



# HIGHER ORDER EFFECTS IN LEPTON-HADRON PRODUCTION PROCESSES

Newport News VA, *01.06.2016*

Accardi, Anderle, de Florian, Ringer, Rotstein, Stratmann, Vogelsang

# ABOUT ME...

2006-2009 — **Bachelor's Degree**: University of Trento. Advisor: Paolo Verrocchio.

2009-2012 — **Master's Degree**: Double degree program between University of Trento and University of Tübingen. Advisors: Werner Vogelsang and Marco Traini.

2013-2016 — **PhD**: University of Tübingen. Advisor: Werner Vogelsang.

## MAIN COLLABORATIONS

- University of Buenos Aires: with Prof. Dr. Daniel de Florian and Yamila Rotstein. Project funded by “*Fondazione Cassa Rurale di Trento*” and Tübingen University.
- Jefferson Laboratory and Hampton University: with Prof. Dr. Alberto Accardi. Project funded by Hampton University.

## MAIN COLLABORATORS

Alberto Accardi, Daniel de Florian, Felix Ringer, Yamila Rotstein, Marco Stratmann, Werner Vogelsang



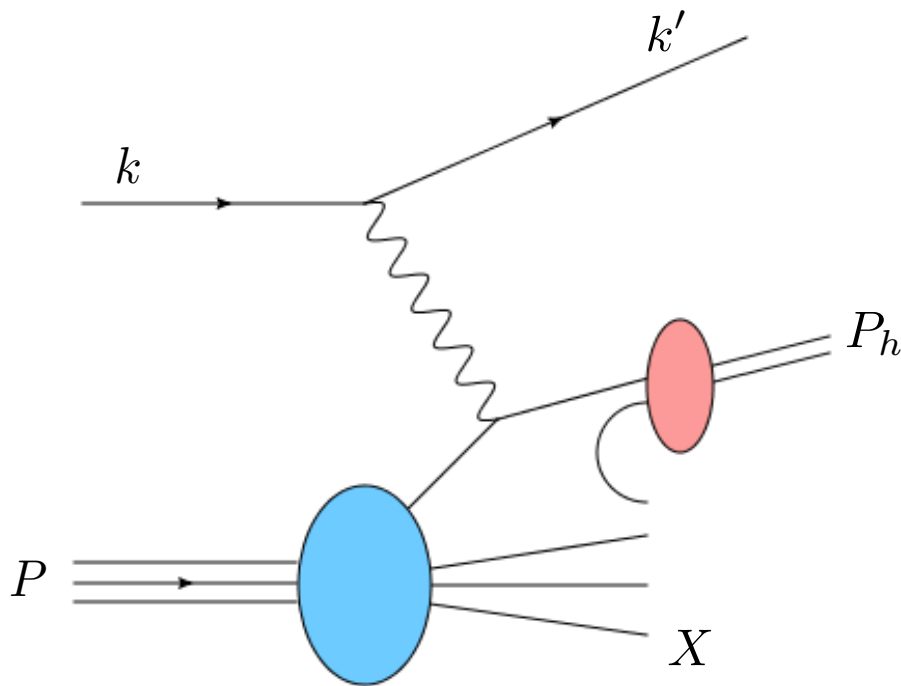
# OUTLINE

- THRESHOLD RESUMMATION FOR SIDIS
- HMC + THRESHOLD RESUMMATION
- TOWARDS A GLOBAL NNLO FF FIT
- CONCLUSIONS



# SEMI-INCLUSIVE DIS

$$\ell(k) p(P) \rightarrow \ell(k') h(P_h) X$$



Define the usual variables:

$$Q^2 \equiv -q^2 = -(k - k')^2$$

$$y \equiv \frac{P \cdot q}{P \cdot k}$$

$$x \equiv \frac{Q^2}{2P \cdot q}$$

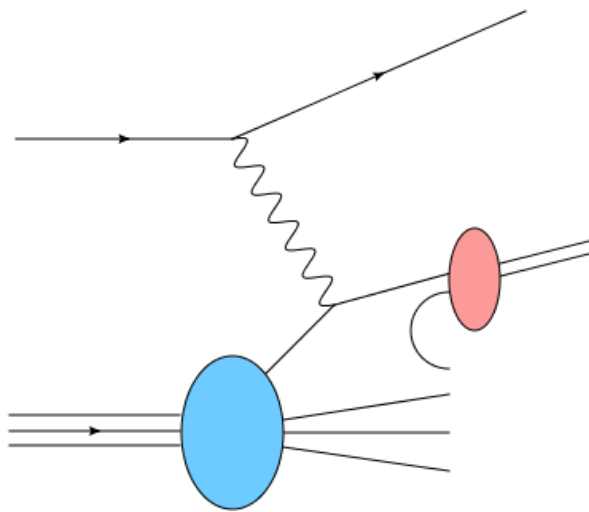
$$z \equiv \frac{P \cdot P_h}{P \cdot q}$$



$$\text{SIDIS} \quad \frac{d^3 \sigma^h}{dx dy dz} = \frac{4\pi\alpha^2}{Q^2} \left[ \frac{1 + (1-y)^2}{2y} \mathcal{F}_T^h(x, z, Q^2) + \frac{1-y}{y} \mathcal{F}_L^h(x, z, Q^2) \right]$$

$$\text{DIS} \quad \frac{d^2 \sigma}{dx dy} = \frac{4\pi\alpha^2}{Q^2} \left[ \frac{1 + (1-y)^2}{2y} \mathcal{F}_T(x, Q^2) + \frac{1-y}{y} \mathcal{F}_L(x, Q^2) \right]$$

$$\mathcal{F}_i^h(x, z, Q^2) = \sum_{f, f'} \int_x^1 \frac{d\hat{x}}{\hat{x}} \int_z^1 \frac{d\hat{z}}{\hat{z}} f\left(\frac{x}{\hat{x}}, \mu^2\right) D_{f'}^h\left(\frac{z}{\hat{z}}, \mu^2\right) C_{f'f}^i\left(\hat{x}, \hat{z}, \frac{Q^2}{\mu^2}, \alpha_s(\mu^2)\right)$$



hard-scattering coefficient function:

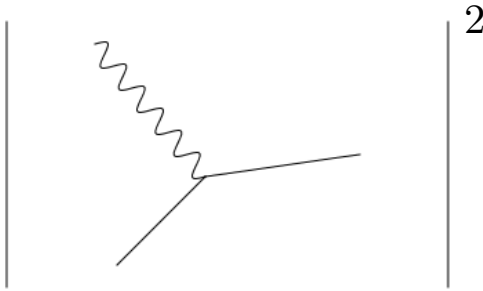
$$C_{f'f}^i = C_{f'f}^{i,(0)} + \frac{\alpha_s(\mu^2)}{2\pi} C_{f'f}^{i,(1)} + \mathcal{O}(\alpha_s^2)$$

Hadron multiplicity  
definition for SIDIS

$$R_{\text{SIDIS}}^h \equiv \frac{d^3 \sigma^h / dx dy dz}{d^2 \sigma / dx dy}$$



LO:

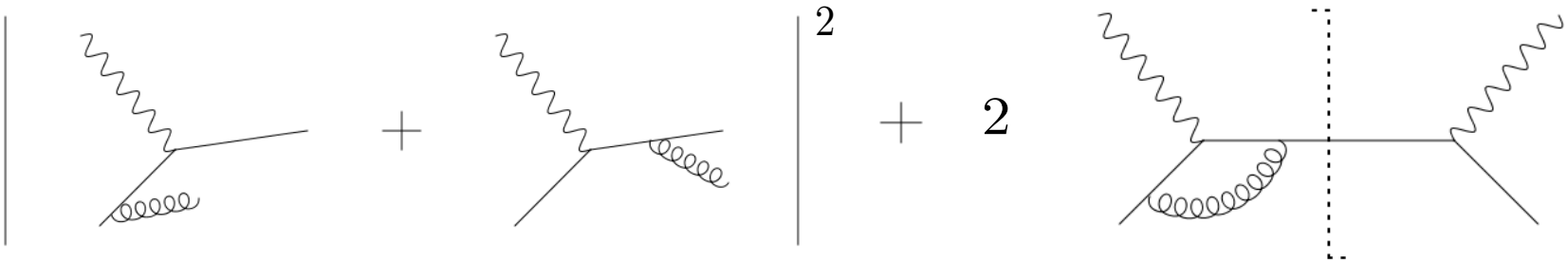


Parton Model:

$$C_{qq}^{T,(0)}(\hat{x}, \hat{z}) = e_q^2 \delta(1 - \hat{x}) \delta(1 - \hat{z})$$

$$C_{qq}^{L,(0)}(\hat{x}, \hat{z}) = 0$$

NLO:



Singularities from the quark propagator

$$G \sim \frac{1}{(p_i - k)^2}$$

$$(p_i - k)^2 = -2|p_i||k|(1 - \cos \theta),$$

- *soft*: cancel for real + virtual corrections
- *collinear*: factorize into PDF/ FF



# NLO COEFFICIENT FUNCTION

*Altarelli et al.; Furmanski, Petronzio; de Florian et al.*

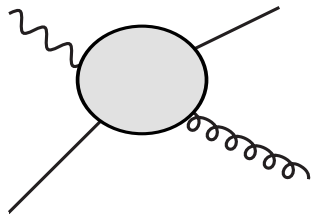
large corrections near threshold  $\hat{x}, \hat{z} \rightarrow 1$

$$C_{qq}^{T,(1)}(\hat{x}, \hat{z}) \sim e_q^2 C_F \left[ -8 \delta(1 - \hat{x}) \delta(1 - \hat{z}) + 2 \delta(1 - \hat{x}) \left( \frac{\ln(1 - \hat{z})}{1 - \hat{z}} \right)_+ + 2 \delta(1 - \hat{z}) \left( \frac{\ln(1 - \hat{x})}{1 - \hat{x}} \right)_+ + \frac{2}{(1 - \hat{x})_+ (1 - \hat{z})_+} \right]$$

$\overline{\text{MS}}$  scheme

$$\int_0^1 dz f(z) \left( \frac{\ln(1 - z)}{1 - z} \right)_+ \equiv \int_0^1 dz (f(z) - f(1)) \frac{\ln(1 - z)}{1 - z}$$

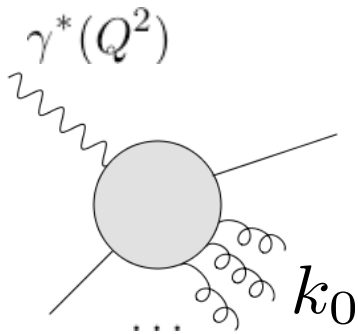




### NLO Threshold Logarithms

coming from emission  
of one soft gluon

$$+ 2 \delta(1 - \hat{x}) \left( \frac{\ln(1 - \hat{z})}{1 - \hat{z}} \right)_+ \text{ and } \hat{x} \leftrightarrow \hat{z}$$



### N<sup>k</sup>LO Threshold Logarithms

coming from emission  
of k soft gluon

$$\alpha_s^k \delta(1 - \hat{z}) \left( \frac{\ln^{2k-1}(1 - \hat{x})}{1 - \hat{x}} \right)_+ \text{ and } \hat{x} \leftrightarrow \hat{z}$$

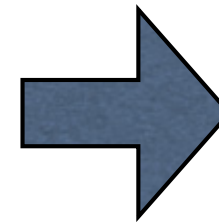
$$\alpha_s^k \left( \frac{\ln^m(1 - \hat{x})}{1 - \hat{x}} \right)_+ \left( \frac{\ln^n(1 - \hat{z})}{1 - \hat{z}} \right)_+$$

$$m + n = 2k - 2$$

They become large for  $\hat{x}, \hat{z} \rightarrow 1$

With a careful  
*kinematic analysis*

$$(1 - \hat{x}) + (1 - \hat{z}) \approx \frac{2k^0}{Q}$$



Large Logs associated  
with  
soft gluon emissions

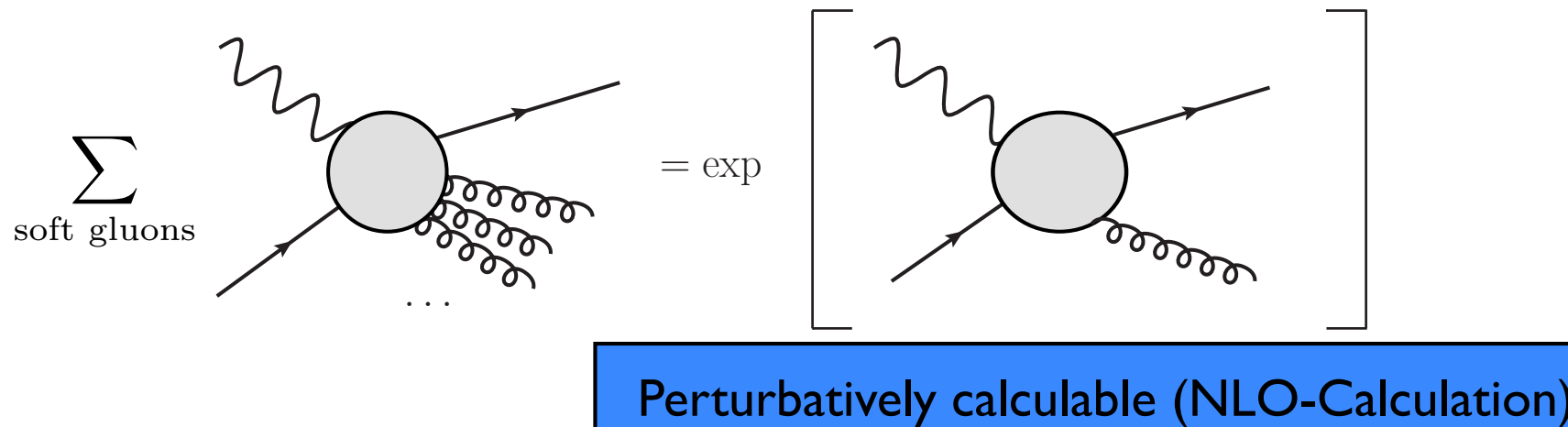
G. F. Sterman and W. Vogelsang, Catani and Trentadue





# THE EXPONENTIATION

The **Resummation** of the *Threshold Logs* occurs via the *exponentiation* of the “single soft emission”-contribution (one-loop contribution in the soft-limit  $k_0 \approx 0$ )



- *Matrix elements* factorize in the soft-limit approximation (eikonal-approximation)
- In order for the *phase space* to factorize we have to move into

$$\delta \left( 1 - k_0 - \sum_{i=1}^n k_i \right) = \frac{1}{2\pi i} \int_C dN e^{N(1-k_0-\sum_{i=1}^n k_i)} \quad \longrightarrow \quad \text{Mellin Space}$$



large corrections near threshold  $\hat{x}, \hat{z} \rightarrow 1$

$$C_{qq}^{T,(1)}(\hat{x}, \hat{z}) \sim e_q^2 C_F \left[ -8 \delta(1 - \hat{x}) \delta(1 - \hat{z}) + 2 \delta(1 - \hat{x}) \left( \frac{\ln(1 - \hat{z})}{1 - \hat{z}} \right)_+ + 2 \delta(1 - \hat{z}) \left( \frac{\ln(1 - \hat{x})}{1 - \hat{x}} \right)_+ + \frac{2}{(1 - \hat{x})_+ (1 - \hat{z})_+} \right]$$



**double-Mellin transform:**

$$\tilde{C}_{qq}^{T,(1)}(N, M) \sim e_q^2 C_F \left[ -8 + \frac{\pi^2}{3} + (\ln \bar{N} + \ln \bar{M})^2 \right]$$

$$\bar{N} = N e^{\gamma_E}, \quad \bar{M} = M e^{\gamma_E}$$

for  $N, M \rightarrow \infty$



# ACCURACY OF RESUMMATION

$$\mathcal{O}(\alpha_s^k) : C_{knm} \times \alpha_s^k \ln^n \bar{N} \ln^m \bar{M}, \quad \text{where } n + m \leq 2k$$

Fixed Order						
LO	1					
NLO	$\alpha_s L^2$	$\alpha_s L$	$\alpha_s$			
NNLO	$\alpha_s^2 L^4$	$\alpha_s^2 L^3$	$\alpha_s^2 L^2$	$\alpha_s^2 L$	$\alpha_s^2$	
...	...	...	...	...	...	
N <sup>k</sup> LO	$\alpha_s^k L^{2k}$	$\alpha_s^k L^{2k-1}$	$\alpha_s^k L^{2k-2}$	$\alpha_s^k L^{2k-3}$	$\alpha_s^k L^{2k-4}$	...

In Mellin Space



# ACCURACY OF RESUMMATION

$$\mathcal{O}(\alpha_s^k) : C_{knm} \times \alpha_s^k \ln^n \bar{N} \ln^m \bar{M}, \quad \text{where } n + m \leq 2k$$

Fixed Order						
Resummation	LO	1				
	NLO	$\alpha_s L^2$	$\alpha_s L$	$\alpha_s$		
	NNLO	$\alpha_s^2 L^4$	$\alpha_s^2 L^3$	$\alpha_s^2 L^2$	$\alpha_s^2 L$	$\alpha_s^2$
	...	...	...	...	...	...
	N <sup>k</sup> LO	$\alpha_s^k L^{2k}$	$\alpha_s^k L^{2k-1}$	$\alpha_s^k L^{2k-2}$	$\alpha_s^k L^{2k-3}$	$\alpha_s^k L^{2k-4}$
	↓		↓		↓	
	LL		NLL		NNLL	



# INVERSE MELLIN TRANSFORMATION

- Minimal Prescription *Catani, Mangano, Nason, Trentadue*

-Double Mellin Inverse Transformation

$$\mathcal{F}_i^h(x, z, Q^2) = \int_{c_N} \frac{dN}{2\pi i} x^{-N} \int_{c_M} \frac{dM}{2\pi i} z^{-M} \tilde{\mathcal{F}}_i^h(N, M, Q^2)$$

-Choose contours to the left of the Landau singularity at

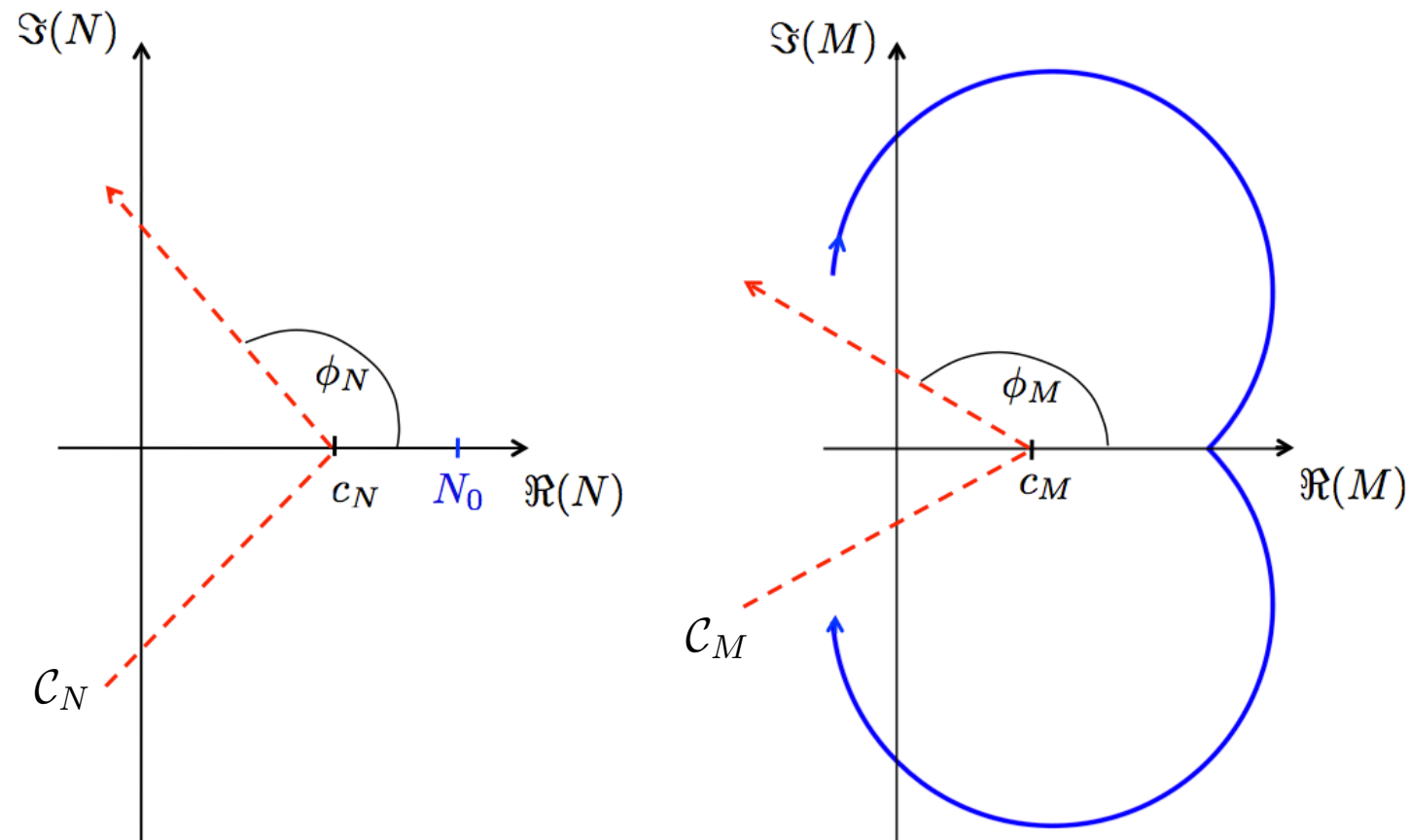
$$\lambda_{NM} = 1 \leftrightarrow NM = e^{1/(\alpha_s b_0) - 2\gamma_E}$$

-Tilted contours in complex plane to increase numerical convergence

$$N = c_N + z_N e^{i\phi_N}$$



# INTEGRATION CONTOURS



tilted contours  $N = c_N + z_N e^{i\phi_N}$

Location of the Landau pole as  $N$  moves along its contour



# COMPASS DATA

SIDIS  $\pi^+$  hadron multiplicities,  
COMPASS kinematics

$$0.041 < x < 0.7$$

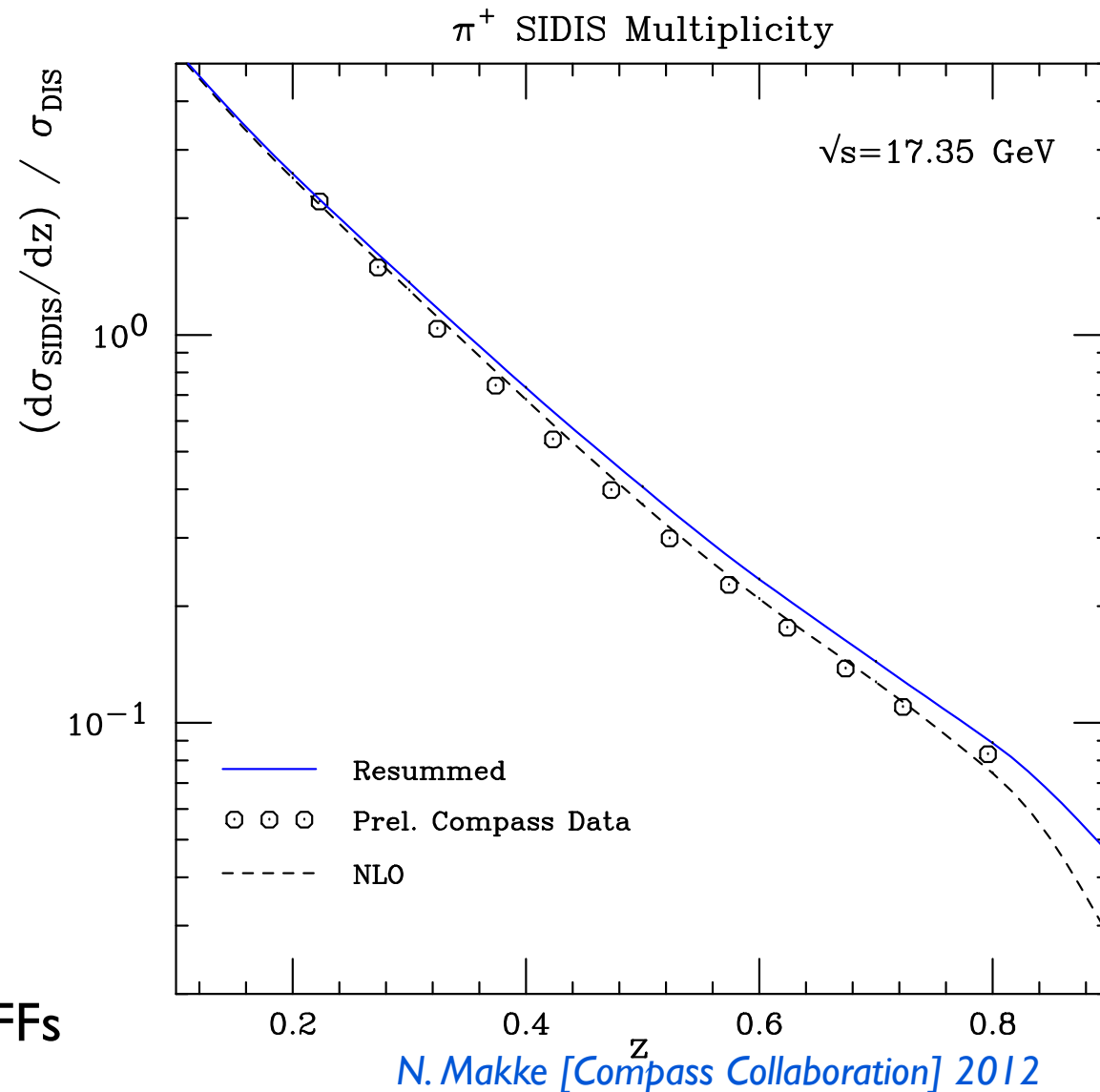
$$0.1 < y < 0.9$$

$$Q^2 > 1 \text{ GeV}^2$$

$$W^2 > 7 \text{ GeV}^2$$

$$R_{\text{SIDIS}}^h \equiv \frac{d^3\sigma^h/dxdydz}{d^2\sigma/dxdy}$$

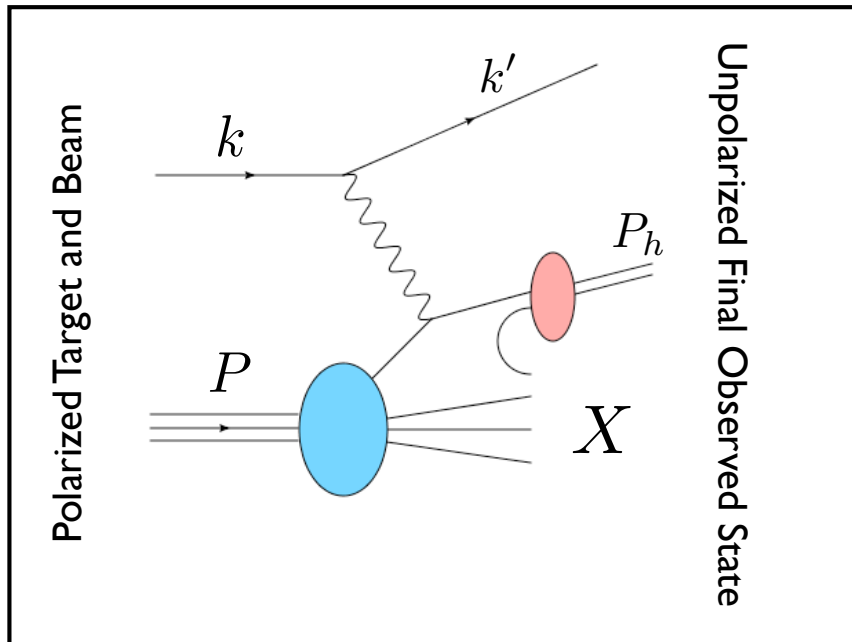
using MSTW08 PDFs and DSS FFs



# POLARIZED SIDIS

Anderle, Ringer Vogelsang, PhysRevD.87. 094021

Longitudinal double-spin asymmetry  $\vec{l}(k)\vec{p}(P) \rightarrow l(k')h(P_h)X$



$$A_1^h(x, z, Q^2) \approx \frac{g_1^h(x, z, Q^2)}{F_1^h(x, z, Q^2)}$$

where  $2F_1^h(x, z, Q^2) = \mathcal{F}_T^h(x, z, Q^2)$

$$2g_1^h(x, z, Q^2) = \sum_{f, f'=q, \bar{q}, g} \int_x^1 \frac{d\hat{x}}{\hat{x}} \int_z^1 \frac{d\hat{z}}{\hat{z}} \Delta f\left(\frac{x}{\hat{x}}, \mu^2\right) \times D_{f'}^h\left(\frac{z}{\hat{z}}, \mu^2\right) \Delta C_{f'f}\left(\hat{x}, \hat{z}, \frac{Q^2}{\mu^2}, \alpha_s(\mu^2)\right)$$

**Polarized PDFs:**  $\Delta f(x, Q^2) \equiv f^+(x, Q^2) - f^-(x, Q^2)$





# RESUMMATION FOR PSIDIS

Since in the threshold limit  $\hat{x}, \hat{z} \rightarrow 1$

$$\Delta C_{qq}^{(1)}(\hat{x}, \hat{z}) \sim C_{qq}^{(1)}(\hat{x}, \hat{z}) \sim e_q^2 C_F \left[ + 2\delta(1 - \hat{x}) \left( \frac{\ln(1 - \hat{z})}{1 - \hat{z}} \right)_+ + 2\delta(1 - \hat{z}) \left( \frac{\ln(1 - \hat{x})}{1 - \hat{x}} \right)_+ + \frac{2}{(1 - \hat{x})_+(1 - \hat{z})_+} - 8\delta(1 - \hat{x})\delta(1 - \hat{z}) \right]$$

*Daniel de Florian, Yamila Rotstein Habarnau*

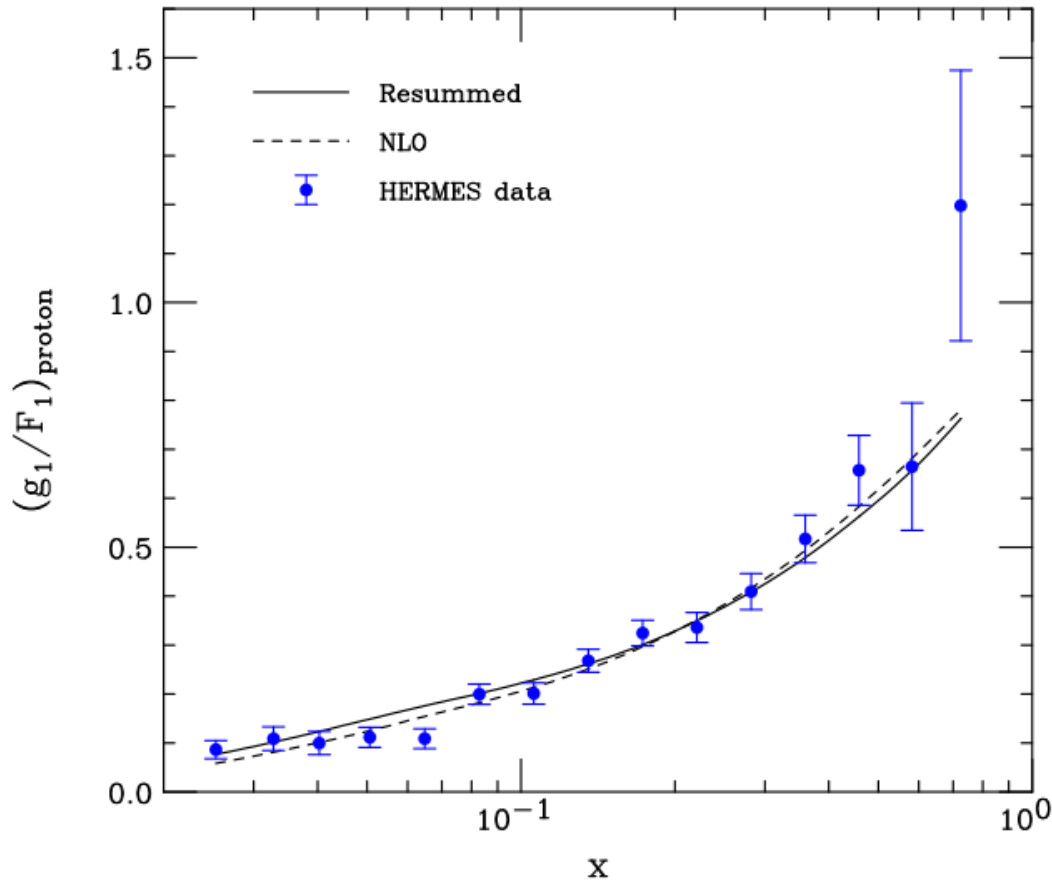
The resummed spin-dependent coefficient function **IDENTICAL** to spin averaged one

$$\Delta \tilde{C}_{qq}^{\text{res}}(N, M, \alpha_s(Q^2)) = \tilde{C}_{qq}^{\text{res}}(N, M, \alpha_s(Q^2))$$

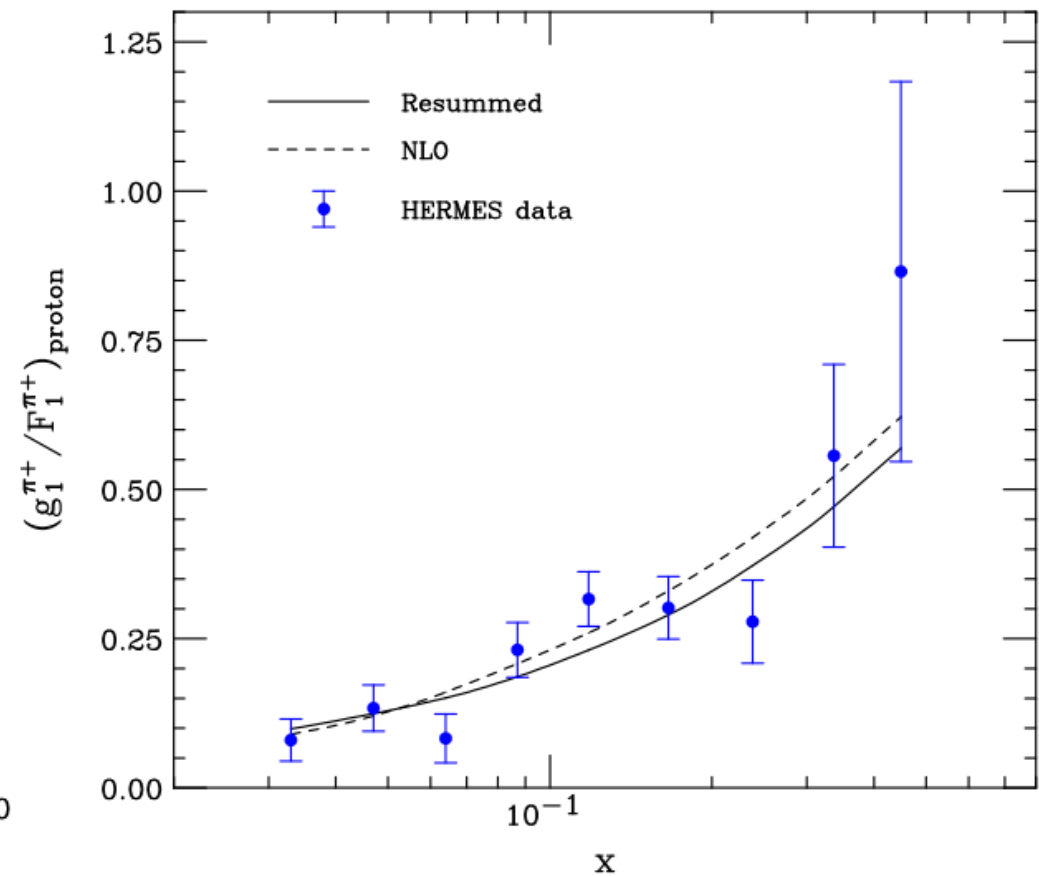


# INCLUSIVE & SI-DIS ASYMMETRIES

DIS



SIDIS



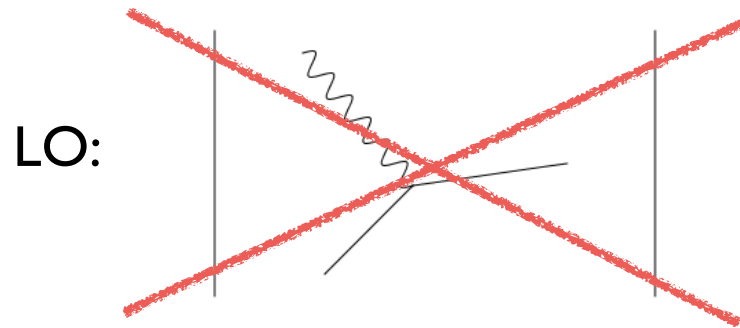
proton target  $0.2 < z < 0.8$

using MRST'02/DSSV PDFs and DSS FFs

*A.Airapetian et al. [Hermes Collaboration] 2005*



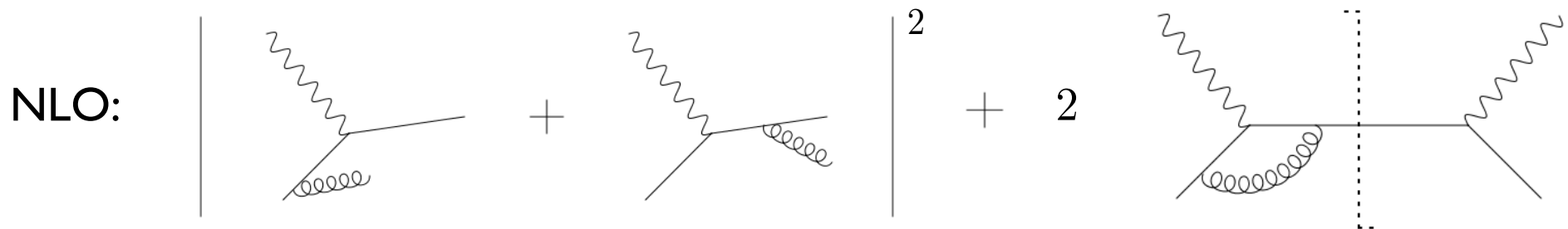
# TOWARDS NNLO $F_L$



Parton Model:

$$C_{qq}^{T,(0)}(\hat{x}, \hat{z}) = e_q^2 \delta(1 - \hat{x}) \delta(1 - \hat{z})$$

$$C_{qq}^{L,(0)}(\hat{x}, \hat{z}) = 0$$



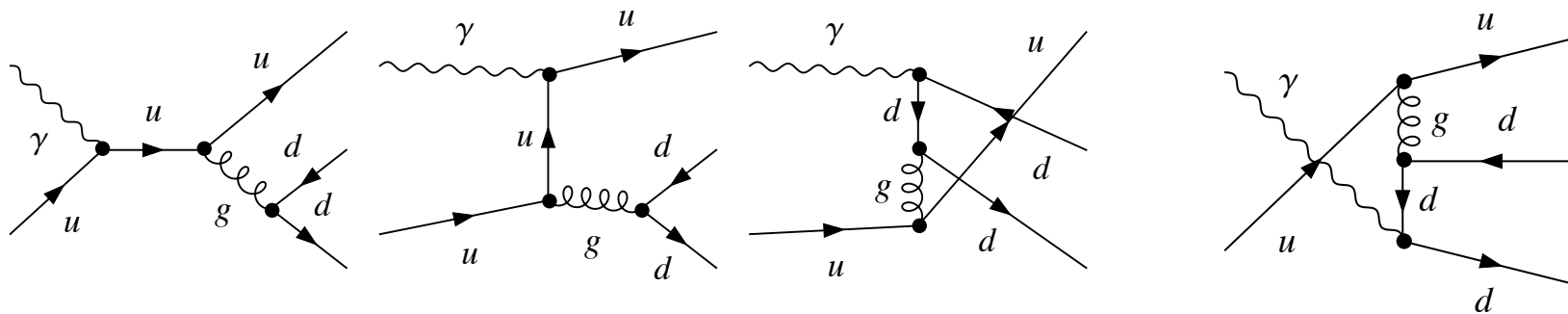
For the Longitudinal Structure Function at NLO, the quark scattering and the gluon-fusion are **Tree-Level diagrams**



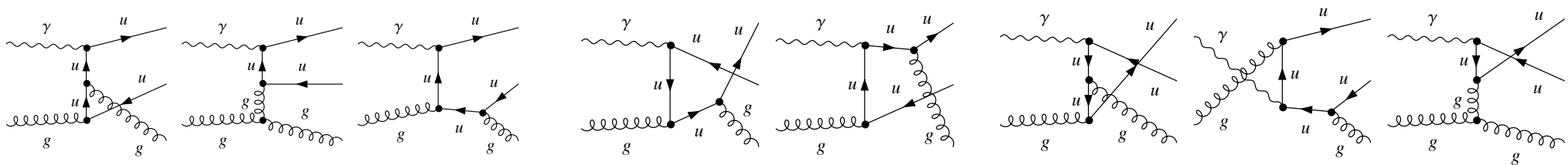
# THE NEW CHANNELS OF NNLO $F_L$

Tree Level diagrams at NNLO:

**QUARK INITIATED**  $\gamma q \rightarrow q' \bar{q}' q$   $q \neq q'$



**GLUON INITIATED**  $\gamma g \rightarrow q \bar{q} g$   $q \neq q'$



# THE CALCULATION

It is a **BRUTE-FORCE** calculation:

PHASE SPACE 2 to 3

$$\int d\text{PS}_3^{\text{DI}} = \frac{1}{(4\pi)^n} \frac{(s - Q^2)^{n-3}}{\Gamma(n-3)} (1-x)^{n-3} \int_0^\pi d\theta \int_0^\pi d\phi (\sin\theta)^{n-3} (\sin\phi)^{n-4} \\ \times \int_0^1 dy \int_0^1 dz y^{(n/2)-2} (1-y)^{n-3} \{z(1-z)\}^{(n/2)-2}$$

- Angular part solvable using known integrals of type: Beenakker, Kuijf, van Neerven, Smith  
(Phys.Rev. D40 (1989) 54-82)

$$\int_0^\pi d\theta \int_0^\pi d\phi \frac{(\sin\theta)^{n-3} (\sin\phi)^{n-4}}{(a + b \cos\theta)^i (A + B \cos\theta + C \cos\phi \sin\theta)^j}$$

- z-Integration remaining can be solved **analytically with many tricks**



# OUTLINE

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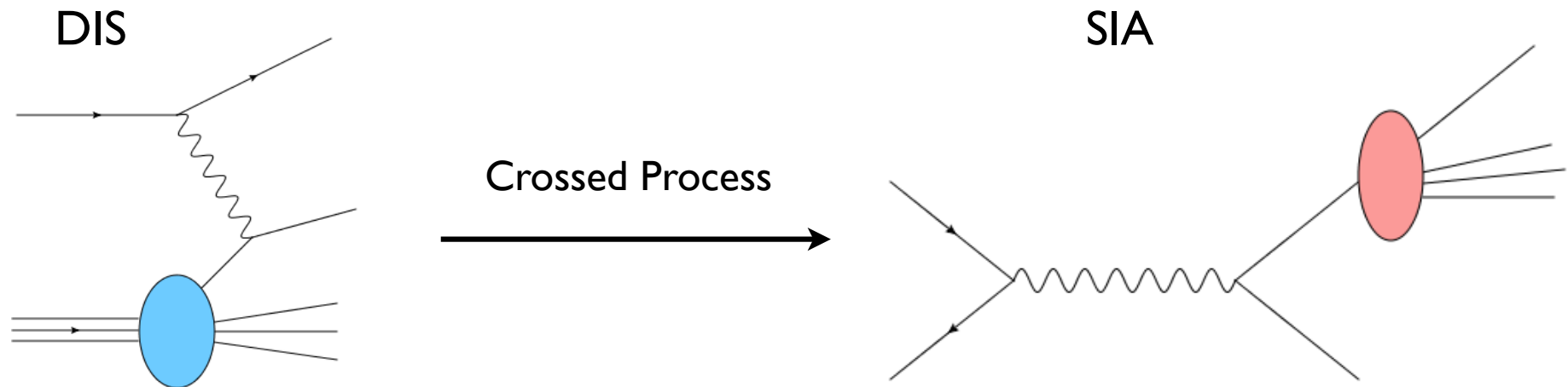


# HMC + THRESHOLD RESUMMATION

*Accardi, Anderle, Ringer (Phys. Rev. D 91, 034008 (2015))*

We consider two corrections on standard pQCD calculation of SIA and DIS:

- Threshold resummation
- Hadron Mass Correction

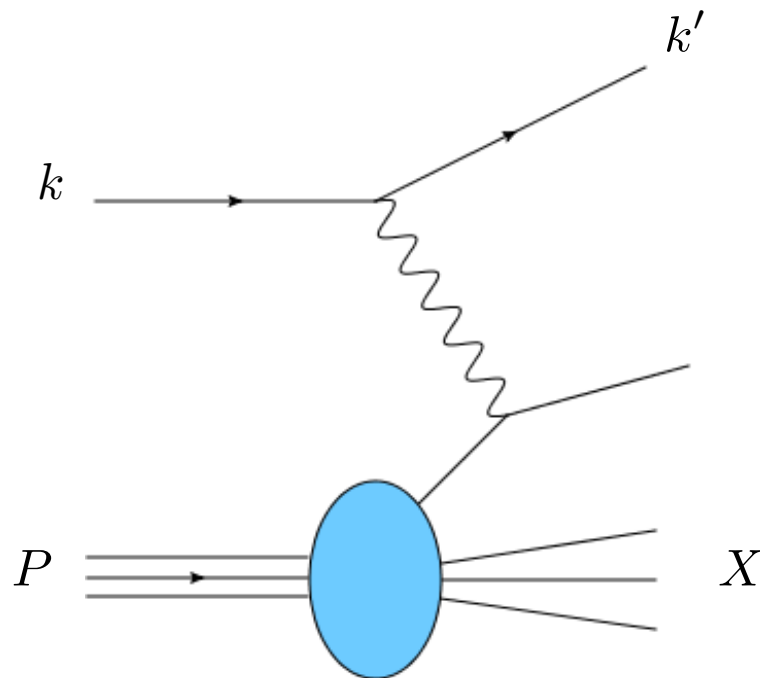


Both corrections become **relevant only** in some kinematical phase space regions



# DEEP INELASTIC SCATTERING

$$l(k)p(P) \rightarrow l(k')X$$



Defined **kinematic** variables:

$$Q^2 \equiv -q^2 = -(k - k')^2 \quad \text{Virtual Photon Energy}$$

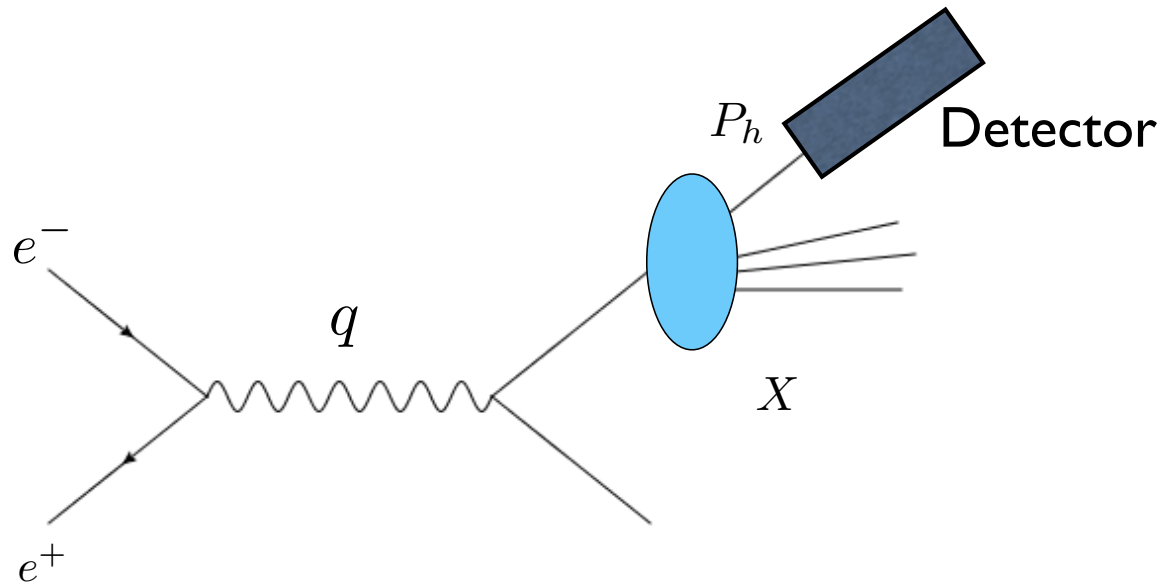
$$y \equiv \frac{P \cdot q}{P \cdot k} \quad \propto \text{to lepton scattered angle}$$

$$x \equiv \frac{Q^2}{2P \cdot q}$$





# ELECTRON-POSITRON ANNHILATION



Defined **kinematic variables** variables:

$$q^2 = Q^2 \quad \text{Virtual Photon Energy}$$

$$x_E \equiv \frac{2P_h \cdot q}{Q^2} = \frac{2E_h}{\sqrt{s}} \quad (\text{c.m.s})$$

Hadron multiplicities

$$R_{e^+e^-}^h \equiv \frac{1}{\sigma^{\text{tot}}} \frac{d^2\sigma^h}{dx_E d\cos\theta}$$



# THRESHOLD RESUMMATION

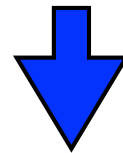
For both DIS and SIA

in Mellin space: exponentiation of the one-loop results

$$C_{\text{res}}^{q,N} = H_q \Delta_q^N J_q^N = H_q \exp \left[ \int_0^1 dx \frac{x^{N-1} - 1}{1-x} \int_{\mu_F^2}^{(1-x)Q^2} \frac{dp^2}{p^2} A_q[\alpha_s(p^2)] + \frac{1}{2} B_q[\alpha_s((1-x)Q^2)] \right]$$

where  $A^{(1)} = C_F$  ,  $A^{(2)} = \frac{1}{2} C_F K = \frac{1}{2} C_F \left[ C_A \left( \frac{67}{18} - \frac{\pi^2}{6} \right) - \frac{5}{9} N_f \right]$  *Catani, Trentadue; Serman*

$$B^{(1)} = -\frac{3}{2} C_F .$$



Threshold Resummation acts for DIS and SIA in the same exact way and is **relevant** for the same Phase Space region:

$$\begin{array}{l} \text{DIS: } x \rightarrow 1 \\ \text{SIA: } x_E \rightarrow 1 \end{array}$$



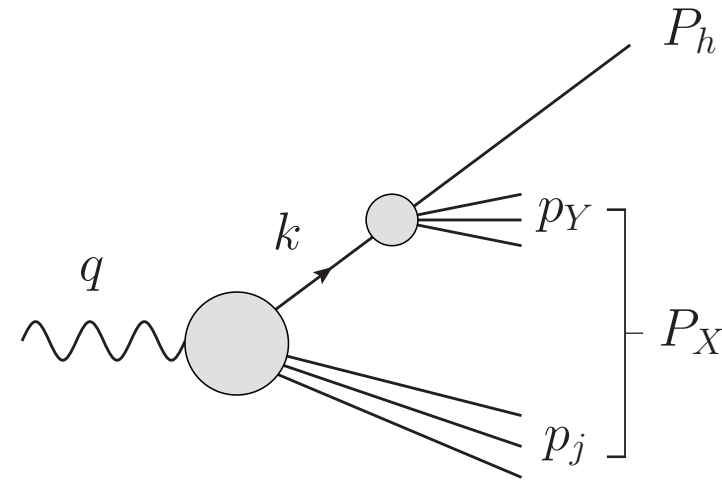
# STUDYING THE KINEMATICS (SIA)

we study the kinematics in the  $\gamma - h$  frame

$$q = q^+ \bar{n} + \frac{Q^2}{2q^+} n$$

$$P_h = P_h^+ \bar{n} + \frac{m_h^2}{2p_h^+} n$$

$$k = k^+ \bar{n} + \frac{k^2 + k_T^2}{2k^+} n + \mathbf{k}_T$$



we work in collinear factorization

$$z = \frac{P_h^+}{k^+}, \quad \mathbf{k}_T = 0$$

Accardi, Qiu

where the light-cone vectors

$$n^\mu = \frac{1}{\sqrt{2}} (1, 0, 0, -1)$$

$$\bar{n}^\mu = \frac{1}{\sqrt{2}} (1, 0, 0, 1)$$

$$n^2 = \bar{n}^2 = 0 \quad n \cdot \bar{n} = 1$$

$$a^+ = a \cdot n \quad a^- = a \cdot \bar{n}$$



The boson fractional momentum in respect to the hadron is not anymore

$$\cancel{x_E = \frac{2q \cdot P_h}{q^2}}$$

but  
for **SIA**

$$P_h^+ / q^+ = \xi_E = \frac{1}{2} x_E \left( 1 + \sqrt{1 - \frac{4}{x_E^2} \frac{m_h^2}{Q^2}} \right)$$

and analogously  
for **DIS**

$$\xi = \frac{2x_B}{1 + \sqrt{1 + 4x_B^2 m_N^2 / Q^2}}$$

$$\cancel{x_B = \frac{Q^2}{2q \cdot P_h}}$$

One should use those variables when calculating structure functions, since they represent the right physical fractional momentum variables

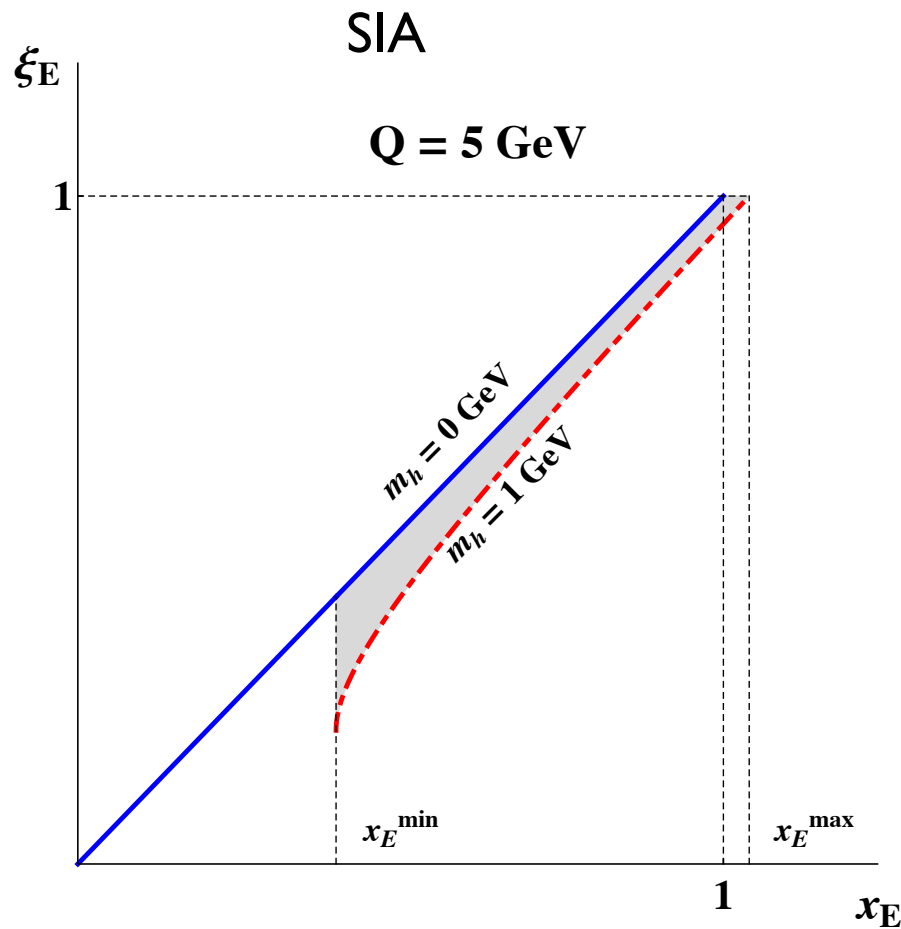
$$\mathcal{F}_i(x_E, Q^2) \rightarrow \mathcal{F}_i(\xi_E, Q^2)$$

$$\mathcal{F}_i(x_B, Q^2) \rightarrow \mathcal{F}_i(\xi, Q^2)$$

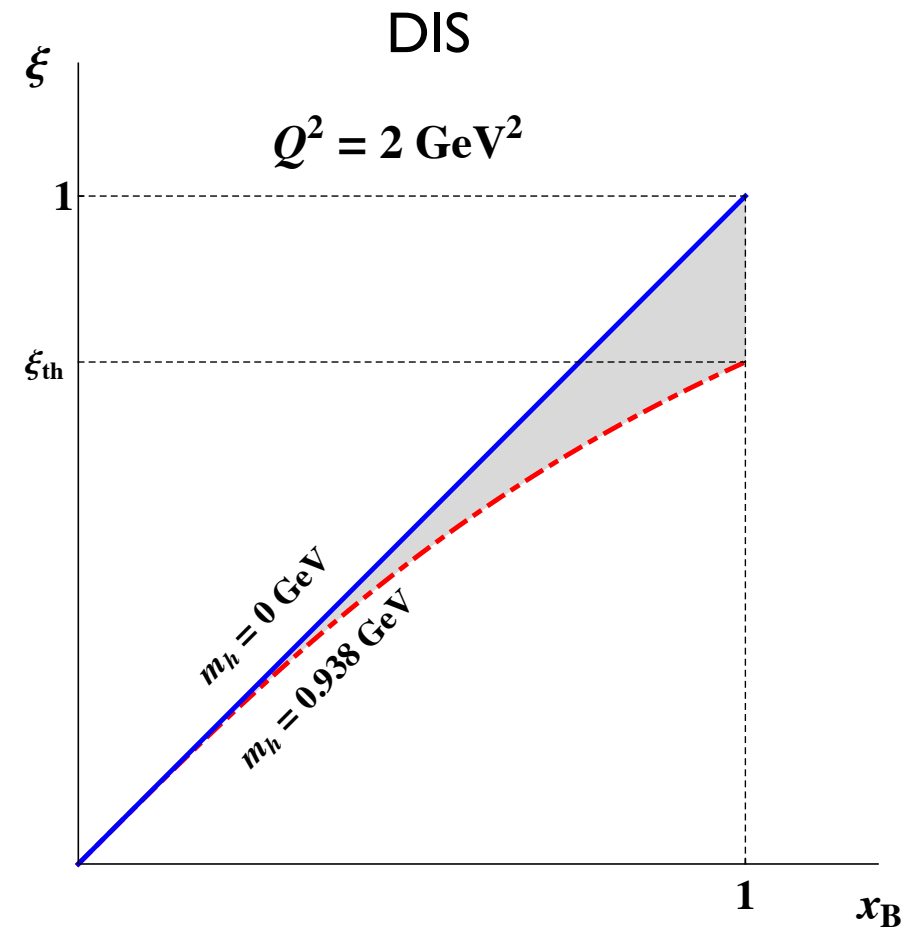
*Albino et al.*



The hadron mass acts kinematically on the two processes in a very different way



Low  $x_E$  effect

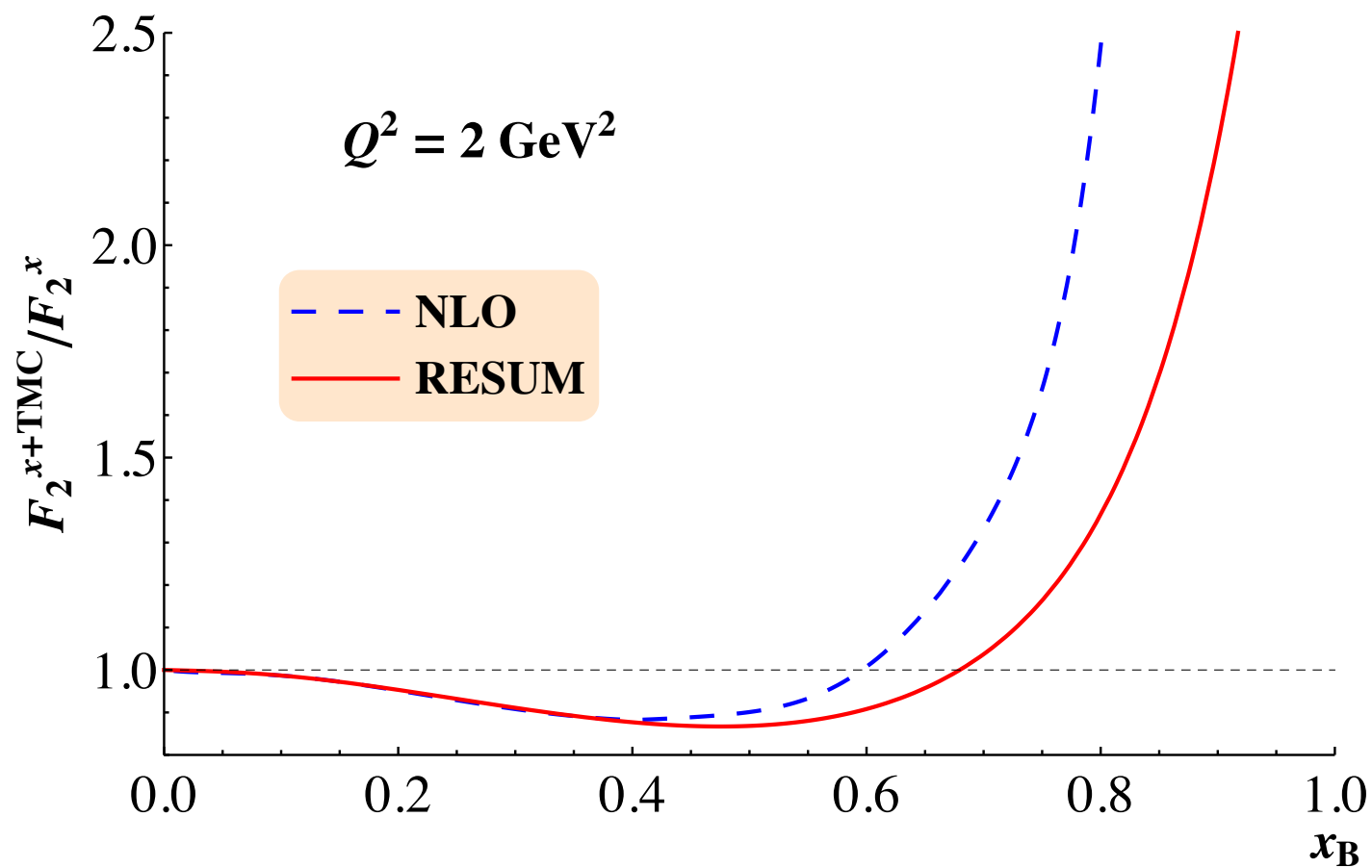


High  $x_B$  effect

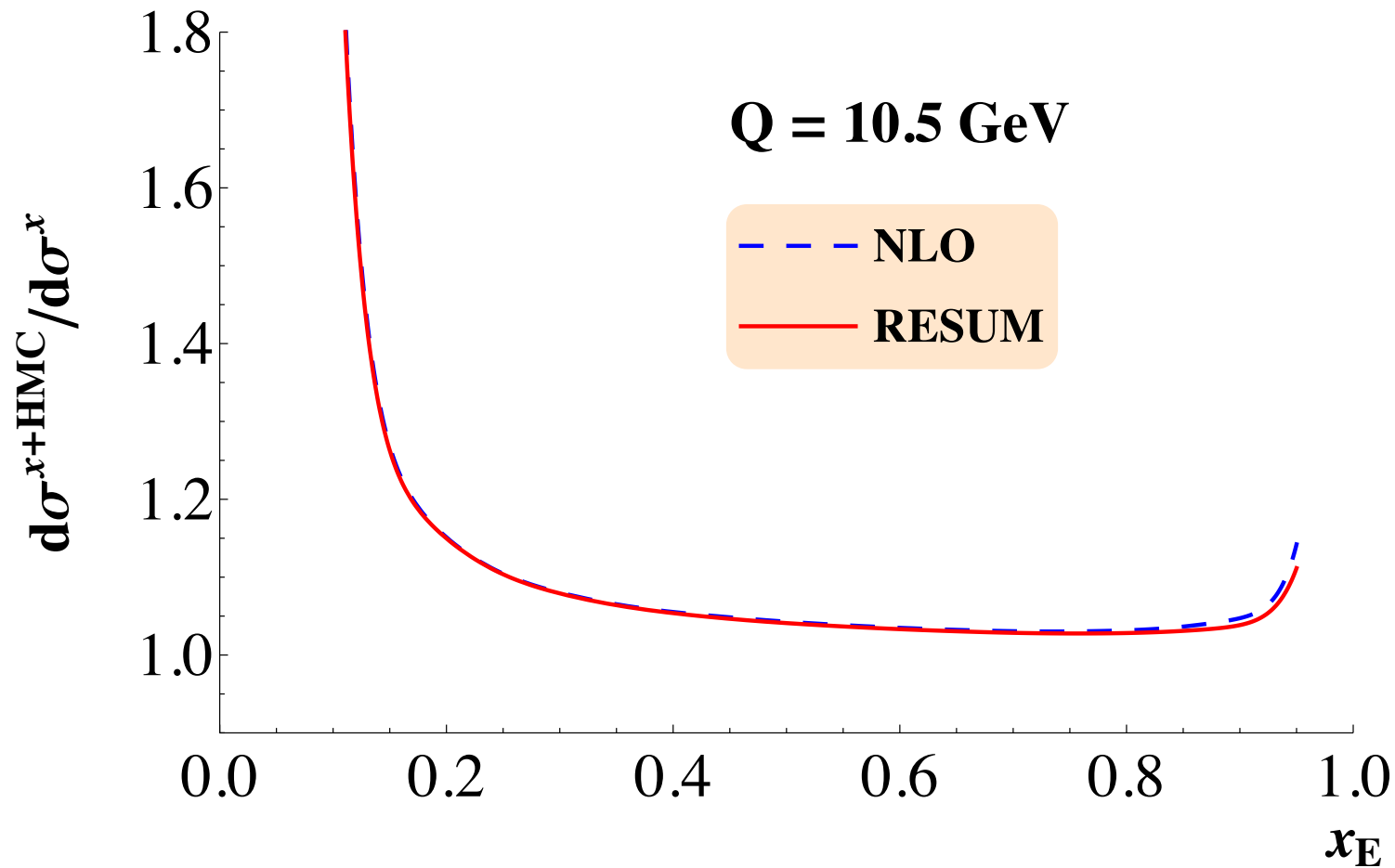
AS RESUMMATION!!!



For DIS the TMC and Threshold Resummation **do not act independently**



No interplay between the two effects is found since they act independently on two different kinematical regions

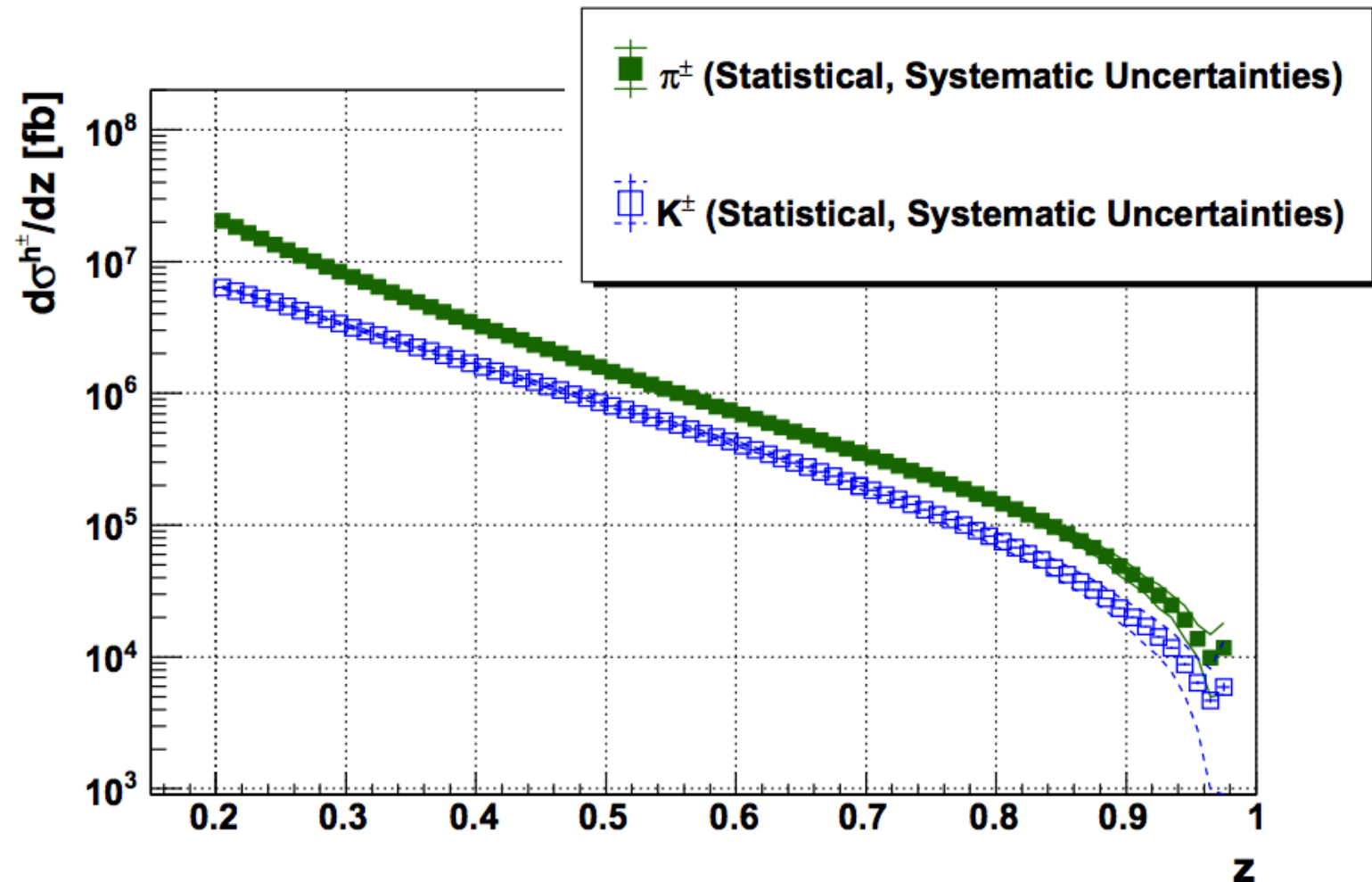


# BELLE AND BABAR DATA

BELLE Kinematics

$$\sqrt{s} = 10.5 \text{ GeV}$$

$$-1 < \cos\theta < 1$$

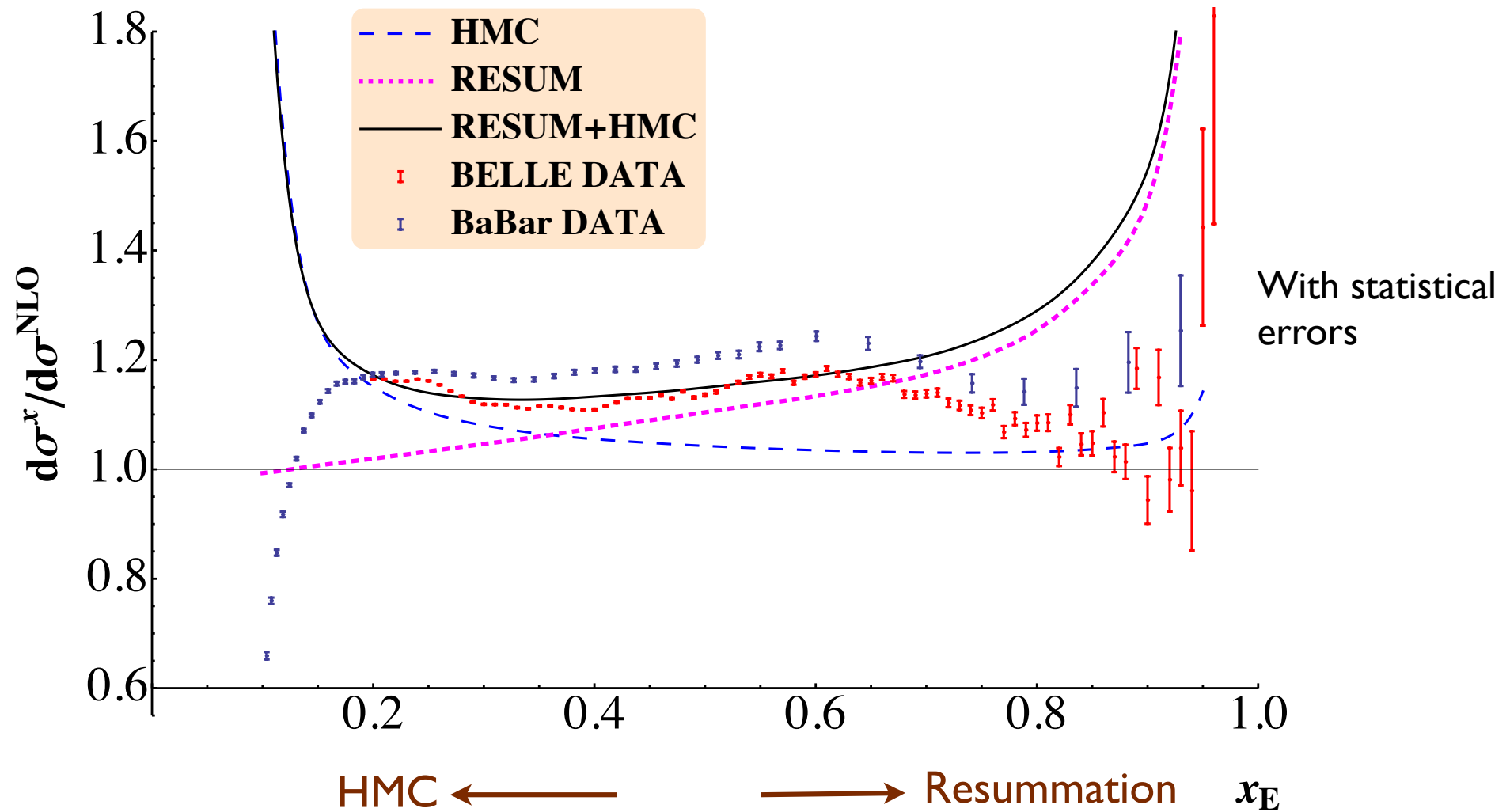


*Belle collaboration arXiv: 1301.6183*





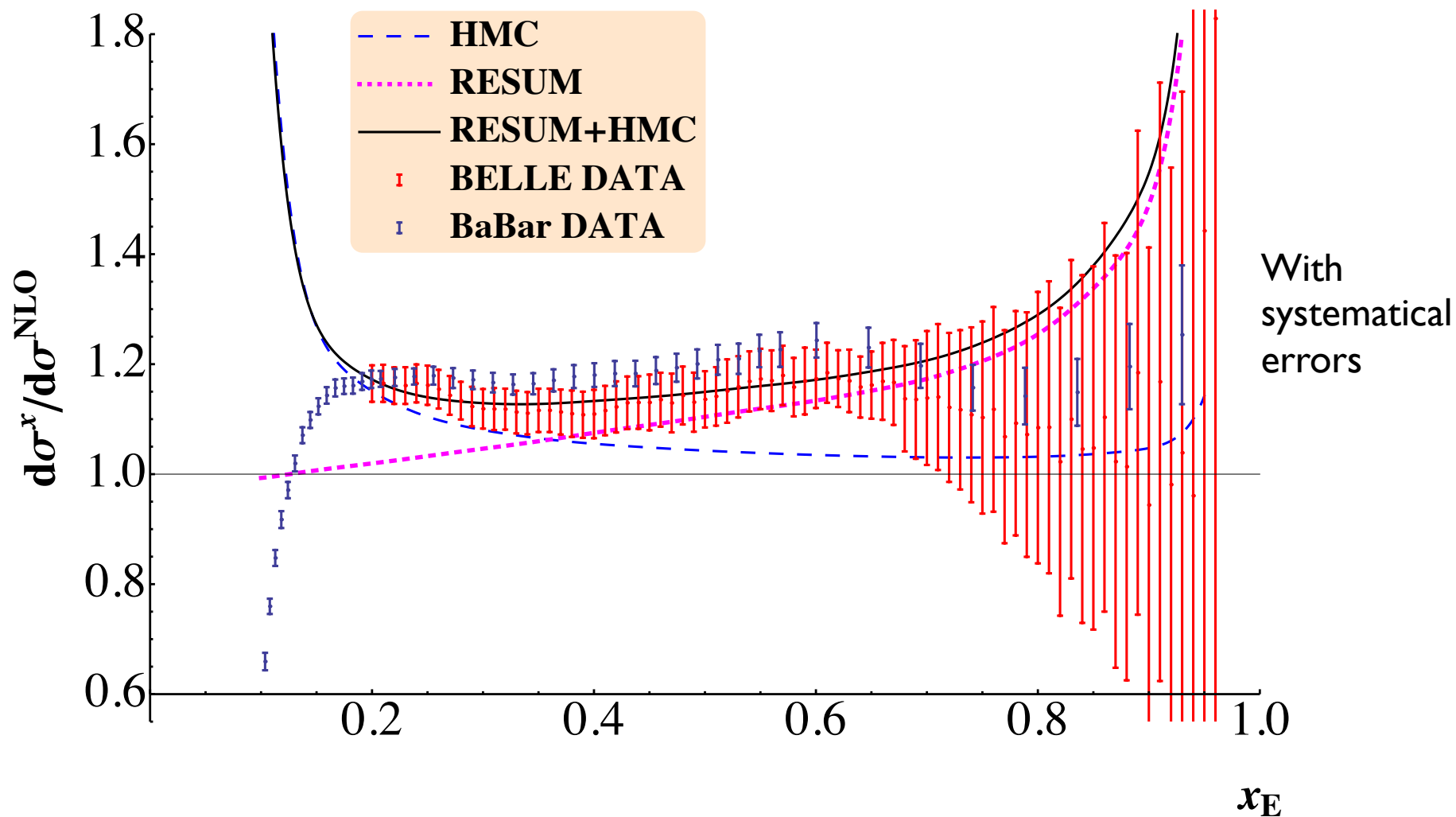
For Kaons one has to take into account HMC



Belle collaboration arXiv: 1301.6183; BaBar collaboration arXiv: 1306.2895



For Kaons one as to take into account HMC



*Belle collaboration arXiv: 1301.6183; BaBar collaboration arXiv: 1306.2895*



# OUTLINE

- THRESHOLD RESUMMATION FOR SIDIS
- HMC + THRESHOLD RESUMMATION
- TOWARDS A GLOBAL NNLO FF FIT
- CONCLUSIONS



# TOWARDS A GLOBAL NNLO FF FIT

*Anderle, Ringer, Stratmann*

Ingredients needed to achieve the goal:

## DATA SETS:

**SI- $e^+e^-$**  → old: TPC(Phys. Rev. Lett 61, 1263 (1998)), SLD(Phys. Rev. D59,052001 (1999)),  
ALEPH(Phys. Lett. B357, 487 (1995)),  
DELPHI(Eur. Phys. J. C5, 585 (1998),Eur. Phys. J.C6, 19 (1999))  
OPAL(Eur. Phys. J. C16, 407 (2000),Eur. Phys. J.C7, 369 (1999)),  
TASSO( Z. Phys.C42, 189 (1989))

**SIDIS** → old: EMC(Z. Phys. C52, 361 (1991)), JLAB(Phys. Rev. Lett. 98, 022001)

**SI-  $p(\text{anti-})p$**  → old: CDF(Phys. Rev. Lett. 61,1819 (1988)), UAI (Nucl. Phys. B335,261 (1990)),  
UA2(Z. Phys. C27, 329 (1985))



# TOWARDS A GLOBAL NNLO FF FIT

*Anderle, Ringer, Stratmann*

Ingredients needed to achieve the goal:

## DATA SETS:

**SI- $e^+e^-$**  → **new:** BaBar(Phys. Rev. D 88, 032011 (2013)), Belle(Phys. Rev. Lett. 111, 062002 (2013))

**SIDIS** → **new:** HERMES(Ph.D. thesis, Erlangen Univ., Germany, September 2005),  
Compass( PoS DIS 2013, 202 (2013)), JLAB@12GeV

**SI-  $p(\text{anti-})p$**  → **new:** Phenix(Phys. Rev. D 76, 051106 (2007)), Alice(Phys. Lett. B 717, 162 (2012).),  
Brahms(Phys. Rev. Lett. 98, 252001 (2007)), Star(Phys. Rev. Lett. 97, 152302 (2006))

**$pp \rightarrow (\text{Jet } h)X$**  → **future:** Star, CMS(JHEP 1210, 087 (2012) ), Alice(arXiv:1408.5723),  
Atlas(Eur. Phys. J. C 71, 1795 (2011))



# TOWARDS A GLOBAL NNLO FF FIT

*Anderle, Ringer, Stratmann*

Ingredients needed to achieve the goal:

## NNLO EVOLUTION KERNELS:

Splitting  
functions



**NNLO-Non Singlet:** Mitov, Moch, Vogt (Phys.Lett. B638 (2006) 61-67)

**NNLO-Singlet:** Moch, Vogt (Phys.Lett. B659 (2008) 290-296)

**NNLO-Singlet:** Almasy, Mitov, Moch, Vogt (Nucl.Phys. B854 (2012)) 133-152)

Both computed in **x-Space** and in **Mellin Space**



# TOWARDS A GLOBAL NNLO FF FIT

*Anderle, Ringer, Stratmann*

Ingredients needed to achieve the goal:

## NNLO COEFFICIENT FUNCTIONS:

SI- $e^+e^-$   $\longrightarrow$  **x-Space** Rijken, van Neerven  
 (Phys.Lett.B386(1996)422, Nucl.Phys.B488(1997)233, Phys.Lett.B392(1997)207)

**Mellin-Space** Mitov, Moch (Nucl.Phys.B751 (2006) 18-52)  
 Blümlein, Ravindran (Nucl.Phys.B749 (2006) 1-24)

SIDIS  $\longrightarrow$  **NOT COMPUTED YET but work in progress**

$\gamma q' \rightarrow q\bar{q}q'$   
 $\gamma g \rightarrow q\bar{q}q'$  Anderle, de Florian, Rotstein, Vogelsang

SI- p(anti-)p  $\longrightarrow$  **NOT COMPUTED YET**

pp  $\rightarrow$  (Jet h)X  $\longrightarrow$  **NOT COMPUTED YET**



# TOWARDS A GLOBAL NNLO FF FIT

*Anderle, Ringer, Stratmann*

Ingredients needed to achieve the goal:

## NNLO COEFFICIENT FUNCTIONS:

**SIDIS** → **Soft gluon Resummed results (can be expanded @ NNLO)**

Anderle, Ringer, Vogelsang ( *Phys.Rev. D87 (2013) 094021,*  
*Phys.Rev. D87 (2013) 3, 034014* )

**SI- p(anti-)p** → **Soft gluon Resummed results (can be expanded @ NNLO)**

Work in progress for  $\frac{d\sigma}{dp_T d\eta}^{(NNLL)}$  Hinderer, Ringer, Sterman, Vogelsang

**pp → (Jet h)X** → **Resummed results (can be expanded @ NNLO)**

Work in progress from T. Kaufmann, Vogelsang





# TOWARDS A GLOBAL NNLO FF FIT

*Anderle, Ringer, Stratmann*

Ingredients needed to achieve the goal:

**NNLO COEFFICIENT FUNCTIONS:**

To include the last processes we need a

**NNLO Mellin Space Fitting Program**



# THE NNLO EVOLUTION CODE “PEGASUS\_FF”

## Existing NNLO Evolution CODES:

**X-SPACE** APFEL(time-like version C/C++, Fortran77, Python)  
BertoneI, Carrazza, Rojo (CERN-PH-TH/2013-209)

**Mellin SPACE** MELA(Fortran77)  
BertoneI, Carrazza, Nocera (CERN-PH-TH-2014-265)

## Newly born:

**Mellin SPACE** Pegasus\_FF (Fortran77) → based on Pegasus(Fortran77)  
Anderle, Ringer, Stratmann Vogt (Comput.Phys.Commun.170:65-92,2005)



# OUR SIA FIT

Parametrization of light patrons FF @  $\mu_0$

$$D_i^h(z, Q_0) = \frac{N_i z^{\alpha_i} (1-z)^{\beta_i} [1 + \gamma_i (1-z)^{\delta_i}]}{B[2 + \alpha_i, \beta_i + 1] + \gamma_i B[2 + \alpha_i, \beta_i + \delta_i + 1]}$$

So that 
$$N_i = \int_0^1 z D_i^h dz$$

**Heavy Quark Treatment:**

**NON PERTURBATIVE INPUT:** at  $\mu > m_q$  the evolution is set to evolve with  $n_f + 1$  for flavours and for the q-heavy quark FF the same functional form as for the light quark is set at  $\mu = m_q$

**Data sets:**

**I5 Data Set:** from Sld, Aleph, Delphi, Opal, Tpc, BaBar, Belle either inclusive, uds tagged, b tagged or c tagged. We use a GLOBAL CUT  $0.075 < z < 0.95$



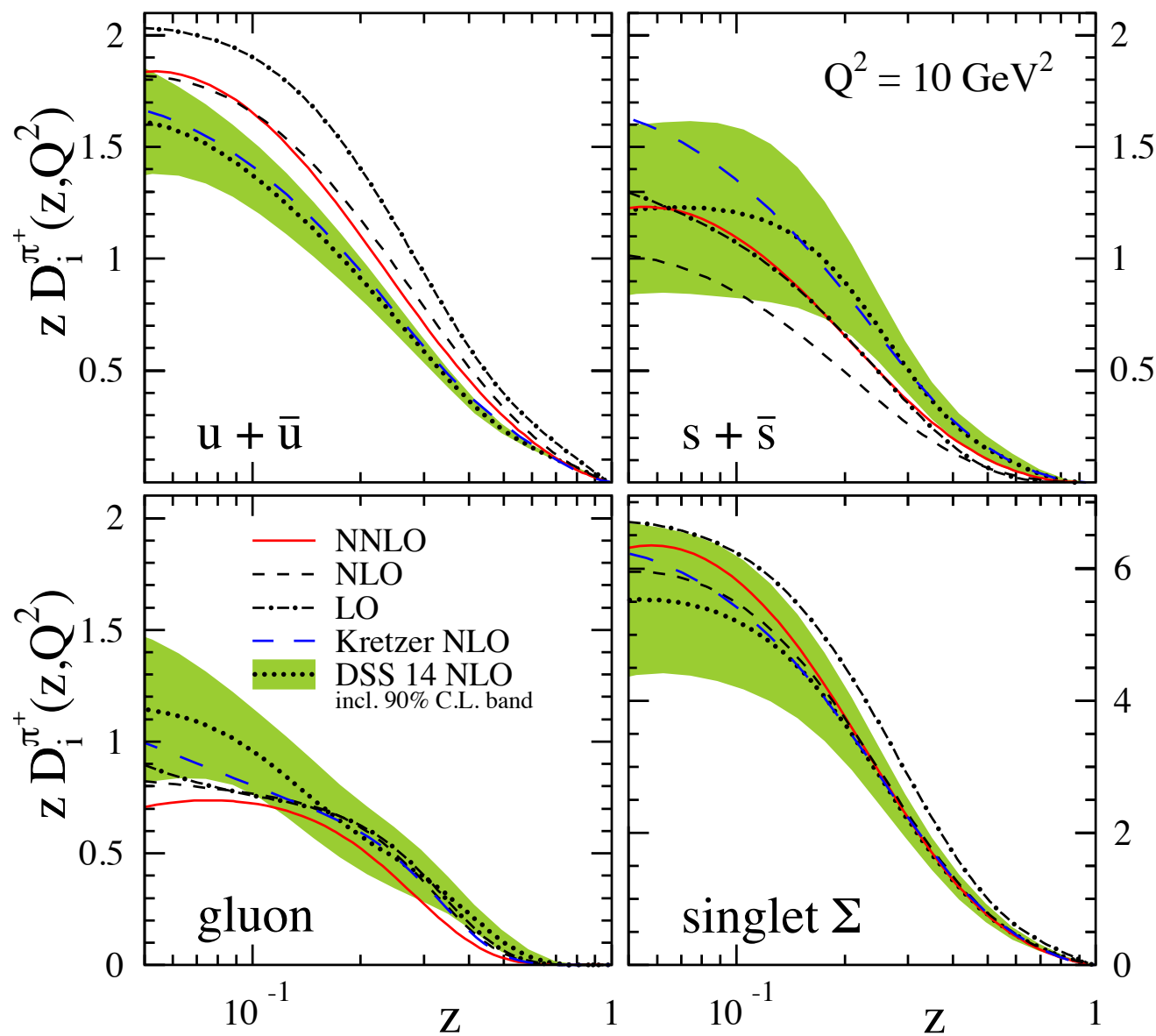
# PARAMETERS FOR $P_{I+}$

With only SIA one needs some extra constrains:

parameter	LO	NLO	NNLO	
$N_{u+\bar{u}}$	0.735	0.572	0.579	<b>5 free param needed</b> charge conjugation and isospin symmetry $D_{u+\bar{u}}^{\pi^\pm} = D_{d+\bar{d}}^{\pi^\pm}$ ,
$\alpha_{u+\bar{u}}$	-0.371	-0.705	-0.913	
$\beta_{u+\bar{u}}$	0.953	0.816	0.865	
$\gamma_{u+\bar{u}}$	8.123	5.553	4.062	
$\delta_{u+\bar{u}}$	3.854	1.968	1.775	
$N_{s+\bar{s}}$	0.243	0.135	0.271	<b>1 free param, 2 fixed by</b> $\alpha_{s+\bar{s}} = \alpha_{u+\bar{u}}, \beta_{s+\bar{s}} = \beta_{u+\bar{u}} + \delta_{u+\bar{u}}$
$\alpha_{s+\bar{s}}$	-0.371	-0.705	-0.913	
$\beta_{s+\bar{s}}$	4.807	2.784	2.640	
$N_g$	0.273	0.211	0.174	<b>2 free param, 1 fixed</b>
$\alpha_g$	2.414	2.210	1.595	
$\beta_g$	8.000	8.000	8.000	
$N_{c+\bar{c}}$	0.405	0.302	0.338	<b>3 free param</b>
$\alpha_{c+\bar{c}}$	-0.164	-0.026	-0.233	
$\beta_{c+\bar{c}}$	5.114	6.862	6.564	
$N_{b+\bar{b}}$	0.462	0.405	0.445	<b>5 free param</b>
$\alpha_{b+\bar{b}}$	-0.090	-0.411	-0.695	
$\beta_{b+\bar{b}}$	4.301	4.039	3.681	
$\gamma_{b+\bar{b}}$	24.85	15.80	11.22	
$\delta_{b+\bar{b}}$	12.25	11.27	9.908	

**TOT = 16 free param**





Kretzer FFS (Phys. Rev. D 62, 054001 (2000))

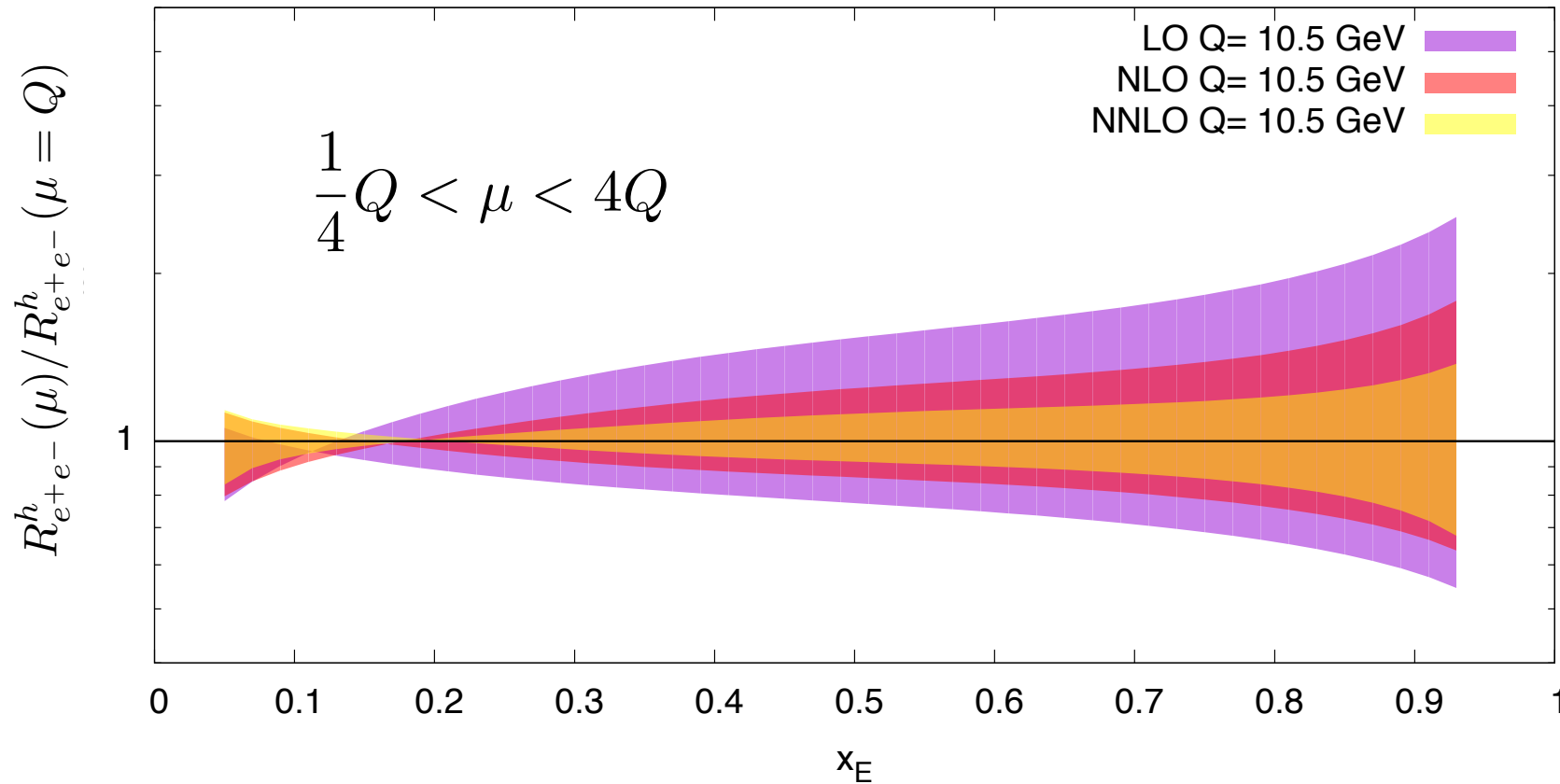
DSS FFS (Phys. Rev. D 91, 014035 (2015))

$$D_{\Sigma}^h = \sum_{i=1}^{n_f} (D_{q_i}^h + D_{\bar{q}_i}^h)$$



# SCALE DEPENDENCE

$e^+ e^- \mu$  scale dependence



Multiplicity  $R_{e^+e^-}^h \equiv \frac{1}{\sigma^{\text{tot}}} \frac{d^2\sigma^h}{dx_E d\cos\theta}$

using input parameter for FF  
of Kretzer (Phys.Rev. D62 (2000) 054001)  
and truncated-solution

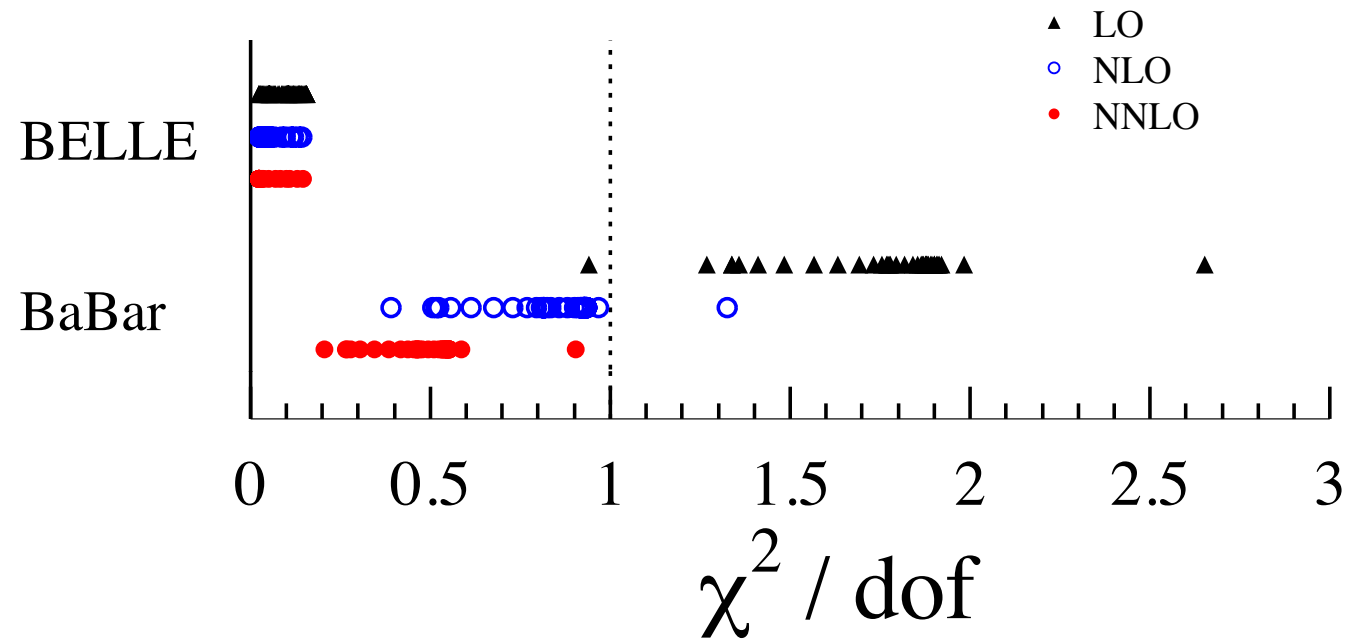


# $\chi^2$ COMPARISON

experiment	data type	# data in fit	$\chi^2$		
			LO	NLO	NNLO
SLD [40]	incl.	23	15.0	14.8	15.5
	<i>uds</i> tag	14	9.7	18.7	18.8
	<i>c</i> tag	14	10.4	21.0	20.4
	<i>b</i> tag	14	5.9	7.1	8.4
ALEPH [41]	incl.	17	19.2	12.8	12.6
DELPHI [42]	incl.	15	7.4	9.0	9.9
	<i>uds</i> tag	15	8.3	3.8	4.3
	<i>b</i> tag	15	8.5	4.5	4.0
OPAL [43]	incl.	13	8.9	4.9	4.8
TPC [44]	incl.	13	5.3	6.0	6.9
	<i>uds</i> tag	6	1.9	2.1	1.7
	<i>c</i> tag	6	4.0	4.5	4.1
	<i>b</i> tag	6	8.6	8.8	8.6
BABAR [10]	incl.	41	108.7	54.3	37.1
BELLE [9]	incl.	76	11.8	10.9	11.0
<b>TOTAL:</b>		288	241.0	190.0	175.2

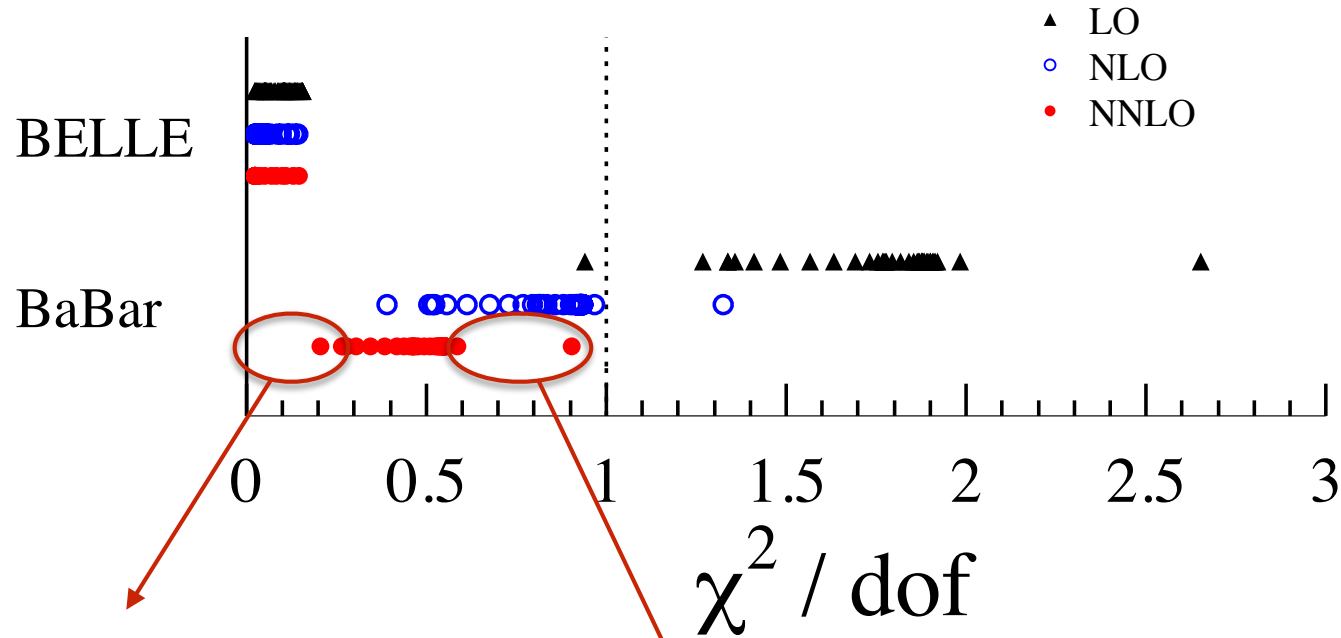


# $\chi^2$ COMPARISON





# $\chi^2$ COMPARISON



Vogt, arXiv:1108.2993 (JHEP);

Kom, Vogt, Yeats, arXiv:1207.5631 (JHEP);

Small  $z$  Logs

$$\alpha_s^k \frac{\ln^{2k}(z)}{z}$$

~~+Hadron Mass Cor.~~

Threshold Logs

$$\alpha_s^k \left( \frac{\ln^{2k-1}(1-x)}{1-x} \right)_+$$



# OUTLINE

- THRESHOLD RESUMMATION FOR SIDIS
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# CONCLUSIONS

- We have extended threshold resummation for SIDIS and begun the calculation of the tree level graphs appearing at NNLO for  $F_L$
- We have presented a framework for combined HMC with Resummation. Future extension to SIDIS
- We have presented our  $e^+e^-$  only FF NNLO fit and its extension to a global fit
- Future resummed FF fit including  $\text{Log}(N)/N$





# THANKS FOR YOUR ATTENTION

ANY QUESTIONS?