Hadron Structure：
*A **La**rge **M**omentum **E**ffective field **T**heory Approach*

Xiaonu Xiong (INFN, Pavia)

Jan. 6th, 2016

Biographical

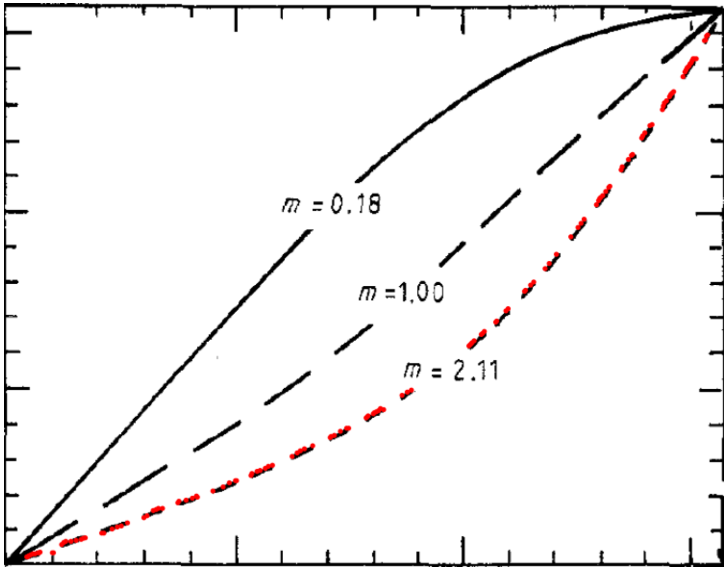
- Xiaonu Xiong
- ***Born:*** 1986-09-20
- ***Current Position:*** postdoc, INFN-section Pavia (2014-)
- ***Ph. D:*** Peking University, supervisor : Xiangdong Ji (2009-2014)
- ***Bachelor:*** Huazhong Normal University (Central China Normal University), Wuhan (2005-2009)
- ***Research Interests:*** spin structure, partonic OAM, parton distributions(PDF, TMD, GPD, DA, Wigner, LaMET), NRQCD
- ***Interests:*** Sci-Fi (*favourite: Hal Clement, Cixin Liu, Stephen Baxter; X-Files, SG1…*), popular science, badminton, ping-pong, playing with Mathematica

- Computer Skills

Mathematica(including HEP package, mathlink, parallelization, my own tools)

C, Python, GSL

My GitHub: https://github.com/ChiMaoShuPhy , part of my Mathematica tools (very recently)



e.g. automatic data extraction from a figure

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- Language: Chinese, English

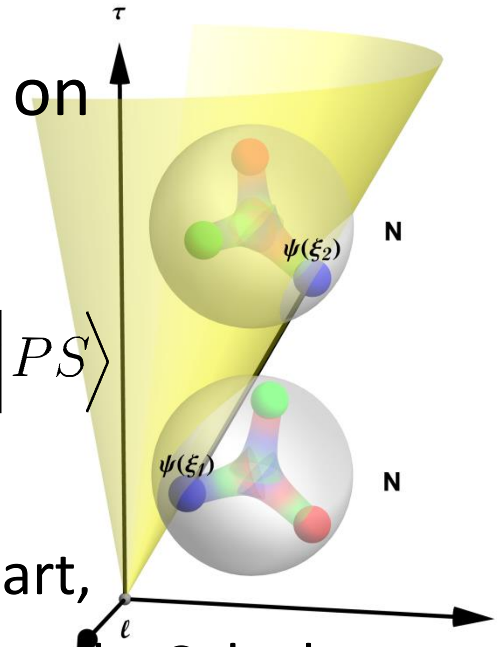
Outline

- LaMET
- NRQCD
- Spin structure and twist-3 GPD
- Research Plan

Research Topics

I. ***L***arge ***M***omentum ***E***ffective field ***T***heory

High-Energy Scattering & Lattice PDF Calculation Approach



- High-Energy scattering: probing physics on the light-cone

$$q(x) = \int \frac{d\xi^-}{2\pi} e^{-ixP^+\xi^-} \left\langle PS \middle| \bar{\psi}(\tfrac{\xi^-}{2}) \gamma^+ \mathcal{L}[\tfrac{\xi^-}{2}; \tfrac{-\xi^-}{2}] \psi(\tfrac{-\xi^-}{2}) \middle| PS \right\rangle$$

- Lattice calculation: $\xi^{\pm} \sim (-i\tau \pm z)$ has imaginary part, lattice can't calculate light-cone correlation directly. Calculate Mellin moments(local operator) instead. But higher-moments (higher order derivative) require fine lattice ⟹ computation cost

X. Ji, PRL. **110** (2013) 262002, Sci.China Phys.Mech.Astron. **57** (2014) 7, 1407-1412

LaMET Approach

- Construct a quasi quantity $\tilde{O}$ that can be directly calculated on lattice (Euclidean)

- $\langle P | \tilde{O} | P \rangle$ depends on the momentum P of the external state (large but finite)

- Extract light-cone(IMF) quantity $\langle P_\infty | O | P_\infty \rangle$ by matching condition (factorization formula)

$$\langle P | \tilde{O} | P \rangle (P) = \langle P_\infty | O | P_\infty \rangle (\mu) \otimes Z(\mu, P) + \mathcal{O}\left(\Lambda_{QCD}^2/P^2, M_N^2/P^2\right)$$

Same IR

UV control, perturbatively calculable

e.g. Apply on PDF

- Quasi-PDF

$$\tilde{q}(x, P^z, \mu) = \int \frac{dz}{2\pi} e^{ixp^z z} \langle P | \bar{\psi}(\tfrac{z}{2}) \gamma^z \mathcal{L}[\tfrac{z}{2}; -\tfrac{z}{2}] \psi(-\tfrac{z}{2}) | P \rangle$$

finite

- calculated on lattice
- Lattice renormalization (**Research Plan**)

- Matching

$$\tilde{q}(x, P^z, \mu) \otimes Z^{(-1)}(x, P^z, \mu)$$

- 1-loop continuum completed
- Lattice perturbation (**Research Plan**)
- Non-perturbative (**Research Plan**)

- Nucleon mass & higher-twist corrections

$$q(x, \mu) = \tilde{q}(y, P^z, \mu) \otimes Z^{(-1)}\left(\frac{x}{y}, P^z, \mu\right)$$

$$+ \mathcal{O}\left(\frac{M_N^n}{(P^z)^n}\right)$$

major correction currently $M_N / P^z \approx 1$

- $n = 2$: scaling factor of $x, \tilde{q}$ $\lambda^{-1}\tilde{q}(\lambda^{-1}x)$, $\lambda \sim 1 + M_N^2/4P_z^2$
- $n > 2$ (**Research Plan, major correction in LaMET**)

$$+ \mathcal{O}\left(\frac{\Lambda_{QCD}^n}{(P^z)^n}\right)$$

Higher twist, Lattice calculable

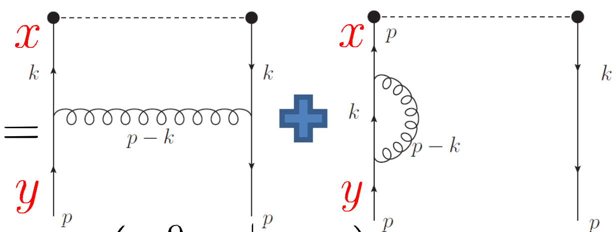
Ultimate goal: direct lattice determination of light-cone distributions

PDF Matching @ 1-loop

- gauge choice: $n \cdot A = 0 \rightarrow \mathcal{P}e^{i\int dn \cdot z\, n \cdot A} = 1$

  IMF: $n \cdot A = A^{+},\ n^2 = 0$, Quasi: $n \cdot A = A^{z},\ n^2 = -1$

$$\mathcal{Q}\left(x, P^z, \mu\right) =$$



- momentum: $P^{\mu} = \left(P^0, \mathbf{0}^{\perp}, P^z\right)$
- quark mass: m regularize collinear divergence
- massless gluon
- transverse cut-off: $\int_0^{\mu} dk_{\perp}$ regularize UV divergence (mimic lattice, breaks Lorentz symmetry, possibly breaks gauge symmetry)

Quasi/LC PDF @ 1-Loop

- Unpolarized (helicity, transversity also completed)

quasi

$$\lim_{\mu \gg P^z} \mathcal{Q}^{(1)}(x, P^z, \mu) = \tilde{q}^{(1)}(x, \mu)$$

$$= \frac{\alpha_S C_F}{2\pi} \begin{cases} -\frac{1+x^2}{1-x} \ln \frac{x}{x-1} - 1 + \frac{\mu}{(1-x)^2 P^z} \,, & x < 0 \,, \\ \frac{1+x^2}{1-x} \ln \frac{(P^z)^2}{m^2} + \frac{1+x^2}{1-x} \ln \frac{4x}{1-x} - \frac{4x}{1-x} + 1 + \frac{\mu}{(1-x)^2 P^z} \,, & 0 < x < 1 \,, \\ -\frac{1+x^2}{1-x} \ln \frac{x-1}{x} + 1 + \frac{\mu}{(1-x)^2 P^z} \,, & x > 1 \,, \end{cases}$$

$$+\delta(x-1) \frac{\alpha_S C_F}{2\pi} \int dy \begin{cases} -\frac{1+y^2}{1-y} \ln \frac{y}{y-1} - 1 + \frac{\mu}{(1-y)^2 P^z} \,, & y < 0 \,, \\ \frac{1+y^2}{1-x} \ln \frac{(P^z)^2}{m^2} + \frac{1+y^2}{1-y} \ln \frac{4y}{1-y} - \frac{4y^2}{1-y} + 1 + \frac{\mu}{(1-y)^2 P^z} \,, & 0 < y < 1 \,, \\ -\frac{1+y^2}{1-y} \ln \frac{y-1}{y} + 1 + \frac{\mu}{(1-y)^2 P^z} \,, & y > 1 \,, \end{cases}$$

LC

$$\lim_{P^z \gg \mu} \mathcal{Q}^{(1)}(x, P^z, \mu) = q^{(1)}(x)$$

$$= \frac{\alpha_S C_F}{2\pi} \begin{cases} 0 \,, & x > 1 \text{ or } x < 0 \,, \\ \frac{1+x^2}{1-x} \ln \frac{\mu^2}{m^2} - \frac{1+x^2}{1-x} \ln (1-x)^2 - \frac{2x}{1-x} \,, & 0 < x < 1 \,, \end{cases}$$

$$+\delta(x-1) \frac{\alpha_S C_F}{2\pi} \int dy \begin{cases} 0 \,, & y > 1 \text{ or } y < 0 \,, \\ -\frac{1+y^2}{1-y} \ln \frac{\mu^2}{m^2} + \frac{1+y^2}{1-y} \ln (1-y)^2 + \frac{2y}{1-y} \,, & 0 < y < 1 \,, \end{cases}$$

- Matching factor (unpolarized PDF)

$$Z^{(1)}\left(\xi,\frac{P^z}{\mu}\right)=\tilde{q}^{(1)}\left(\xi,P^z\right)-q^{(1)}(\xi,\mu)=\left(\frac{1+x^2}{1-x}\right)_+\ln\frac{\mu^2}{\left(P^z\right)^2}+\cdots$$

No $\ln(m)$, no IR pole: quasi/LC have same IR, matching UV.

Transfer momentum dependence into cut-off UV scale dependence

- Vector current conservation

$$\int dx\,\tilde{q}^{(1)}(x)+\int dy\,\delta\tilde{Z}_F(y)=0$$ ⇒ gauge symmetry preserved

$$\int d\xi\,Z^{(1)}\left(\xi,\frac{P^z}{\mu}\right)=0$$ ⇒ Forms a plus-distribution

X. Xiong, X. Ji, J.-H. Zhang, Y. Zhao, Phys. Rev. D **90**, 014051 (2014)
J.-W. Qiu, M.-Y. Qing arXiv:1404.6860 [hep-ph]

- Quark GPD, DA (1-loop)

$$\tilde{H}^{(1)}(x,\xi,t,P^z)\vee H^{(1)}(x,\xi,t,\mu) =$$

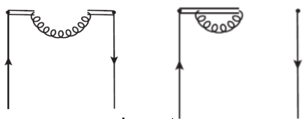
$$\frac{\alpha_S C_F}{2\pi}\begin{cases} \cdots + \frac{\mu}{(1-x)^2 p^z} \vee 0 & x<-\xi \\ \frac{x+\xi}{2\xi(1+\xi)}(1+\frac{2\xi}{1-x})\ln\frac{P_z^2}{-t}\vee\ln\frac{\mu^2}{-t} + \cdots + \frac{\mu}{(1-x)^2 P^z} & -\xi<x<\xi \\ \frac{1+x^2-2\xi^2}{(1-x)(1-\xi^2)}\ln\frac{P_z^2}{-t}\vee\ln\frac{\mu^2}{-t} + \cdots + \frac{\mu}{(1-x)^2 P^z} & \xi<x<1 \\ \cdots + \frac{\mu}{(1-x)^2 P^z} \vee 0 & x>1, \end{cases}$$

polynomiality of ln(…)

$$\tilde{E}^{(1)}(x,\xi,t,\mu)=E^{(1)}(x,\xi,t,\mu) =$$

$$\frac{\alpha_S C_F}{2\pi}\frac{m^2}{-t}\begin{cases} \frac{2(x-\xi)}{1+\xi}\ln\left(\frac{-t}{m^2}\right)+\cdots & -\xi<x<\xi \\ \frac{4(x+\xi^2)}{1-\xi^2}\ln\left(\frac{-t}{m^2}\right)+\cdots & \xi<x<1 \\ 0 & \text{otherwise}, \end{cases}$$

X. Ji, A. Schäfer, X. Xiong, J.-H. Zhang 2015
X. Xiong, J.-H. Zhang 2015

- Quark TMD: Soft-factor subtraction, cancels double pole ($\mu/\left(P^z(1-2x)^2\right)$ from )
- Twsit-3 PDF: $e(x)\sim\bar{\psi}(0)\mathcal{L}^\dagger\mathcal{L}\psi(z)$, $g_T(x)\sim\bar{\psi}(z)\mathcal{L}^\dagger\gamma^\perp\gamma^5\mathcal{L}\psi(0)$

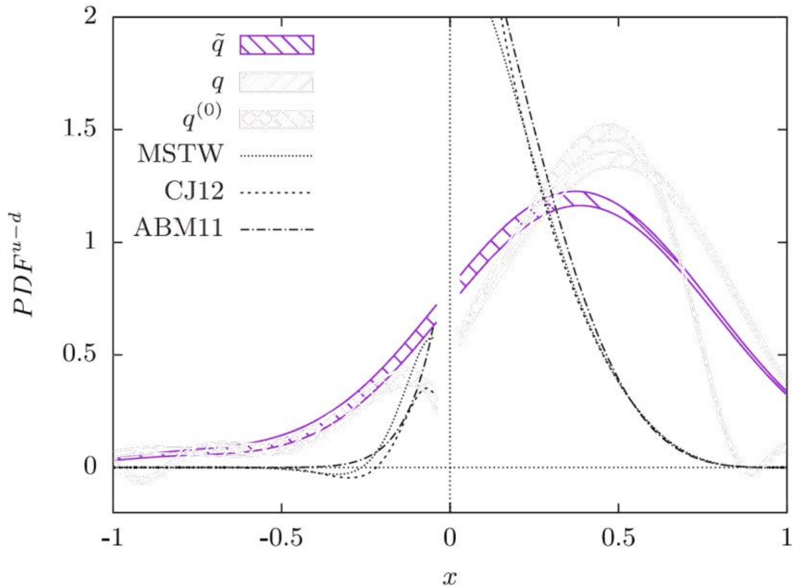
$$h_L(x)\sim\bar{\psi}(z)\mathcal{L}^\dagger i\sigma^{+-}\gamma^5\mathcal{L}\psi(0)$$

LC, quasi share same IR, matching UV

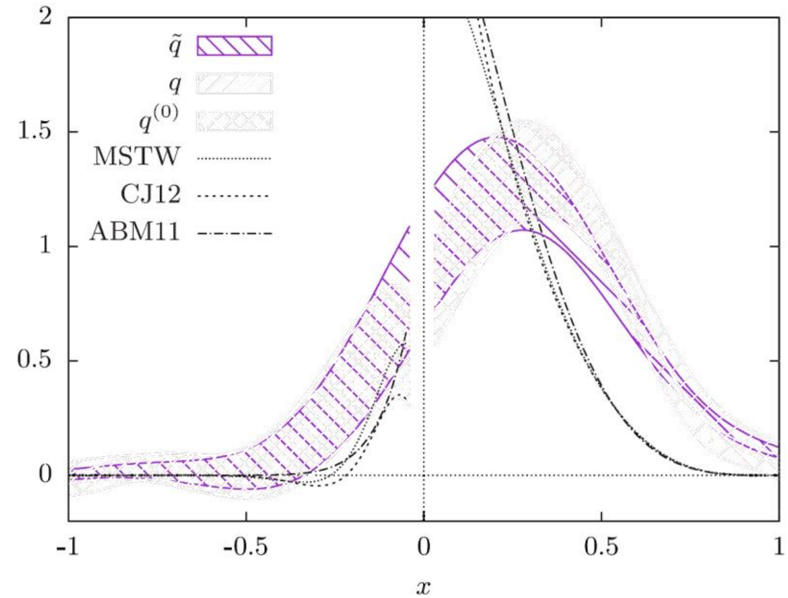
Lattice Quasi PDF Result

- C. Alexandrou, K. Cichy, V. Drach, E. Garcia-Ramos, K. Hadjiyiannakou, Karl Jansen, F. Steffens, and C. Wiese, PRD **92**, 014502 (2015)

$P^z = 0.98\text{GeV}$



$P^z = 1.47\text{GeV}$

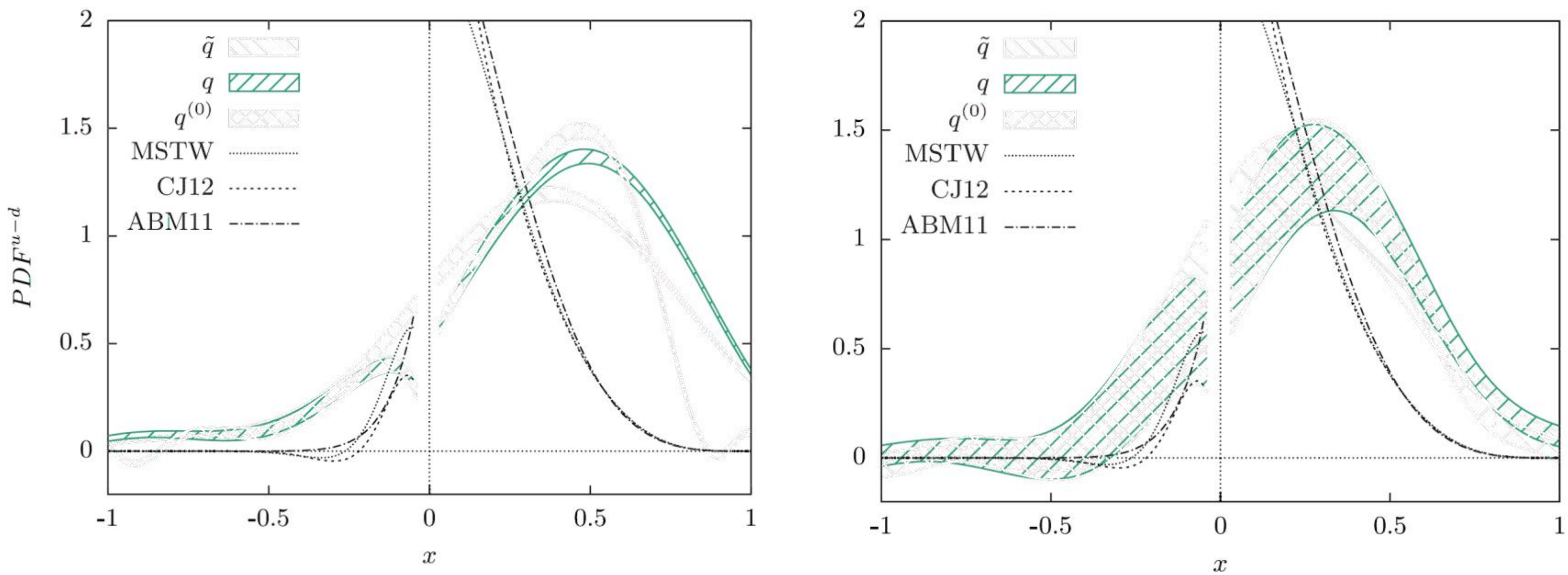


$P^z \approx M_P$

Lattice Quasi PDF Result + $\mathcal{O}(\alpha_s)$ matching

- C. Alexandrou, K. Cichy, V. Drach, E. Garcia-Ramos, K. Hadjiyiannakou, Karl Jansen, F. Steffens, and C. Wiese, PRD **92**, 014502 (2015)

$P^z = 0.98\text{GeV}$ $P^z = 1.47\text{GeV}$

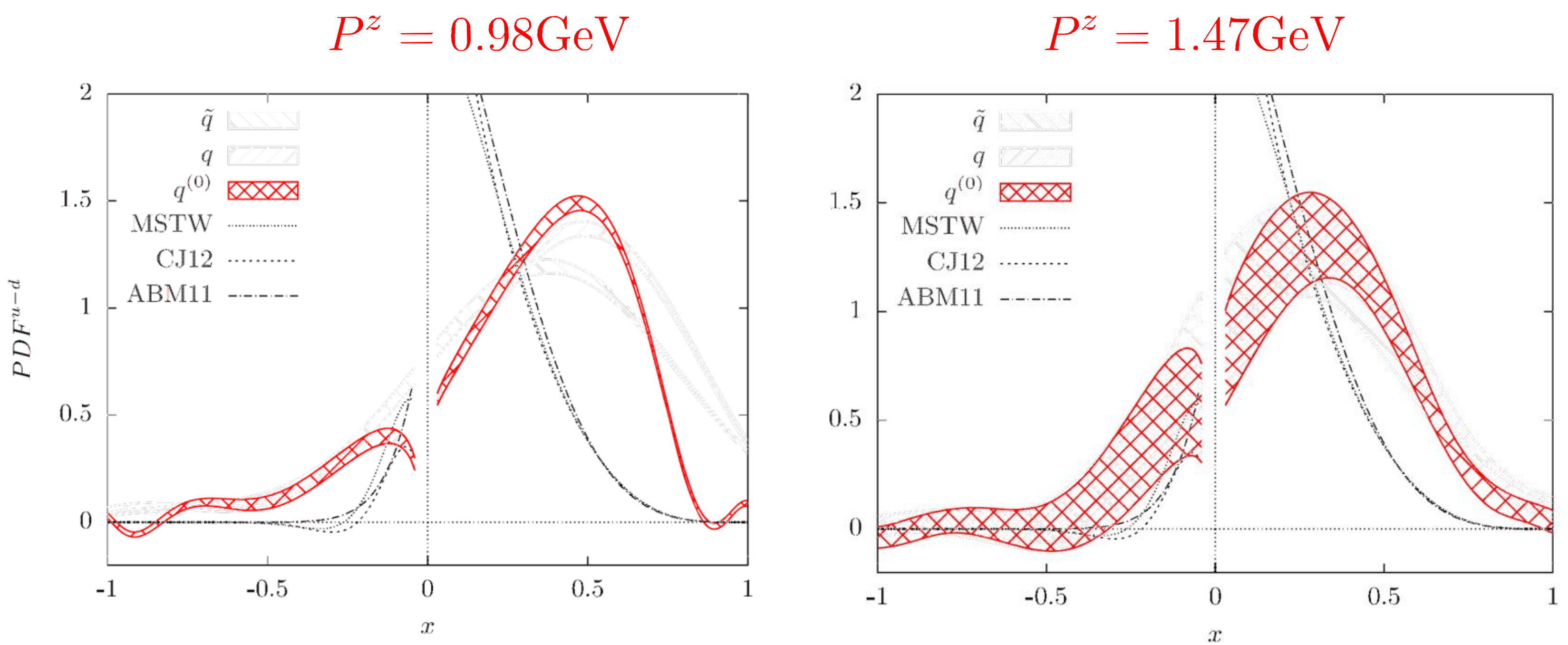


Perturbative matching pushes $\tilde{q}$ to unphysical region ???

➡ need a test to understand the role of perturbative matching

Lattice quasi PDF Results
$+\mathcal{O}(\alpha_s)$ matching+mass corrections

- C. Alexandrou, K. Cichy, V. Drach, E. Garcia-Ramos, K. Hadjiyiannakou, Karl Jansen, F. Steffens, and C. Wiese, PRD **92**, 014502 (2015)



Nucleon mass correction must be considered when $P^z \approx M_N$, a major correction in quasi PDF

Non-perturbative Test

Dec. 2015

- Motivation: Test LaMET non-perturbatively, understand the role of perturbative matching
- Theoretical laboratory: 2-D Large N_c QCD

  a. exactly solvable

  b. no physical gluon in 2-D, simple Fock state wave function for mesons

  c. IMF: 't Hooft Equation

light-cone wave function

$\frac{g^2 N_c}{4\pi}$

$$\left(\frac{m^2-2f}{x}+\frac{m^2-2f}{1-x}-M^2\right)\phi_+(x)=2f\,⨍_0^1 dy\frac{\phi_+(y)}{(x-y)^2}$$

Quasi: Bethe–Salpeter Equation

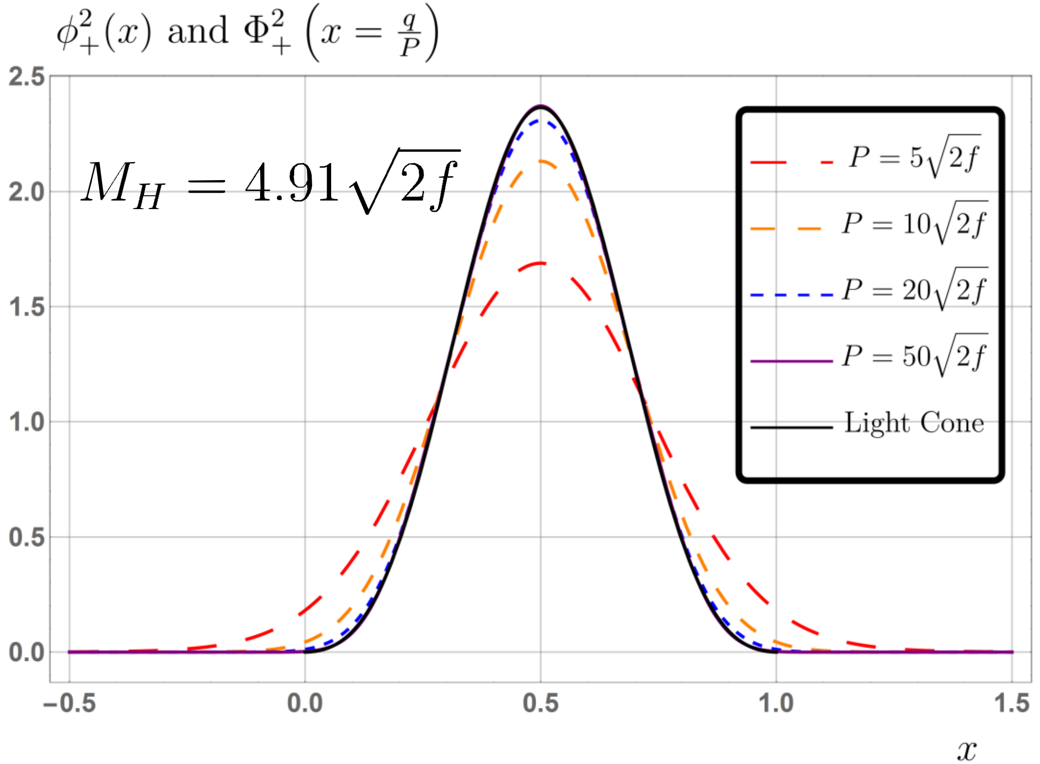
wave function, finite momentum

$$(\omega(q)+\omega(P-q)\mp P^0)\,\Phi_\pm(P,q)=f⨍\frac{dk}{(q-k)^2}\left(\xi_1(q,k)\Phi_\pm(P,q)\right.$$
$$\left.-\,(\xi_2(q,k)\,\Phi_\mp(P,q)\right)$$

d. Numerical Solution

Effective expansion in f/m_q^2, $m_q^2 = 8.9f \gg f$

first heavy quark limit, then large N_c limit: test perturbative matching



$P = 20\sqrt{2f}$ is already a good approximation to light-cone

$P = 50\sqrt{2f}$ almost same as light-cone

- With the wave functions all solved, one can calculate PDF, GPD…, then compare LC and quasi

- Understand the role of matching and test perturbative matching (why matching pushes quasi PDF to unphysical region):
    1-loop: trivial matching $Z^{(1)}\left(\frac{x}{y}\right) = \delta\left(1-\frac{x}{y}\right)$
    2-loop: much fewer diagram (no non-planner diagrams in large N_c system, under working)

Research Topics

II. ***NRQCD***

Heavy Meson Distribution Amplitudes

with LaMET

- Definition

$$-if_{J/\psi}P^{+}\Phi_{J/\psi}(x)=\int\frac{d\xi^{-}}{2\pi}\,e^{i\left(x-\frac{1}{2}\right)p^{+}\xi^{-}}\left\langle\eta_{c}\right|\bar{\psi}\left(\frac{\xi^{-}}{2}\right)\gamma^{+}\mathcal{L}[\tfrac{\xi^{-}}{2};\tfrac{-\xi^{-}}{2}]\psi\left(\frac{-\xi^{-}}{2}\right)\left|0\right\rangle$$

$$-if_{J/\psi}P^{z}\tilde{\Phi}_{J/\psi}(x)=\int\frac{dz}{2\pi}\,e^{-i\left(x-\frac{1}{2}\right)p^{z}z}\left\langle\eta_{c}\left(P^{z}\right)\right|\bar{\psi}\left(\tfrac{z}{2}\right)\gamma^{z}\mathcal{L}[\tfrac{z}{2};\tfrac{-z}{2}]\psi\left(\tfrac{-z}{2}\right)\left|0\right\rangle$$

- NRQCD refactorization of heavy meson DAs

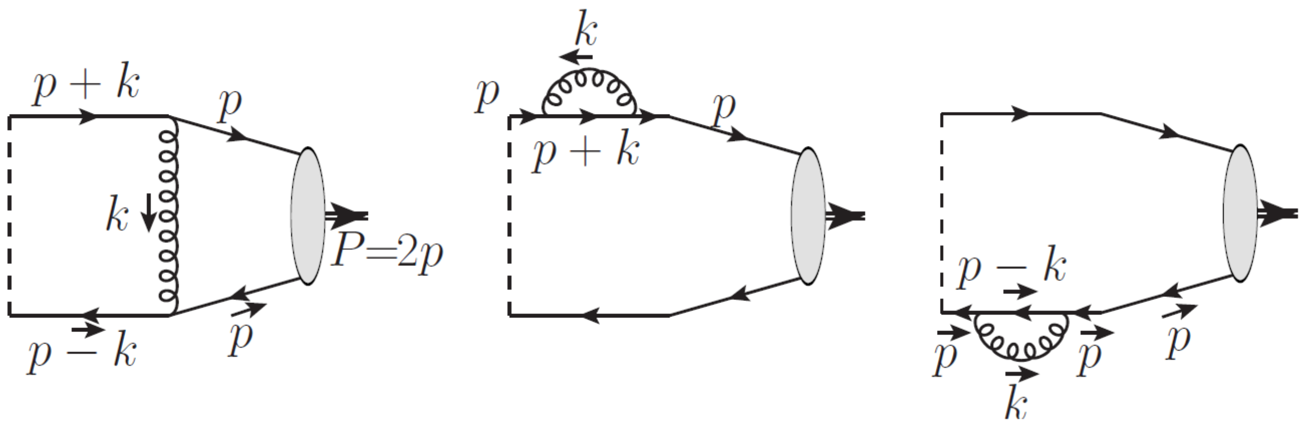
$$\Phi\left(x,\mu\right)=\sum_{n}\left(\left\langle H\left|\mathcal{O}_{n}^{NRQCD}\right|0\right\rangle\right)\left(\phi^{n}\left(x,\mu\right)\right)$$

$$\tilde{\Phi}(x,\mu,P^{z})=\sum_{n}\left(\left\langle H\left|\mathcal{O}_{n}^{NRQCD}\right|0\right\rangle\right)\left(\tilde{\phi}^{n}(x,\mu,P^{z})\right)$$

Perturbatively calculable coefficient function (UV), compare quasi v.s IMF ➡ P^z needed to recover LCDA

NR behavior, same IR between quasi and IMF ($v_{\mathrm{rel.}}^{n}$) expansion, n=0,1,.. : s,p wave DA

- s-wave DA @ 1-loop



- DR(IR) + Cut-off(UV) Hybrid regularization

$$\left(\frac{\mu^2 e^{\gamma_E}}{4\pi}\right)^{\epsilon}\int d^{2-2\epsilon}k_\perp = \left(\frac{\mu^2 e^{\gamma_E}}{4\pi}\right)^{\epsilon}\int_0^{\Lambda} dk_\perp k_\perp^{1-2\epsilon}$$

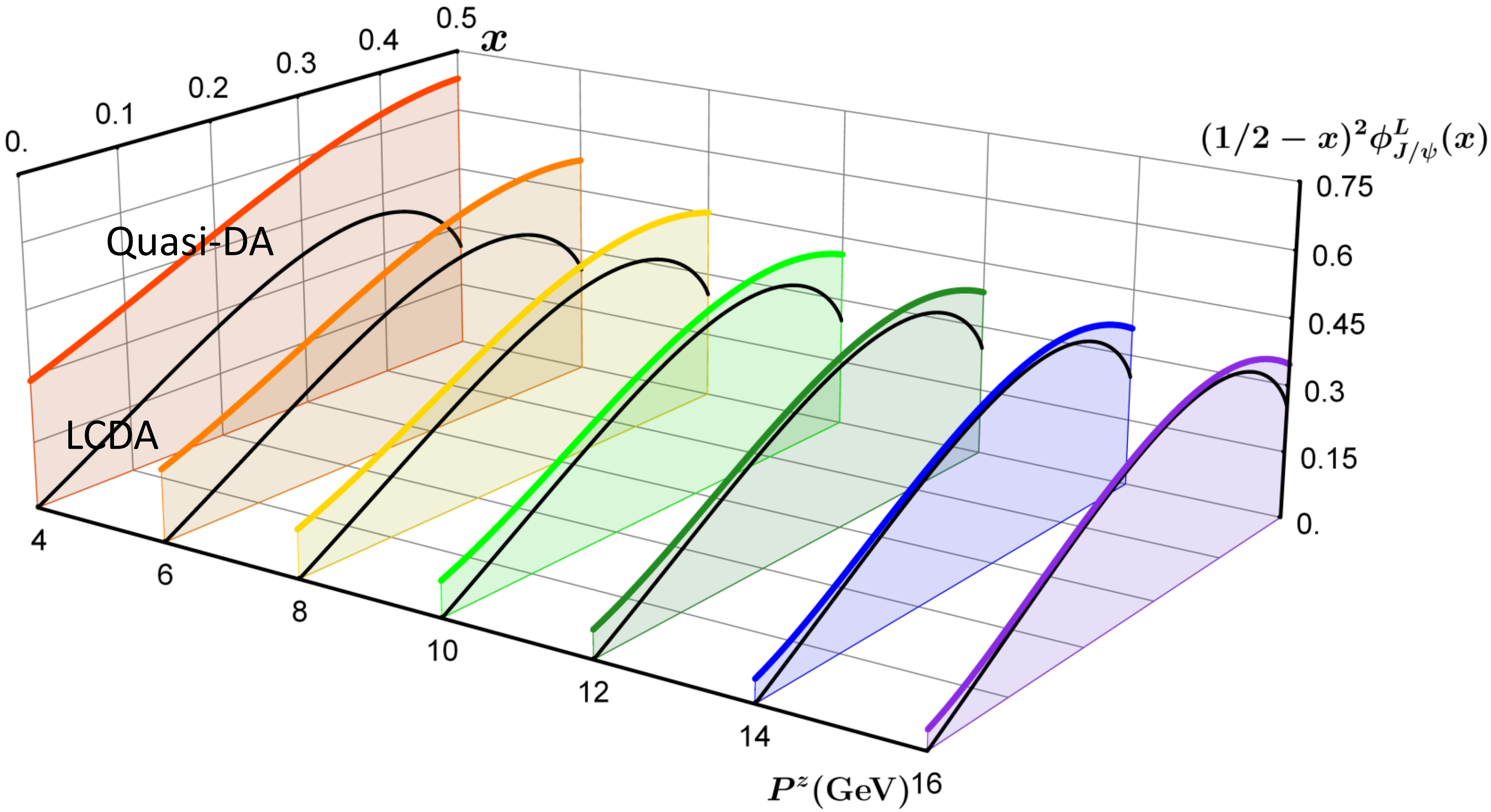
- all $\epsilon^{-1}+\ln\mu^2$ are cancelled and $(1-2x)^{-1},(1-2x)^{-2}$ are regularized to +,++ distributions, no IR pole

➡ NRQCD factorize IR into long range matrix element

- Numerical Results of DA @ 1-loop

charmonium: $J/\psi^L, J/\psi^T, \eta_c$'s s-wave $\tilde{\phi}(x,\mu,p^z)$, $\phi(x,\mu)$

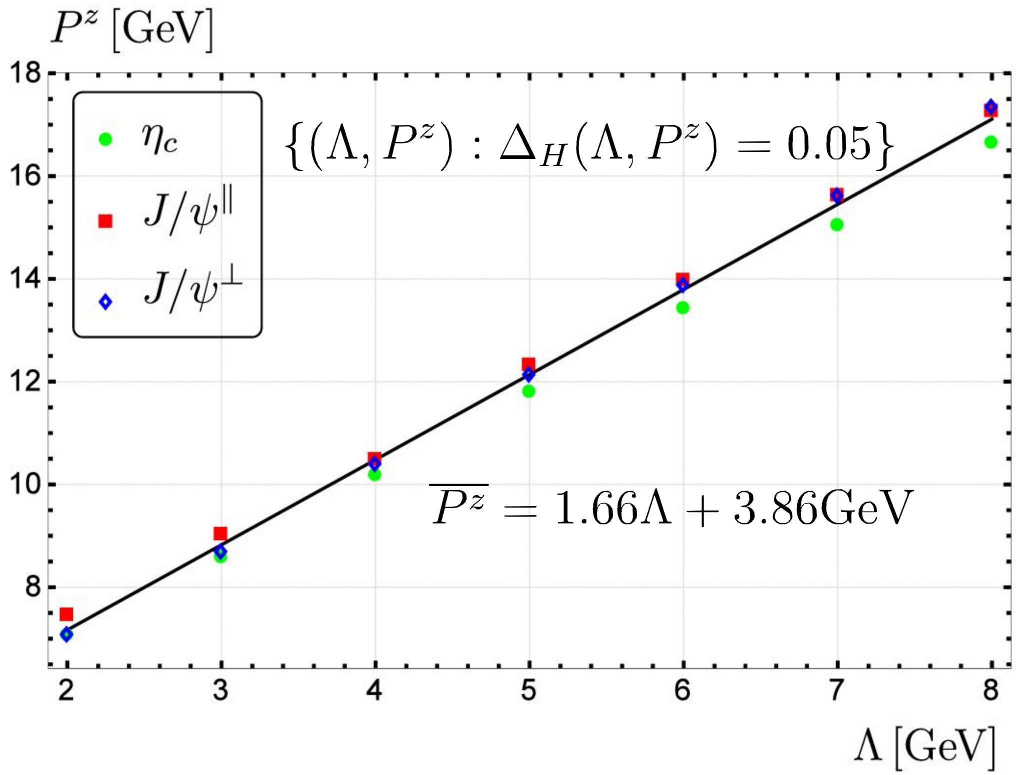
e.g. $\phi^L_{J/\psi}: m_c = 1.4\mathrm{GeV}, \Lambda = 3\mathrm{GeV}$



Yu. Jia, X. Xiong, arXiv:1511.04430 [hep-ph]

- *Degree of Resemblance*

$$\Delta_H(P^z) = \frac{\int_0^{\frac{1}{2}} dx\ (1-2x)^4 \left(\tilde{\phi} - \phi\right)^2}{\int_0^{\frac{1}{2}} dx\ (1-2x)^4\, \phi^2}$$



- Provide some information on setting lattice spacing parameter and estimating the correction needed

Heavy Meson Fragmentation Function

Dec. 2015

not LaMET

- Motivation: First η_c production differential cross section measurement on LHC recently, help to understand LHC data

- Definition

$$\mathcal{D}_{q\to h}(z) = \frac{z}{4N_c}\sum_X \int \frac{d\xi^-}{2\pi} e^{izk^+\xi^-} \mathrm{Tr}\left\{\left\langle h, X \left| \bar{\psi}(0)\,\mathcal{L}[0;\infty] \right| 0 \right\rangle \gamma^+ \left\langle 0 \left| \mathcal{L}[\infty,\xi^-]\psi\left(\xi^-\right) \right| h, X \right\rangle\right\}$$

$$\mathcal{D}_{g\to h}(z) = \frac{-z}{2k^+\left(N_c^2-1\right)}\sum_X \int \frac{d\xi^-}{2\pi} e^{izk^+\xi^-} \left\langle h, X \left| F^{+\rho}(0)\mathcal{L}[0;\infty] \right| 0 \right\rangle \left\langle 0 \left| \mathcal{L}[\infty,\xi^-] F_\rho^+\left(\xi^-\right) \right| h, X \right\rangle$$

- Similar NRQCD refactorization as DA

$$\mathcal{D}(x,\mu) = \sum_n \left[\left\langle H \left| \mathcal{O}_n^{NRQCD} \right| 0 \right\rangle\right]\left[D^n(x,\mu)\right]$$

- $\mathcal{O}(\alpha_s)\left(1+\mathcal{O}\left(v^2\right)\right)$ FF(complete) + scale evolution(under working)

Research Topics

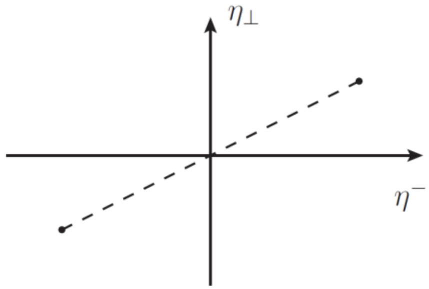
III. ***Nucleon spin structure and Tomography(Ph. D period)***

Partonic Spin Structure

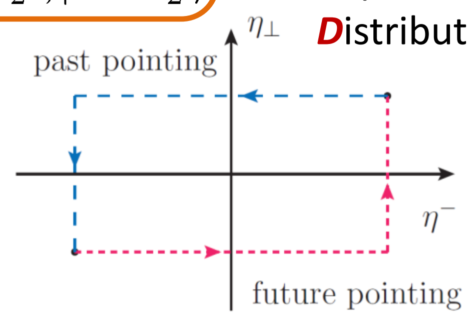
- Wigner distribution: quantum phase-space distribution (**most complete information of a nucleon**)

$$W^q(x,\boldsymbol{k}_\perp,\boldsymbol{b}_\perp)=\int\frac{d^2\boldsymbol{\Delta}_\perp}{(2\pi)^2}e^{-i\boldsymbol{\Delta}_\perp\cdot\boldsymbol{b}_\perp}\int\frac{d\eta^- d^2\boldsymbol{\xi}_\perp}{(2\pi)^3}e^{-ik\eta}\left\langle P+\frac{\boldsymbol{\Delta}_\perp}{2}\right|\bar{\psi}\left(\frac{\eta}{2}\right)\gamma^+\mathcal{L}\left[\frac{\eta}{2},-\frac{\eta}{2}\right]\psi\left(\frac{-\eta}{2}\right)\left|P-\frac{\boldsymbol{\Delta}_\perp}{2}\right\rangle$$

Generalized
Transverse
Momentum
dependent
Distribution



L_{FS}



L_{LC}

$$\frac{\left\langle P,S\left|\int d^3\vec{r}\,\overline{\psi}\left(\vec{r}\right)\gamma^+\left(\vec{r}_\perp\times i\vec{D}_\perp\right)\psi\left(\vec{r}\right)\right|P,S\right\rangle}{2}=\int dx\int d^2\vec{b}_\perp d^2\vec{k}_\perp\left(\vec{b}_\perp\times\vec{k}_\perp\right)W_{FS}\left(x,\vec{k}_\perp,\vec{b}_\perp\right)$$

$$\frac{\left\langle P,S\left|\int d^3\vec{r}\,\overline{\psi}\left(\vec{r}\right)\gamma^+\left(\vec{r}_\perp\times i\vec{\partial}_\perp\right)\psi\left(\vec{r}\right)\right|P,S\right\rangle}{2}=\int dx\int d^2\vec{b}_\perp d^2\vec{k}_\perp\left(\vec{b}_\perp\times\vec{k}_\perp\right)W_{LC}\left(x,\vec{k}_\perp,\vec{b}_\perp\right)$$

X. Ji, X. Xiong, F. Yuan, PRL 2012

- Gauge Inv. OAM

  ~ twist-3 GPD, measurable

  moments reduce to local operator (lattice calculable)

Transverse gluon: 3-particle correlation(**twist-3**)

$$\frac{1}{n}\sum_i \overline{\psi}(0)\gamma^+\left(iD^+\right)^i\left(\vec{r}_\perp \times i\vec{D}_\perp\right)\left(iD^+\right)^{n-1-i}\psi(0)$$

$r_\perp \overset{\text{F.T.}}{\leftrightarrow} \Delta_\perp$: **GPD**

$$= \int dx \int dy \frac{1}{n}\sum_{k=0}^{n-1} x^{n-1-k}(x-y)^k H_D^{q(3)}(x,y,0,0)$$

Twist-3 GPD

- Canonical OAM

  can be made gauge inv. through GIE, then measurable

$$\bar{\psi}\, r^\perp \times \left[i\tilde{\partial}^\perp = iD^\perp + \int^{\xi^-} d\eta^- L_{[\xi^-,\eta^-]} g F^{+\perp}\left(\eta^-,\xi_\perp\right) L_{[\eta^-,\xi^-]}\right]\psi$$

  but highly non-local

X. Ji, X. Xiong, F. Yuan, PRD 2013

Research Plan

- **Parton OAM measurement:**

1. Identify OAM piece in spin decomposition
2. GTMD parameterization

Wigner distribution　　GTMD

$$\int d^2\boldsymbol{k}_\perp d^2\boldsymbol{b}_\perp\, \boldsymbol{b}_\perp \times \boldsymbol{k}_\perp W^{[+]}(x,\boldsymbol{k}_\perp,\boldsymbol{b}_\perp) = \int d^2\boldsymbol{k}_\perp \frac{\boldsymbol{k}_\perp}{2M^2} F_{1,4}$$

$$\int \frac{d\xi^- d^2\boldsymbol{\xi}^\perp}{(2\pi)^3} e^{-ixP^+\xi^- + ik_\perp\cdot\boldsymbol{\xi}_\perp} \langle P',\lambda \left| \bar{\psi}\left(\tfrac{\xi}{2}\right)\gamma^+\psi\left(-\tfrac{\xi}{2}\right)\right| P,\lambda\rangle$$

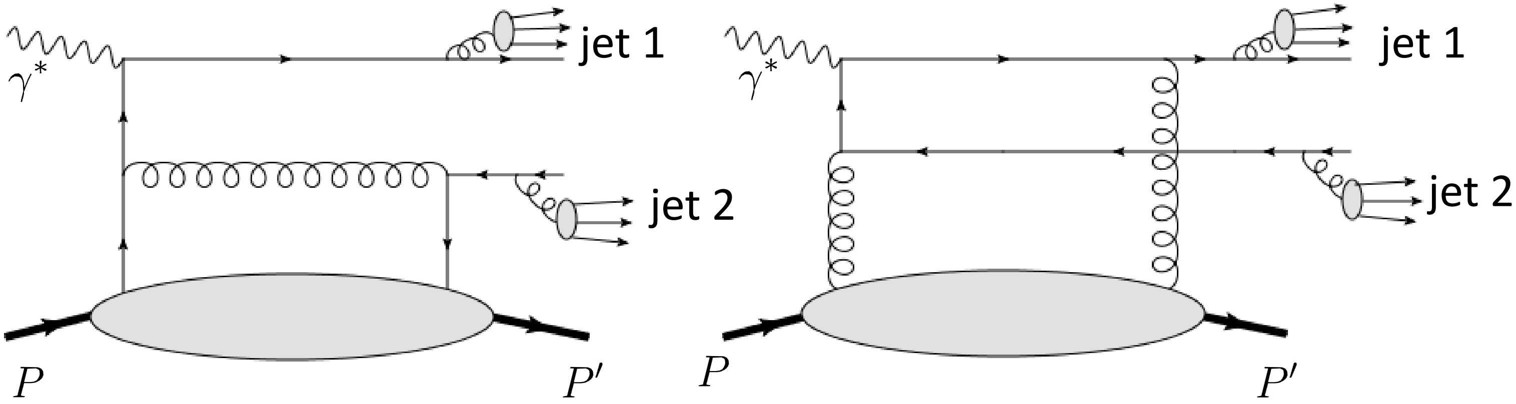
$$= \bar{u}_\lambda(P')\gamma^+ u_\lambda(P) F_{1,1} + \bar{u}_\lambda(P') \frac{\boldsymbol{S}\cdot\boldsymbol{k}_\perp \times (\boldsymbol{P}'_\perp - \boldsymbol{P}_\perp)}{M^2} u_\lambda(P) F_{1,4}$$

spin asymmetry　　exclusive process

3. Look for spin asymmetry in a polarized exclusive process measuring $F_{1,4}$

S. Lituri et al. 2014, 2015

4. Possible processes: di-jet production



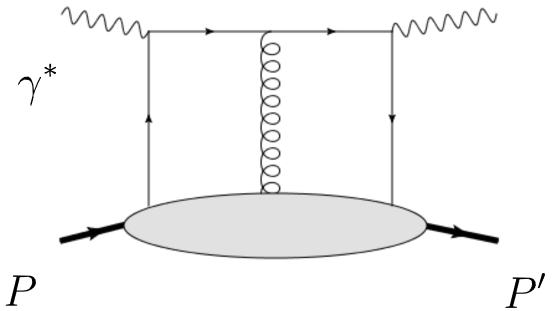
- **Twist-3 GPD**

1. parton OAM is related to twist-3 GPD

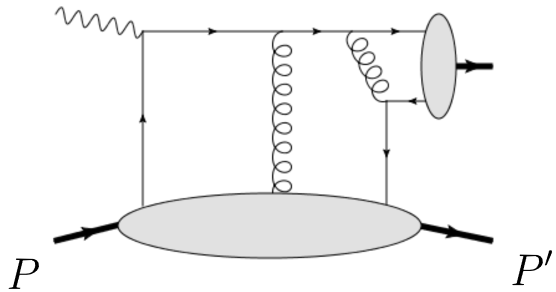
$$L_q \sim \int \frac{d\lambda}{2\pi} \int \frac{d\mu}{2\pi} e^{i\lambda(x-y)} e^{i\mu y} \left\langle P', S \left| \bar{\psi}(0)\gamma^+ iD^\perp(\mu n)\psi(\lambda n) \right| P, S \right\rangle$$

$$= \frac{i\epsilon^{\perp\alpha}}{2} \Delta_\alpha H_D^{q(3)}(x, y, \eta, t) \bar{U}(P')\gamma^+\gamma_5 U(P) + \cdots$$

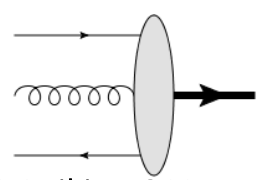
2. measuring twist-3 GPD via exclusive process (3-parton correlation)



DVCS



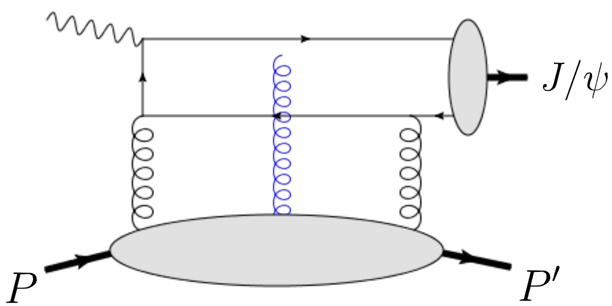
exclusive meson production, spin density matrix: 15 independent observables

but twist-3 effects in DA  should be considered Trans. pol. meson: I.V. Anikin , O.V. Teryaev, PLB 554 (2003) 51–63

Using NRQCD to solve it e.g. J/ψ?

- **Gluon GPD, TMD**

1. gluon's multi-dimensional distribution inside nucleon
2. gluon GPD provides information of gluon's contribution to nucleon spin



e.g. J/ψ production: dominated by gluon contribution

3. Extend to gluon twist-3 GPD and OAM

- **LaMET, Lattice QCD**

1. Nucleon mass corrections:

   major correction in LaMET, boost lattice calculation's performance (computation cost $\sim a^{-7}$ [CP-PACS, JLQCD])

2. Lattice perturbation matching & non-perturbative matching:

   continuum, perturbative matching doesn't work well

3. LaMET application on TMD & GPD and gluon distribution.

- **NRQCD…**


Thanks

Backup Slides

LaMET Approach

- Construct a quasi quantity $\tilde{O}$ that can be directly calculated on lattice (Euclidean)

- $\langle P | \tilde{O} | P \rangle$ depends on the momentum P of the external state (large but finite)

- Extract light-cone(IMF) quantity $\langle P_\infty | O | P_\infty \rangle$ by matching condition (fractorization formula)

$$\langle P | \tilde{O} | P \rangle (P) = \boxed{Z(\mu, P)} \otimes \langle P_\infty | O | P_\infty \rangle (\mu) + \mathcal{O}\left(P^{-n}\right)$$

UV controlled,
perturbatively calculable

Scattering Experiments vs. Quasi Lattice Calculation

	High-Energy Scattering	Quasi Lattice Calculation
"observables"	Cross section	Quasi-quantities
Scale	Large momentum transfer (Q).	Hadron momentum (P).
Factorization	$\sigma = \sigma_H \left(x, Q^2\right) \otimes f\left(x, Q^2\right) + \mathcal{O}\left(\left(Q\right)^{-n}\right)$	$\tilde{f}\left(P^z\right) = Z\left(\frac{P^z}{\mu}\right) \otimes f\left(\mu\right) + \mathcal{O}\left(\left(P^z\right)^{-n}\right)$

- Space like correlation function $\neq$ static, does not depend on time

$$\langle q_1 | e^{iHt} \bar{\psi}\left(\tfrac{z}{2}\right) \gamma^z \Gamma \mathcal{L}[\tfrac{z}{2}; \tfrac{-z}{2}] \psi\left(\tfrac{-z}{2}\right) e^{-iHt} | q_2 \rangle = e^{i(E_1 - E_2)t} \langle \cdots \rangle$$

Forward case: no time dependence

Off-forward case: fixed time

Light-cone case:

$$H = P^- \to e^{i\left(P_1^- - P_2^-\right)\xi^+} \sim 1 + \mathcal{O}\left(\frac{m^2 \xi^+}{P^+}\right)$$

Mass and Higher-Twist Correction

- Mass correction at $\mathcal{O}(M^2/(P^z)^2)$

$$\tilde{q}(x,P^z,\mu)=\int\frac{dz}{2\pi}e^{ixp^z z}\langle P|\bar{\psi}(\tfrac{z}{2})\gamma^z\mathcal{L}[\tfrac{z}{2};-\tfrac{z}{2}]\psi(-\tfrac{z}{2})|P\rangle$$

series expansion

$$\langle P|\bar{\psi}(0)\gamma^z\mathcal{L}[0,z]\psi(z)|P\rangle=\frac{1}{2P^z}\sum_n\frac{(-iz)^n}{(n)!}\langle P|\bar{\psi}(0)\gamma^z(iD^z)^n\psi(0)|P\rangle$$

ignore trace of operator (higher-twist correction)

e.g. $\langle P\left|g^{zz}\bar{\psi}(0)(iD^z)^i(\gamma^\mu iD_\mu)(iD^z)^{n-i}\psi(0)\right|P\rangle\sim\mathcal{O}\left(\frac{\Lambda^n_{QCD}}{(P^z)^n}\right)$

gives

$$\langle P|\bar{\psi}(0)\gamma^z(iD^z)^n\psi(0)|P\rangle=2a_n\left[P^{(\mu_0}\cdots P^{\mu_n)}-\mathrm{tr}\left(P^{(\mu_0}\cdots P^{\mu_n)}\right)\right]\Big|_{\mu_i=z}$$

Mellin moments of quasi PDF $\int dx x^n\tilde{q}(x)$

the trace of matrix element is

$$\mathrm{tr}\left(P^{(\mu_0}\cdots P^{\mu_n)}\right)=\sum_{i=1}^{n}\frac{g^{\mu_0\mu_i}P^2}{4}P^{(\mu_1}\cdots P^{\mu_{i-1}}\cdots P^{\mu_{i+1}}\cdots P^{\mu_n)}+\mathcal{O}\left(\frac{M^4}{(P^z)^4}\right)$$

taking $\mu_i = z$ gives

$$\mathrm{tr}\left(\cdots\right)=-n\frac{M^2}{4\left(P^z\right)^2}\left(P^z\right)^{n+1}$$

Therefore

$$\frac{1}{2P^z}\sum_n\frac{(-iz)^n}{(n)!}\langle P\left|\bar{\psi}\left(0\right)\gamma^z\left(iD^z\right)^n\psi\left(0\right)\right|P\rangle$$

$$=\sum_n\frac{(-iz)^n}{n!}2a_n\left(P^z\right)^n\left[1+n\frac{M^2}{4\left(P^z\right)^2}\right]+\mathcal{O}\left(\frac{\Lambda_{QCD}^2}{\left(P^z\right)^2},\frac{M^4}{\left(P^z\right)^4}\right)$$

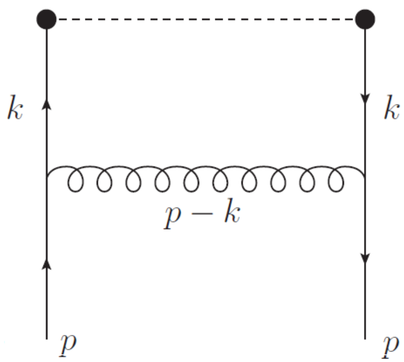
$$=\sum_n\frac{(-i\lambda z)^n}{n!}2a_n\left(P^z\right)^n+\mathcal{O}\left(\frac{\Lambda_{QCD}^2}{\left(P^z\right)^2},\frac{M^4}{\left(P^z\right)^4}\right)$$

with $\lambda = 1+\frac{M^2}{4\left(P^z\right)^2}$

Fourier Transform to x space

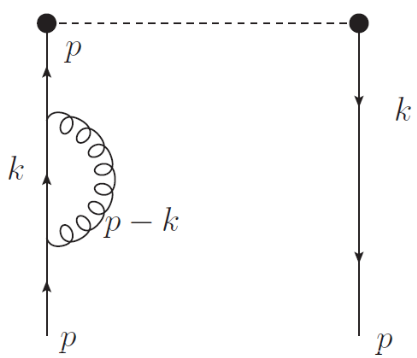
$$\tilde{q}(x,P^z,\mu)\to\lambda^{-1}\tilde{q}\left(\lambda^{-1}x\right)$$

Feynman Diagram ($A^z = 0$)

- 

$$\mathcal{Q}(x, P^z, \mu) \sim \int d^4k\, q\,(k, P^z)\,\delta\left(x - \frac{k^z}{P^z}\right)$$

+



$$\delta\mathcal{Z}_F\,(P^z, \mu)\,\delta(x-1) \sim \int d^4k\,\delta z_F\,(k, P^z, \mu)\,\delta\,(x-1)$$

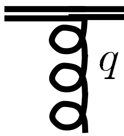
=

$$\mathcal{Q}^{(1)}(x, P^z, \mu)$$

Gauge Invariance

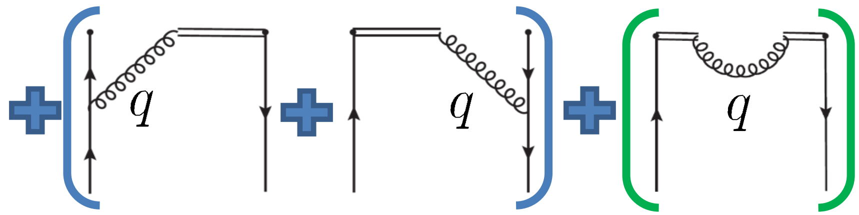
- Preserved by gauge link.
- $n \cdot A = 0$ And Feynman gauge and gauge

$$D_n^{\mu\nu}(q) = \frac{-i}{q^2}\left(g^{\mu\nu} - \frac{q^\mu n^\nu + n^\mu q^\nu}{n \cdot q} + n^2 \frac{q^\mu q^\nu}{n \cdot q^2}\right)$$

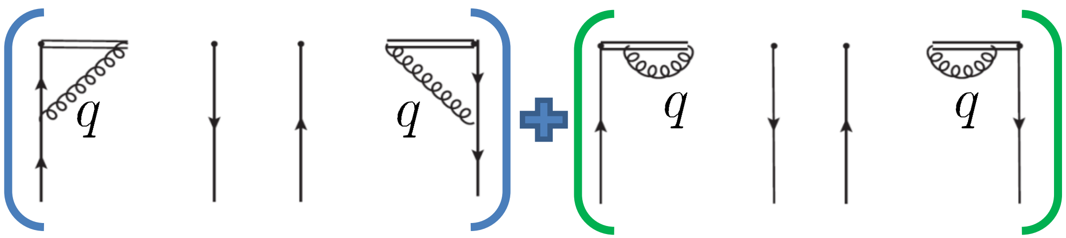


$$\sim \frac{n^\mu}{n \cdot q \pm i\epsilon}$$

$$D_F^{\mu\nu}(q) = \frac{-i g^{\mu\nu}}{q^2}$$



for $q^{(1)}(x)$



for $\delta Z_F^{(1)} \delta(1-x)$

E.g.1 PDF

- Definition

$$q(x)=\int\frac{d\xi^-}{2\pi}\,e^{-ixp^+\xi^-}\left\langle PS\left|\bar{\psi}(\tfrac{\xi^-}{2})\gamma^+\mathcal{L}[\tfrac{\xi^-}{2};-\tfrac{\xi^-}{2}]\psi(-\tfrac{\xi^-}{2})\right|PS\right\rangle$$

$$\tilde{q}(x)=\int\frac{dz}{2\pi}\,e^{ixp^z z}\left\langle PS\left|\bar{\psi}(\tfrac{z}{2})\gamma^z\mathcal{L}[\tfrac{z}{2};-\tfrac{z}{2}]\psi(-\tfrac{z}{2})\right|PS\right\rangle$$

pure spatial correlation

directly calculated on lattice, no prob. int..

- Moments

$$q^n=\int dx\,x^{n-1}q(x)=\frac{1}{(p^+)^n}\left\langle PS\left|\bar{\psi}(0)\left(i\overleftrightarrow{D}^+\right)^{n-1}\gamma^+\psi(0)\right|PS\right\rangle\sim(P^+)^n$$

$$\tilde{q}^n=\int dx\,x^{n-1}\tilde{q}(x)=\frac{1}{(p^z)^n}\left\langle PS\left|\bar{\psi}(0)\left(i\overleftrightarrow{D}^z\right)^{n-1}\gamma^z\psi(0)\right|PS\right\rangle\sim(P^z)^n$$

recover L.C. moments when boost to IMF

with higher twist correction

Matching Condition

- Lattice “cross section” factorization

$$\tilde{q}(x) = \int_{-1}^{1} \frac{dy}{|y|} Z\left(\frac{x}{y}\right) q(y)$$

- Perturbative expansion

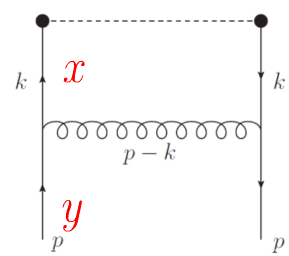
$$\tilde{q}(x) = \tilde{q}^{(1)}(x) + \delta\tilde{Z}_F^{(1)}\delta(1-x) \qquad q(x) = \tilde{q}^{(1)}(x) + \delta Z_F^{(1)}\delta(1-x)$$

$$\delta\tilde{Z}_F^{(1)}\delta(1-x) + \tilde{q}^{(1)}(x)$$

$$= \int_0^1 \frac{dy}{y}\delta\left(\frac{x}{y}-1\right)\left[\delta Z_F^{(1)}\delta(1-y) + q^{(1)}(y)\right] + \int_0^1 \frac{dy}{y} Z^{(1)}\left(\frac{x}{y}, \frac{P^z}{\mu}\right)\delta(1-y)$$

$$= \delta Z_F \delta(1-x) + q^{(1)}(x) + Z^{(1)}\left(x, \frac{P^z}{\mu}\right).$$

- Matching factor



$$\mathcal{O}(\alpha_s^0): \quad Z^{(0)}\left(\xi, \frac{p^z}{\mu}\right) = \delta(1-\xi), \quad \xi = \frac{x}{y}$$

$$\mathcal{O}(\alpha_s): \quad Z^{(1)}\left(\xi, \frac{p^z}{\mu}\right) = \tilde{q}^{(1)}(\xi, p^z) - q^{(1)}(\xi, \mu) + \left[\delta\tilde{Z}_F(p^z) - \delta Z_F(\mu)\right]\delta(1-\xi)$$

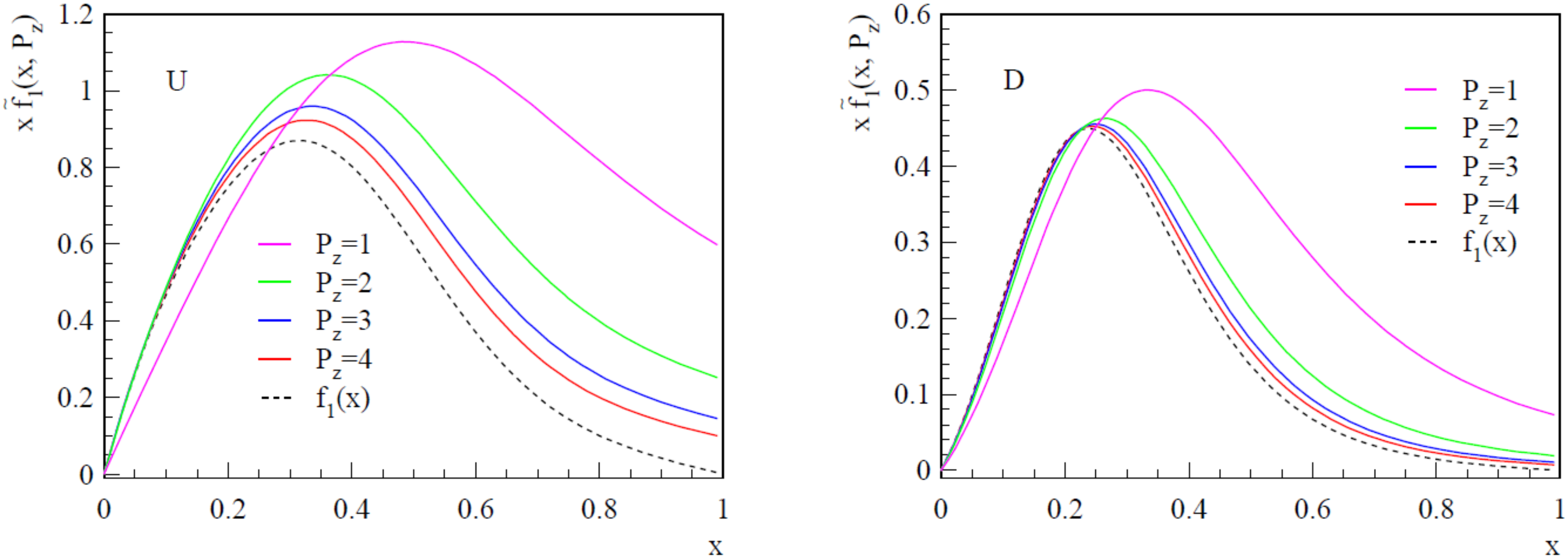
- Dim Reg. v.s. Cut-off Reg.

	DR	**Cut-off**
symmetry	**preserved**	broken
non-Abelian	**suitable**	not suitable
complicity	**low**	high
γ_5 ambiguity	NDR/HVDR	no
power divergence	no	**preserved**
other	**higher loop**	**mimic lattice**

Diquark Model Results of quasi PDF

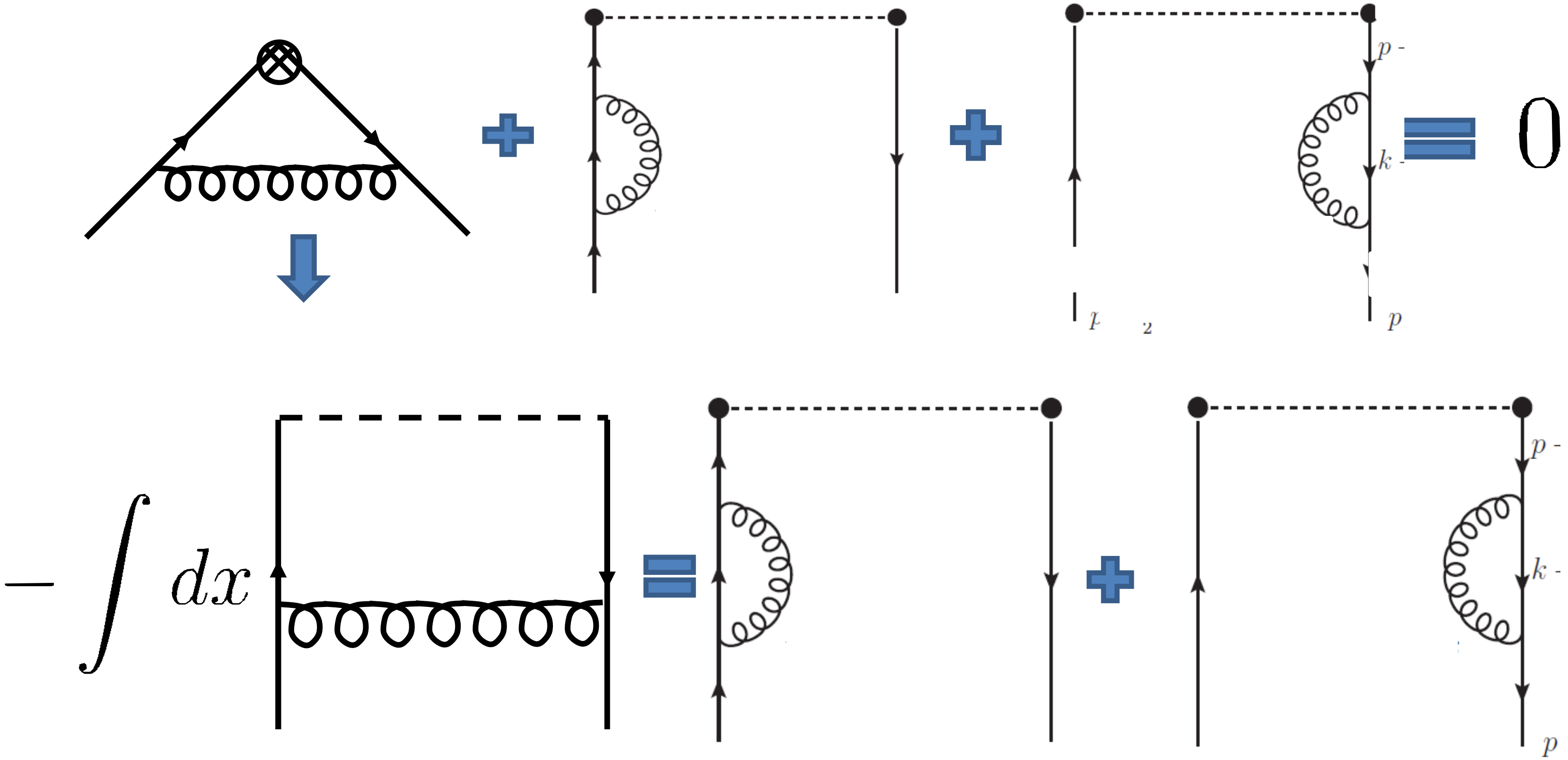
- L. Gamberg, Z. B. Kang, I. Vitev and H. Xing, PLB **743**, 112 (2015)

$$\mu^2 = 0.3\mathrm{GeV}^2$$



Gauge Invariance

- Start from vector current conservation



$$q(x) - \delta Z_F \delta\left(x-1\right) = q(x) - \delta\left(x-1\right)\int dy q(y)$$

k^-(LC)/ k^0(Qujasi)-integral

Performed by Cauchy residue theorem

quark, gluon propagator (linear in k^-,quadratic in k^0)

$$k^2-m^2+i\epsilon=2k^+k^--\boldsymbol{k}_\perp^2-m^2+i\epsilon \qquad (p-k)^2+i\epsilon=2\left(P^+-k^+\right)\left(P^--k^-\right)-\boldsymbol{k}_\perp^2+i\epsilon$$

	$P^- + \frac{-(\boldsymbol{P}_\perp-\boldsymbol{k}_\perp)^2+i\epsilon}{2(1-x)P^+}$	$\frac{\boldsymbol{k}_\perp^2+m^2-i\epsilon}{2xP^+}$	$\int dk^-[\cdots]$
$x<0$	$+$	$+$	0
$0<x<1$	$+$	$-$	$\neq 0$
$x>1$	$-$	$-$	0

$$k^2-m^2+i\epsilon=\left(k^0\right)^2-\boldsymbol{k}_\perp^2-(k^z)^2-m^2+i\epsilon \quad (p-k)^2+i\epsilon=\left(P^0-k^0\right)^2-(P^z-k^z)^2-\boldsymbol{k}_\perp^2+i\epsilon$$

always one k^0 pole on upper/lower plane

Calculation Example

- Feynman Part $D_n^{\mu\nu}(q) = \frac{-i}{q^2}\left(g^{\mu\nu} - \frac{q^\mu n^\nu + n^\mu q^\nu}{n\cdot q} + n^2 \frac{q^\mu q^\nu}{n\cdot q^2}\right)$

$$q^{(1)}(x) = \frac{1}{P^z}\int \frac{d^4k}{(2\pi)^4}\bar{u}(P)\left(-ig_s \mathrm{t}_a \gamma^\mu\right)\frac{i}{\not{k} - m + i\epsilon}\gamma^z \frac{i}{\not{k} - m + i\epsilon}\left(-ig_s \mathrm{t}_\mathrm{b}\gamma^\nu\right)$$

$$\times \frac{-ig_{\mu\nu}}{(P-k)^2 + i\epsilon}\delta\left(k^z - xP^z\right)u(P) + \cdots$$

$$= \int \frac{d^2k_\perp}{(2\pi)^4}\frac{g_s^2 C_F \pi P^z}{\sqrt{k_\perp^2 + (1-x)^2 P_z^2}\left[2P^0\sqrt{k_\perp^2 + (1-x)^2 P_z^2} - (1-2x)P_z^2 + m^2 - P_0^2\right]}$$

$$- \frac{g_s^2 C_F \pi P^z}{\sqrt{k_\perp^2 + x^2 P_z^2 + m^2}\left[2P^0\sqrt{k_\perp^2 + x^2 P_z^2 + m^2} + 2xP_z^2 + m^2 + P_0^2 - P_z^2\right]}$$

$$\sim \frac{P^z}{\sqrt{P_z^2 + m^2}}\ln\frac{\sqrt{P_z^2 + m^2}\sqrt{\mu^2 + (1-x)^2 P_z^2} - (1-x)P_z^2}{\sqrt{P_z^2 + m^2}\sqrt{(1-x)^2 P_z^2} - (1-x)P_z^2} + \cdots$$

- $P^z \to \infty$

$$q(x) \to \int_0^{\mu} \frac{d^2 k_\perp}{(2\pi)^4} \begin{cases} \frac{2g_s^2 C_F \pi}{k_\perp^2 + m^2(1-x)^2} & 0 < x < 1 \\ \mathcal{O}\left(\frac{1}{P^z}\right)^n & \text{Otherwise} \end{cases}$$

$$= \begin{cases} \frac{g_s^2 C_F}{8\pi^2} \boxed{\ln\left[\frac{\mu^2 + m^2(1-x)^2}{m^2(1-x)^2}\right]} & 0 < x < 1 \\ 0 & \text{Otherwise} \end{cases}$$

$$= q_{LC}(x)$$

Can be calculated directly using light-cone coordinates

Same collinear, different UV → perturbative matching

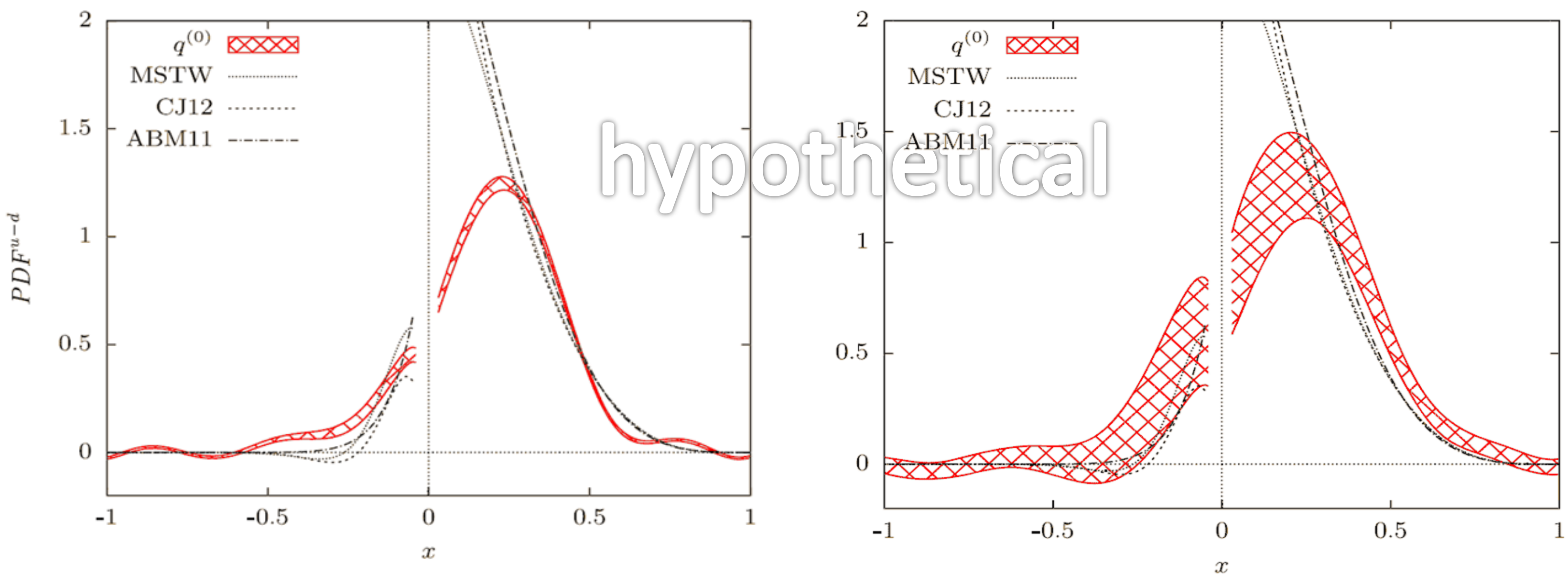
- $\mu \to \infty$

$$q(x) \to \begin{cases} \frac{g_s^2 C_F}{8\pi^2} \boxed{\ln\left[\frac{(P^z)^2}{m^2}\right]} + \text{non-ln}\left(\frac{P^z}{m}\right) \text{ terms} & 0 < x < 1 \\ \text{non-ln}\left(\frac{P^z}{m}\right) \text{ terms} & \text{Otherwise} \end{cases}$$

$$= q_{quasi}(x)$$

Lattice quasi PDF Results
+ mixed momentum setup

- C. Alexandrou, K. Cichy, V. Drach, E. Garcia-Ramos, K. Hadjiyiannakou, Karl Jansen, F. Steffens, and C. Wiese, PRD **92**, 014502 (2015)



$$q^{(0)} = \mathcal{O}\left(\frac{M_N}{P^z}\right) + Z^{-1} \otimes \int \frac{dz}{2\pi} e^{iP^z z} \langle P | \cdots z, 0 \cdots | P \rangle$$

$P^z = 1.96\text{GeV}$

$P^z = 0.98, 1.47\text{GeV}$

- Conventions

$$p^\mu = \left(p^0, \mathbf{0}^\perp, p^z\right), \ \Delta^\mu = \left(\Delta^0, \Delta^1, 0, \Delta^z\right), \quad x = \frac{k^z}{p^z}, \quad \xi = \frac{\Delta^z}{p^z}, \quad t = \Delta^2$$

kinematic constrain

$$\left(p \pm \frac{\Delta}{2}\right)^2 = m^2, \ \Delta^2 = t, \ \Delta^1 \geq 0$$

$$\Rightarrow \quad 0 \leq \xi \leq \frac{1}{2p^z}\sqrt{\frac{-t\left((p^z)^2 + m^2 - \frac{t}{4}\right)}{m^2 - \frac{t}{4}}} \quad \xrightarrow{p^z \to \infty} \quad 0 \leq \xi \leq \sqrt{\frac{t}{t - 4m^2}}$$

- Properties of GPD

  Forward limit : $H(x,0,0) = f(x)$

  Polynomiality: Lorentz symmetry

GPD matching @ one loop

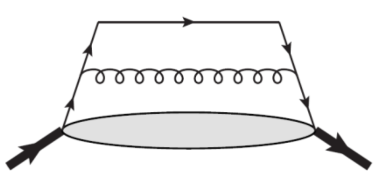
- Unpol.

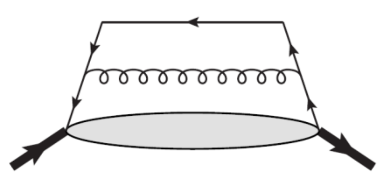
$$
Z_H^{(1)}(\eta,\zeta,\mu/p^z)/C_F = \begin{cases}
\frac{(\zeta^2+\eta)\ln\frac{\zeta+\eta}{\eta-\zeta}}{2\zeta(\zeta^2-1)} + \frac{(-2\zeta^2+\eta^2+1)\ln\frac{(\eta-1)^2}{\eta^2-\zeta^2}}{2(\zeta^2-1)(\eta-1)} + \frac{\mu}{p^z(1-\eta)^2} & \eta < -\zeta \\
\frac{\eta+\zeta}{2\zeta(1+\zeta)}\left(1+\frac{2\zeta}{1-\eta}\right)\ln\frac{p_z^2}{\mu^2} + \frac{1+\eta^2-2\zeta^2}{2(1-\eta)(1-\zeta^2)}\left(\ln[4(1-\eta)^2] - \ln\frac{\zeta-\eta}{\eta+\zeta}\right) & \\
+\frac{\eta+\zeta^2}{2\zeta(1-\zeta^2)}\ln\left[4(\zeta^2-\eta^2)\right] + \frac{\eta+\zeta}{(1+\zeta)(\eta-1)} + \frac{\mu}{p^z(1-\eta)^2} & -\zeta < \eta < \zeta \\
\frac{1+\eta^2-2\zeta^2}{(1-\eta)(1-\zeta^2)}\ln\frac{p_z^2}{\mu^2} + \frac{1+\eta^2-2\zeta^2}{2(1-\eta)(1-\zeta^2)}\left(\ln[16(\eta^2-\zeta^2)] + 2\ln(1-\eta)\right) & \\
-\frac{\eta+\zeta^2}{2\zeta(1-\zeta^2)}\ln\frac{\eta-\zeta}{\eta+\zeta} - \frac{2(\eta-\zeta^2)}{(1-\eta)(1-\zeta^2)} + \frac{\mu}{p^z(1-\eta)^2} & \zeta < \eta < 1 \\
-\frac{(\zeta^2+\eta)\ln\frac{\zeta+\eta}{\eta-\zeta}}{2\zeta(\zeta^2-1)} - \frac{(-2\zeta^2+\eta^2+1)\ln\frac{(\eta-1)^2}{\eta^2-\zeta^2}}{2(\zeta^2-1)(\eta-1)} + \frac{\mu}{p^z(1-\eta)^2} & \eta > 1.
\end{cases}
$$

$$
\eta = \frac{x}{y},\ \zeta = \frac{\xi}{y}
$$

$$
\begin{aligned}
\frac{1}{|y|}Z_H^{(1)}\left(\frac{x}{y},\frac{\xi}{y},\frac{\mu}{p^z}\right)/C_F = \frac{1}{y}\Big[& F_1\left(\frac{x}{y},\frac{\xi}{y},\frac{\mu}{p^z}\right)\theta(x<-\xi)\theta(x<y) \\
& + F_2\left(\frac{x}{y},\frac{\xi}{y},\frac{\mu}{p^z}\right)\theta(-\xi<x<\xi)\theta(x<y) \\
& + F_3\left(\frac{x}{y},\frac{\xi}{y},\frac{\mu}{p^z}\right)\theta(\xi<x<y) + F_4\left(\frac{x}{y},\frac{\xi}{y},\frac{\mu}{p^z}\right)\theta(x>\xi)\theta(x>y)\Big]
\end{aligned}
$$

- Antiquark's contribution also should be included





$$
x \to -x, y \to -y
$$

- Forward limit

  first take forward limit $\xi, t \to 0$

  then $m \to 0$ recover PDF from an finite $t,\ m$ result

$\xi, t \to 0$ and $m \to 0$ DO NOT commute

e.g.

$$\ln\left(m^2 - \frac{t}{4}\right) \to \ln\left(-\frac{t}{4}\right)$$

$$\ln\left(m^2 - \frac{t}{4}\right) \to \ln\left(m^2\right)$$

E.g.2: GPD

- Definition

$$P^z \int \frac{dz}{2\pi} e^{-ixp^z z} \langle p + \tfrac{\Delta}{2}, S \left| \bar{\psi}(-\tfrac{z}{2}) \gamma^z \mathcal{L}[-\tfrac{z}{2}; \tfrac{z}{2}] \psi(\tfrac{z}{2}) \right| p - \tfrac{\Delta}{2}, S \rangle$$

$$= \mathcal{H}(x, \xi, \Delta^2) \bar{U}(p + \tfrac{\Delta}{2}) \gamma^z U(p - \tfrac{\Delta}{2}) + \mathcal{E}(x, \xi, \Delta^2) \bar{U}(p + \tfrac{\Delta}{2}) \frac{i\sigma^{z\rho}\Delta_\rho}{2m} U(p - \tfrac{\Delta}{2})$$

- Convention

$$p^\mu = \left(p^0, \mathbf{0}^\perp, p^z\right), \ \Delta^\mu = \left(\Delta^0, \Delta^1, 0, \Delta^z\right), \ x = \frac{k^z}{p^z}, \ \ \xi = \frac{\Delta^z}{p^z}, \ \ t = \Delta^2$$

- Tree level: $H^{(0)}(x, \xi, t) = \delta(x-1), \ E^{(0)}(x, \xi, t) = 0$

- Properties of GPD

  Forward limit : $H(x, 0, 0) = f(x)$

  Polynomiality: Lorentz symmetry

- Polynomiality

taking moments of $\int dx\ x^n \int \frac{dz}{2\pi} e^{-ixp^z z} \langle p+\frac{\Delta}{2} | \bar{\psi}(-\frac{z}{2})\gamma^z \mathcal{L}[-\frac{z}{2};\frac{z}{2}]\psi(\frac{z}{2}) | p-\frac{\Delta}{2}\rangle$

$$n_{\mu_0} n_{\mu_1} \cdots n_{\mu_n} \left\langle P+\frac{\Delta}{2} \right| \bar{\psi}(0)\,\gamma^{\mu_0} i \overleftrightarrow{D}^{\mu_1} \cdots i \overleftrightarrow{D}^{\mu_n} \psi(0) \left| P-\frac{\Delta}{2} \right\rangle$$

$$\sim C(t)\,(n\cdot P)\cdots(n\cdot P)\,(n\cdot\Delta)\cdots(n\cdot\Delta) \sim \sum_i C_i(t)\xi^i$$

In 1-loop GPD, only H, E's $\ln\left(\frac{\mu^2}{-t}\right), \ln\left(\frac{P_z^2}{-t}\right)$ terms satisfy polynomiality (transverse cut-off breaks Lorentz Symmetry, but $\ln(\mu^2)$ terms are the same as DR)

- Meson DA from GPD

$$\langle q_1 | \bar{\psi}(\tfrac{z}{2})\gamma^z\gamma^5 \mathcal{L}[\tfrac{z}{2};\tfrac{-z}{2}]\psi(\tfrac{-z}{2}) | q_2 \rangle$$

crossing symmetry

$$\langle q_1\,\bar{q}_2 | \bar{\psi}(\tfrac{z}{2})\gamma^z\gamma^5 \mathcal{L}[\tfrac{z}{2};\tfrac{-z}{2}]\psi(\tfrac{-z}{2}) | 0 \rangle$$

+,++ Distribution

- +-distribution

$$\int_0^{\frac{1}{2}} dx \left[\frac{f(x)}{\frac{1}{2}-x}\right]_+ g(x) = \int_0^{\frac{1}{2}} dx \, \frac{f(x)\left[g(x) - g\left(\frac{1}{2}\right)\right]}{\frac{1}{2}-x}$$

in DR

$$\int_0^{\frac{1}{2}} dx g(x) \left|\frac{1}{2}-x\right|^{-1-2\epsilon} = \int_0^{\frac{1}{2}} dx g(x) \left\{\left(-\frac{1}{2\epsilon} - \ln 2\right)\delta\left(x-\frac{1}{2}\right) + \left[\left(\frac{1}{2}-x\right)^{-1}\right]_+\right\}$$

- ++-distribution

$$\int_0^{\frac{1}{2}} dx \left[\frac{f(x)}{\left(\frac{1}{2}-x\right)^2}\right]_{++} g(x) = \int_0^{\frac{1}{2}} dx \, \frac{f(x)\left[g(x) - g'(\frac{1}{2})(x-\frac{1}{2}) - g\left(\frac{1}{2}\right)\right]}{\left(\frac{1}{2}-x\right)^2}$$

in DR

$$\int_0^{\frac{1}{2}} dx g(x) \left|\frac{1}{2}-x\right|^{-2-2\epsilon} = \int_0^{\frac{1}{2}} dx g(x) \left\{\left(-\frac{1}{2\epsilon} - \ln 2\right)\delta'\left(x-\frac{1}{2}\right) - 2\delta\left(x-\frac{1}{2}\right) + \left[\left(\frac{1}{2}-x\right)^{-2}\right]_{++}\right\}$$

- GPD Polynomiality Results

$$H^{n+1}(\xi,t) = \sum_{i=0}^{[n/2]} (2\xi)^i A^q_{n+1,2i}(t) + \mathrm{mod}\ (n,2)(2\xi)^{n+1} C^q_{n+1}(t)$$

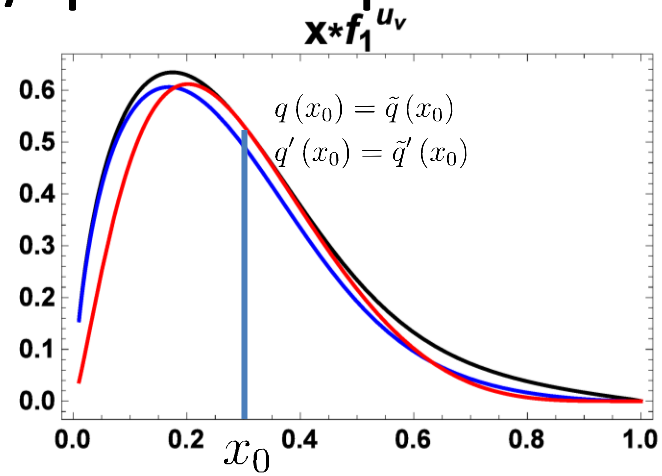
$$E^{n+1}(\xi,t) = \sum_{i=0}^{[n/2]} (2\xi)^i B^q_{n+1,2i}(t) - \mathrm{mod}\ (n,2)(2\xi)^{n+1} C^q_{n+1}(t)$$

$$\begin{aligned} H^{n+1}(\xi,t) =& \frac{C_F\alpha_s}{2\pi}\ln\left(\frac{\mu^2}{-t}\right)\begin{cases} \frac{1}{2k^2+3k+1}\sum_{i=0}^{k}\xi^{2i} & n=2k \\ \frac{1}{2k^2+5k+3}\sum_{i=0}^{k}\xi^{2i} & n=2k+1 \end{cases} \\ &+\frac{C_F\alpha_s}{2\pi}\ln\left(\frac{\mu^2}{-t}\right)\sum_{i=0}^{n}\begin{pmatrix} n \\ i \end{pmatrix}(-\xi)^{n-i}(1+\xi)^{i+1}\int_0^1 d\chi\,\frac{\chi^{i+1}}{(1-\chi)_+} \\ &+\frac{C_F\alpha_s}{2\pi}\ln\left(\frac{\mu^2}{-t}\right)\sum_{i=0}^{n}\begin{pmatrix} n \\ i \end{pmatrix}\xi^{n-i}(1-\xi)^{i+1}\int_0^1 d\chi\,\frac{\chi^{i+1}}{(1-\chi)_+} \\ &-\frac{C_F\alpha_s}{2\pi}\ln\left(\frac{\mu^2}{m^2}\right)\int_0^1 d\chi\left[(1-\chi)+\frac{2\chi}{(1-\chi)_+}\right] \end{aligned}$$

$$\begin{aligned} E^{n+1}(\xi,t) =& \frac{C_F\alpha_s}{2\pi}\frac{m^2}{-t}\ln\left(\frac{-t}{m^2}\right)\begin{cases} \frac{2(4k+3)}{(2k+1)(k+1)}\sum_{i=1}^{k}\xi^{2i}+\frac{2}{(k+1)} & n=2k \\ \frac{2(4k+5)\xi^2}{(2k+3)(k+1)}\sum_{i=0}^{k}\xi^{2i}+\frac{4}{2k+3} & n=2k+1 \end{cases} \\ &-\frac{C_F\alpha_s}{2\pi}\ln\left(\frac{\mu^2}{m^2}\right)\int_0^1 d\chi\left[(1-\chi)+\frac{2\chi}{(1-\chi)_+}\right] \end{aligned}$$

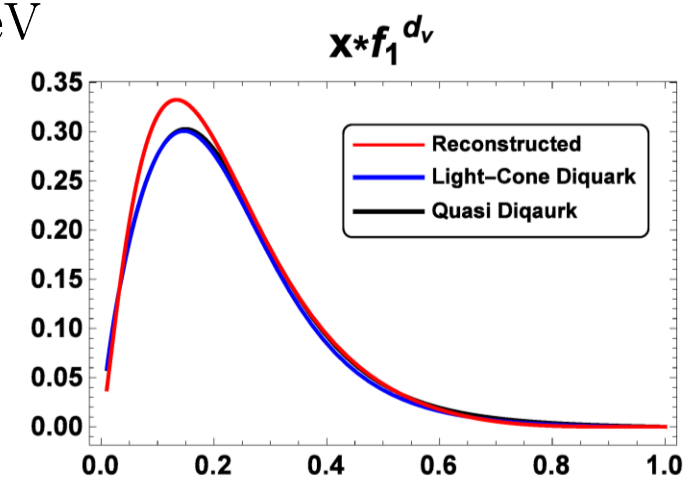
High-x behavior of PDF

- LC/quasi di-quark model test



$\mu = 2\text{GeV}$

$P^z = 5\text{GeV}$



$$q(x;\alpha,\beta,p_i) = x^{\alpha}(1-x)^{\beta} \times \left(1 + p_1 x^{1/2} + p_2 x + p_3 x^{3/2}\right)$$

reconstructed pdf

$$\int_{x_0}^{1} dx\, x^{n-1} q(x;\alpha,\beta,p_i) = q^n(\mu) - \int_0^{x_0} dx\, x^{n-1} \tilde{q}(x,\mu)$$

moments from lattice, Test: from LC model

quasi pdf from lattice, Test: from quasi model

Twist-3 GPDs (longitudinally polarized nucleon)

- D-type

$$\int \frac{d\lambda}{2\pi} \int \frac{d\mu}{2\pi} e^{i\lambda(x-y)} e^{i\mu y} \left\langle P', S \left| \bar{\psi}(0)\gamma^+ iD^\perp(\mu n)\psi(\lambda n) \right| P, S \right\rangle$$
$$= \frac{i\epsilon^{\perp\alpha}}{2} \Delta_\alpha H_D^{q(3)}(x, y, \eta, t) \bar{U}(P')\gamma^+\gamma_5 U(P) + \cdots$$

- F-type

$$\int \frac{d\lambda}{2\pi} \int \frac{d\mu}{2\pi P^+} e^{i\lambda(x-y)} e^{i\mu y} \left\langle P', S \left| \bar{\psi}(0)\gamma^+ gF^{+\perp}(\mu n)\psi(\lambda n) \right| P, S \right\rangle$$
$$= \frac{\epsilon^{\perp\alpha}}{2} \Delta_\alpha H_F^{q(3)}(x, y, \eta, t) \bar{U}(P')\gamma^+\gamma_5 U(P) + \cdots$$

- Canonical

$$\int \frac{d\lambda}{2\pi} \int \frac{d\mu}{2\pi} e^{i\lambda(x-y)} e^{i\mu y} \left\langle P', S \left| \bar{\psi}(0)\gamma^+ i\tilde{\partial}^\perp(\mu n)\psi(\lambda n) \right| P, S \right\rangle$$
$$= \frac{i\epsilon^{\perp\alpha}}{2} \Delta_\alpha \tilde{H}_q^{(3)}(x, y, \eta, t) \bar{U}(P')\gamma^+\gamma_5 U(P) + \cdots$$

- Relation between OAM distributions and twist-3 GPD (longitudinally polarized nucleon)

$$L_q^n = \int dx \int dy \, \frac{1}{n} \sum_{k=0}^{n-1} x^{n-1-k} (x-y)^k H_D^{q(3)}(x,y,0,0)$$

$$l_q(x) = \tilde{H}_q^{(3)}(x,0,0)$$

$$l_{q,\mathrm{pot}}^n = -\int dx \int dy \, \frac{1}{n} \sum_{k=0}^{n-1} x^{n-1-k} (x-y)^k \, \mathrm{P.V.} \frac{1}{y} H_F^{q(3)}(x,y,0,0)$$

$$L_{q/g}(x) = l_{q/g}(x) + l_{q/g,\mathrm{pot}}(x)$$

$$H_D^{q/g(3)}(x,y,0,0) = -\mathrm{P.V.} \tfrac{1}{y} H_F^{q,g(3)} + \delta(y) \tilde{H}_{q,g}^{(3)}(x,0,0)$$

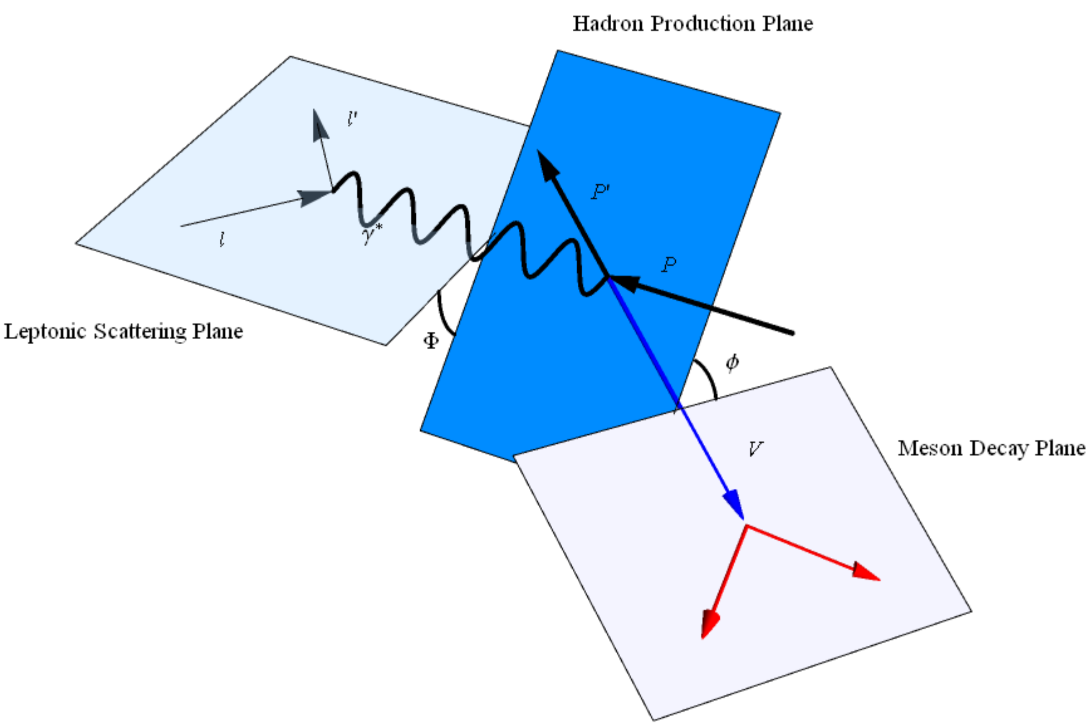
- Spin Density Matrix Elements (SDMEs)

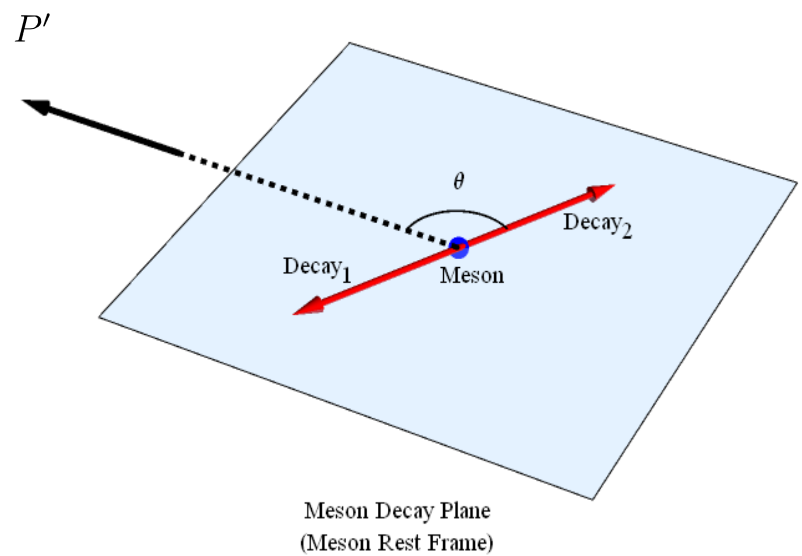
  The angle (Φ, θ, ϕ) distribution of meson decay can be expressed in SDMEs:

Wigner rotation

$$\frac{\mathrm{d}N}{\mathrm{d}\cos\theta\,\mathrm{d}\phi} \equiv W(\cos\theta, \phi) \propto \sum_{\lambda_V,\lambda'_V} D^{1*}_{\lambda_V,0}(\phi,\theta,-\phi)\,\varrho(V)_{\lambda_V,\lambda'_V}\,D^{1}_{\lambda'_V,0}(\phi,\theta,-\phi)$$

Spin density matrix of vector meson





- The meson SDME and photon SDME are related through

$$\varrho(V)_{\lambda_V\lambda'_V} = \sum_{\lambda_{N'}\lambda_N,\lambda'_\gamma\lambda_\gamma} T_{\lambda_V\lambda_{N'};\lambda_\gamma\lambda_N}\,\varrho(\gamma^*)_{\lambda_\gamma\lambda'_\gamma}\,T^*_{\lambda'_V\lambda_{N'};\lambda'_\gamma\lambda_N}$$

$$T_{\lambda_V\lambda_{N'};\lambda_\gamma\lambda_N} = (-1)^{\lambda_{\gamma^*}}\langle V\lambda_V;P'\lambda'_N|J^\mu|P\lambda\rangle\epsilon_\mu^{\lambda_{\gamma^*}}$$

GPD encoded

- Experimentally, 15 independent observables

$$r^{04}_{\lambda_V\lambda'_V} = \frac{(\varrho^0_{\lambda_V\lambda'_V}+\varepsilon r\varrho^4_{\lambda_V\lambda'_V})}{(1+\varepsilon r)}$$

0=unpol.;

$$r^{\alpha}_{\lambda_V\lambda'_V} = \begin{cases} \frac{\varrho^\alpha_{\lambda_V\lambda'_V}}{(1+\varepsilon\tilde{R})} & \alpha = 1,2,3 \\ \frac{\sqrt{r}\varrho^\alpha_{\lambda_V\lambda'_V}}{(1+\varepsilon r)} & \alpha = 5,6,7,8 \end{cases}$$

1,2=trans. pol. ; 3=circular pol. 4=log. pol.;

5—8=interference between 0,1,2,3 and 4