Two-photon exchange corrections in elastic lepton-proton interaction

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Outline

forward virtual Compton scattering

lepton

\[ l = e, \mu \]

hadronic state

\[ X = p, \pi N \]

Kinematics

forward \( Q^2 = 0 \)
calculable

small \( Q^2 \)
proton structure functions

larger \( Q^2 \)
disp. rel.
1γ approximation

\[ \Gamma^\mu (Q^2) = \gamma^\mu F_D(Q^2) + \frac{i\sigma^{\mu\nu} q_\nu}{2M} F_P(Q^2) \]

Dirac and Pauli form factors

\[ \nu = \frac{s - u}{4} \quad s = (p + k)^2 \]
\[ u = (k - p')^2 \quad Q^2 = -(k - k')^2 \]

photon-proton vertex

momentum transfer

crossing symmetric variable

\[ T = \frac{e^2}{Q^2} \left( \bar{u}(k', h') \gamma^\mu u(k, h) \right) \cdot \left( \bar{N}(p', \lambda') \Gamma^\mu (Q^2) N(p, \lambda) \right) \]
Form factors in $1\chi$ approximation

Sachs electric and magnetic form factors

\[ G_E = F_D - \tau F_P, \quad G_M = F_D + F_P \]

kinematic variables

\[ \tau = \frac{Q^2}{4M^2}, \quad \epsilon = \frac{\nu^2 - \tau(1 + \tau)}{\nu^2 + \tau(1 + \tau)} \]

Rosenbluth separation

\[ \frac{d\sigma^{\text{unpol}}}{d\Omega} \sim \sigma_R = \tau \left( G_M^2(Q^2) + \frac{\epsilon}{\tau} G_E^2(Q^2) \right) \]

Rosenbluth slope is sensitive to corrections beyond $1\chi$
Form factors in 1\(\chi\) approximation

Sachs electric and magnetic form factors

\[ G_E = F_D - \tau F_P, \quad G_M = F_D + F_P \]

Polarization transfer

\[ \vec{e} + p \rightarrow \vec{e} + \vec{p} \]

realized in 2000

\[ P_T \sim G_E(Q^2)G_M(Q^2) \]

\[ P_L \sim G_M^2(Q^2) \]

\[ \frac{P_T}{P_L} \sim \frac{G_E(Q^2)}{G_M(Q^2)} \]
The early measurements of the form factors of the neutron have been used to measure polarization and beam-target asymmetry, both techniques that provide complete kinematical matching to the four-momentum transfer $Q^2$. The striking feature of the results of the GEp(3) experiment is possibly the continued, strong and almost linear decrease of $\mu_p G_{Ep}/G_{Mp}$ at $Q^2\approx 9 \text{GeV}^2$ versus $Q^2$. The GEp(3) overlap with $Q^2\approx 9 \text{GeV}^2$ is in good agreement with the two sur-rect of the form factors of the neutron charge measured at MIT-Bates laboratory in the late 80's using the exclusive $\mu p \rightarrow e^+ e^- \gamma p$ reaction \cite{107}. The advantage of using a complex light target like $\text{Mn}(\vec{e}, \vec{e}')_p$ vs. the deuteron target is that theoretical calculations predict the extracted neutron form factor results to be insensitive to effects like, final state interaction (FSI), meson exchange currents (MEC), isobar configurations (IC), and to the dependence of effects like, final state interaction (FSI), meson exchange currents (MEC), isobar configurations (IC), and to the dependence of $Q^2$.

The results of the three JLab GEp experiments are presented in figs. 8, 9, 10, 11, and a7. The parameter fit gives nine q.\cite{44}.

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Proton form factors puzzle

Polarization transfer
JLab (Hall A, C)

Rosenbluth separation
SLAC, JLab (Hall A, C)

A possible explanation
two-photon exchange

\[ \frac{\mu_{p}}{G_{Ep}} / G_{Mp} \]

\[ Q^2 \ (GeV^2) \]

V. Punjabi et al. (2015)
Beam normal spin asymmetry

Vanishing in $1\chi$ approximation

Clear evidence of $2\chi$
Form factors and size
form factor in atoms and nuclei
Fourier transform of charge distribution

How accurate do we know the proton size?
Proton charge radius

electric charge radius

\[ < r_E^2 > \equiv -\frac{1}{6} \frac{dG_E(Q^2)}{dQ^2} \bigg|_{Q^2=0} \]

- ep elastic scattering

\[ r_E = 0.879 \pm 0.008 \text{ fm} \]

J. C. Bernauer et al. (2014)
Proton form factors and size

hydrogen spectroscopy

S state has finite wave function at origin

μH is sensitive to charge distribution
Proton charge radius

electric charge radius

\[ <r_E^2> = \left. -\frac{1}{6} \frac{dG_E(Q^2)}{dQ^2} \right|_{Q^2=0} \]

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  \( r_E = 0.879 \pm 0.008 \text{ fm} \)

- atomic spectroscopy

H, D spectroscopy

\( r_E = 0.8758 \pm 0.0077 \text{ fm} \)

CODATA 2010

\[ \Delta E_{nS} \sim m_r^3 <r_E^2> \]

μH Lamb shift

\( r_E = 0.8409 \pm 0.0004 \text{ fm} \)

CREMA (2010, 2013)
Proton radius puzzle

electric charge radius

\[ < r_E^2 > = - \frac{1}{6} \left. \frac{dG_E(Q^2)}{dQ^2} \right|_{Q^2=0} \]

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\[ \mu \text{H Lamb shift} \]

7 \sigma difference!
µH Lamb shift and 2\( \chi \)

**2P-2S transition in µH**

Discrepancy \( 310 \, \mu eV \)

µH uncertainty \( 2.5 \, \mu eV \)

---

The obtained experimental values for the Lamb shift and the HFS \[ 24, 25, 226 \],

\[
E_{\text{exp}}^{\text{LS}} = \frac{1}{4} h \nu_t + \frac{3}{4} h \nu_s + E_{\text{HFS}}(2P_3) - \frac{1}{8} E_{\text{HFS}}(2P_3) = 202(3706(23) \, \text{meV},
\]

\[
E_{\text{exp}}^{\text{HFS}}(2S) = h \nu_s + h \nu_t + E_{\text{HFS}}(2P_3) - E_{\text{HFS}}(2P_3) = 22(8089(51) \, \text{meV},
\]

thus rely on the theoretical calculation of the fine and hyperfine splittings of the 2\( P \)-levels \[ 227 \].
μH Lamb shift and $2\chi$

$\Delta E^{2\gamma}_{2P-2S} = 33 \pm 2 \, \mu eV$


Discrepancy $310 \, \mu eV$

μH uncertainty $2.5 \, \mu eV$
µH HFS and 2χ

Impressive 1 ppm accuracy requires improvement on 2χ

<table>
<thead>
<tr>
<th></th>
<th>$10^3 \Delta$</th>
<th>relative uncertainty</th>
</tr>
</thead>
<tbody>
<tr>
<td>$X=p$</td>
<td>-6.51</td>
<td>140 ppm</td>
</tr>
<tr>
<td>$X=\pi N,\ldots$ (polarizability)</td>
<td>0.373</td>
<td>92 ppm</td>
</tr>
<tr>
<td>total</td>
<td>-6.137</td>
<td>168 ppm</td>
</tr>
</tbody>
</table>

A. Antognini (BVR47@PSI 2016)
Scattering experiments and $2\chi$

$\sqrt{<r_E^2>}$, fm

MAMI scatt Bernauer et al
JLAB scatt Zhan et al
H&D spectr CODATA 2010

PSI, $\mu$H Antognini et al.

$2\chi$ is not fully accounted in scattering experiments

$\sigma^{\text{exp}} \equiv \sigma_{1\gamma}(1 + \delta_{\text{rad}} + \delta_{\text{soft}} + \delta_{2\gamma})$

charge radius only slightly depends on $2\chi$
magnetic radius significantly depends on $2\chi$

J. C. Bernauer et al. (2014)
Scattering experiments and $2\chi$

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$\mu p$ elastic scattering is planned by MUSE@PSI(2017-18)

$2\chi$ correction in MUSE ?

J. C. Bernauer et al. (2014)
$k' = k = (m,0,0,0)$

$p' = p = (M,0,0,0)$

forward scattering at zero energy
(atomic correction)
Lamb shift $2\gamma$ correction. Forward VVCS

Shift of S energy level $2\gamma$ correction

$$\Delta E_{nS}^{2\gamma} \sim f_+ |\psi_n(0)|^2$$

$f_+ -$ unpolarized $2\gamma$ amplitude

$2\gamma$ blob - forward virtual Compton scattering

photon energy

$$\nu_\gamma = \frac{p \cdot q}{M}$$

photon virtuality

$$Q^2 = -q^2$$

Forward VVCS tensor

$$M^{\mu\nu} = M_S^{\mu\nu} + M_A^{\mu\nu}$$

$M_S^{\mu\nu} \sim T_1(\nu_\gamma, Q^2), T_2(\nu_\gamma, Q^2)$

spin-independent amplitudes

$M_A^{\mu\nu} \sim S_1(\nu_\gamma, Q^2), S_2(\nu_\gamma, Q^2)$

spin-dependent amplitudes
Forward VVCS. Dispersion relations

Optical theorem
relates Compton amplitudes
to proton structure functions

\[ \text{Im } T_1 \sim F_1 \quad \text{Im } T_2 \sim F_2 \quad \text{Im } S_1 \sim g_1 \quad \text{Im } S_2 \sim g_2 \]

Fixed-\(Q^2\) dispersion relations

Dis. rel. for amplitude \(T_1\) requires subtraction function

\[
T_1^{\text{subt}}(0, Q^2) \equiv T_1(0, Q^2) - T_1^{\text{Born}}(0, Q^2)
\]

Unsubtracted disp. rel. works for \(T_2, S_1, S_2, \nu\gamma S_2\)
Empirical estimate of subtraction function

High-energy behavior of $T_1$ in Regge theory

$$T_1^R(\nu_\gamma, Q^2) \sim \sum_{\alpha_0 > 0} \frac{\gamma_{\alpha_0}(Q^2)}{\sin \pi \alpha_0} \left\{ (\nu_0 - \nu_\gamma - i\varepsilon)^{\alpha_0} + (\nu_0 + \nu_\gamma - i\varepsilon)^{\alpha_0} \right\}$$

$$+ \sum_{\alpha_0 > 1} \frac{\alpha_0 \nu_0 \gamma_{\alpha_0}(Q^2)}{\sin \pi (\alpha_0 - 1)} \left\{ (\nu_0 - \nu_\gamma - i\varepsilon)^{\alpha_0-1} + (\nu_0 + \nu_\gamma - i\varepsilon)^{\alpha_0-1} \right\}$$


Evaluate dispersion relation for $T_1(\nu_\gamma, Q^2) - T_1^R(\nu_\gamma, Q^2)$

$$T_{1\text{subt}}(0, Q^2) = T_1^R(0, Q^2) \frac{\alpha}{M} F_D^2(Q^2) + \frac{2\alpha}{M} \int_{\nu_{\text{thr}}}^{\infty} F_1(\nu_\gamma, Q^2) - F_1^R(\nu_\gamma, Q^2) \frac{1}{\nu_\gamma} d\nu_\gamma$$
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$$+ \sum_{\alpha_0 > 1} \frac{\alpha_0 \nu_0 \gamma_{\alpha_0}(Q^2)}{\sin \pi (\alpha_0 - 1)} \{ (\nu_0 - \nu_\gamma - i\varepsilon)^{\alpha_0-1} + (\nu_0 + \nu_\gamma - i\varepsilon)^{\alpha_0-1} \}$$


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Donnachie-Landshoff and Bosted-Christy fits at low $Q^2$

Forthcoming JLab data will improve fits around $W^2 \sim 10$ GeV$^2$
Empirical estimate of subtraction function

Empirical result vs. theoretical predictions

Expected low-$Q^2$ behavior

$T_{1}^{\text{subt}}(0, Q^2) = \beta_M Q^2 + O(Q^4)$

satisfied within $1.5\sigma$

O. Tomalak and M. Vanderhaeghen (2016)
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O. Tomalak and M. Vanderhaeghen (2016)

empirical estimate connected to p.d.g. value of magnetic polarizability

$$\Delta E_{2S}^{\text{subt}}(\mu H) \approx 2.3 \pm 1.3 \, \mu eV$$

slightly smaller than Birse et al.

$$\Delta E_{2S}^{\text{subt}}(\mu H) \approx 4.2 \pm 1.0 \, \mu eV$$
forward scattering

\[ k' = k \]
\[ p' = p \]

non-forward scattering

\[ k' \neq k \]
\[ p' \neq p \]
Structure amplitudes

Electron scattering is described by 3 structure amplitudes

\[ T^{\text{non-flip}} \sim G_M(\nu, Q^2), F_2(\nu, Q^2), F_3(\nu, Q^2) \]

Muon scattering requires lepton helicity-flip amplitudes

\[ T^{\text{flip}} \sim F_4(\nu, Q^2), F_5(\nu, Q^2), F_6(\nu, Q^2) \]

Discrete symmetries

\[ \begin{align*}
Q^2 &= -(k - k')^2 \\
S &= (p + k)^2 \\
u &= \frac{s - u}{4} \\
u &= \frac{s - u}{4} \\
\epsilon &= \frac{u}{4}
\end{align*} \]

Photon polarization parameter

Momentum transfer

Crossing symmetric variable

Goldberger et al. (1957)


**2\(\chi\) correction to cross-section**

Leading 2\(\chi\) contribution to cross section - interference term

\[
\delta_{2\gamma} = \frac{2}{G_M^2 + \frac{\varepsilon}{\tau_p} G_E^2} \left\{ G_M \Re G_1^{2\gamma} + \frac{\varepsilon}{\tau_p} G_E \Re G_2^{2\gamma} + \frac{1 - \varepsilon}{1 - \varepsilon_0} \left( \frac{\varepsilon_0}{\tau_p} \frac{\nu}{M^2} G_E \Re G_4^{2\gamma} - G_M \Re G_3^{2\gamma} \right) \right\}
\]

O. Tomalak and M. Vanderhaeghen (2014)

\[
\tau = \frac{Q^2}{4M^2}
\]

\[
G_1 = G_M + \frac{\nu}{M^2} F_3 + \frac{m^2}{M^2} F_5
\]

\[
G_2 = G_M - (1 - \tau) F_2 + \frac{\nu}{M^2} F_3
\]

\[
G_3 = \frac{\nu}{M^2} F_3 + \frac{m^2}{M^2} F_5
\]

\[
G_4 = F_4 + \frac{\nu}{M^2 (1 + \tau)} F_5
\]

\[
\varepsilon_0 = \frac{2m^2}{Q^2}
\]

\(\varepsilon\) in range \((\varepsilon_0, 1)\)

or \((1, \varepsilon_0)\)

**2\(\chi\) correction is given by amplitudes real parts**
\[ k' \neq k \]
\[ p' \neq p \]

non-forward scattering
proton state
Box diagram model

The one-photon exchange on-shell vertex

\[ \Gamma^\mu(Q^2) = \gamma^\mu F_D(Q^2) + \frac{i\sigma^{\mu\nu}q_\nu}{2M} F_P(Q^2) \]


Point-like couplings

Dipole FFs for \( G_M, G_E \)

Dirac and Pauli form factors

IR divergencies are subtracted


\[ F_D(Q^2) = 1 \quad F_P(Q^2) = \mu_P - 1 \]
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unsubtracted disp. rel. in ep scattering disagree with model
2γ in e−p elastic scattering

box diagram model vs. unsubtr. dis. rel.

$Q^2 = 0.05 \text{ GeV}^2$

$Q^2 = 1 \text{ GeV}^2$
2γ in e−p elastic scattering

box diagram model vs. unsubtr. dis. rel.

unsubtracted vs. subtracted disp. rel.

Proton only partially accounts for 2γ at low ε and large Q²

O. T. and M. Vanderhaeghen (2015)
CLAS data and $2\gamma$

box diagram model vs. subtracted dis. rel.

$Q^2 \approx 0.206$ GeV$^2$

$Q^2 \approx 0.85$ GeV$^2$

$Q^2 \approx 1.45$ GeV$^2$

$2\gamma$ data from $e^+p/e^-p$ cross section ratio

$$R = \frac{\sigma(e^+p)}{\sigma(e^-p)} \approx 1 - 2\delta_{2\gamma}$$

data points compatible with zero

CLAS data in agreement with $2\gamma$ fit
$k' = k$
$p' = p$

forward scattering

$k' \neq k$
$p' \neq p$

near-forward scattering
account for inelastic $2\gamma$
Low-$Q^2$ inelastic $2\chi$ correction (e$^-p$)

TPE blob - near-forward virtual Compton scattering

$Q, Q^2 \ln^2 Q^2, Q^2 \ln Q^2$ terms reproduced

$\delta_{2\gamma} = \int d\nu_\gamma d\tilde{Q}^2 (\omega_1(\nu_\gamma, \tilde{Q}^2) \cdot F_1(\nu_\gamma, \tilde{Q}^2) + \omega_2(\nu_\gamma, \tilde{Q}^2) \cdot F_2(\nu_\gamma, \tilde{Q}^2))$

unpolarized proton structure

M. E. Christy, P. E. Bosted (2010)

O. T. and M. Vanderhaeghen (2016)

$2\chi$ at large $\varepsilon$ agrees with empirical fit

$rE$ extraction ✓
Low-$Q^2$ inelastic $2\gamma$ correction ($e^-p$)

Comparison with low $Q^2$ measurements

CLAS data in agreement with Born + inelastic $2\gamma$

VEPP-3 data in agreement with Born $2\gamma$ only

$\varepsilon \approx 0.9$
MUSE estimates ($\mu^- p$)

proton box diagram model + inelastic $2\gamma$

$\delta_{2\gamma}, \%$

$k = 115 \text{ MeV}$

$k = 210 \text{ MeV}$

O. T. and M. Vanderhaeghen (2014, 2016)
MUSE estimates ($\mu^- p$)  
proton box diagram model + inelastic $2\gamma$

expected muon over electron ratio

O. T. and M. Vanderhaeghen (2014, 2016)

small inelastic $2\gamma$

small $2\gamma$ uncertainty

MUSE experiment can test $r_E$ extraction

K. Mesick talk (PAVI 2014), MUSE TDR (2016)
\[ k' \neq k \quad p' \neq p \]

near-forward scattering
elastic + inelastic

\[ X = p + \pi N \]

non-forward scattering
disp. rel.
\[ Q^2 = m^2_\pi \left( \frac{2M + m_\pi}{M + m_\pi} \right)^2 \sim 3.5m^2_\pi \]
πN in dispersive framework (e⁻p)

unsubtracted disp. rel.

other partial waves important

\[ Q^2 = 0.05 \text{ GeV}^2 \]

Preliminary

\[ \delta_{27}, \% \]

\[ 0 \quad 0.2 \quad 0.4 \quad 0.6 \quad 0.8 \]

\[ 0 \quad 0.1 \quad 0.2 \]

\[ \begin{array}{c}
\text{P}_{33}, \text{ unsubtracted disp. rel.} \\
\Delta, \text{ low-Q}^2 \\
\text{inelastic low-Q}^2 
\end{array} \]

l

\[ \gamma \quad \pi \quad \gamma \]

l

l

P

N

P

P_{33}
$\pi N$ in dispersive framework ($e^-p$)

$P_{33}$ contributes up to $1/2$ of inelastic $2\chi$

$Q^2 = 0.05 \text{ GeV}^2$
Conclusions

- Forward limit of $2\gamma$ in $lp$ scattering
- Proton T1 subtraction function estimated from data
- Subtracted disp. rel. formalism for ep scattering
- Theoretical estimates for $2\gamma$ (ep and $\mu p$)
- First estimates for $\pi N$ channel in disp. rel.

Outlook

- Application to forthcoming high-precision HFS exp.
- Extraction of magnetic radius accounting for $2\gamma$
- Comparison with VEPP-3, CLAS, OLYMPUS
Thanks for your attention !!!
Fixed-$Q^2$ dispersion relation framework

2$\gamma$ corrections

$f(z)$

analyticity

on-shell one-photon amplitudes

exp. data/phenomenology

unitarity

amplitudes: imaginary parts

DR

amplitudes: real parts

cross section correction

$\Re \mathcal{F}(\nu) = \frac{2\nu}{\pi} \mathcal{P} \int_{\nu_{min}}^{\infty} \frac{\Im \mathcal{F}(\nu' + i0)}{\nu'^2 - \nu^2} d\nu'$

Proton intermediate state is outside physical region

Analytical continuation for arbitrary FFs parametrization is found
Hadronic model vs. dispersion relations

- Imaginary parts are the same
- Real parts are the same for all $F_{DFP}$ amplitudes

\[
\begin{align*}
&G_M \quad F_2 \quad F_3 \quad F_5 \\
&G_M + \frac{\nu}{M^2} F_3 \quad F_5
\end{align*}
\]

Fixed-$Q^2$ subtracted dispersion relation works for all amplitudes

- Calculation based on DR for ep scattering
  - for amplitudes $G_1, G_2$ unsubtracted DR can be used
  - for amplitude $F_3$ subtracted DR should be used
  - subtraction point $\Re F_3^{FPFP}(\nu_0, Q^2)$ fixed from $\delta_{2\gamma}(\nu_0, Q^2)$ data
T₁ subtraction function TPE correction

Subtraction function contributes only to $\mathcal{F}_4$ amplitude

$$\delta_{2\gamma,0}^{\text{subt}} \approx -\frac{Q^2m^2}{\omega} \int_0^\infty f \left( x, \frac{Q^2}{m^2} \right) \beta \left( \frac{Q^2(x - 1)}{4} \right) dx$$

In the limit of small electron mass TPE correction vanishes

Valid only for small $Q^2$

For enhanced at HE function

$$\delta_{2\gamma,0}^{\text{subt}} \approx -\frac{3Q^2m^2}{2\pi\omega} \int_0^\infty \beta \left( \tilde{Q}^2 \right) \frac{d\tilde{Q}^2}{\tilde{Q}^2}$$
πN TPE contribution in dispersive framework (ep)

account of $P_{33}$ channel decreases uncertainty with subtracted DR
Proton form factors problem

Polarization transfer
JLab (Hall A, C)

Rosenbluth separation
SLAC, JLab (Hall A, C)

A possible explanation - two-photon exchange