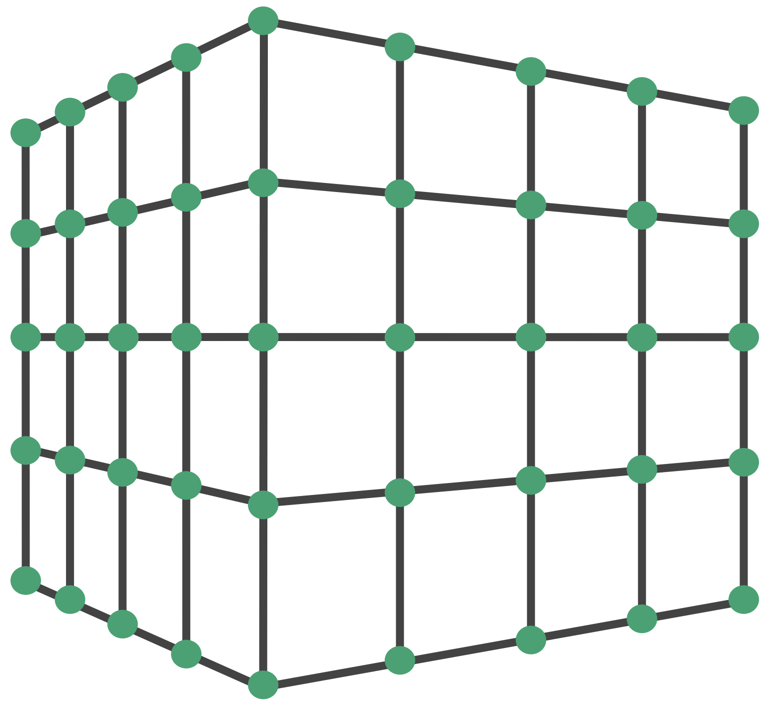
... however...

Rotational symmetry broken



Mixing patterns under renormalisation more complicated on the lattice

Rotational symmetry broken



Mixing patterns under renormalisation more complicated on the lattice

Mixing between operators of different mass dimension: power-divergent mixing

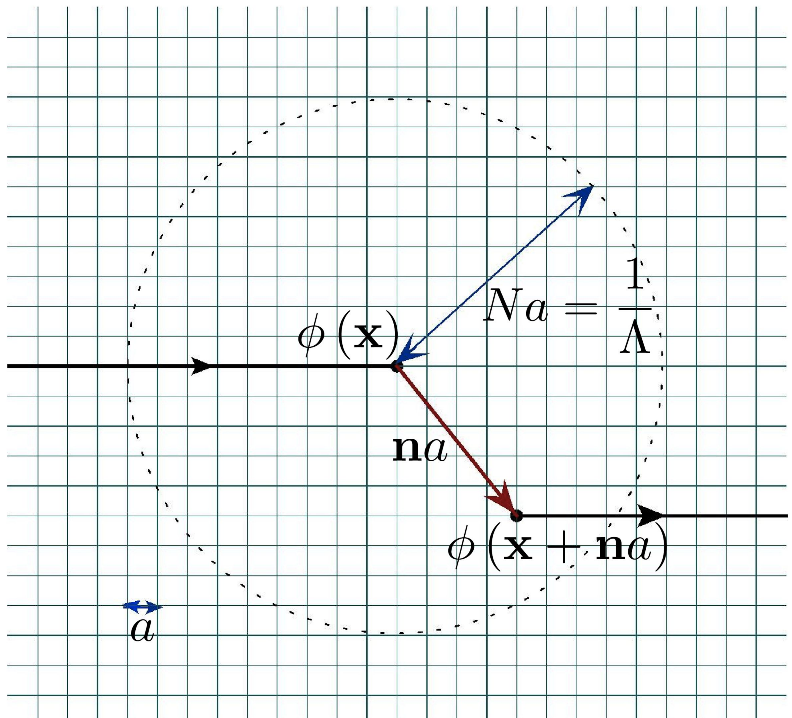
$$\overline{\psi}\gamma_4\gamma_5 \overleftrightarrow{D}_4 \overleftrightarrow{D}_4 \psi \sim \frac{1}{a^2}\overline{\psi}\gamma_4\gamma_5\psi$$

Power-divergent mixing restricts lattice calculations to first four moments

Detmold *et al.*, Eur. Phys. J. C 3 (2001) 1
Detmold *et al.*, Phys. Rev. D 68 (2001) 034025
Detmold *et al.*, Mod. Phys. Lett. A 18 (2003) 2681

3. A solution:
The gradient flow

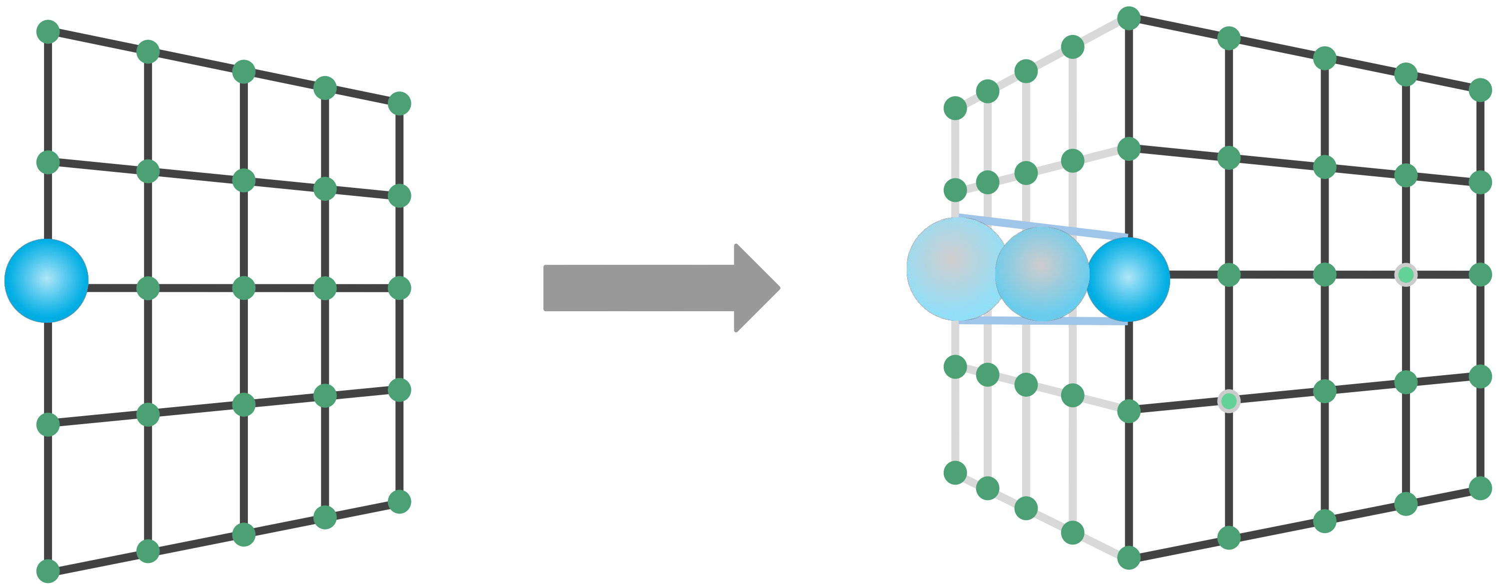
“Smearing” partially restores rotational symmetry: **suppresses operator mixing**



Davoudi & Savage, Phys. Rev. D 86 (2012) 054505

Construct operators with improved continuum limits

Gradient flow: deterministic evolution in new parameter - flow time



Drives fields to minimise action - removes UV fluctuations

Renormalised boundary theory remains finite

Narayanan & Neuberger, JHEP 0603 (2006) 064
Lüscher, Commun. Math. Phys. 293 (2010) 899

Lüscher & Weisz, JHEP 1102 (2011) 51
Luscher, JHEP 04 (2013) 123

Some technical comments:

Four-dimensional smearing in Euclidean spacetime

Status unclear for nonrenormalisable theories

Perhaps best to think of this as just a tool

Some technical comments:

Four-dimensional smearing in Euclidean spacetime

Under investigation

Status unclear for nonrenormalisable theories

Perhaps best to think of this as just a tool

Up for debate?

Scalar field theory

$$\frac{\partial}{\partial \tau}\overline{\phi}(\tau,x) = \partial^2 \overline{\phi}(\tau,x) \qquad \overline{\phi}(\tau=0,x) = \phi(x) \qquad \widetilde{\overline{\phi}}(\tau,p) = e^{-\tau p^2}\widetilde{\phi}(p)$$

CJM & K. Orginos, PRD 91 (2015) 074513

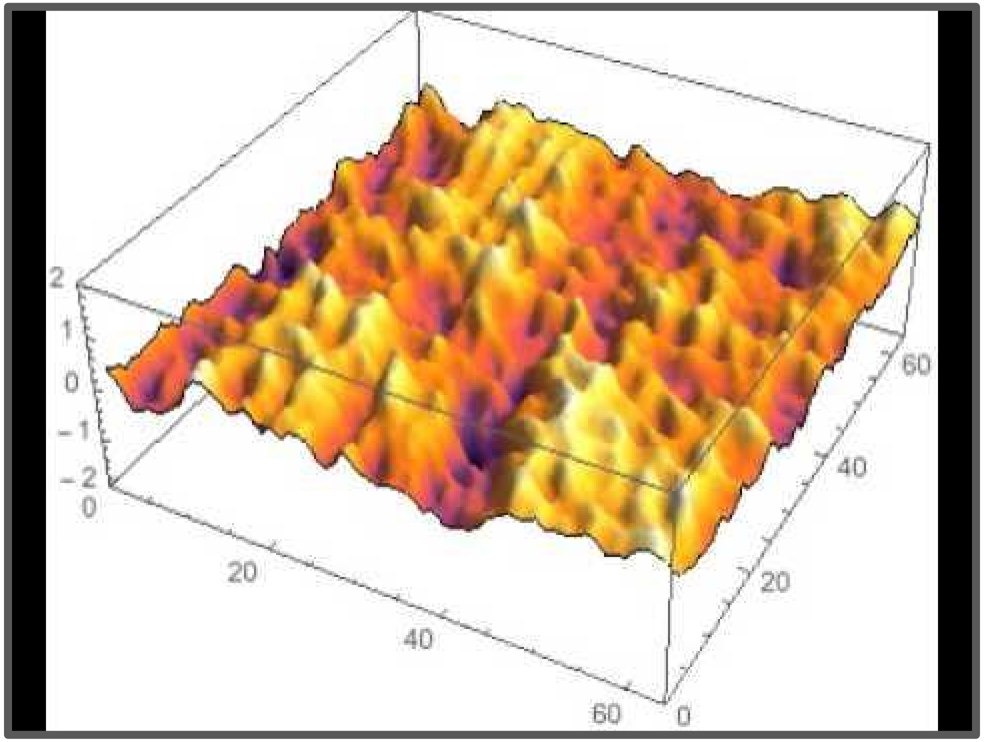
Exact solution possible with Dirichlet boundary conditions

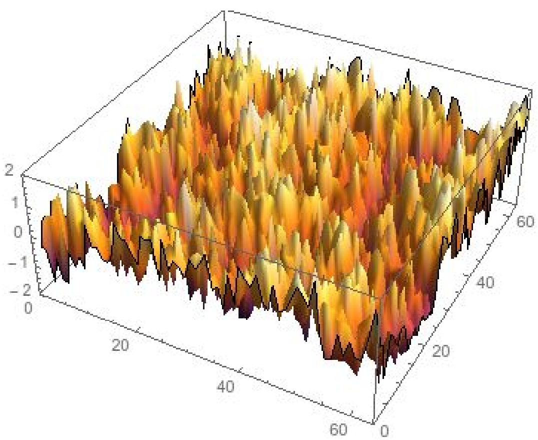
$$\overline{\phi}(\tau,x) = \int \mathrm{d}^4 y \int \frac{\mathrm{d}^4 p}{(2\pi)^4} e^{ip\cdot(x-y)} e^{-\tau p^2}\phi(y) = \frac{1}{16\pi^2\tau^2}\int \mathrm{d}^4 y\, e^{-(x-y)^2/(4\tau)}\phi(y)$$

Smearing radius $s_{\mathrm{rms}} = \sqrt{8\tau}$

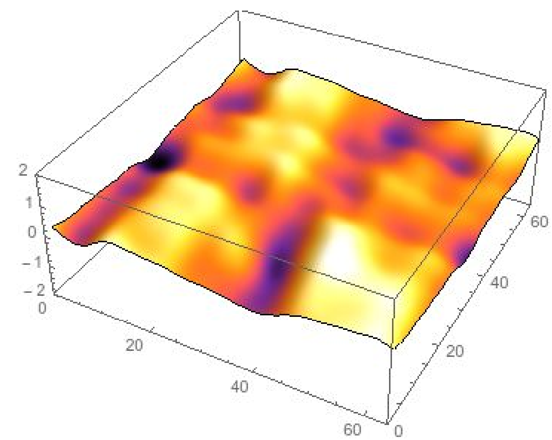
Interactions occur at zero flow time (*i.e.* in the original “boundary” theory)

Lüscher & Weisz, JHEP 1102 (2011) 51
Makino & Suzuki, arXiv:1410.7538





Flow-time



CJM, PoS(Lattice2015) 052

Gradient flow in QCD

$$\frac{\partial}{\partial\tau}B_\mu(\tau,x) = D_\nu\Big(\partial_\nu B_\mu - \partial_\mu B_\nu + [B_\nu, B_\mu]\Big) \qquad D_\mu = \partial_\mu + [B_\mu, \cdot\,]$$

$$\frac{\partial}{\partial\tau}\chi(\tau,x) = D^F_\mu D^F_\mu \chi(\tau,x) \qquad D^F_\mu = \partial_\mu + B_\mu$$

Dirichlet boundary conditions

$$B_\mu(\tau=0,x) = A_\mu(x) \qquad \chi(\tau=0,x) = \psi(x)$$

Tree-level expansion

$$B_\mu(\tau,x) = \int \mathrm{d}^4y \Big\{ K_\tau(x-y)_{\mu\nu} A_\nu(y) + \int_0^\tau \mathrm{d}\sigma\, K_{\tau-\sigma}(x-y)_{\mu\nu} R_\nu(\sigma,y) \Big\}$$

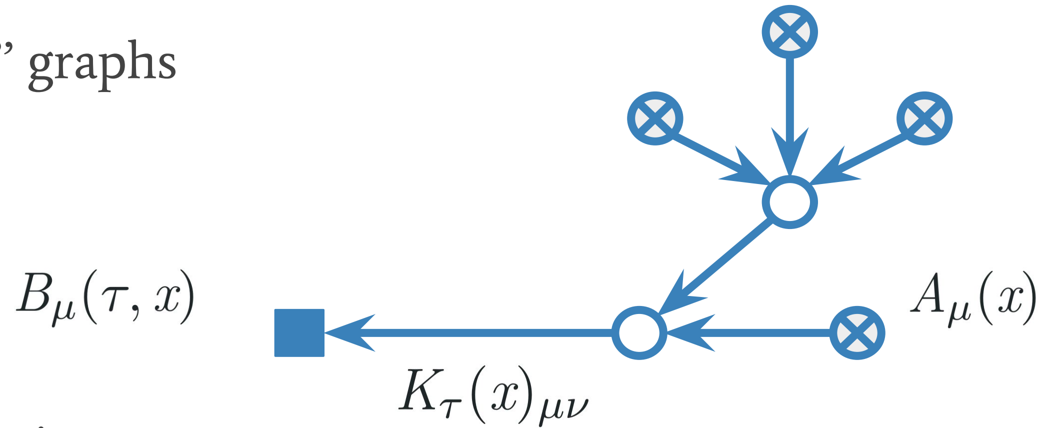
"Flow propagator"

$$K_\tau(x)_{\mu\nu} = \int \frac{\mathrm{d}^4p}{(2\pi)^4}\frac{e^{ipx}}{p^2}\Big\{(\delta_{\mu\nu}p^2 - p_\mu p_\nu)e^{-\tau p^2} + p_\mu p_\nu\Big\}$$

$$R_\mu(\tau,x) = 2[B_\nu, \partial_\nu B_\mu] - [B_\nu, \partial_\mu B_\nu] - [B_\mu, \partial_\nu B_\nu] + [B_\nu, [B_\nu, B_\mu]]$$

Lüscher & Weisz, JHEP 1102 (2011) 51
Luscher, JHEP 04 (2013) 123

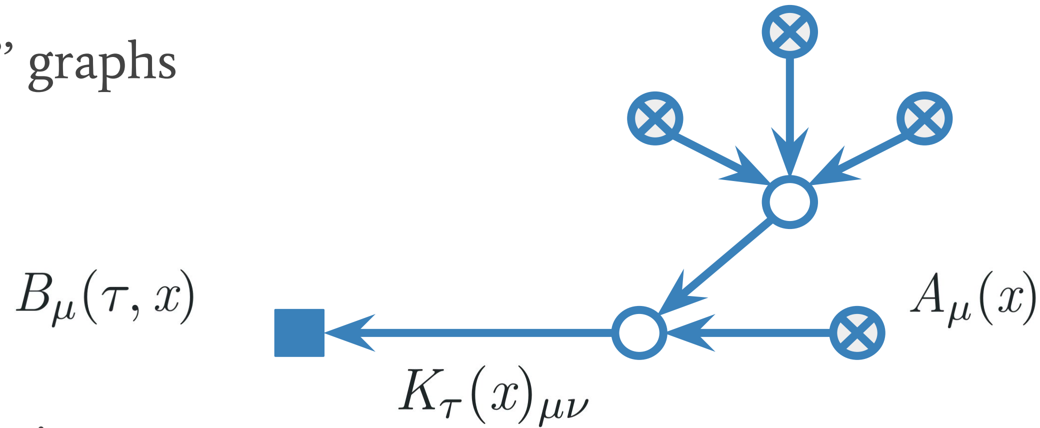
Directed ``tree" graphs



Two-point function

$$\left\langle B^a_\mu(\tau, x) B^b_\nu(\sigma, y) \right\rangle =$$

Directed ``tree” graphs



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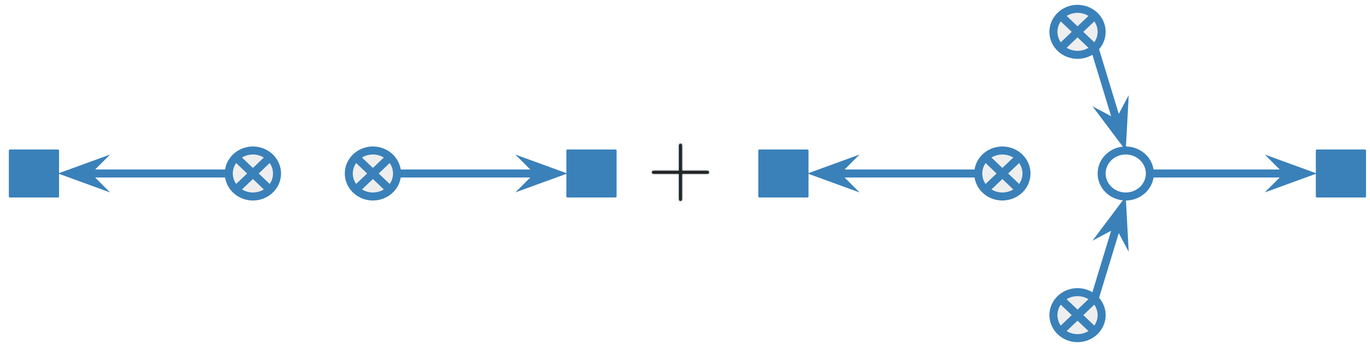

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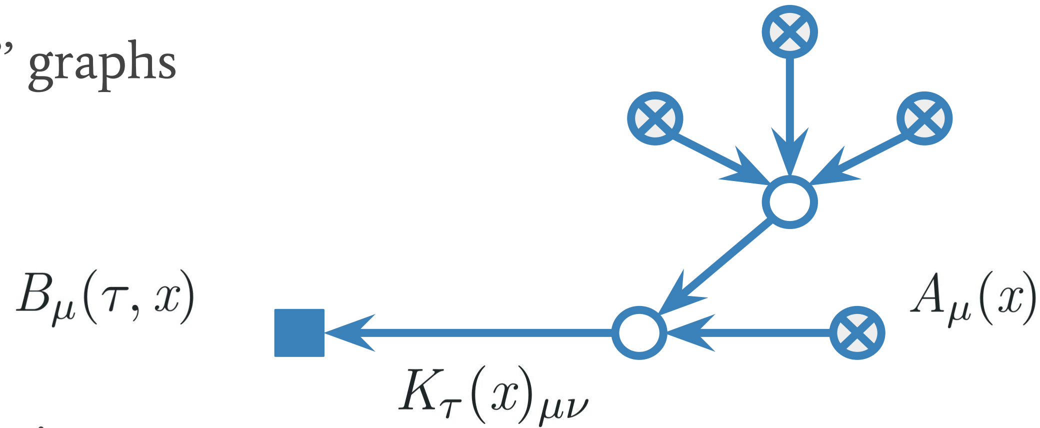

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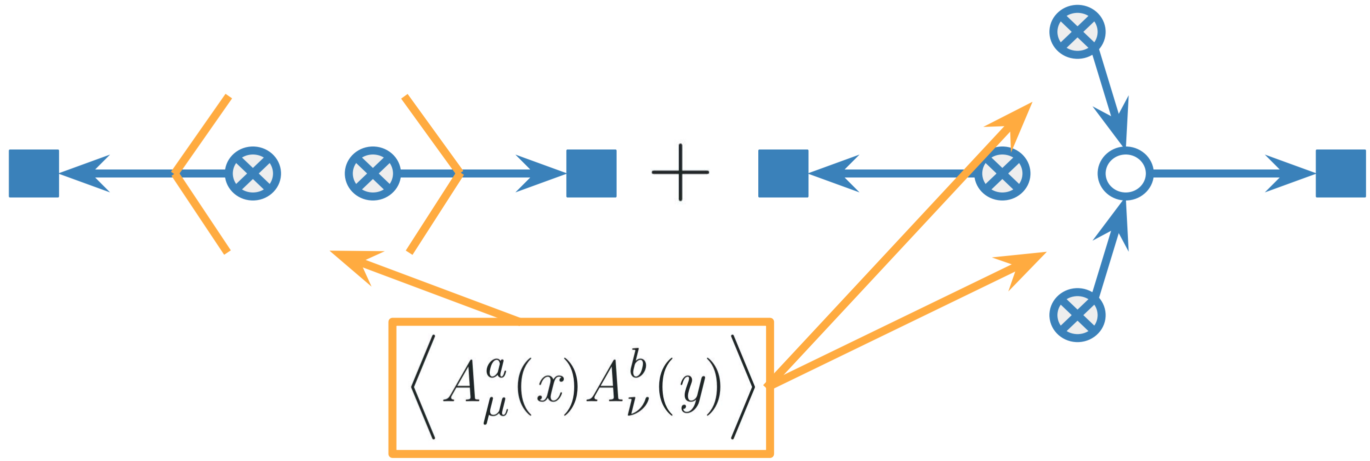

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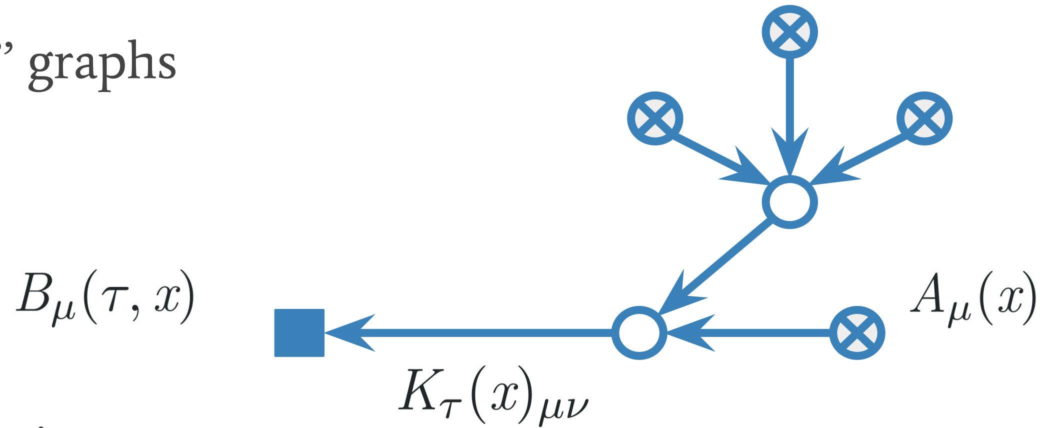

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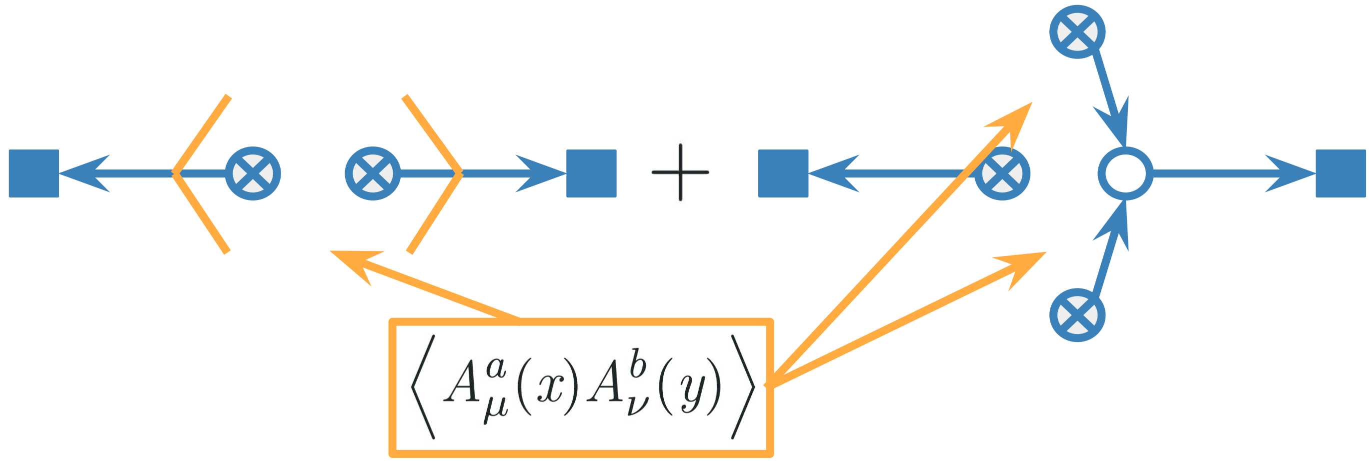
$\left\langle B^a_\mu(\tau, x) B^b_\nu(\sigma, y) \right\rangle =$



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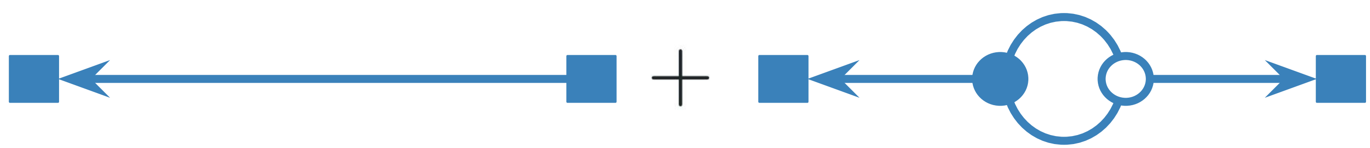

$$\left\langle \widetilde{B}^a_\mu(t, p) \widetilde{B}^b_\nu(s, q) \right\rangle = (2\pi)^4 \delta(p+q) \delta^{ab} \delta_{\mu\nu} \frac{e^{-(s+t)p^2}}{p^2} + \mathcal{O}(g^2)$$

Smearing removes power-divergent mixing in the continuum, at the expense of introducing a new scale

CJM, PoS(Lattice2015) 052
CJM & K. Orginos, PRD 91 (2015) 074513
CJM & K. Orginos, PoS(Lattice2014) 330

4. No free lunch:
The gradient flow scheme

Consider twist-2 operators

$$\mathcal{T}_{\mu_1\ldots\mu_n}(x) = \phi(x)\partial_{\mu_1}\ldots\partial_{\mu_n}\phi(x) - \text{traces}$$

Example: continuum matrix element

$$\langle\,\Omega\,|\,\phi^2(0)\cdot\phi(0)\partial_\mu\partial_\nu\phi(0)\,|\,\Omega\,\rangle = 0$$

On the lattice

$$\left\langle\Omega|\phi^2(0)\cdot\phi(0)\nabla_\mu\nabla_\nu\phi(0)|\Omega\right\rangle = -\frac{\delta_{\mu\nu}}{32a^2} + \mathcal{O}(a^0,\lambda)$$

With smeared degrees of freedom

$$\langle\,\Omega\,|\,\overline{\phi}^2(\tau,0)\cdot\overline{\phi}(\tau,0)\nabla_\mu\nabla_\nu\overline{\phi}(\tau,0)\,|\,\Omega\,\rangle = -\frac{\delta_{\mu\nu}}{256\pi^2\tau} + \mathcal{O}(a^0,\lambda)$$

Two approaches to our new scale:

“Smeared Operator Product Expansion” (sOPE)

Small flow-time expansion

sOPE

Replace local operators in the OPE

$$\mathcal{O}(x) \overset{x\to 0}{\sim} \sum_k c_k(x,\mu)\mathcal{O}_{\rm R}^{(k)}(0,\mu) + \ldots$$

with locally-smeared operators

$$\mathcal{O}(x) \overset{x\to 0}{\sim} \sum_k d_k(x,\mu,\tau)\overline{\mathcal{O}}^{(k)}(0,\mu,\tau) + \ldots$$

CJM, PoS(Lattice2015) 052
CJM & K. Orginos, PRD 91 (2015) 074513
CJM & K. Orginos, PoS(Lattice2014) 330

Small flow-time expansion

Relate locally-smeared operators to local operators

$$\overline{\mathcal{O}}(\tau) = C(\mu, \tau)\mathcal{O}_{\mathrm{R}}(\mu) + \ldots$$

Case-study: determine heavy-light quark bilinear renormalisation parameters

$$\Sigma_\Gamma^{\mathrm{latt}}(a, M_Q) \equiv \langle\Omega|\overline{Q}\,\Gamma q \cdot \overline{Q}\,\Gamma q\,|\Omega\rangle^{\mathrm{latt}}$$

$$\Sigma_\Gamma^{\mathrm{smear}}(\tau, a, M_Q) \equiv \langle\Omega|\overline{Q}\,\Gamma q(\tau) \cdot \overline{Q}\,\Gamma q(\tau)\,|\Omega\rangle^{\mathrm{latt}}$$

Continuum matrix element $\Sigma_\Gamma^{\mathrm{R}}(\mu, M_Q) = \lim_{a\to 0} Z_\Gamma^2(\mu, a, M_Q)\Sigma_\Gamma^{\mathrm{latt}}(a, M_Q)$

Case-study: determine heavy-light quark bilinear **renormalisation parameters**

$$\Sigma_\Gamma^{\mathrm{latt}}(a, M_Q) \equiv \langle \Omega | \overline{Q}\,\Gamma q \cdot \overline{Q}\,\Gamma q\, |\Omega\rangle^{\mathrm{latt}}$$

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Continuum matrix element $\quad \Sigma_\Gamma^{\mathrm{R}}(\mu, M_Q) = \lim_{a\to 0} \boxed{Z_\Gamma^2(\mu, a, M_Q)} \Sigma_\Gamma^{\mathrm{latt}}(a, M_Q)$

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Continuum matrix element $\Sigma_\Gamma^{\mathrm{R}}(\mu, M_Q) = \lim_{a\to 0} Z_\Gamma^2(\mu, a, M_Q)\Sigma_\Gamma^{\mathrm{latt}}(a, M_Q)$

Small flow-time expansion

$$\Sigma_\Gamma(\tau, M_Q) = C(\mu, \tau)\Sigma_\Gamma^{\mathrm{R}}(\mu, M_Q)$$

Case-study: determine heavy-light quark bilinear renormalisation parameters

$$\Sigma_\Gamma^{\rm latt}(a, M_Q) \equiv \langle\Omega|\overline{Q}\,\Gamma\, q \cdot \overline{Q}\,\Gamma\, q\,|\Omega\rangle^{\rm latt}$$

$$\Sigma_\Gamma^{\rm smear}(\tau, a, M_Q) \equiv \langle\Omega|\overline{Q}\,\Gamma\, q(\tau) \cdot \overline{Q}\,\Gamma\, q(\tau)\,|\Omega\rangle^{\rm latt}$$

Continuum matrix element $\Sigma_\Gamma^{\rm R}(\mu, M_Q) = \lim_{a\to 0} Z_\Gamma^2(\mu, a, M_Q)\Sigma_\Gamma^{\rm latt}(a, M_Q)$

↑ Small flow-time expansion

$$\Sigma_\Gamma(\tau, M_Q) = C(\mu, \tau)\Sigma_\Gamma^{\rm R}(\mu, M_Q)$$

↑

$$\Sigma_\Gamma(\tau, M_Q) = \lim_{a\to 0} \Sigma_\Gamma^{\rm smear}(\tau, a, M_Q)$$

Case-study: matrix elements of heavy-light quark bilinears

$$\Sigma_\Gamma^{\mathrm{latt}}(a, M_Q) \equiv \langle \Omega | \overline{Q}\,\Gamma q \cdot \overline{Q}\,\Gamma q \, | \Omega \rangle^{\mathrm{latt}}$$

$$\Sigma_\Gamma^{\mathrm{smear}}(\tau, a, M_Q) \equiv \langle \Omega | \overline{Q}\,\Gamma q(\tau) \cdot \overline{Q}\,\Gamma q(\tau) \, | \Omega \rangle^{\mathrm{latt}}$$

Continuum matrix element $\Sigma_\Gamma^{\mathrm{R}}(\mu, M_Q) = \lim_{a \to 0} Z_\Gamma^2(\mu, a, M_Q) \Sigma_\Gamma^{\mathrm{latt}}(a, M_Q)$

Small flow-time expansion

$$\Sigma_\Gamma(\tau, M_Q) = C(\mu, \tau) \Sigma_\Gamma^{\mathrm{R}}(\mu, M_Q)$$

$$\Sigma_\Gamma(\tau, M_Q) = \lim_{a \to 0} \Sigma_\Gamma^{\mathrm{smear}}(\tau, a, M_Q)$$

Defines renormalisation scheme: gradient flow scheme

$$Z_\Gamma^{\mathrm{GF}}(\mu, a, M_Q) = \sqrt{\frac{\lim_{a \to 0} \Sigma_\Gamma^{\mathrm{smear}}(\tau, a, M_Q)}{C(\mu, \tau)} \frac{1}{\Sigma_\Gamma^{\mathrm{latt}}(a, M_Q)}}$$

Gradient flow scheme

$$Z_\Gamma^{\mathrm{GF}}(\mu, a, M_Q) = \sqrt{\frac{\lim_{a\to 0} \Sigma_\Gamma^{\mathrm{smear}}(\tau, a, M_Q)}{C(\mu, \tau)} \frac{1}{\Sigma_\Gamma^{\mathrm{latt}}(a, M_Q)}}$$

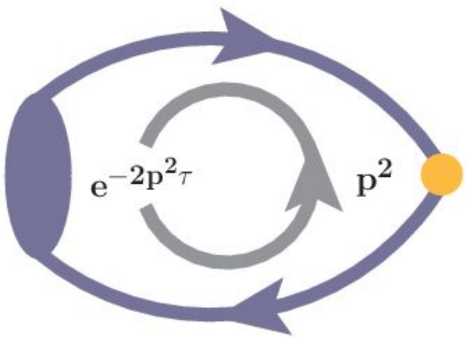
Gradient flow scheme

$$Z_\Gamma^{\mathrm{GF}}(\mu, a, M_Q) = \sqrt{\frac{\lim_{a\to 0}\Sigma_\Gamma^{\mathrm{smear}}(\tau, a, M_Q)}{C(\mu, \tau)} \frac{1}{\Sigma_\Gamma^{\mathrm{latt}}(a, M_Q)}}$$

$$\Sigma_\Gamma^{\text{smear}}(\tau, a, M_Q)$$

Calculate with smeared fermions

Controls all divergences



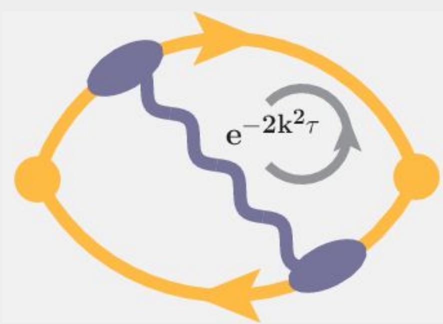
But...

Requires fermion renormalisation

Complicates perturbation theory

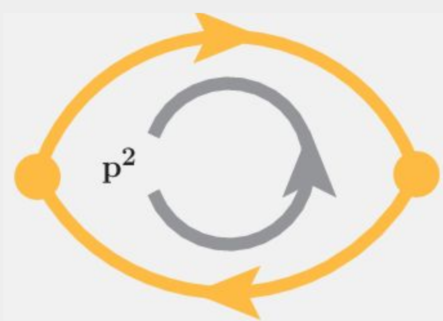
Calculate with smeared gauge fields

Controls all gluonic divergences



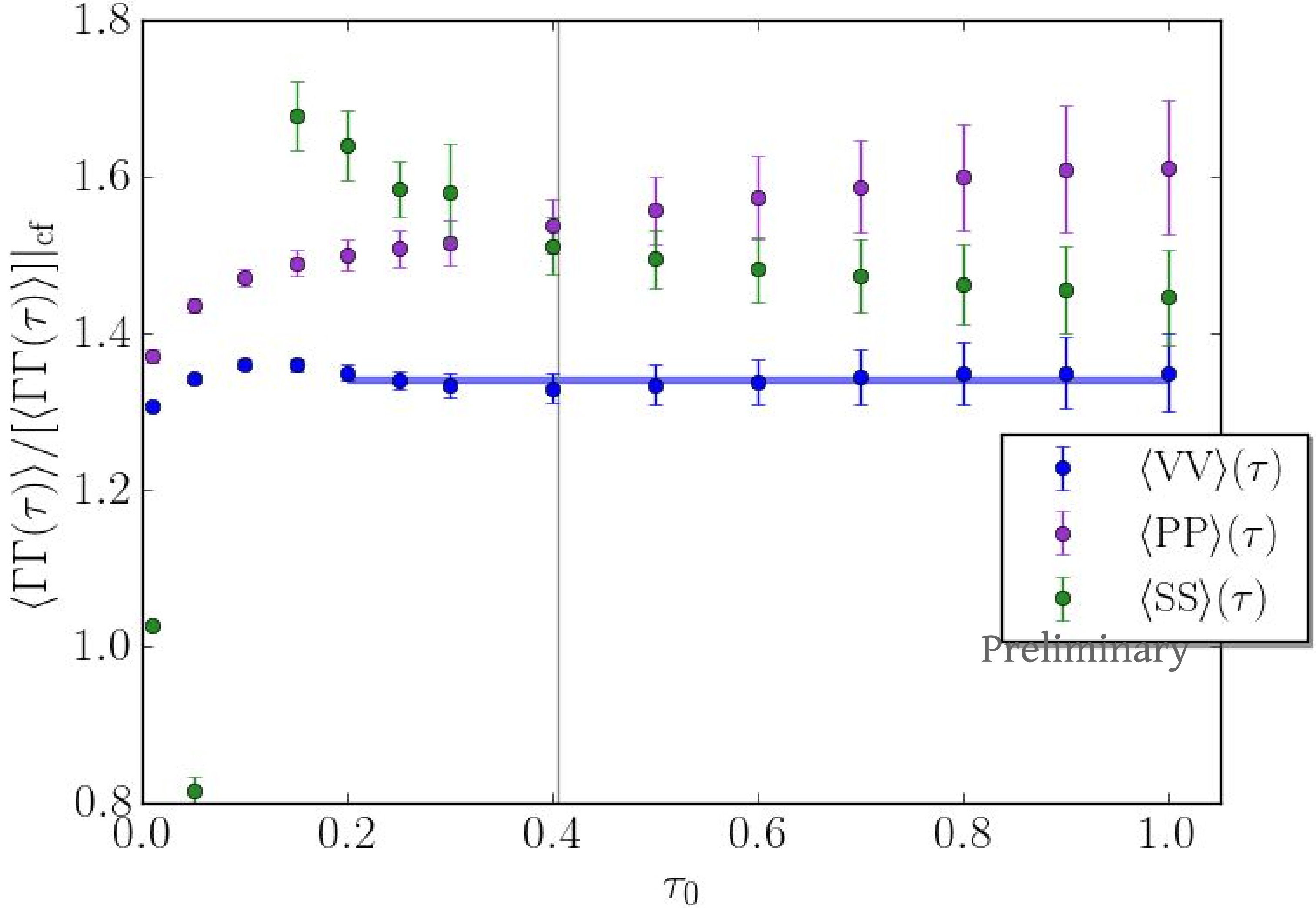
But...

Does not remove tree-level divergence



Calculate with smeared fermions: [JLab Theory Seminar 12/9/2016]

$$Z_\Gamma^{\rm GF}(\mu, a, M_Q) = \sqrt{\frac{\lim_{a\to 0} \Sigma_\Gamma^{\rm smear}(\tau, a, M_Q)}{C(\mu,\tau)} \frac{1}{\Sigma_\Gamma^{\rm latt}(a, M_Q)}}$$



Update: calculate with smeared gauge fields

Remove tree-level divergence by taking continuum limit of

$$\frac{\Sigma_\Gamma^{\text{smear}}(\tau, a, M_Q)}{\Sigma_\Gamma^{\text{tree}}(\tau, a, M_Q)}$$

1. Fix physical volume

2. Tune bare coupling at different lattice sizes

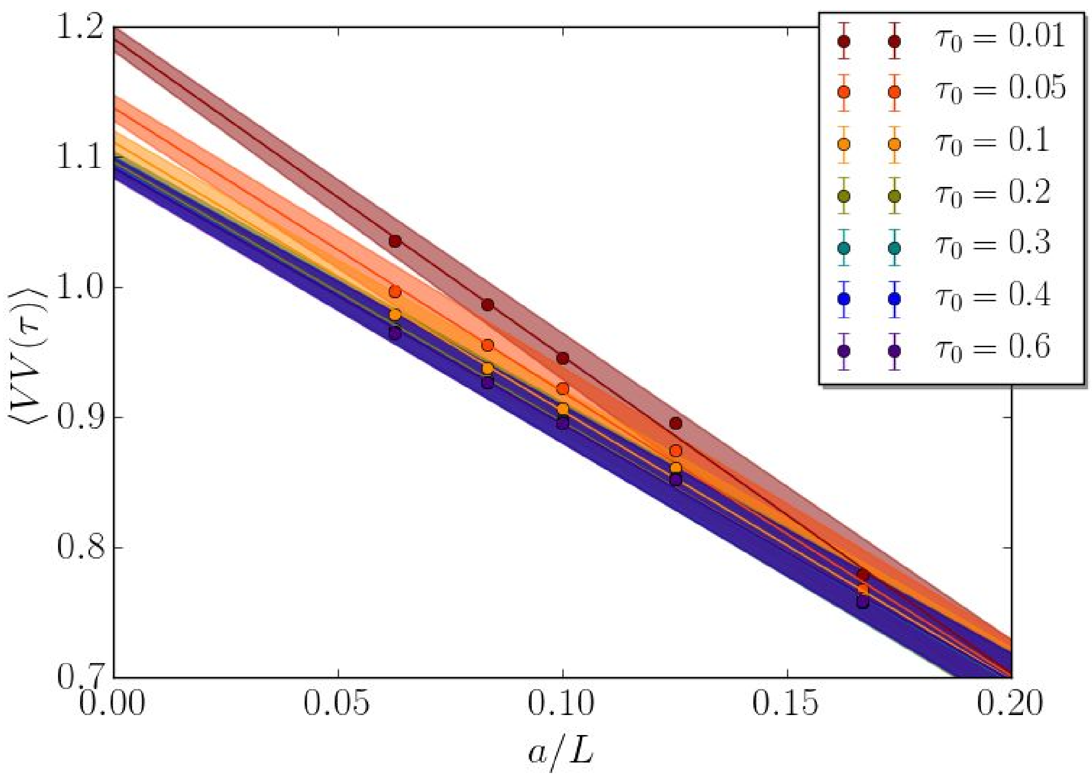
3. Fix flow time in physical units

Update: calculate with smeared gauge fields

Remove tree-level divergence by taking continuum limit of

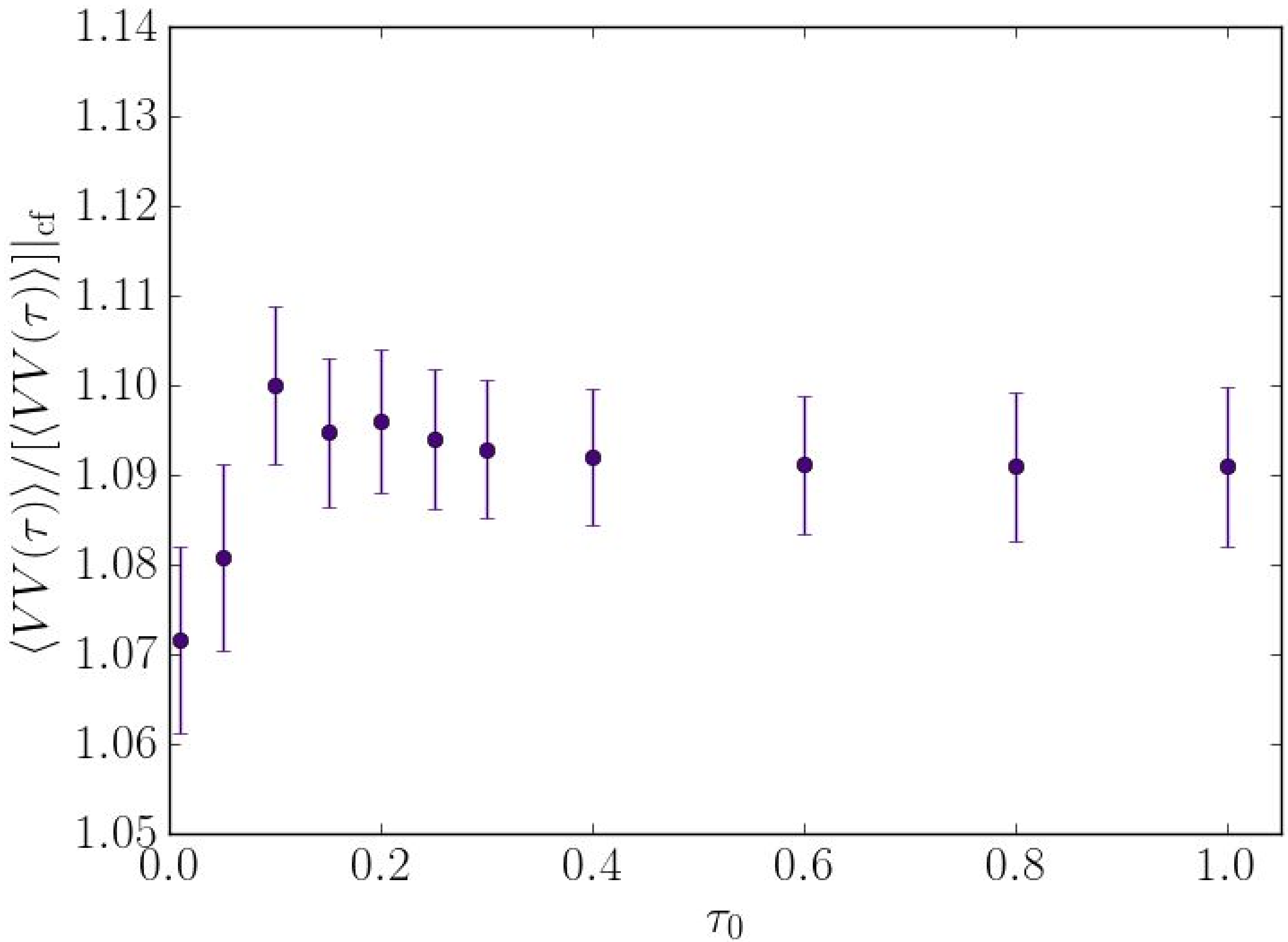
$$\frac{\Sigma_{\Gamma}^{\text{smear}}(\tau, a, M_Q)}{\Sigma_{\Gamma}^{\text{tree}}(\tau, a, M_Q)}$$

1. Fix physical volume
2. Tune bare coupling at different lattice sizes
3. Fix flow time in physical units



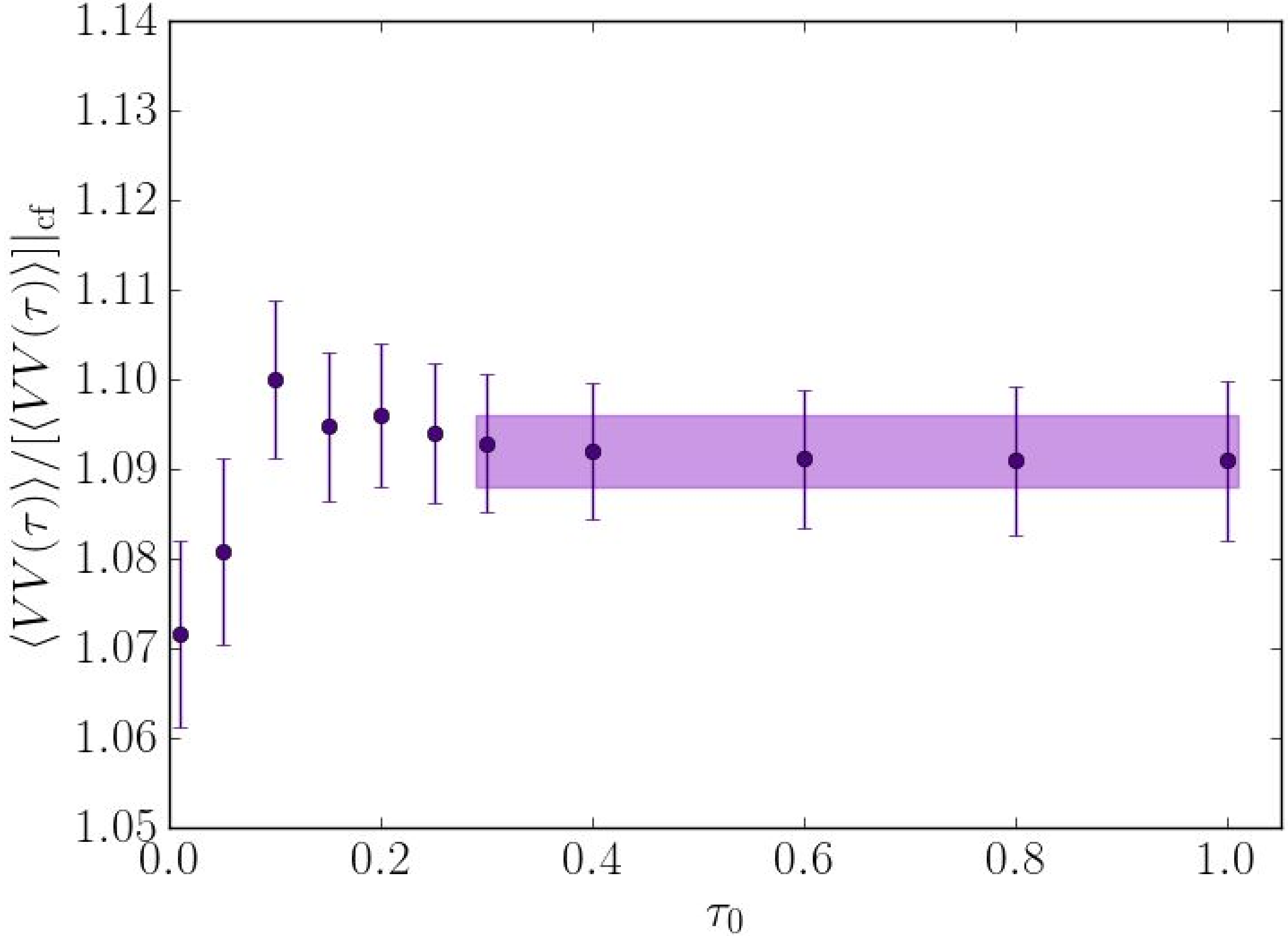
Update: calculate with smeared gauge fields

$$Z_\Gamma^{\rm GF}(\mu, a, M_Q) = \sqrt{\frac{\lim_{a\to 0} \Sigma_\Gamma^{\rm smear}(\tau, a, M_Q)}{C(\mu, \tau)} \frac{1}{\Sigma_\Gamma^{\rm latt}(a, M_Q)}}$$



Update: calculate with smeared gauge fields

$$Z_\Gamma^{\mathrm{GF}}(\mu, a, M_Q) = \sqrt{\frac{\lim_{a\to 0} \Sigma_\Gamma^{\mathrm{smear}}(\tau, a, M_Q)}{C(\mu, \tau)} \frac{1}{\Sigma_\Gamma^{\mathrm{latt}}(a, M_Q)}}$$



Gradient flow scheme

$$Z_\Gamma^{\mathrm{GF}}(\mu, a, M_Q) = \sqrt{\frac{\lim_{a\to 0} \Sigma_\Gamma^{\mathrm{smear}}(\tau, a, M_Q)}{C(\mu, \tau)} \frac{1}{\Sigma_\Gamma^{\mathrm{latt}}(a, M_Q)}}$$

5. Looking forward

Direct determination of PDFs

Via gradient flow procedure

Determine moments

Hypercubic symmetry = power-divergent mixing

Restricts us to lowest moments of PDFs or GPDs

Gradient flow removes power-divergent mixing

Gradient flow scheme relates matrix elements at non-zero and zero flow times

Will allow determination of higher moments

- involves only local operators
- analysis a well-established procedure

Direct determination of PDFs

X. Ji & J.-H. Zhang, PRD 92 (2015) 034006
X. Ji, Sc. China (2014)
X. Ji, PRL 110 (2013) 262002

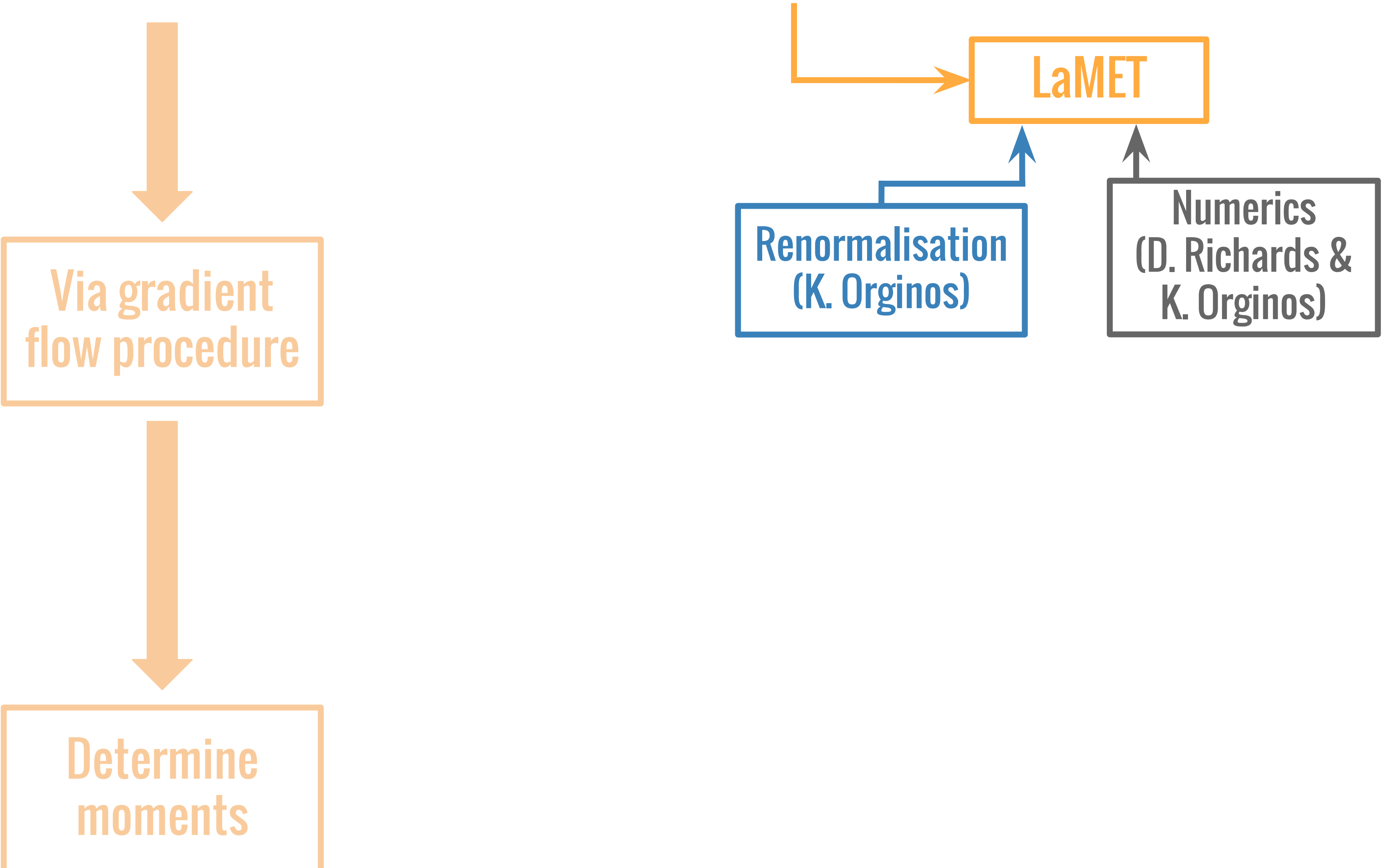
LaMET

with David Richards, Kostas Orginos, Carl Carlson

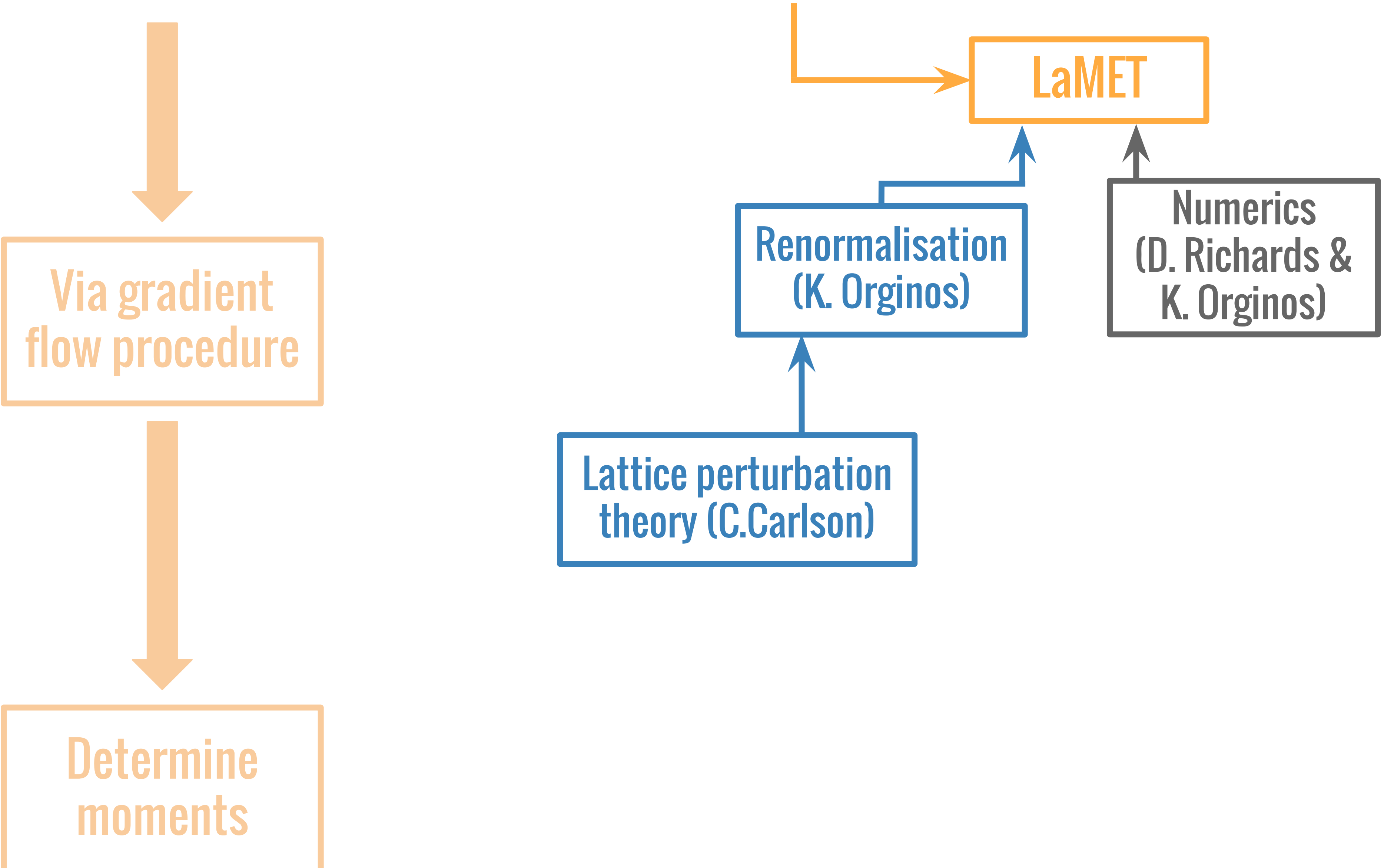
Via gradient flow procedure

Determine moments

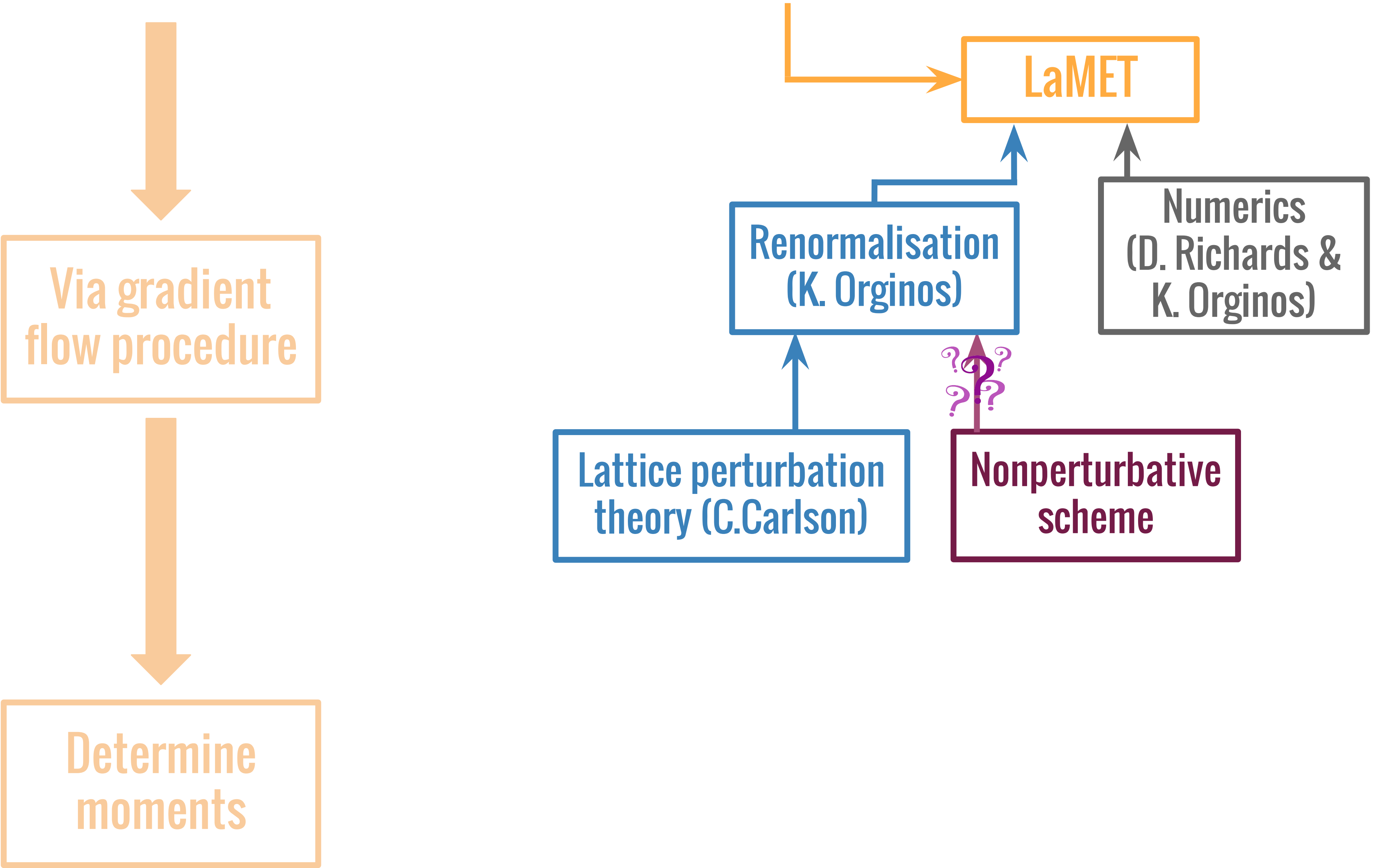
Direct determination of PDFs



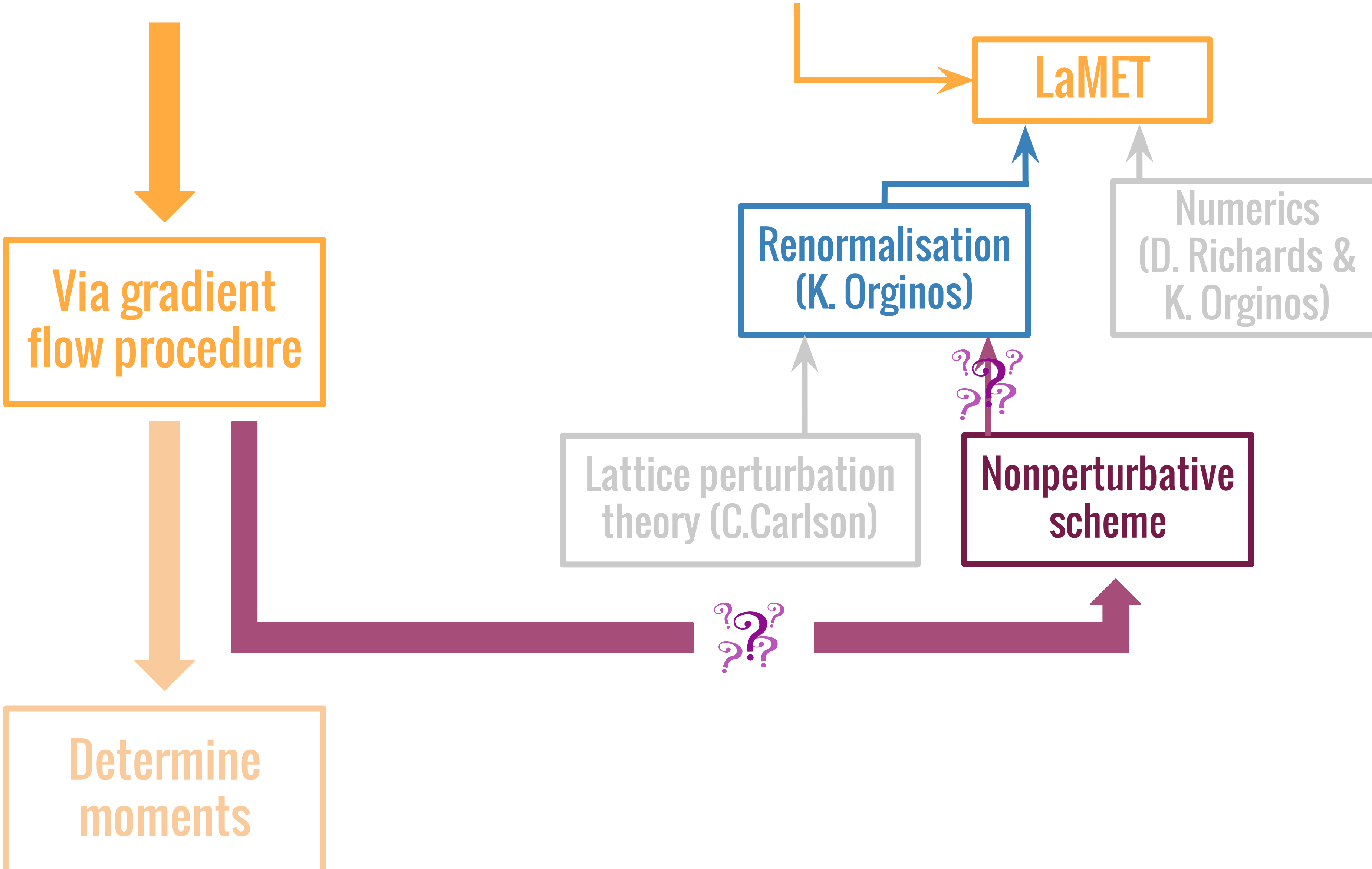
Direct determination of PDFs



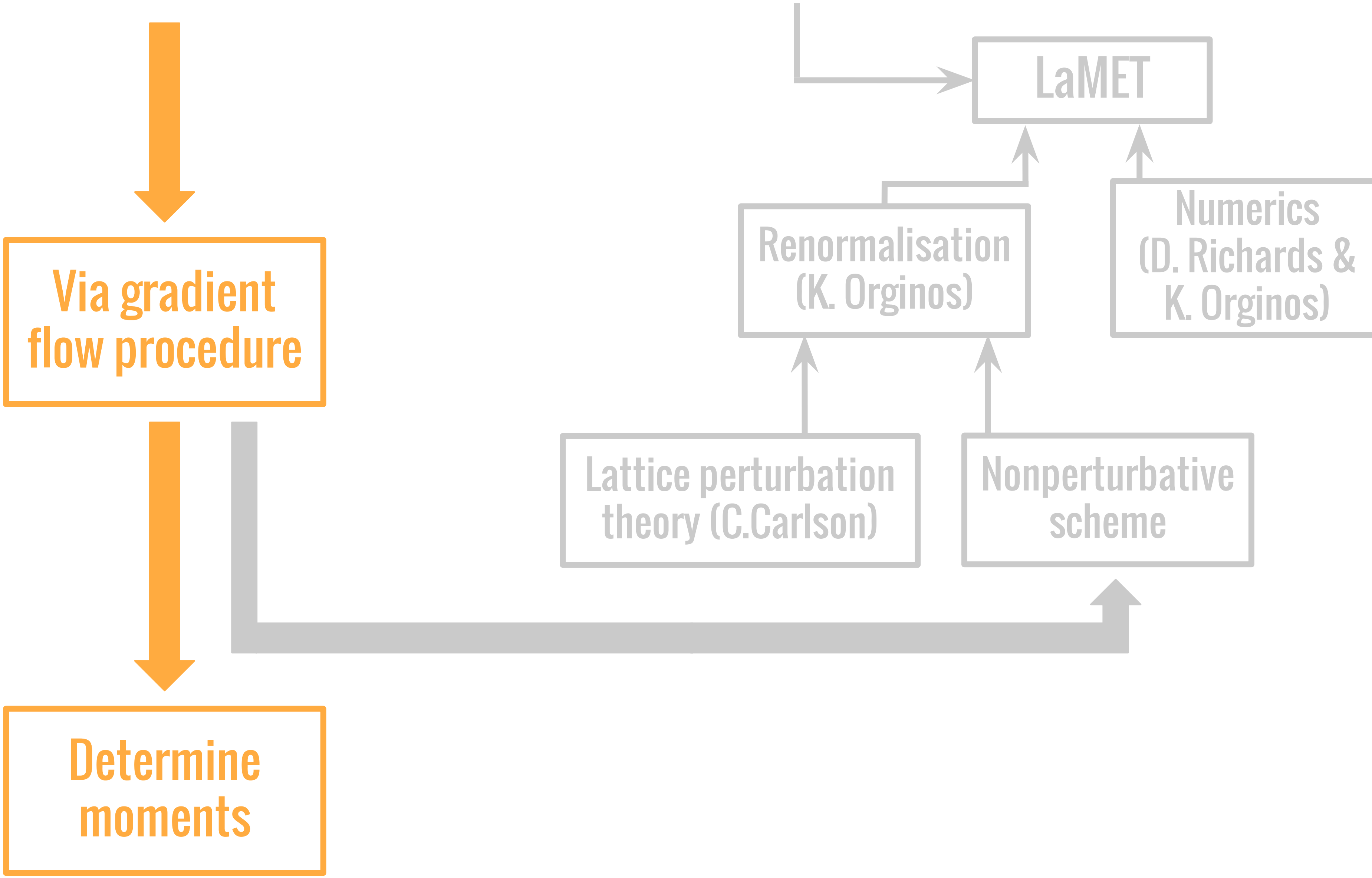
Direct determination of PDFs



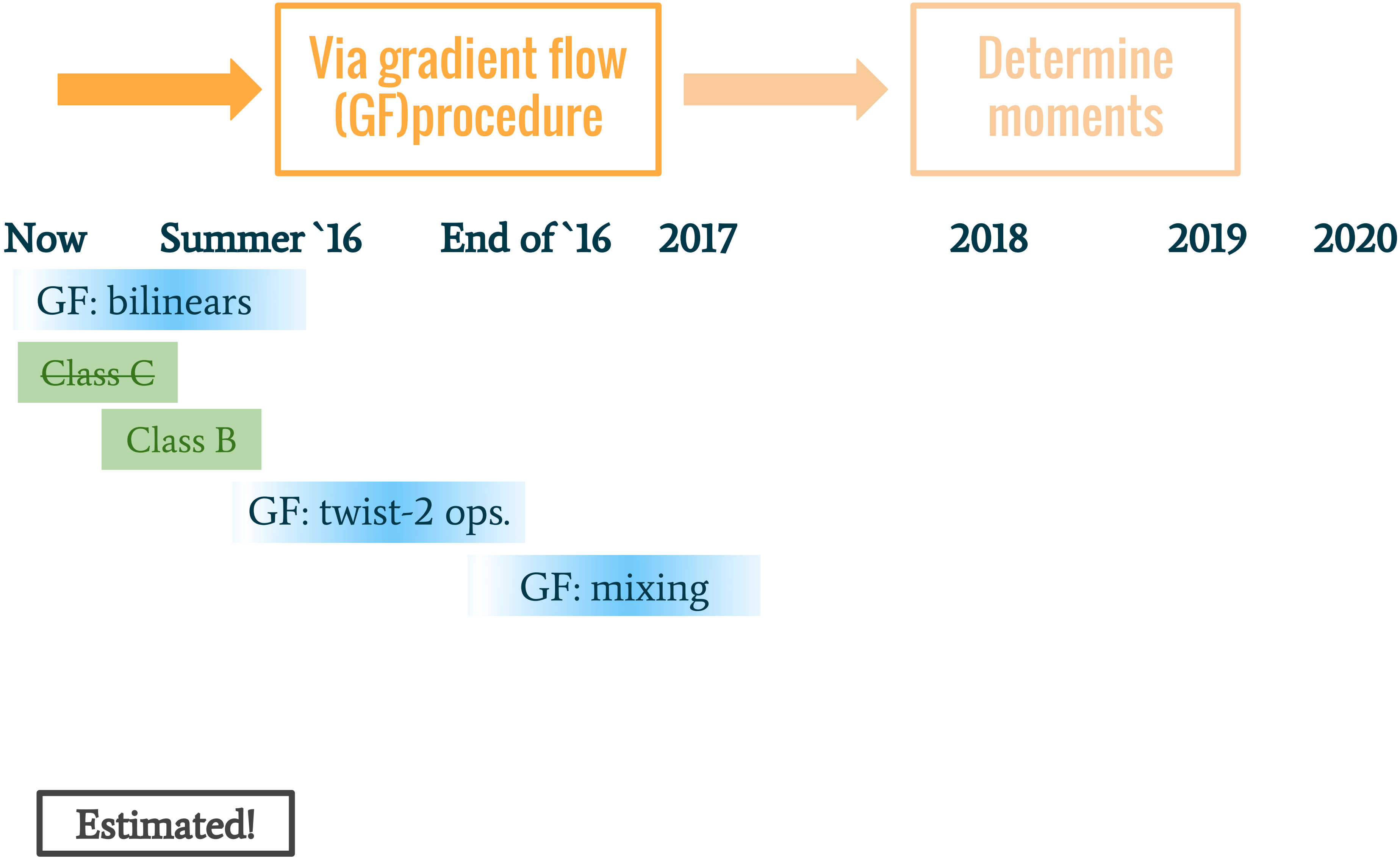
Direct determination of PDFs



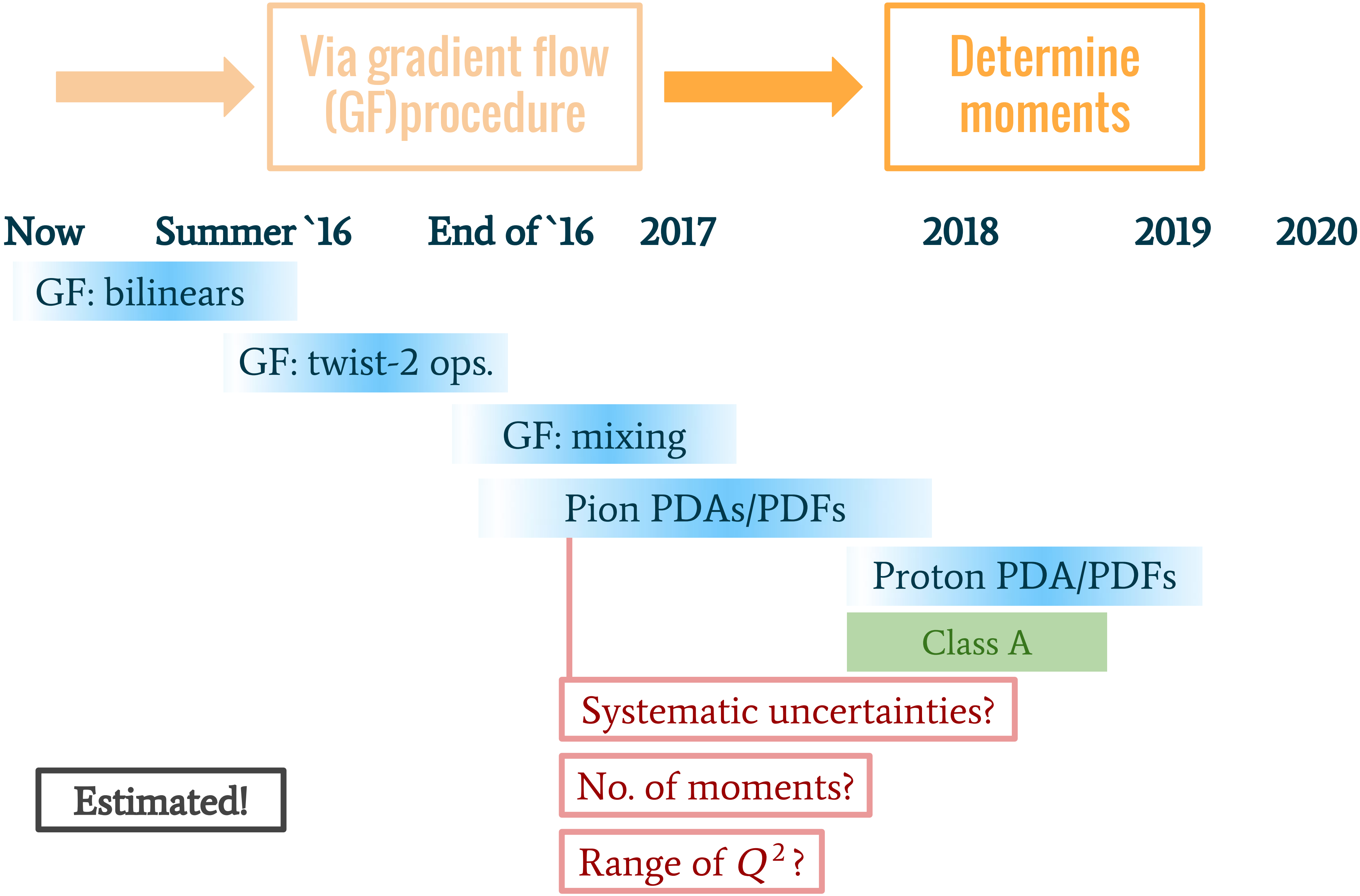
Direct determination of PDFs



Direct determination of PDFs



Direct determination of PDFs



Direct determination of PDFs

→ Via gradient flow (GF)procedure → Determine moments

Now | Summer `16 | End of `16 | 2017 | 2018 | 2019 | 2020

GF: bilinears

GF: twist-2 ops.

GF: mixing

Pion PDAs/PDFs

Proton PDA/PDFs

Spin physics...

Sea quarks...

Tomography...

Estimated!

Pion PDAs/PDFs

Proton PDA/PDFs

Calculations

1. Low moments of spin-(in)dependent structure functions
   - operators that do not mix
   - operators that mix

Example tests:
- **operators in the same lattice irrep have the same renormalisation**
- **operators in the same continuum irrep and different lattice irreps give the same result**

Physics

1. Basic tests
   - verification of basic implementation via comparison with previous results
   - verification that gradient flow removes mixing

Pion PDAs/PDFs

Proton PDA/PDFs

Calculations

1. Low moments of spin-(in)dependent structure functions
   - operators that do not mix
   - operators that mix

2. Higher moments

Questions:
- **how many moments?**
- **can full PDF be reconstructed?**
- **can high moments constrain global fits?**

Physics

1. Basic tests
   - verification of basic implementation via comparison with previous results
   - verification that gradient flow removes mixing

2. First ever such calculation

a) comparison with theory
   - nonrelativistic quark models at m_q
   - tests of ``Borromean" picture of baryonic di-quark correlations in Dyson-Schwinger formalism
   - predictions from AdS/QCD

b) experiment
   - global fits
   - JLab 12 GeV deep valence region

c) constrain high-x regimes of PDFs

Tomography...

Calculations

Physics

3. Low moments of spin-(in)dependent generalised structure functions on unquenched configurations

4. Higher moments

3. Comparisons with:
   - quenched lattice calculations for first three moments of GPDs
   - experimental data
   - Ji sum rule for total quark angular momentum

4. First ever such calculation...

Corollaries

Calculations

Physics

a) Gluon operators and mixing — Largely unstudied in lattice QCD

b) Twist-3 contributions (*e.g.* to g_2)

c) Nonperturbative improvement — Improvement in systematic uncertainties.

d) Nonperturbative Wilson coefficients

Summary

Gradient flow - tool to remove power-divergent mixing

Gradient flow scheme - operator renormalisation

Enable lattice calculations of moments of PDFs and GPDs

Thank you

chris.monahan@rutgers.edu

Update: calculate with smeared gauge fields

1. Choose some bare coupling and lattice size *L/a*

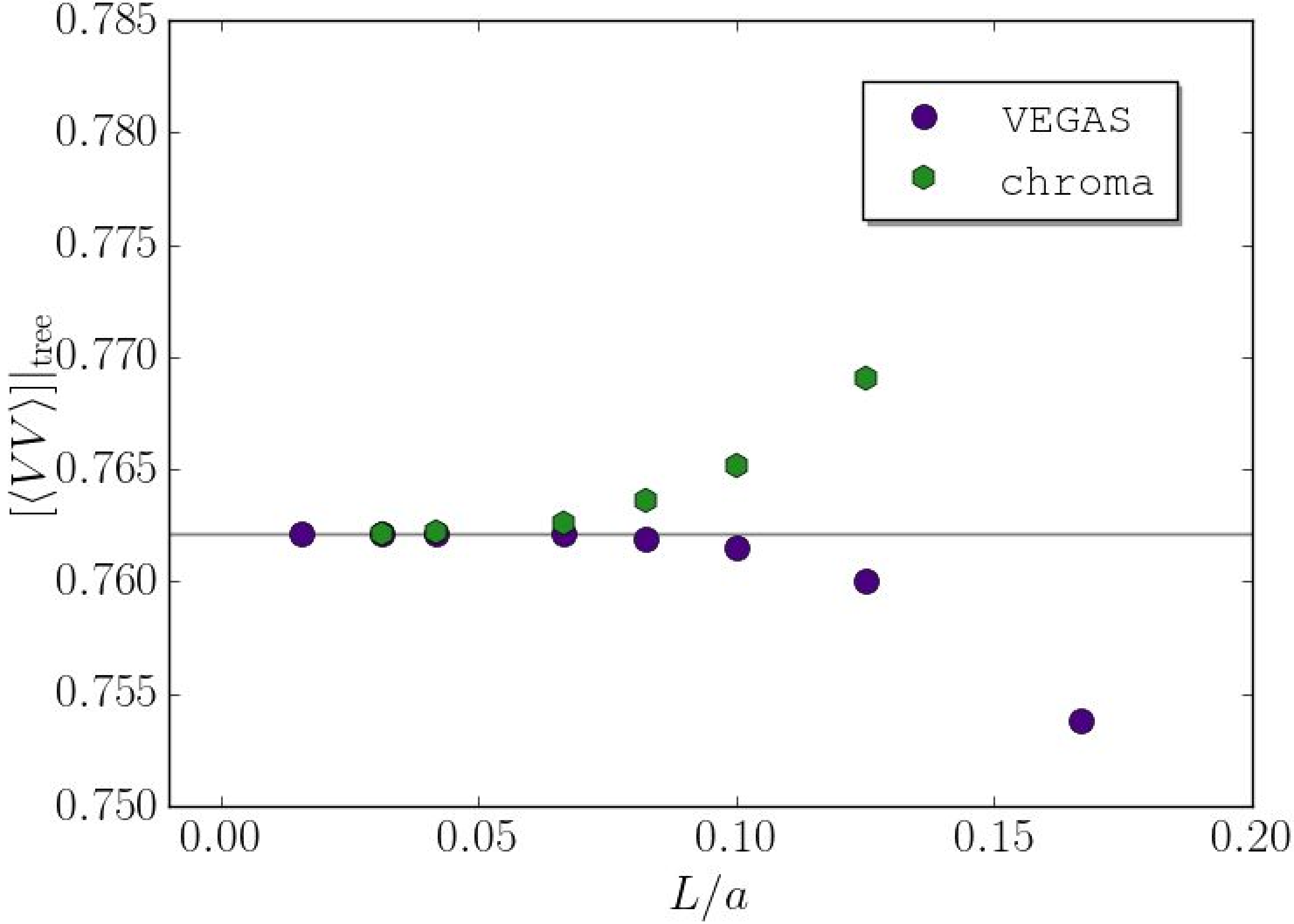
2. Measure finite volume renormalised coupling

3. Determine the matrix element at some flow time *t*

4. Fix renormalised coupling, which fixes the physical box size, and tune bare coupling to match at new lattice spacing *L'/a*

5. Determine matrix element at fixed physical flow time by choosing flow time in lattice units by fixing product $t' = (m_{crit})^2/(m_{crit}')^2$, where the critical mass is calculated at two loops in "cactus-improved" lattice perturbation theory

6. Repeat steps 3-5 to take continuum limit



VEGAS
chroma
$[\langle VV \rangle]|_{\mathrm{tree}}$
L/a
0.785
0.780
0.775
0.770
0.765
0.760
0.755
0.750
0.00
0.05
0.10
0.15
0.20

Lattice determinations: nucleon structure

Meson distribution amplitudes

- quenched — Martinelli & Sachrajda, PLB 1 (1987) 184; Martinelli & Sachrajda, NPB 306 (1988) 805
- unquenched — Best et al, PRD 56 (1997) 2743

Nucleon

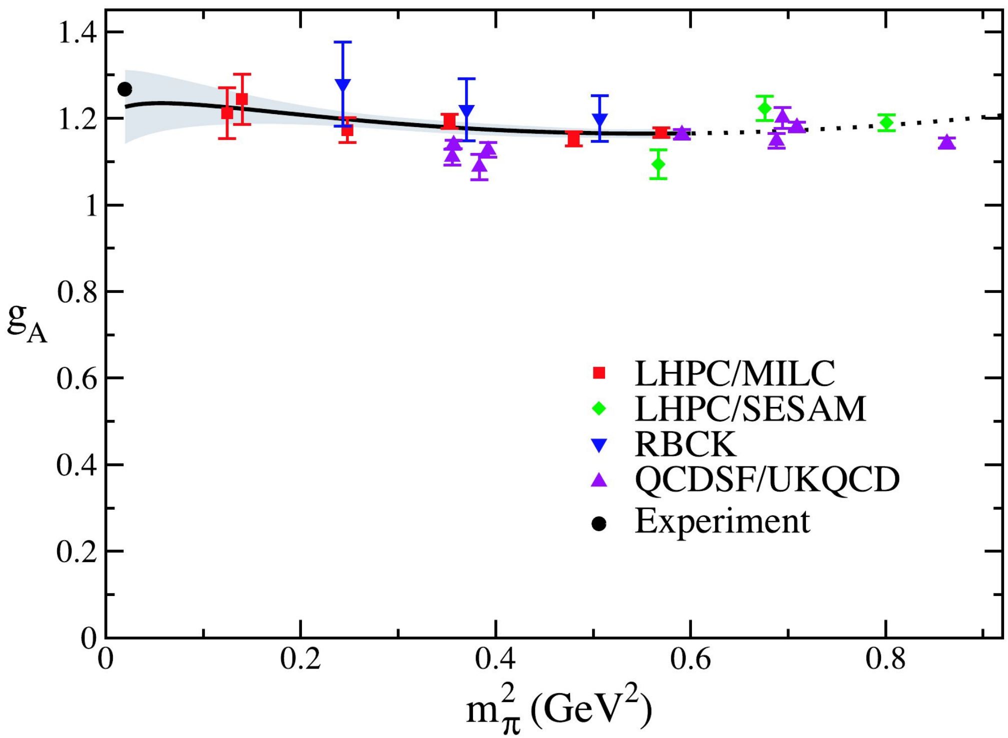
- axial charge — Edwards et al, PRL 96 (2006) 052001; Capitani et al, PRD 86 (2012) 074502; Horsley *et al*, PLB 732 (2014) 41
- unpolarised — Gockeler et al, PRD 53 (1996) 2317
- polarised — Gockeler *et al*, PRD 53 (1996) 2317
- higher twist contributions — Capitani *et al*, NPB (Proc. Suppl.) 79 (1999) 179
- transverse momentum distributions — Y. Zhao, arXiv/1506.08832; Musch *et al*, PRD 83 (2011) 094507
- generalised parton distributions — Hagler et al, PRL 93 (2004) 112001; Gockeler et al, PRL 92 (2004) 042002

W. Bietenholz et al, PoS LATTTICE(2009) 138

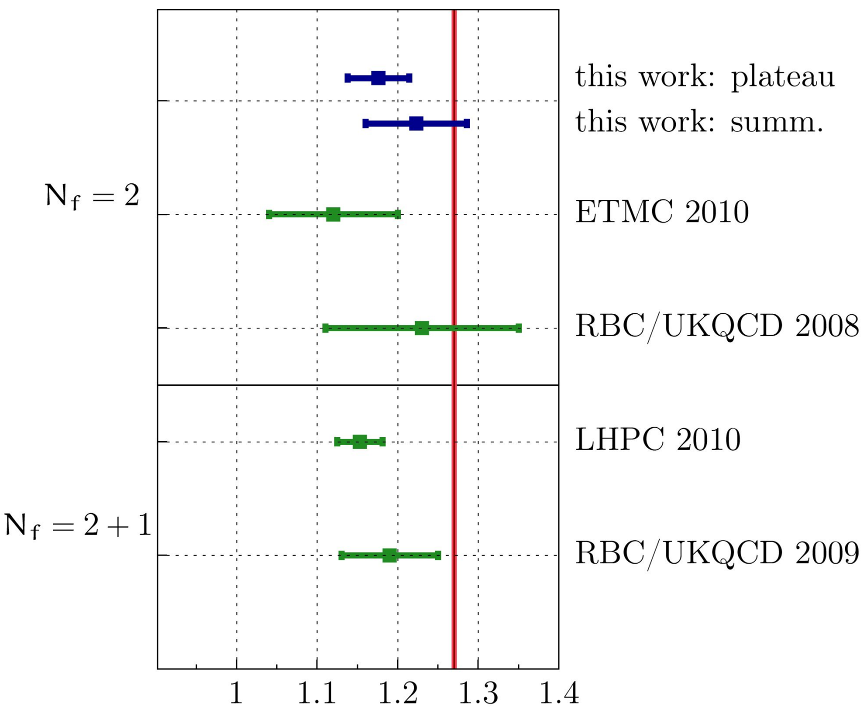
Nucleon axial charge

$$\langle x^0 \rangle_{\Delta q} = \int_0^1 \mathrm{d}x \, [\Delta q(x) + \Delta \bar{q}(x)]$$

$$\Delta q(x) = q_\uparrow(x) - q_\downarrow(x)$$



Edwards *et al*, Phys. Rev. Lett. 96 (2006) 052001



Capitani *et al*, Phys. Rev. D 86 (2012) 074502

Direct determination of PDFs: LaMET

Relate PDFs

$$q(x,\mu^2) = \int \frac{\mathrm{d}\xi^-}{4\pi} e^{-ix\xi^- P^+} \langle P|\overline{\psi}(\xi^-)\gamma^+ e^{-ig\int_0^{\xi^-} \mathrm{d}\eta^- A^+(\eta^-)}\psi(0)|P\rangle$$

X. Ji et al, PRD 91 (2015) 074009
X. Ji, Sc. China (2014)
X. Ji, PRL 110 (2013) 262002

to "quasi"-distributions

$$\overline{q}(x,\mu^2,P^z) = \int \frac{\mathrm{d}z}{4\pi} e^{izk^z} \langle P|\overline{\psi}(z)\gamma^z e^{-ig\int_0^z \mathrm{d}z' A^z(z')}\psi(0)|P\rangle + \mathcal{O}\left(\Lambda^2_{\mathrm{QCD}}/(P^z)^2, M^2/(P^z)^2\right)$$

via a factorisation formula

$$\overline{q}(x,\mu^2,P^z) = \int_x^1 \frac{\mathrm{d}y}{y} Z\left(\frac{x}{y},\frac{\mu}{P^z}\right) q(y,\mu^2) + \mathcal{O}\left(\Lambda^2_{\mathrm{QCD}}/(P^z)^2, M^2/(P^z)^2\right)$$

Requires renormalisation of nonlocal operators

X. Ji & J.-H. Zhang, PRD 92 (2015) 034006
X. Ji et al, arXiv/1506.00248
X. Xiong et al, PRD 90 (2014) 014051

Some progress towards this via HQET at NLO

- relation to OPE-based approaches?

W. Detmold & C.J.D. Lin, PRD 73 (2006) 014501

Initial lattice studies at a single lattice spacing

C. Alexandrou et al, PRD 92 (2015) 014502
H.-W. Lin et al, PRD 91 (2014) 054510

Moments of quark density

$$\langle x^n \rangle_q = \int_0^1 \mathrm{d}x\, x^n (q(x) + (-1)^{n+1} \bar{q}(x)) \qquad q = q_\uparrow + q_\downarrow$$

helicity

$$\langle x^n \rangle_{\Delta q} = \int_0^1 \mathrm{d}x\, x^n (\Delta q(x) + (-1)^n \Delta \bar{q}(x)) \qquad \Delta q = q_\uparrow - q_\downarrow$$

and transversity

$$\langle x^n \rangle_{\delta q} = \int_0^1 \mathrm{d}x\, x^n (\delta q(x) + (-1)^{n+1} \delta \bar{q}(x)) \qquad \delta q = q_\top - q_\perp$$

Odd moments related to spin-independent structure functions

$$\int_0^1 \mathrm{d}x\, x^{n-1} F_1(x, Q^2) = \frac{1}{2} c_n^{(q)}(Q^2/\mu^2) \sum_f e_f^2 \langle x^{n-1} \rangle_{q_f}(\mu)$$

$$\int_0^1 \mathrm{d}x\, x^{n-2} F_2(x, Q^2) = c_n^{(q)}(Q^2/\mu^2) \sum_f e_f^2 \langle x^{n-1} \rangle_{q_f}(\mu)$$

Even moments related to spin-dependent structure function

$$\int_0^1 \mathrm{d}x\, x^n g_1(x, Q^2) = \frac{1}{2} c_n^{(\Delta q)}(Q^2/\mu^2) \sum_f e_f^2 \langle x^n \rangle_{\Delta q_f}(\mu)$$

Moments are related to matrix elements of local operators

$$\mathcal{O}^{(q_f)}_{\{\mu_1\ldots\mu_n\}} = \left(\frac{i}{2}\right)^{n-1} \overline{\psi}^f \gamma_{\{\mu_1} \overset{\leftrightarrow}{D}_{\mu_2} \cdots \overset{\leftrightarrow}{D}_{\mu_n\}} \psi^f$$

$$\mathcal{O}^{(\Delta q_f)}_{\{\sigma\mu_1\ldots\mu_n\}} = \left(\frac{i}{2}\right)^{n} \overline{\psi}^f \gamma_5 \gamma_{\{\sigma} \overset{\leftrightarrow}{D}_{\mu_1} \cdots \overset{\leftrightarrow}{D}_{\mu_n\}} \psi^f$$

$$\mathcal{O}^{(\delta q_f)}_{\mu\{\nu\mu_1\ldots\mu_n\}} = \left(\frac{i}{2}\right)^{n} \overline{\psi}^f \gamma_5 \sigma_{\mu\{\nu} \overset{\leftrightarrow}{D}_{\mu_1} \cdots \overset{\leftrightarrow}{D}_{\mu_n\}} \psi^f$$

Via

$$2\langle x^{n-1}\rangle_{q_f} P_{\mu_1} \cdots P_{\mu_n} = \frac{1}{2}\sum_S \langle P,S|\mathcal{O}^{(q_f)}_{\{\mu_1\ldots\mu_n\}}|P,S\rangle$$

$$\frac{2}{n+1}\langle x^n\rangle_{\Delta q_f} S_{\{\sigma} P_{\mu_1} \cdots P_{\mu_n\}} = -\langle P,S|\mathcal{O}^{(\Delta q_f)}_{\{\sigma\mu_1\ldots\mu_n\}}|P,S\rangle$$

$$\frac{2}{m_N}\langle x^n\rangle_{\delta q_f} S_{[\mu} P_{\{\nu]} P_{\mu_1} \cdots P_{\mu_n\}} = \langle P,S|\mathcal{O}^{(\delta q_f)}_{\mu\{\nu\mu_1\ldots\mu_n\}}|P,S\rangle$$

For Euclidean lattice operators

$$\mathcal{O}^{(q_f)}_{\{\mu_1\ldots\mu_n\}} = \overline{\psi}^f \gamma_{\{\mu_1} \overleftrightarrow{D}_{\mu_2} \cdots \overleftrightarrow{D}_{\mu_n\}} \psi^f$$

$$\mathcal{O}^{(5)}_{\{\sigma\mu_1\ldots\mu_n\}} = \overline{\psi}^f \gamma_{\{\sigma}\gamma_5 \overleftrightarrow{D}_{\mu_1} \cdots \overleftrightarrow{D}_{\mu_n\}} \psi^f$$

Lie in same O(4) irrep, but inequivalent reps of H(4)

$$\mathcal{O}^{(q_f)}_{\{14\}} \qquad \mathcal{O}^{(q_f)}_{\{44\}} - \frac{1}{3}\sum_{i=1}^{3} \mathcal{O}^{(q_f)}_{\{ii\}}$$

Lie in same H(4) irrep

$$\mathcal{O}^{(5)}_{\{14\}} \qquad \mathcal{O}^{(5)}_{\{24\}}$$

See, for example,
Gockeler et al, PRD 54 (1996) 5705

Second moment operator

$$\mathcal{O}^{(q_f)}_{\{114\}} - \frac{1}{2}\left(\mathcal{O}^{(q_f)}_{\{224\}} + \mathcal{O}^{(q_f)}_{\{334\}}\right)$$

Third moment operator

$$\mathcal{O}^{(q_f)}_{\{1144\}} + \mathcal{O}^{(q_f)}_{\{2233\}} - \mathcal{O}^{(q_f)}_{\{1133\}} - \mathcal{O}^{(q_f)}_{\{2244\}}$$

which mixes with

$$\overline{\psi}^f \sigma_{[\mu\{\nu}\gamma_5 \overleftrightarrow{D}_{\mu_1]} \overleftrightarrow{D}_{\mu_2\}} \psi^f$$

JLab 12GeV physics

Study deep valence region of PDFs, $x > 0.5$. Proton well constrained for $x < 0.8$, but no free neutron targets limits precision for neutrons above $x \sim 0.5$, due to ignorance of nuclear modification effects. Answer question: why is d quark distribution softer than expected from flavour symmetry?

Polarisation asymmetry asymmetry unknown in neutrons for $x > 0.6$. Though there exist rigorous QCD predictions.

Large-x distributions relevant to high energy collider backgrounds: high-x uncertainties at lower Q^2 feed into low-x region at higher Q^2 via perturbative evolution.

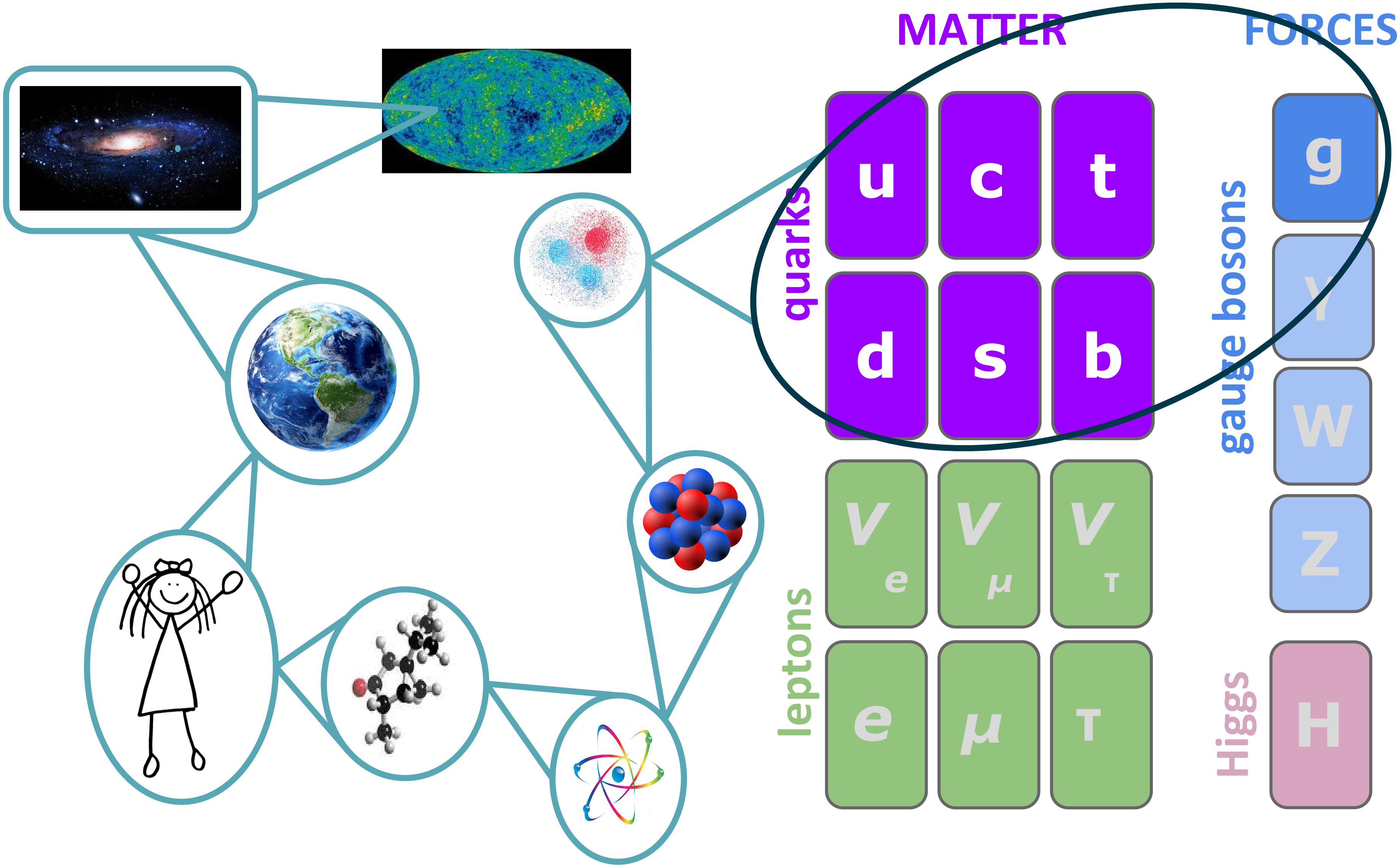
Proton and neutron polarisation asymmetries. Both at $x > 0.4$ for $Q^2 \sim 8\text{-}9$ GeV and for $x < 0.95$ in resonance region, $Q^2 \sim 2\text{-}7$ GeV

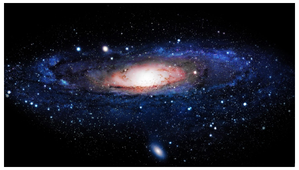
JLab 12GeV physics

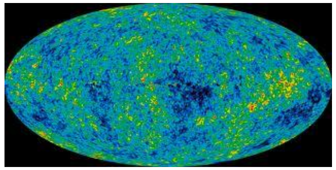
Ji sum rule - vector GPDs yield total contribution of quark OAM to nucleon spin. Cannot measure sum directly, but constrain models of GPDs that predict sum rule values.

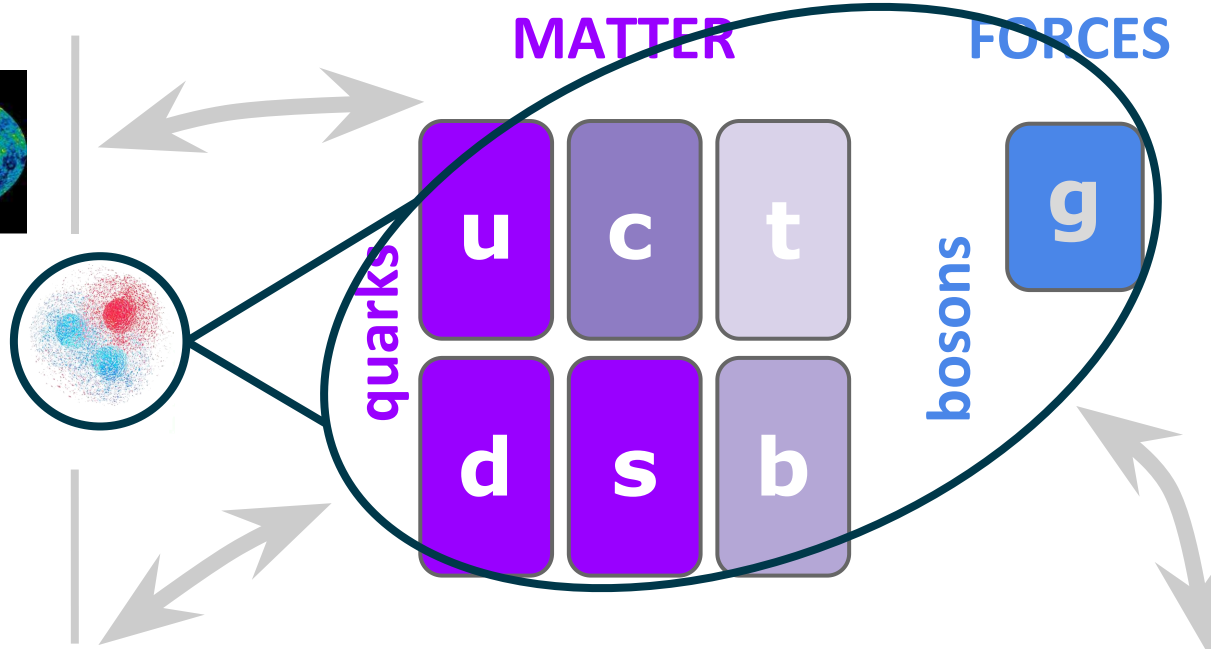
Expect strong correlation between transverse size and longitudinal momentum: large x -> soft t dependence (small in transverse direction) & small x -> stiff t dependence

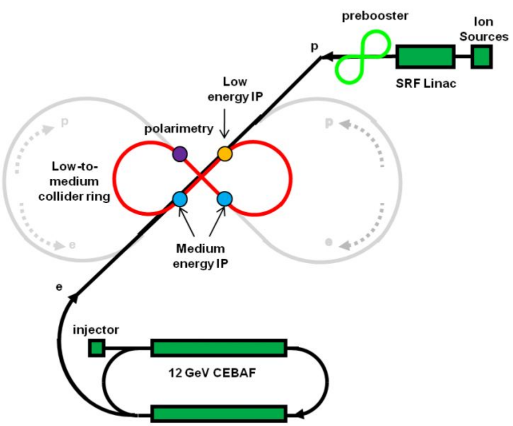
Test factorisation in DVCS and high P_T meson production


MATTER
FORCES
quarks
u
c
t
d
s
b
leptons
ν e
ν μ
ν τ
e
μ
τ
gauge bosons
g
γ
W
Z
Higgs
H

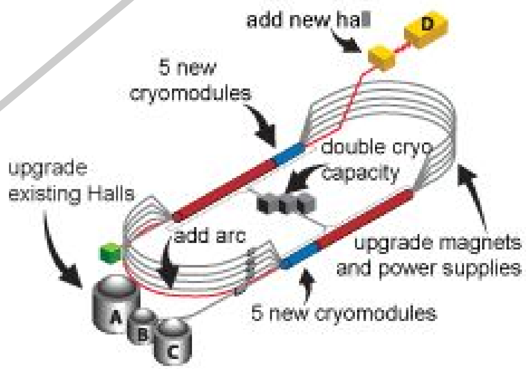




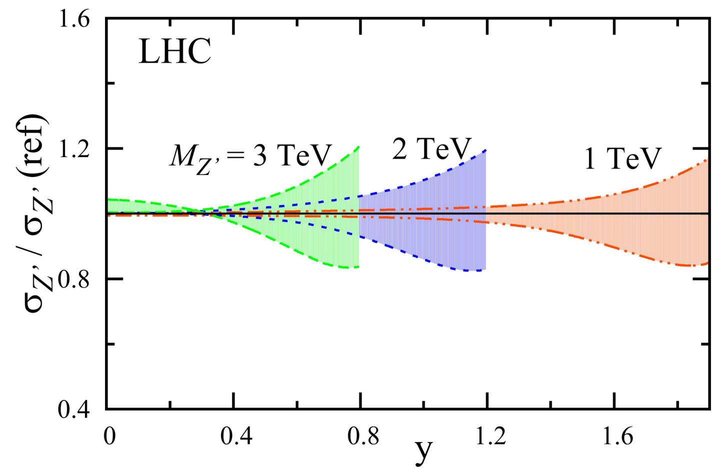




MEIC @ JLab



JLab 12 GeV



Brady *et al.*, JHEP 1206 (2012) 019