JLab
Theory Center Seminar

20 April, 2016

Two-photon exchange corrections in elastic lepton-proton interaction

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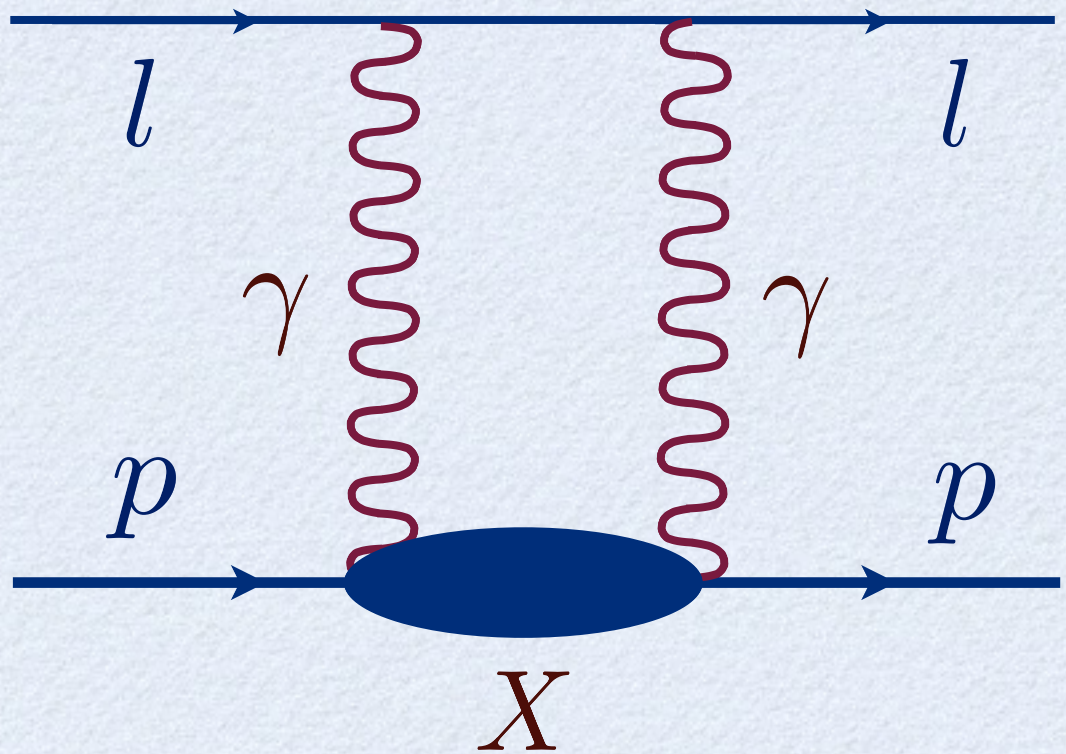
Outline

forward virtual Compton scattering

+

lepton

$l = e,\ \mu$



hadronic state

$X = p,\ \pi N$

Kinematics

forward $Q^2=0$

calculable

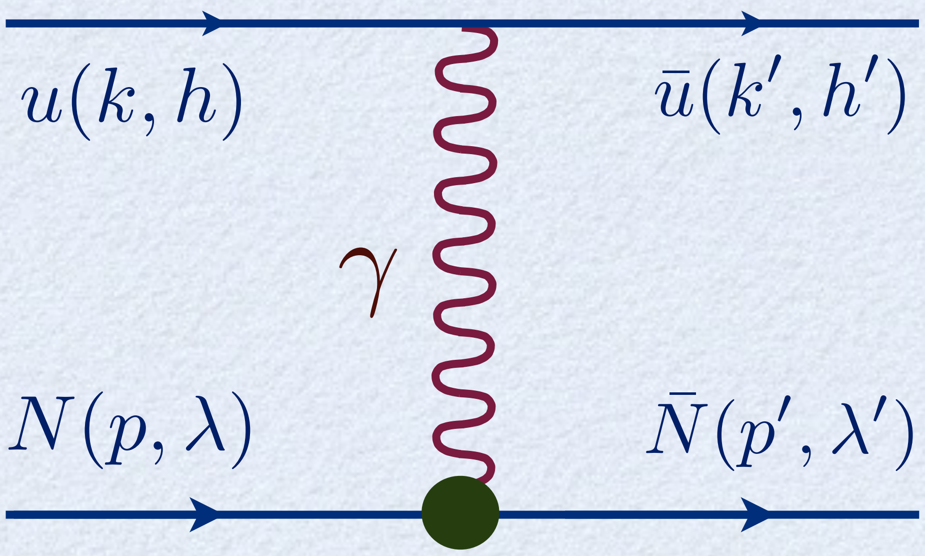
small Q^2

proton
structure functions

larger Q^2

disp. rel.

1ɣ approximation



photon-proton vertex

$$\Gamma^\mu(Q^2) = \gamma^\mu F_D(Q^2) + \frac{i\sigma^{\mu\nu} q_\nu}{2M} F_P(Q^2)$$

Dirac and Pauli form factors

crossing symmetric variable

$$\nu = \frac{s-u}{4} \qquad s = (p+k)^2 \qquad u = (k-p')^2$$

momentum transfer

$$Q^2 = -(k-k')^2$$

l-p amplitude

$$T = \frac{e^2}{Q^2} \left(\bar{u}\left(k',h'\right)\gamma_\mu u\left(k,h\right)\right) \cdot \left(\bar{N}\left(p',\lambda'\right)\Gamma^\mu(Q^2) N\left(p,\lambda\right)\right)$$

Form factors in 1ɣ approximation

Sachs electric and magnetic form factors

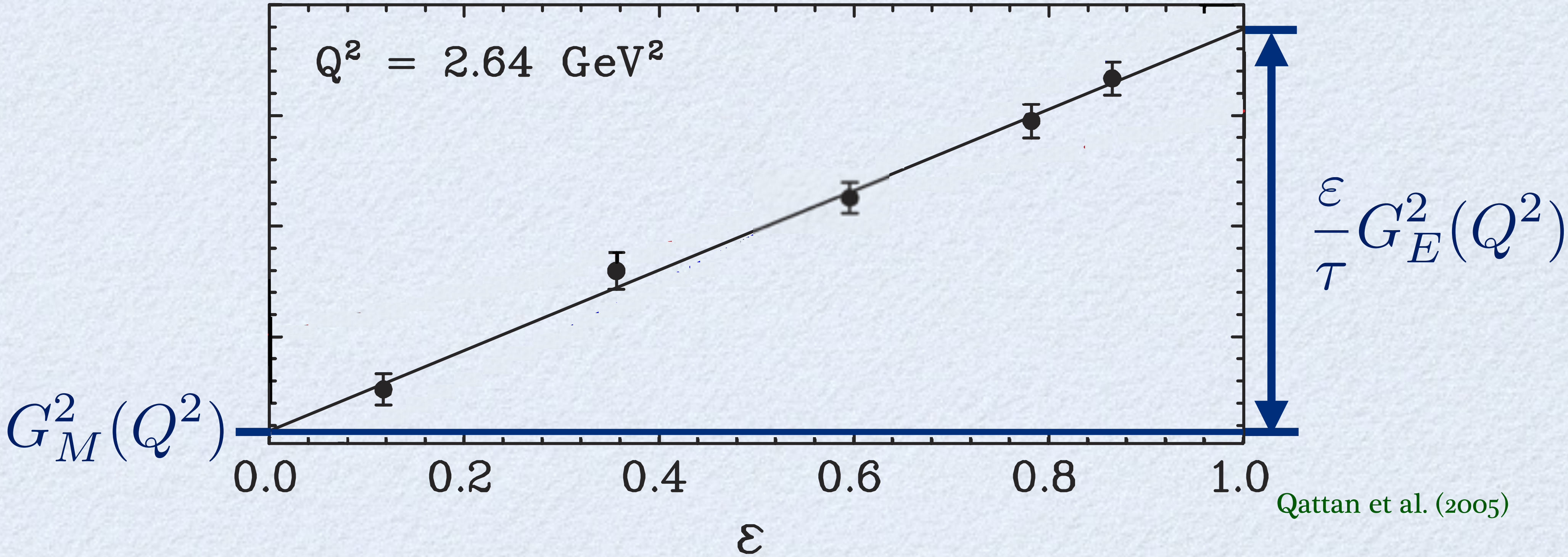
$$G_E = F_D - \tau F_P, \qquad G_M = F_D + F_P$$

kinematic variables

$$\tau = \frac{Q^2}{4M^2}, \qquad \epsilon = \frac{\nu^2 - \tau(1+\tau)}{\nu^2 + \tau(1+\tau)}$$

Rosenbluth separation

$$\frac{d\sigma^{\text{unpol}}}{d\Omega} \sim \sigma_R = \tau \left(G_M^2(Q^2) + \frac{\varepsilon}{\tau} G_E^2(Q^2) \right)$$



Rosenbluth slope is sensitive to corrections beyond 1ɣ

Form factors in 1ɣ approximation

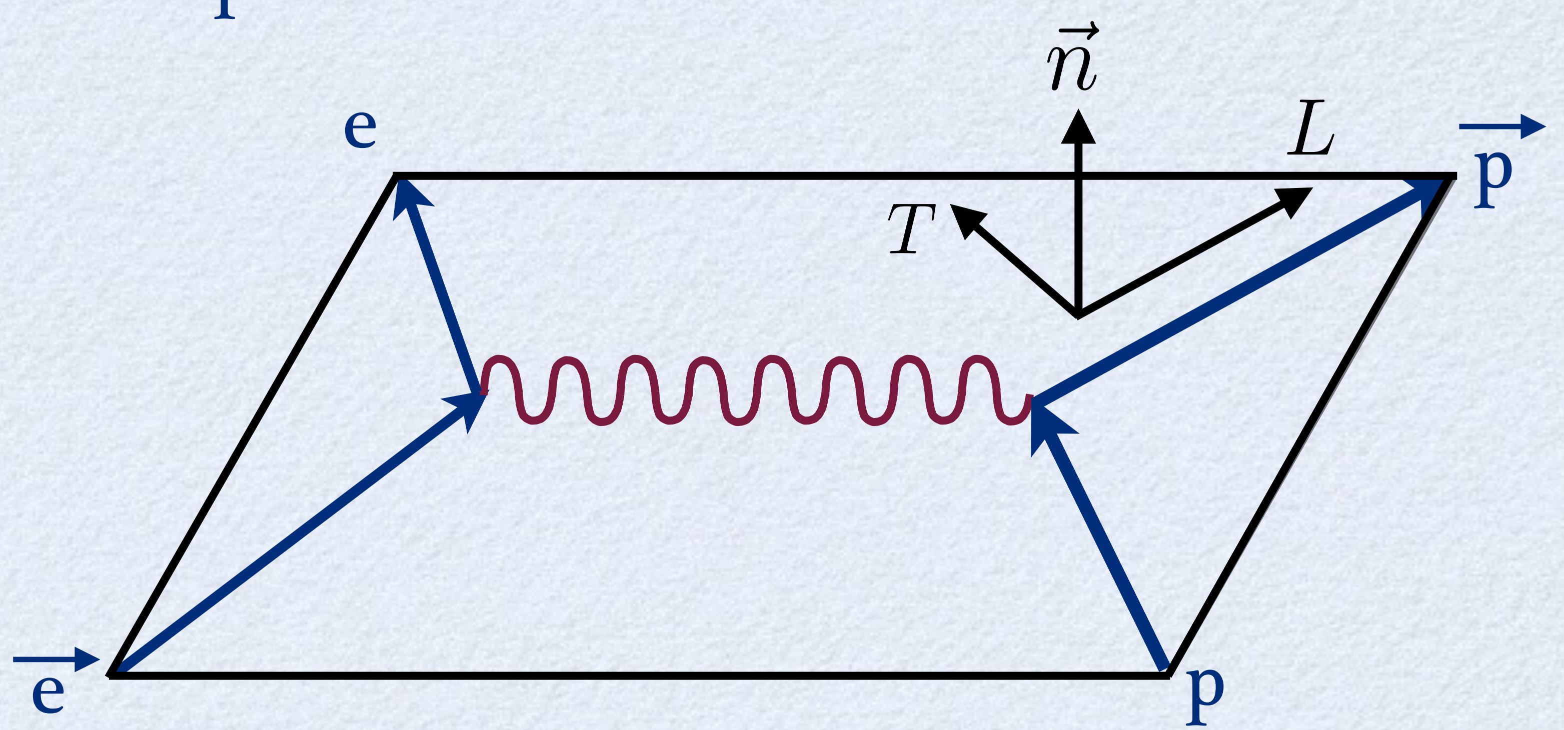
Sachs electric and magnetic form factors

$$G_E = F_D - \tau F_P, \qquad G_M = F_D + F_P$$

Polarization transfer

$\vec{e}$ + p -> e + $\vec{p}$

realized in 2000



$$P_T \sim G_E(Q^2) G_M(Q^2)$$

$$P_L \sim G_M^2(Q^2)$$

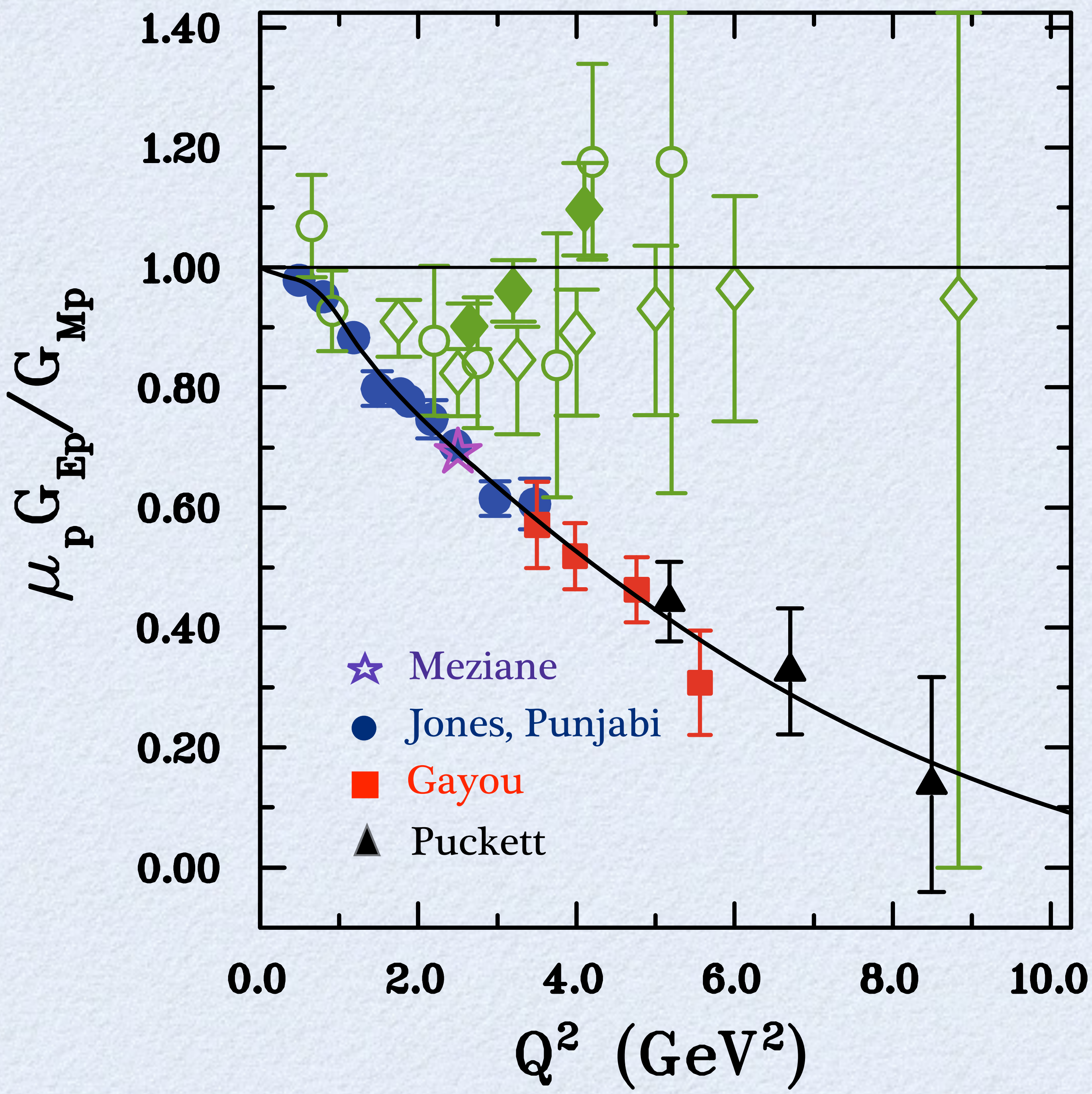
$$\frac{P_T}{P_L} \sim \frac{G_E(Q^2)}{G_M(Q^2)}$$

Proton form factors puzzle

Polarization transfer
JLab (Hall A, C)

vs.

Rosenbluth separation
SLAC, JLab (Hall A, C)



V. Punjabi et al. (2015)

Proton form factors puzzle

Polarization transfer
JLab (Hall A, C)

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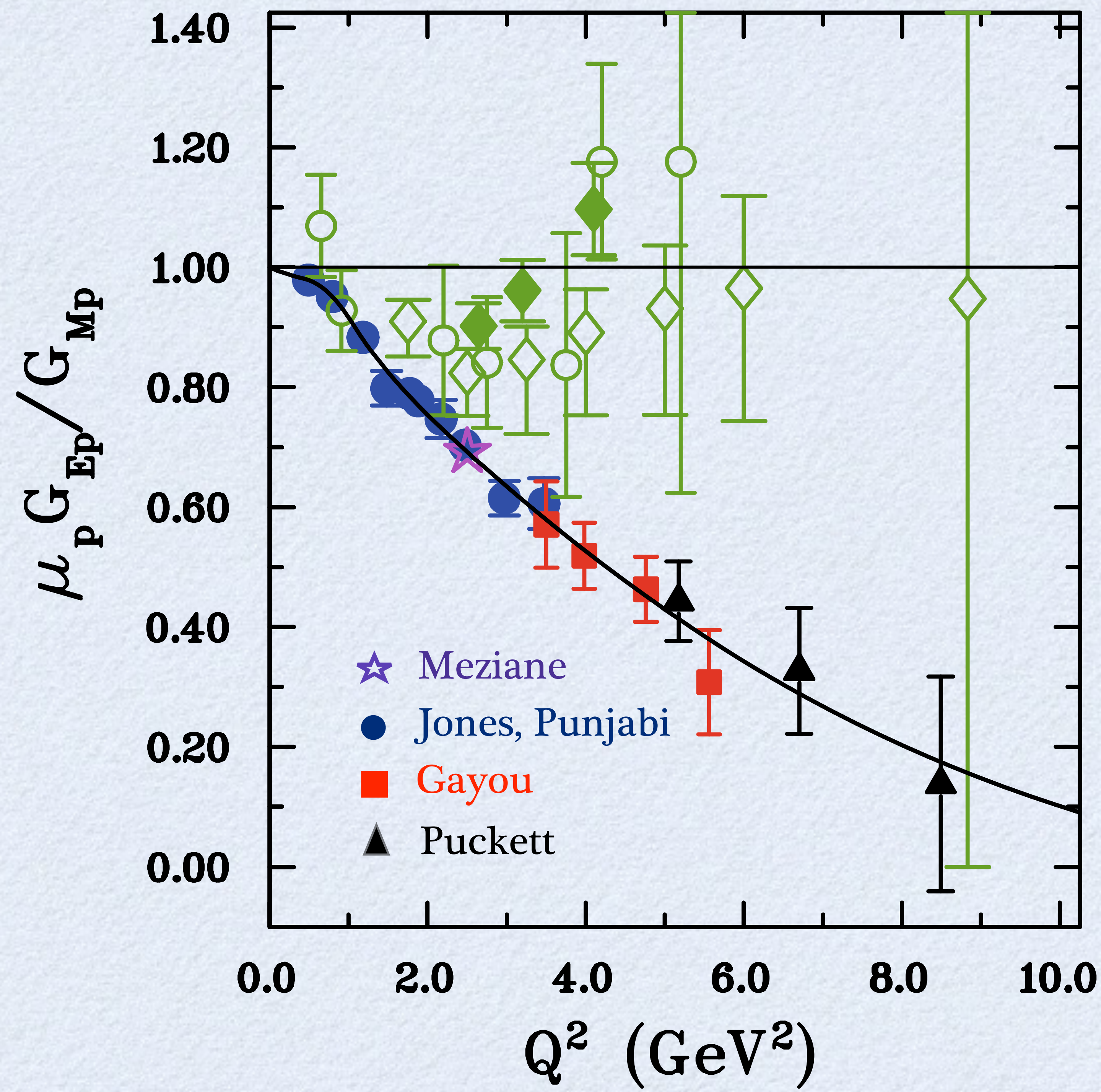
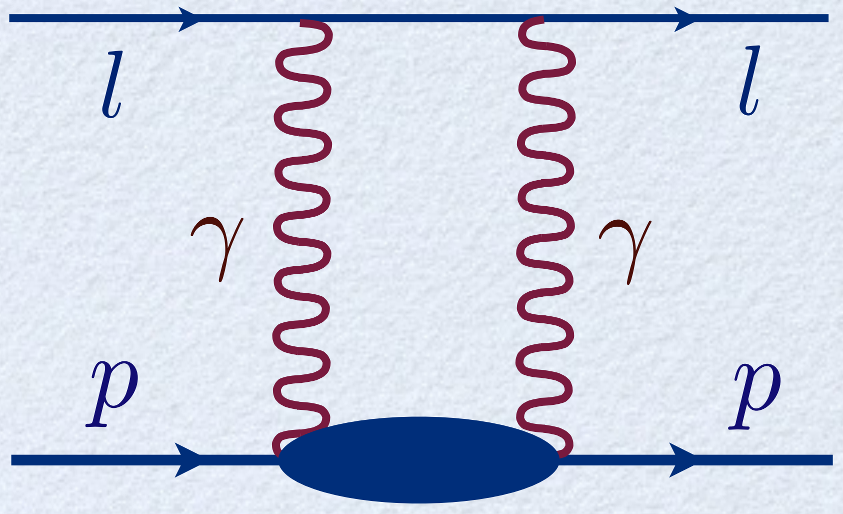


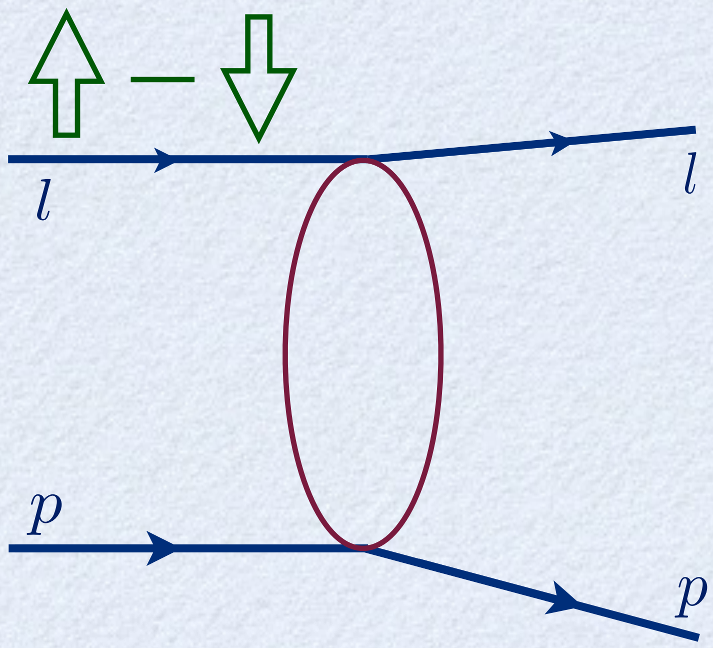


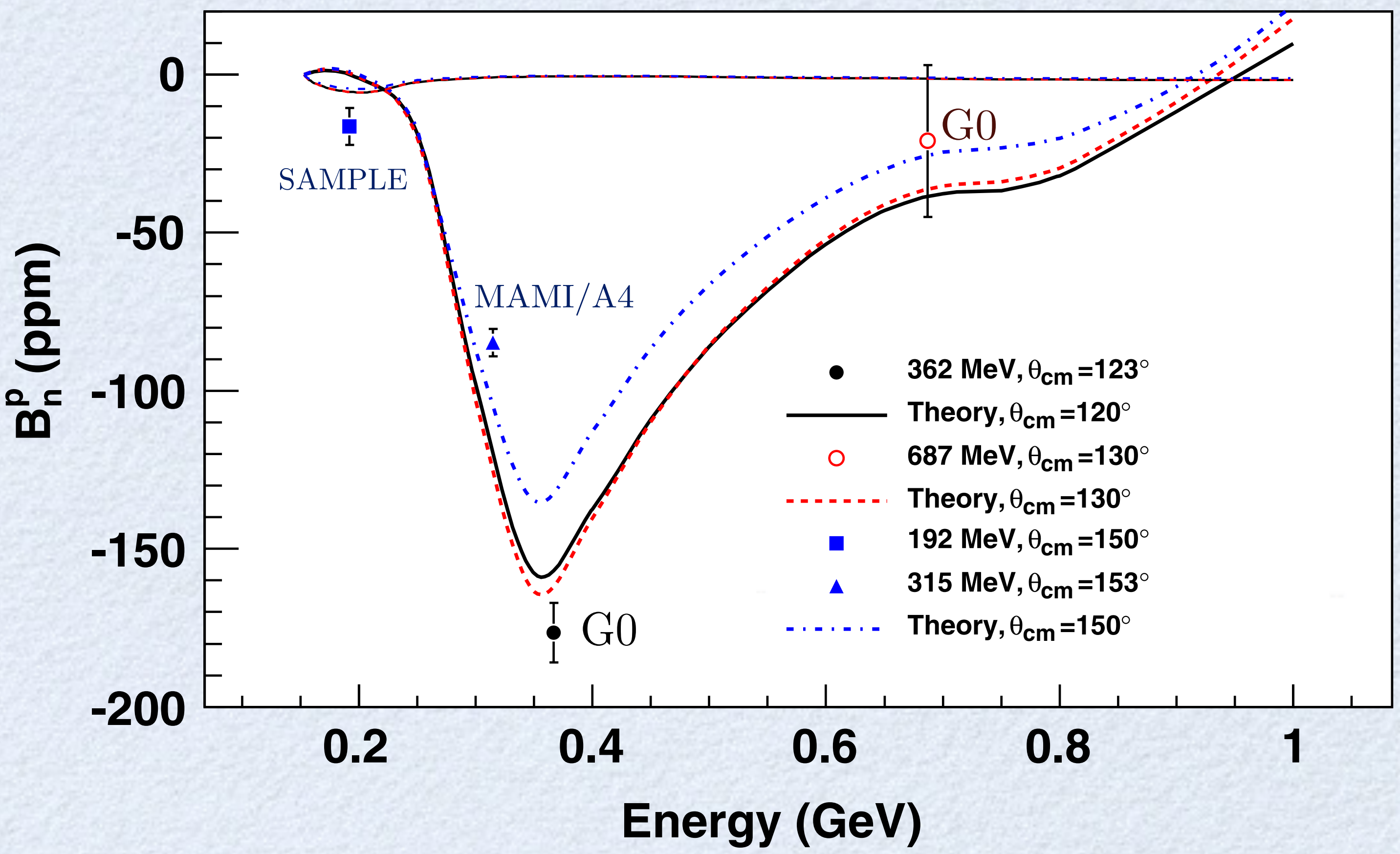
A possible explanation
two-photon exchange

V. Punjabi et al. (2015)

Beam normal spin asymmetry

Vanishing in 1ɣ approximation



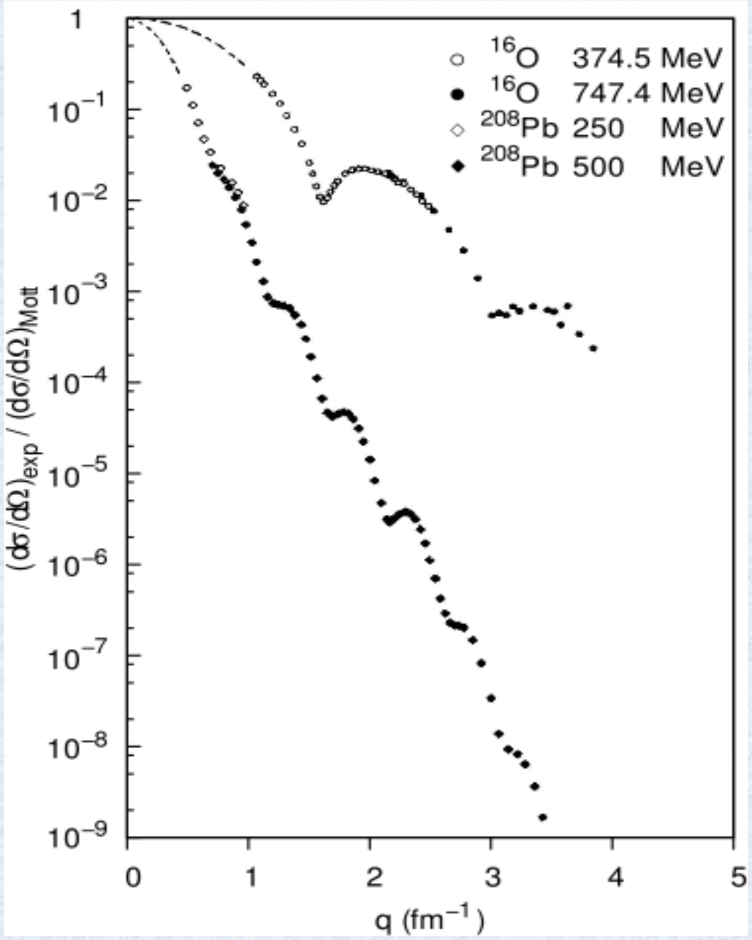


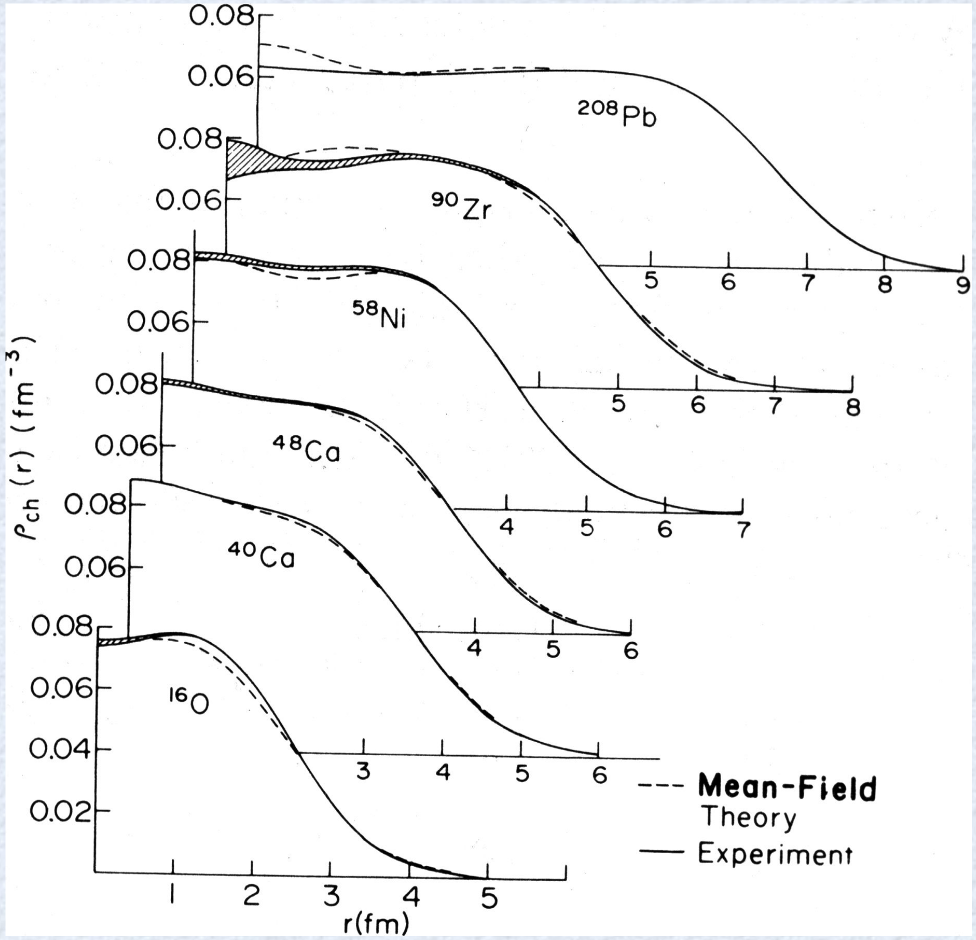
D. Androic et al. (2011)

Clear evidence of 2ɣ

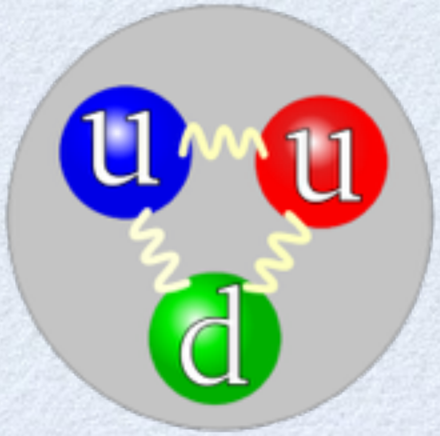
Form factors and size

form factor in atoms and nuclei
Fourier transform of charge distribution





How accurate do we know the proton size ?



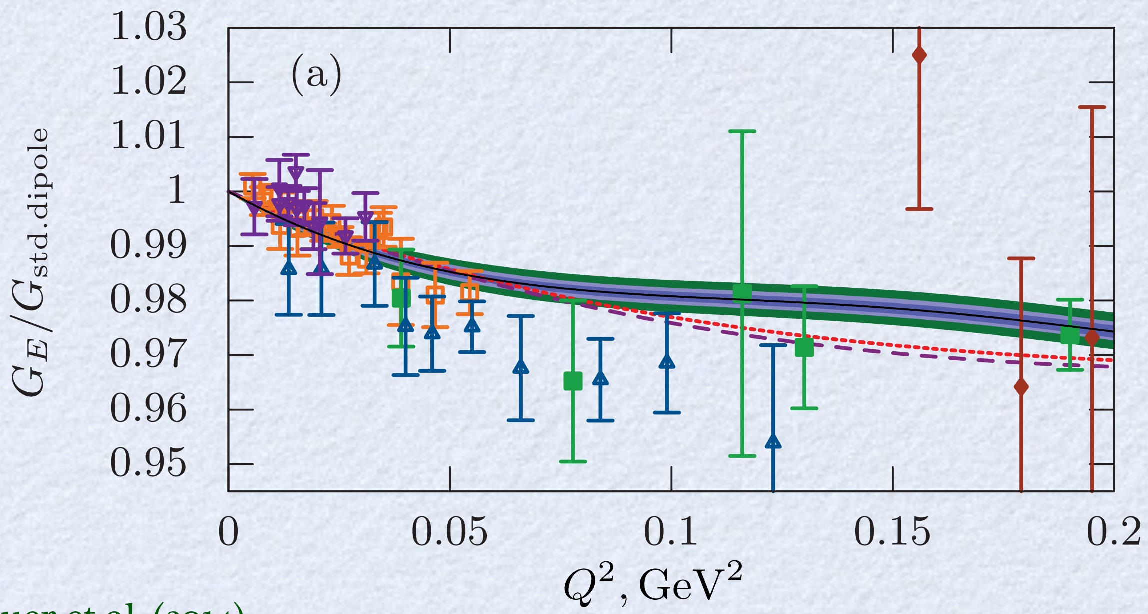
Proton charge radius

electric charge radius

$$< r_E^2 > \equiv -\frac{1}{6} \left. \frac{\mathrm{d}G_E(Q^2)}{\mathrm{d}Q^2} \right|_{Q^2=0}$$

- ep elastic scattering

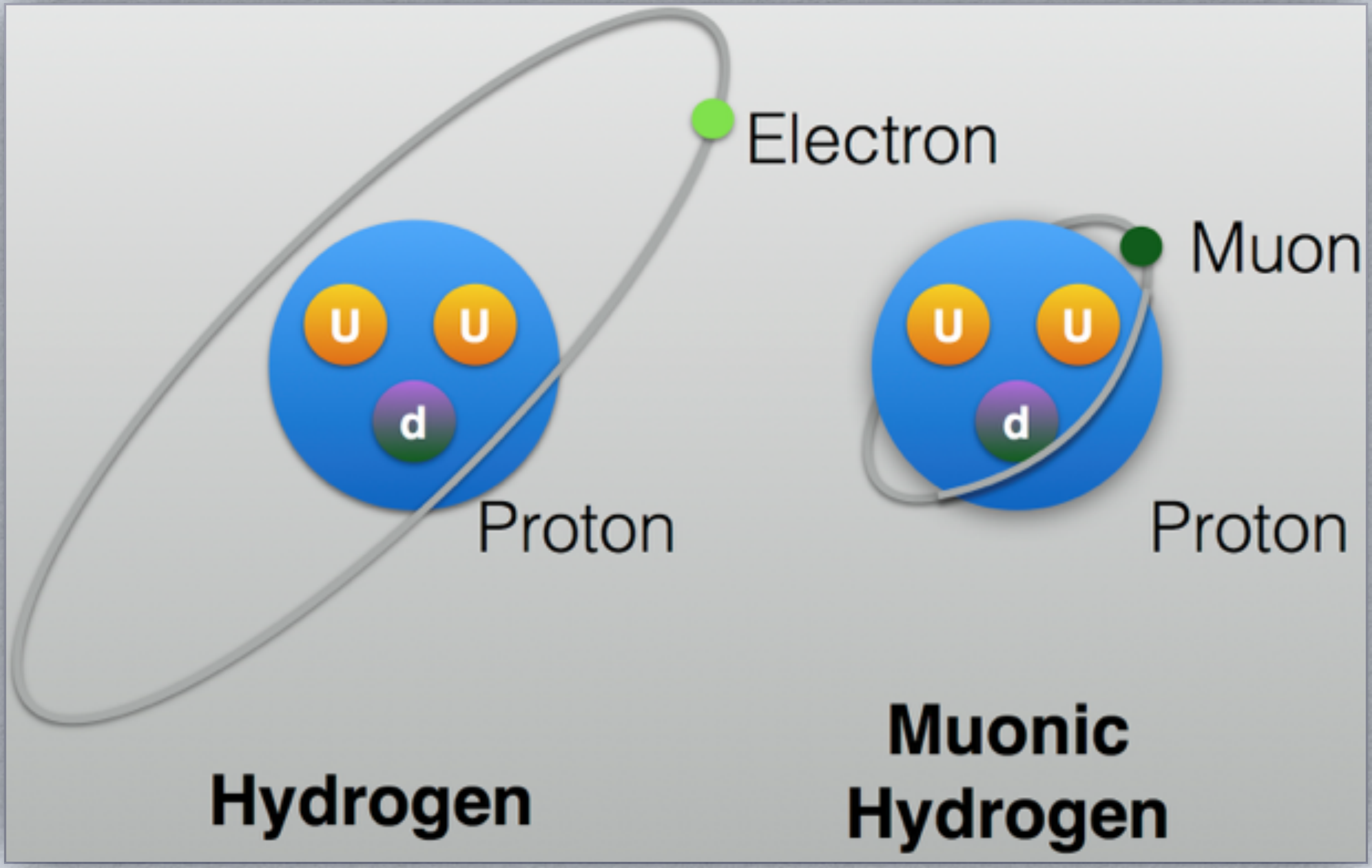
$$r_E = 0.879 \pm 0.008 \text{ fm}$$



J. C. Bernauer et al. (2014)

Proton form factors and size

hydrogen spectroscopy



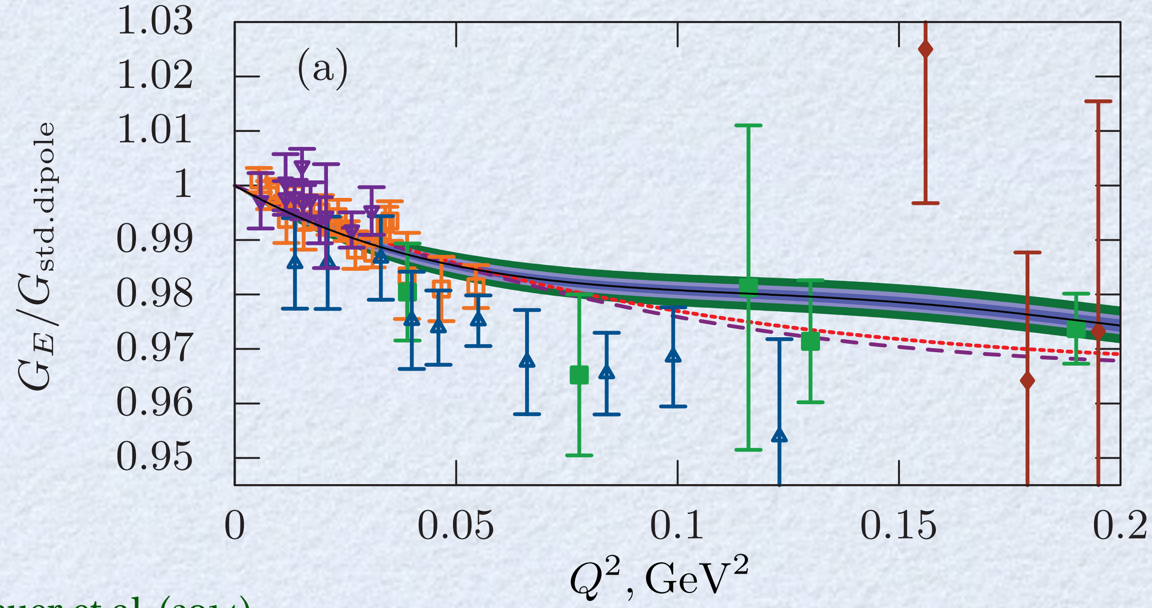
S state has finite wave function at origin

μH is sensitive to charge distribution

Proton charge radius

electric charge radius

$$< r_E^2 > \equiv -\frac{1}{6}\frac{dG_E(Q^2)}{dQ^2}\bigg|_{Q^2=0}$$



- ep elastic scattering

$$r_E = 0.879 \pm 0.008 \text{ fm}$$

J. C. Bernauer et al. (2014)

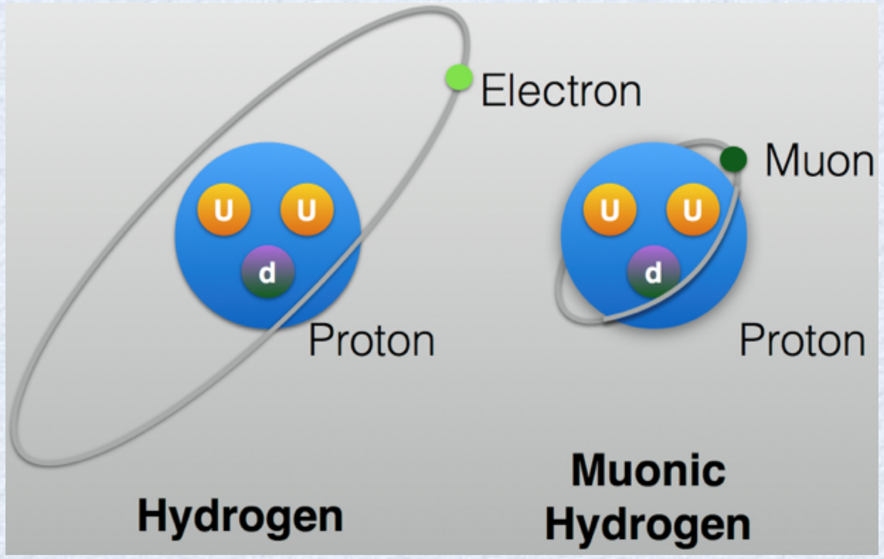
- atomic spectroscopy

$$\Delta E_{nS} \sim m_r^3 < r_E^2 >$$

H, D spectroscopy

$$r_E = 0.8758 \pm 0.0077 \text{ fm}$$

CODATA 2010



μH Lamb shift

$$r_E = 0.8409 \pm 0.0004 \text{ fm}$$

CREMA (2010, 2013)

Proton radius puzzle

electric charge radius

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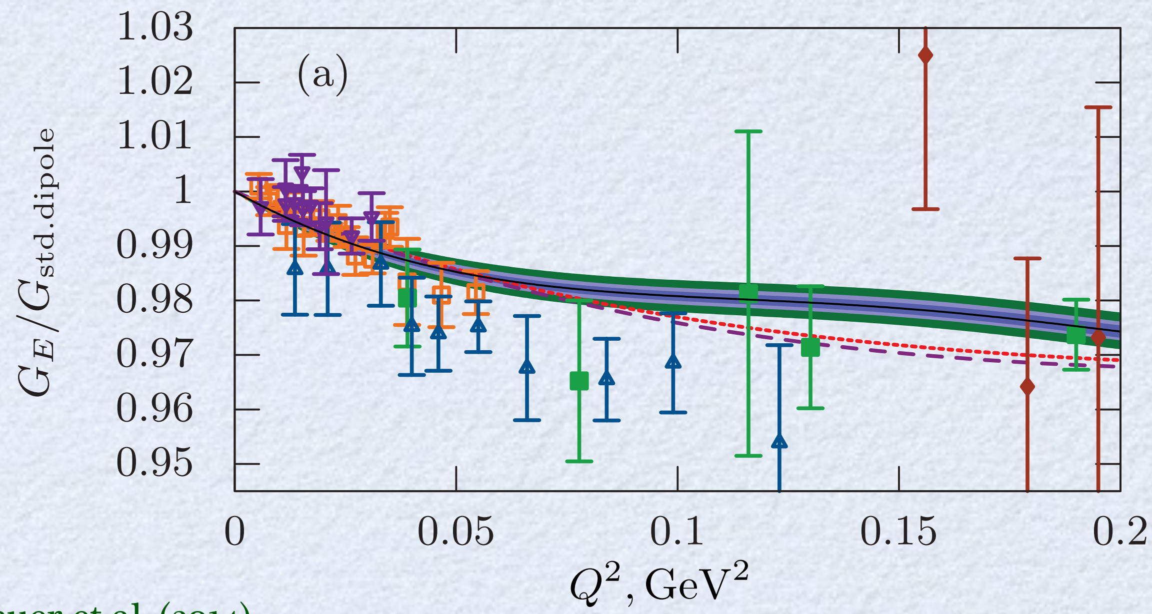

- ep elastic scattering

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J. C. Bernauer et al. (2014)

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$$\Delta E_{\rm nS} \sim m_r^3 < r_E^2 >$$

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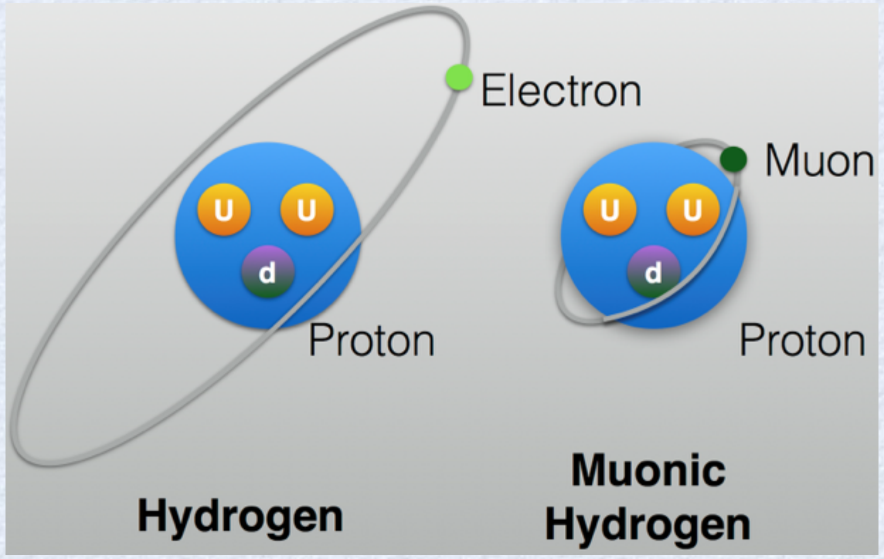


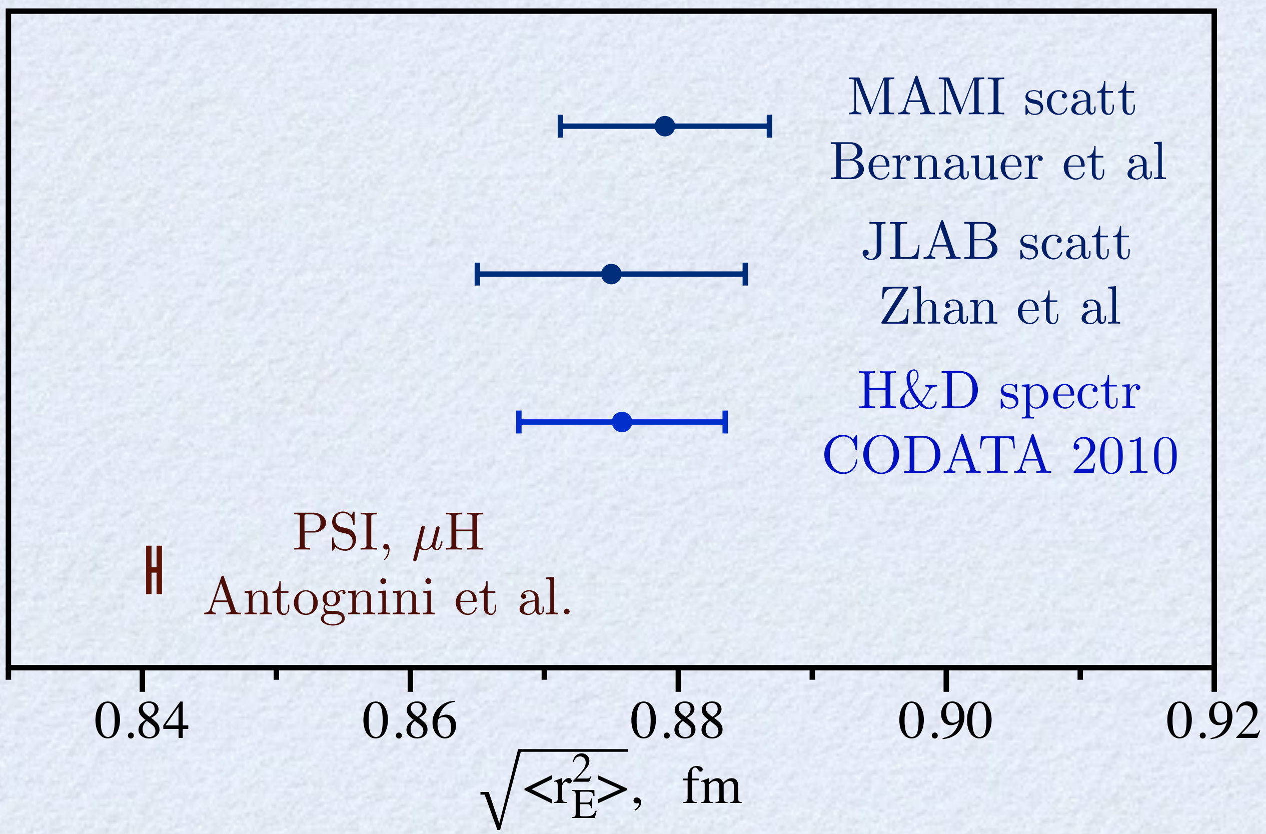
μH Lamb shift

$$r_E = 0.8409 \pm 0.0004 \text{ fm}$$

CREMA (2010, 2013)

7 σ difference !

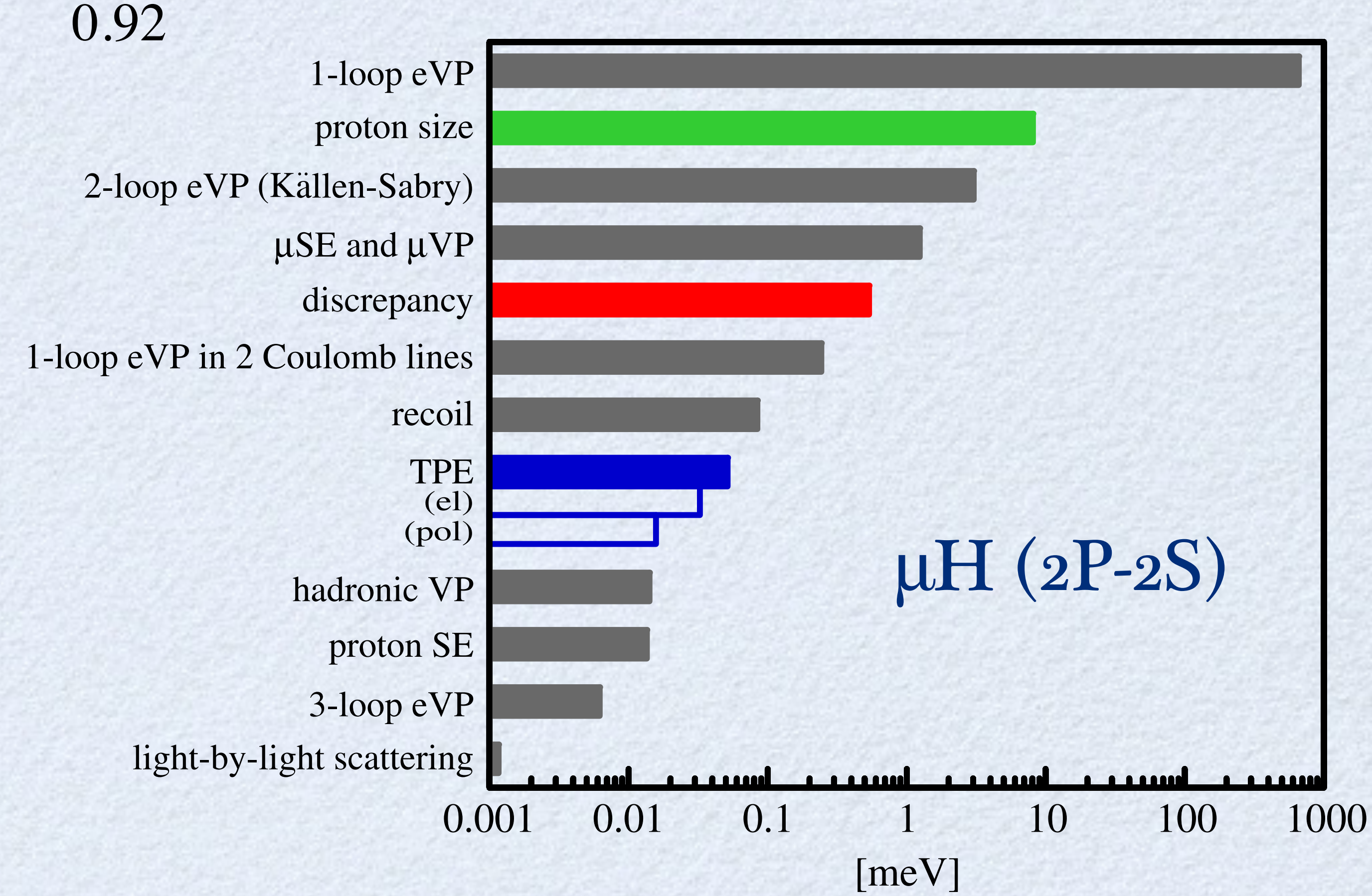
μH Lamb shift and 2ɣ



2P-2S transition in μH

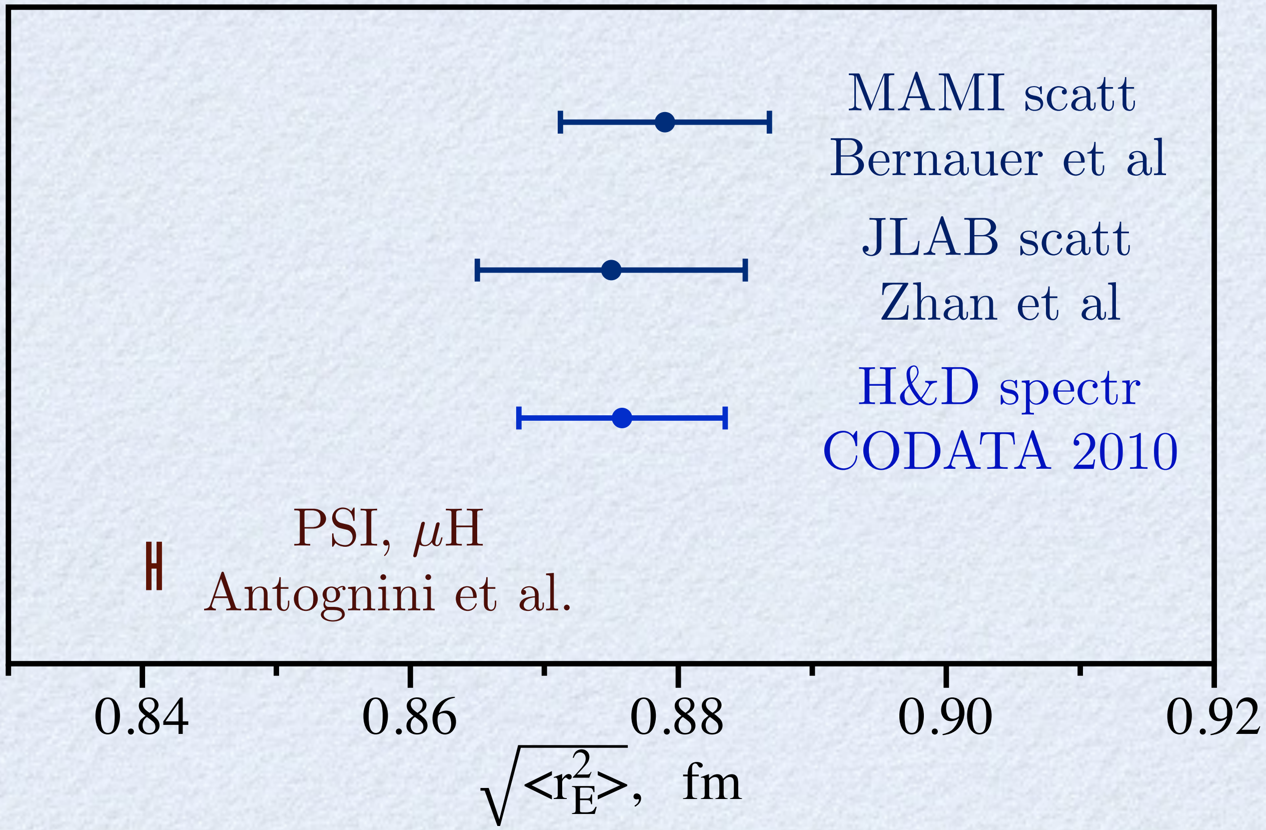
Discrepancy 310 μeV

μH uncertainty 2.5 μeV



A. Antognini et al. (2013)

μH Lamb shift and 2ɣ



2P-2S transition in μH

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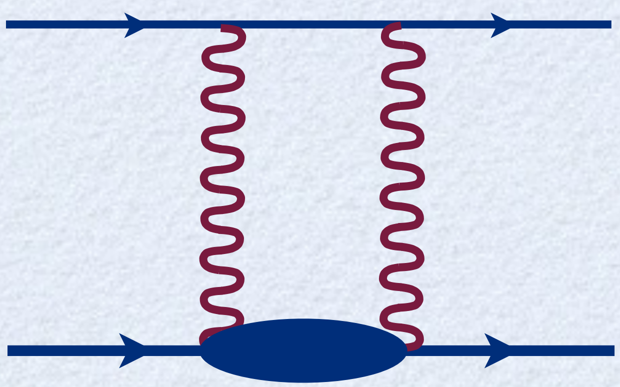
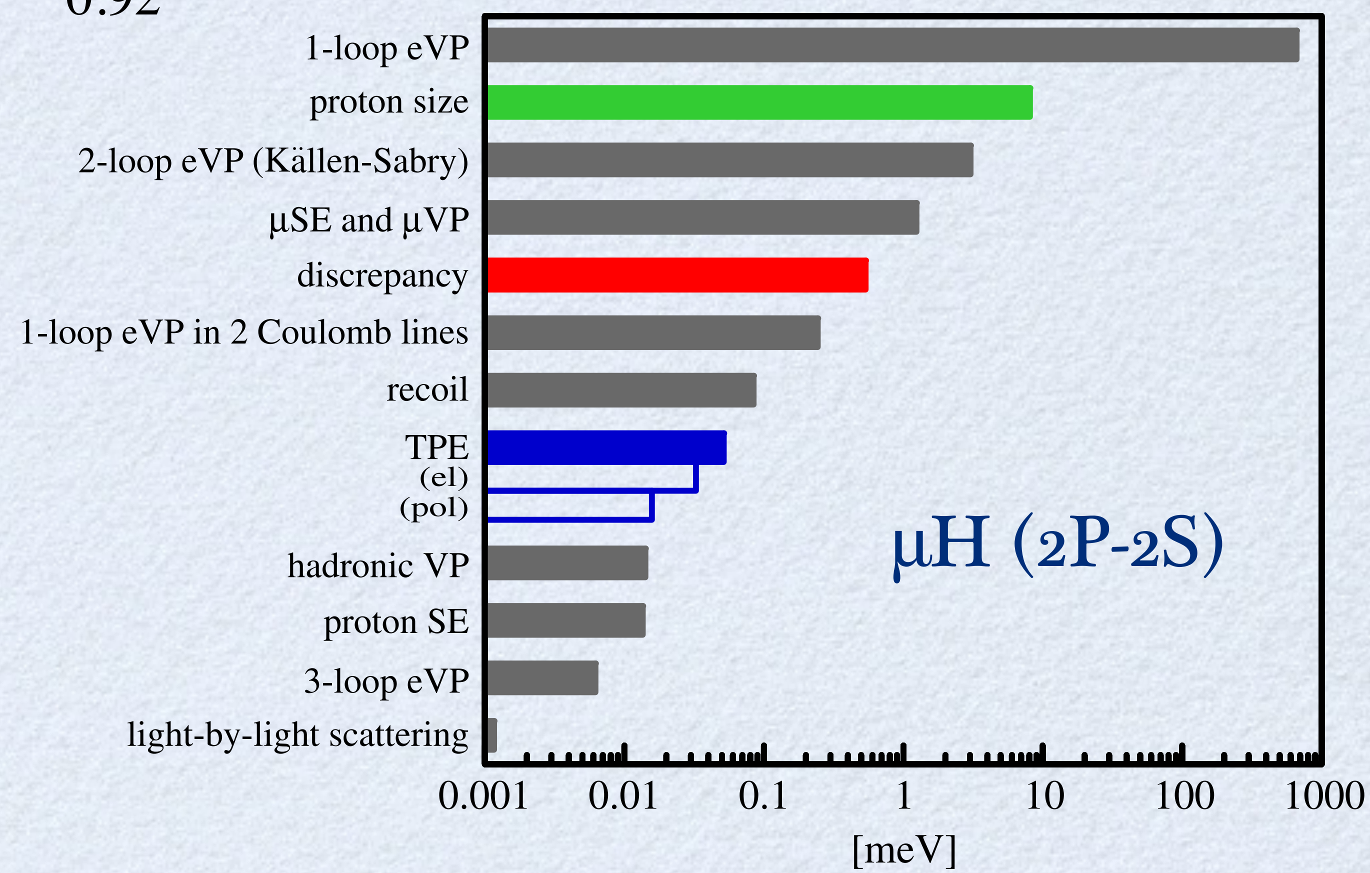
2ɣ hadronic correction



$$\Delta E^{2\gamma}_{2\mathrm{P}-2\mathrm{S}} = 33 \pm 2\ \mu\mathrm{eV}$$

C. Carlson, M. Vanderhaeghen (2011) +
M. Birse, J. McGovern (2012)

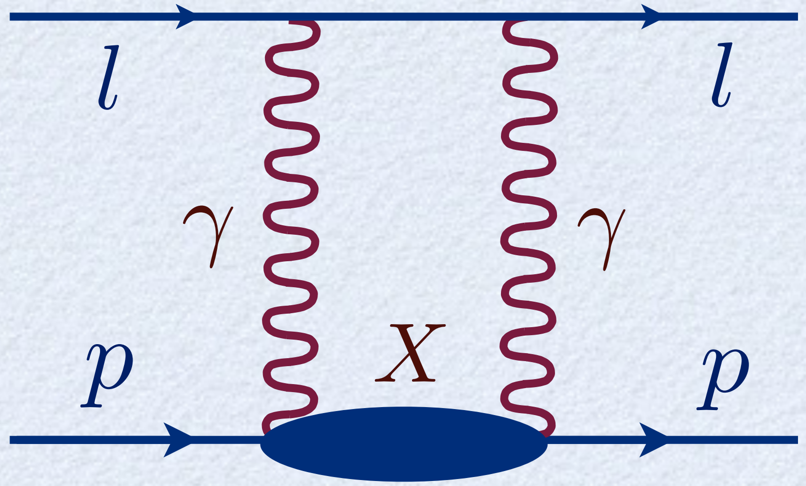


A. Antognini et al. (2013)

μH HFS and 2ɣ

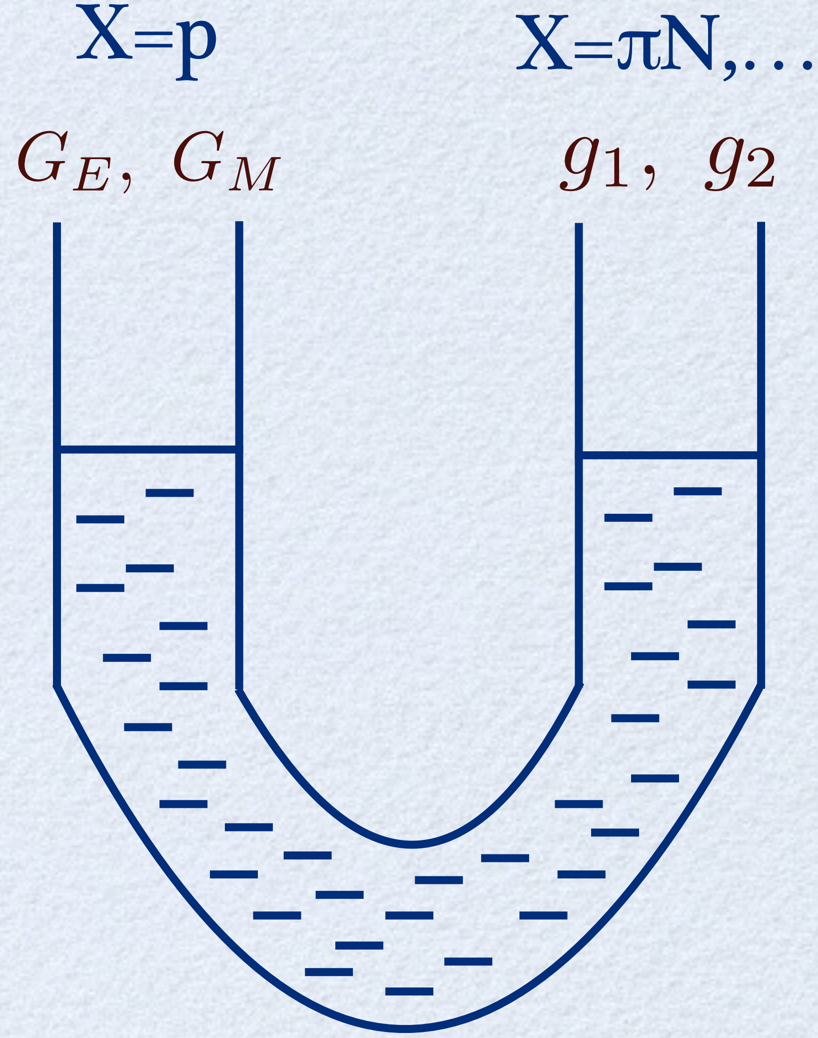
forthcoming
1S-HFS measurement in μH
with 1 ppm accuracy

A. Antognini (BVR47@PSI 2016)



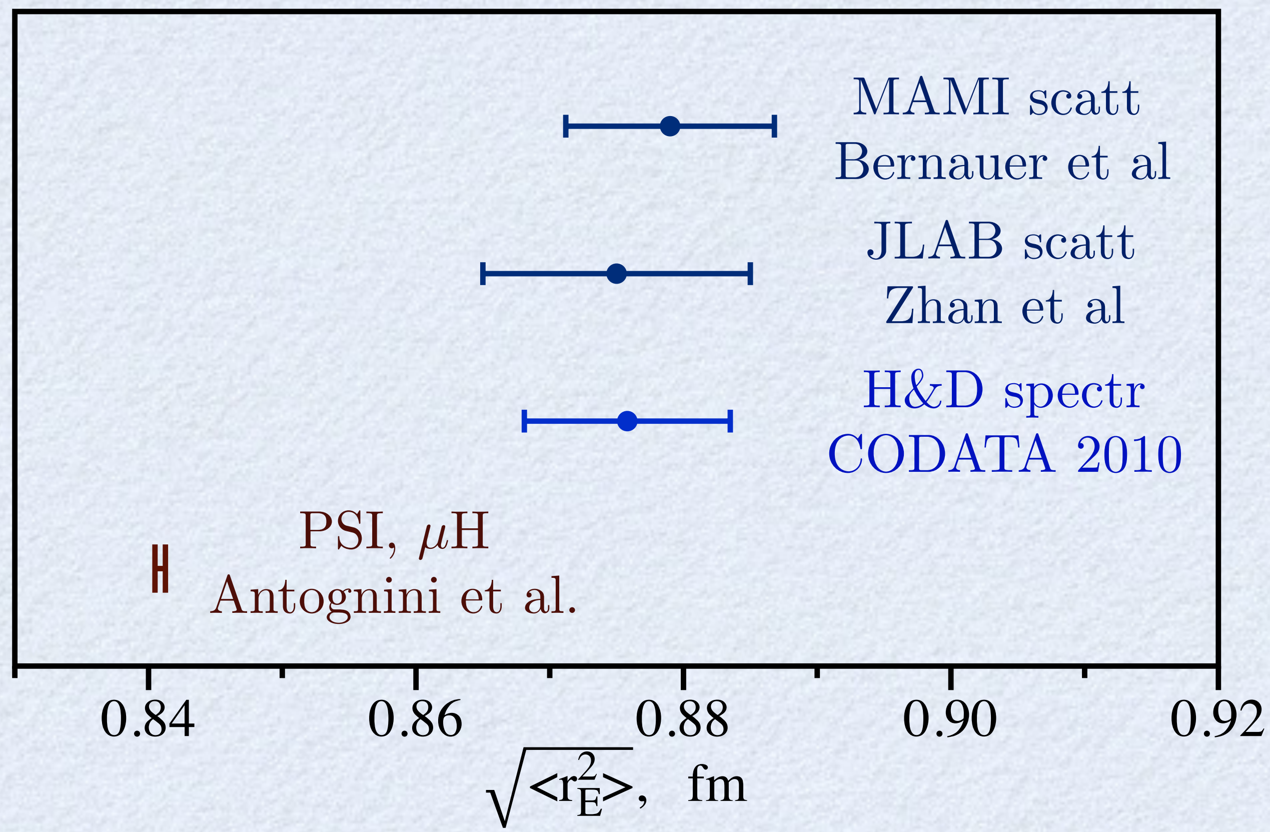
	$10^3\Delta$	relative uncertainty
X=p	-6.51	140 ppm
X=πN,…(polarizability)	0.373	92 ppm
total	-6.137	168 ppm

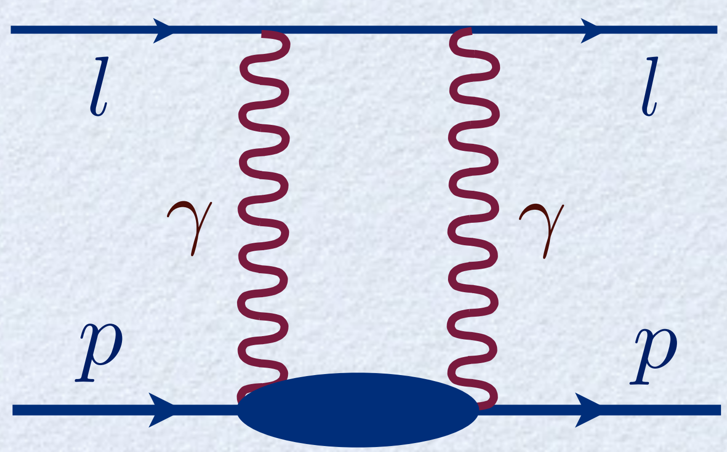
uncertainty balance



Impressive 1 ppm accuracy requires improvement on 2ɣ

Scattering experiments and 2ɣ





dominant uncertainty
in Lamb shift

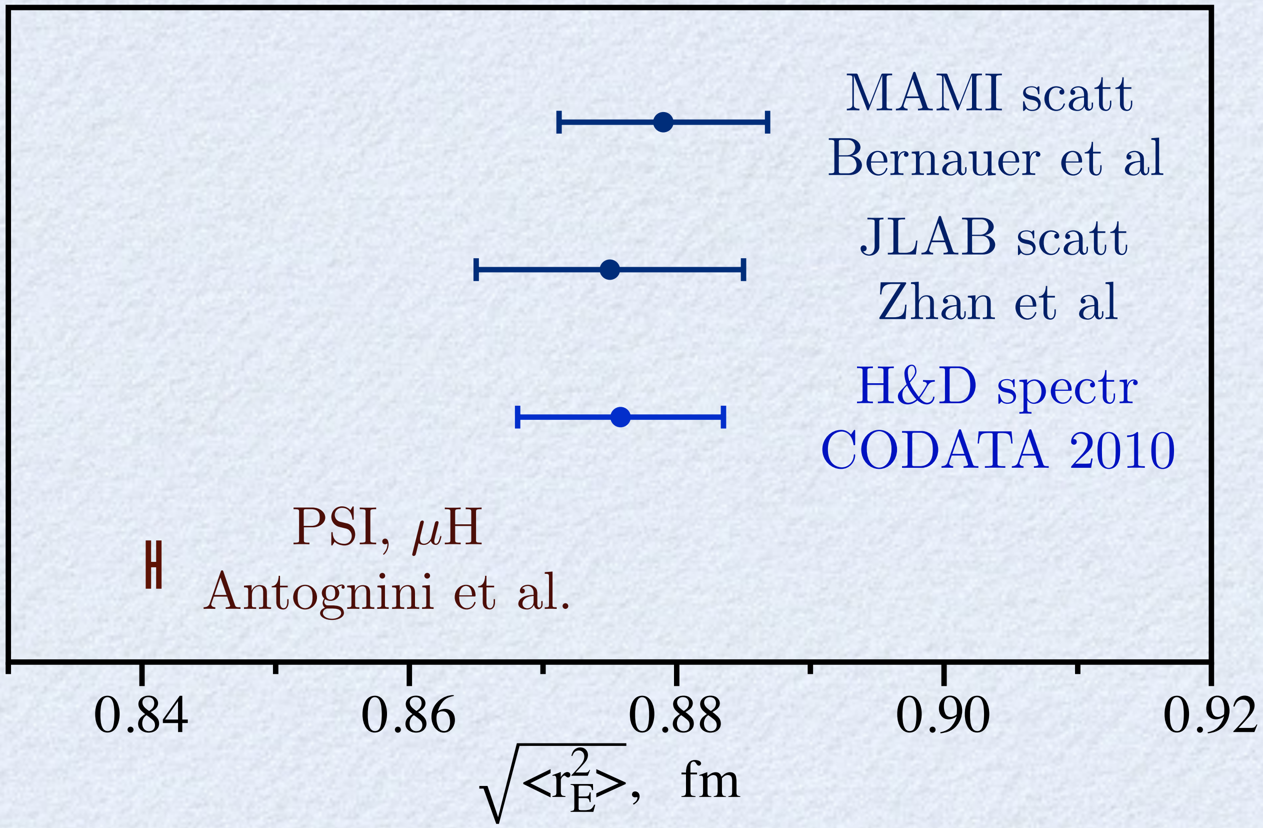
2ɣ is not fully accounted in scattering experiments

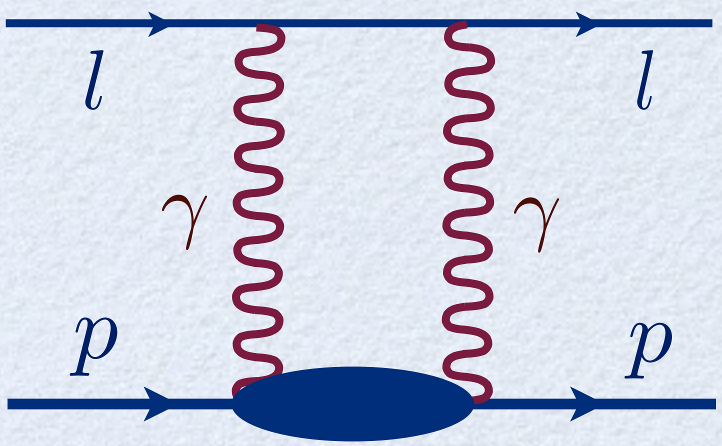
$$\sigma^{\text{exp}} \equiv \sigma_{1\gamma}(1 + \delta_{\text{rad}} + \delta_{\text{soft}} + \delta_{2\gamma})$$

charge radius only slightly depends on 2ɣ
magnetic radius significantly depends on 2ɣ

J. C. Bernauer et al. (2014)

Scattering experiments and 2γ





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$$\sigma^{\exp} \equiv \sigma_{1\gamma}(1 + \delta_{\rm rad} + \delta_{\rm soft} + \delta_{2\gamma})$$

charge radius only slightly depends on 2γ
magnetic radius significantly depends on 2γ

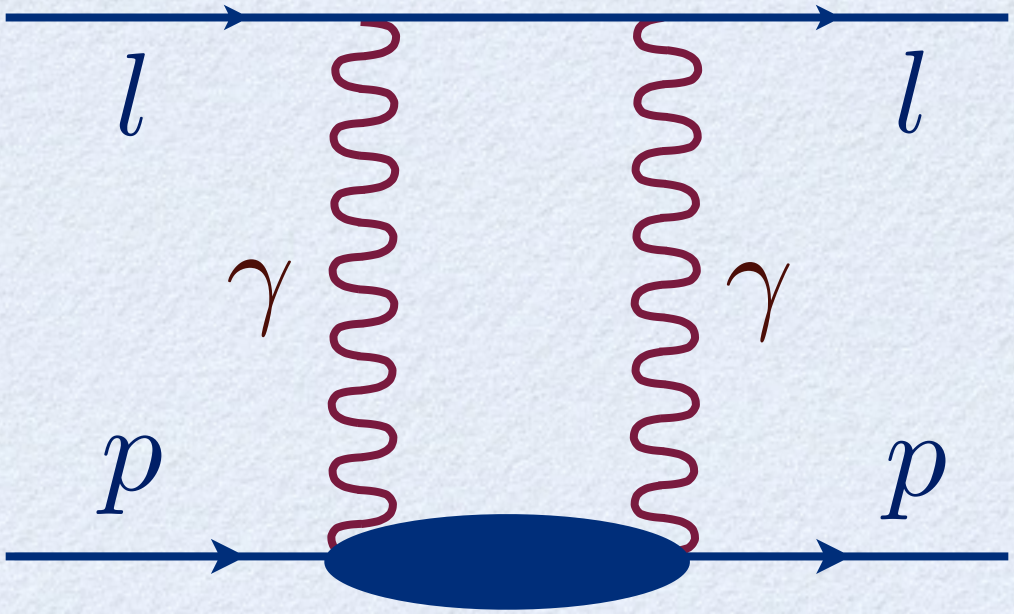
J. C. Bernauer et al. (2014)

μp elastic scattering is planned by MUSE@PSI(2017-18)

2γ correction in MUSE ?

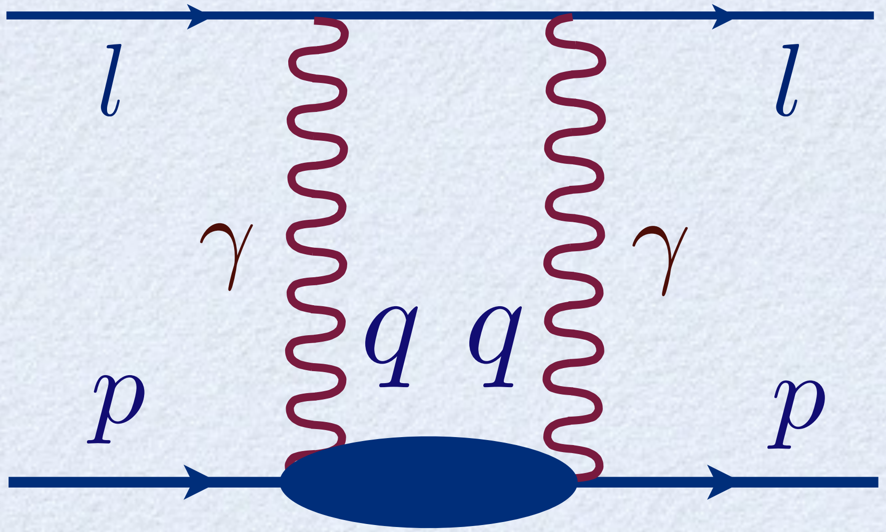
k′=k=(m,0,0,0)

p′=p=(M,0,0,0)



forward scattering
at zero energy
(atomic correction)

Lamb shift 2ɣ correction. Forward VVCS



Shift of S energy level 2ɣ correction

$$\Delta E^{2\gamma}_{\mathrm{nS}} \sim f_{+} |\psi_n(0)|^2$$

f_{+} - unpolarized 2ɣ amplitude

2ɣ blob - forward virtual Compton scattering

photon energy $\nu_\gamma = \frac{p \cdot q}{M}$ photon virtuality $Q^2 = -q^2$

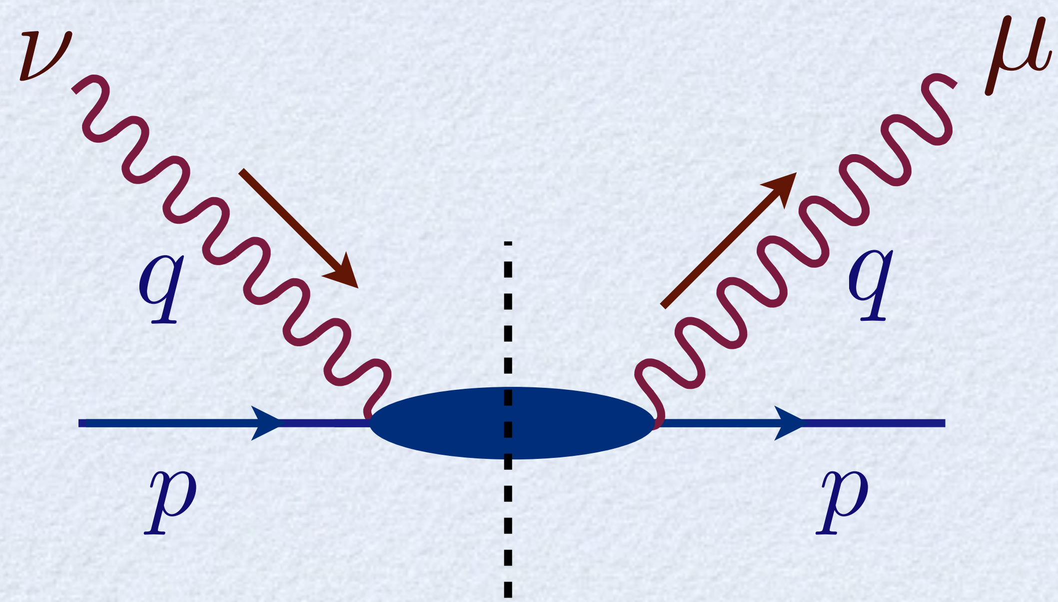
Forward VVCS tensor

$$M^{\mu\nu} = M^{\mu\nu}_{\mathrm{S}} + M^{\mu\nu}_{\mathrm{A}}$$

$M^{\mu\nu}_{\mathrm{S}} \sim \mathrm{T}_1(\nu_\gamma, Q^2), \mathrm{T}_2(\nu_\gamma, Q^2)$ spin-independent amplitudes

$M^{\mu\nu}_{\mathrm{A}} \sim \mathrm{S}_1(\nu_\gamma, Q^2), \mathrm{S}_2(\nu_\gamma, Q^2)$ spin-dependent amplitudes

Forward VVCS. Dispersion relations



Optical theorem

relates Compton amplitudes

to proton structure functions

$\mathrm{Im}\ \mathrm{T}_1 \sim F_1$ $\mathrm{Im}\ \mathrm{T}_2 \sim F_2$ $\mathrm{Im}\ \mathrm{S}_1 \sim g_1$ $\mathrm{Im}\ \mathrm{S}_2 \sim g_2$

Fixed-Q^2 dispersion relations

Dis. rel. for amplitude T_1 requires subtraction function

$$\mathrm{T}_1^{\mathrm{subt}}(0,Q^2) \equiv \mathrm{T}_1(0,Q^2) - \mathrm{T}_1^{\mathrm{Born}}(0,Q^2)$$

Unsubtracted disp. rel. works for $\mathrm{T}_2,\ \mathrm{S}_1,\ \mathrm{S}_2,\ \nu_\gamma \mathrm{S}_2$

Empirical estimate of subtraction function

High-energy behavior of T_1 in Regge theory

$$\mathrm{T}_1^{\mathrm{R}}(\nu_\gamma, Q^2) \sim \sum_{\alpha_0 > 0} \frac{\gamma_{\alpha_0}(Q^2)}{\sin \pi\alpha_0} \left\{ (\nu_0 - \nu_\gamma - i\varepsilon)^{\alpha_0} + (\nu_0 + \nu_\gamma - i\varepsilon)^{\alpha_0} \right\}$$

$$+ \sum_{\alpha_0 > 1} \frac{\alpha_0 \nu_0 \gamma_{\alpha_0}(Q^2)}{\sin \pi(\alpha_0 - 1)} \left\{ (\nu_0 - \nu_\gamma - i\varepsilon)^{\alpha_0 - 1} + (\nu_0 + \nu_\gamma - i\varepsilon)^{\alpha_0 - 1} \right\}$$

G. Gasser, H. Leutwyler et al. (1974, 2015) M. Gorchtein et al. (2013) I. Caprini (2016)

Evaluate dispersion relation for $\mathrm{T}_1(\nu_\gamma, Q^2) - \mathrm{T}_1^{\mathrm{R}}(\nu_\gamma, Q^2)$

$$\mathrm{T}_1^{\mathrm{subt}}(0, Q^2) = \mathrm{T}_1^{\mathrm{R}}(0, Q^2) + \frac{\alpha}{M} F_D^2(Q^2) + \frac{2\alpha}{M} \int_{\nu_{\mathrm{thr}}}^{\infty} \frac{F_1(\nu_\gamma, Q^2) - F_1^R(\nu_\gamma, Q^2)}{\nu_\gamma} \mathrm{d}\nu_\gamma$$

Empirical estimate of subtraction function

High-energy behavior of T_1 in Regge theory

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$$+ \sum_{\alpha_0>1} \frac{\alpha_0 \nu_0 \gamma_{\alpha_0}(Q^2)}{\sin \pi(\alpha_0 - 1)} \left\{(\nu_0 - \nu_\gamma - i\varepsilon)^{\alpha_0 - 1} + (\nu_0 + \nu_\gamma - i\varepsilon)^{\alpha_0 - 1}\right\}$$

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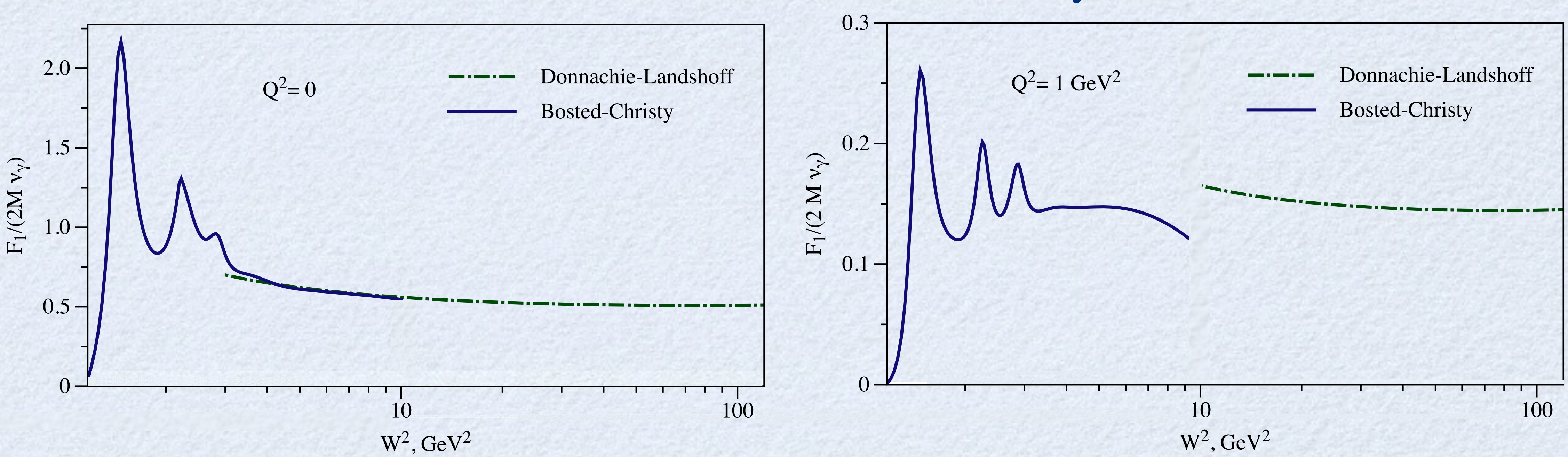
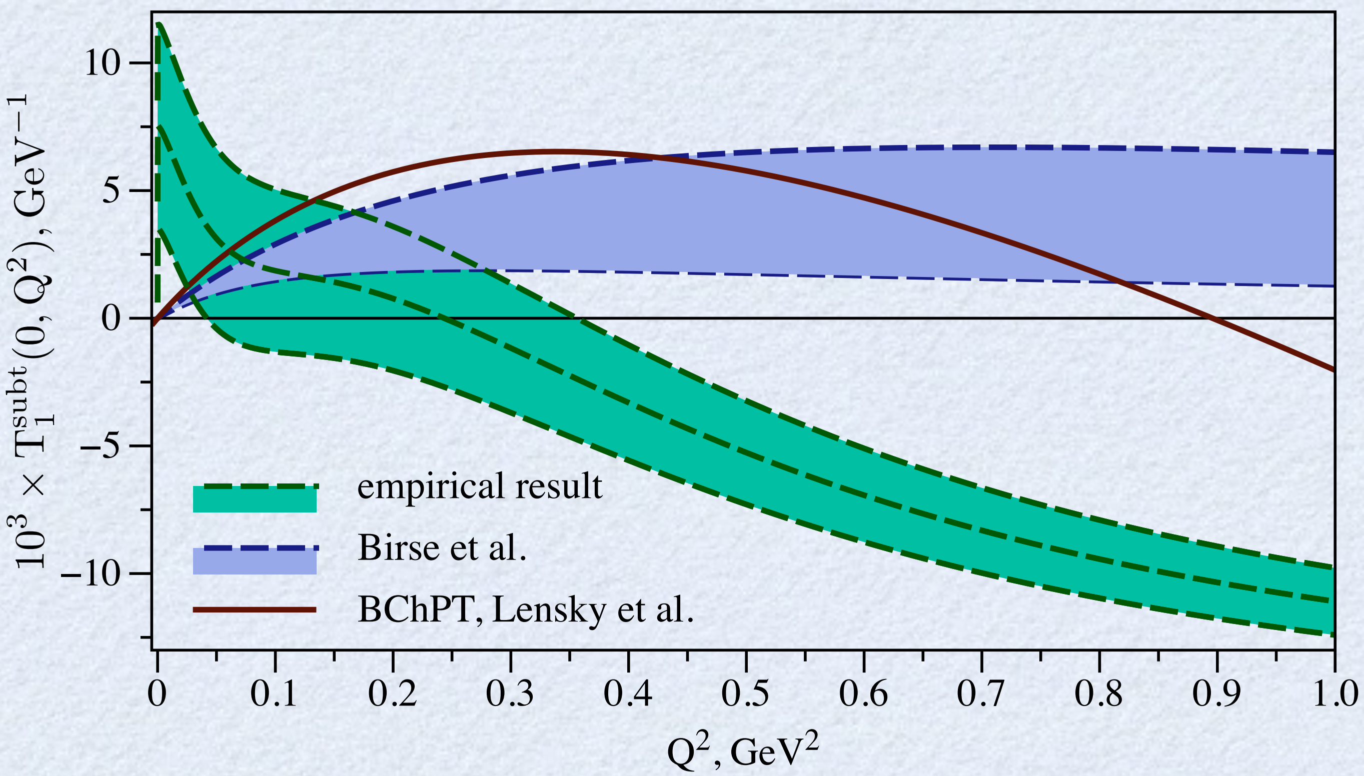
Donnachie-Landshoff and Bosted-Christy fits at low Q^2



Forthcoming JLab data will improve fits around W^2~10 GeV^2

Empirical estimate of subtraction function

Empirical result vs. theoretical predictions



expected low-Q² behavior

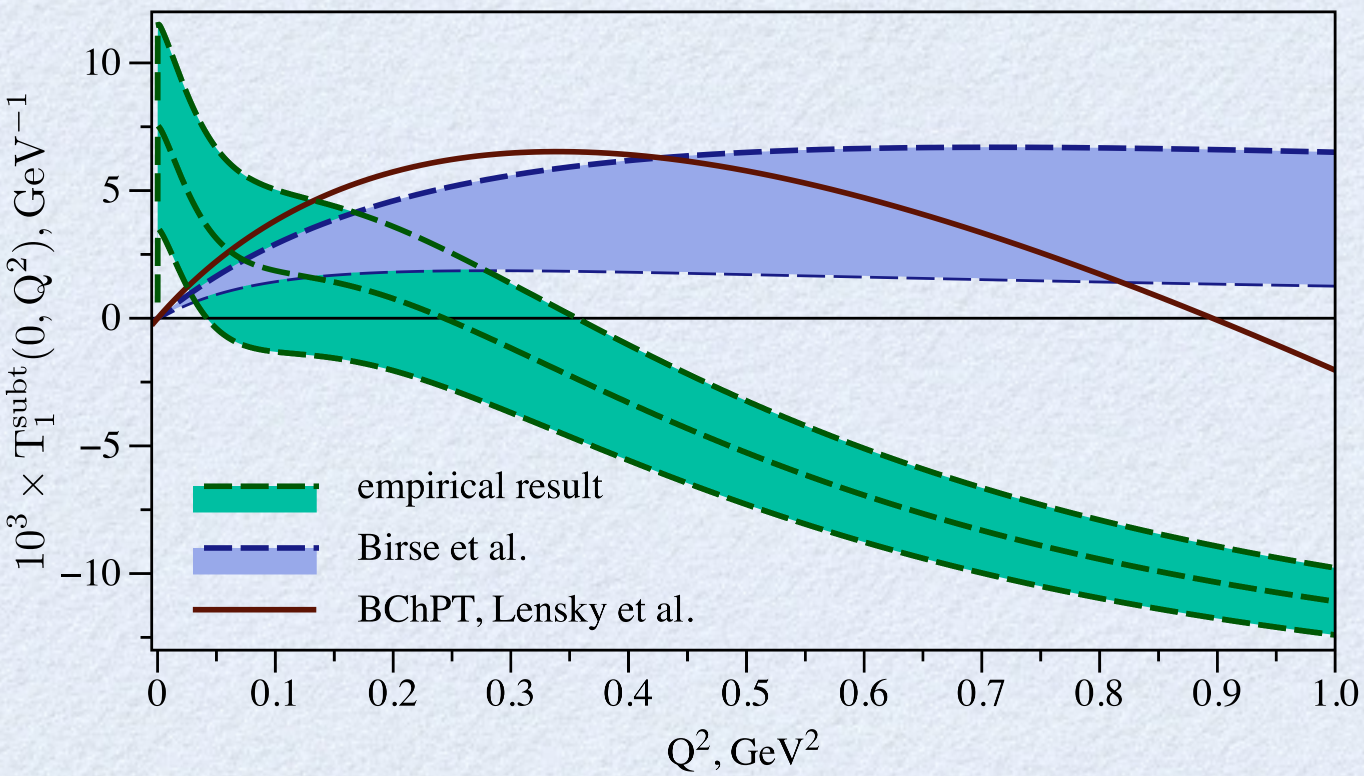
$$T_1^{subt}(0, Q^2) = \beta_M Q^2 + O(Q^4)$$

satisfied within 1.5σ

O. Tomalak and M. Vanderhaeghen (2016)

Empirical estimate of subtraction function

Empirical result vs. theoretical predictions



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O. Tomalak and M. Vanderhaeghen (2016)

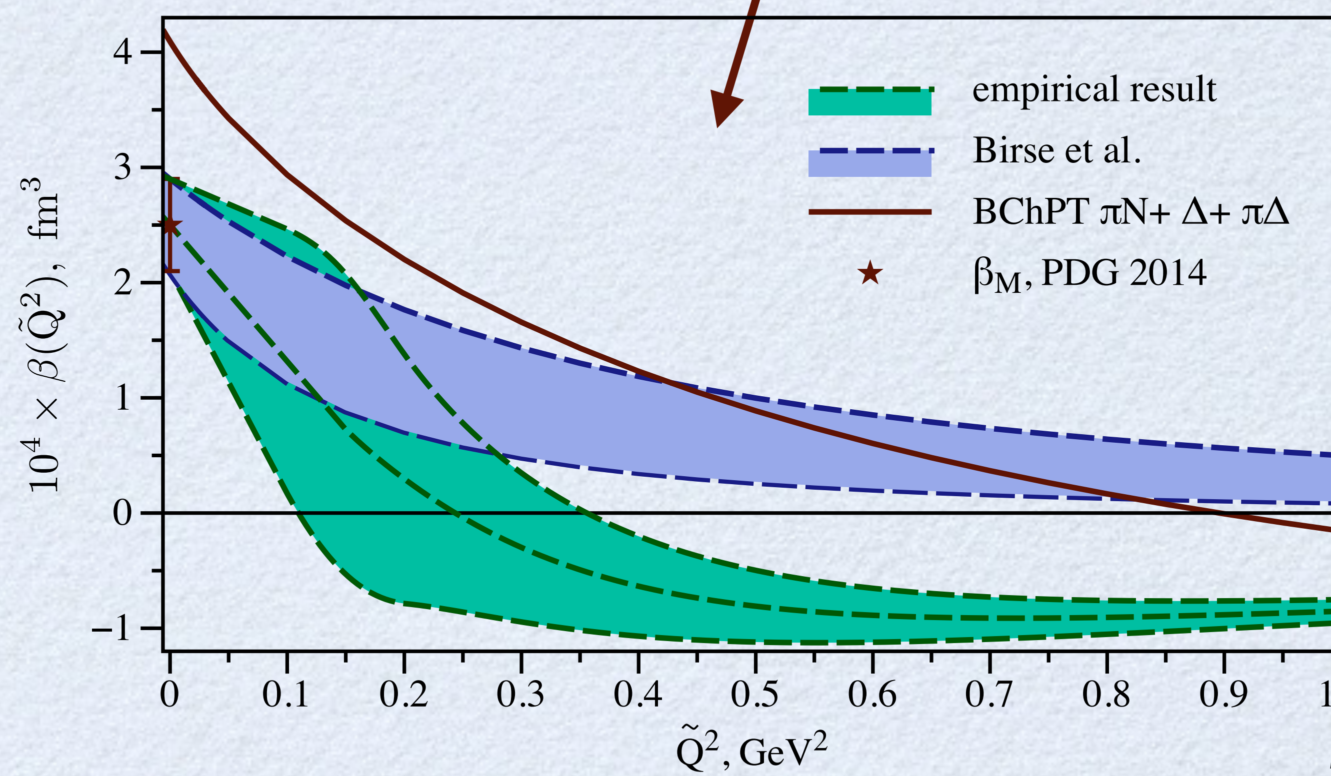
$$\beta(Q^2) \equiv \frac{\mathrm{T}_1^{\mathrm{subt}}(0, Q^2)}{Q^2}$$

empirical estimate connected to p.d.g. value of magnetic polarizability

$$\Delta E_{2S}^{\mathrm{subt}}(\mu H) \approx 2.3 \pm 1.3\ \mu\mathrm{eV}$$

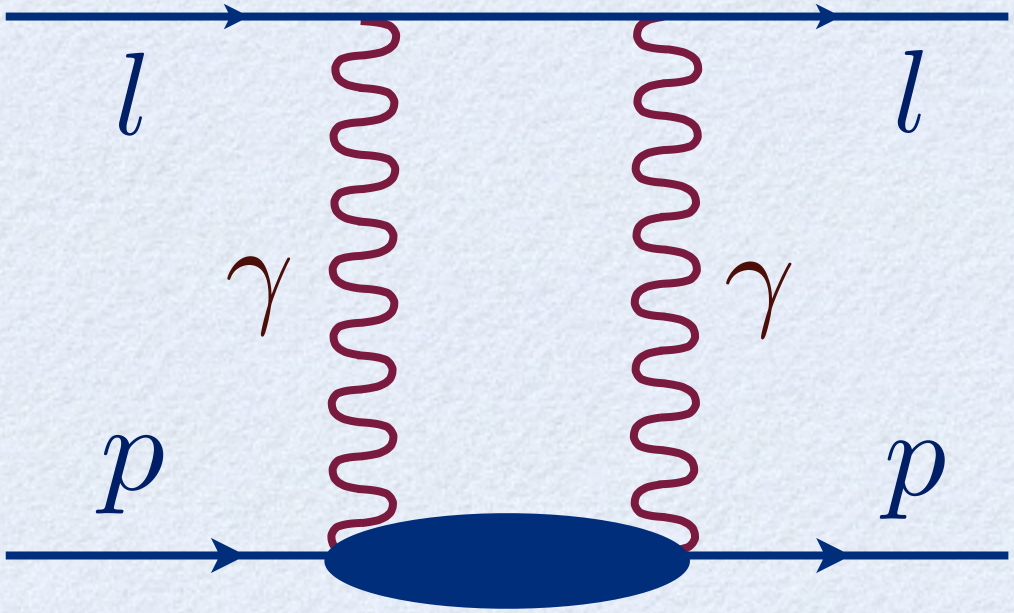
slightly smaller than Birse et al.

$$\Delta E_{2S}^{\mathrm{subt}}(\mu H) \approx 4.2 \pm 1.0\ \mu\mathrm{eV}$$



k′=k

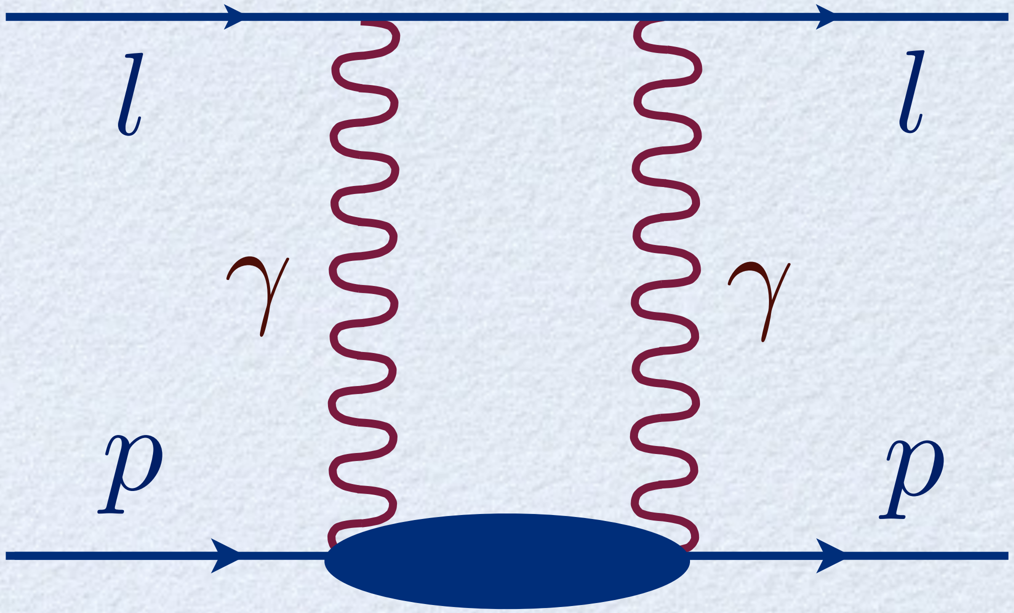
p′=p



forward scattering

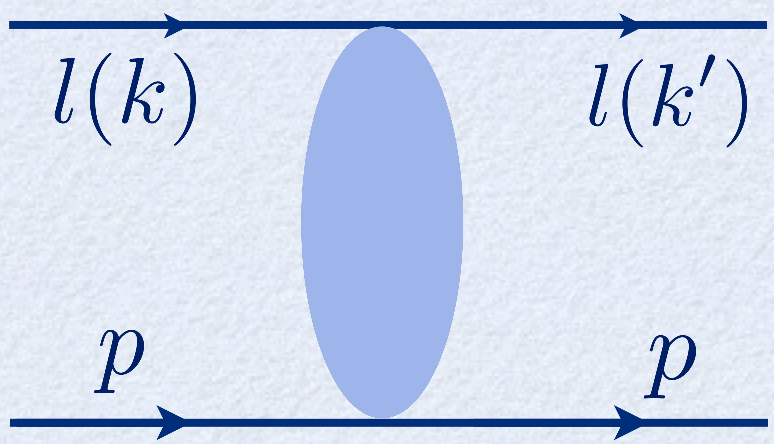
k′≠k

p′≠p



non-forward scattering

Structure amplitudes



$Q^2 = -(k-k')^2$ momentum transfer

$s = (p+k)^2$

$u = (k-p')^2$

$\nu = \frac{s-u}{4}$ crossing symmetric variable

ϵ photon polarization parameter

Discrete symmetries $\longrightarrow$ 6 structure amplitudes

Goldberger et al. (1957)

Electron scattering is described by 3 structure amplitudes

$$T^{\text{non-flip}} \sim \mathcal{G}_M(\nu, Q^2), \mathcal{F}_2(\nu, Q^2), \mathcal{F}_3(\nu, Q^2)$$

P.A.M. Guichon and M. Vanderhaeghen (2003)

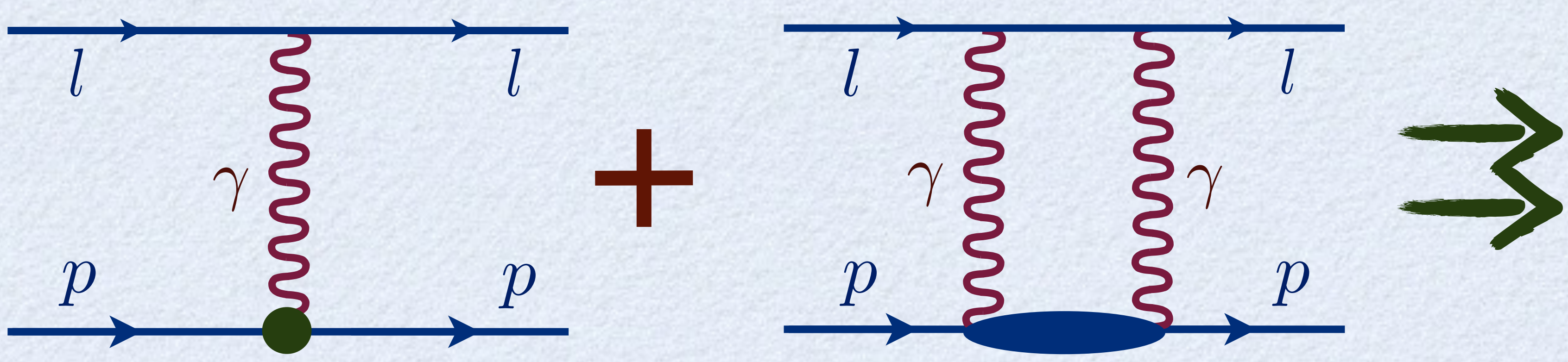
Muon scattering requires lepton helicity-flip amplitudes

$m_l \neq 0 \longrightarrow$ $T^{\text{flip}} \sim \mathcal{F}_4(\nu, Q^2), \mathcal{F}_5(\nu, Q^2), \mathcal{F}_6(\nu, Q^2)$

M. Gorchtein, P.A.M. Guichon and M. Vanderhaeghen (2004)

2ɣ correction to cross-section

Leading 2ɣ contribution to cross section - interference term



$$\delta_{2\gamma} = \frac{2\Re\left(\sum_{\rm spin} T^{2\gamma}(T^{1\gamma})^*\right)}{\sum_{\rm spin} |T^{1\gamma}|^2}$$

$$\delta_{2\gamma} = \frac{2}{G_M^2 + \frac{\varepsilon}{\tau_P} G_E^2}\left\{ G_M \Re\mathcal{G}_1^{2\gamma} + \frac{\varepsilon}{\tau_P} G_E \Re\mathcal{G}_2^{2\gamma} + \frac{1-\varepsilon}{1-\varepsilon_0}\left(\frac{\varepsilon_0}{\tau_P}\frac{\nu}{M^2} G_E \Re\mathcal{G}_4^{2\gamma} - G_M \Re\mathcal{G}_3^{2\gamma}\right)\right\}$$

O. Tomalak and M. Vanderhaeghen (2014)

$$\tau = \frac{Q^2}{4M^2}$$

$$\mathcal{G}_1 = \mathcal{G}_M + \frac{\nu}{M^2}\mathcal{F}_3 + \frac{m^2}{M^2}\mathcal{F}_5$$

$$\mathcal{G}_2 = \mathcal{G}_M - (1-\tau)\mathcal{F}_2 + \frac{\nu}{M^2}\mathcal{F}_3$$

$$\mathcal{G}_3 = \frac{\nu}{M^2}\mathcal{F}_3 + \frac{m^2}{M^2}\mathcal{F}_5$$

$$\mathcal{G}_4 = \mathcal{F}_4 + \frac{\nu}{M^2(1+\tau)}\mathcal{F}_5$$

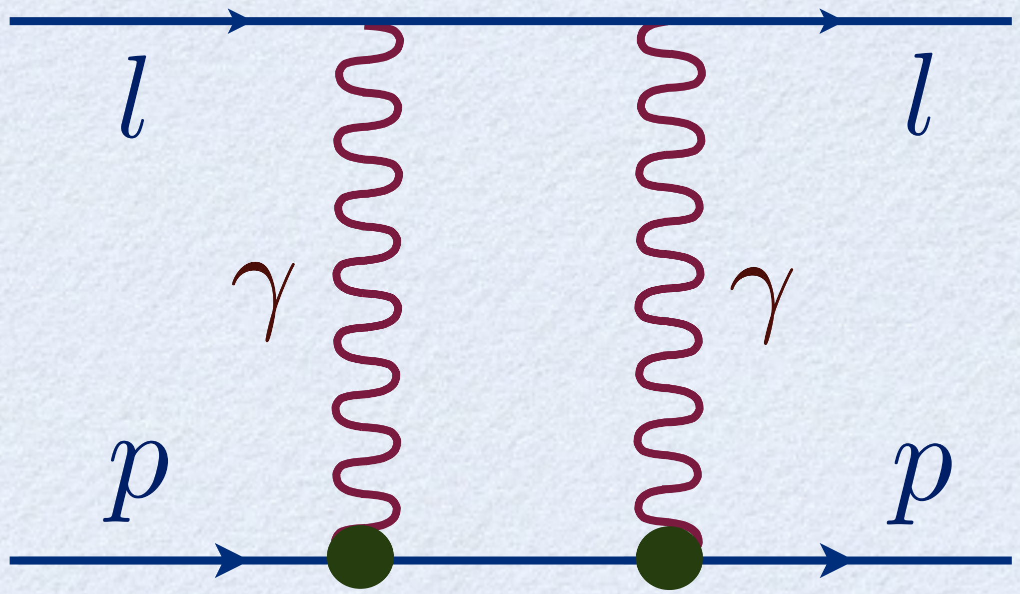
$$\varepsilon_0 = \frac{2m^2}{Q^2}$$

ε in range (ε₀, 1) or (1, ε₀)

2ɣ correction is given by amplitudes real parts

k′≠k

p′≠p



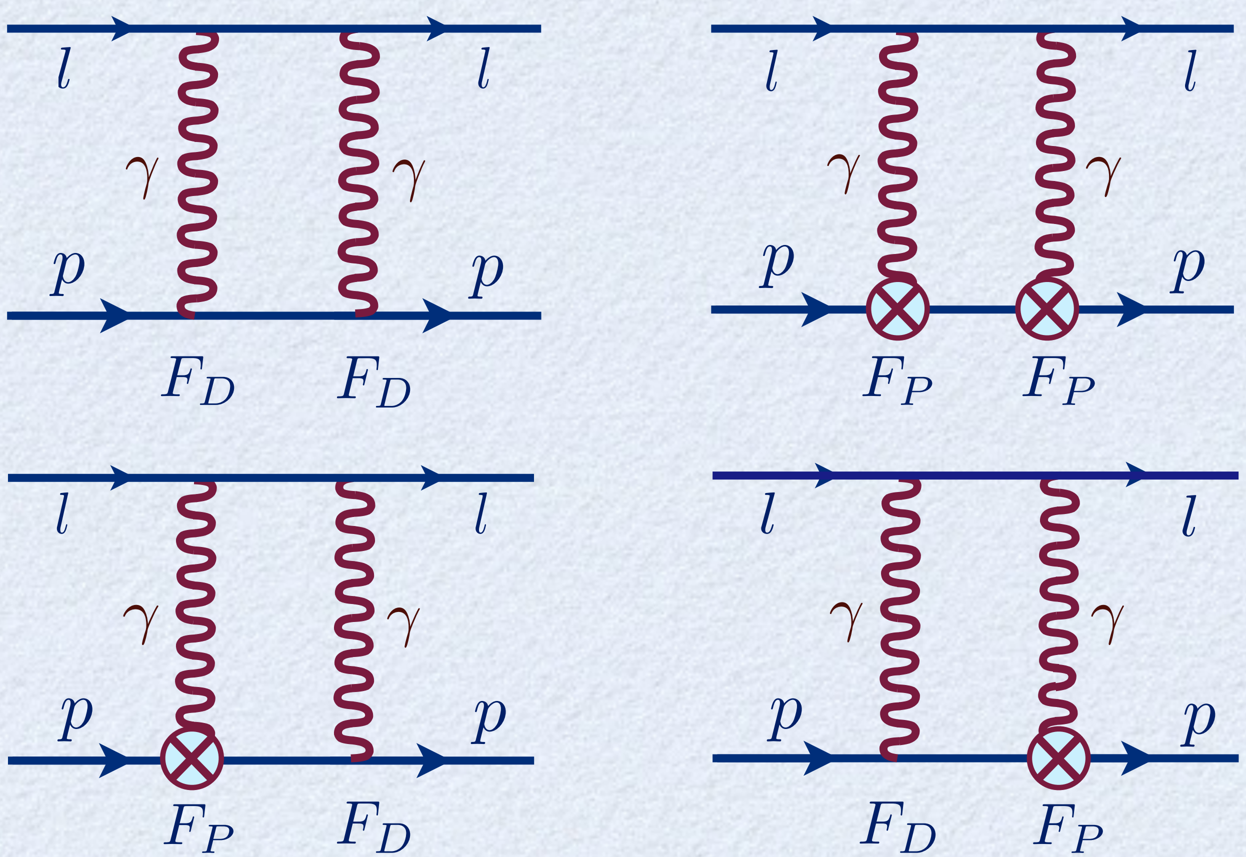
non-forward scattering
proton state

Box diagram model

The one-photon exchange on-shell vertex

$$\Gamma^\mu(Q^2) = \gamma^\mu F_D(Q^2) + \frac{i\sigma^{\mu\nu}q_\nu}{2M} F_P(Q^2)$$

P. G. Blunden, W. Melnitchouk, and J. A. Tjon (2003)



F_D F_P

Dirac and Pauli form factors

IR divergencies are subtracted

L.C. Maximon and J. A. Tjon (2000)

Point-like couplings → $F_D(Q^2) = 1$ $F_P(Q^2) = \mu_P - 1$

Dipole FFs for G_M, G_E

Box diagram model

The one-photon exchange on-shell vertex

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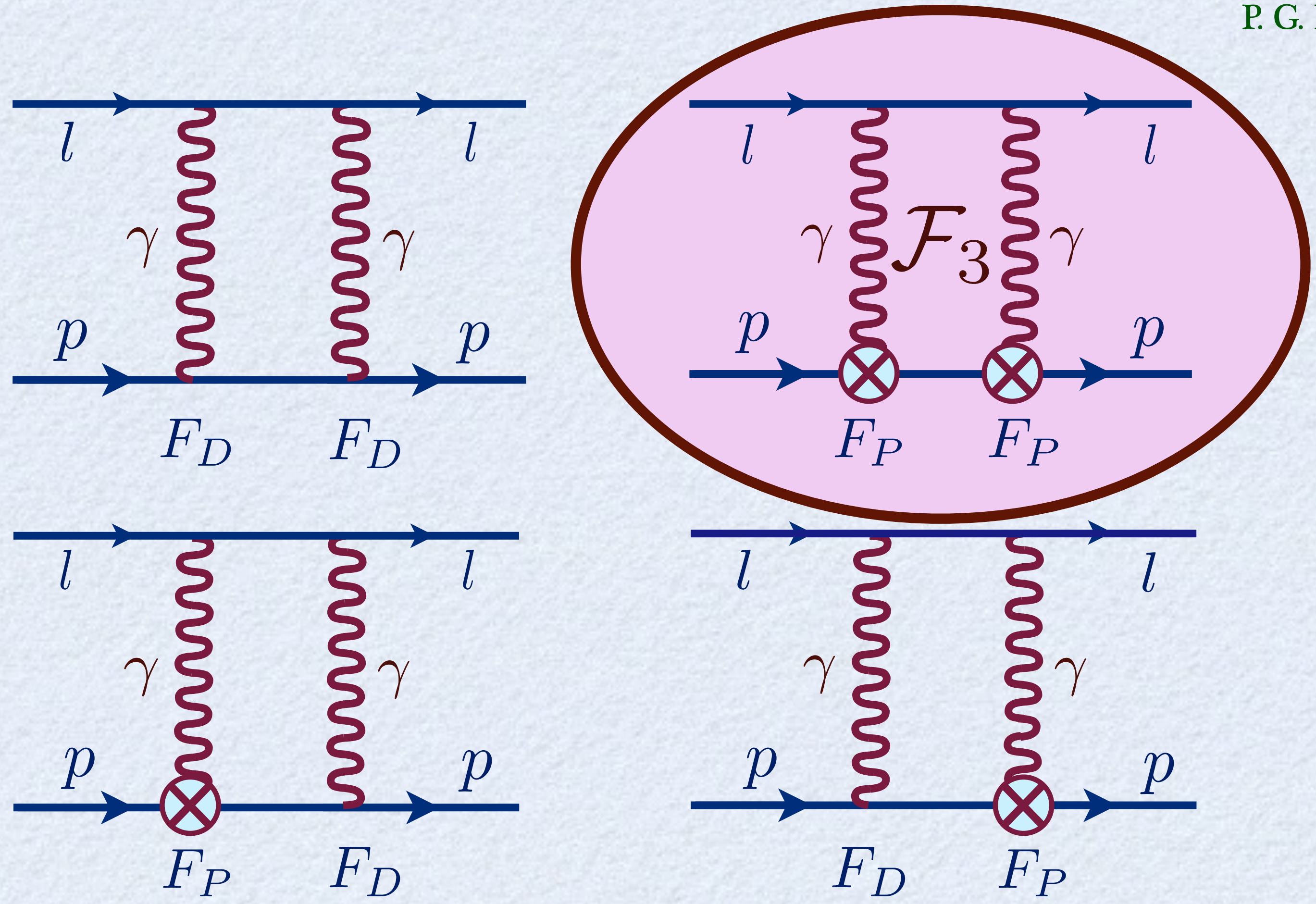
P. G. Blunden, W. Melnitchouk, and J. A. Tjon (2003)



$F_D \quad F_P$

Dirac and Pauli form factors

IR divergencies are subtracted

L.C. Maximon and J. A. Tjon (2000)

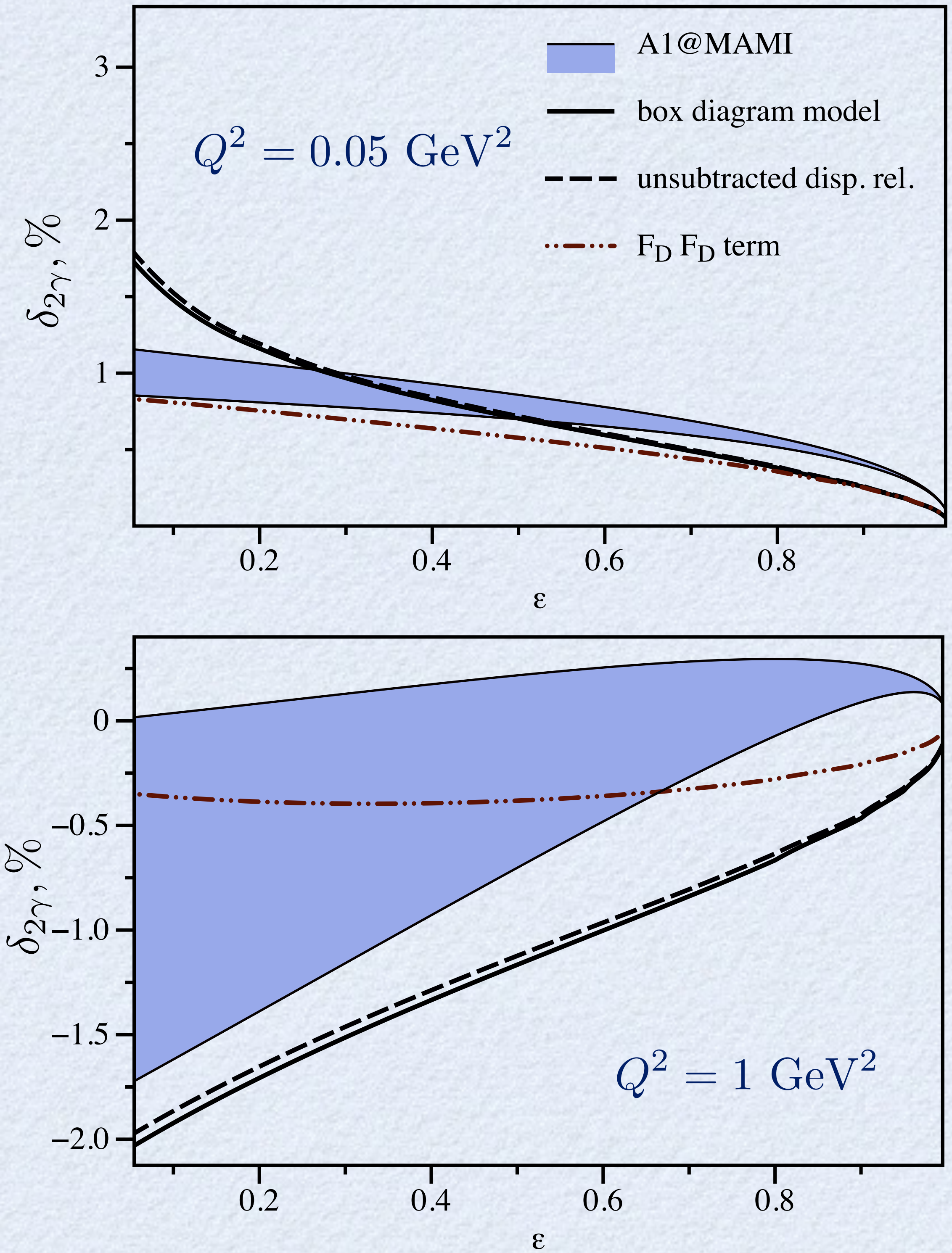
Point-like couplings → $F_D(Q^2) = 1 \qquad F_P(Q^2) = \mu_P - 1$

Dipole FFs for G_M, G_E

unsubtracted disp. rel. in ep scattering disagree with model

2ɣ in e⁻p elastic scattering

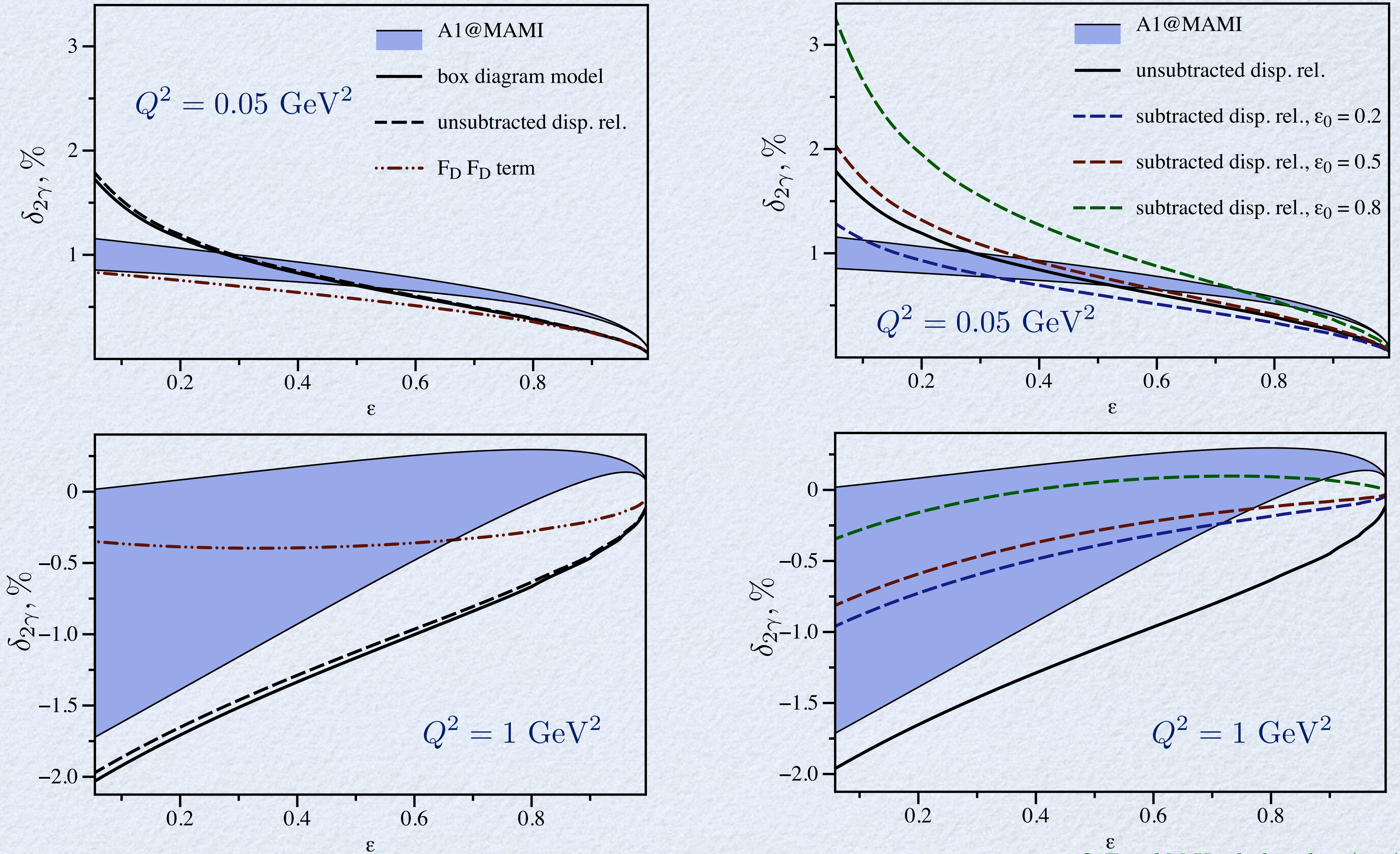
box diagram model vs. unsubtr. dis. rel.



2ɣ in e⁻p elastic scattering

box diagram model vs. unsubtr. dis. rel.

unsubtracted vs. subtracted disp. rel.

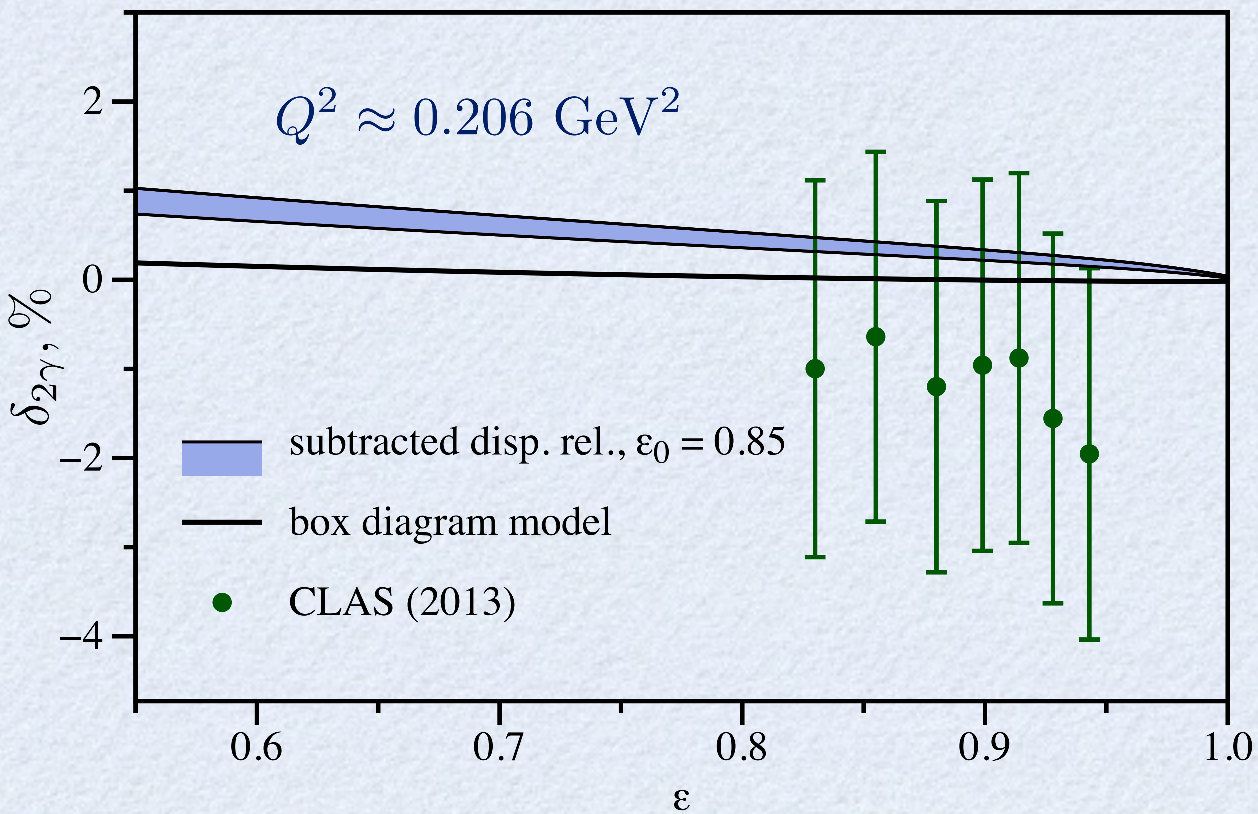


O. T. and M. Vanderhaeghen (2015)

Proton only partially accounts for 2ɣ at low ε and large Q²

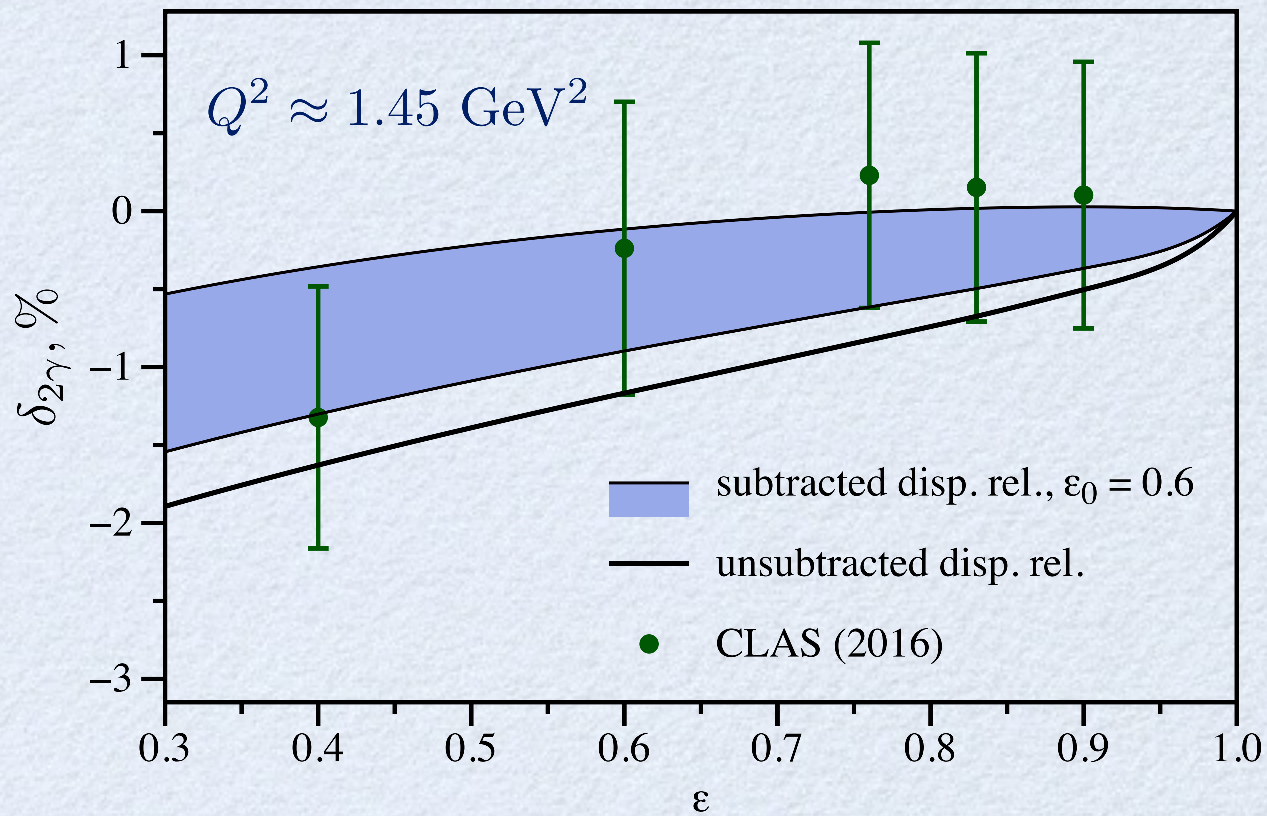
CLAS data and 2ɣ

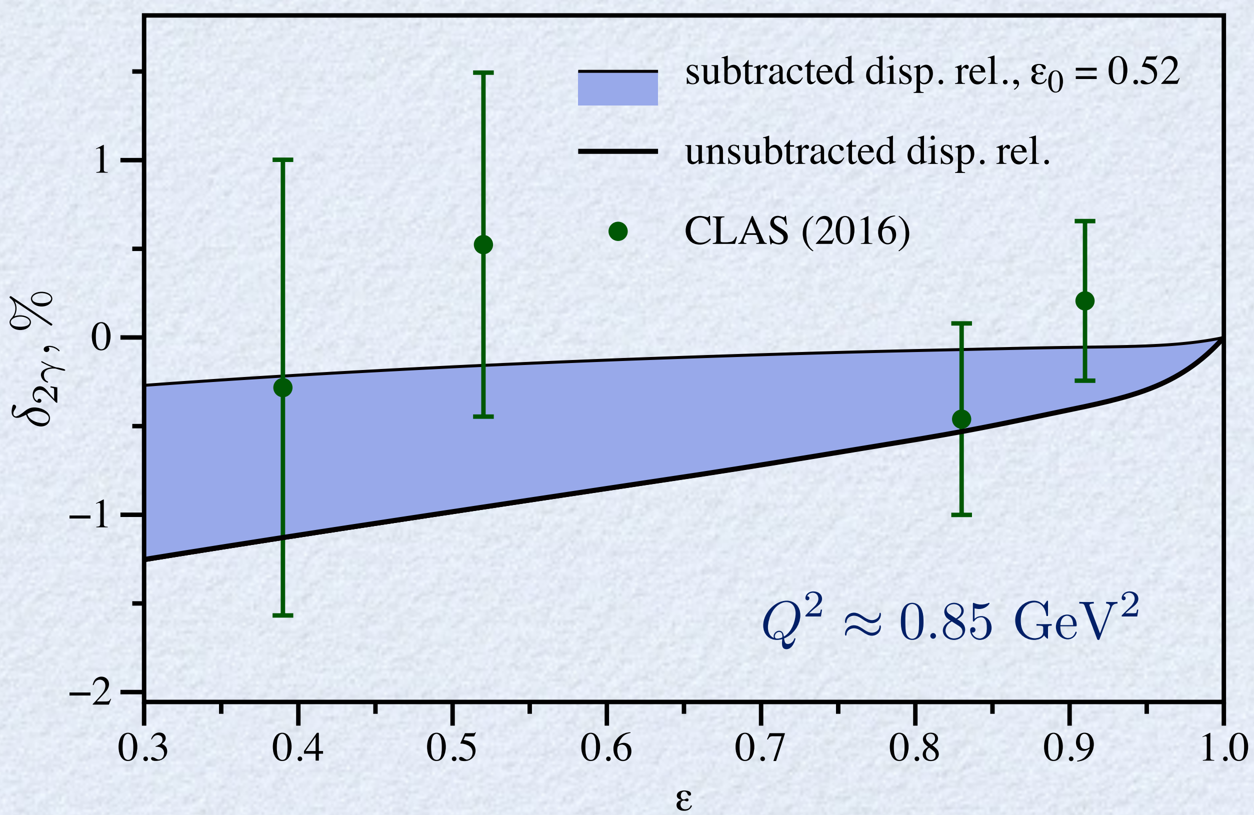
box diagram model vs. subtracted dis. rel.



2ɣ data from
e^+p/e^-p cross section ratio

$$R = \frac{\sigma(e^+p)}{\sigma(e^-p)} \approx 1 - 2\delta_{2\gamma}$$





data points compatible with zero

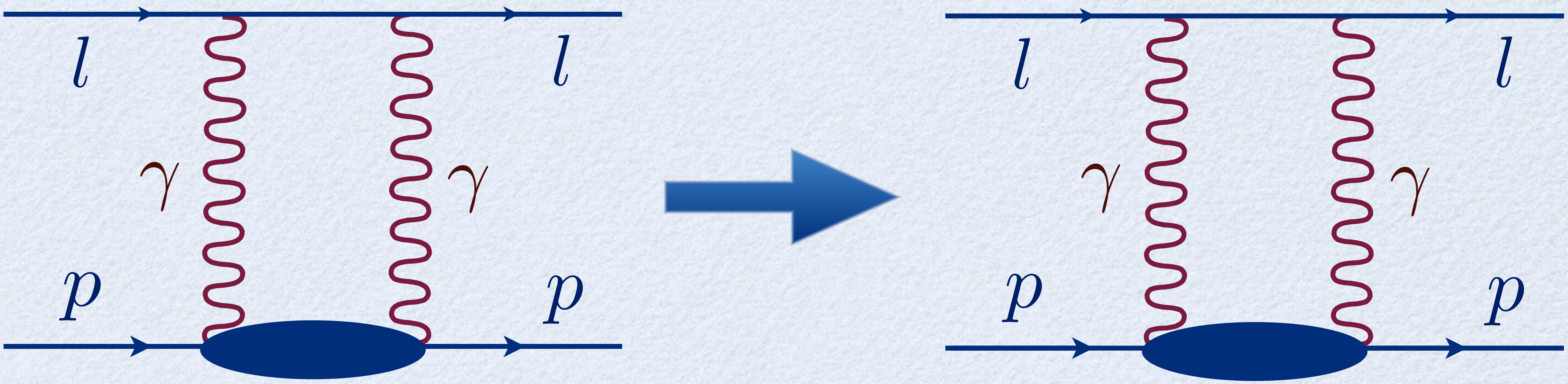
CLAS data
in agreement with 2ɣ fit

k′=k

p′=p

k′≠k

p′≠p

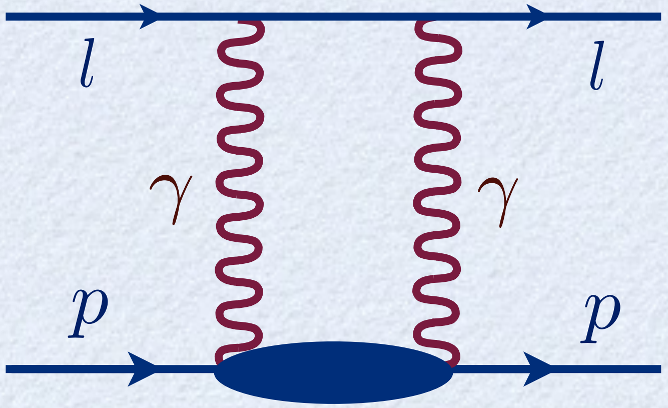


forward scattering

near-forward scattering

account for inelastic 2γ

Low-Q^2 inelastic 2ɣ correction (e^-p)



TPE blob - near-forward virtual Compton scattering

Q, $Q^2 \ln^2 Q^2$, $Q^2 \ln Q^2$ terms reproduced

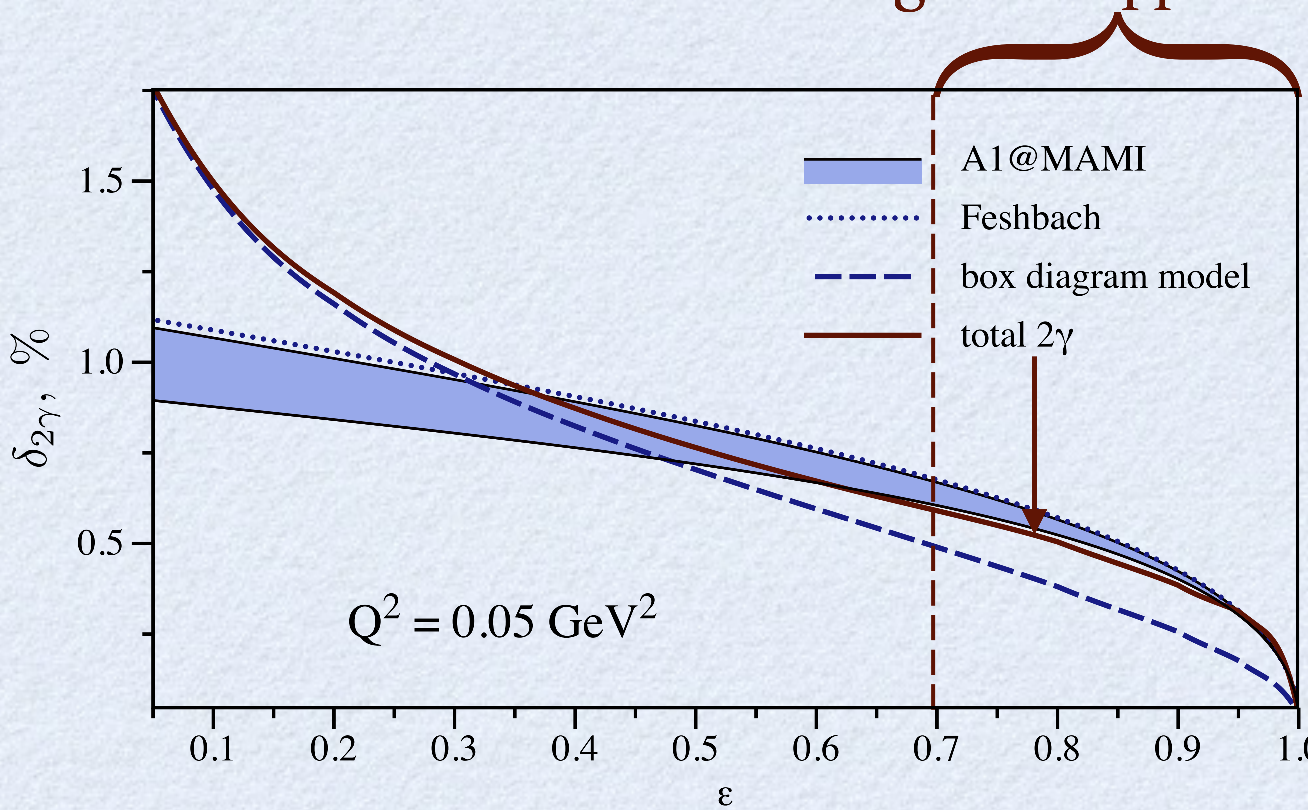
R. W. Brown (1970), M. Gorchtein (2013)

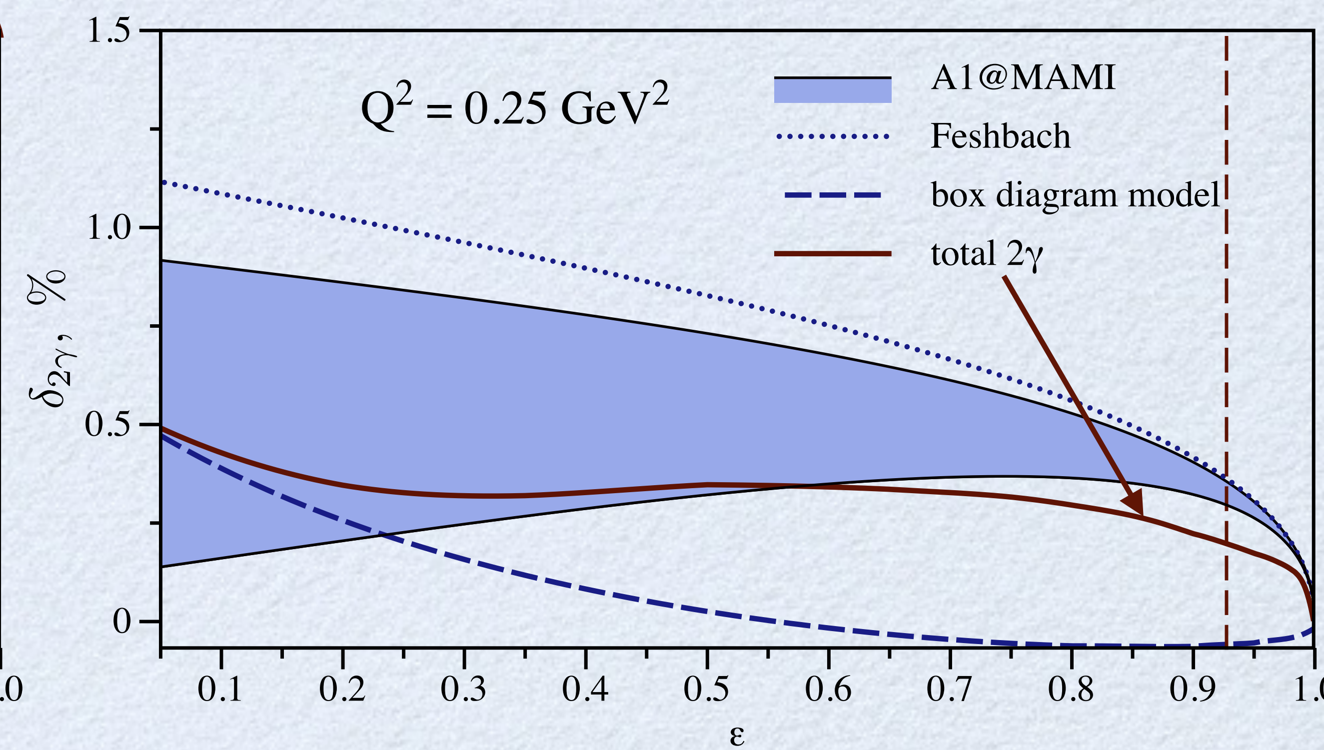
unpolarized proton structure

M. E. Christy, P. E. Bosted (2010)

$$\delta_{2\gamma} = \int d\nu_\gamma d\tilde{Q}^2 (\omega_1(\nu_\gamma, \tilde{Q}^2) \cdot F_1(\nu_\gamma, \tilde{Q}^2) + \omega_2(\nu_\gamma, \tilde{Q}^2) \cdot F_2(\nu_\gamma, \tilde{Q}^2))$$

region of applicability





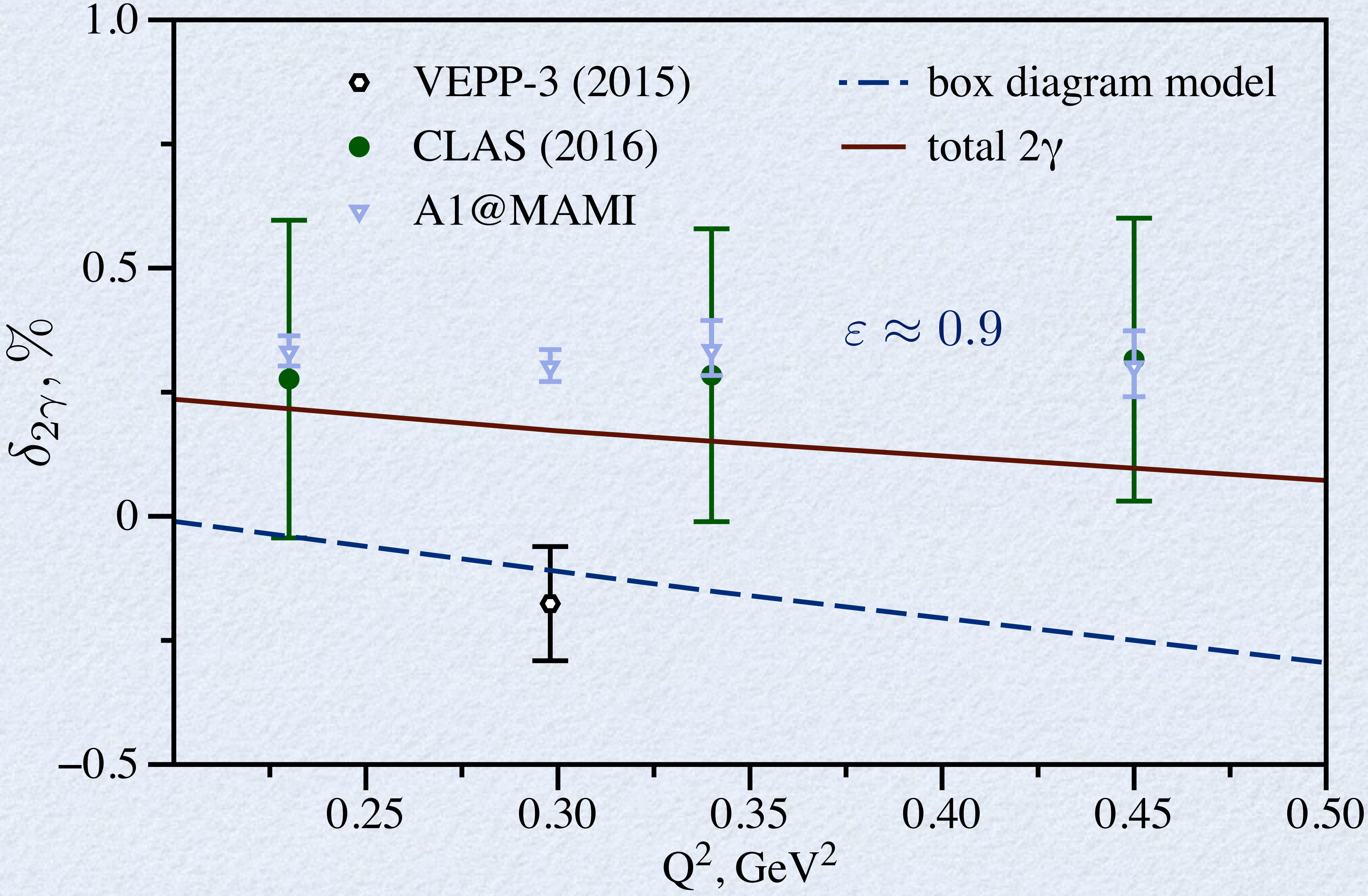
O. T. and M. Vanderhaeghen (2016)

2ɣ at large ε agrees with empirical fit

r_E extraction ✓

Low-Q^2 inelastic 2ɣ correction (e^-p)

comparison with low Q^2 measurements

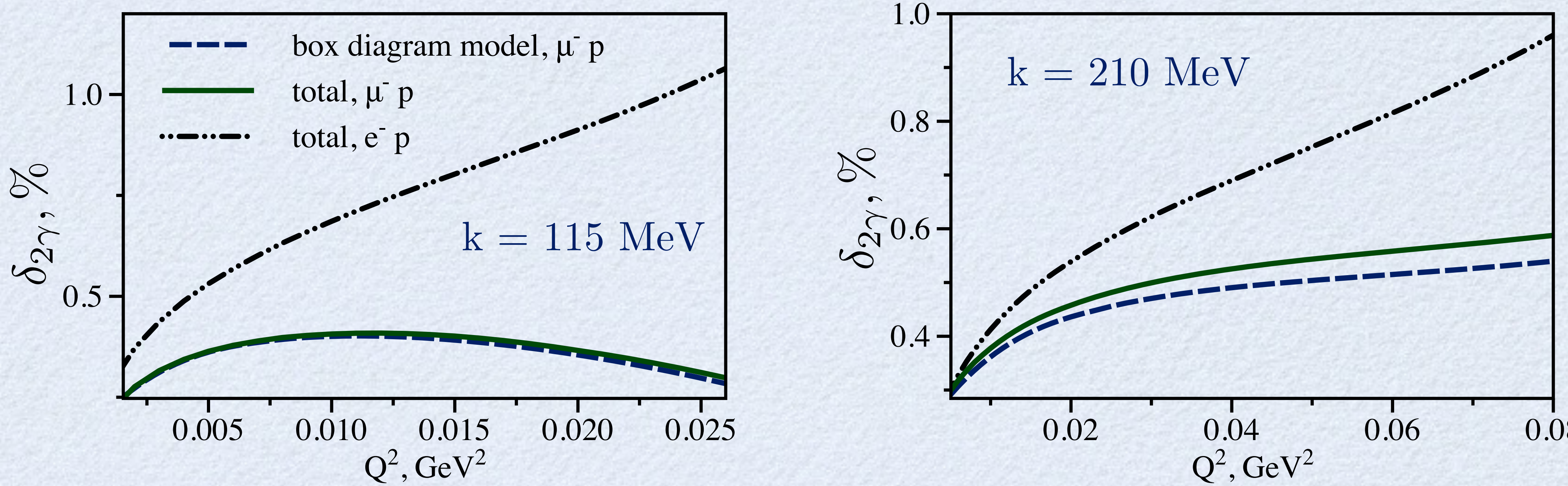


CLAS data in agreement with Born + inelastic 2ɣ

VEPP-3 data in agreement with Born 2ɣ only

MUSE estimates (μ^-p)

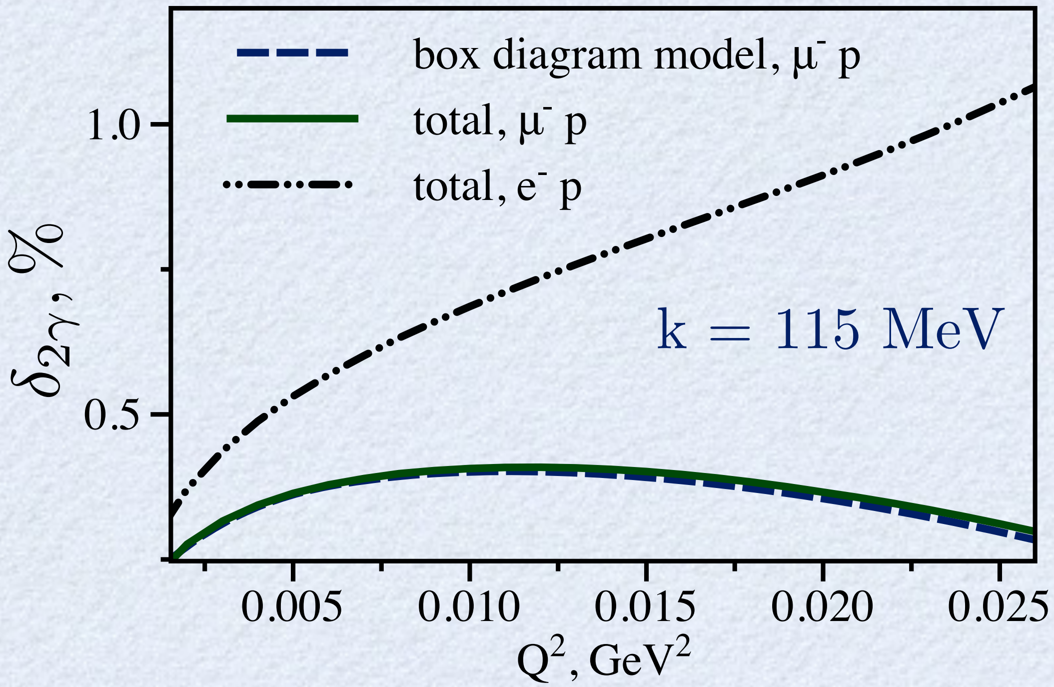
proton box diagram model + inelastic 2γ

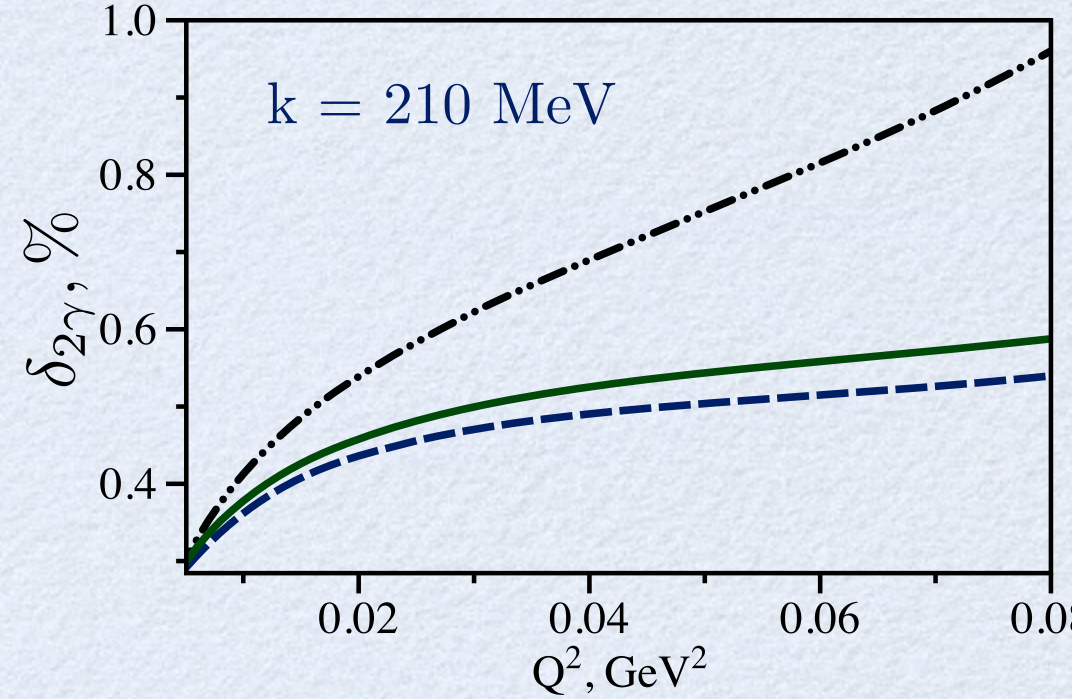


O. T. and M. Vanderhaeghen (2014, 2016)

MUSE estimates (μ^-p)

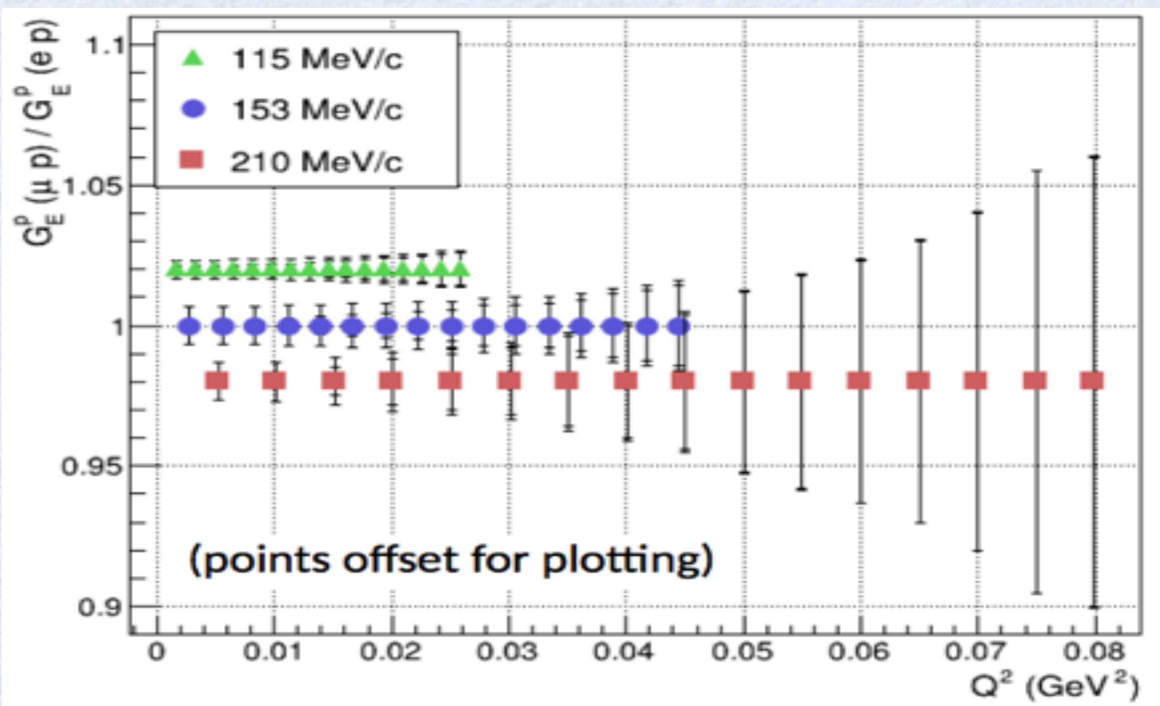
proton box diagram model + inelastic 2ɣ





O. T. and M. Vanderhaeghen (2014, 2016)

expected muon over electron ratio



K. Mesick talk (PAVI 2014), MUSE TDR (2016)

small inelastic 2ɣ

↓ ↓ ↓

small 2ɣ uncertainty

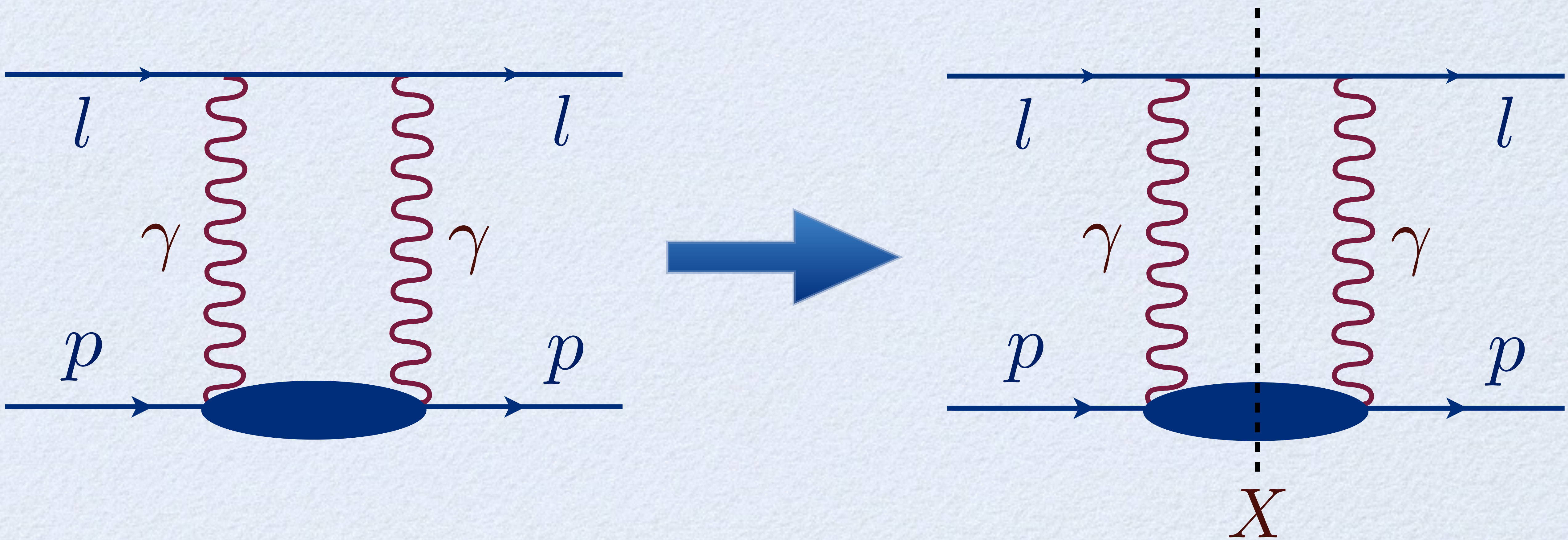
MUSE experiment can test r_E extraction

k′≠k

p′≠p

k′≠k

p′≠p



near-forward scattering

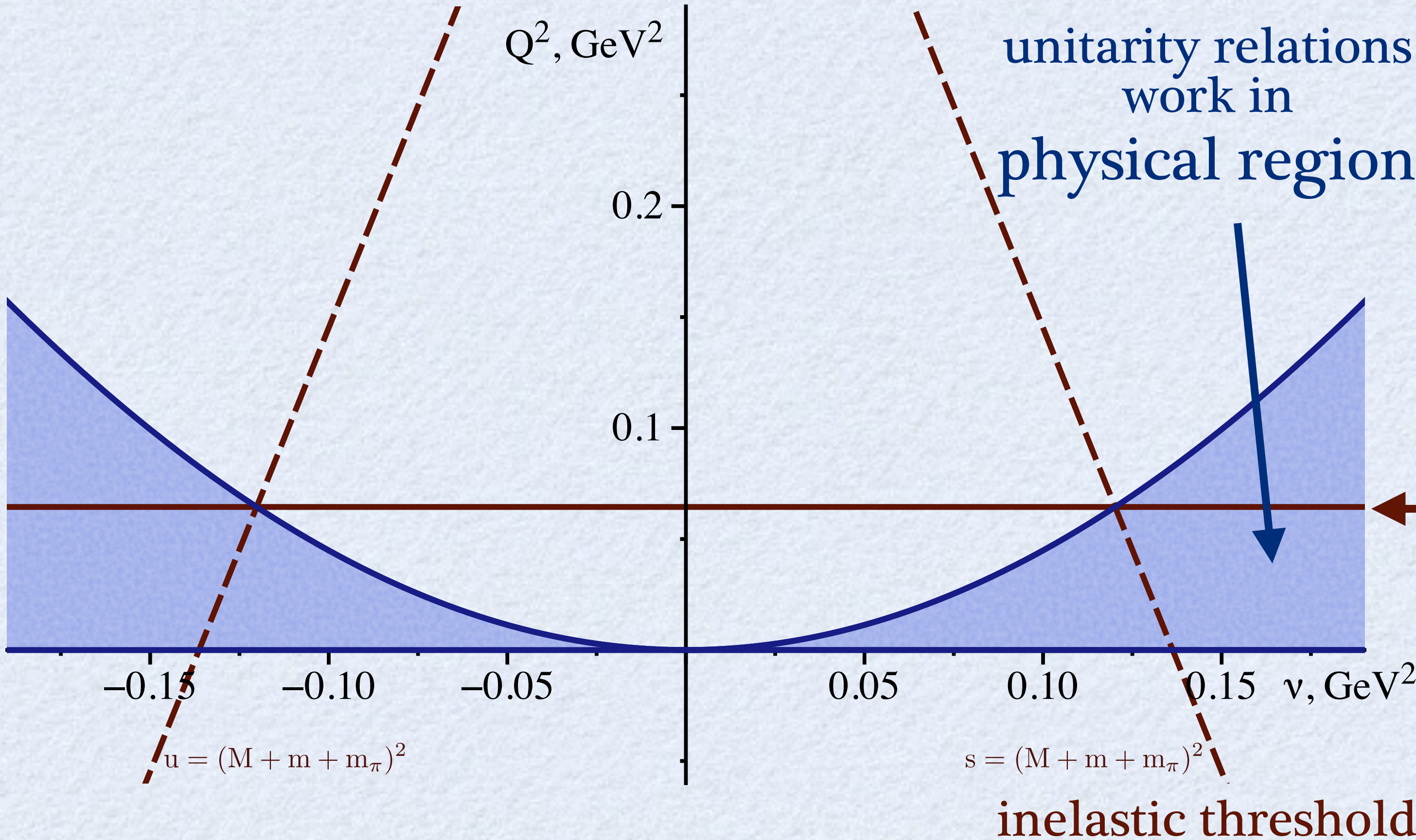
elastic + inelastic

non-forward scattering

disp. rel.

$$X = p + \pi N$$

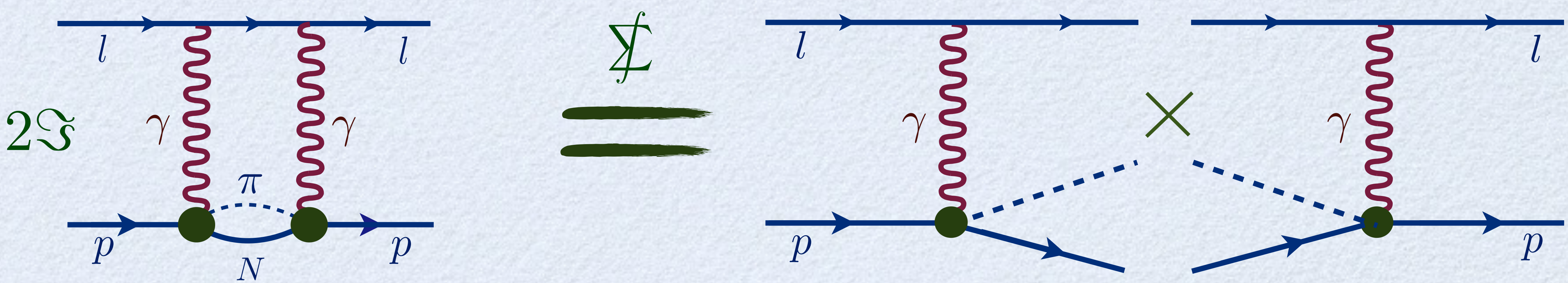
πN in dispersive framework (e$^-$p)



intersection of phys. region and inelastic threshold

$$Q^2 = m_\pi^2 \left(\frac{2M + m_\pi}{M + m_\pi}\right)^2 \sim 3.5 m_\pi^2$$

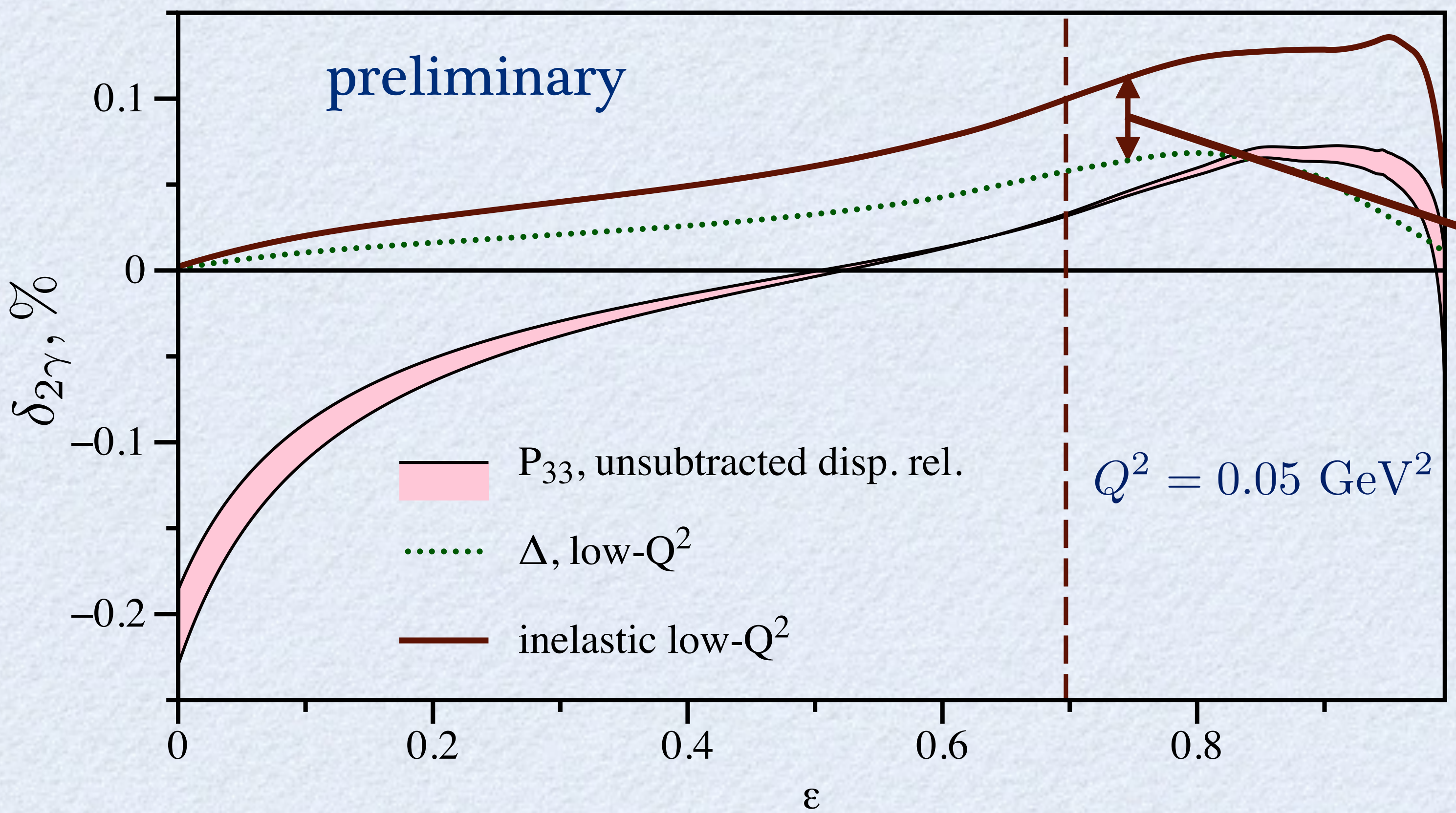
Unitarity relations



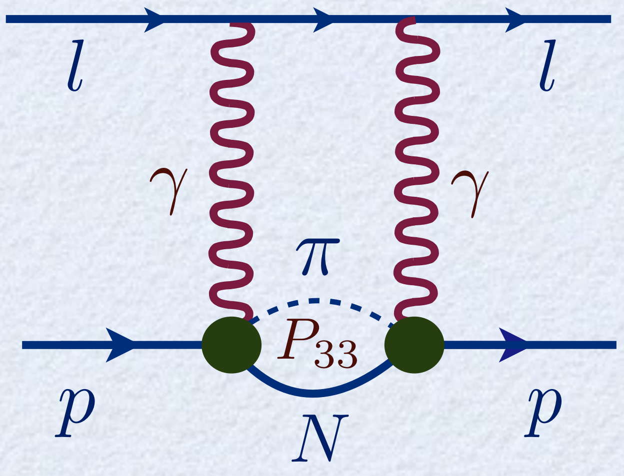
Pion electroproduction amplitudes are taken from MAID

πN in dispersive framework (e^-p)

unsubtracted disp. rel.

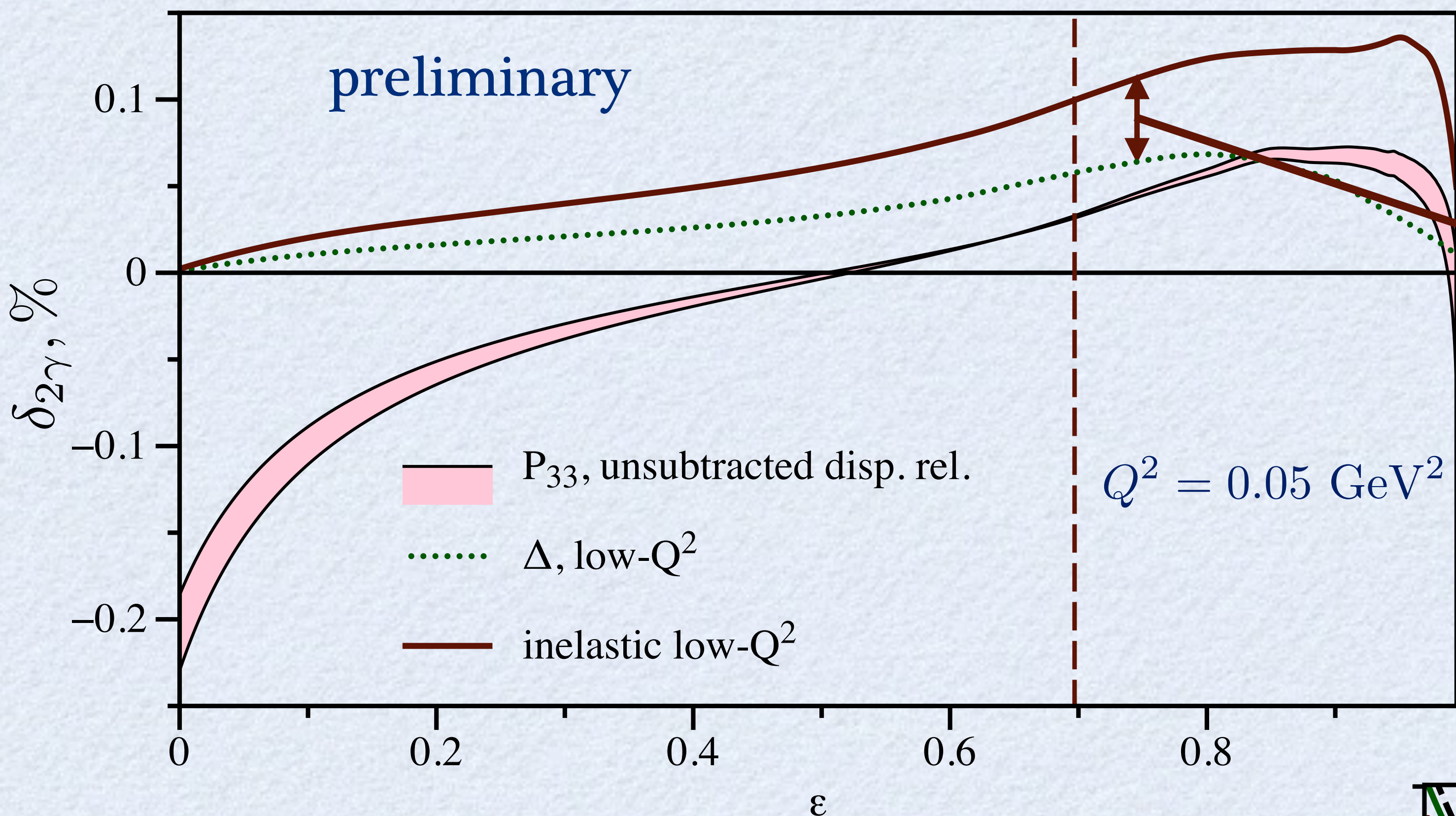


other partial waves important

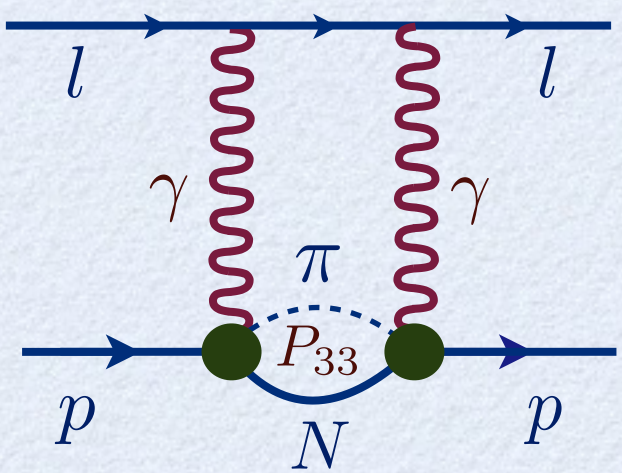


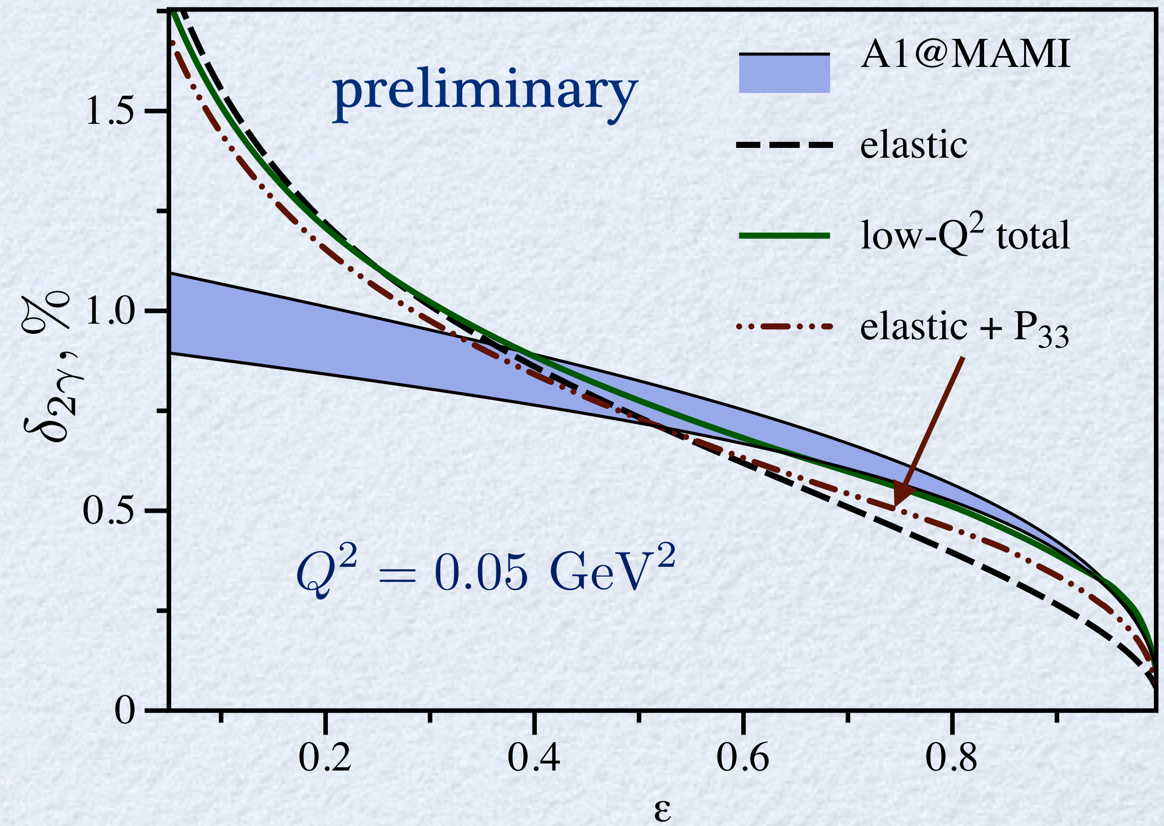
πN in dispersive framework (e⁻p)

unsubtracted disp. rel.



other partial waves important





P_{33} contributes up to 1/2 of inelastic 2γ

Conclusions

- Forward limit of 2γ in lp scattering
- Proton T1 subtraction function estimated from data
- Subtracted disp. rel. formalism for ep scattering
- Theoretical estimates for 2γ (ep and μp)
- First estimates for πN channel in disp. rel.

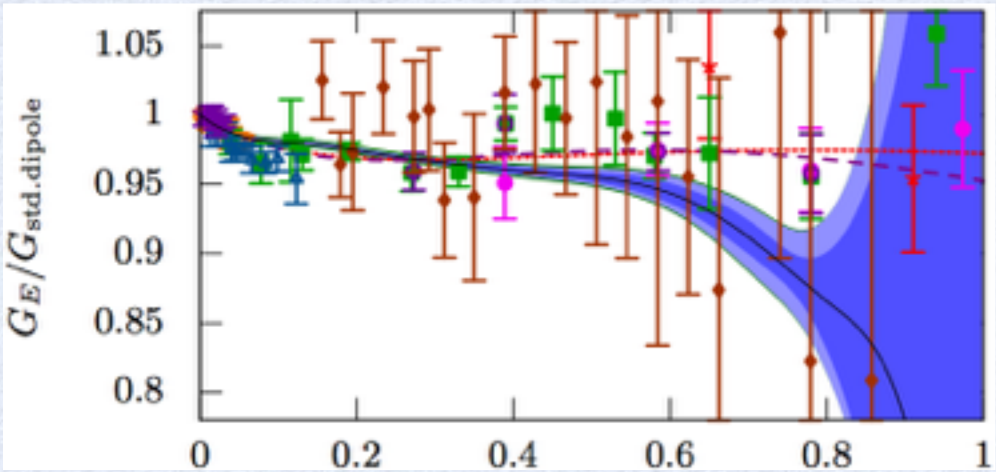
Outlook

- Application to forthcoming high-precision HFS exp.
- Extraction of magnetic radius accounting for 2γ
- Comparison with VEPP-3, CLAS, OLYMPUS

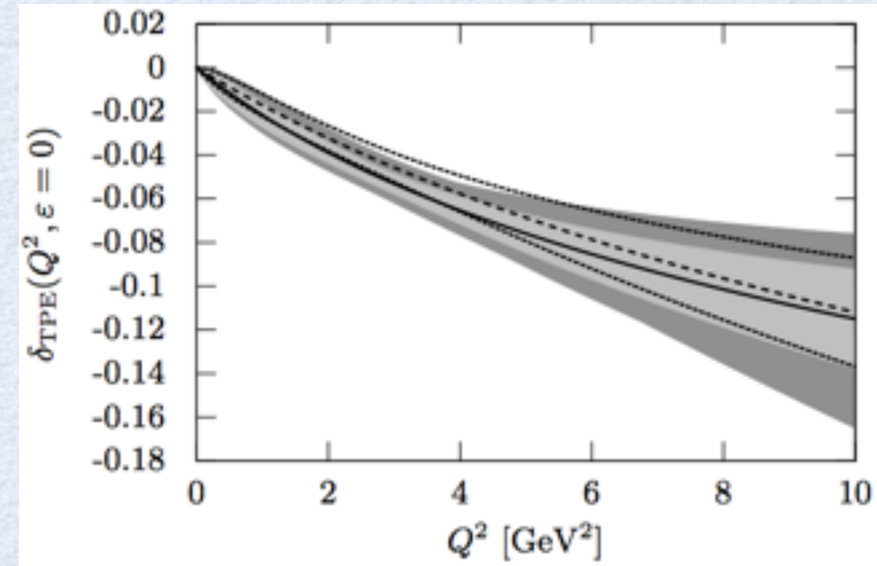
Thanks for your attention !!!

Fixed-Q^2 dispersion relation framework

2γ corrections



on-shell one-photon amplitudes



$f(z)$

analyticity

exp. data/phenomenology

cross section correction

unitarity

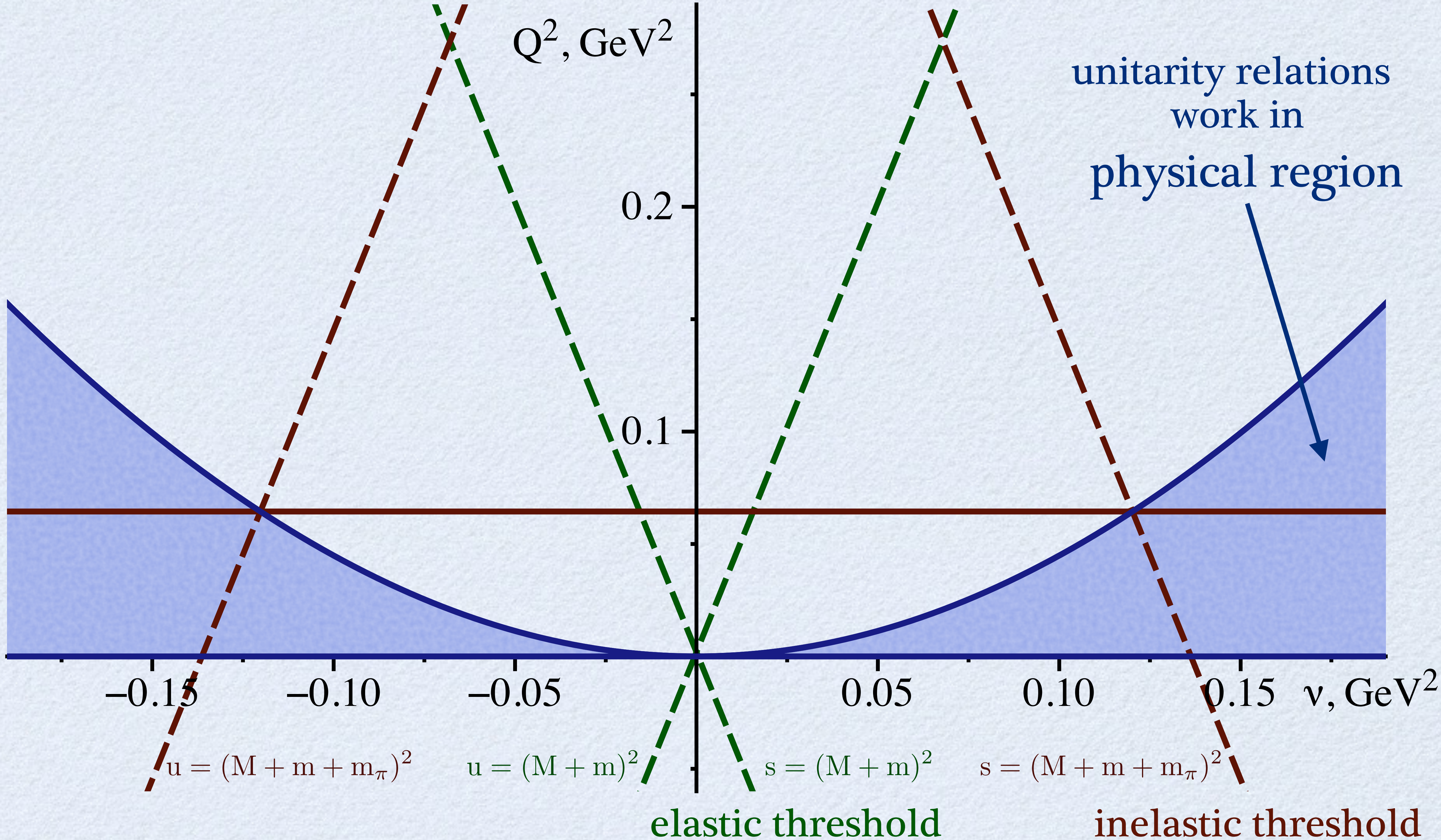
$$\Re \mathcal{F}(\nu) = \frac{2\nu}{\pi} \mathcal{P} \int_{\nu_{min}}^{\infty} \frac{\Im \mathcal{F}(\nu' + i0)}{\nu'^2 - \nu^2} d\nu'$$

DR

amplitudes: imaginary parts $\longmapsto$ amplitudes: real parts

D. Borisyuk, A. Kobushkin (2008)

Mandelstam plot (ep)



Proton intermediate state is outside physical region

Analytical continuation for arbitrary FFs parametrization is found

O. Tomalak and M. Vanderhaeghen (2015)

Hadronic model vs. dispersion relations

- Imaginary parts are the same
- Real parts are the same for

all $F_D F_D$ amplitudes

$F_D F_P$ amplitudes: $\mathcal{G}_M \quad \mathcal{F}_2 \quad \mathcal{F}_3 \quad \mathcal{F}_5$

$F_P F_P$ amplitudes: $\mathcal{F}_2 \quad \mathcal{G}_M + \frac{\nu}{M^2}\mathcal{F}_3 \quad \mathcal{F}_5$

Fixed-Q^2 subtracted dispersion relation works for all amplitudes

- Calculation based on DR for ep scattering

- for amplitudes $\mathcal{G}_1, \ \mathcal{G}_2$ unsubtracted DR can be used

- for amplitude $\mathcal{F}_3$ subtracted DR should be used

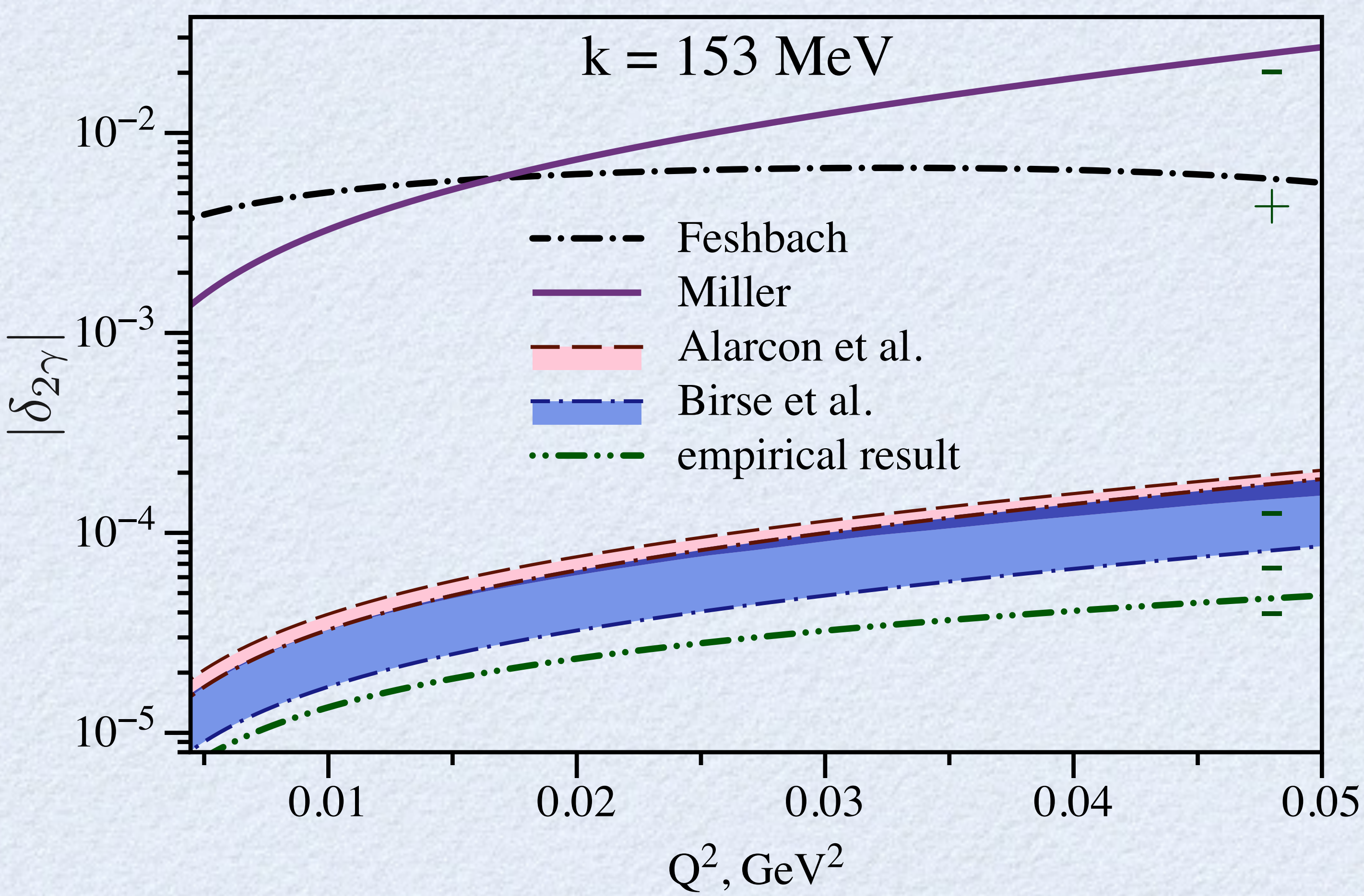
- subtraction point $\Re\mathcal{F}_3^{F_P F_P}(\nu_0, Q^2)$ fixed from $\delta_{2\gamma}(\nu_0, Q^2)$ data

T_1 subtraction function TPE correction

Subtraction function contributes only to $\mathcal{F}_4$ amplitude

$$\delta_{2\gamma,0}^{\text{subt}} \approx -\frac{Q^2 m^2}{\omega}\int\limits_0^\infty f\left(x,\frac{Q^2}{m^2}\right)\beta\left(\frac{Q^2\left(x-1\right)}{4}\right)\mathrm{d}x$$

In the limit of small electron mass TPE correction vanishes



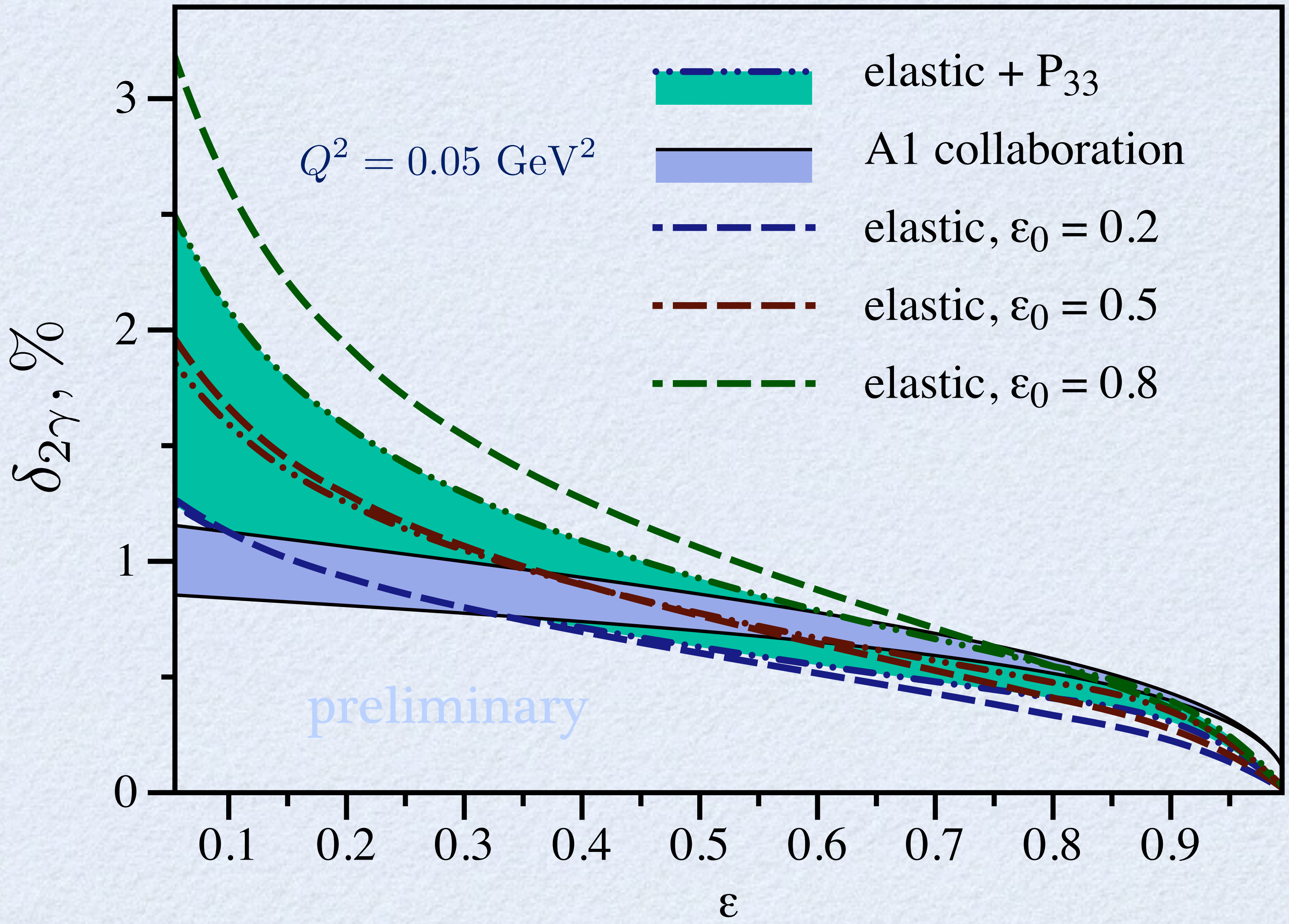
Valid only for small Q^2

For enhanced at HE function

$$\delta_{2\gamma,0}^{subt} \approx -\frac{3Q^2m^2}{2\pi\omega}\int\limits_0^\infty \beta\left(\tilde{Q}^2\right)\frac{\mathrm{d}\tilde{Q}^2}{\tilde{Q}^2}$$

πN TPE contribution in dispersive framework (ep)

subtracted DR



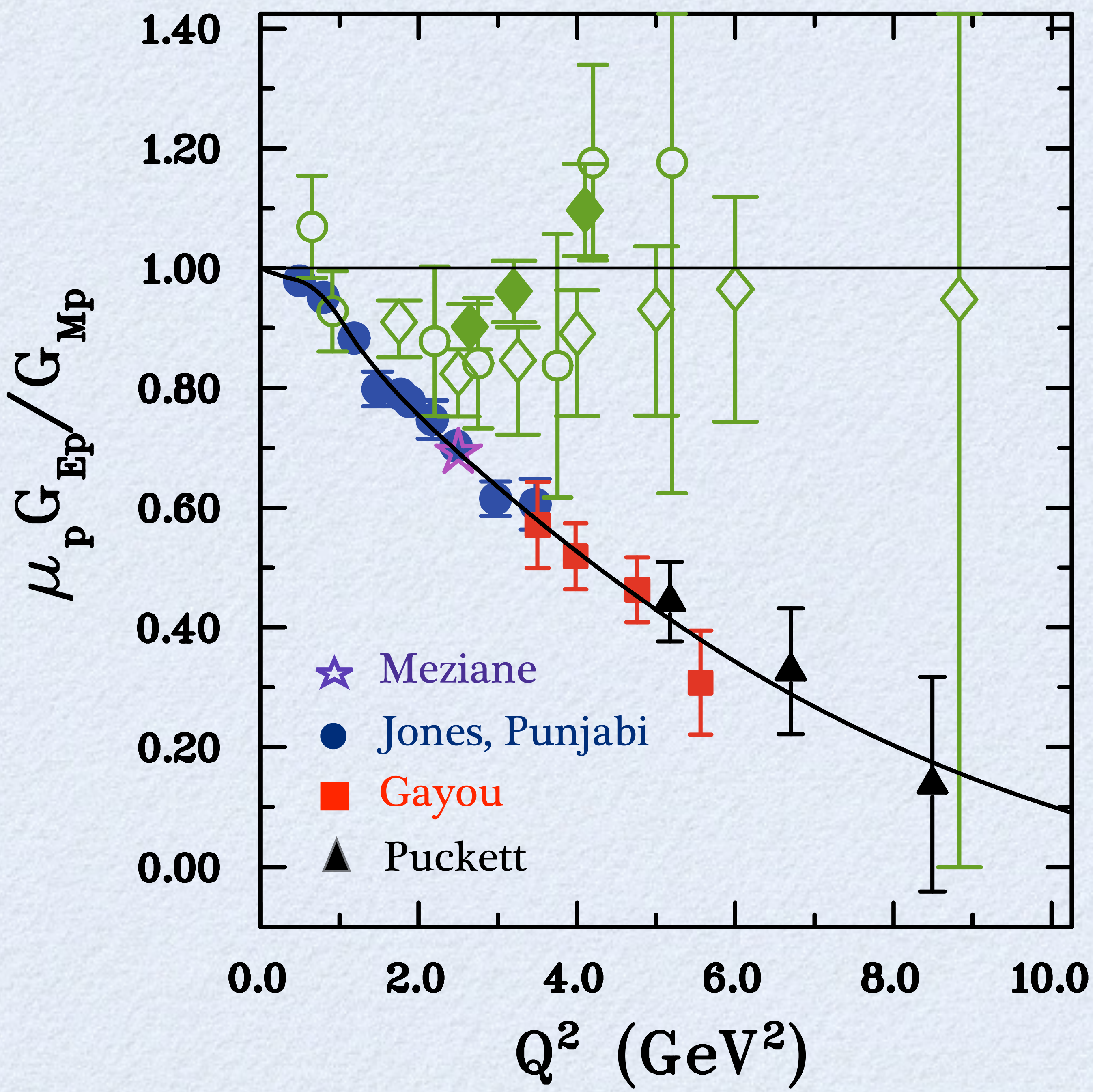
account of P_{33} channel decreases uncertainty with subtracted DR

Proton form factors problem

Polarization transfer
JLab (Hall A, C)

vs.

Rosenbluth separation
SLAC, JLab (Hall A, C)



◇ Andivahis et al. (1994)

○ Christy et al. (2004)

◆ Quattan et al. (2005)

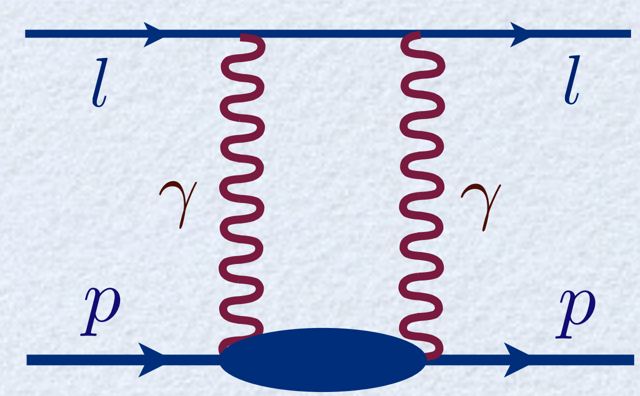
GEpI — Jones et al. (2000), Punjabi et al. (2005)

GEpII — Gayou et al. (2002)

GEpIII — Puckett et al. (2010)

GEp2γ — Meziane et al. (2011)

fit — Gayou et al. (2002)



V. Punjabi et al. (2015)

A possible explanation - two-photon exchange