

# Linearly Polarized Gluons in $J/\psi$ and $\Upsilon$ Production

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# Outline

Gluon TMDs

Quarkonium Models

$J/\psi$  and  $\Upsilon$  production

Conclusion

# Parton Distribution Functions (PDFs)

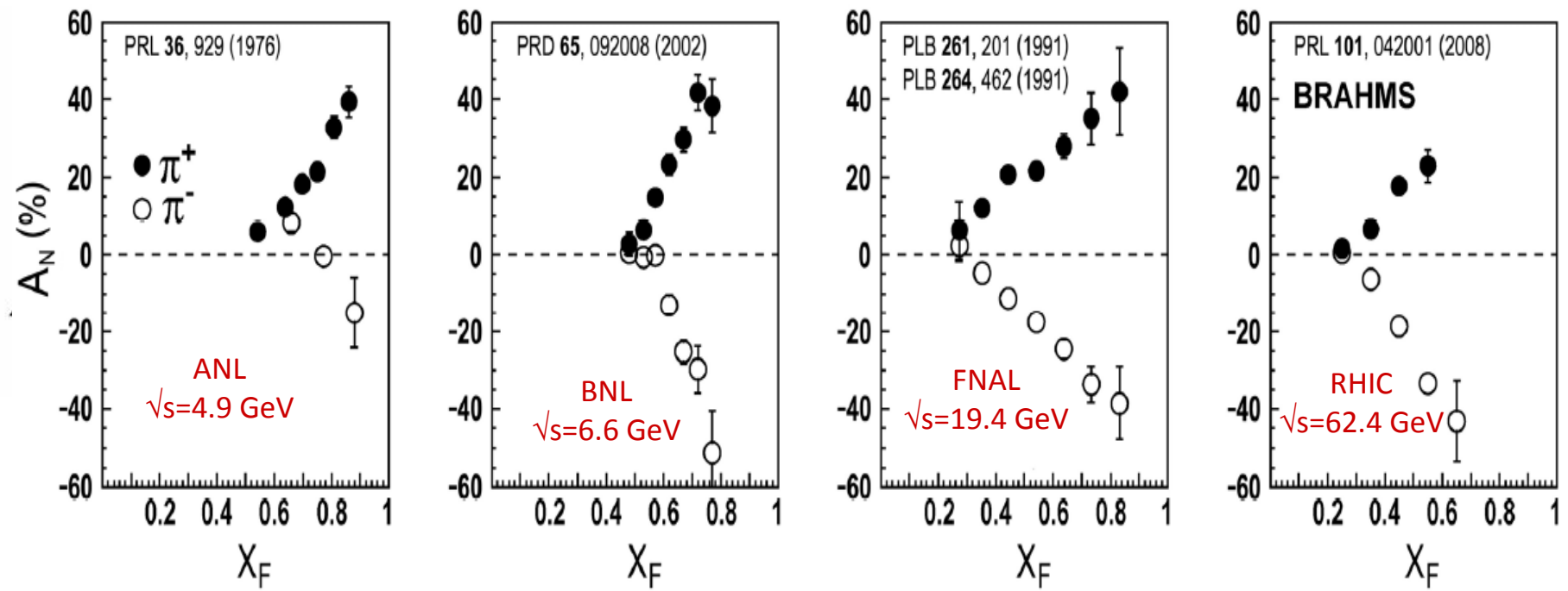


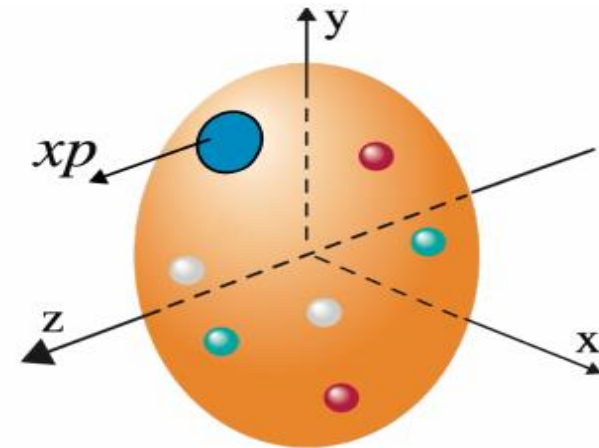
Figure by C. Aidala

- In 1990, D. Sivers proposed that SSA can be explained by considering correlation between transverse momentum of parton and polarization of hadron.

$$f(x) \rightarrow f(x, k_{\perp})$$

$$f_{a/p\uparrow}(x, \mathbf{k}_{\perp}) = f_{a/p}(x, \mathbf{k}_{\perp}) + \frac{1}{2} \Delta_{a/p}^N(x, \mathbf{k}_{\perp}) \hat{\mathbf{S}} \cdot (\hat{\mathbf{P}} \times \hat{\mathbf{k}}_{\perp})$$

# Transverse Momentum Dependent (TMD) Distributions

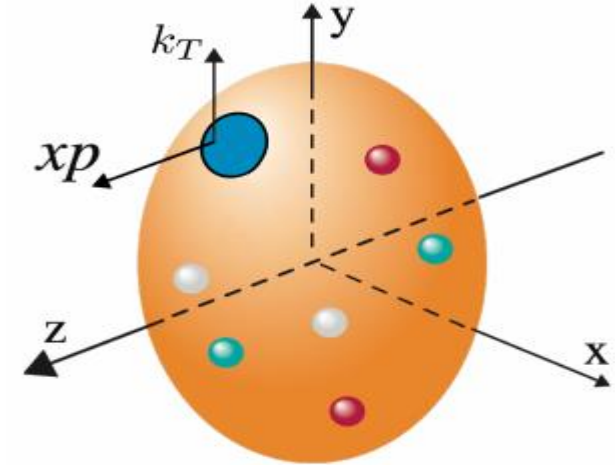


1D

Collinear pdf  $f(x)$

Universality

$$lp \rightarrow lX$$



3D

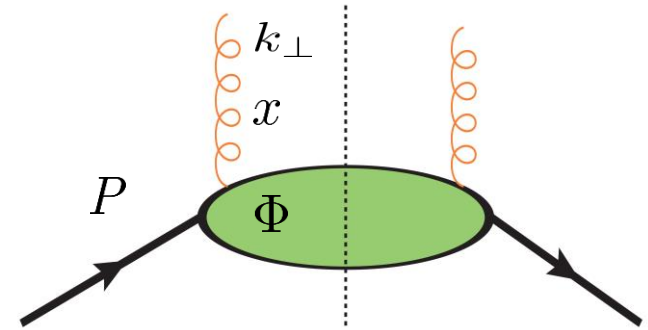
TMD pdf  $f(x, k_{\perp})$

Non-trivial Universality

$$lp \rightarrow lhX$$

$$pp \rightarrow hX$$

# Gluon Correlator



$$\Phi_g^{\mu\nu}(x, \mathbf{k}_\perp) = \frac{n_\rho n_\sigma}{(k \cdot n)^2} \int \frac{d(\lambda \cdot P) d^2 \lambda_T}{(2\pi)^3} e^{ik \cdot \lambda} \langle P | \text{Tr}[F^{\mu\rho}(\lambda) W(\lambda, 0) F^{\nu\sigma}(0)] | P \rangle_{LF}$$

P. J. Mulders and J. Rodrigues PRD 63 094021(2001)

Parameterization of gluon correlator at “Leading Twist” is


$$= -\frac{1}{2x} \left\{ g_T^{\mu\nu} f_1^g(x, \mathbf{k}_\perp^2) - \left( \frac{k_\perp^\mu k_\perp^\nu}{M_h^2} + g_T^{\mu\nu} \frac{\mathbf{k}_\perp^2}{2M_h^2} \right) h_1^{\perp g}(x, \mathbf{k}_\perp^2) \right\}$$

Unpolarized gluon TMD

Linearly polarized gluon TMD

# Leading Twist Gluon TMDs

	Gluons	Unpolarized	Circularly	Linearly	
Target					
Unpolarized		$f_1^g$		$h_1^{\perp g}$	Boer-Mulders
Longitudinal			$g_{1L}^g$	$h_{1L}^{\perp g}$	Kotzinian-Mulders
Transverse		$f_{1T}^{\perp g}$	$g_{1T}^g$	$h_{1T}^g, h_{1T}^{\perp g}$	



Sivers Pretzelosity

D. Boer et al arXiv 1507.05267

# Linearly Polarized Gluons

★ No experimental investigation has been carried out to extract the  $h_1^{\perp g}$  until now .

★ Theoretical upper bound  $\frac{\mathbf{k}_{\perp}^2}{2M_h^2} |h_1^{\perp g}(x, \mathbf{k}_{\perp}^2)| \leq f_1^g(x, \mathbf{k}_{\perp}^2)$   
D. Boer et al. PRL 106 132001 (2011)

## In proton-proton collision

★  $pp \rightarrow \gamma\gamma X$  at RHIC Qiu, Schlegel, Vogelsang, PRL 107, 062001 (2011)

★  $pp \rightarrow \Upsilon\gamma X$  at LHC Dunnen, Lansberg, Pisano, Schlegel, PRL 112, 212001 (2014)

★  $pp \rightarrow HX$  D. Boer et al. PRL 108, 032002 (2012)

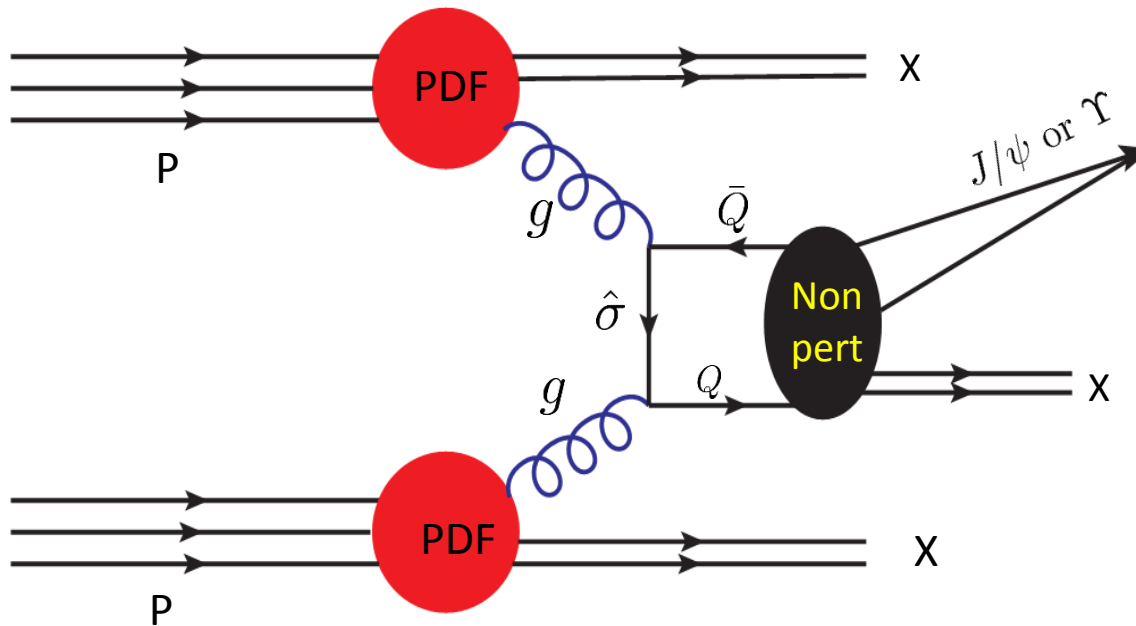
★  $pp \rightarrow \eta_{c,b}$  or  $\chi_{c,b} + X$  at LHCb and AFTER D. Boer, C. Pisano, PRD 86, 094007 (2012)

## In electron-proton scattering

★  $ep \rightarrow eQ\bar{Q}X$  or  $e + jet + jet + X$  C. Pisano et al. JHEP 10 (2013) 024

$$pp \rightarrow J/\psi \text{ or } \Upsilon + X$$

# Quarkonium Models



$$\sigma_{J/\psi, \Upsilon} = \hat{\sigma} \times \text{Nonperturbative factor}$$

Color Singlet Model (CSM)

Color Octet Model (COM)

Color Evaporation Model (CEM)

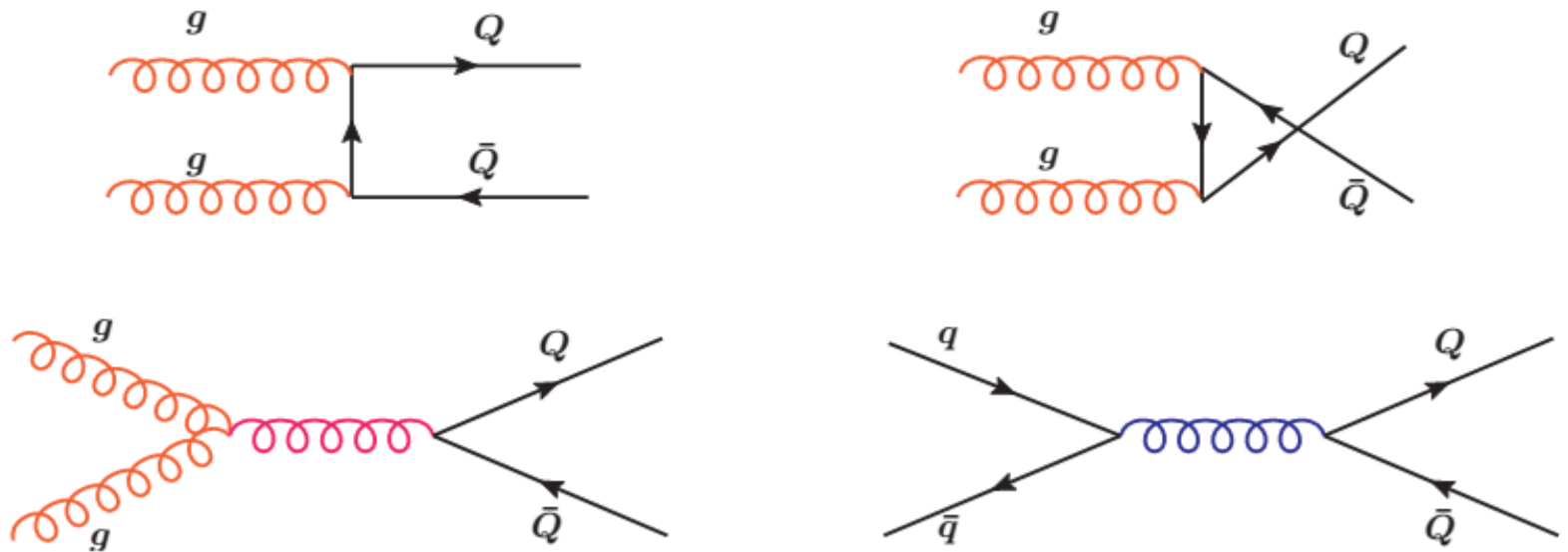


# $pp \rightarrow J/\psi \text{ or } \Upsilon + X$

The cross section for Quarkonium production in CEM is

$$\sigma = \frac{\rho}{9} \int_{2m_Q}^{2m_{Q\bar{q}}} dM \frac{d\hat{\sigma}_{Q\bar{Q}}}{dM}$$

Where  $m_Q = m_c(m_b)$  and  $m_{Q\bar{q}} = m_D(m_B)$  for charmonium (bottomonium)



# $pp \rightarrow J/\psi \text{ or } \Upsilon + X$

Using QCD factorization theorem

$$d\sigma = \frac{\rho}{9} \int dx_a dx_b d^2\mathbf{k}_{\perp a} d^2\mathbf{k}_{\perp b} \left\{ \Phi_g^{\mu\nu}(x_a, \mathbf{k}_{\perp a}) \Phi_{g\mu\nu}(x_b, \mathbf{k}_{\perp b}) d\hat{\sigma}^{gg \rightarrow Q\bar{Q}} + \left[ \Phi^q(x_a, \mathbf{k}_{\perp a}^2) \Phi^{\bar{q}}(x_b, \mathbf{k}_{\perp b}^2) + \Phi^{\bar{q}}(x_a, \mathbf{k}_{\perp a}^2) \Phi^q(x_b, \mathbf{k}_{\perp b}^2) \right] d\hat{\sigma}^{q\bar{q} \rightarrow Q\bar{Q}} \right\}$$

Convolution of TMDs is

$$C[wh_1^{\perp g} h_1^{\perp g}] = \int d^2\mathbf{k}_{\perp a} \int d^2\mathbf{k}_{\perp b} \delta^2(\mathbf{k}_{\perp a} + \mathbf{k}_{\perp b} - \mathbf{q}_T) wh_1^{\perp g}(x_a, \mathbf{k}_{\perp a}^2) h_1^{\perp g}(x_b, \mathbf{k}_{\perp b}^2)$$

$$\text{where } w = \frac{1}{2M^4} [(\mathbf{k}_{\perp a} \cdot \mathbf{k}_{\perp b})^2 - \frac{1}{2} \mathbf{k}_{\perp a}^2 \mathbf{k}_{\perp b}^2]$$

$$\int d^2\mathbf{q}_T (\mathbf{q}_T^2)^\alpha C[wh_1^{\perp g} h_1^{\perp g}] = 0 \quad \text{Model Independent property}$$

$\alpha = 0, 1$

$\alpha = 0 \implies$  Linearly polarized gluons do not affect the  $\mathbf{q}_T$ -integrated cross section

$\alpha = 1 \implies$  At least two nodes in  $\mathbf{q}_T$

# TMDs Parameterization

TMDs exhibit Gaussian distribution

$$f_1^g(x, \mathbf{k}_\perp^2, Q^2) = f_1^g(x, Q^2) \frac{1}{\pi \langle k_\perp^2 \rangle} e^{-\mathbf{k}_\perp^2 / \langle k_\perp^2 \rangle}$$

$$h_1^{\perp g}(x, \mathbf{k}_\perp^2) = \frac{M_h^2 f_1^g(x, Q^2)}{\pi \langle k_\perp^2 \rangle^2} \frac{2(1-r)}{r} e^{1 - \mathbf{k}_\perp^2 \frac{1}{r \langle k_\perp^2 \rangle}}$$

$$0 < r < 1$$

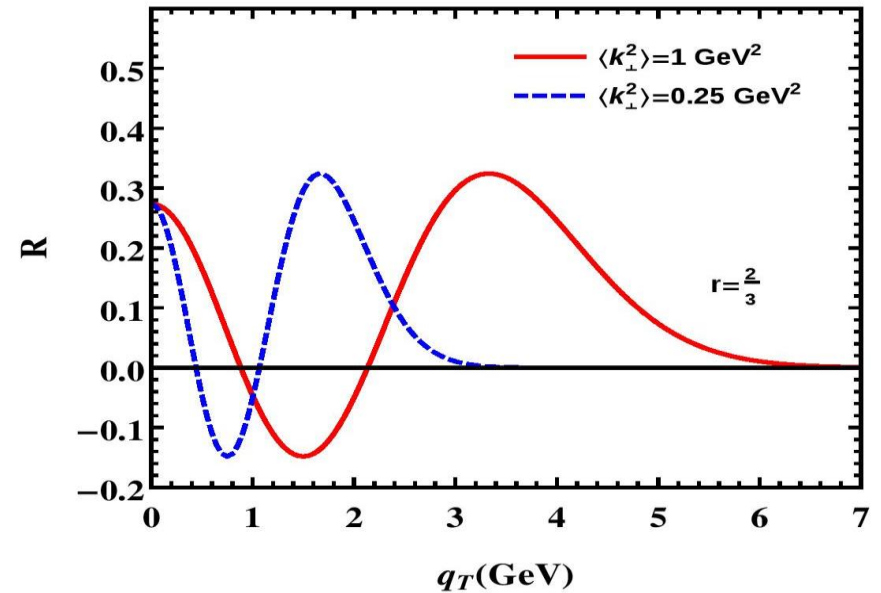
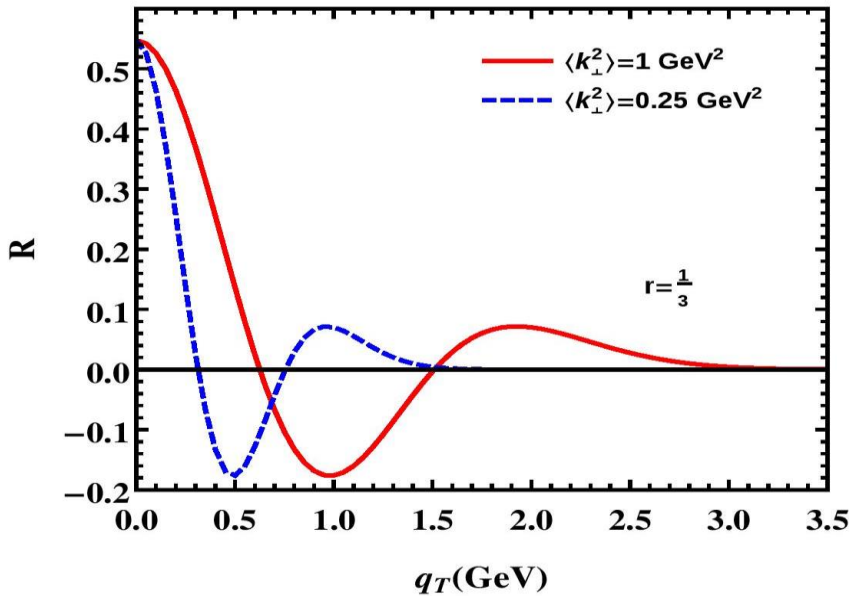
Evolution in collinear PDFs

No Evolution in  $k_\perp$

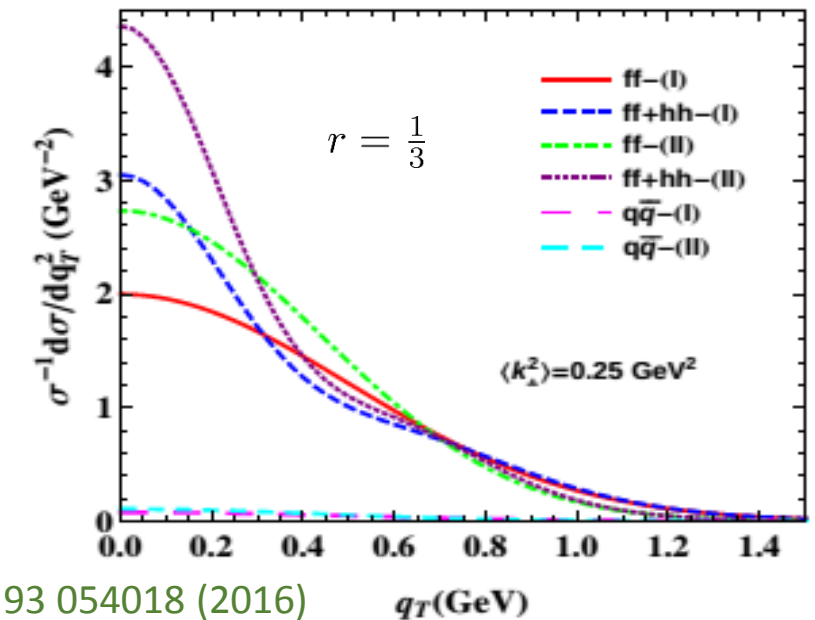
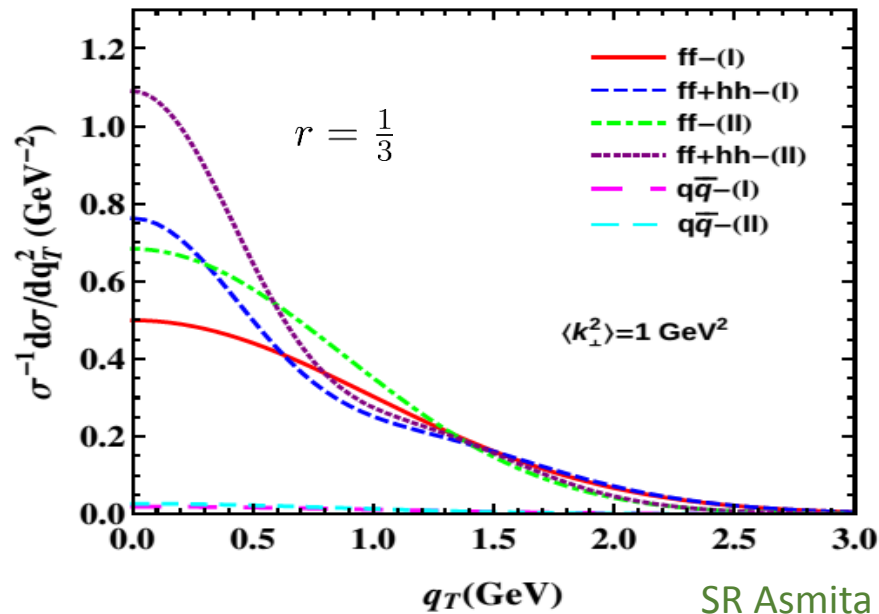
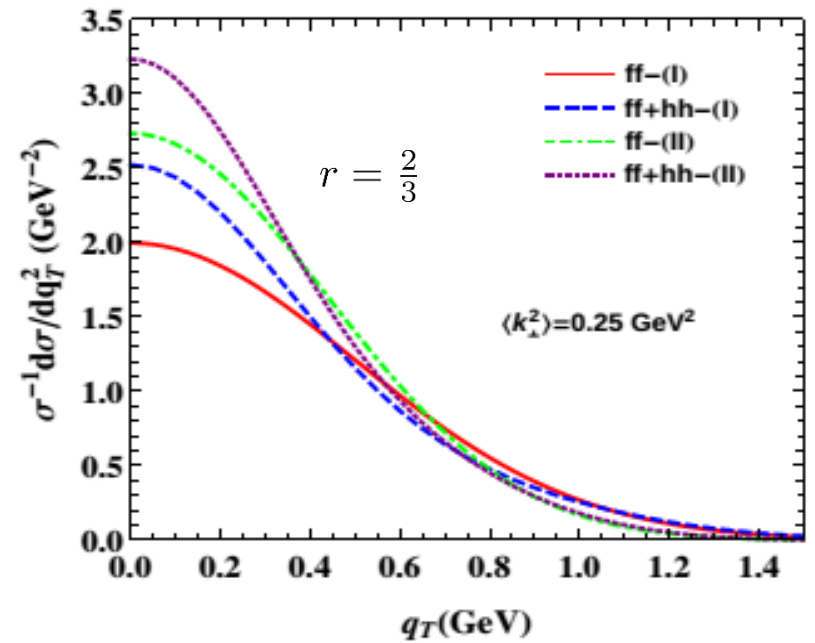
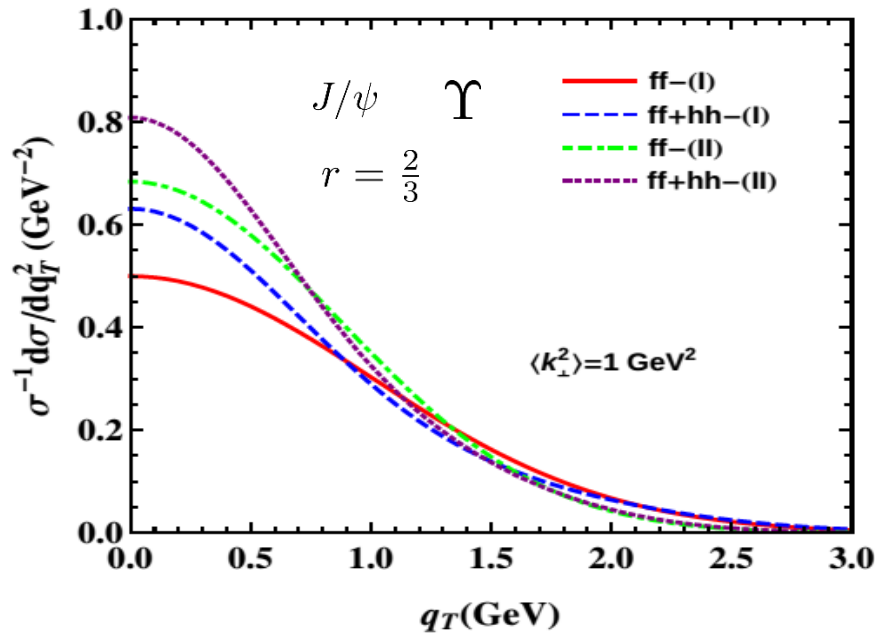
DGLAP Evolution

# $pp \rightarrow J/\psi \text{ or } \Upsilon + X$

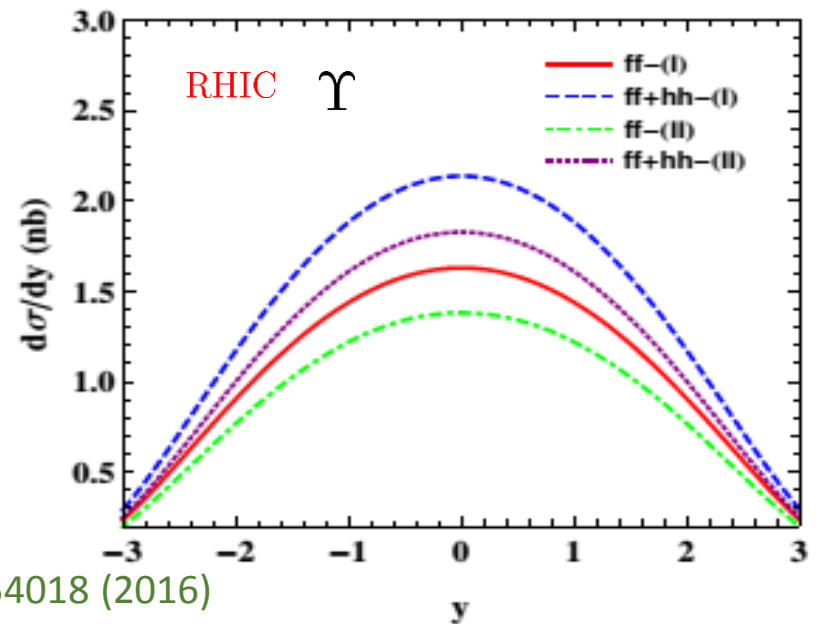
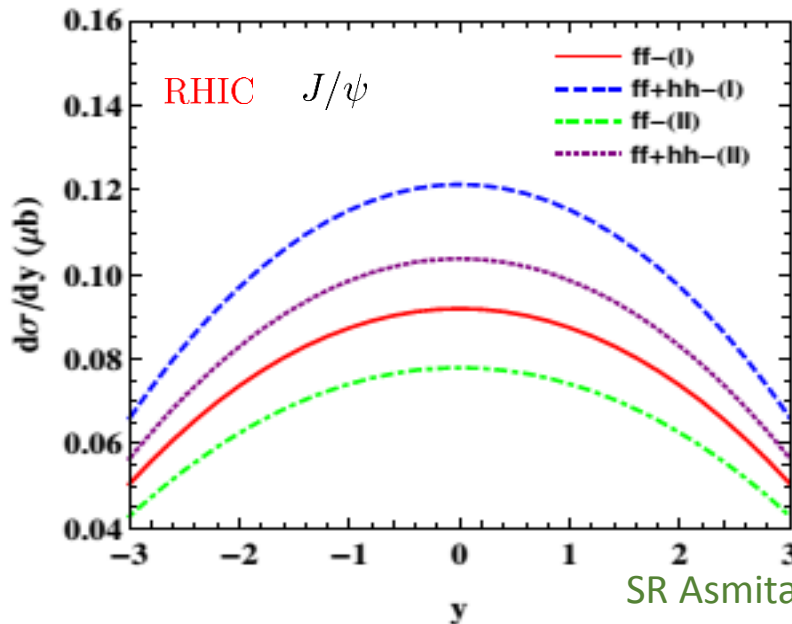
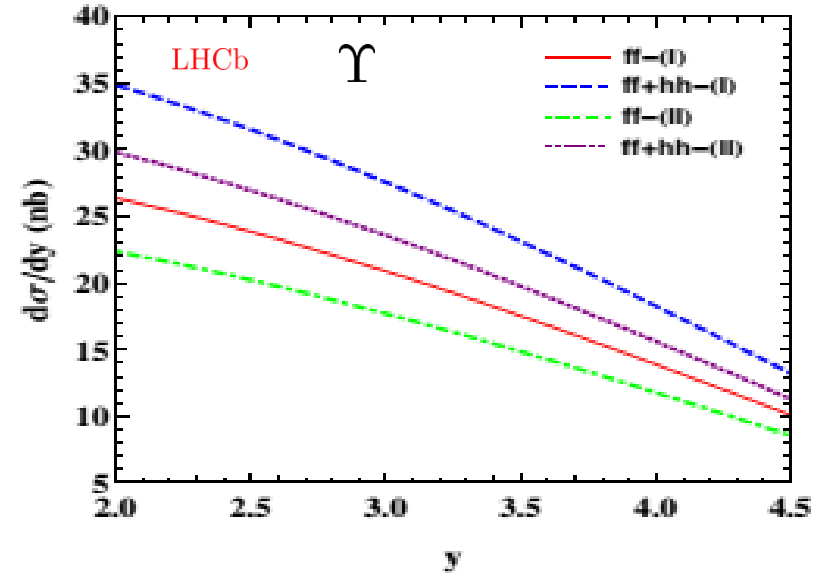
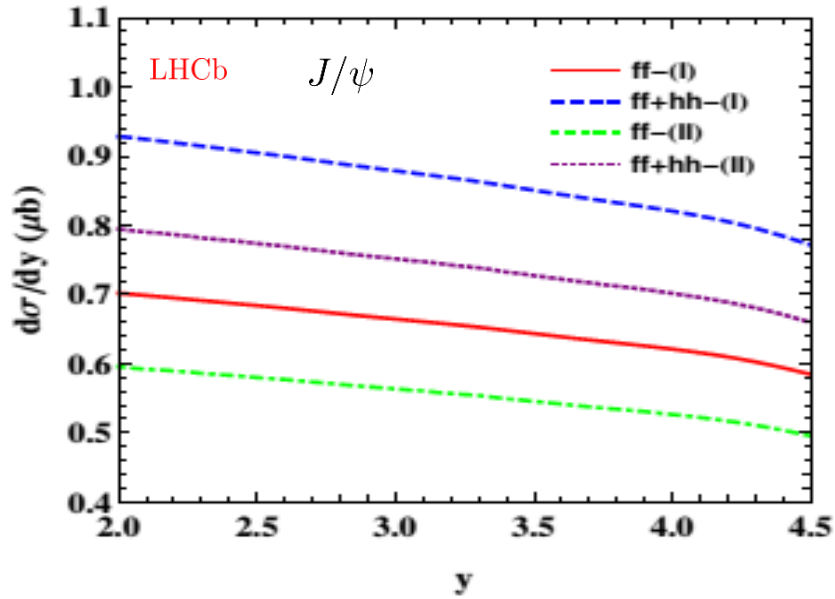
Define  $R(r, \mathbf{q}_T) = \frac{C[wh_1^{\perp g} h_1^{\perp g}]}{C[f_1 f_1]}$



# $Q_T$ Spectrum in DGLAP Evolution



# Rapidity Spectrum in DGLAP Evolution



# TMD Evolution

- ▶ In Collins-Soper-Sterman (CSS) Evolution formalism, TMD pdfs depends on  $\mu$  and  $\zeta$
- ▶ Collins-Soper equation and RGE for TMD

$$\frac{d}{d \ln \sqrt{\zeta}} \ln f(x, b_{\perp}, \mu, \zeta) = K(b_{\perp}, \mu)$$

$$\frac{d}{d \ln \mu} \ln f(x, b_{\perp}, \mu, \zeta) = \gamma_f(b_{\perp}, \zeta) \quad \text{No } x \text{ integral as in DGLAP}$$

$$\frac{d}{d \ln \mu} K(b_{\perp}, \mu) = -\gamma_k(\mu)$$

J. Collins, Foundations of Perturbative QCD, 2011  
S. M. Aybat et al. PRD 85 034043 (2012)

# TMD Evolution

## CSS TMD Evolution

J. Collins, Foundations of Perturbative QCD, 2011

$$f(x, b_{\perp}, Q_f, \zeta) = f(x, b_{\perp}, Q_i, \zeta_0) R_{pert}(Q_f, Q_i, b_*) R_{NP}(Q_f, Q_i, b_{\perp})$$

$$R_{pert}(Q_f, Q_i, b_*) = \exp \left\{ - \int_{Q_i}^{Q_f} \frac{d\mu}{\mu} \left( A \log \left( \frac{Q_f^2}{\mu^2} \right) + B \right) \right\}$$

$$R_{NP} = \exp \left\{ - \left[ \frac{g_2}{2} \log \frac{Q_f}{2Q_0} + \frac{g_1}{2} \left( 1 + 2g_3 \log \frac{10xx_0}{x_0 + x} \right) \right] b_{\perp}^2 \right\}$$

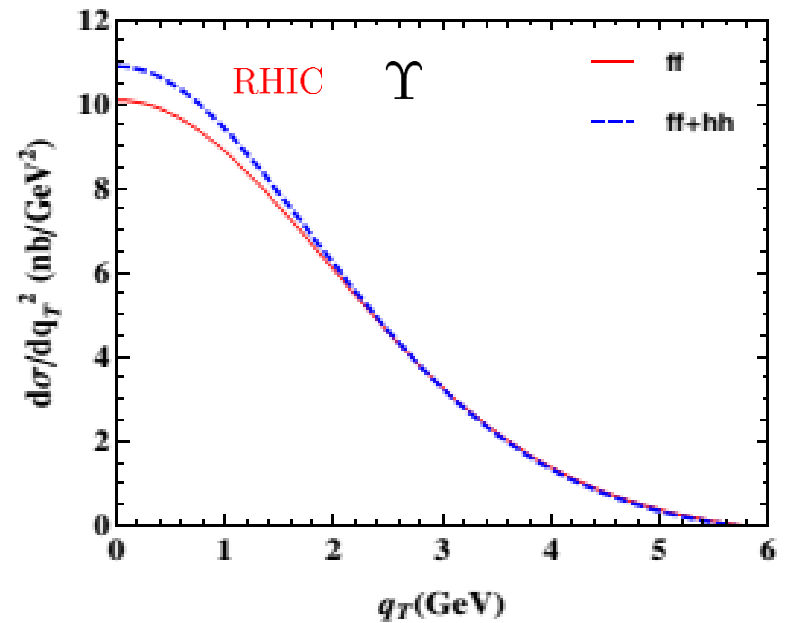
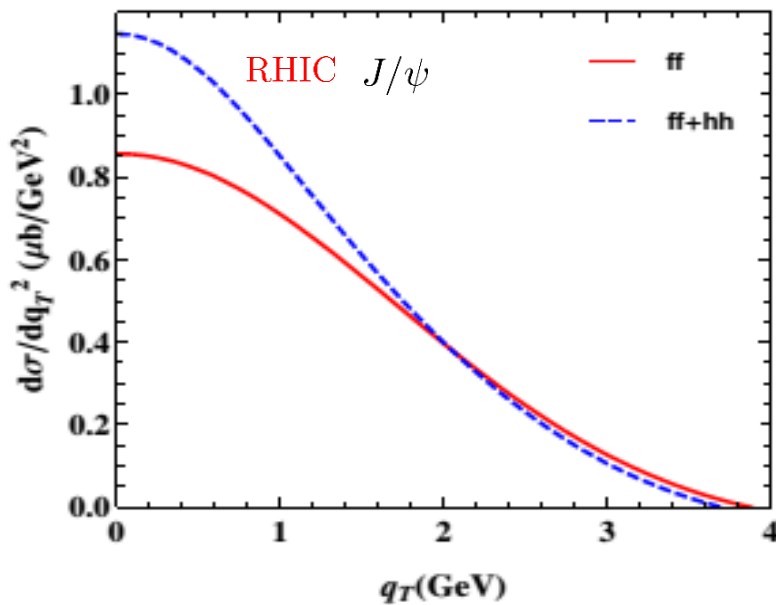
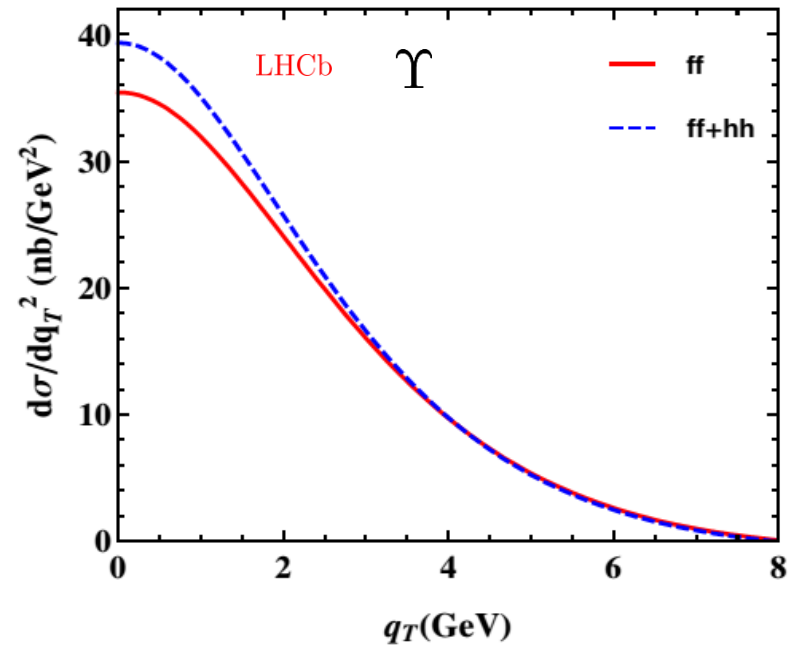
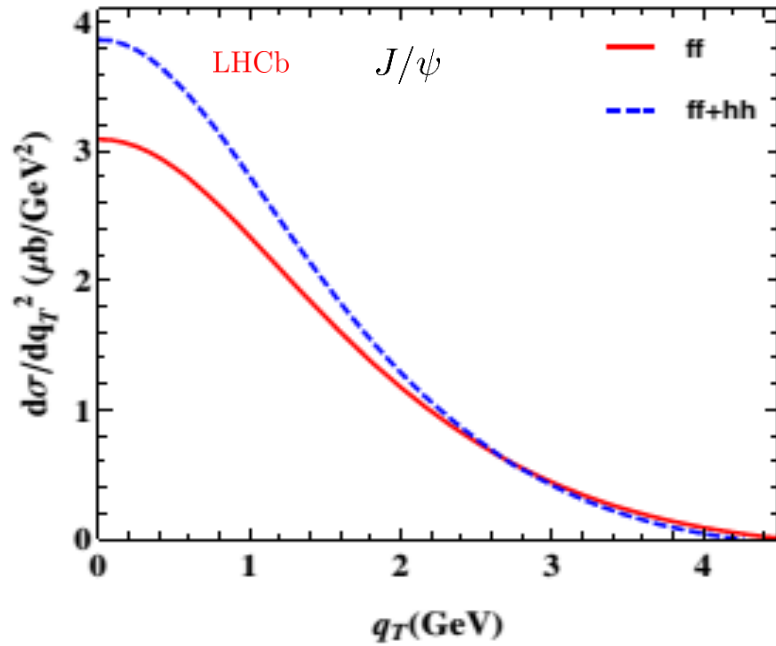
S. M. Ayabt et al. PRD 83 114042 (2011)

where  $Q_i = \frac{2e^{-0.577}}{b_*} = \sqrt{\zeta_0}$ ,  $Q_f = Q$  and  $\zeta = Q_f^2$

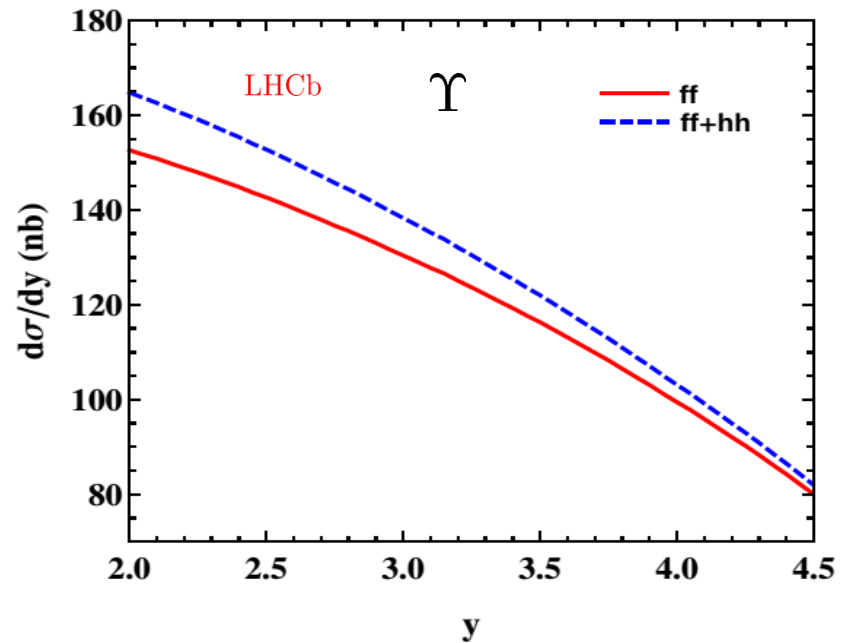
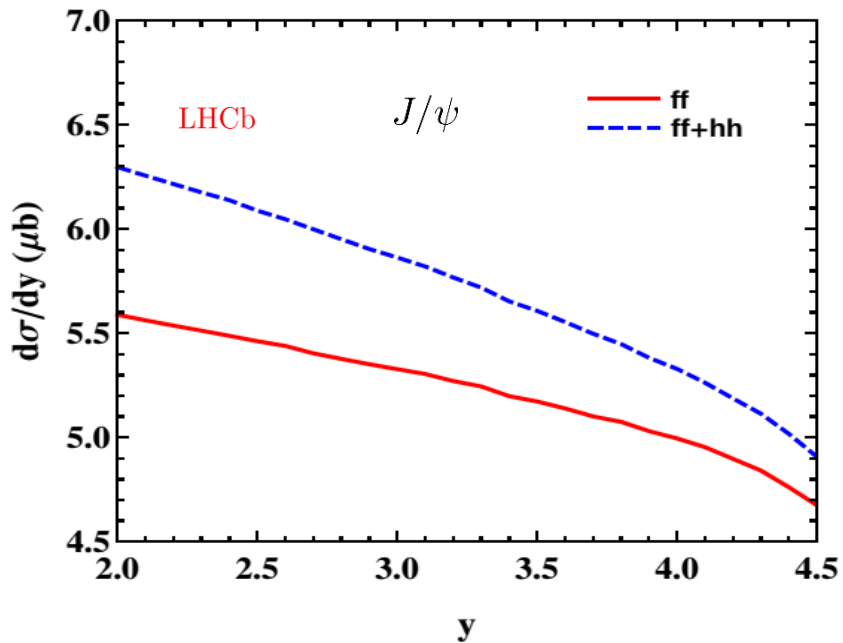
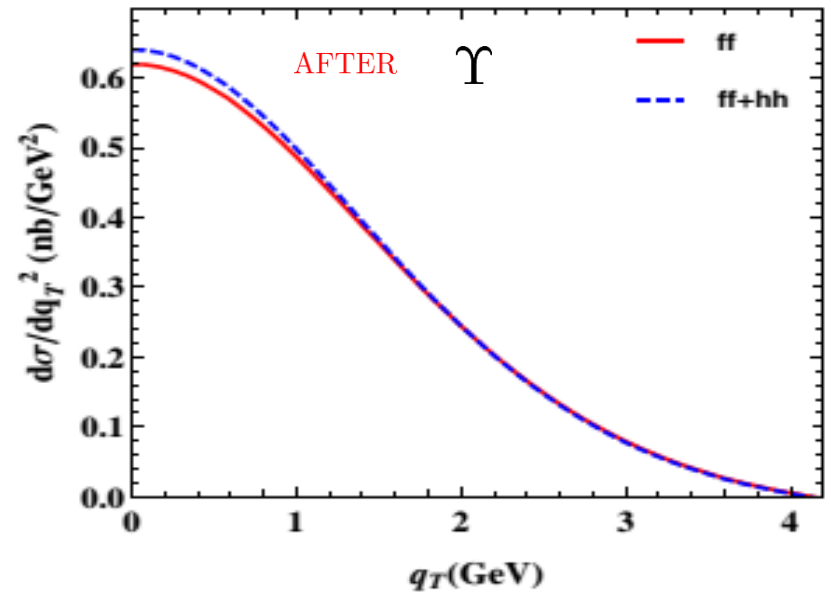
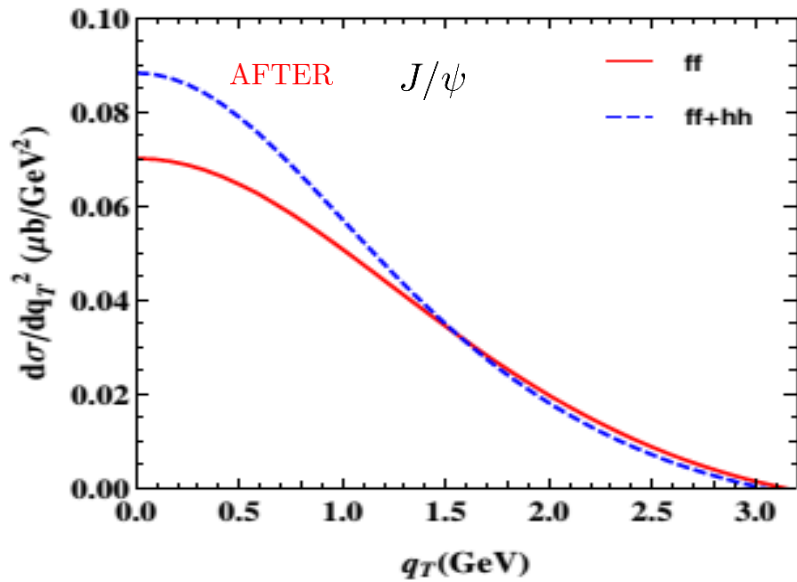
$$b_* = \frac{b_{\perp}}{\sqrt{1 + \left(\frac{b_{\perp}}{b_{\max}}\right)^2}}, \quad A = \sum_{n=1}^{\infty} \left( \frac{\alpha_s(\mu)}{\pi} \right)^n A_n, \quad B = \sum_{n=1}^{\infty} \left( \frac{\alpha_s(\mu)}{\pi} \right)^n B_n$$



# $q_T$ Spectrum in TMD Evolution



# Results in TMD Evolution



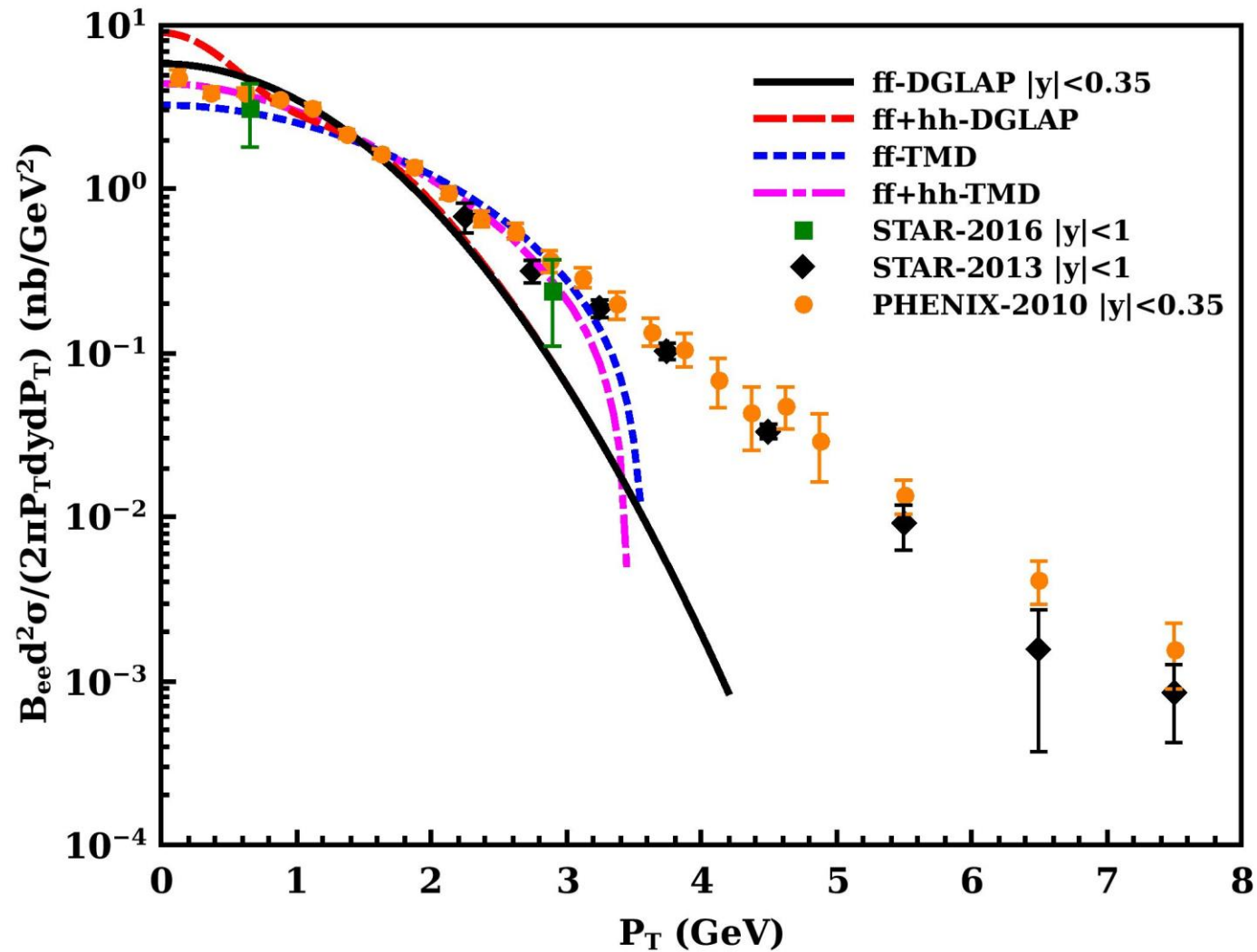
# Conclusion

- ★ The  $q_T$  and  $y$  distributions of quarkonium can be modulated by the presence of linearly polarized gluons in unpolarized proton proton collision.
- ★ Hence, the production of quarkonium is a promising process to probe the  $h_1^\perp g$
- ★ No angular analysis is needed to probe linearly polarized gluons

# Comments & Suggestions

Thank You

# Matching with Experimental data



# Differential Cross Section in DGLAP approach

$$\begin{aligned} \frac{d^2\sigma^{ff}}{dydq_T^2} &= \frac{\beta^2\rho}{36s\pi^2} \int_{4m_Q^2}^{4m_{Q\bar{q}}^2} dM^2 \int d\phi_{q_T} \int dk_{\perp a} k_{\perp a} \int d\phi_{k_{\perp a}} e^{-\Delta\beta} \\ &\times \left\{ f_1^g(x_a) f_1^g(x_b) \hat{\sigma}^{gg \rightarrow Q\bar{Q}}(M^2) + \frac{1}{2} \sum_q \left[ f_1^q(x_a) f_1^{\bar{q}}(x_b) + f_1^{\bar{q}}(x_a) f_1^q(x_b) \right] \right. \\ &\quad \left. \times \hat{\sigma}^{q\bar{q} \rightarrow Q\bar{Q}}(M^2) \right\} \end{aligned}$$

$$\begin{aligned} \frac{d^2\sigma^{hh}}{dydq_T^2} &= \frac{\beta^4\rho(1-r)^2}{18sr^2\pi^2} \int_{4m_Q^2}^{4m_{Q\bar{q}}^2} dM^2 \int d\phi_{q_T} \int dk_{\perp a} k_{\perp a} \int d\phi_{k_{\perp a}} \\ &\times \left[ \frac{1}{2} k_{\perp a}^4 - \frac{1}{2} k_{\perp a}^2 q_T^2 - q_T k_{\perp a}^3 \cos(\phi_{k_{\perp a}} - \phi_{q_T}) + q_T^2 k_{\perp a}^2 \cos^2(\phi_{k_{\perp a}} - \phi_{q_T}) \right] \\ &\times e^{[2 - \frac{\beta}{r}\Delta]} f_1^g(x_a) f_1^g(x_b) \hat{\sigma}^{gg \rightarrow Q\bar{Q}}(M^2) \end{aligned}$$

where  $\Delta = 2k_{\perp a}^2 + q_T^2 - 2q_T k_{\perp a} \cos(\phi_{k_{\perp a}} - \phi_{q_T})$  and  $\beta = \frac{1}{\langle k_{\perp}^2 \rangle}$

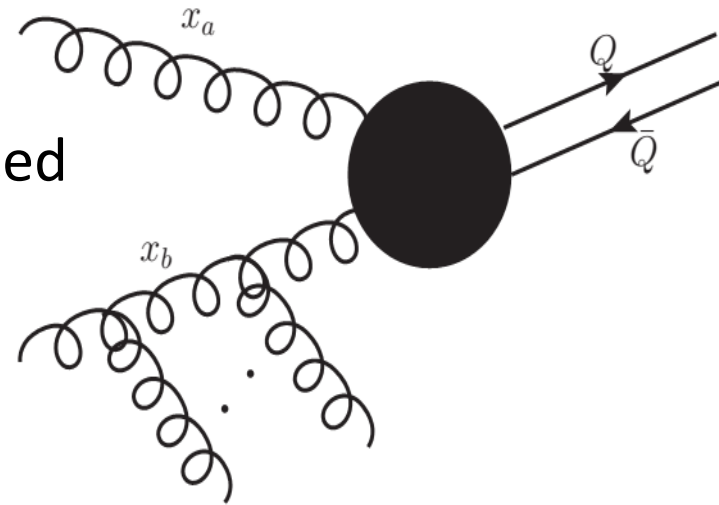
# Differential Cross Section in TMD Evolution approach

$$\frac{d^2\sigma^{ff}}{dydq_T^2} = \frac{\rho}{36s} \int_{4m_Q^2}^{4m_{Q\bar{q}}^2} dM^2 \int_0^\infty b_\perp db_\perp J_0(q_T b_\perp) f_1^g(x_a, c/b_*) f_1^g(x_b, c/b_*) \hat{\sigma}^{gg \rightarrow Q\bar{Q}}(M^2) \exp\left\{-2 \int_{c/b_*}^Q \frac{d\mu}{\mu} \left(A \log\left(\frac{Q^2}{\mu^2}\right) + B\right)\right\} \exp\left\{-\left[0.184 \log \frac{Q}{2Q_0} + 0.332\right] b_\perp^2\right\}$$

$$\frac{d^2\sigma^{hh}}{dydq_T^2} = \frac{\rho C_A^2}{36s\pi^2} \int_{4m_Q^2}^{4m_{Q\bar{q}}^2} dM^2 \int_0^\infty b_\perp db_\perp J_0(q_T b_\perp) \alpha_s^2(c/b_*) \hat{\sigma}^{gg \rightarrow Q\bar{Q}}(M^2) \int_{x_a}^1 \frac{dx_1}{x_1} \left(\frac{x_1}{x_a} - 1\right) f_1^g(x_1, c/b_*) \int_{x_b}^1 \frac{dx_2}{x_2} \left(\frac{x_2}{x_b} - 1\right) f_1^g(x_2, c/b_*) \exp\left\{-2 \int_{c/b_*}^Q \frac{d\mu}{\mu} \left(A \log\left(\frac{Q^2}{\mu^2}\right) + B\right)\right\} \exp\left\{-\left[0.184 \log \frac{Q}{2Q_0} + 0.332\right] b_\perp^2\right\}$$

# Resummation of Sudakov logarithms

The region of low  $q_T$  is strongly influenced by initial state gluon radiation showers



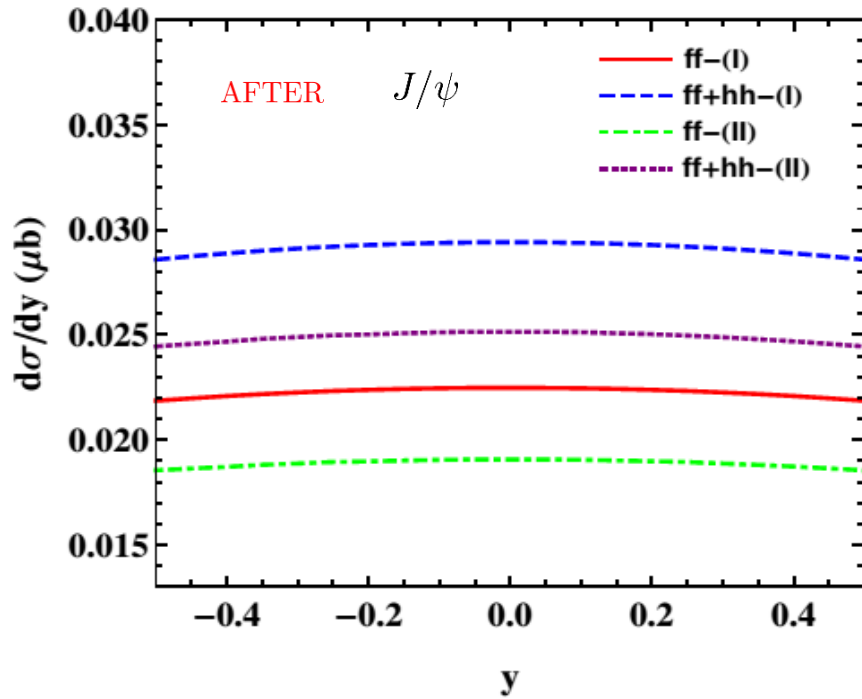
These additional gluon radiation from initial state partons leads to logarithmic corrections for each gluon radiation of the form

$$\alpha_s \log\left(\frac{Q^2}{q_T^2}\right)$$

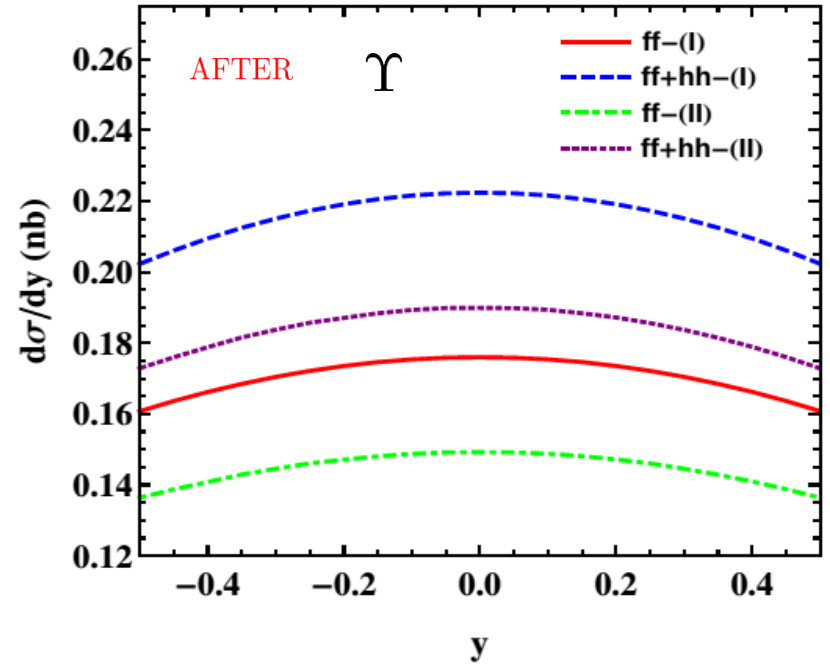
Collins Soper Sterman (CSS) Evolution formalism has been used to resum the large logarithmic terms to all order in  $\alpha_s$

$$f_1^g.$$





$$r = \frac{1}{3}$$



$$\langle k_{\perp}^2 \rangle = 1 \text{ GeV}^2$$

