Meson electro/photo-production from QCD

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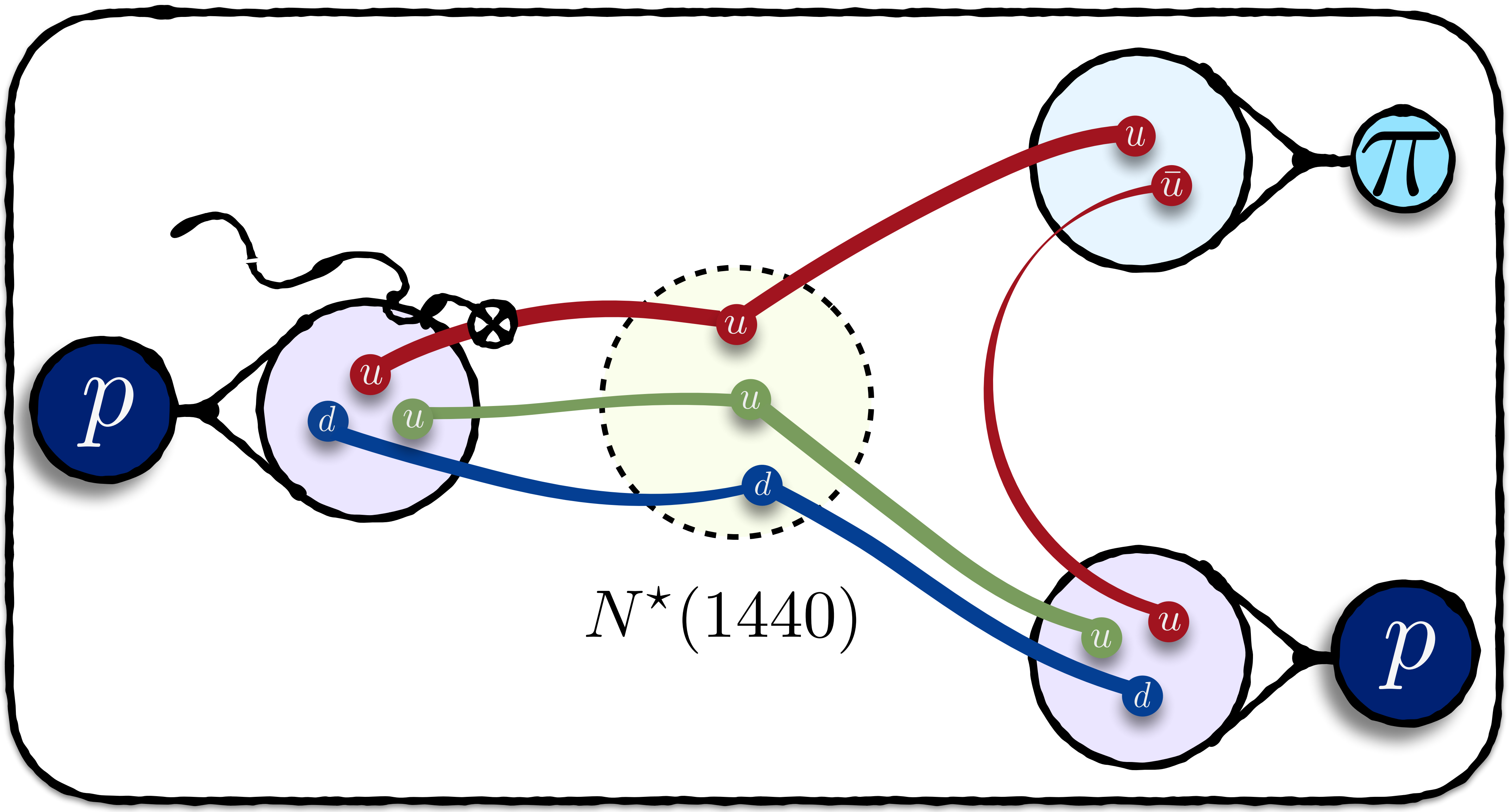
Jefferson Lab

February 2016

Questions to answer:

- What is possible from lattice QCD regarding electro/photo-production? ☐
- How can lattice and experiment compare to each other? ☐
- What is the possibility/timescale for extending this to baryons? ☐
- What does lattice *"need"* from experiment? ☐

Working definition



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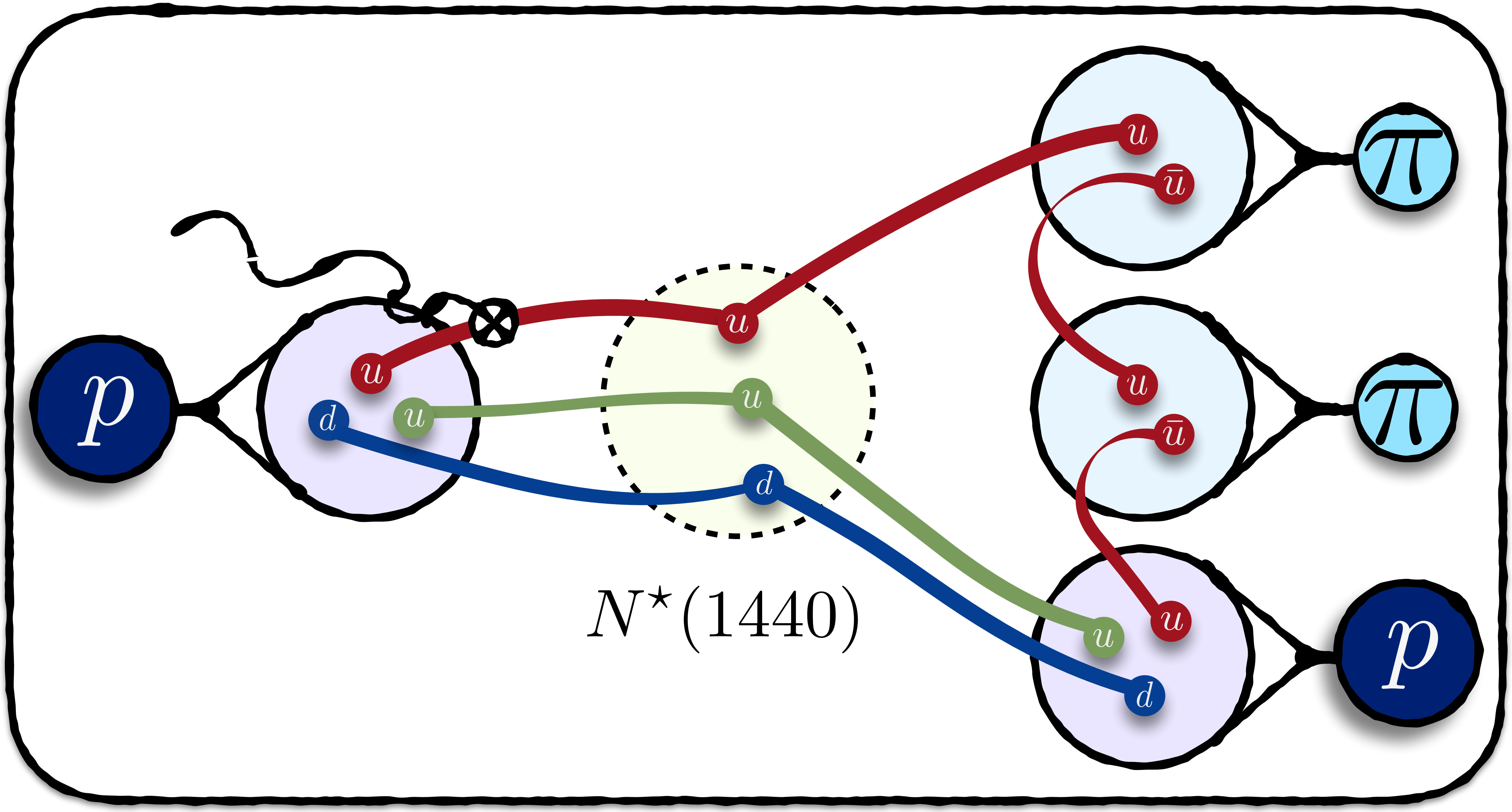
Working definition



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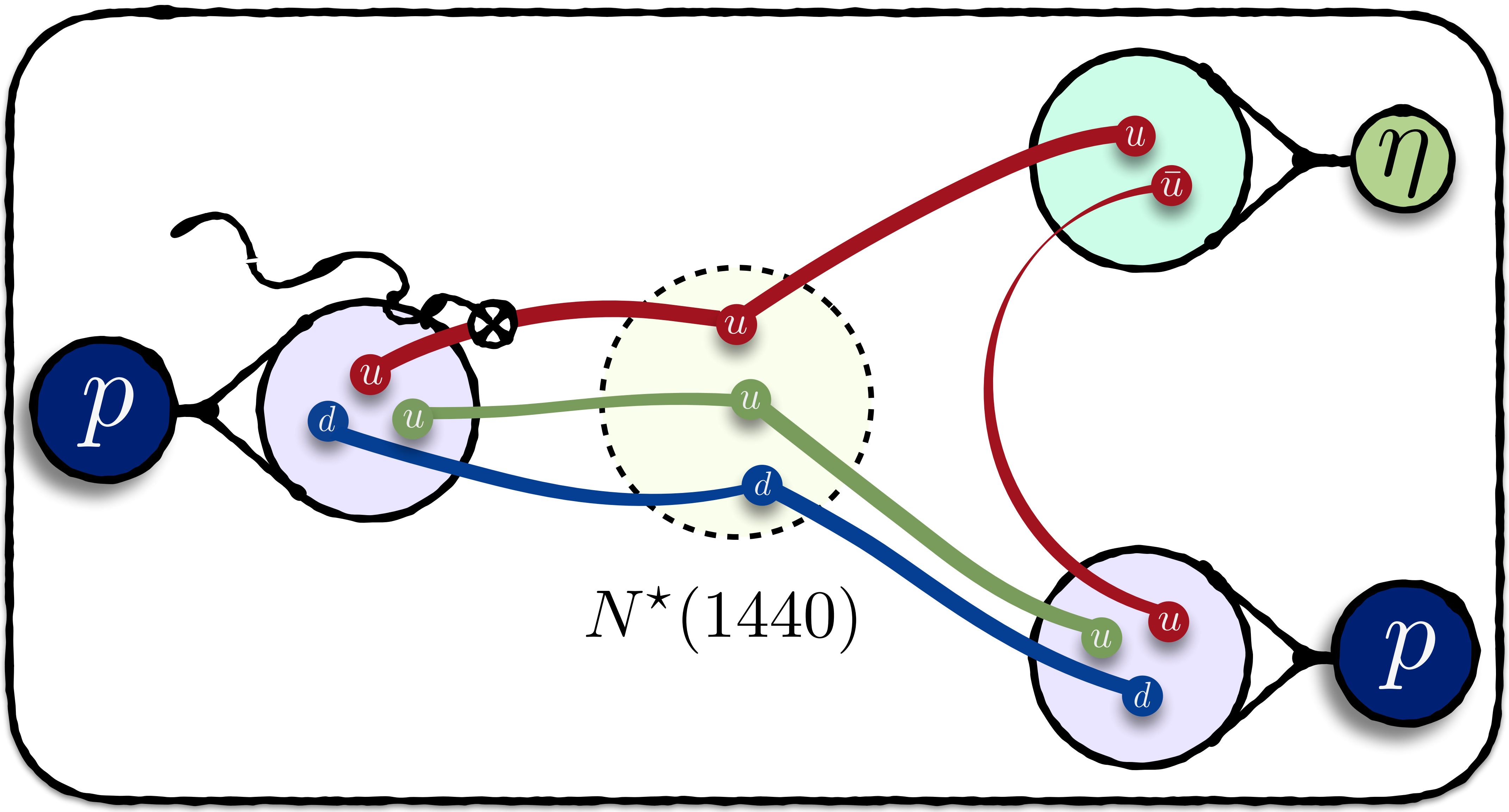
Working definition



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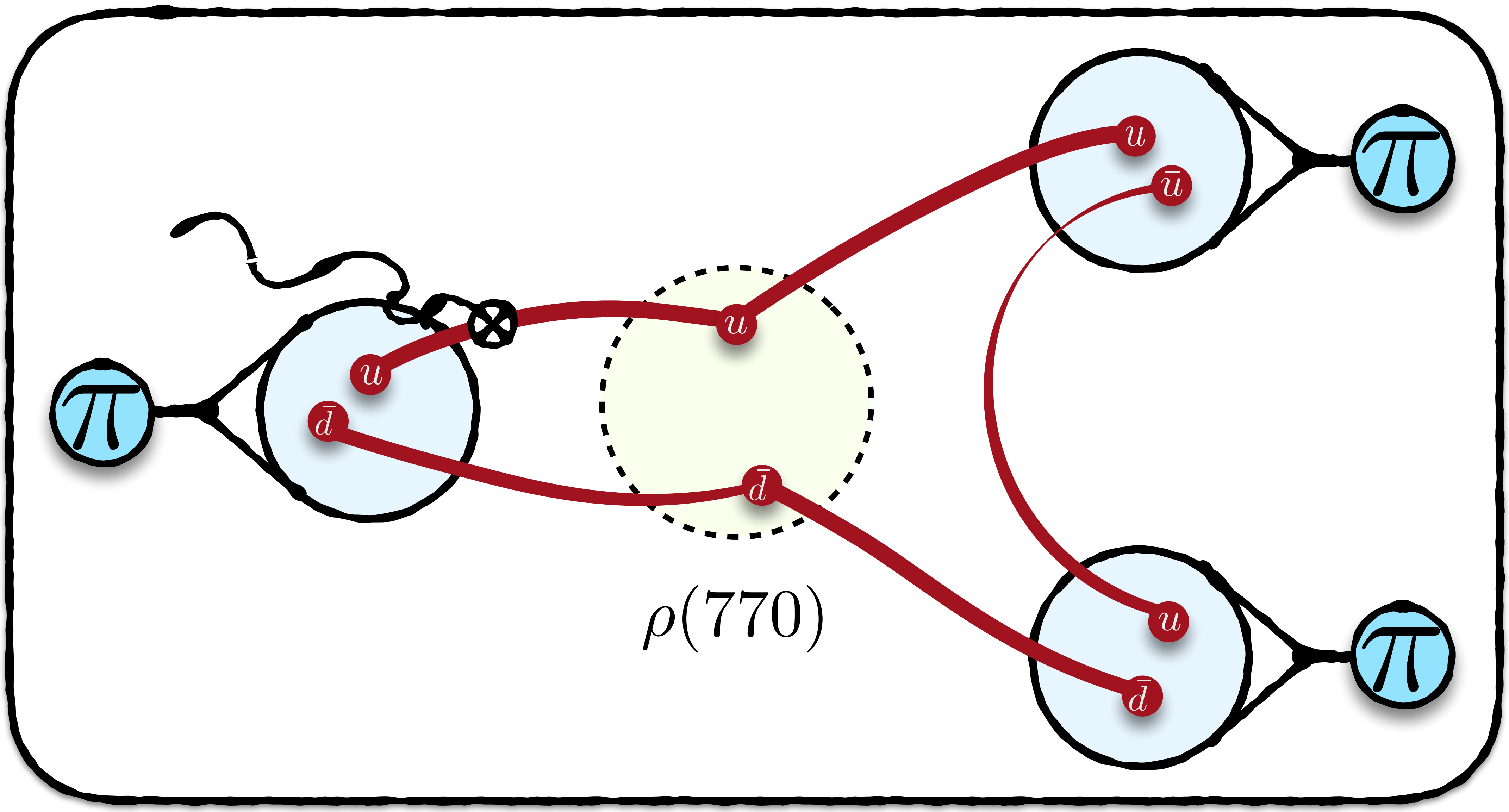
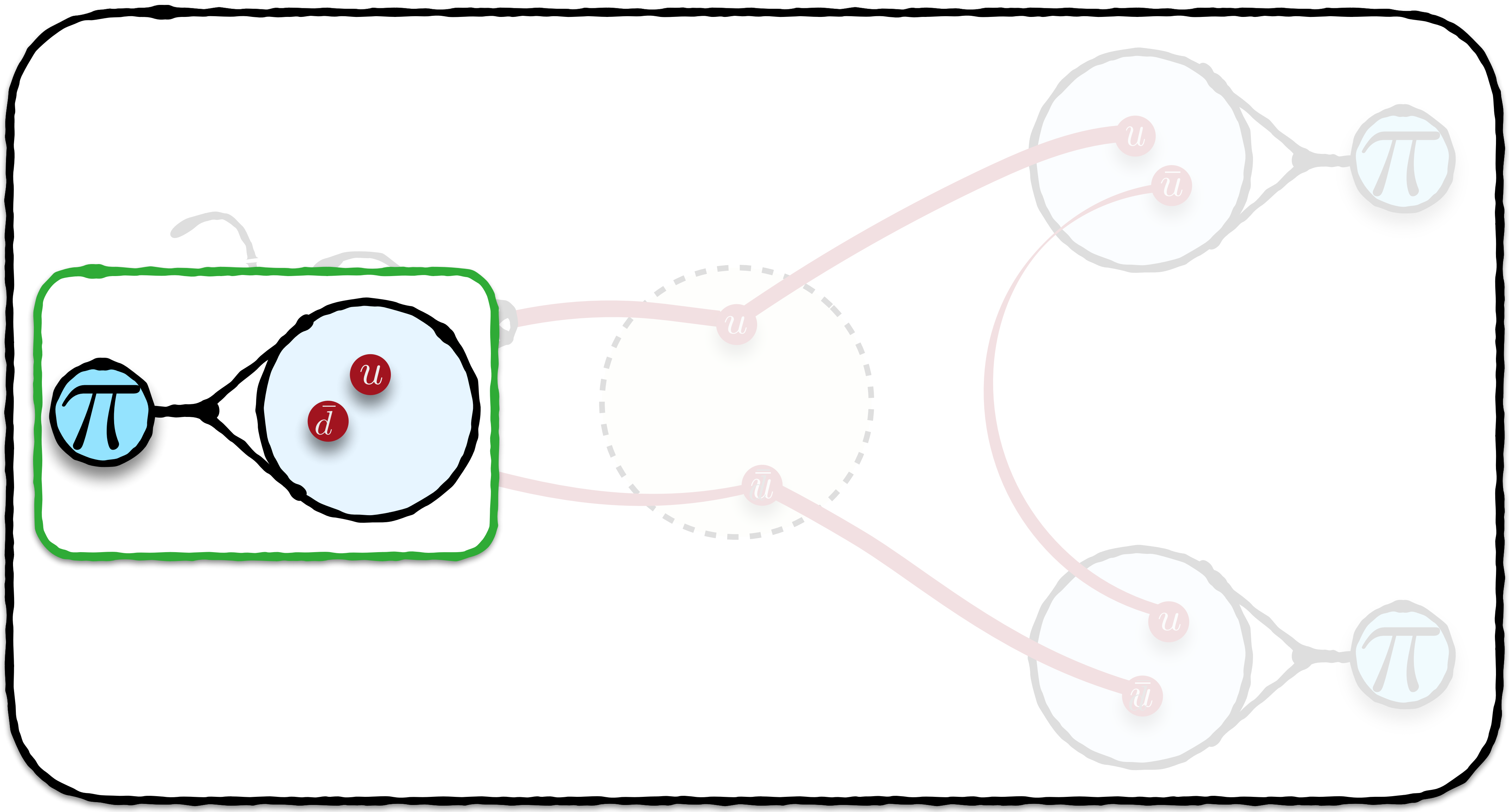
Working definition



Importance

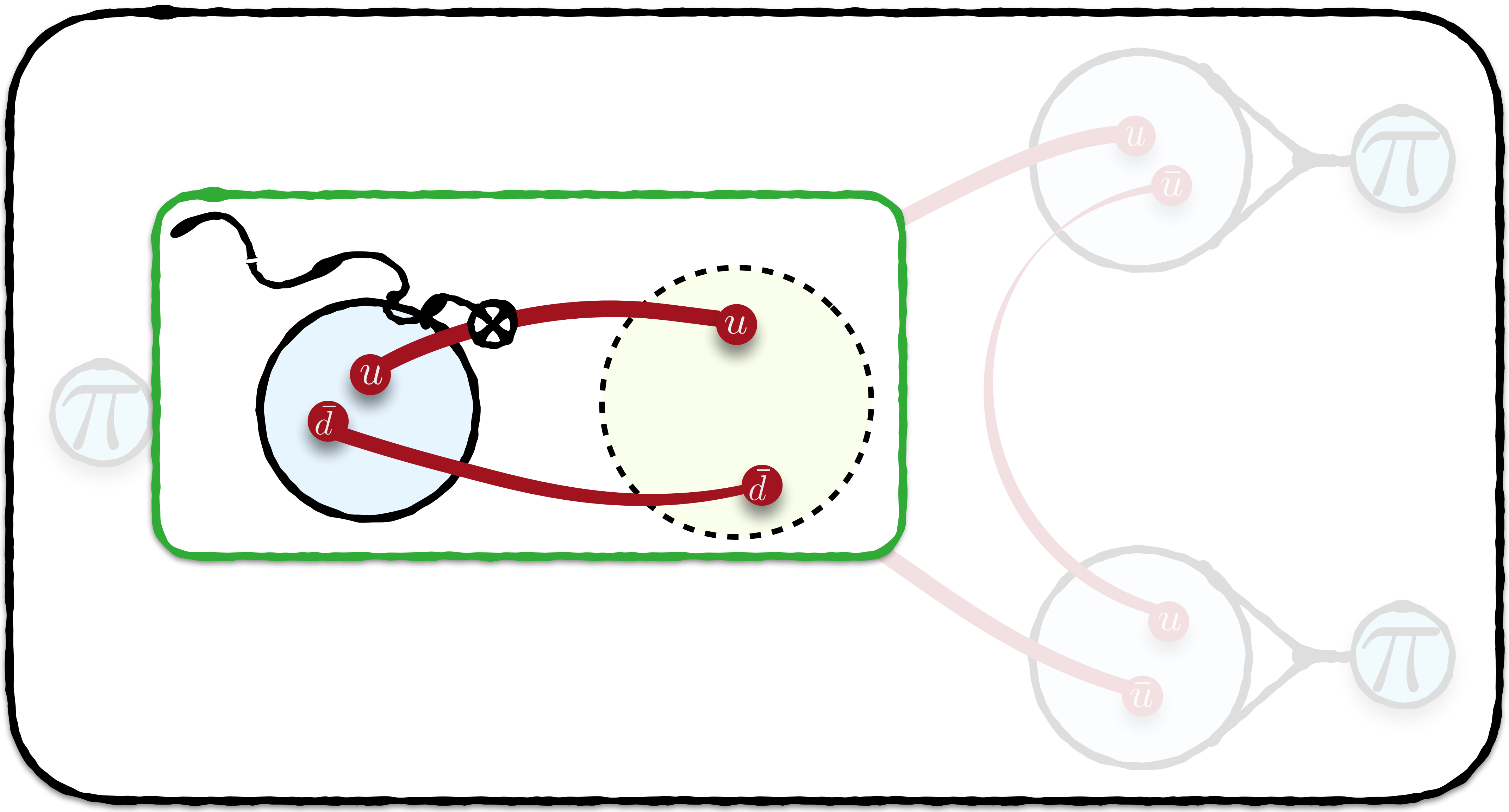
- Access the excited spectrum of QCD
- Test our understanding of QCD
- Probe the inner structure and shape of hadrons
- Test the limits of the standard model
- …

Why lattice?



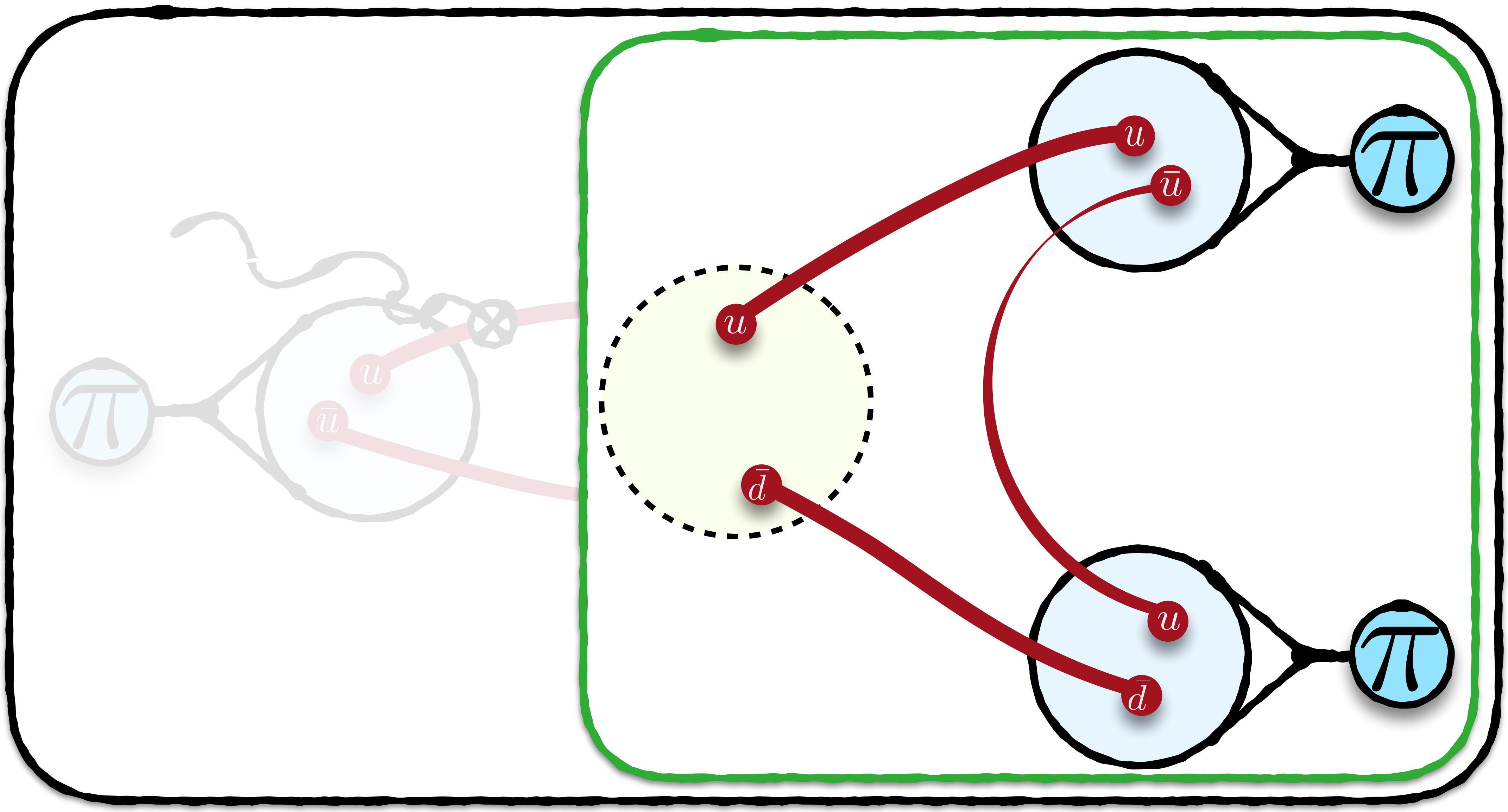
- QCD-stable states are generated exactly

Why lattice?



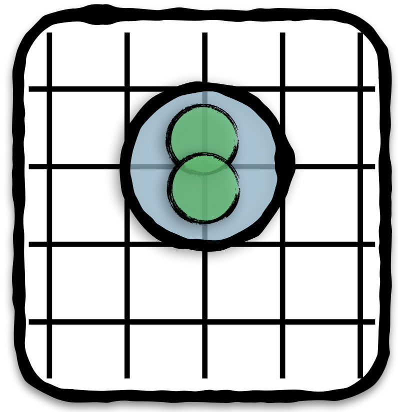
- QCD-stable states are generated exactly
- QED / weak sector can be treated perturbatively or non-perturbatively

Why lattice?

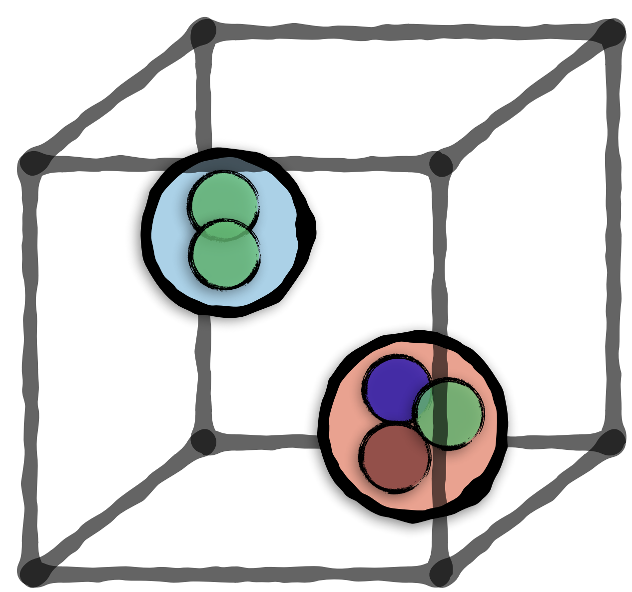


- QCD-stable states are generated exactly
- QED / weak sector can be treated perturbatively or non-perturbatively
- Resonance are generated and decay

Lattice QCD

- Lattice spacing:  $\} \, a \sim 0.1 \text{ fm}$

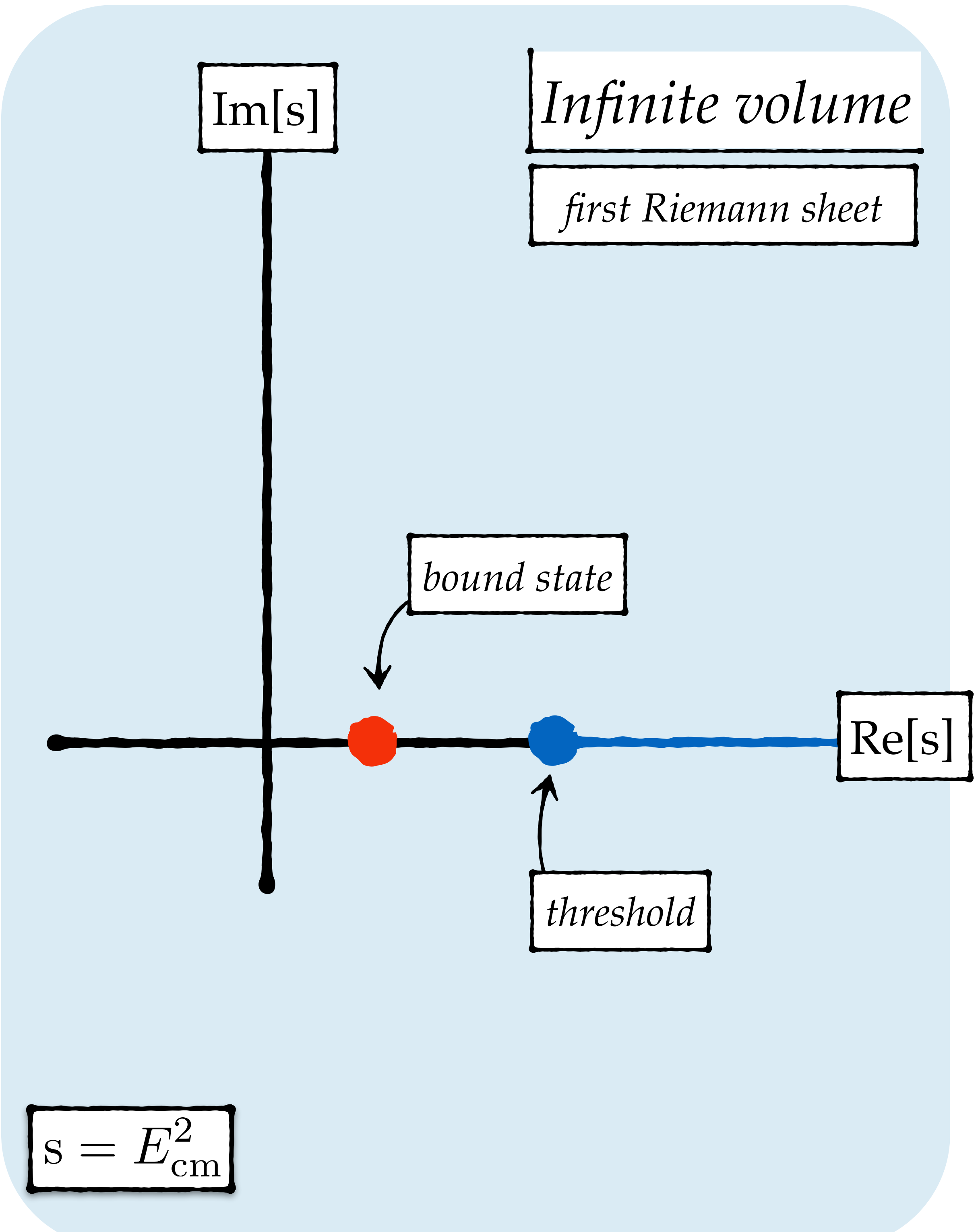
- Wick rotation [Euclidean spacetime]: $t_M \to i t_E$

- Finite volume: 

- Quark masses: $m_q \to m_q^{\text{phys.}}$

Have we '*mangled*' QCD too much?

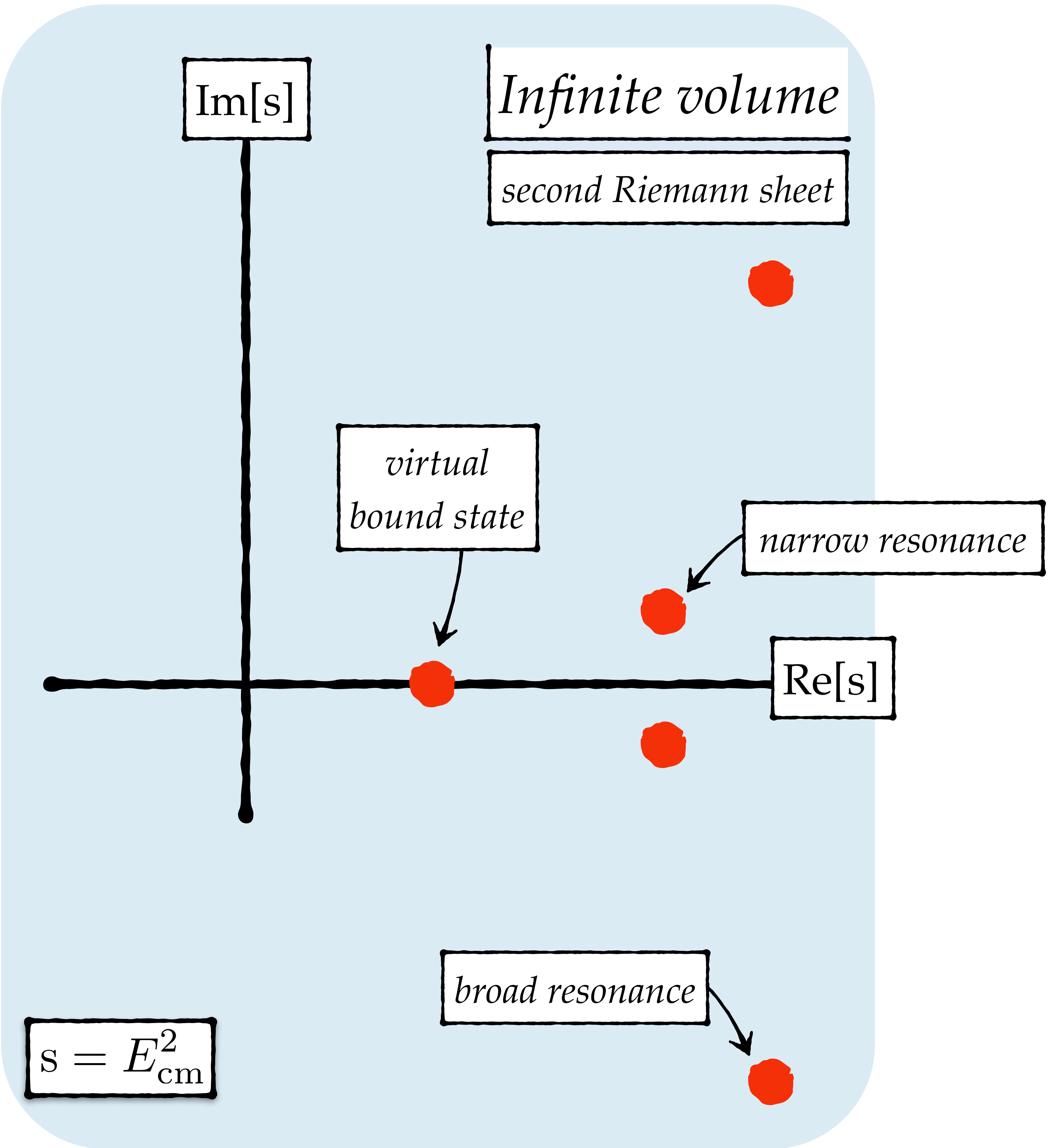
Finite vs. infinite volume spectrum



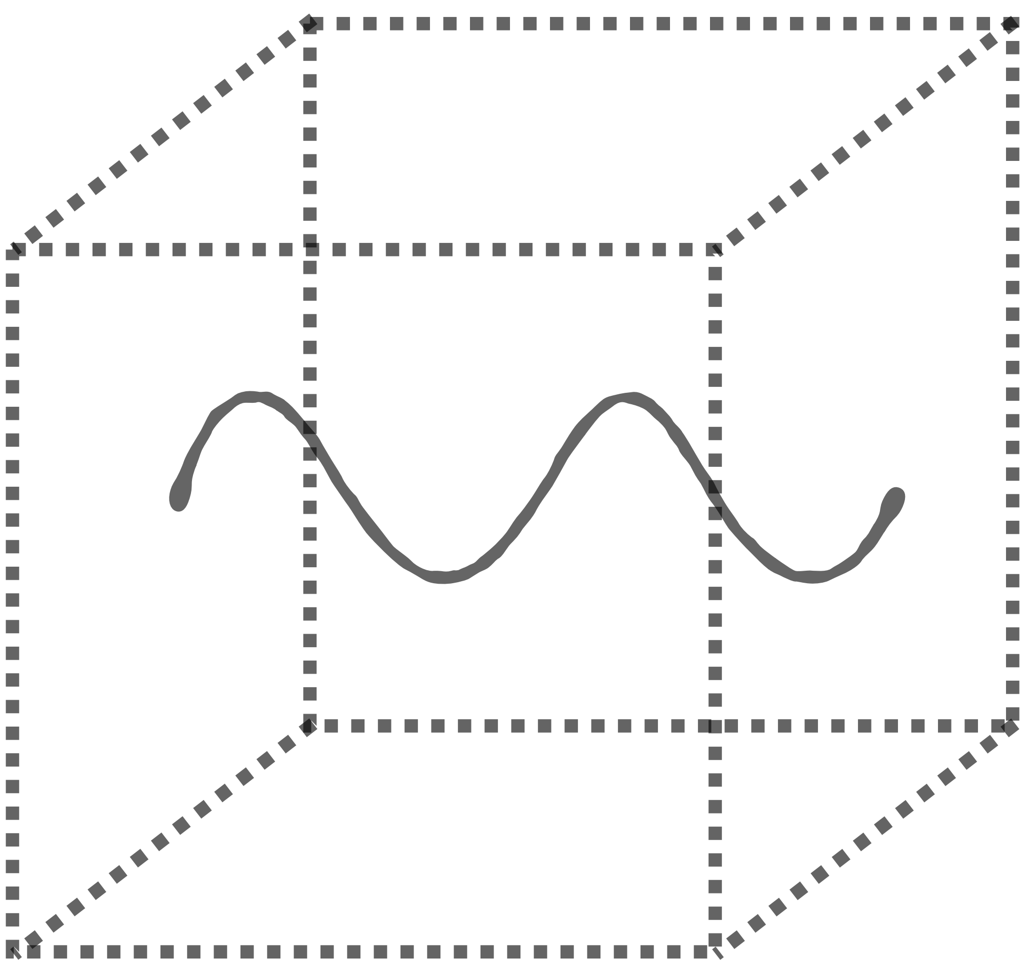
Consider any QCD channel with definite quantum numbers:

- angular momentum - J
- parity - P
- isospin - I
- …

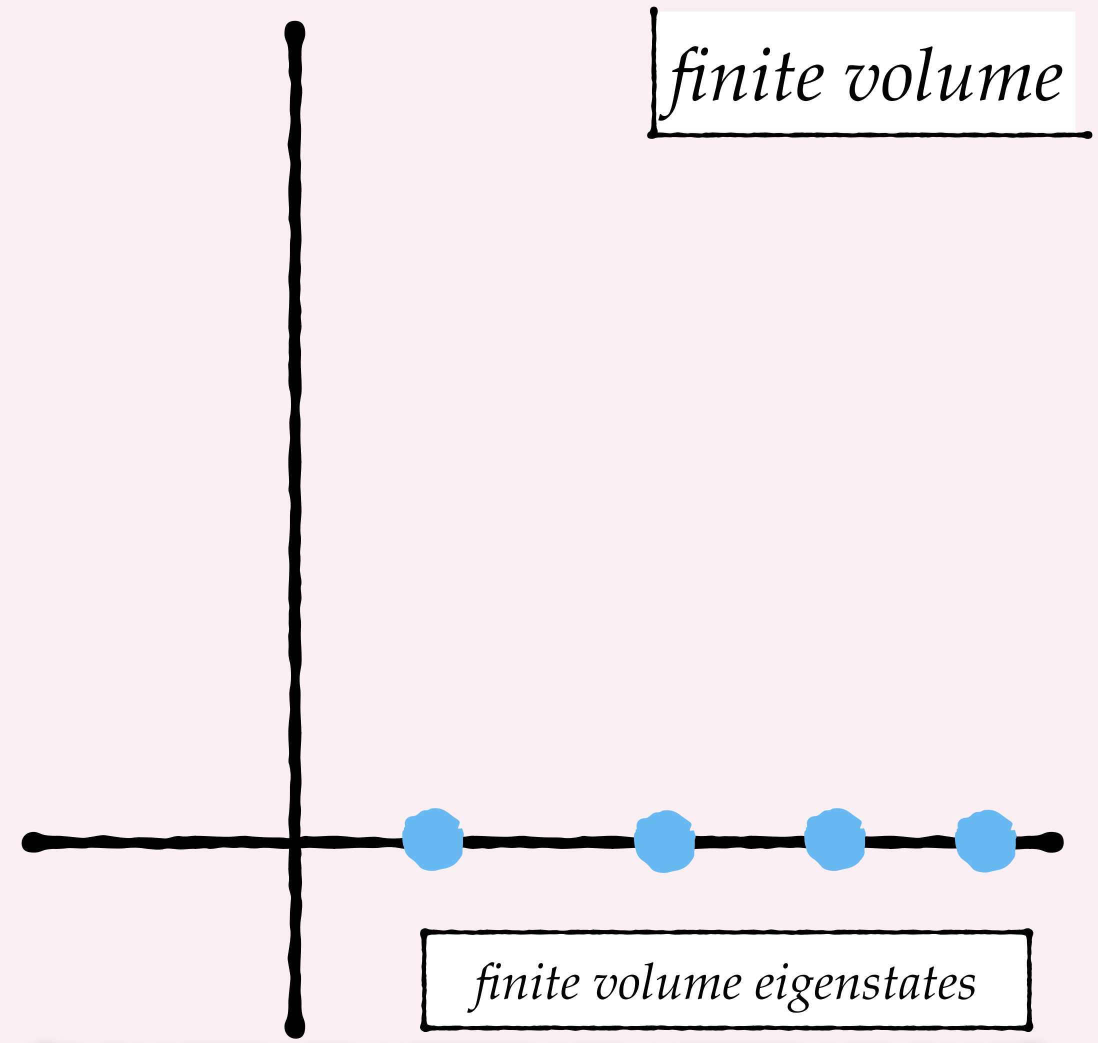
Finite vs. infinite volume spectrum



Finite vs. infinite volume spectrum



"only a finite number of modes can exist in a finite volume"



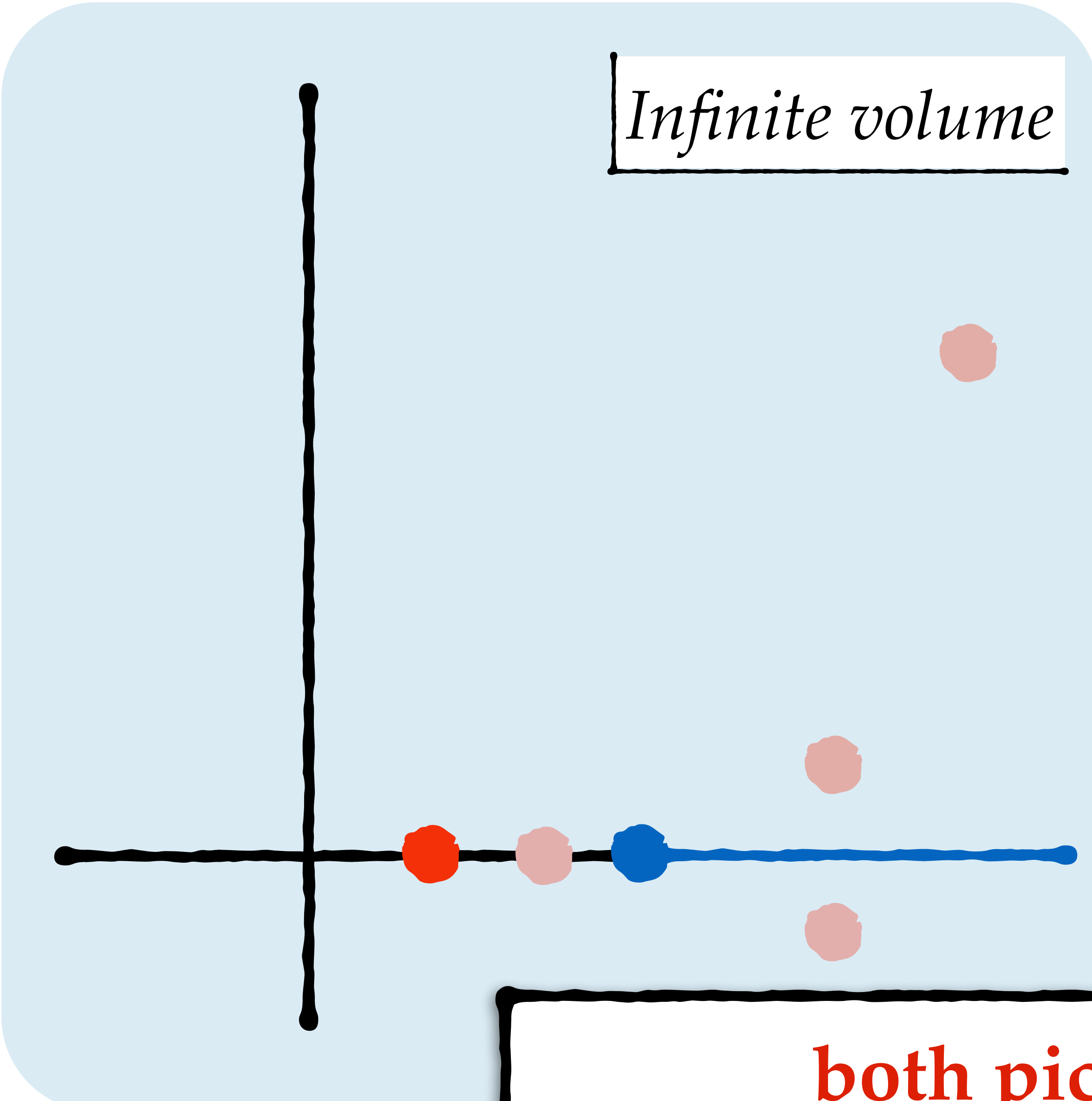
no continuum of states

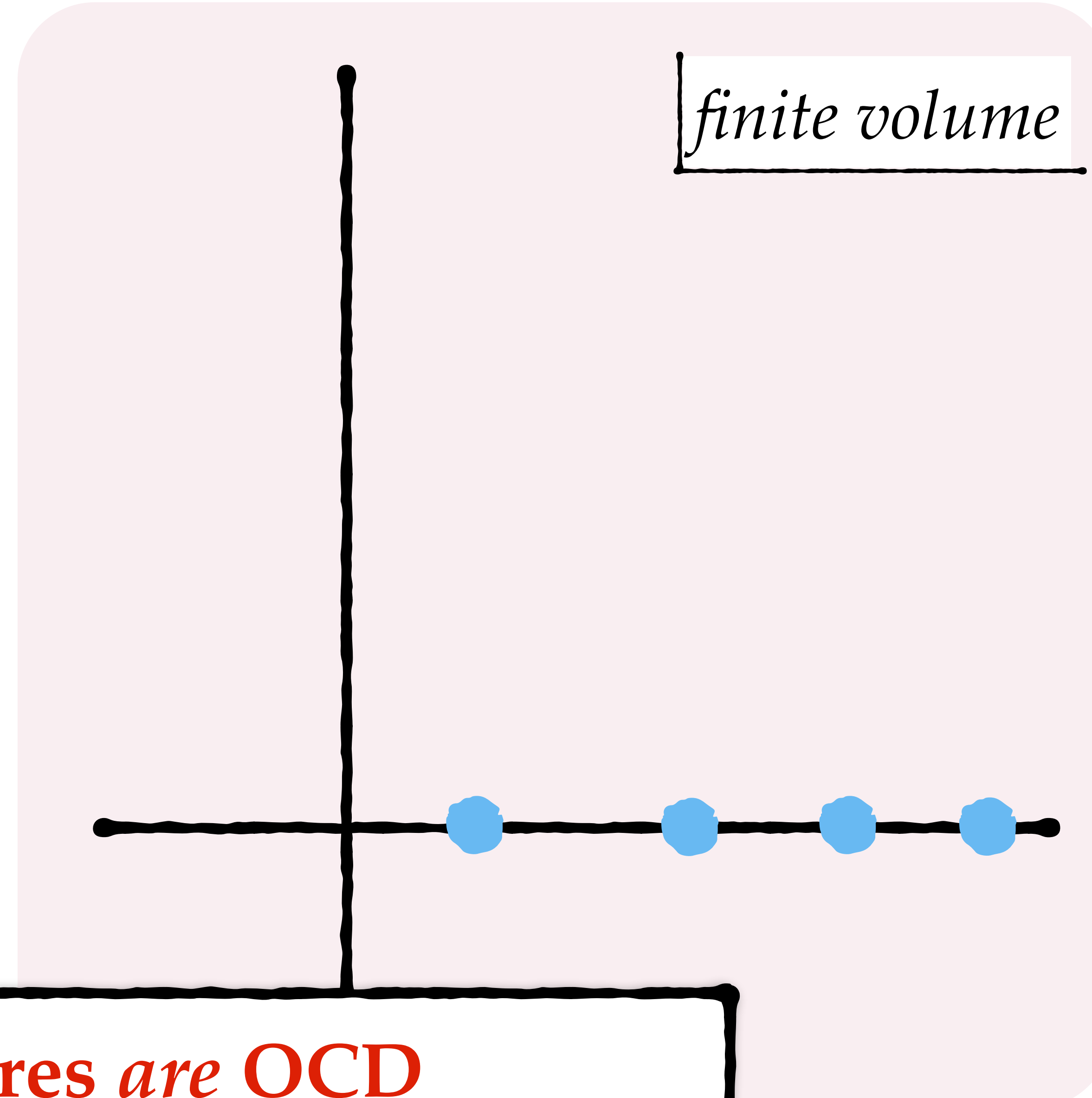
→ no cuts

→ no sheet structure

→ no resonances

Finite vs. infinite volume spectrum





both pictures *are* QCD

the connection is perhaps not obvious since we have historically been "confined" to thinking about infinite volume physics

Questions to answer:

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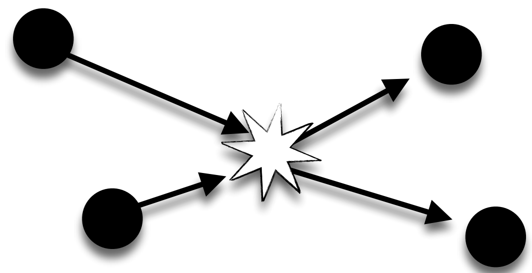
Experiment

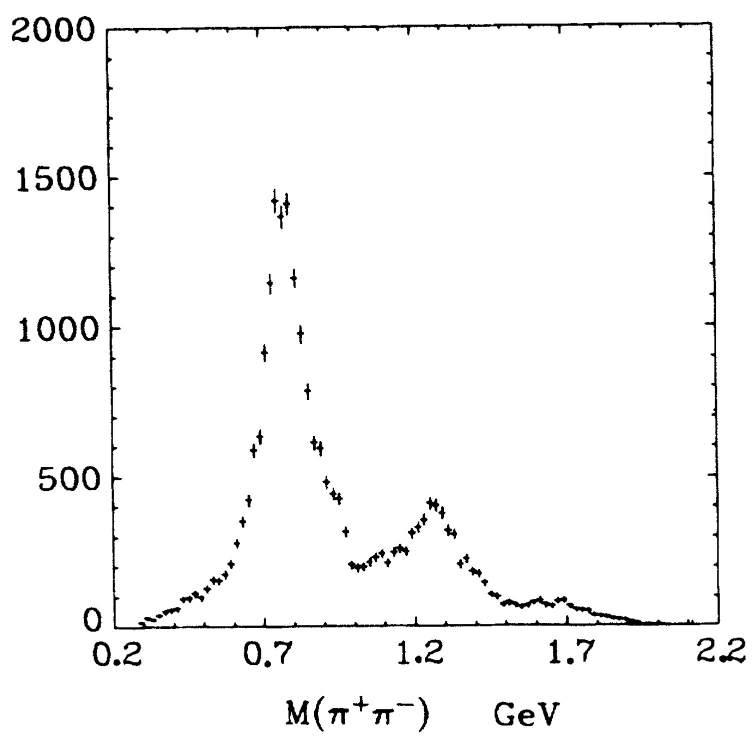


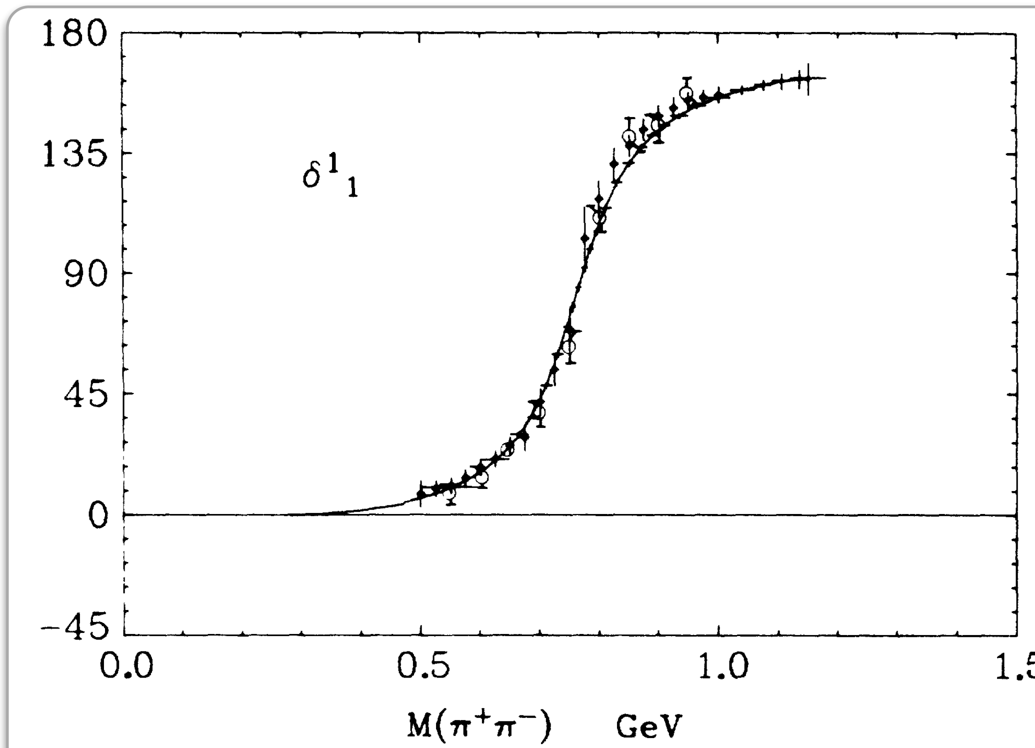
scattering data

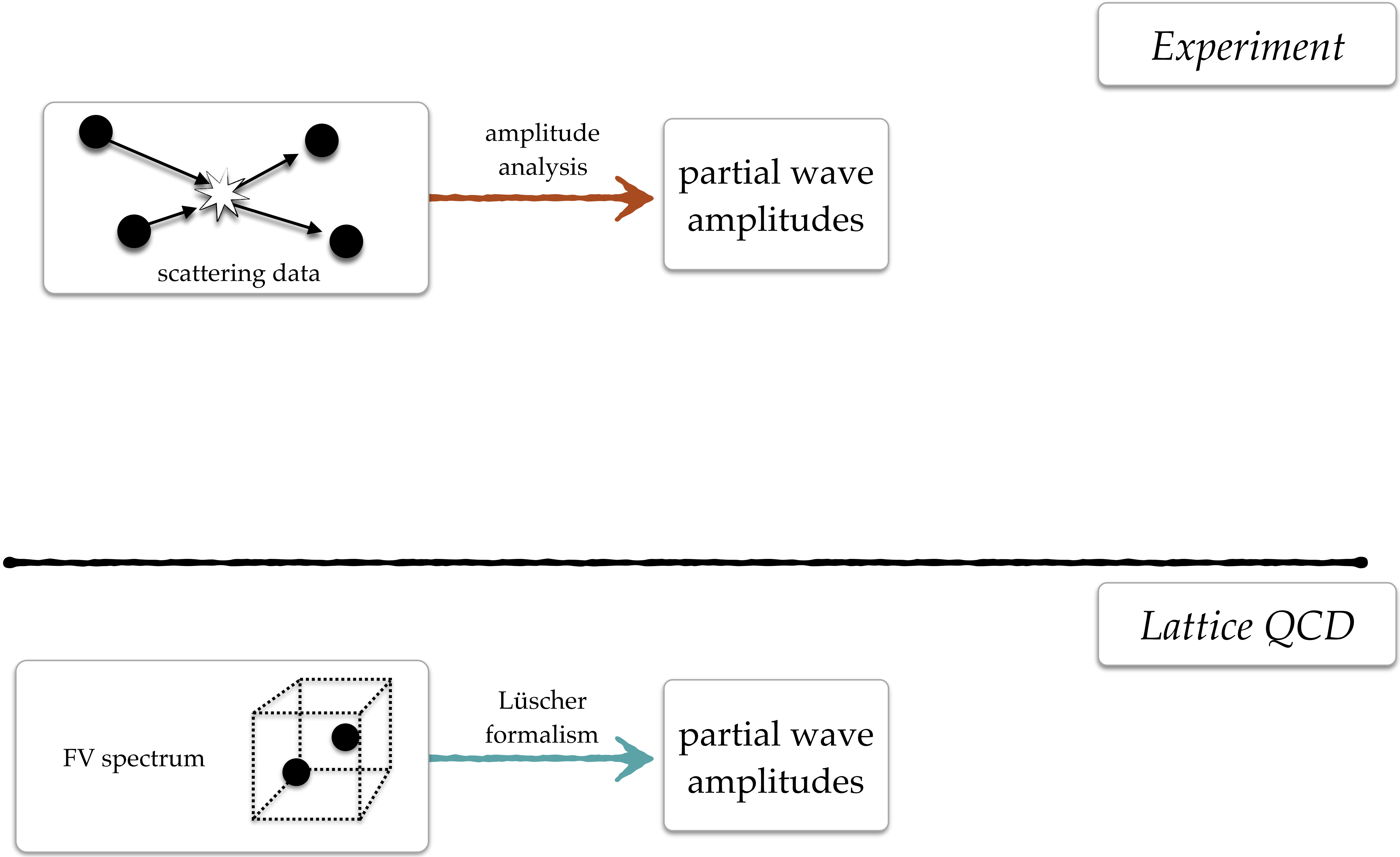
amplitude analysis

partial wave amplitudes

Protopopescu et al. (1972)






Experiment
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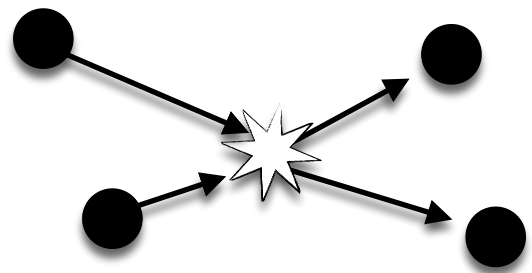
Lattice QCD
FV spectrum
Lüscher formalism
partial wave amplitudes

Experiment



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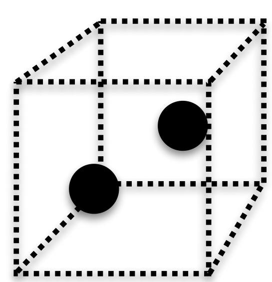
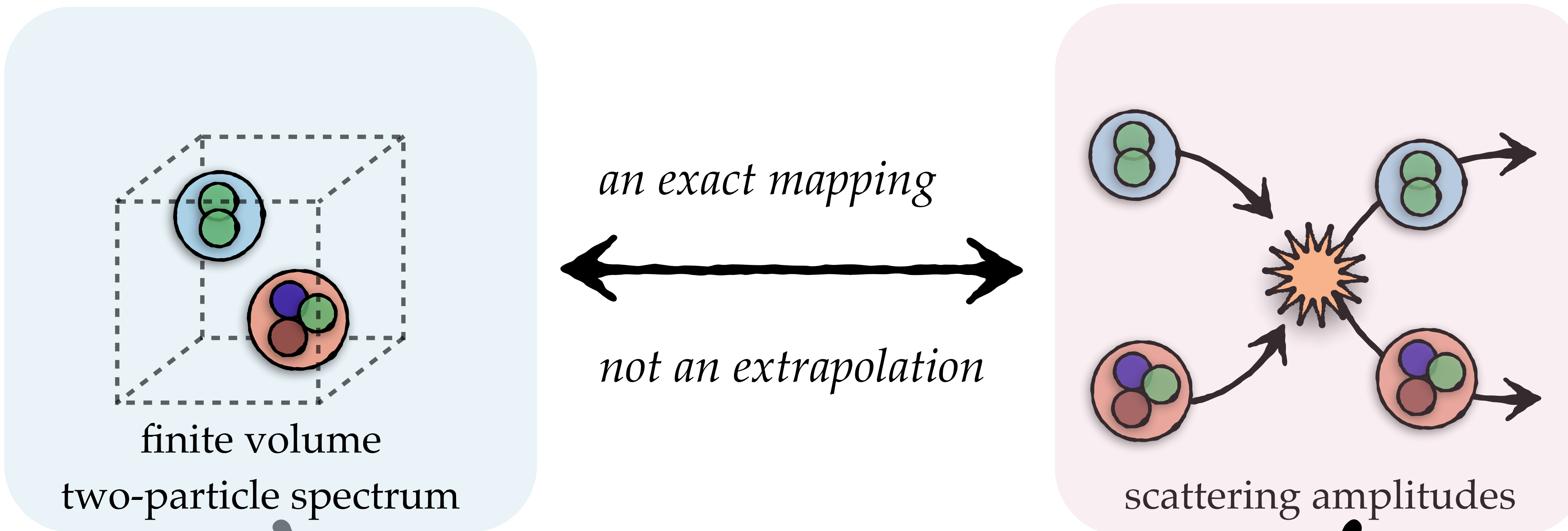


Lüscher formalism

partial wave amplitudes

determining the spectrum is a science of its own, but lets assume it's done properly

Lüscher formalism



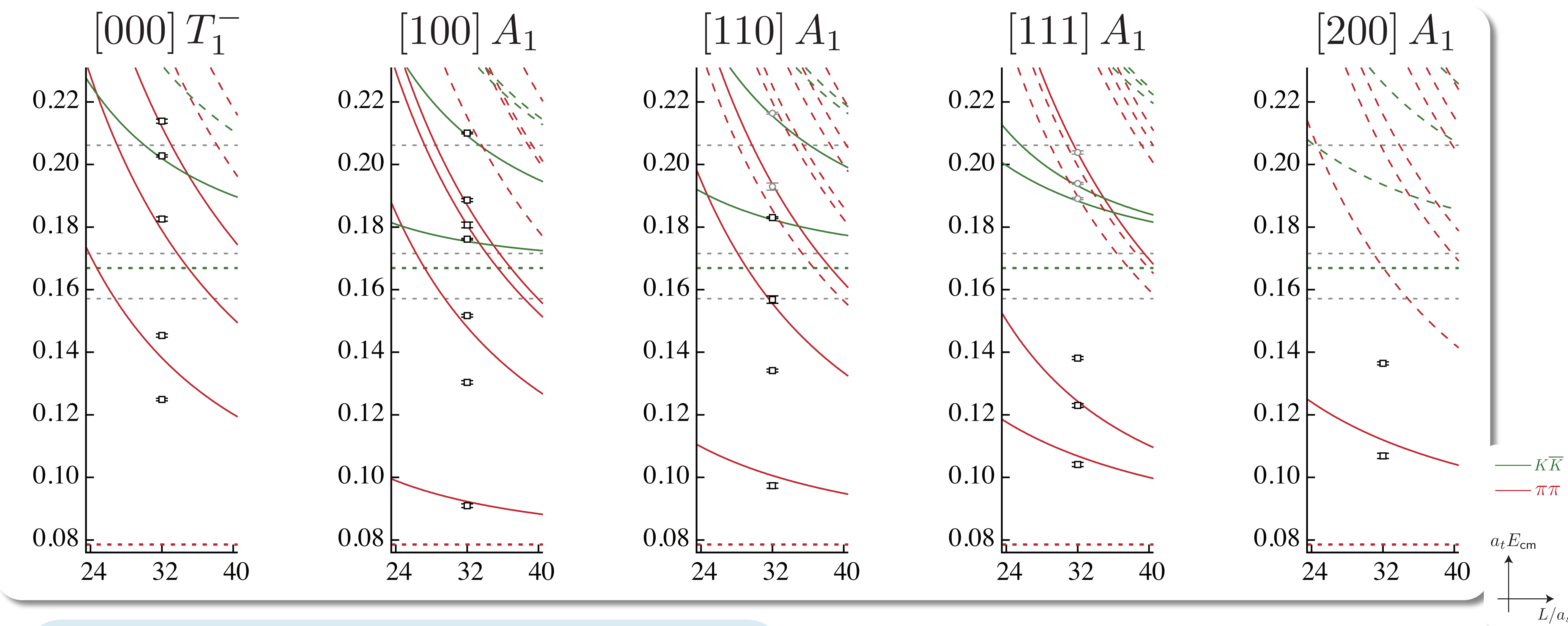
poles satisfy: $\det[F^{-1}(P,L) + \mathcal{M}(P)] = 0$

the most general two-particle quantization condition in a finite volume **[RB (2014)]**

$\pi\pi$ scattering

(I=1 channel)

A subset of the spectrum:



Wilson Thomas Dudek Edwards

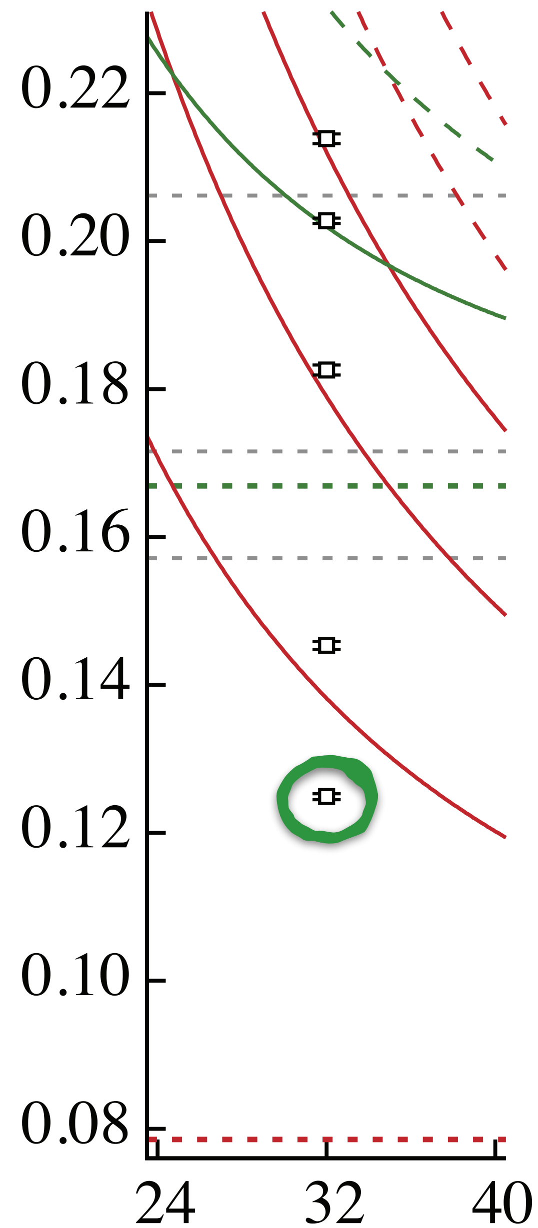
Hadron Spectrum Collaboration

Wilson, RB, Dudek, Edwards & Thomas (2015)

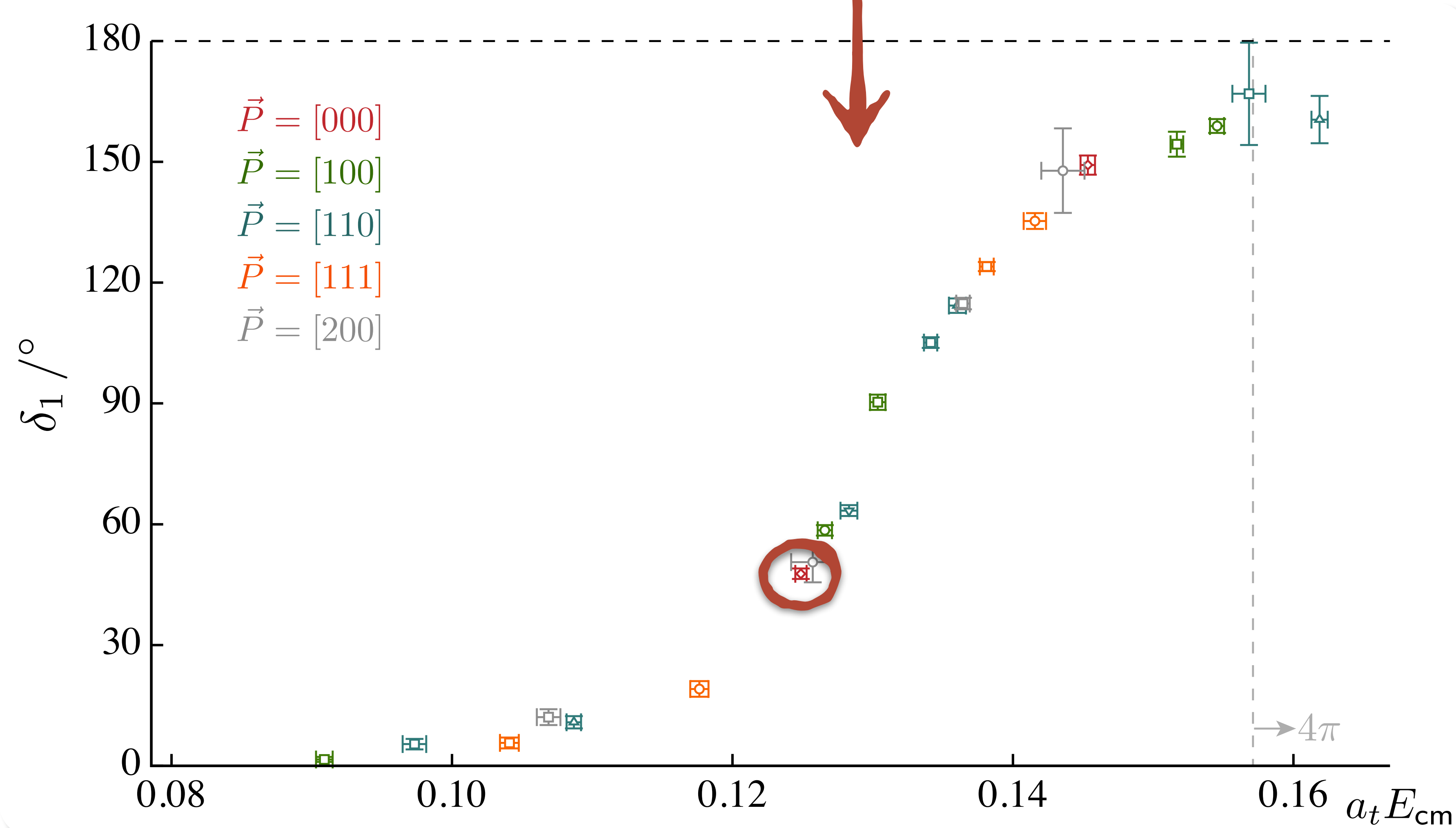
$\pi\pi$ scattering

(I=1 channel)

$[000]\ T_1^-$



$$\det[F^{-1}(P,L) + \mathcal{M}(P)] = 0$$

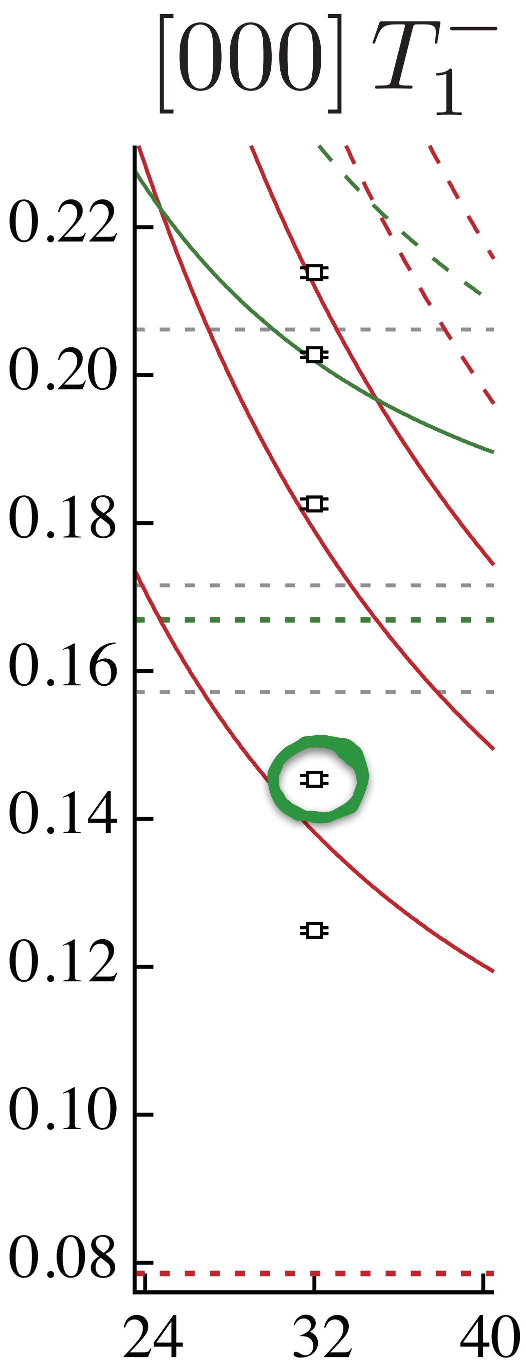


$$\mathcal{M} \propto \frac{1}{\cot \delta_1 - i}$$

Wilson, RB, Dudek, Edwards & Thomas (2015)

$\pi\pi$ scattering

(I=1 channel)



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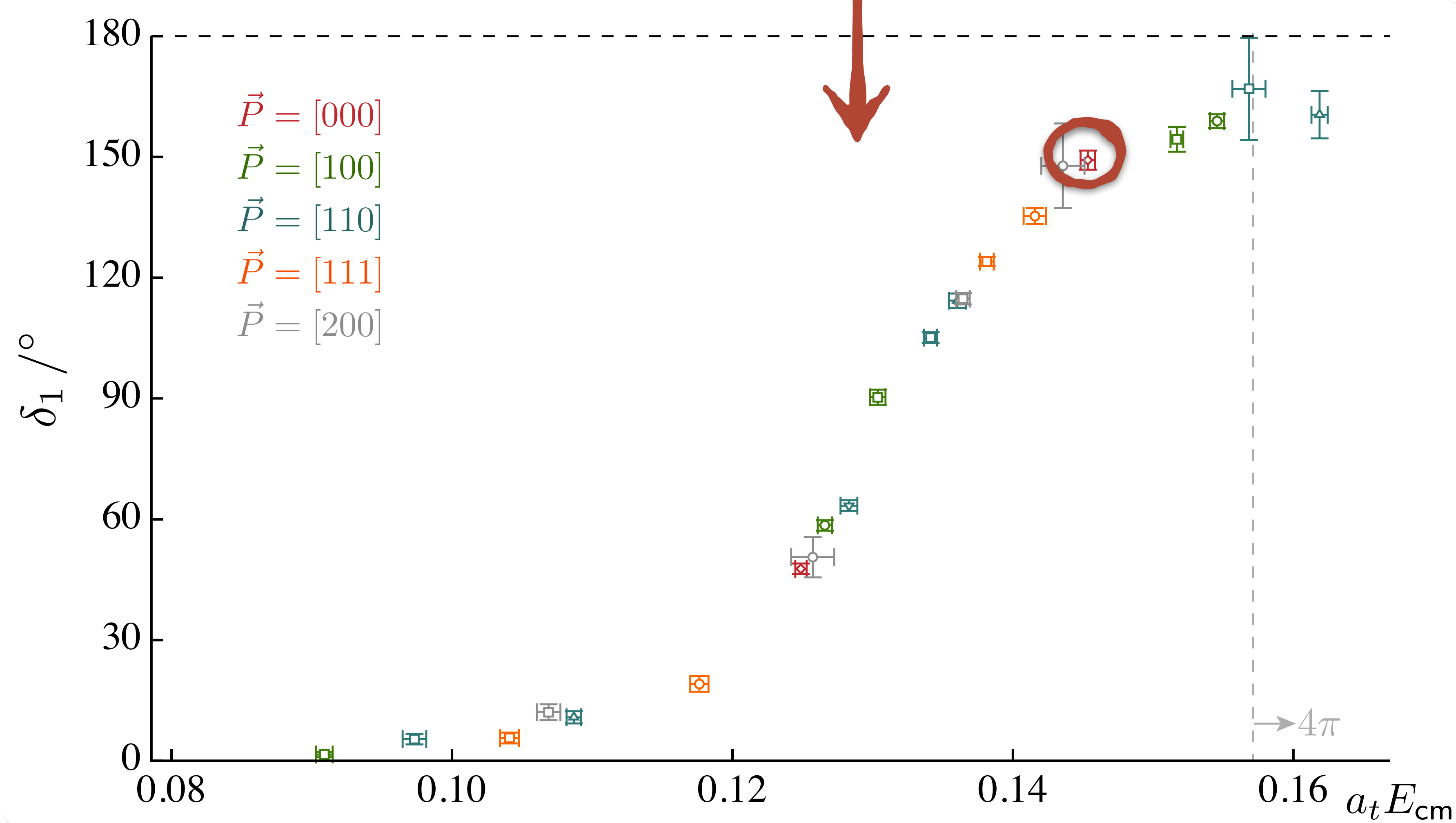

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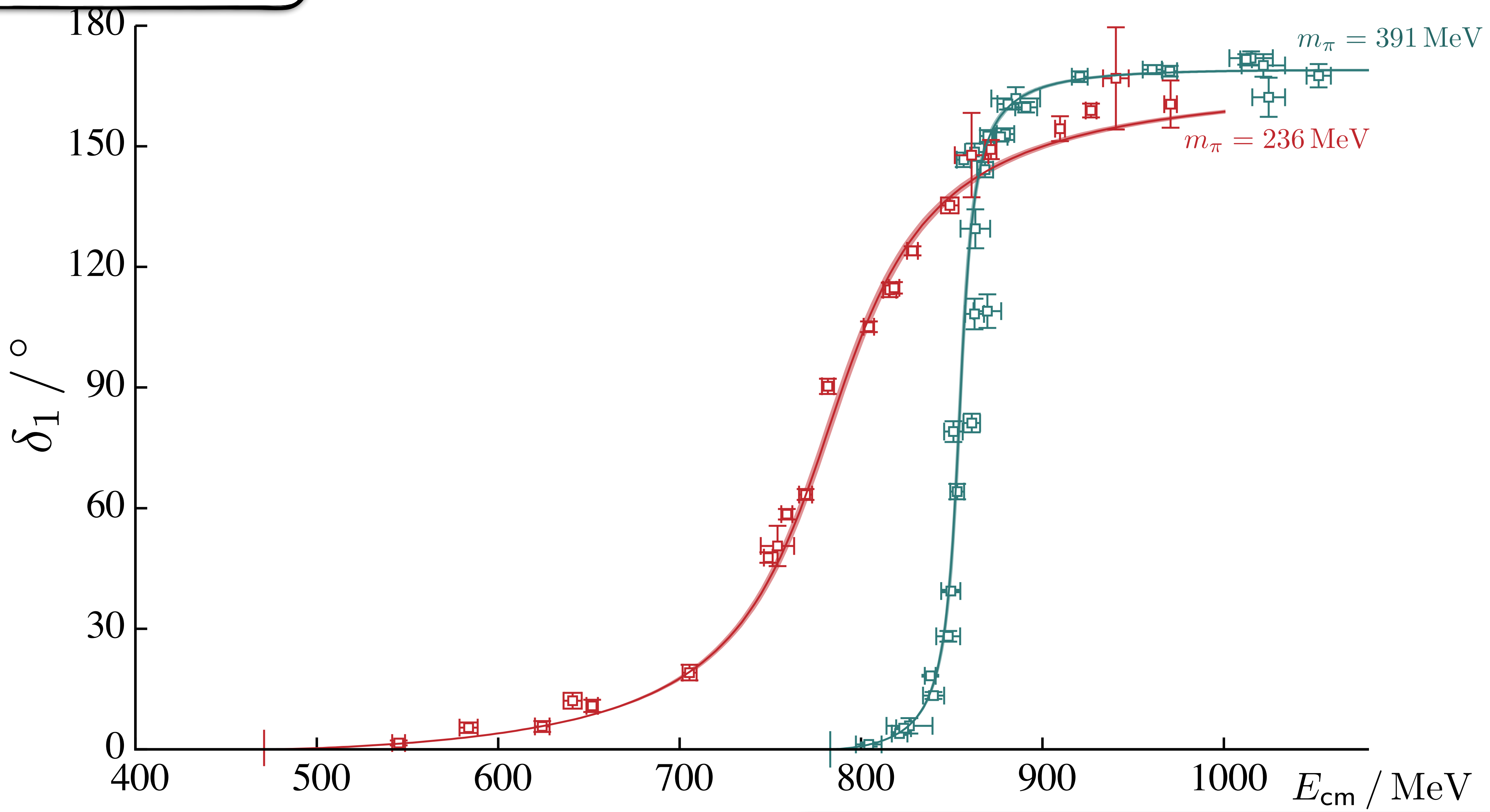
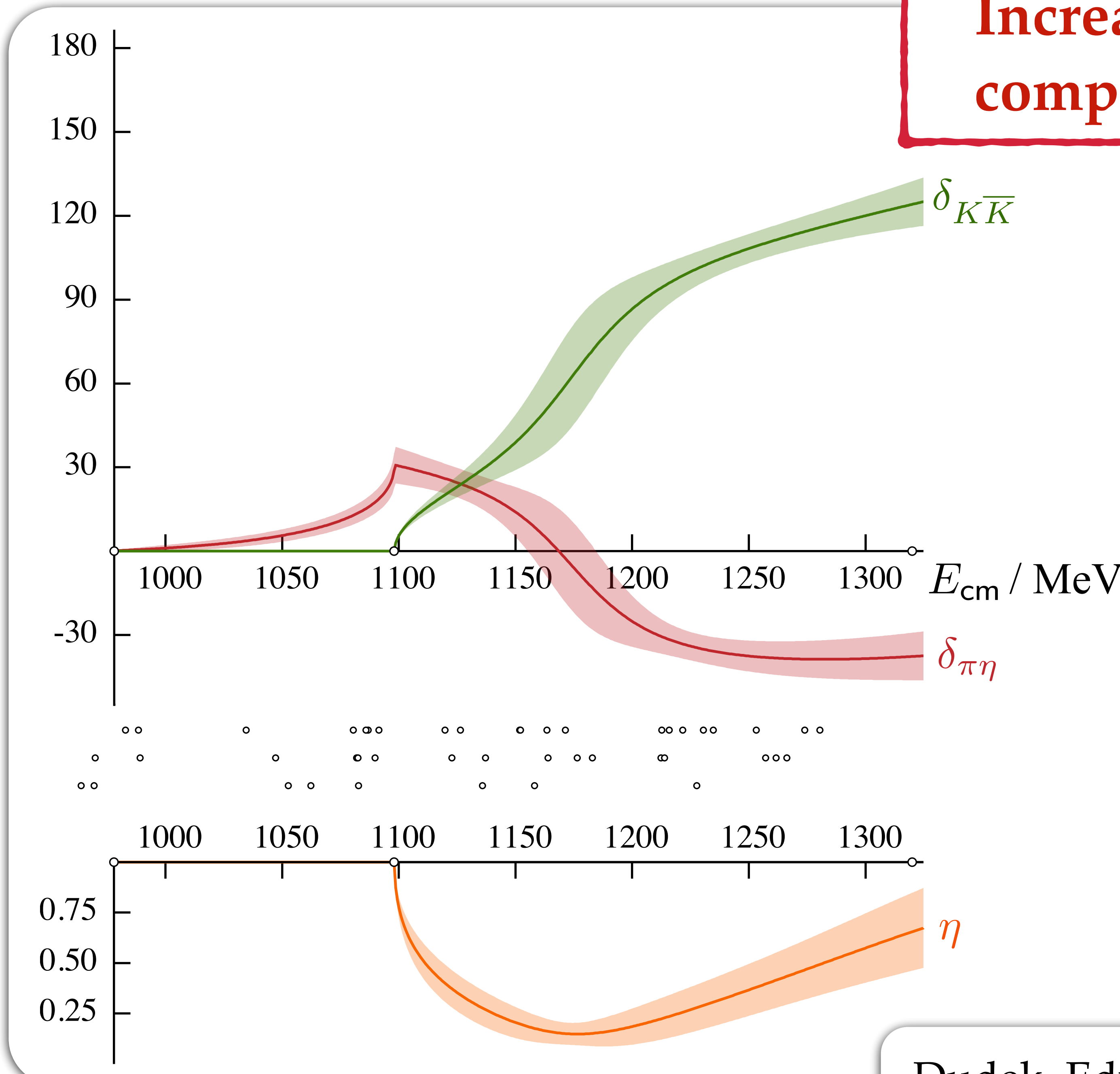
HadSpec Collaboration



Dudek, Edwards & Thomas (2012)
Wilson, RB, Dudek, Edwards & Thomas (2015)

$\pi\eta$–$K\overline{K}$ scattering

(I=1 channel)



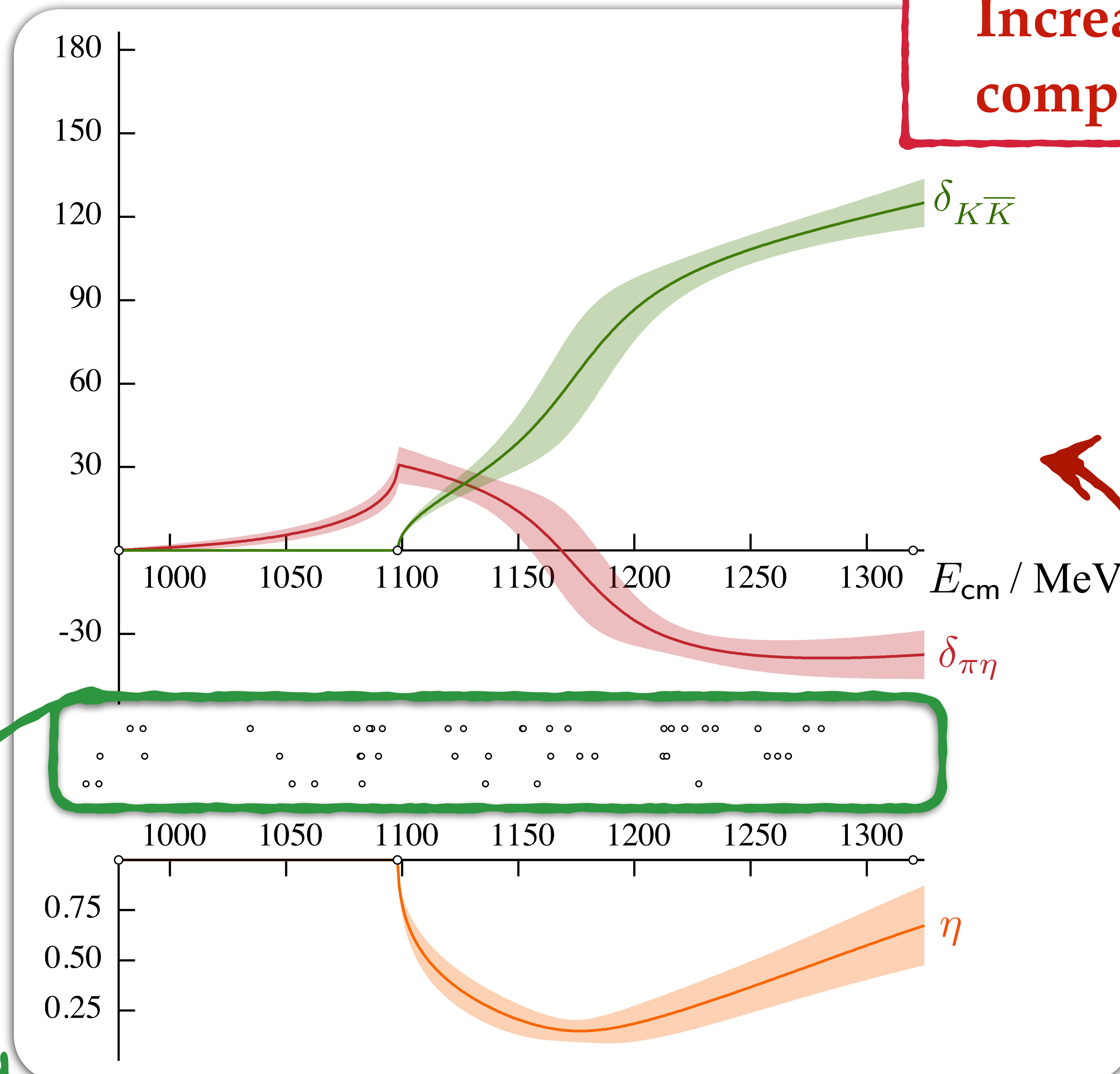
Increasingly complex systems

Dudek, Edwards & Wilson (2016)

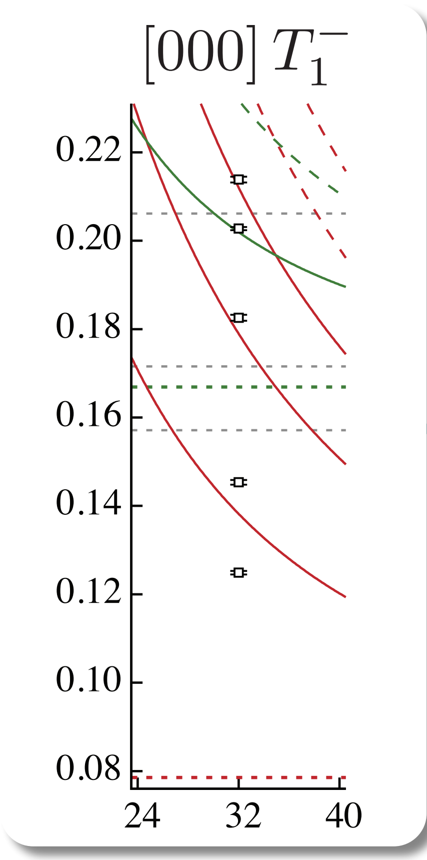
$\pi\eta$–$K\overline{K}$ scattering

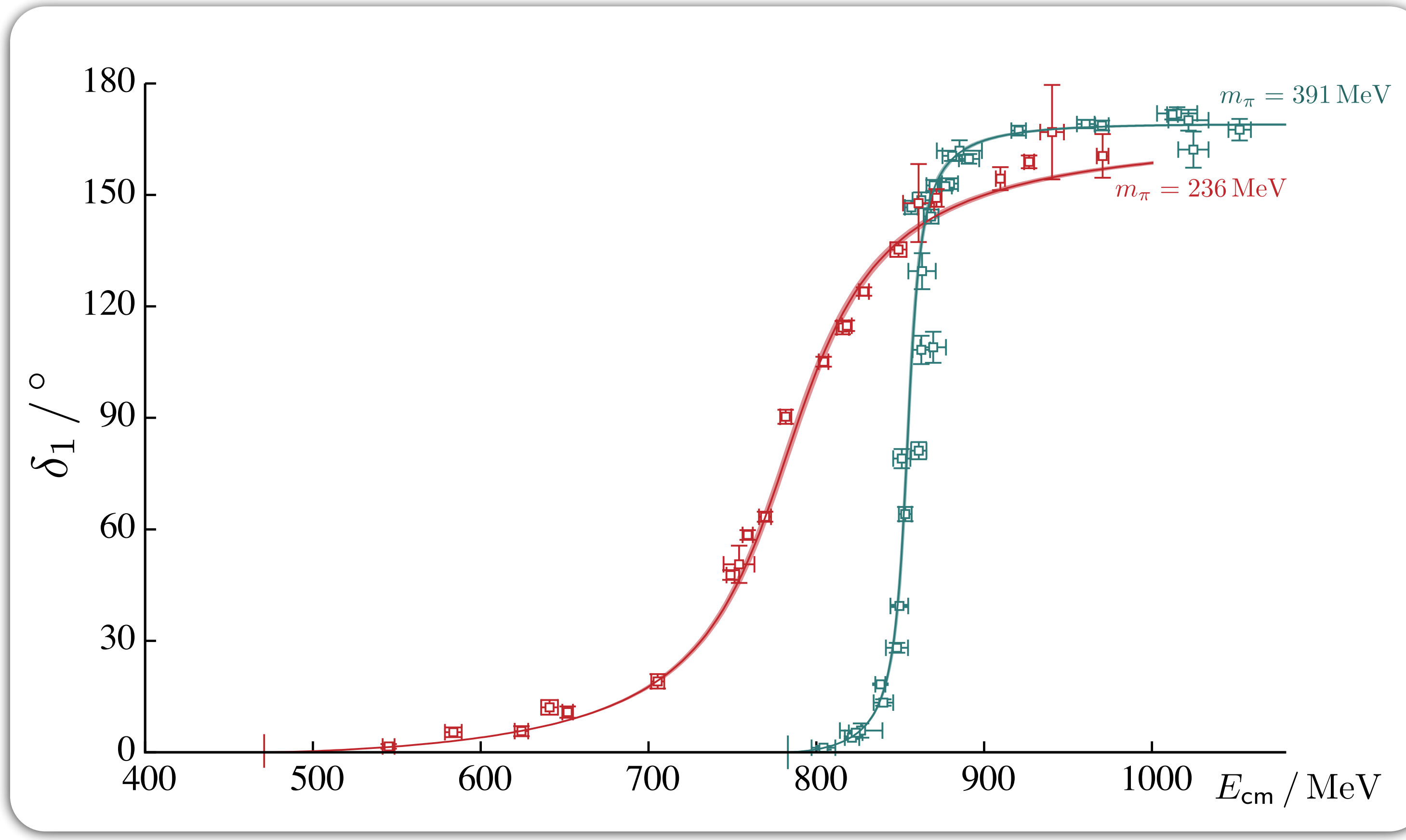
(I=1 channel)

Increasingly complex systems



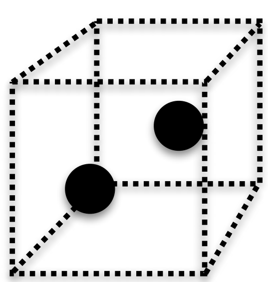
$$\det\left[\begin{pmatrix} F_{\pi\pi} & \\ & F_{K\overline{K}} \end{pmatrix}^{-1} + \begin{pmatrix} \mathcal{M}_{\pi\pi,\pi\pi} & \mathcal{M}_{\pi\pi,K\overline{K}} \\ \mathcal{M}_{\pi\pi,K\overline{K}} & \mathcal{M}_{K\overline{K},K\overline{K}} \end{pmatrix}\right] = 0$$





Lattice QCD

FV spectrum



Lüscher formalism

partial wave amplitudes

Some comments

- Model independent, **universal**, parametrization-independent, and exact

$$\det[F^{-1}(P,L) + \mathcal{M}(P)] = 0$$

RB (Jan 2014)

- Other on-going efforts:
  - Model dependent
  - process-dependent
  - only suitable for few partial waves
  - parametrization dependent
  - low-energy approximations
  - ignore spin
  - ignore three-body

⋮

you get the idea

From Extraction of Nucleon Resonances to LQCD

T.-S. H. Lee,[1] Jia-Jun Wu,[2] and Hiroyuki Kamano[3]

[1]*Physics Division, Argonne National Laboratory, Argonne, Illinois 60439, USA*
[2]*Special Research Center for the Subatomic Structure of Matter (CSSM), School of Chemistry and Physics, University of Adelaide Adelaide 5005, Australia*
[3]*Research Center for Nuclear Physics, Osaka University, Ibaraki, Osaka 567-0047, Japan*

Feb 2016

Finite-volume Hamiltonian method for coupled channel interactions in lattice QCD

Jia-Jun Wu,[1] T.-S. H. Lee,[1] A. W. Thomas,[2,3] and R. D. Young[2,3]

[1]*Physics Division, Argonne National Laboratory, Argonne, Illinois 60439, USA*
[2]*Special Research Center for the Subatomic Structure of Matter (CSSM), School of Chemistry and Physics, University of Adelaide Adelaide 5005, Australia*
[3]*ARC Center of Excellence for Particle Physics at Terascale, School of Chemistry and Physics, University of Adelaide Adelaide 5005,*

Nov 2014

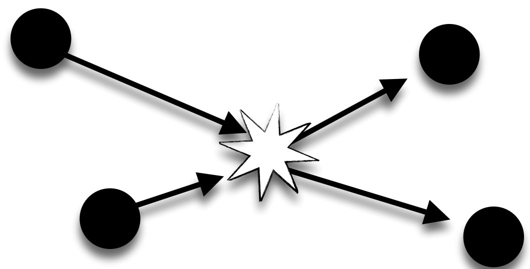
Finite-volume matrix Hamiltonian model for a $\Delta \to N\pi$ system

J. M. M. Hall,[1] A. C.-P. Hsu,[1] D. B. Leinweber,[1] A. W. Thomas,[1,2] and R. D. Young[1,2]

[1]*Special Research Centre for the Subatomic Structure of Matter (CSSM), School of Chemistry and Physics, University of Adelaide 5005, Australia*
[2]*ARC Centre of Excellence for Particle Physics at the Terascale, School of Chemistry and Physics, University of Adelaide 5005, Australia*

May 2013

Experiment



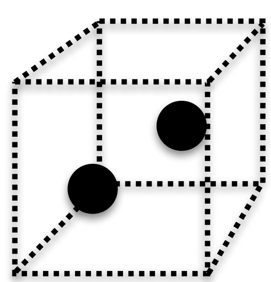
scattering data

amplitude analysis

partial wave amplitudes

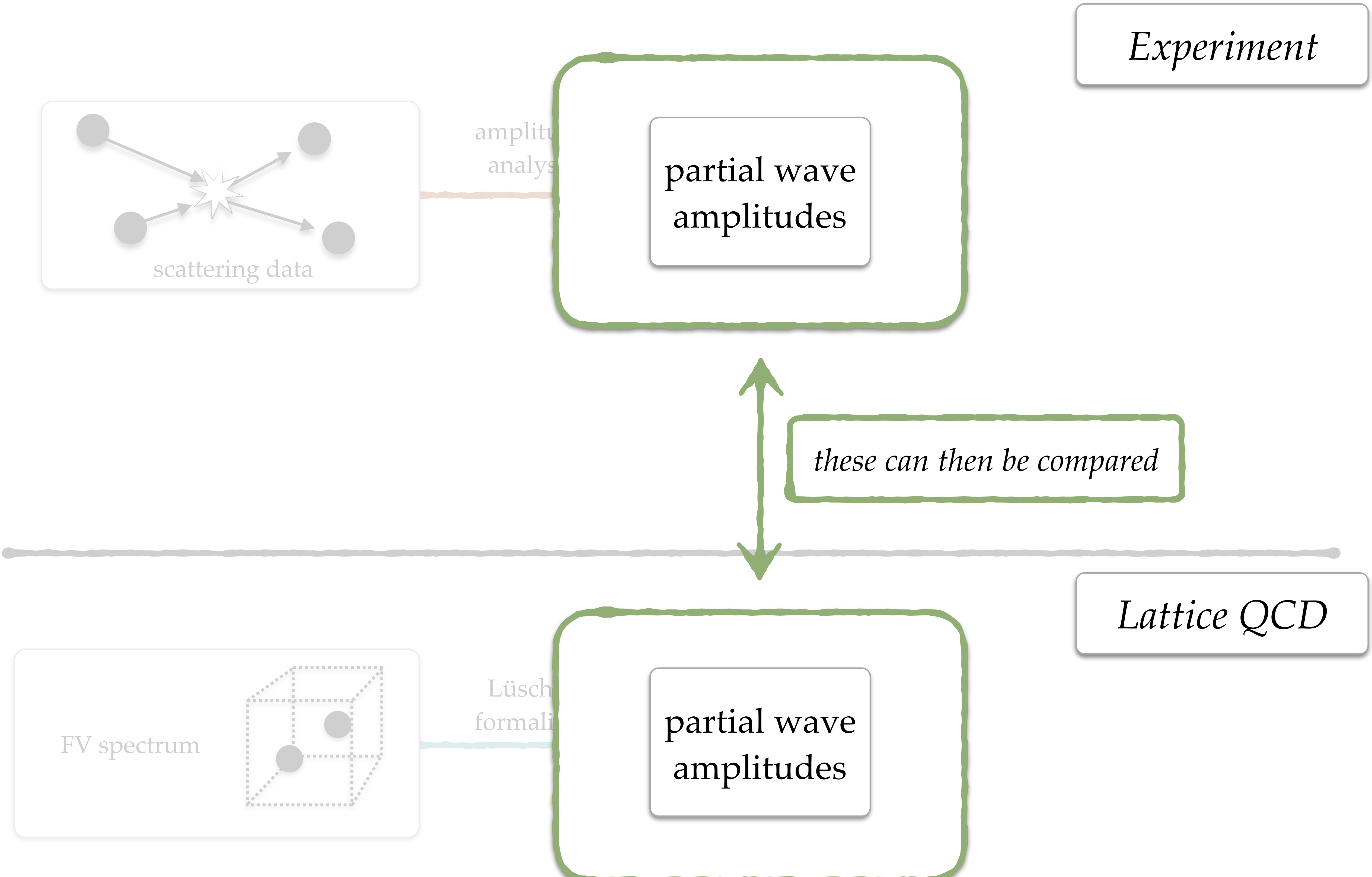
Lattice QCD

FV spectrum



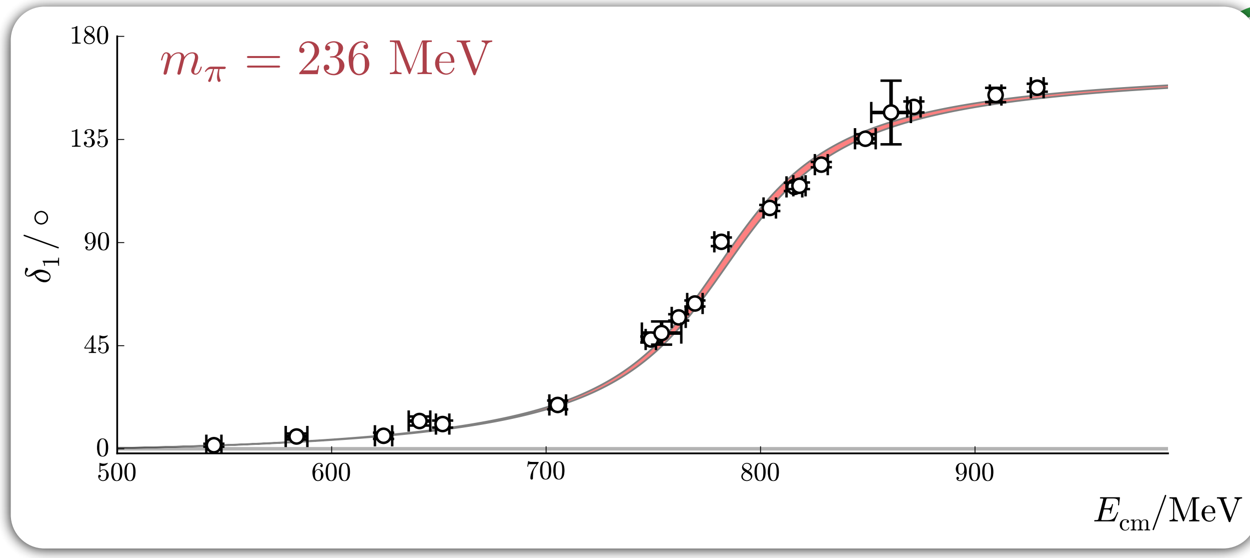
Lüscher formalism

partial wave amplitudes

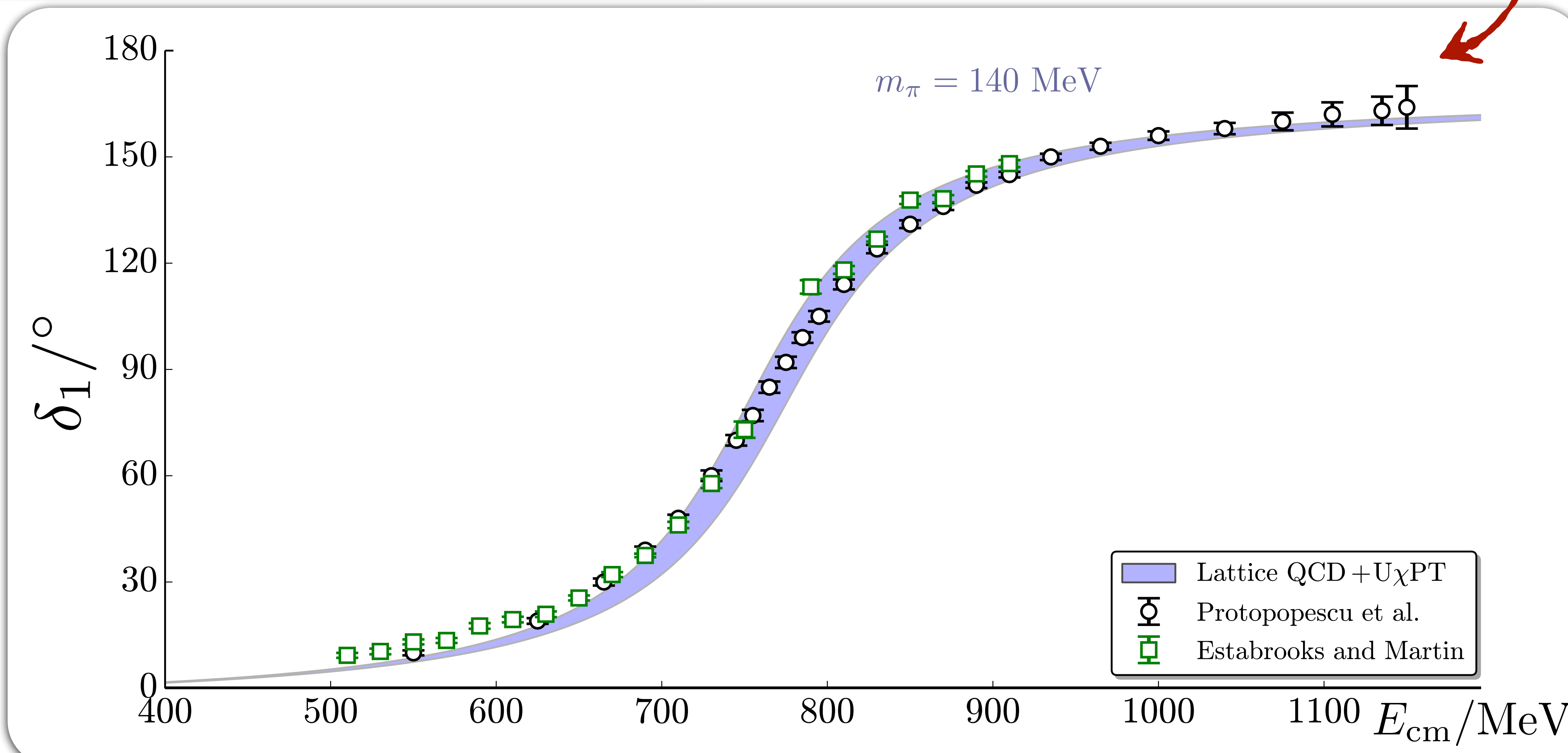


Experiment
scattering data
partial wave amplitudes
these can then be compared
Lattice QCD
FV spectrum
partial wave amplitudes

Comparing with experiment

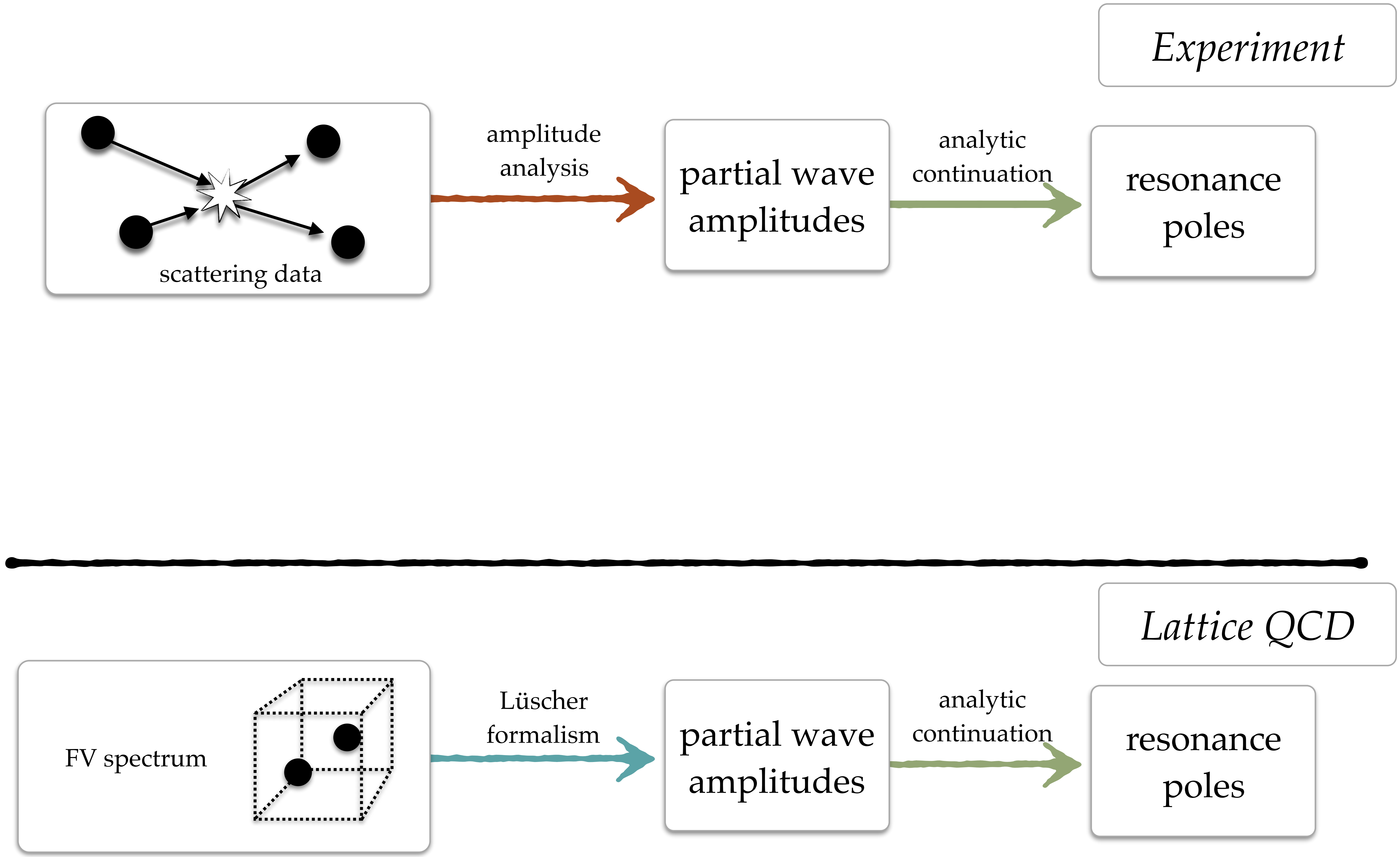


$$\det\left[F^{-1} + \mathcal{M}\left(m_\pi, \{\alpha\}\right)\right] = 0$$



First chiral extrapolation of a resonant amplitude

Bolton, RB & Wilson (2015)



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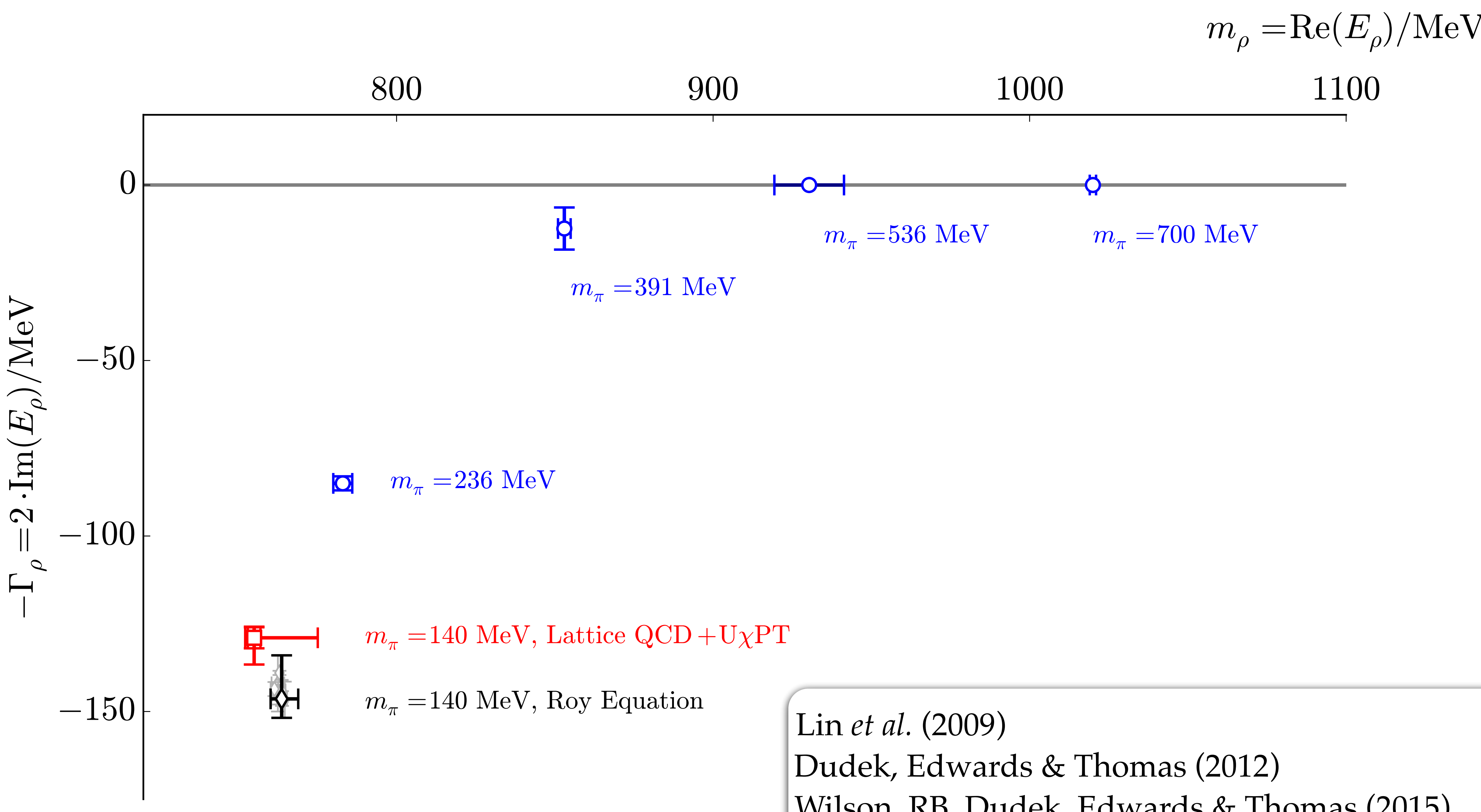
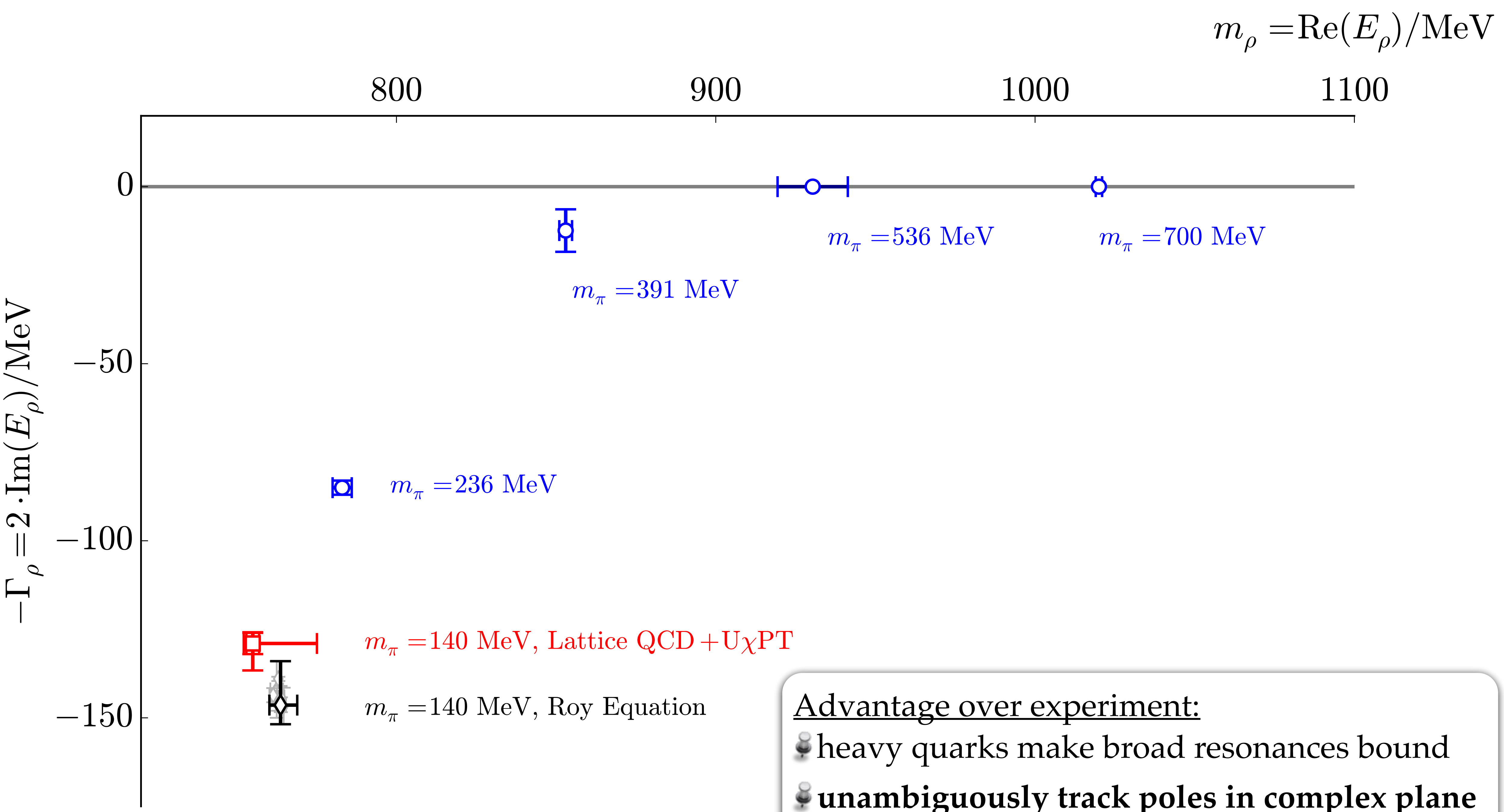
Quark-mass dependence of poles



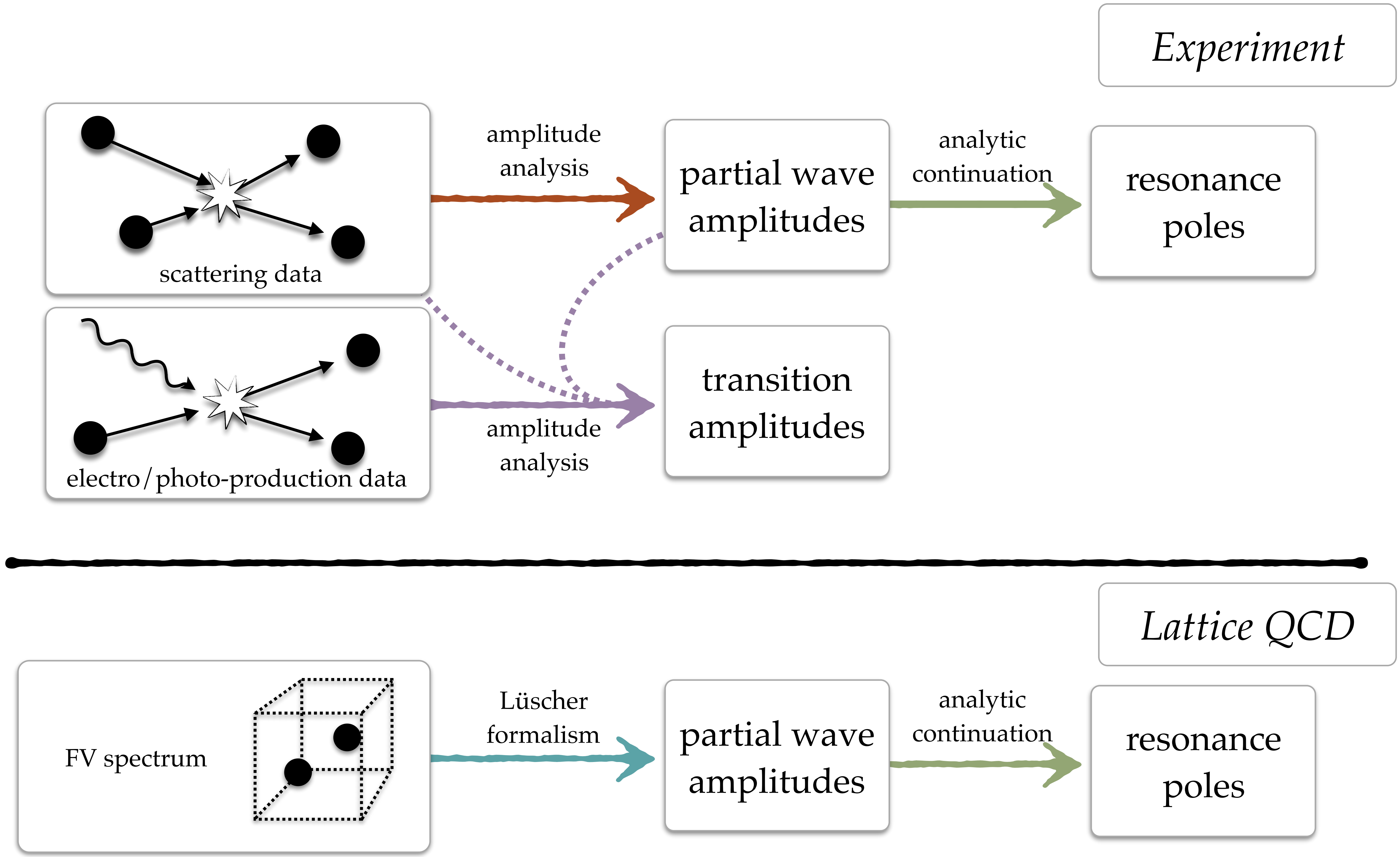
Lin *et al.* (2009)
Dudek, Edwards & Thomas (2012)
Wilson, RB, Dudek, Edwards & Thomas (2015)
Bolton, RB & Wilson (2015)

Quark-mass dependence of poles



Advantage over experiment:

- heavy quarks make broad resonances bound
- **unambiguously track poles in complex plane**



Experiment
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electro/photo-production data
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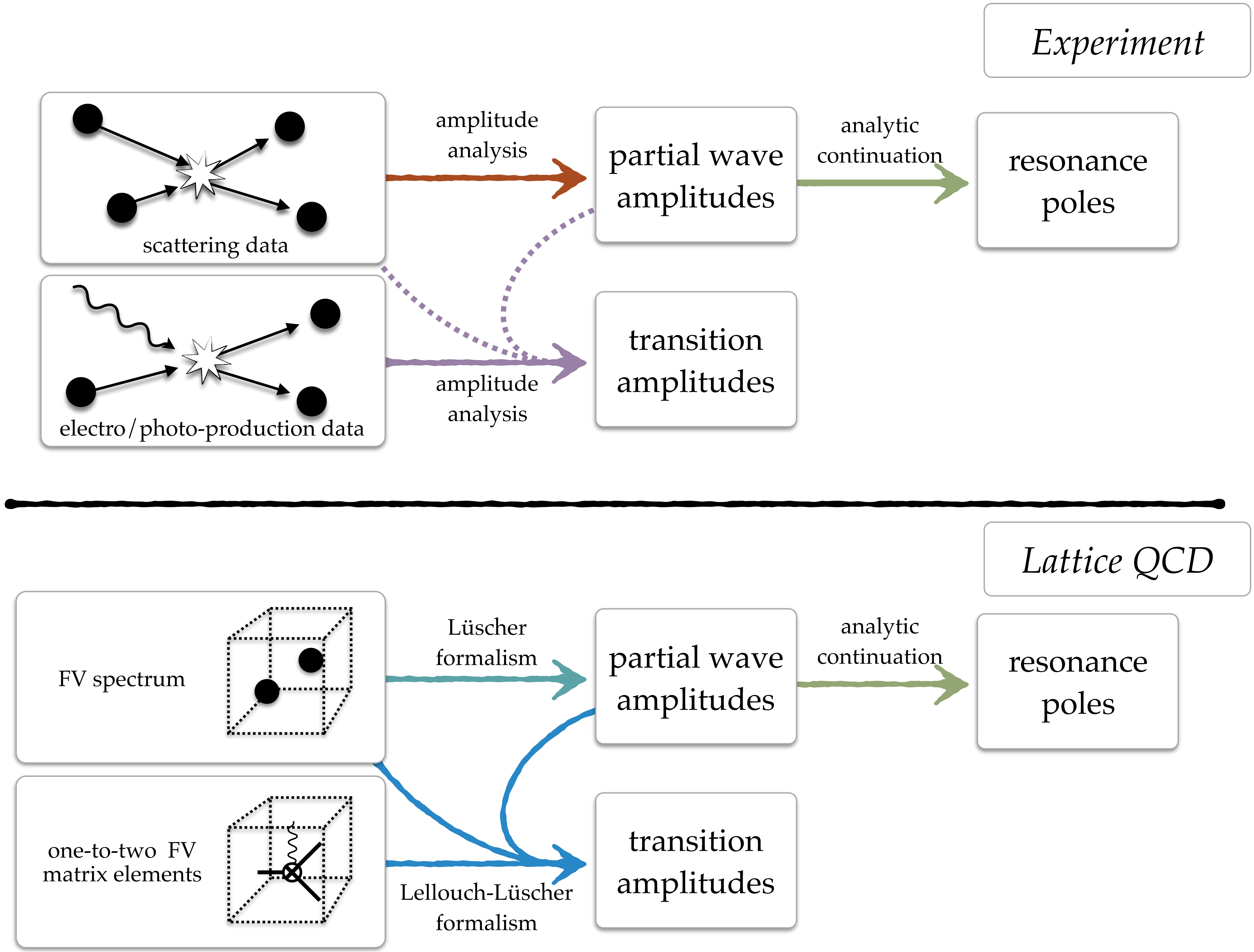
Experiment
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one-to-two FV matrix elements
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Lellouch-Lüscher formalism
partial wave amplitudes
transition amplitudes
analytic continuation
resonance poles

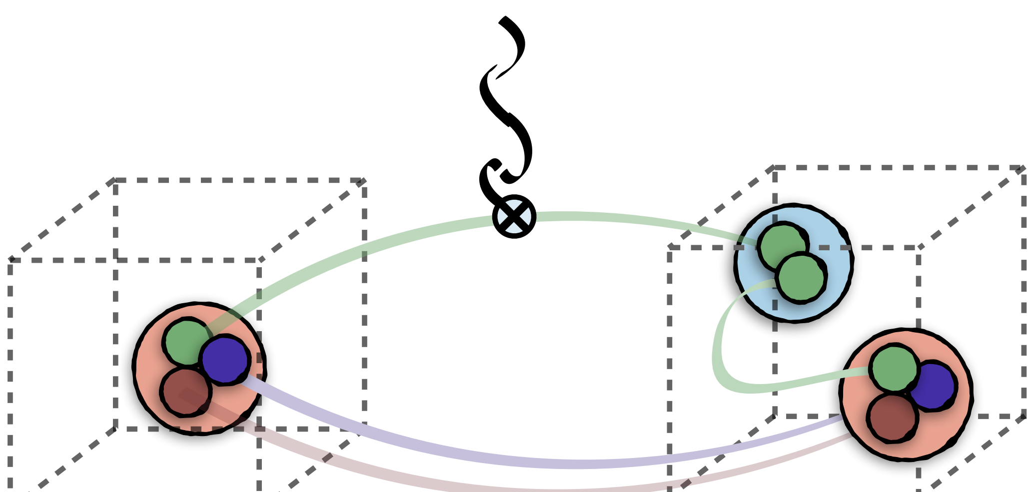
Matrix elements

1) Access matrix elements:

$$C^{3pt.}_{\mathbf{2}\to\mathbf{1}\mathcal{J}} = \langle \mathcal{O}_1(\delta t)\mathcal{J}(t)\mathcal{O}_2^\dagger(0)\rangle \longrightarrow \langle \mathbf{1}|\mathcal{J}|\mathbf{2}\rangle_L Z_1 Z_2^* e^{-(\delta t - t)E_1} e^{-tE_2} + \cdots$$

2) Interpret matrix elements:

$$\boxed{\left|\langle \mathbf{2}|\mathcal{J}|\mathbf{1}\rangle_L\right|^2 = \mathcal{H}\ \mathcal{R}\ \mathcal{H}}$$



RB, Hansen & Walker-Loud (2014)
RB & Hansen (2015)
RB & Hansen (2015)

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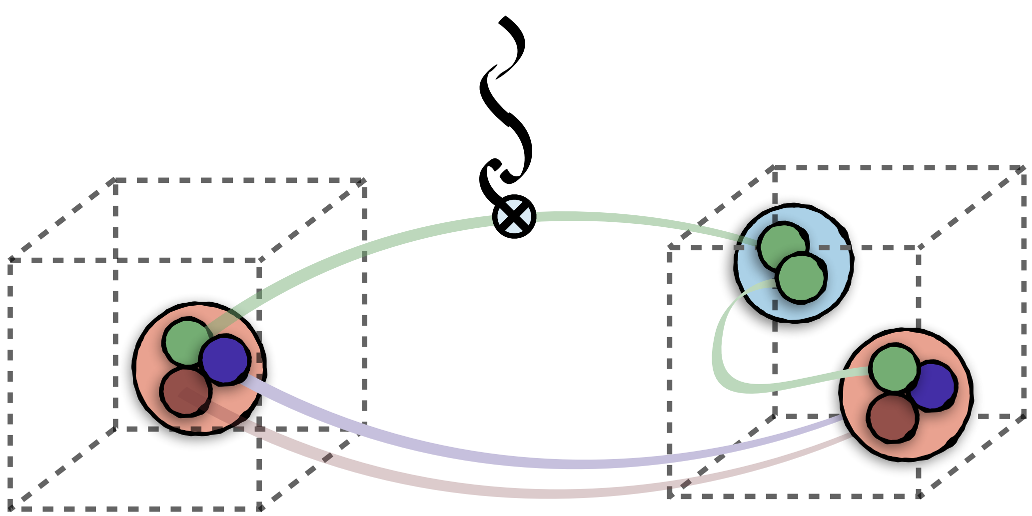
$$\boxed{\left|\langle \mathbf{2}|\mathcal{J}|\mathbf{1}\rangle_L\right|^2 = \mathcal{H}\,\underline{\mathcal{R}}\,\mathcal{H}}$$

known finite volume function

$$\mathcal{R}\left(E_{\mathbf{2}}, L, \delta, \frac{\partial\delta}{\partial E_{\mathbf{2}}}\right)$$



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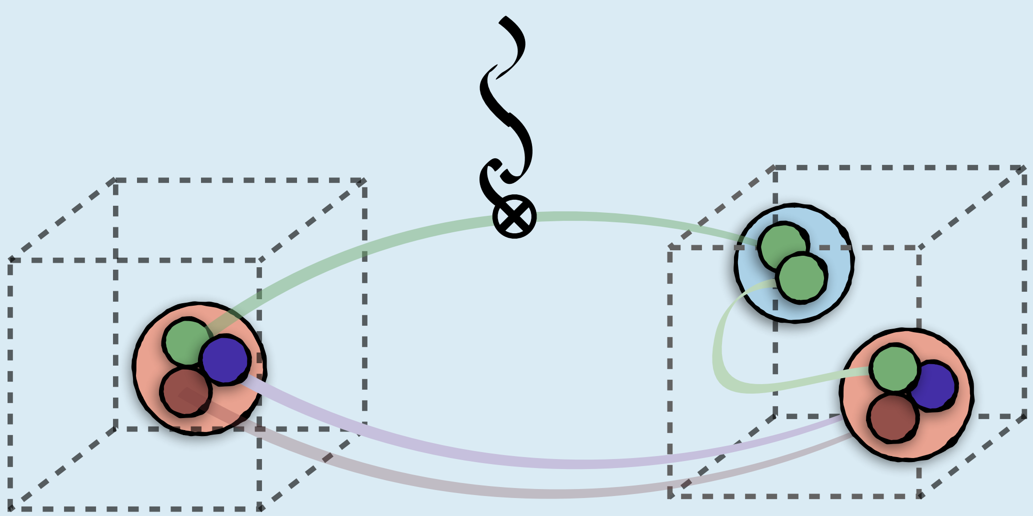
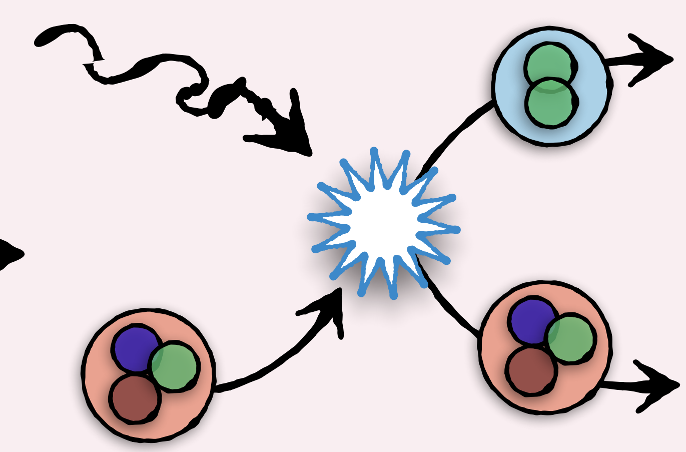
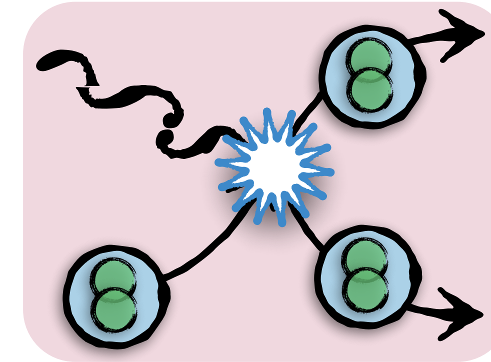


finite volume matrix elements



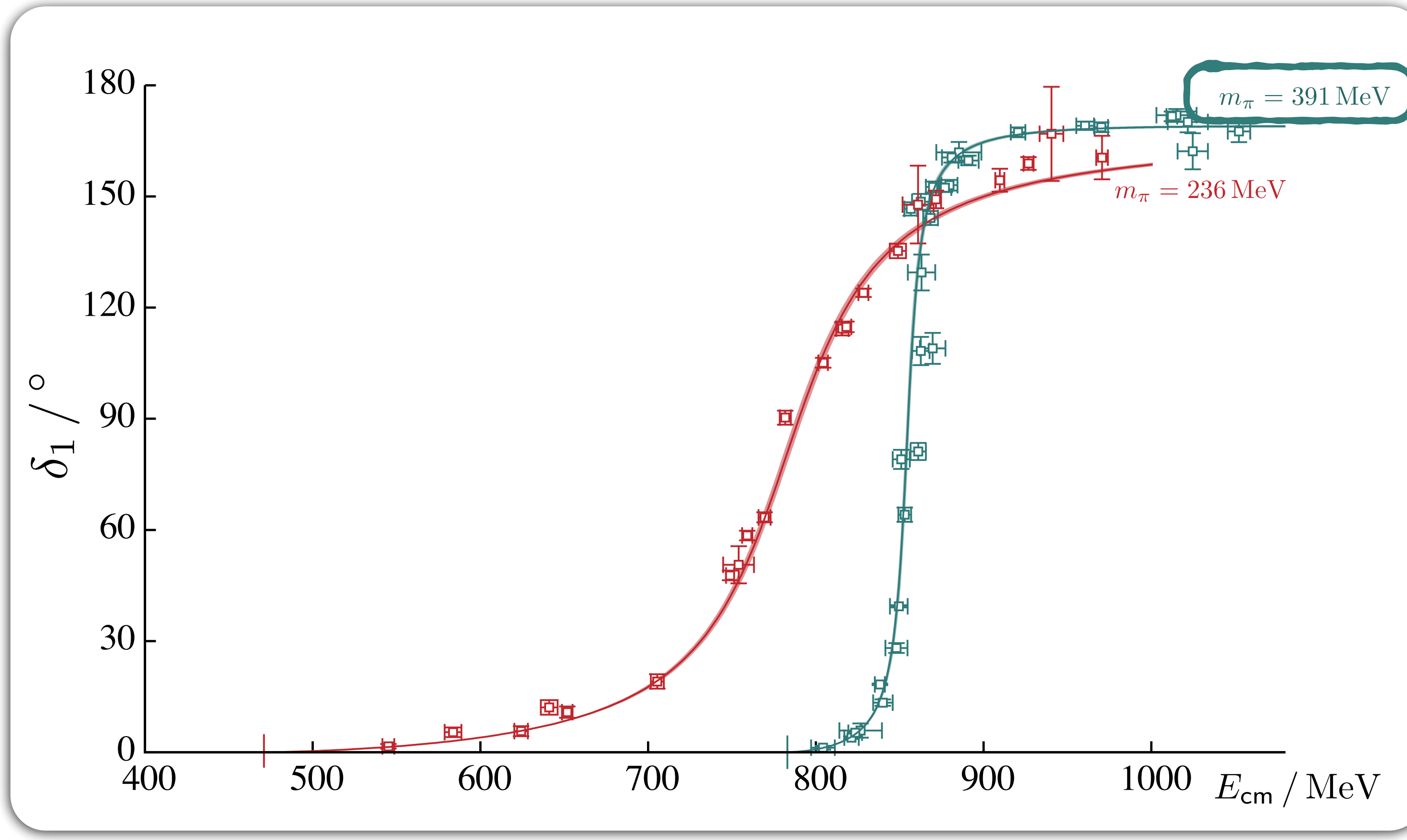
one-to-two transition amplitudes

$\pi\gamma^*$-to-$\pi\pi$



Exploratory $\pi\gamma^*$-to-$\pi\pi$ / $\pi\gamma^*$-to-ρ calculation:

- $m_\pi = 391$ MeV

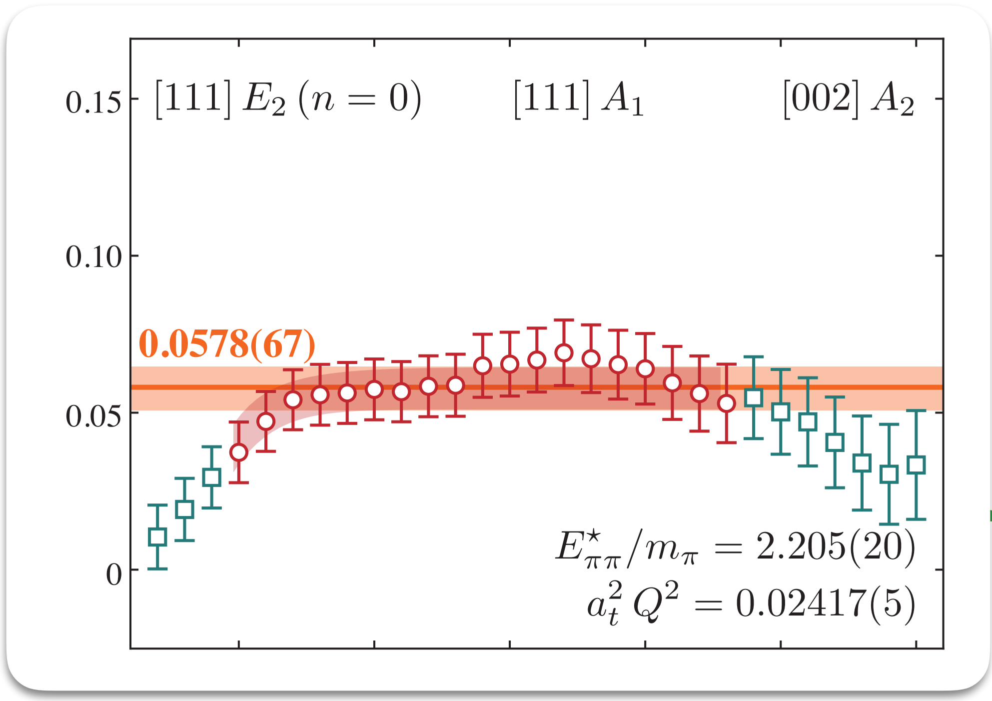


- Matrix element determined in **42** kinematic point: $(E_{\pi\pi}, Q^2)$
- Lorentz decomposition:

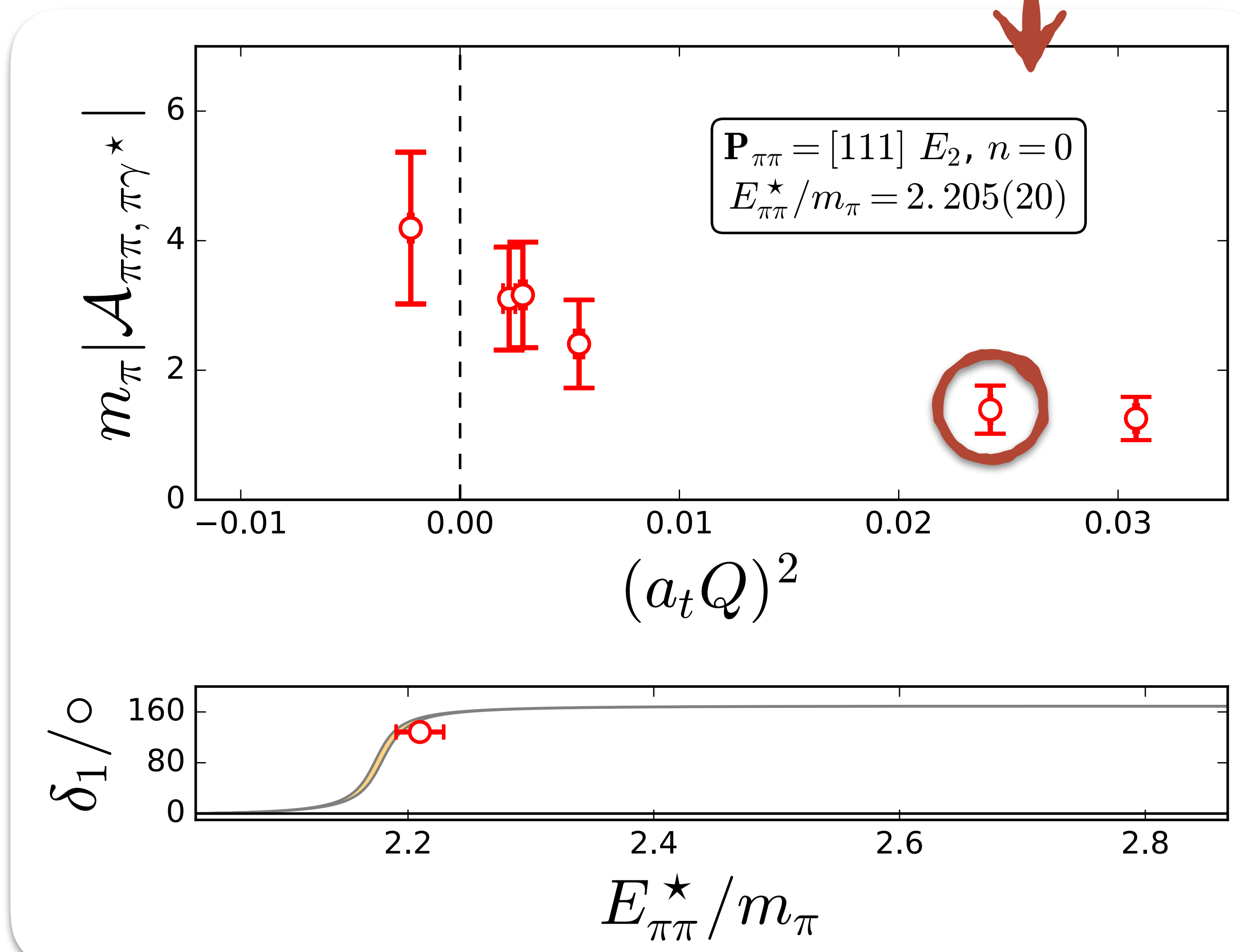
$$\mathcal{H}^{\mu}_{\pi\pi,\pi\gamma^\star} = \epsilon^{\mu\nu\alpha\beta}\, P_{\pi,\nu}\, P_{\pi\pi,\alpha}\, \epsilon_\beta(\lambda_{\pi\pi}, \mathbf{P}_{\pi\pi})\, \frac{2}{m_\pi} \mathcal{A}_{\pi\pi,\pi\gamma^\star}$$

ππ/ρ polarization *ππ/ρ helicity* *Lorentz scalar*

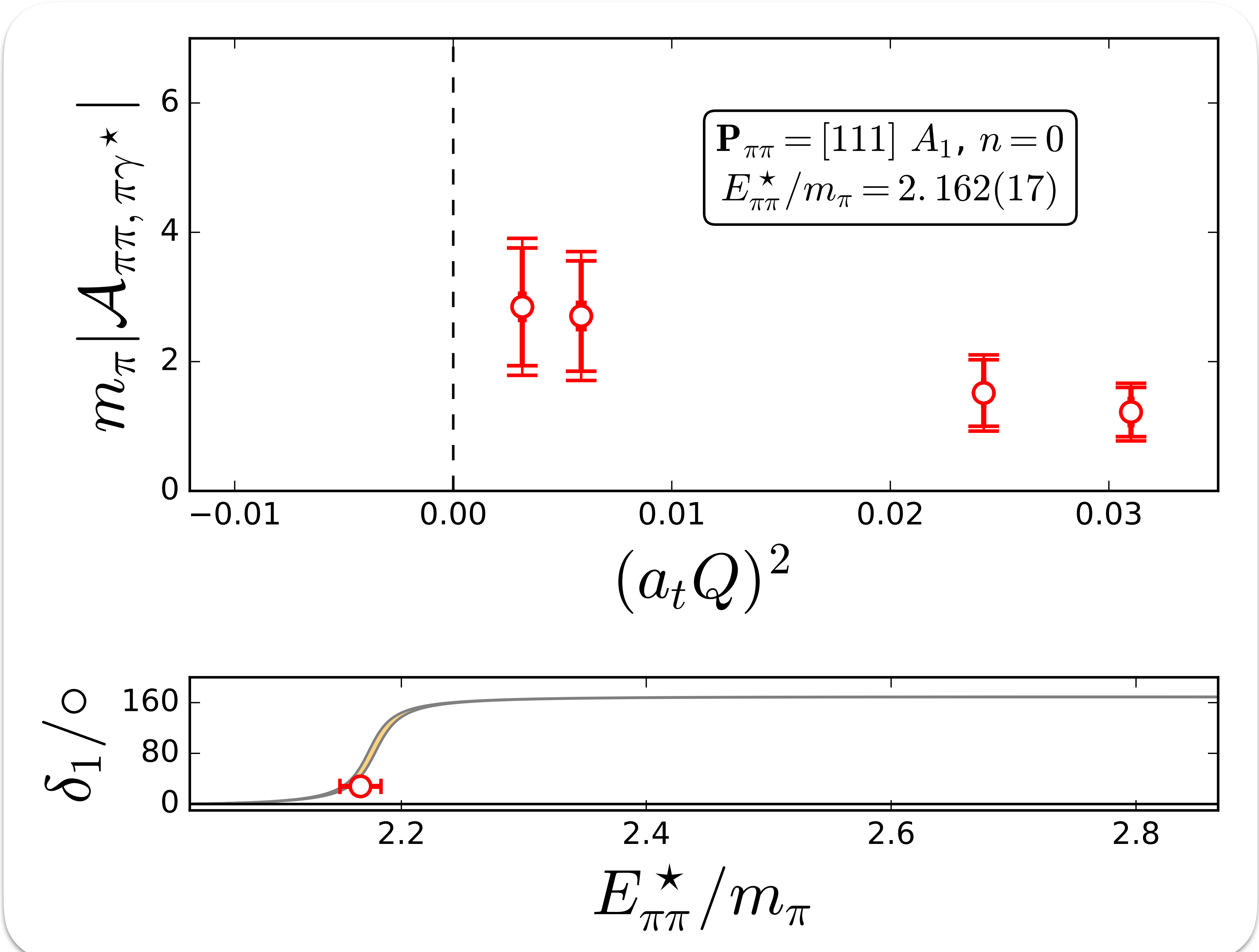
$\pi\gamma^*$-to-$\pi\pi$ amplitude



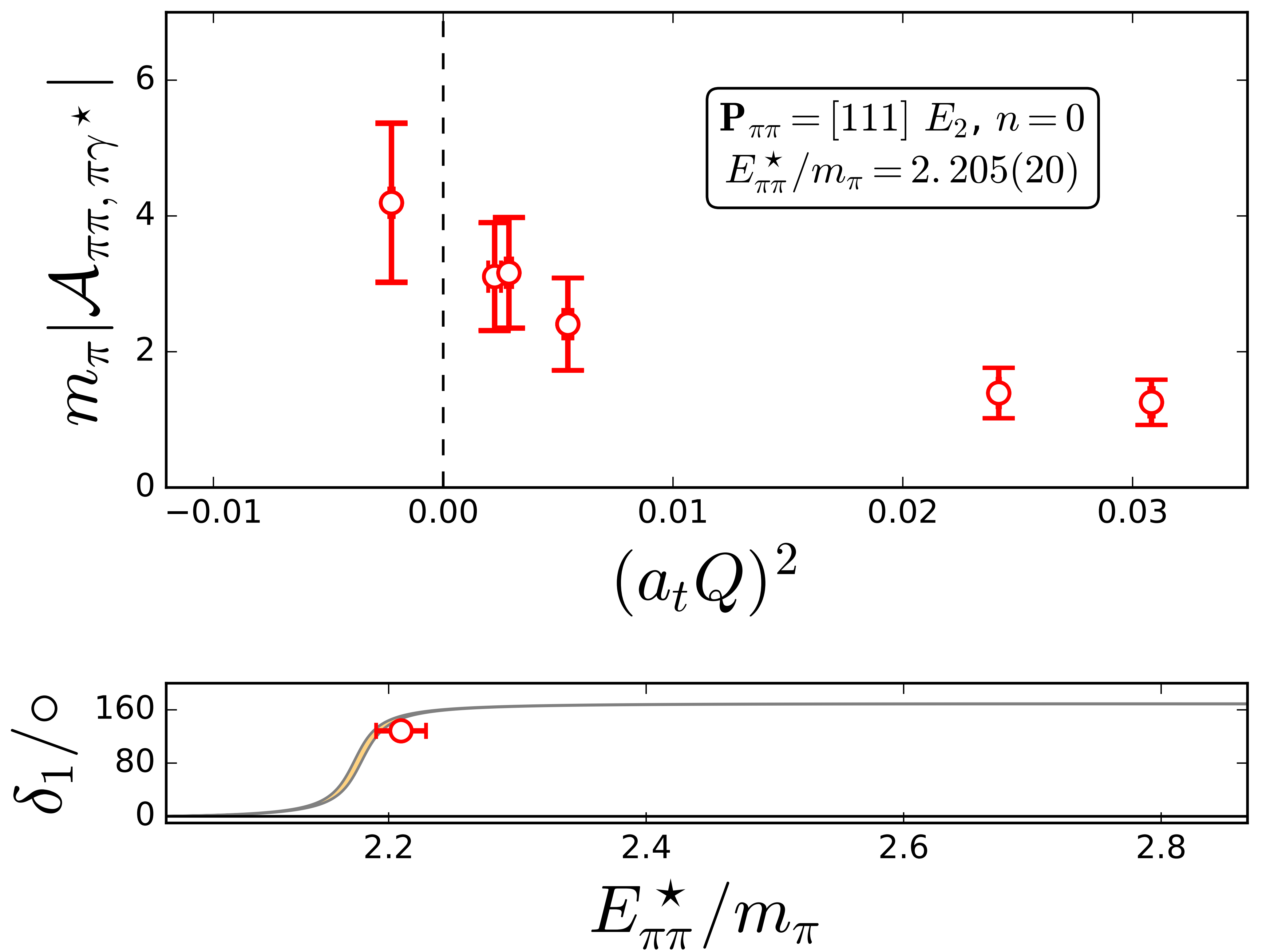
$$\left|\langle \mathbf{2}|\mathcal{J}|\mathbf{1}\rangle_L\right|^2 = \mathcal{H}\,\mathcal{R}\,\mathcal{H}$$



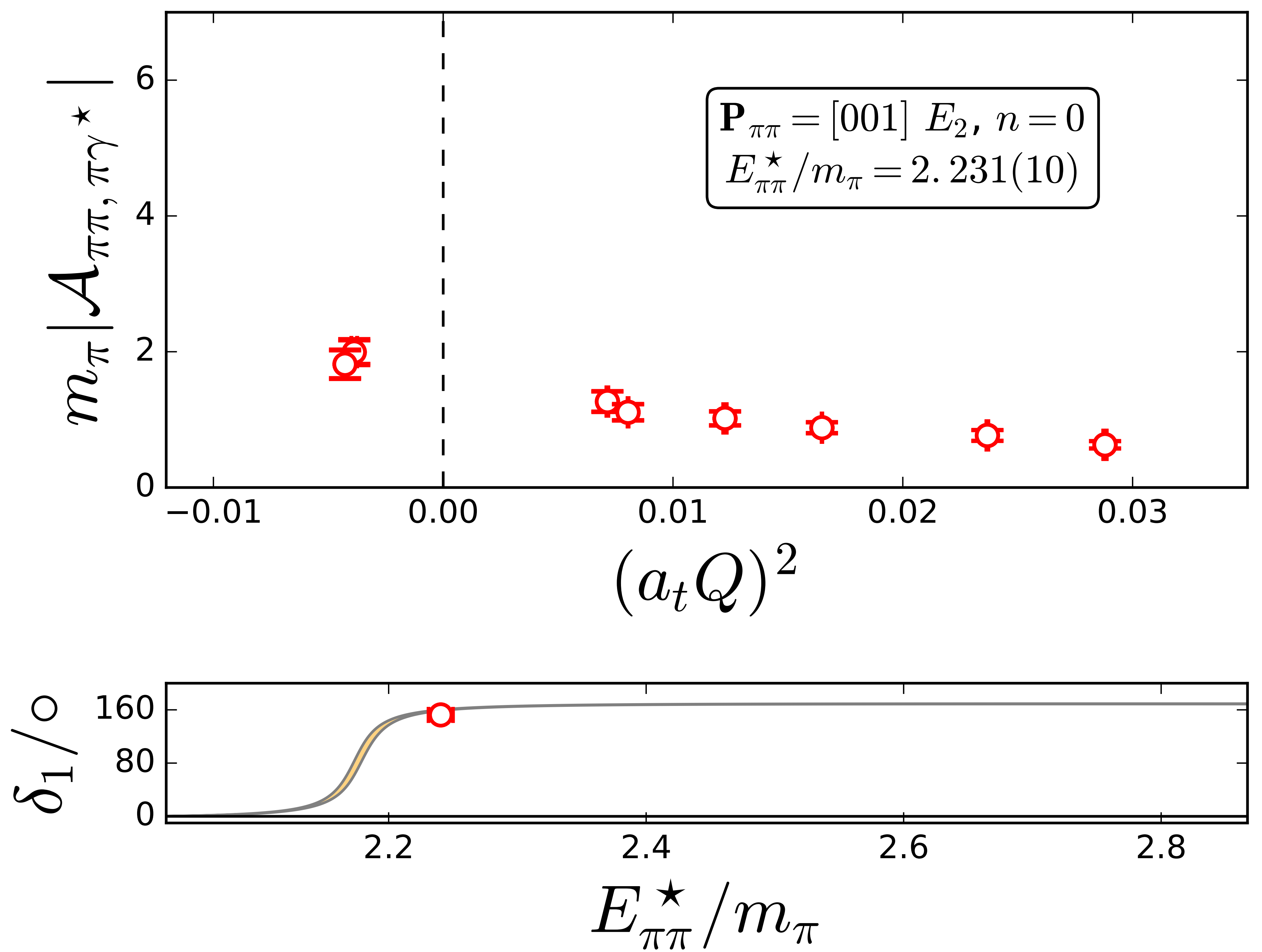
$\pi\gamma^*$-to-$\pi\pi$ amplitude



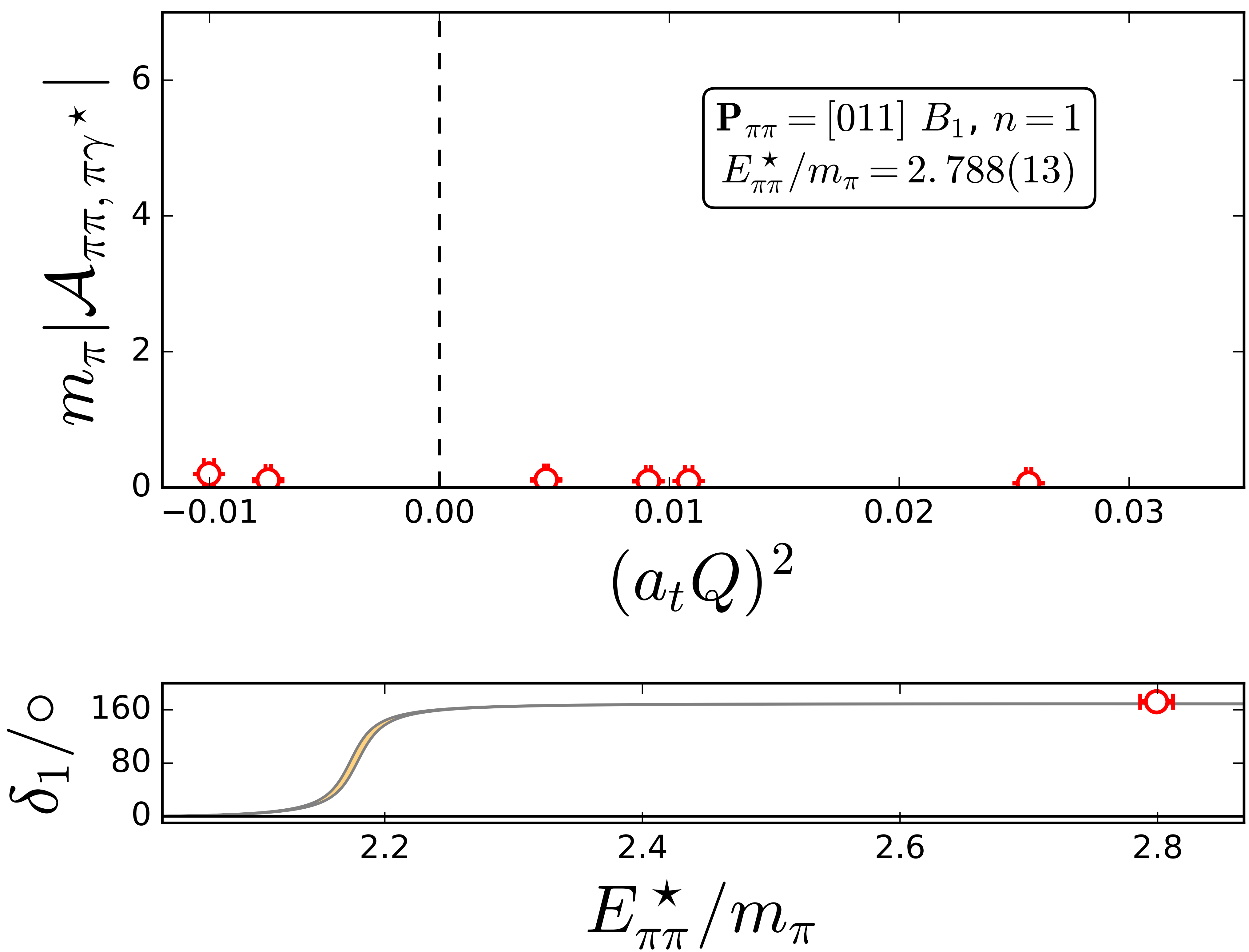
$\pi\gamma^*$-to-$\pi\pi$ amplitude



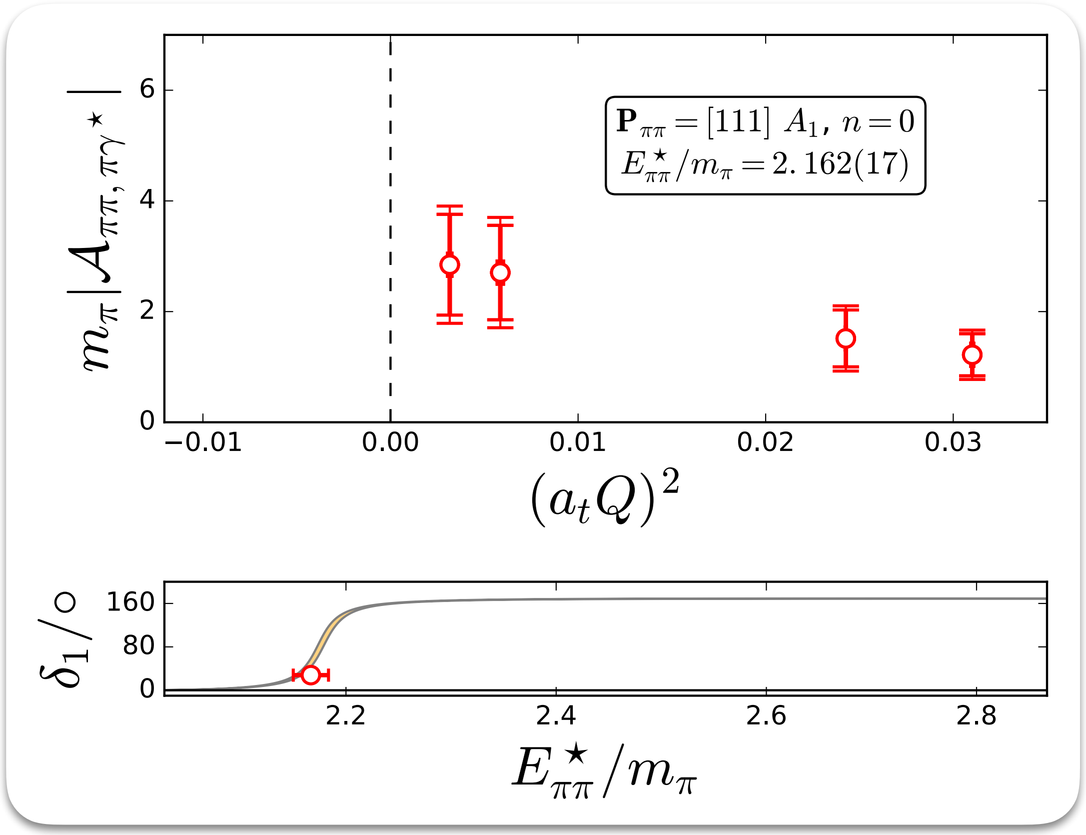
$\pi\gamma^*$-to-$\pi\pi$ amplitude

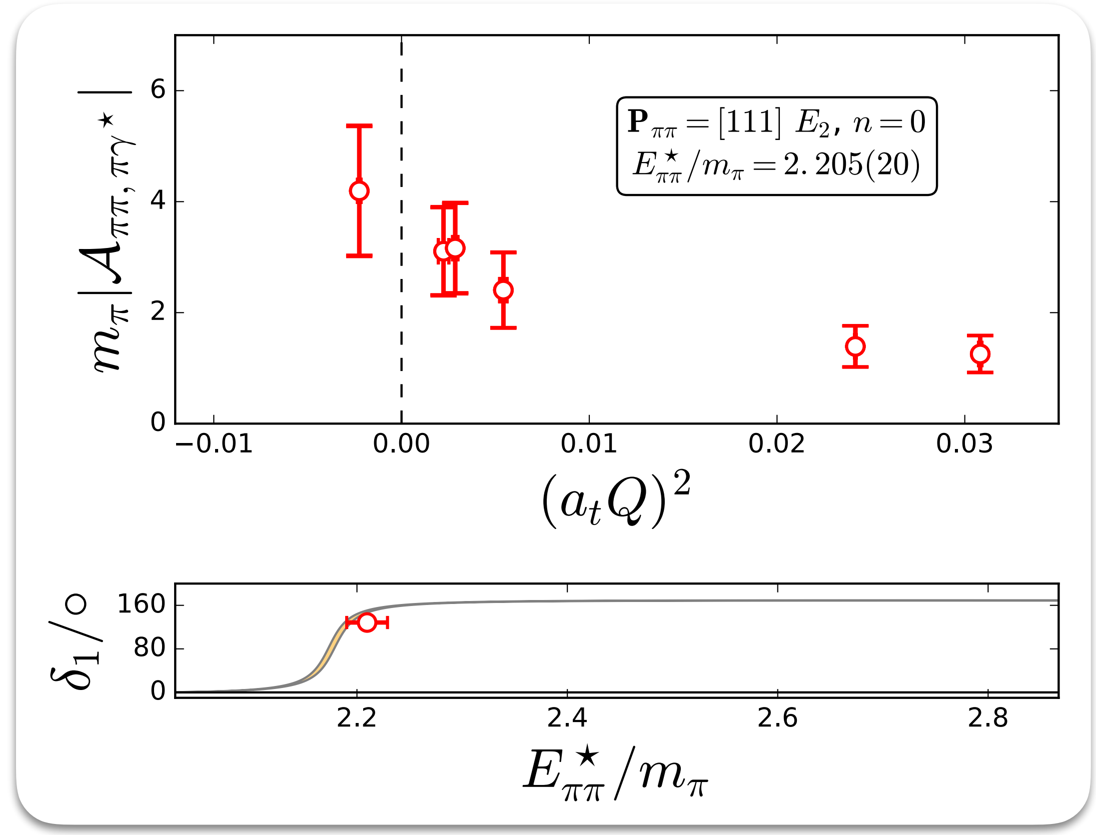


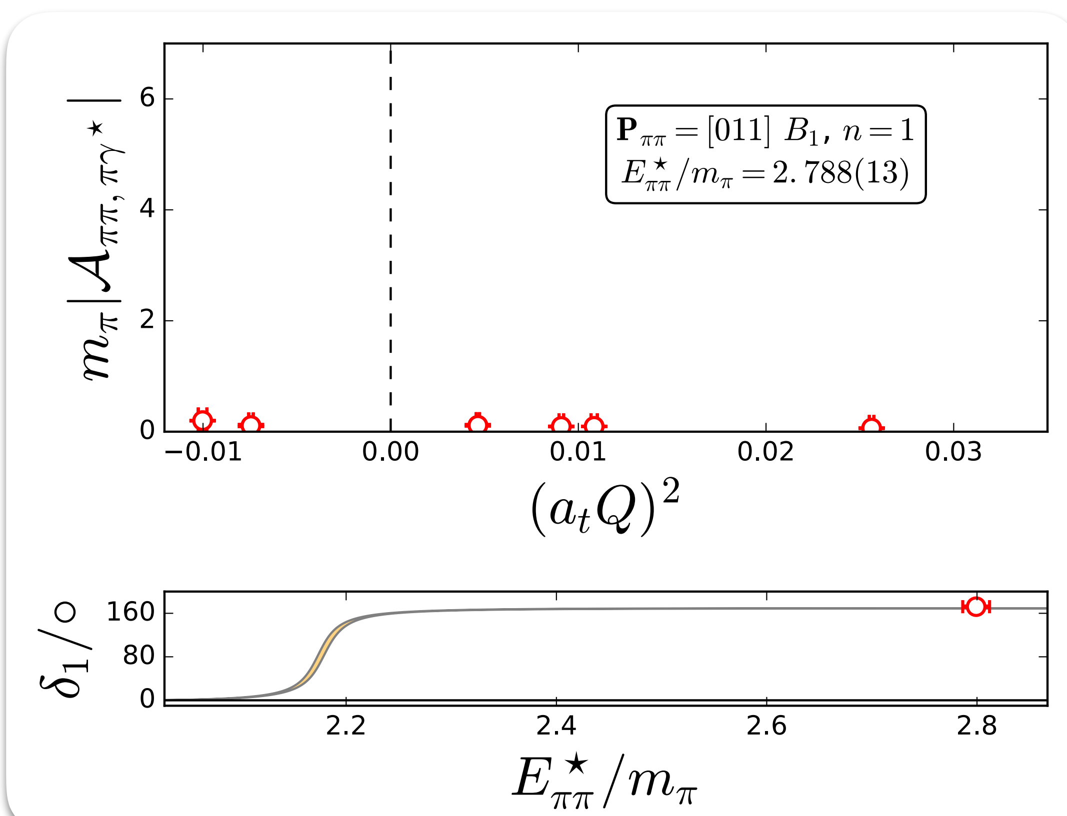
$\pi\gamma^*$-to-$\pi\pi$ amplitude

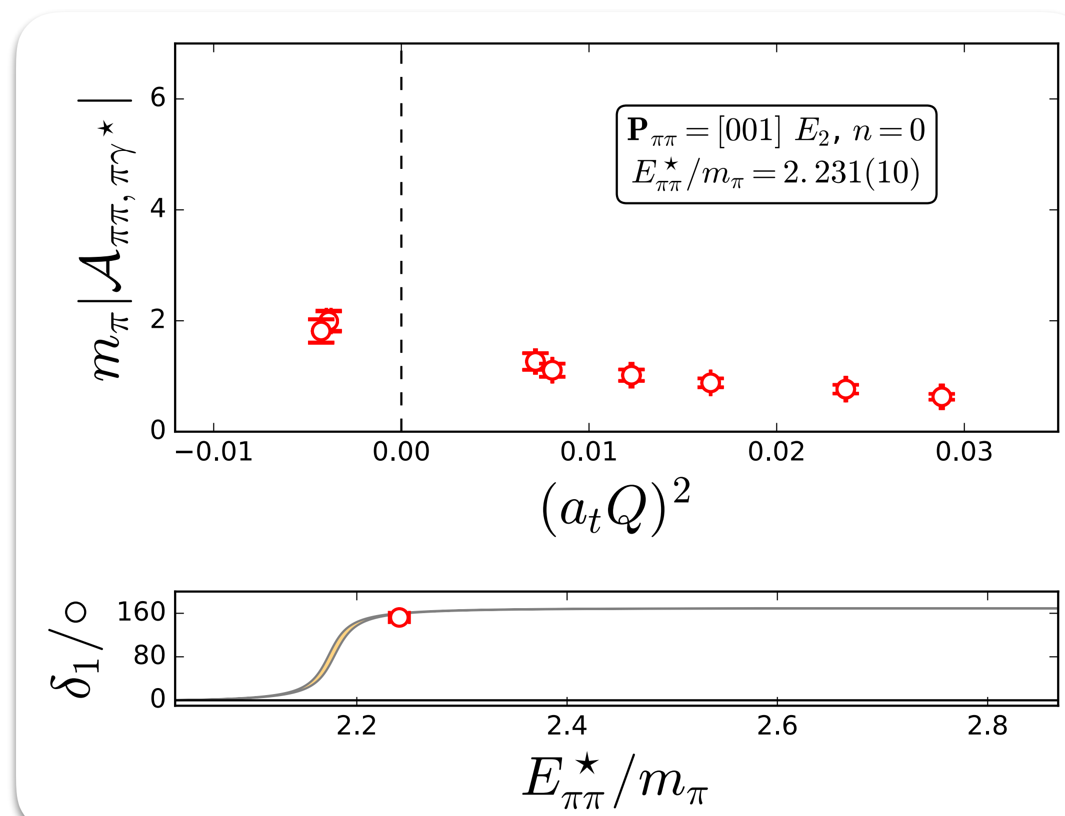


$\pi\gamma^*$-to-$\pi\pi$ amplitude

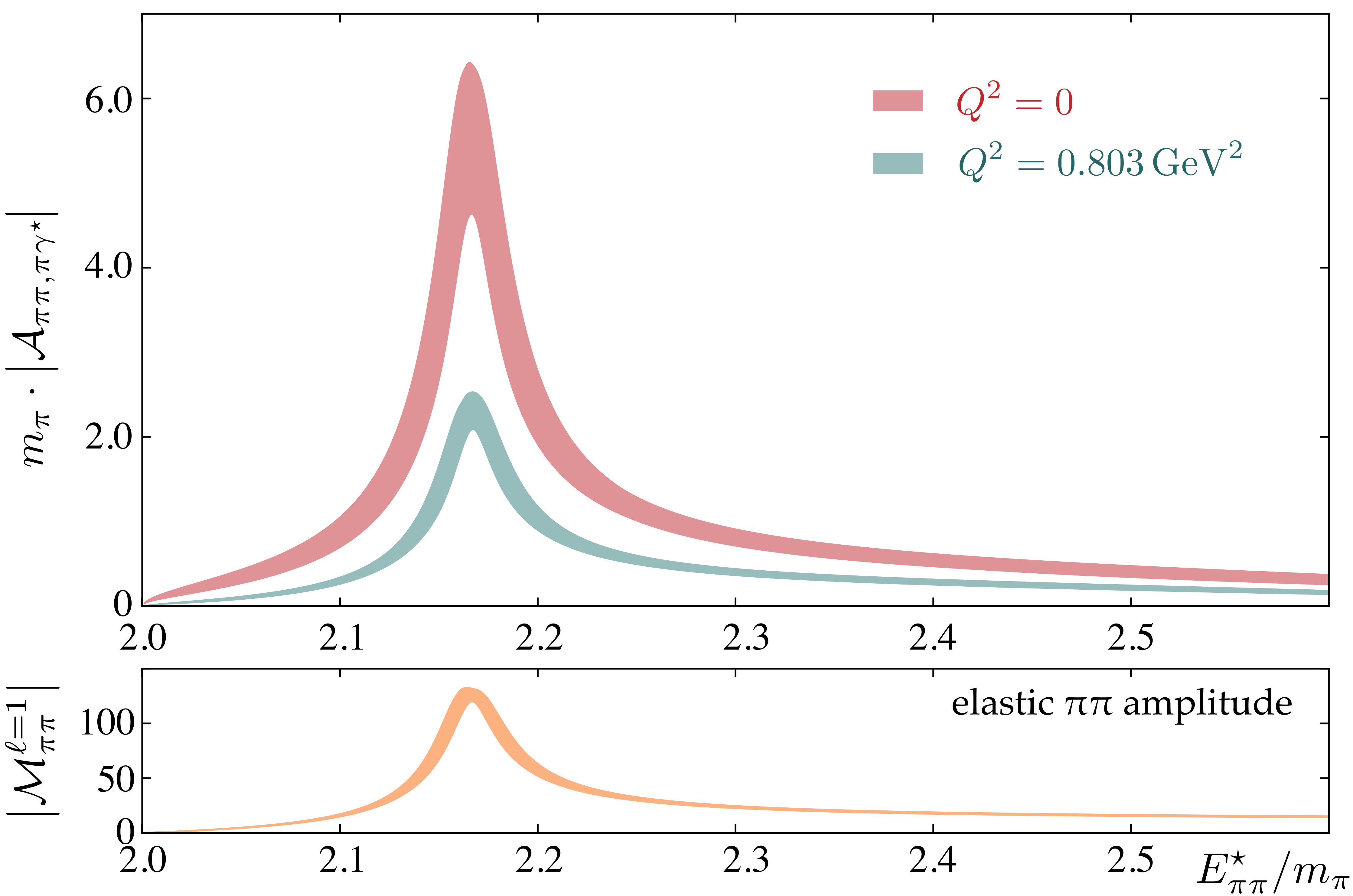




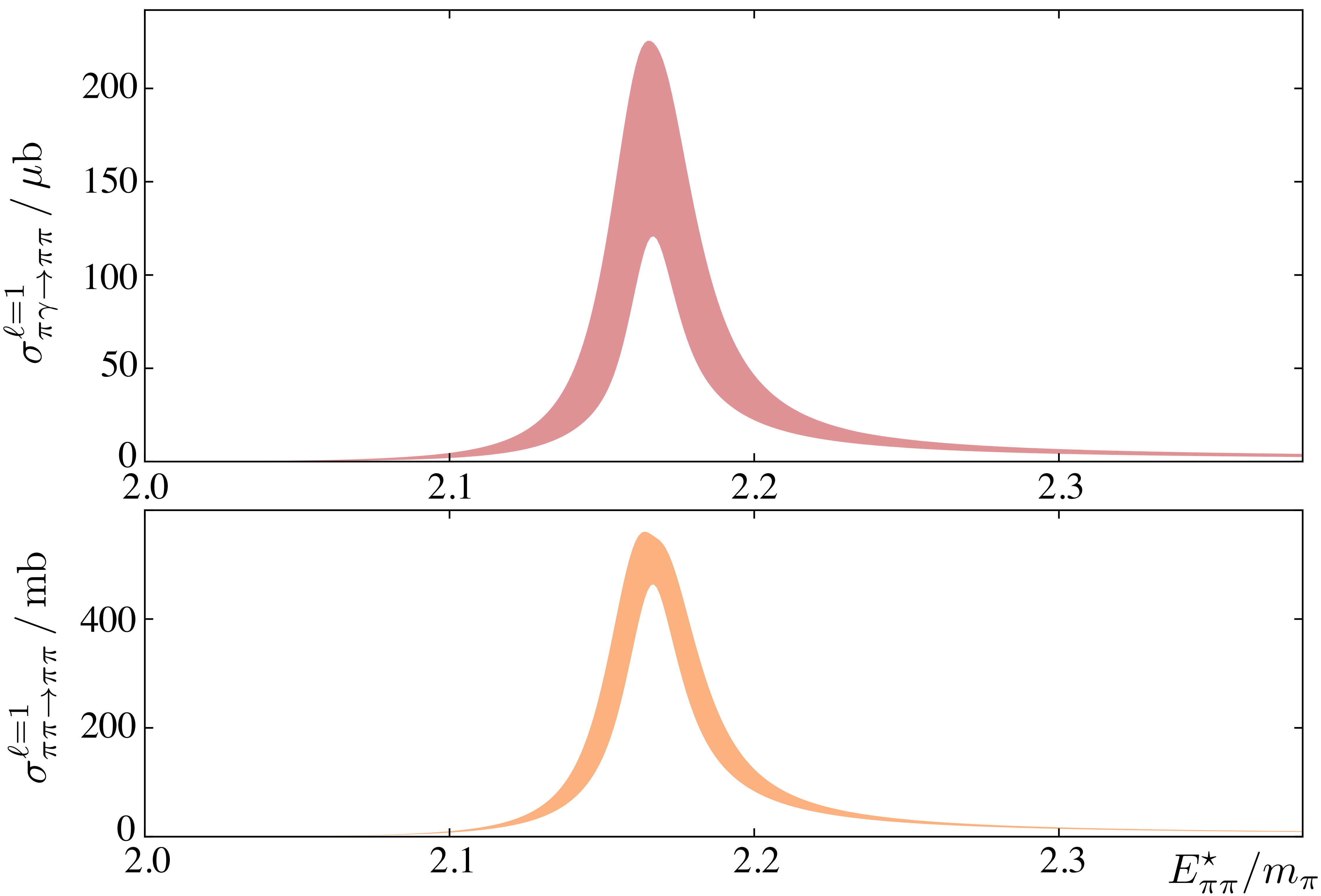


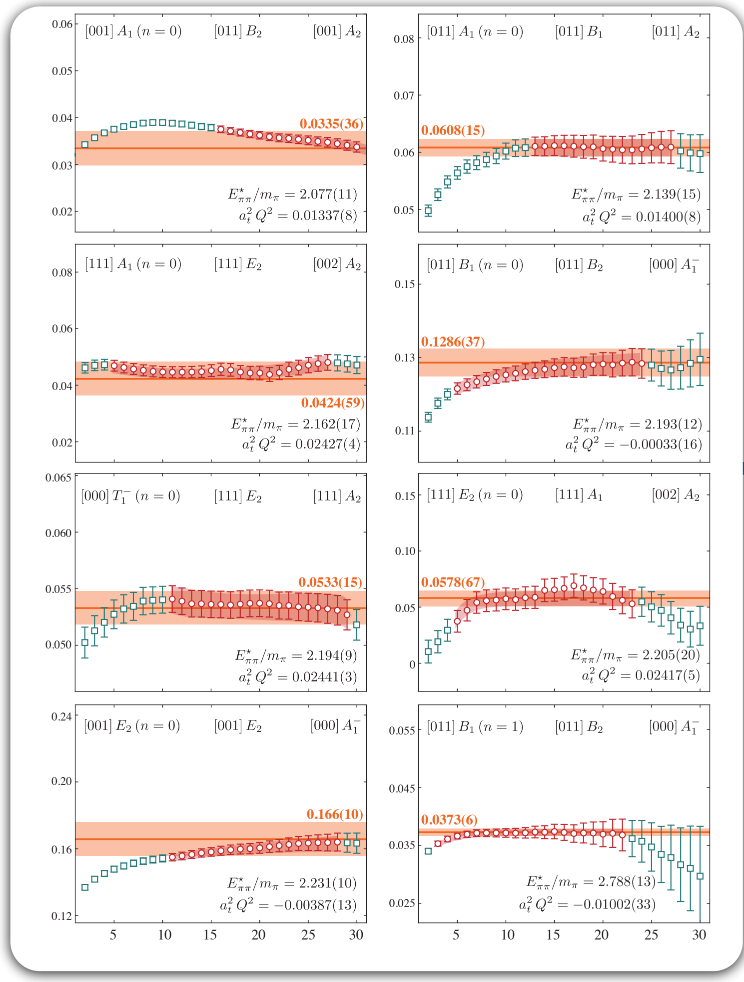


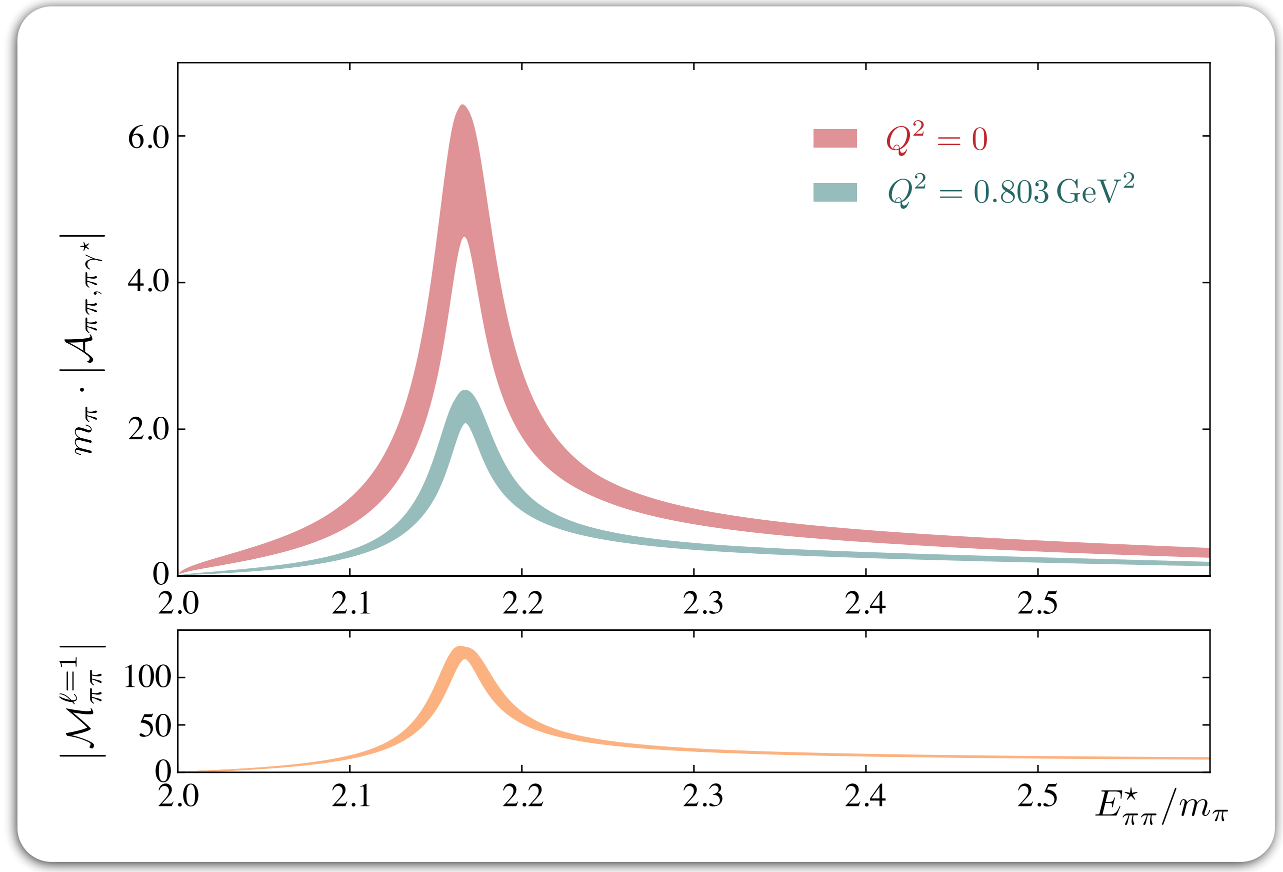
$\pi\gamma^*$-to-$\pi\pi$ amplitude



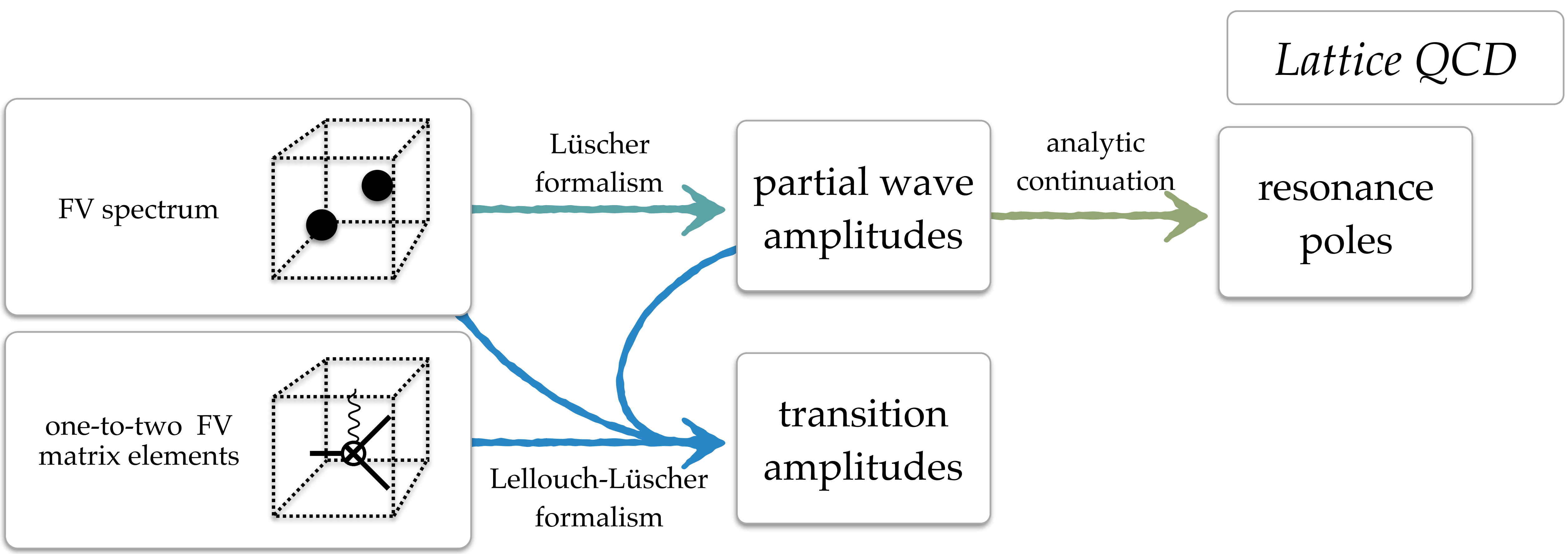
$\pi\gamma^*$-to-$\pi\pi$ cross section



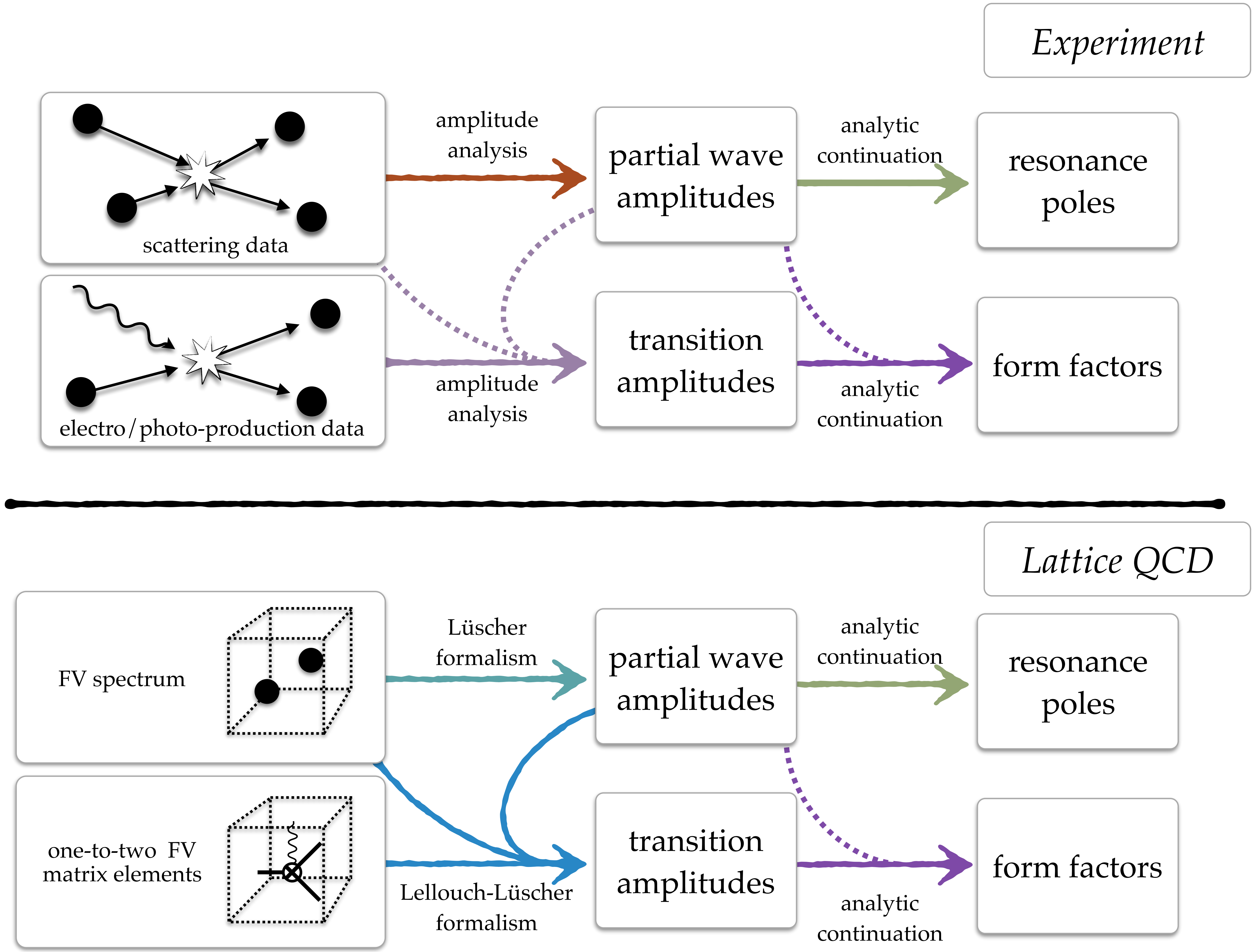




$Q^2 = 0$
$Q^2 = 0.803\,\mathrm{GeV}^2$
$m_\pi \cdot |\mathcal{A}_{\pi\pi,\pi\gamma^\star}|$
$|\mathcal{M}^{\ell=1}_{\pi\pi}|$
$E^\star_{\pi\pi}/m_\pi$



Lattice QCD
FV spectrum
one-to-two FV
matrix elements
Lüscher
formalism
partial wave
amplitudes
analytic
continuation
resonance
poles
transition
amplitudes
Lellouch-Lüscher
formalism

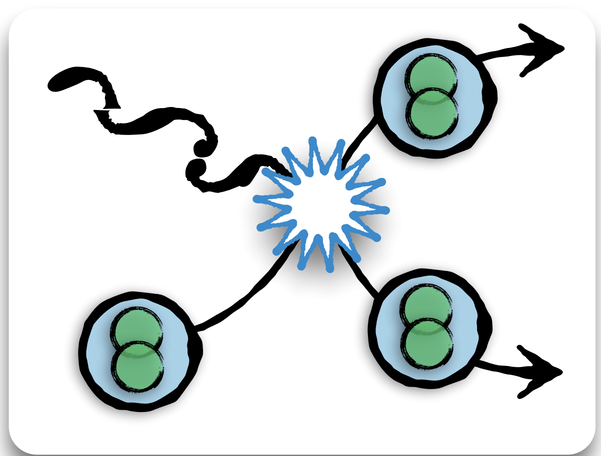


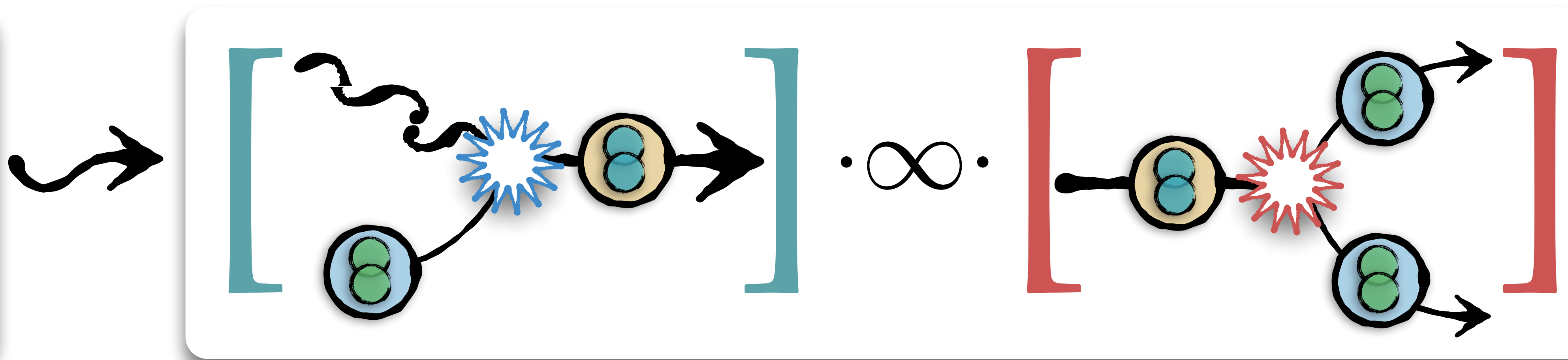
Experiment
scattering data
amplitude analysis
partial wave amplitudes
analytic continuation
resonance poles
electro/photo-production data
amplitude analysis
transition amplitudes
analytic continuation
form factors
Lattice QCD
FV spectrum
Lüscher formalism
partial wave amplitudes
analytic continuation
resonance poles
one-to-two FV matrix elements
Lellouch-Lüscher formalism
transition amplitudes
analytic continuation
form factors

Form factor at ϱ pole

- Near the ϱ-pole, the $\pi\gamma^*$-to-$\pi\pi$ diverges

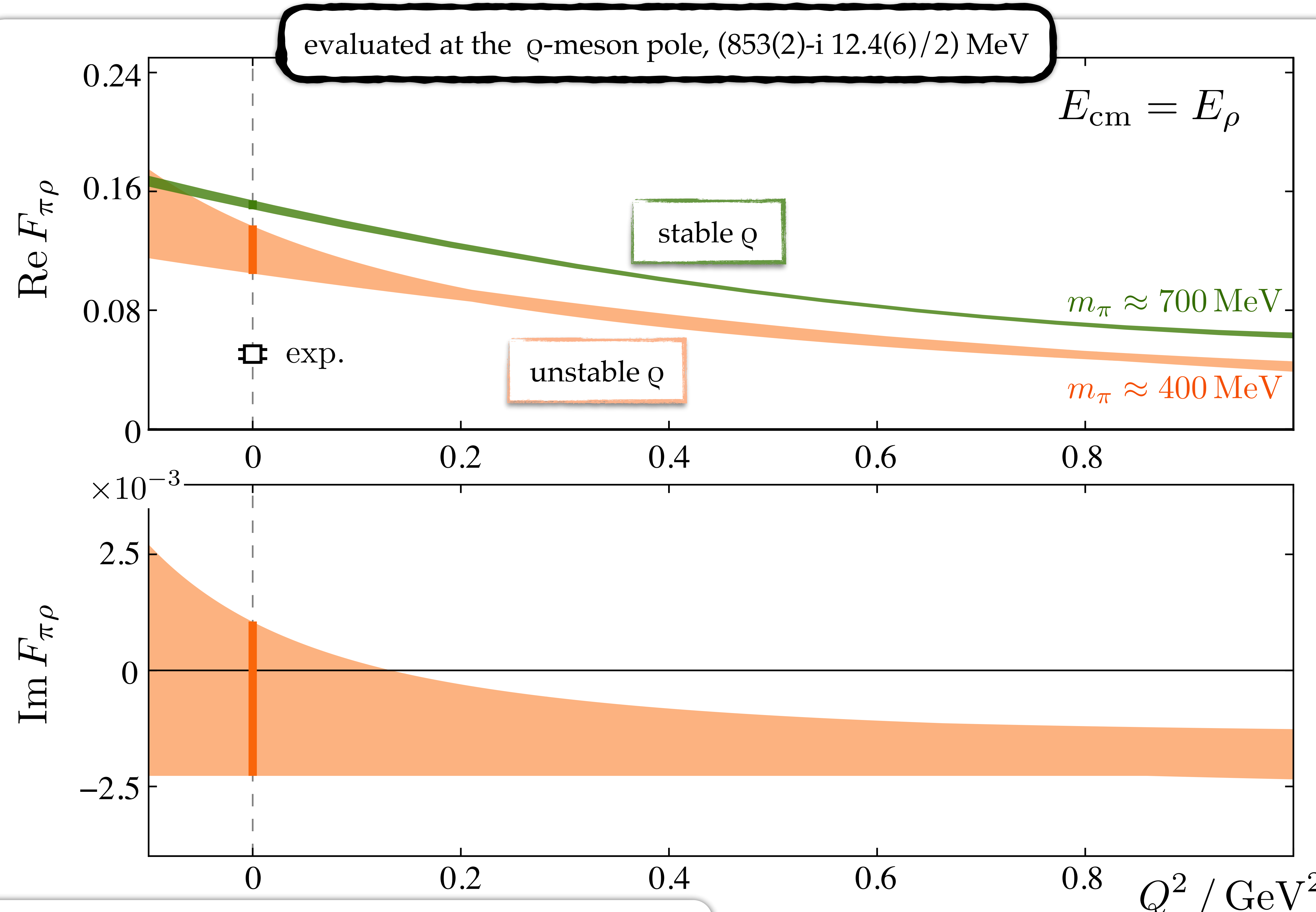
- The residue encodes the $\pi\gamma^*$-to-ϱ form factor





$$\mathcal{A}_{\pi\pi,\pi\gamma^\star}(E_{\pi\pi},Q^2) = \underline{F(E_{\pi\pi},Q^2)} \times \left[\frac{1}{\cot\delta_1(E_{\pi\pi}) - i}\right] \times \sqrt{\frac{16\pi}{\underline{q_{\pi\pi}\,\Gamma(E_{\pi\pi})}}}$$

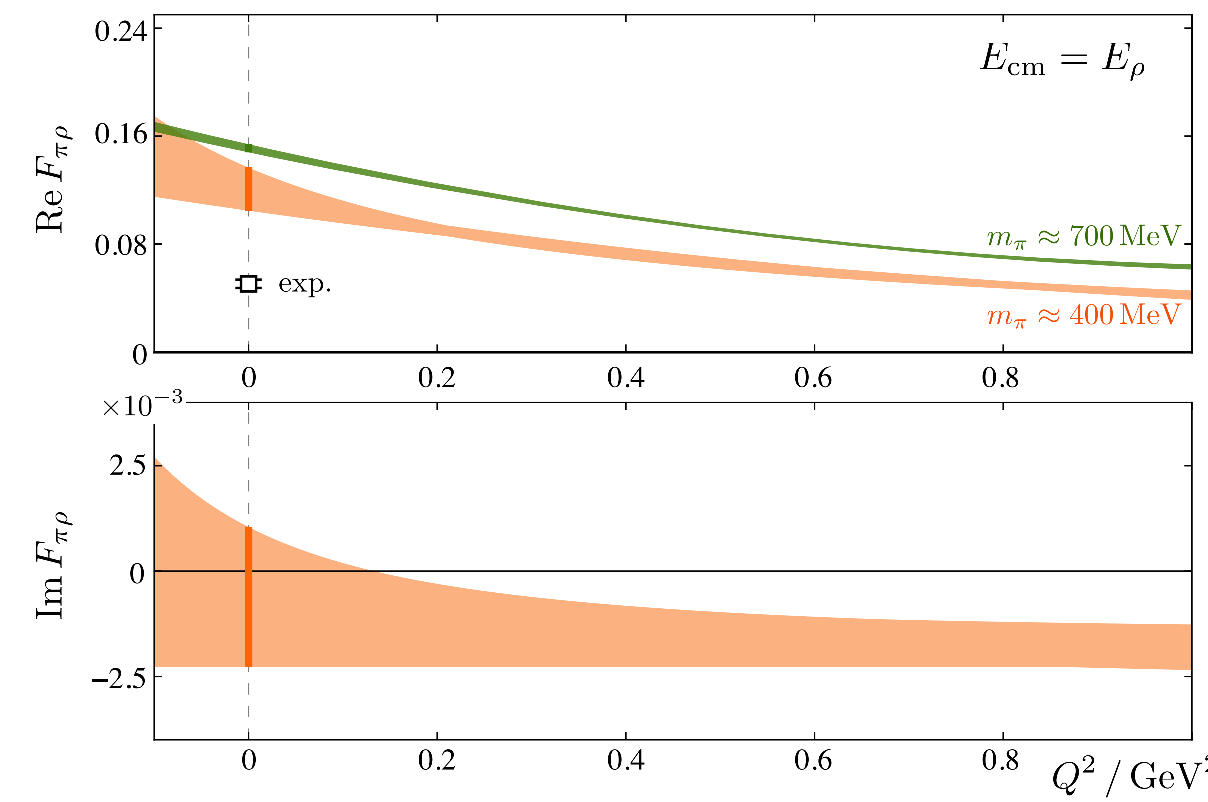
Form factor at ϱ pole



Shultz, Dudek, & Edwards (2014)
RB, Dudek, Edwards, Shultz, Thomas & Wilson (2015)

Some comments

- only lattice calculation of a resonance form factor
- all previous studies ignore resonant nature - it was not known how to
- these include…



PRL **114,** 132002 (2015) PHYSICAL REVIEW LETTERS week ending 3 APRIL 2015

Lattice QCD Evidence that the $\Lambda(1405)$ Resonance is an Antikaon-Nucleon Molecule

Jonathan M. M. Hall,[1] Waseem Kamleh,[1] Derek B. Leinweber,[1,*] Benjamin J. Menadue,[1,2] Benjamin J. Owen,[1] Anthony W. Thomas,[1,3] and Ross D. Young[1,3]

[1]*Special Research Centre for the Subatomic Structure of Matter (CSSM), Department of Physics, University of Adelaide, Adelaide, South Australia 5005, Australia*

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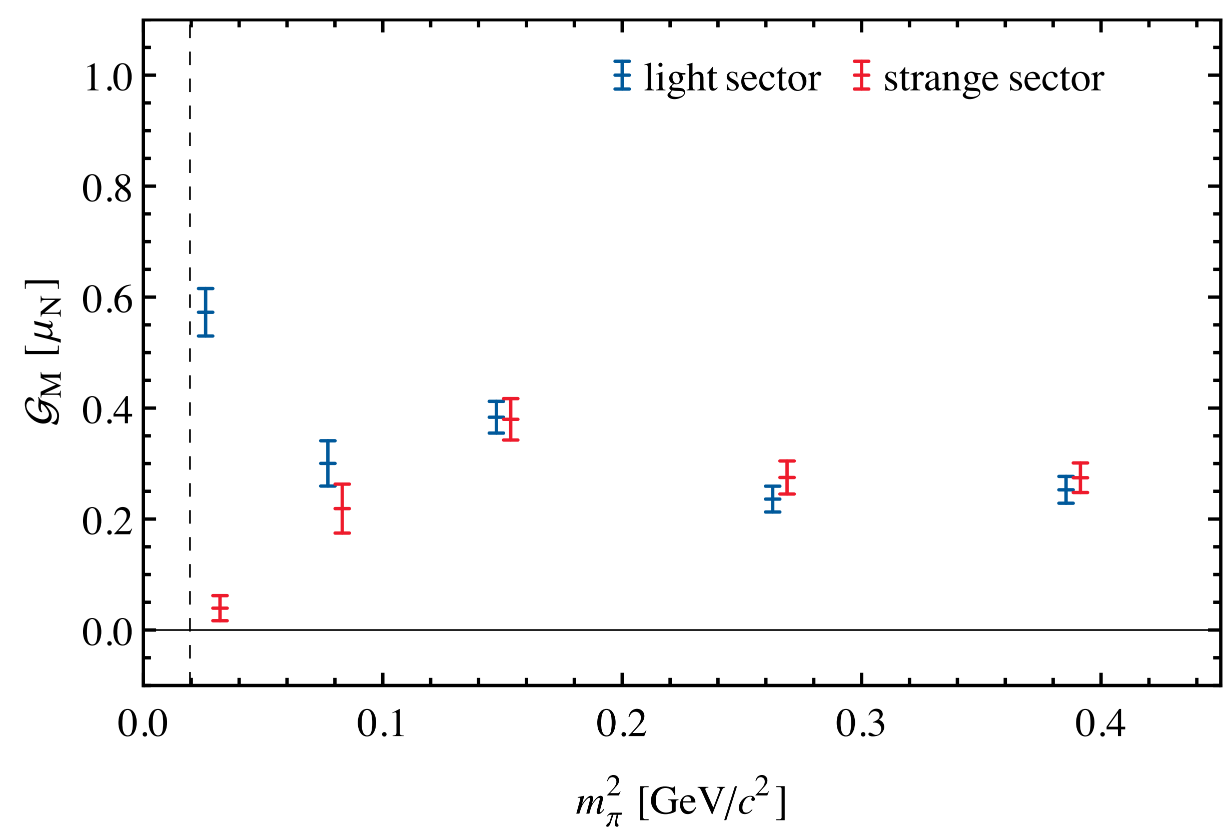
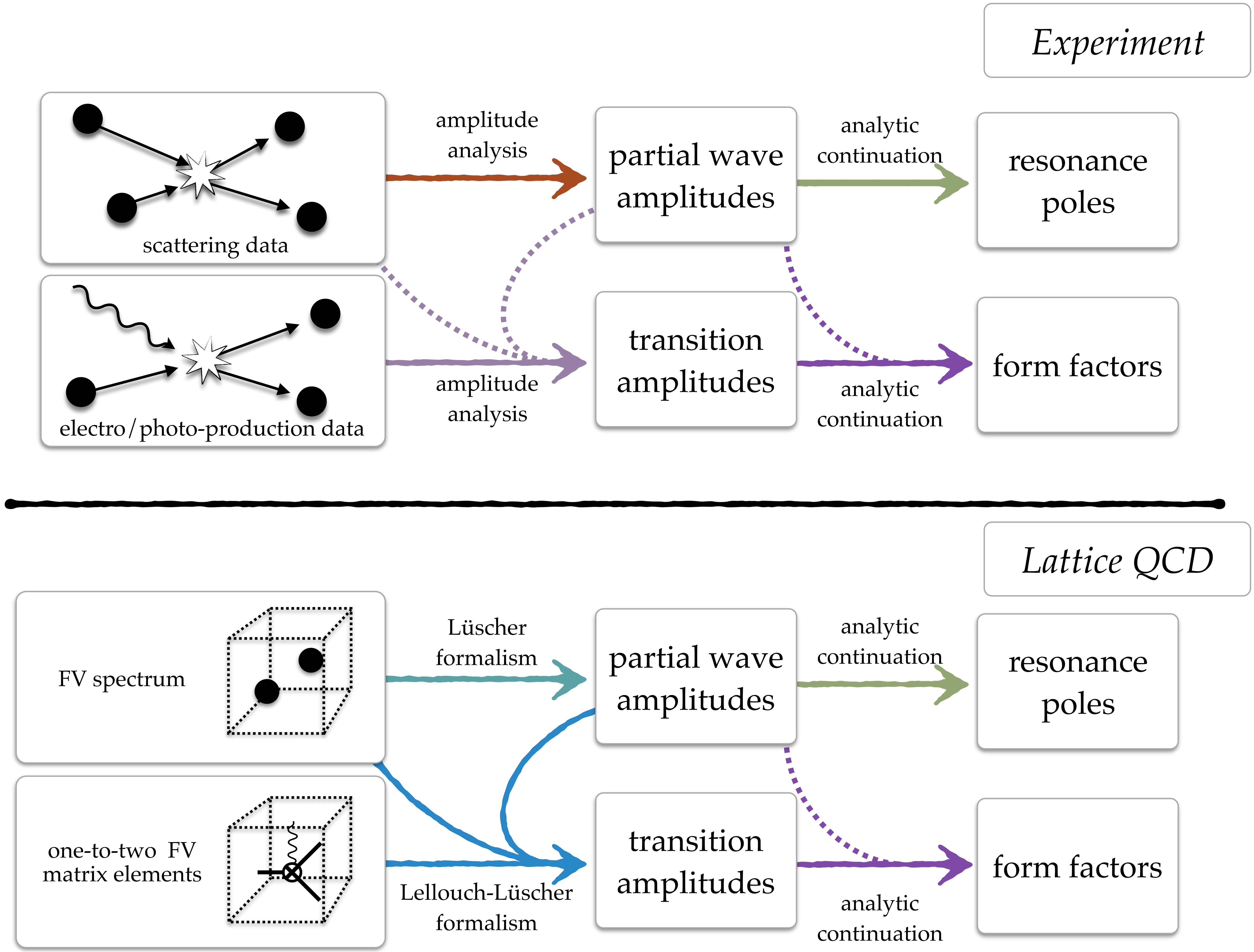
(Received 12 November 2014; revised manuscript received 10 February 2015; published 1 April 2015)

No, there is no evidence *yet* from lattice QCD that this is in fact true!




Experiment
scattering data
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Questions to answer:

- What is possible from lattice QCD regarding electro/photo-production? ☑
- How can lattice and experiment compare to each other? ☑
- What is the possibility/timescale for extending this to baryons? ☐
- What does lattice *"need"* from experiment? ☐

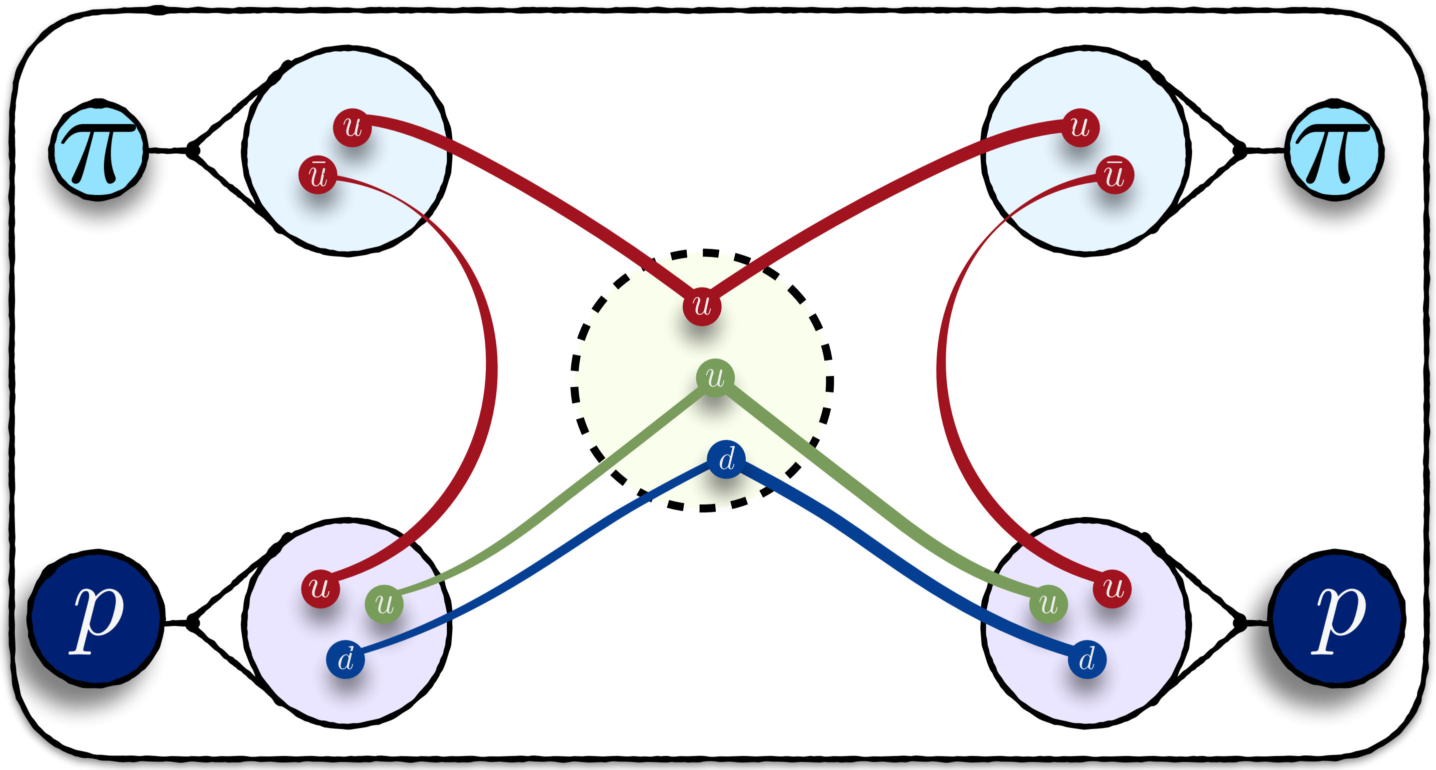
Questions to answer:

- What is possible from lattice QCD regarding electro / photo-production?
- How can lattice and experiment compare to each other?
- What is the possibility / timescale for extending this to baryons?
- Why are baryons harder to study via lattice QCD?
- What are we actively doing towards this goal?

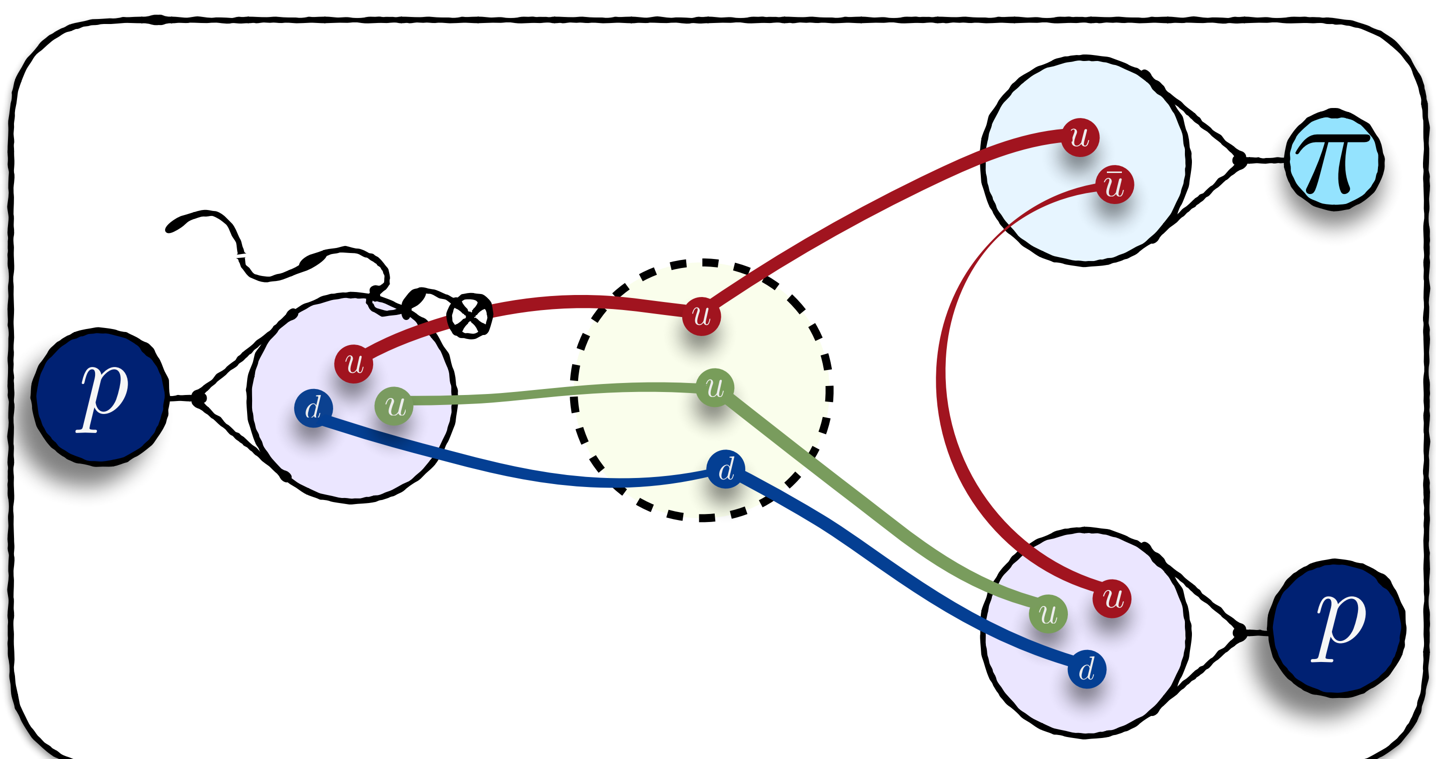
First things first

[no baryonic resonances from QCD, yet!]

First, we must calculate:

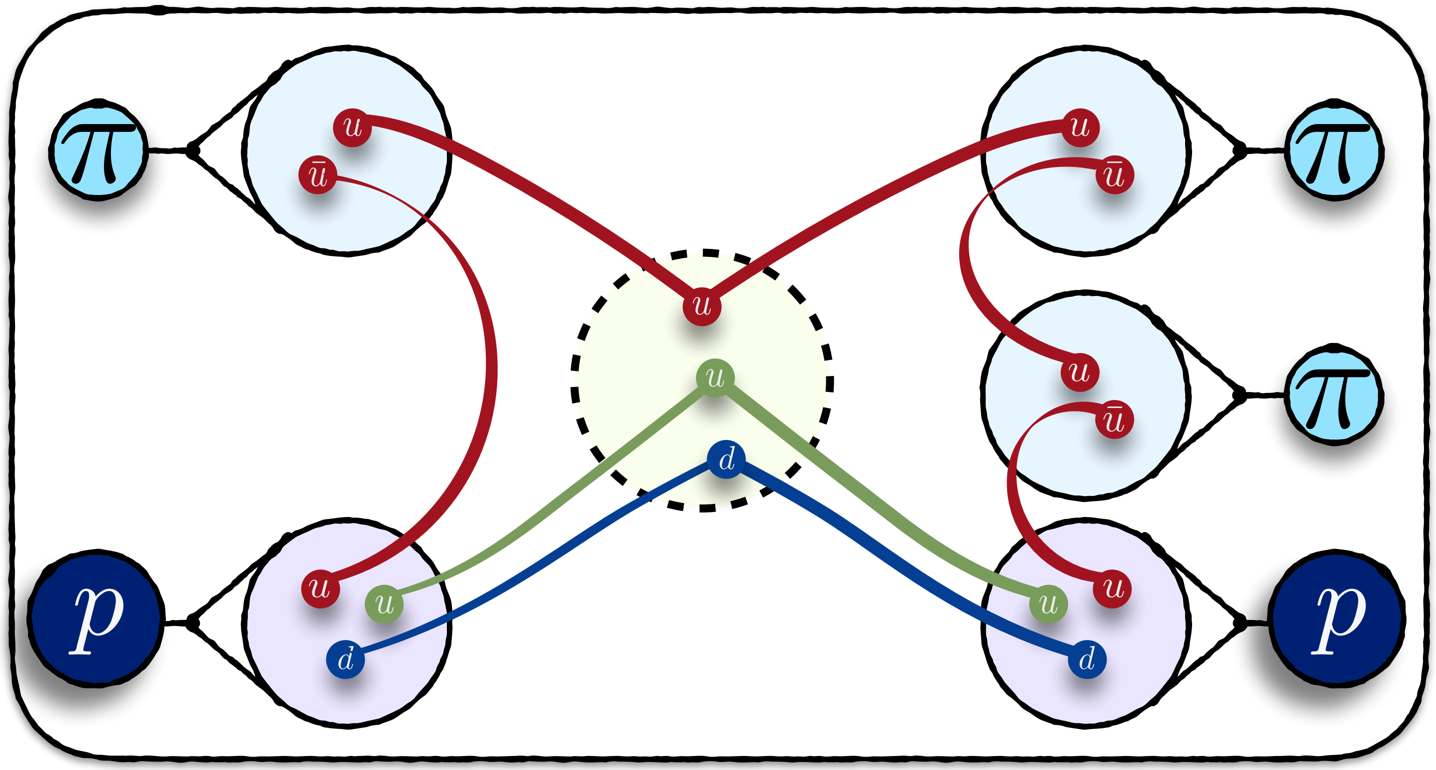


Before we study:



Challenges - three-body

Cannot ignore:



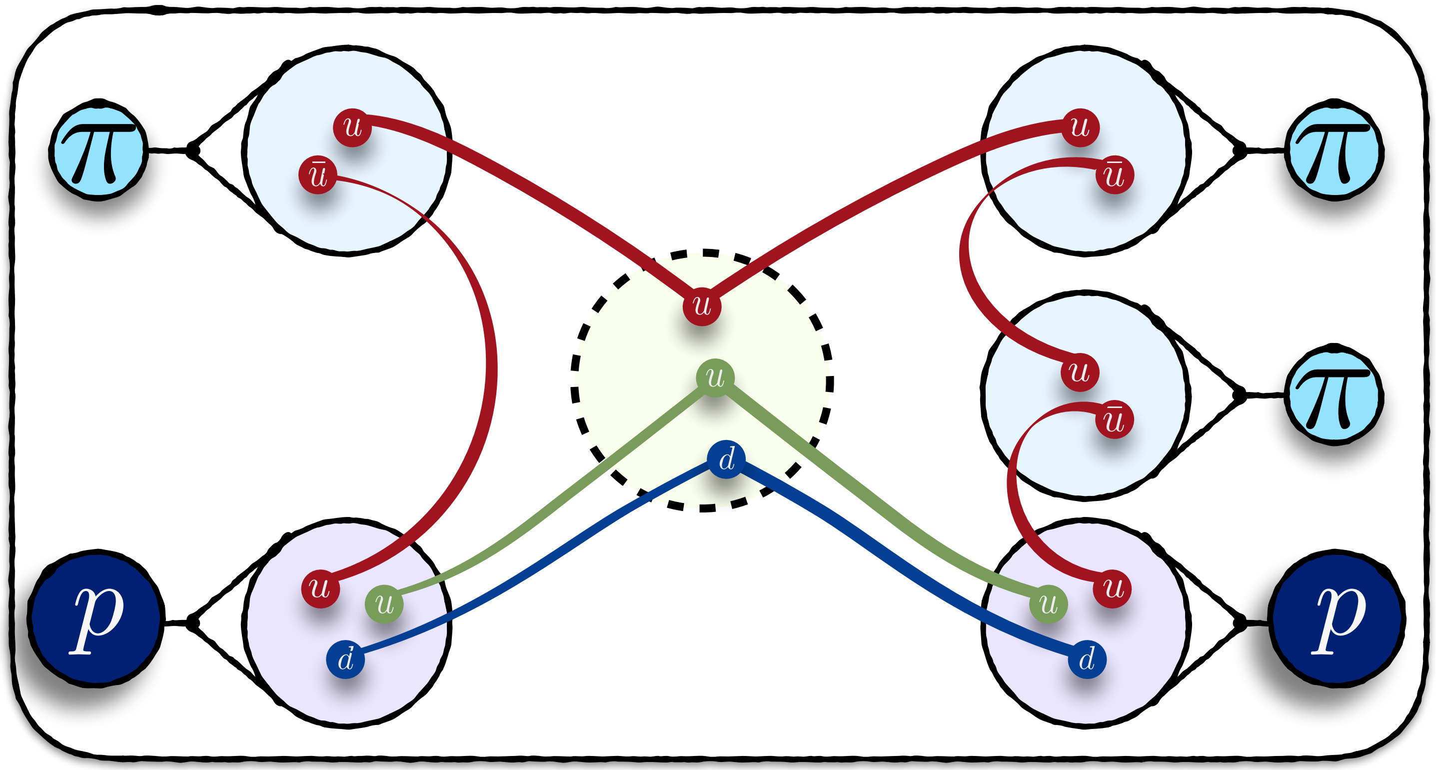
~30-40%

[More contractions, more channels, etc.]

[Formal open question, Harder to analyze]

Challenges - three-body

Cannot ignore:



~30-40%

More contractions, more channels, etc.

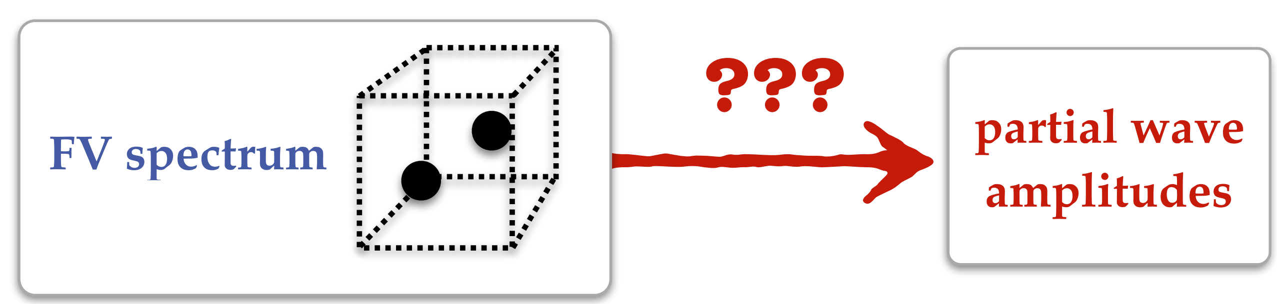
Formal open question, Harder to analyze

Sharpe, Hansen, RB

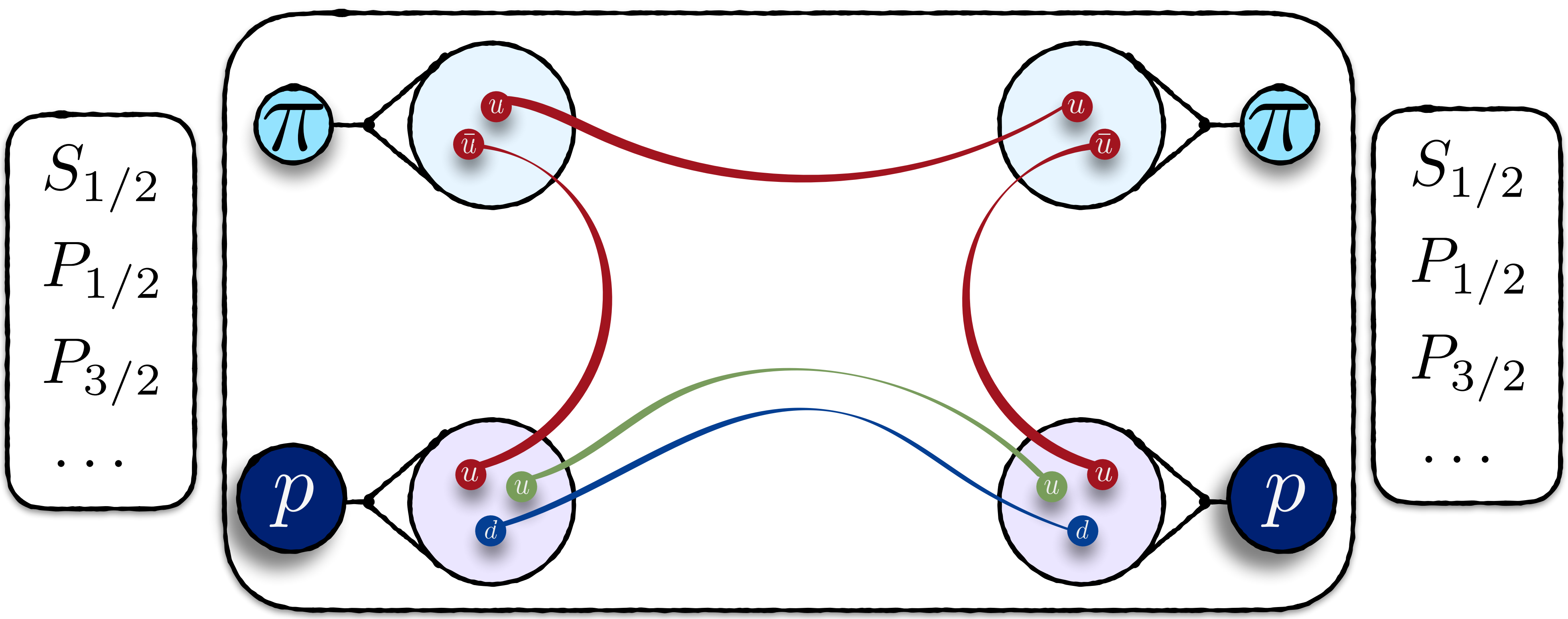
had we had the spectrum, nobody knows what it means!

obtaining FV spectrum is harder, but doable

Robert



Challenges - more partial waves and mixing



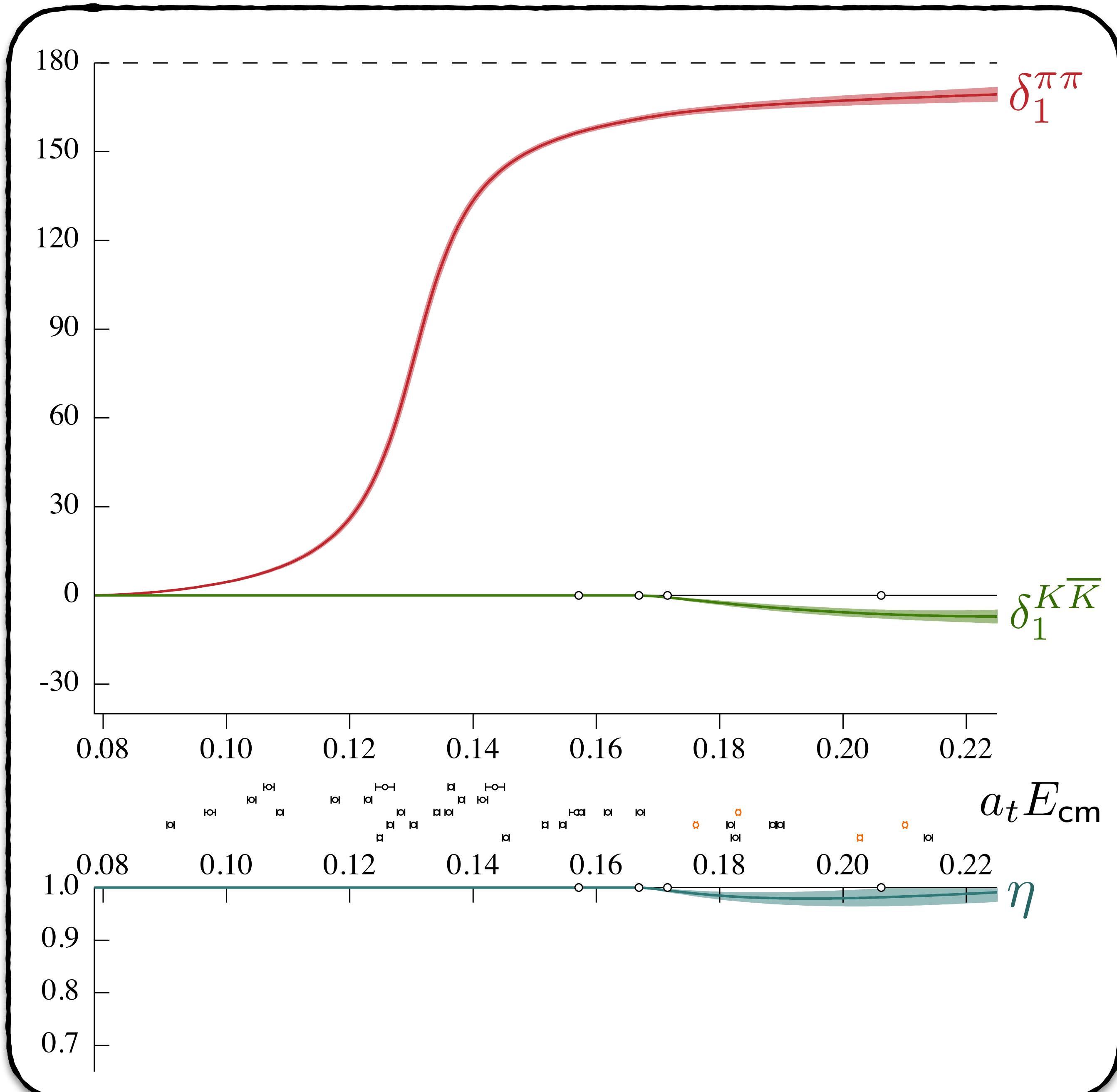
$$\det\left[\begin{pmatrix} F_{S_{1/2},S_{1/2}} & F_{S_{1/2},P_{1/2}} & F_{S_{1/2},P_{3/2}} & \\ F_{P_{1/2},S_{1/2}} & F_{P_{1/2},P_{1/2}} & F_{P_{1/2},P_{3/2}} & \\ F_{P_{3/2},S_{1/2}} & F_{P_{3/2},P_{1/2}} & F_{P_{3/2},P_{3/2}} & \\ & & & \ddots \end{pmatrix}^{-1} + \begin{pmatrix} \mathcal{M}_{S_{1/2}} & & & \\ & \mathcal{M}_{P_{1/2}} & & \\ & & \mathcal{M}_{P_{3/2}} & \\ & & & \ddots \end{pmatrix}\right] = 0$$

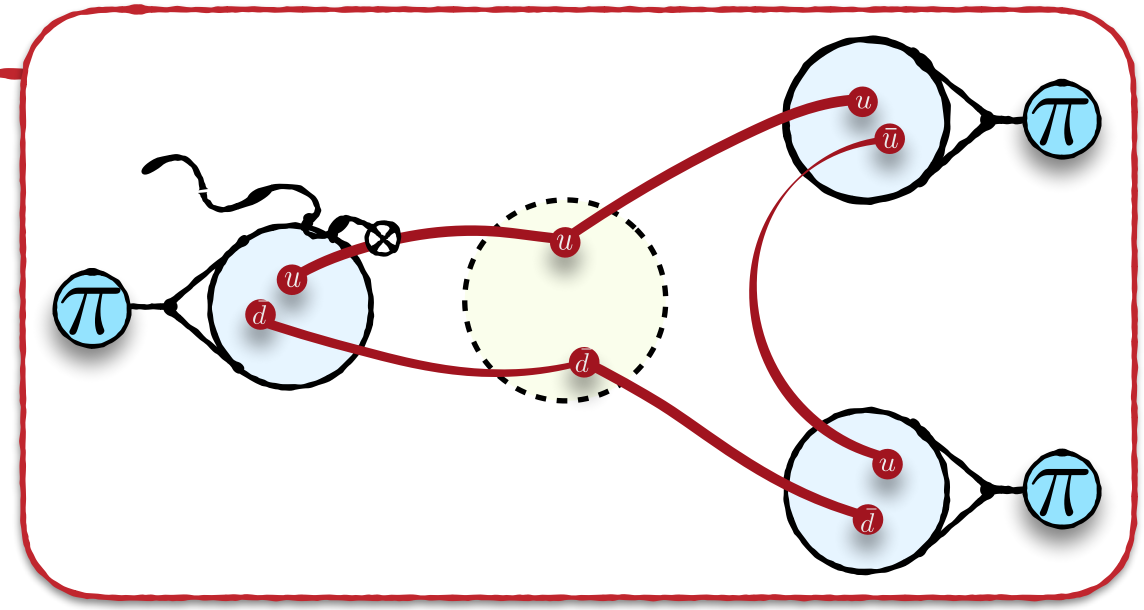
formally addressed - RB (2014)

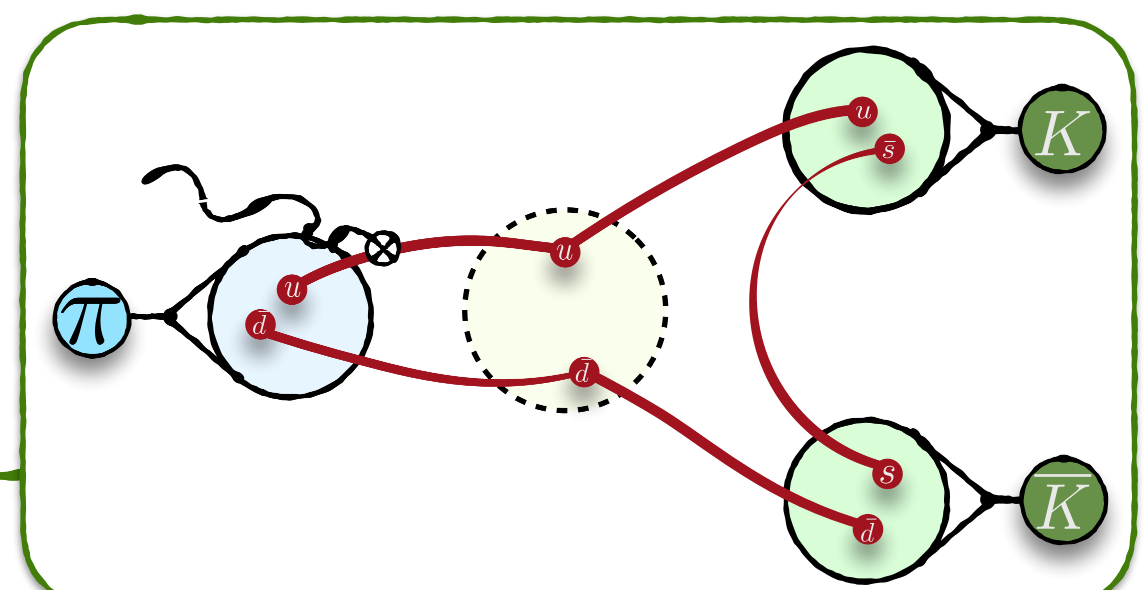
currently being implemented - Jo & Dave Wilson

Expectations for the future

More mesons: reproducing calculation using m_π~236 MeV, including coupled channels



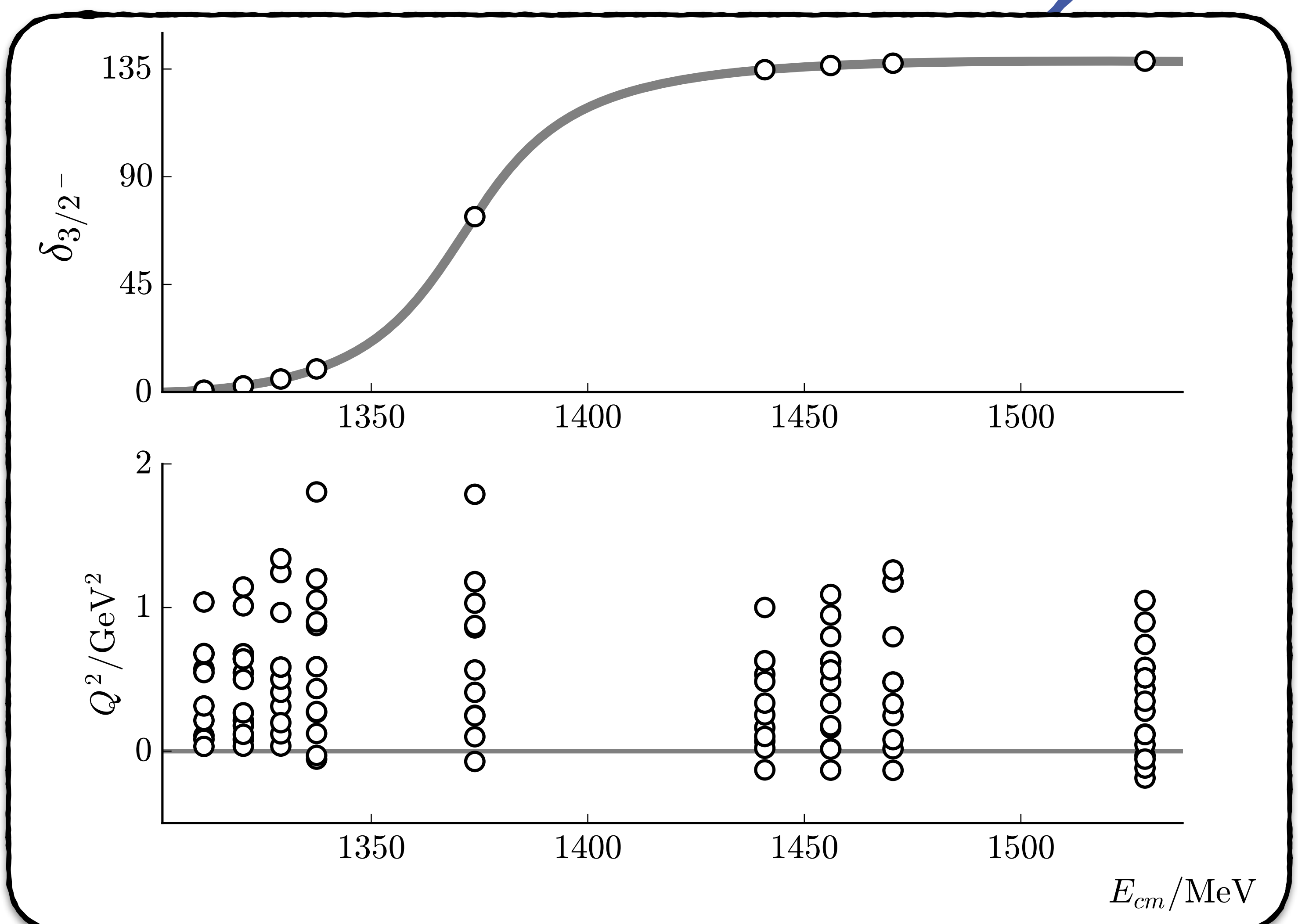


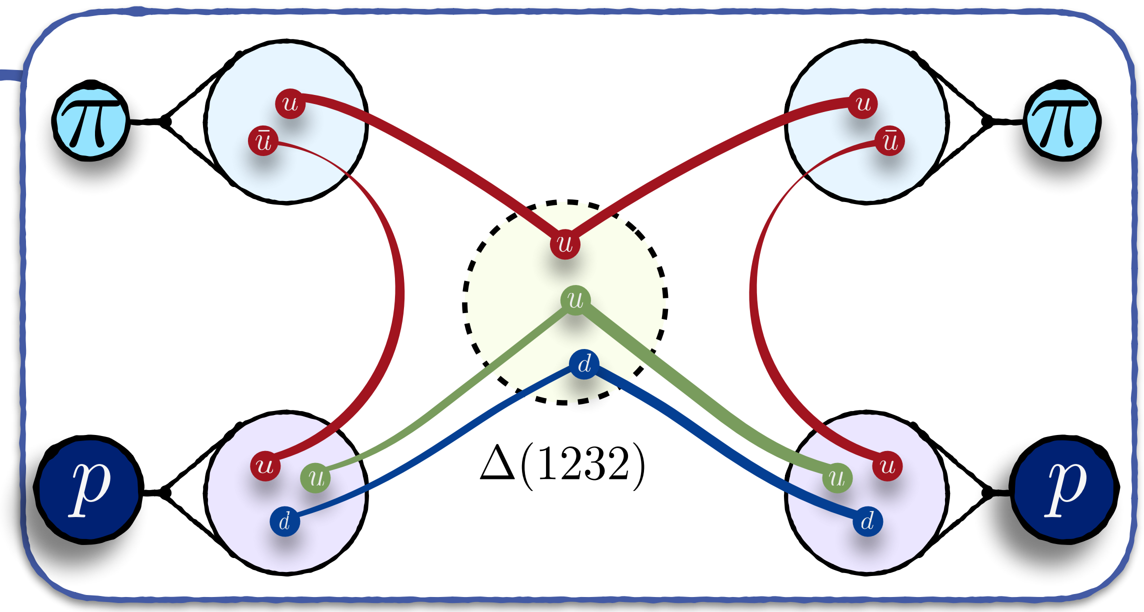


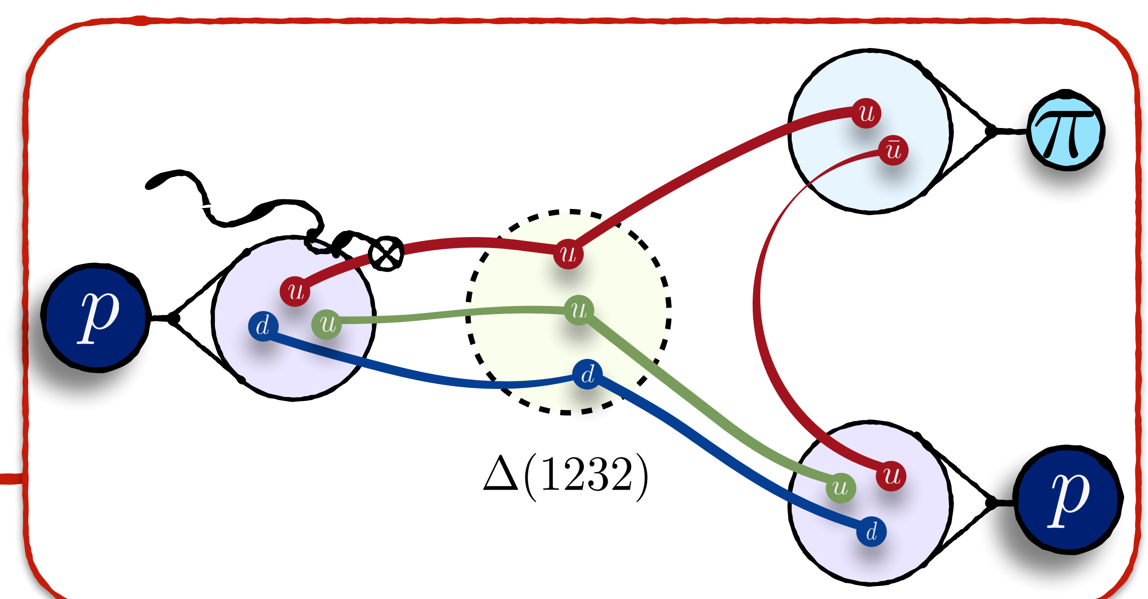
Expectations for the future

Kinematic expectations for $m_\pi \sim 236$ MeV, single volume $L/a_s=32$

Second volume, $L/a_s=40$, is underway







three amplitude/form factors:

- G_{M1} - magnetic dipole
- G_{E2} - electric quadrupole
- G_C - Coulomb quadrupole

Questions to answer:

- What is possible from lattice QCD regarding electro/photo-production?
- How can lattice and experiment compare to each other?
- What is the possibility/timescale for extending this to baryons?
- What does lattice *"need"* from experiment?

What does lattice *"need"* from experiment?

- Absolutely nothing:
  - Lattice QCD is fully predictive
  - No inputs or approximations needed - [except quark masses]

What does lattice *"need"* from experiment?

- Absolutely nothing:
    - Lattice QCD is fully predictive
    - No inputs or approximations needed - [except quark masses]
- Support:
    - Shortage of **people power**!
        - Need more people:
            - developing: formalism, code, amplitude analysis, etc.
                - join effort with JPAC for amplitude analysis…
            - performing calculations, analysis, etc.
            - optimizing code
    - Will need more computer capabilities

Questions to answer:

- What is possible from lattice QCD regarding electro/photo-production?
- How can lattice and experiment compare to each other?
- What is the possibility/timescale for extending this to baryons?
- What does lattice *"need"* from experiment?

Collective achievement / on-going efforts

paving the way

Formal achievements:

Coupled-channels	2012, 2014	
Meson electro/photo-production	2014, 2015	
Elastic resonant form-factors	2015	
Three-body - Sharpe, Hansen, RB	under way	

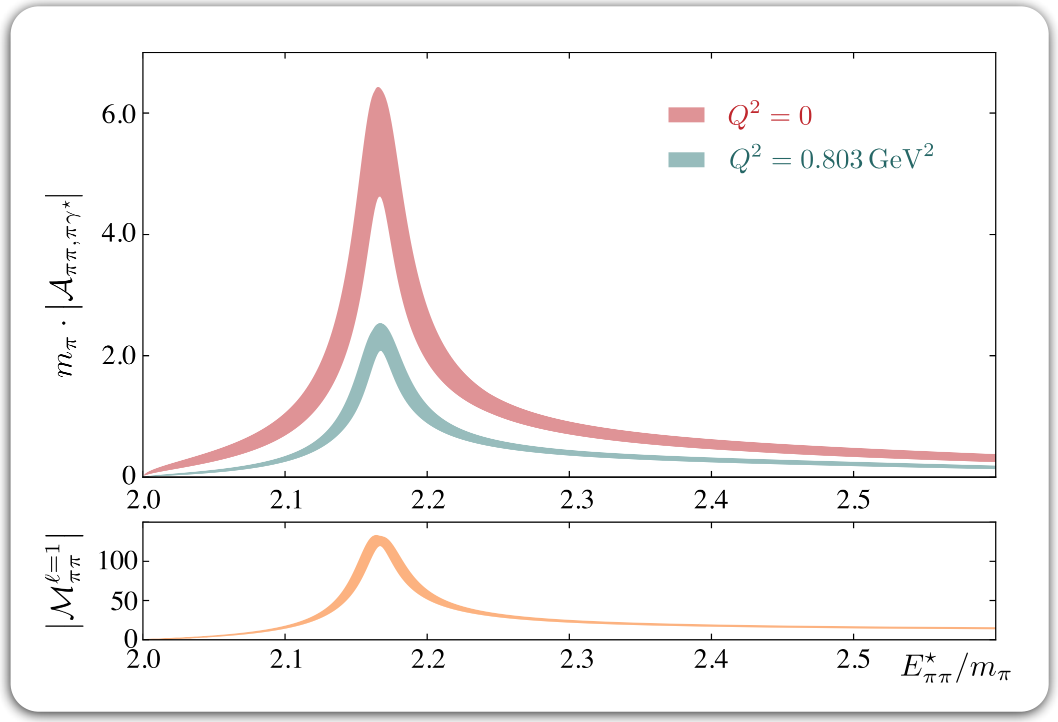
Numerical achievements:

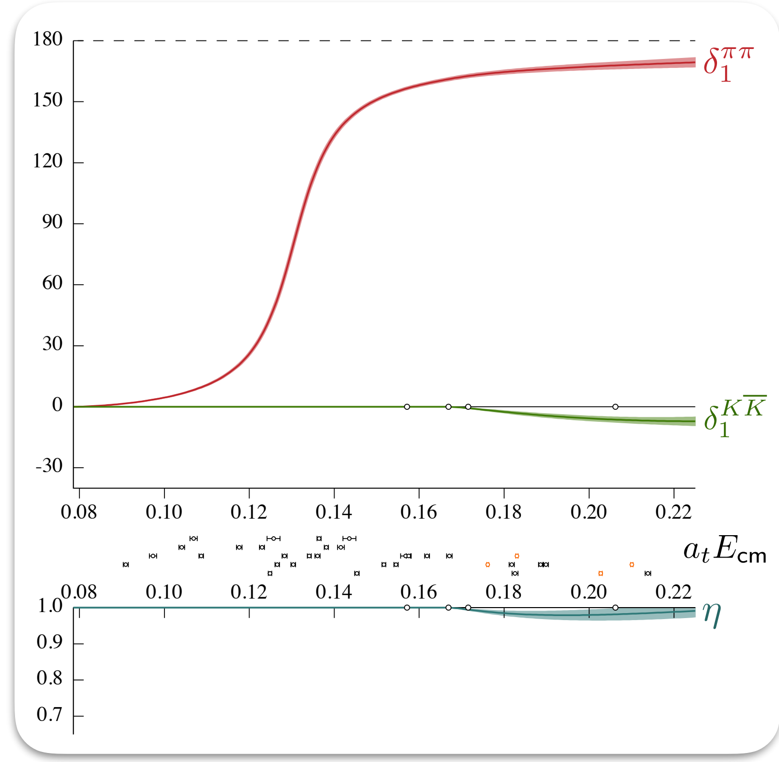
First weakly coupled, two-channels	2014	~~RB~~
First resonant form-factor	2015	
First chiral extrapolation of resonant amplitude	2015	
First strongly coupled, two/three-channels	2016	~~RB~~
TOP SECRET	under way	

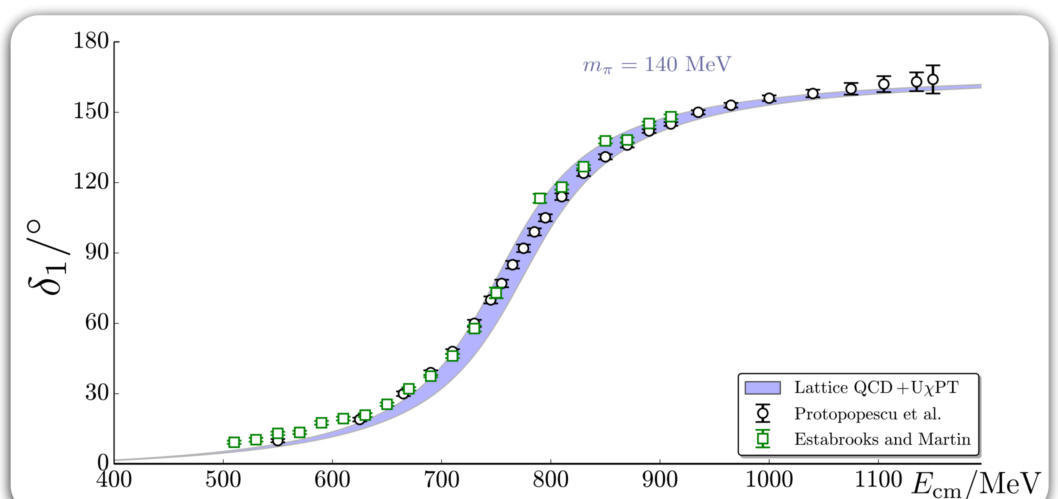
Code development - Robert

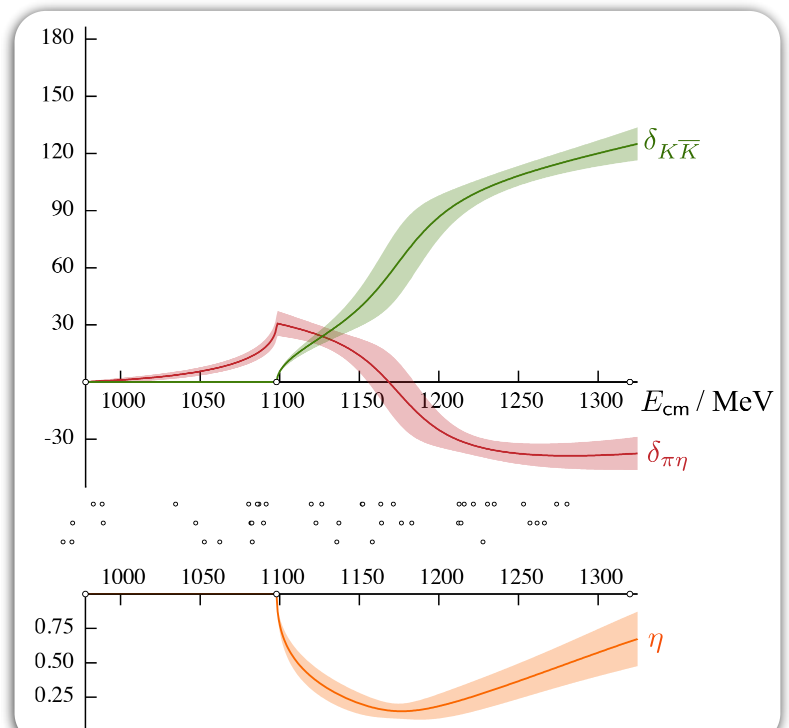
Analysis development [e.g., spin particles]- Jo & Dave Wilson

The big picture!









each one of these was 'impossible' 3 years ago…

Collaborators

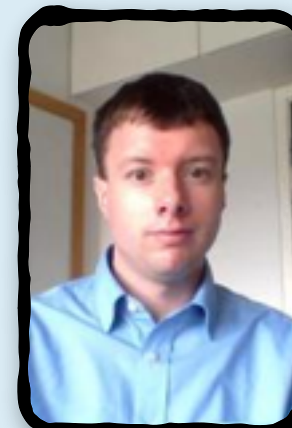
formalism

Hansen

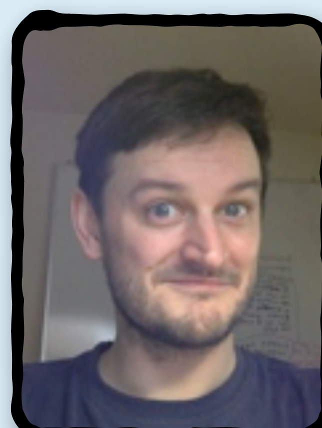
Sharpe

numerics

Wilson



Thomas



Dudek

Edwards

software

Joo

Winter

HadSpec Collaboration

Back-up slides

$\pi\gamma^*$-to-$\pi\pi$

(more details)

Lorentz decomposition:

$$\mathcal{H}^{\mu}_{\pi\pi,\pi\gamma^\star} = \epsilon^{\mu\nu\alpha\beta}\, P_{\pi,\nu}\, P_{\pi\pi,\alpha}\, \epsilon_\beta(\lambda_{\pi\pi}, \mathbf{P}_{\pi\pi})\, \frac{2}{m_\pi} \mathcal{A}_{\pi\pi,\pi\gamma^\star}$$

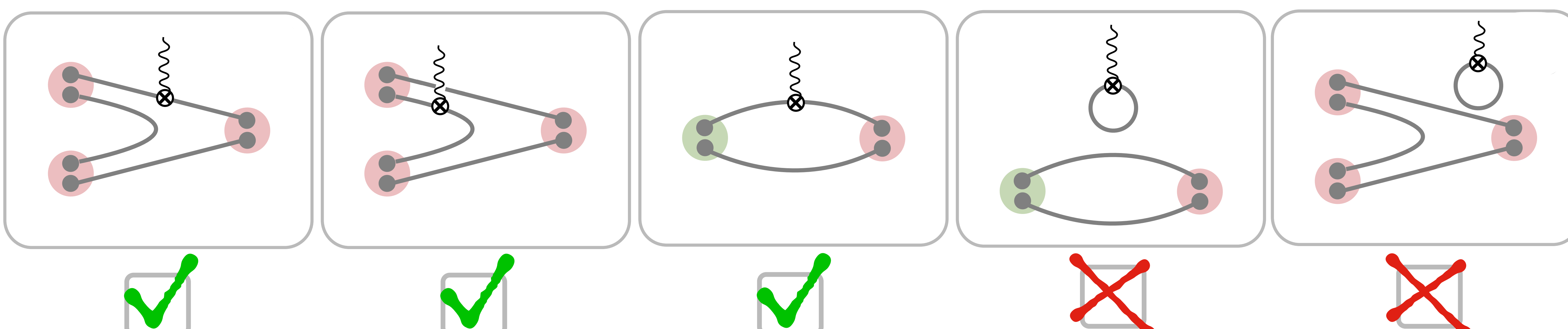
$\pi\pi/\rho$ polarization *$\pi\pi/\rho$ helicity* *Lorentz scalar*

Approximations:

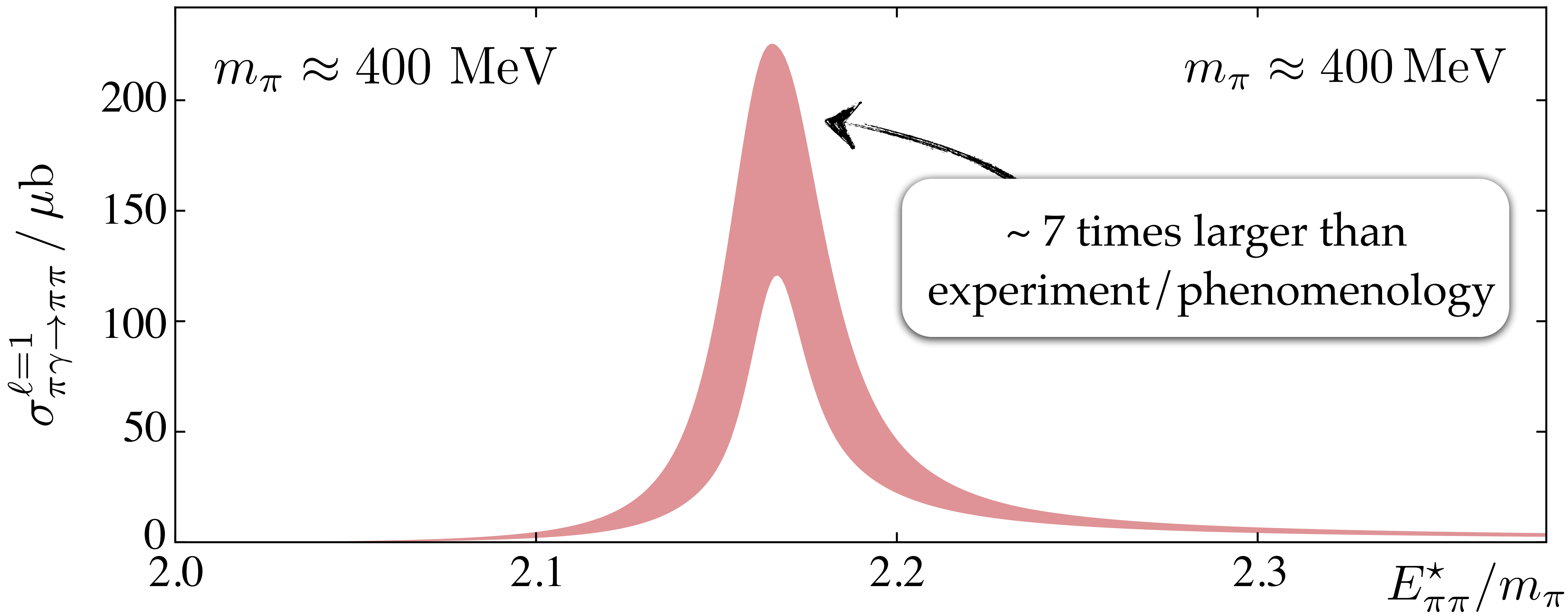
F-wave $\pi\gamma^*$-to-$\pi\pi$ is ignored

kinematically and dynamically suppressed

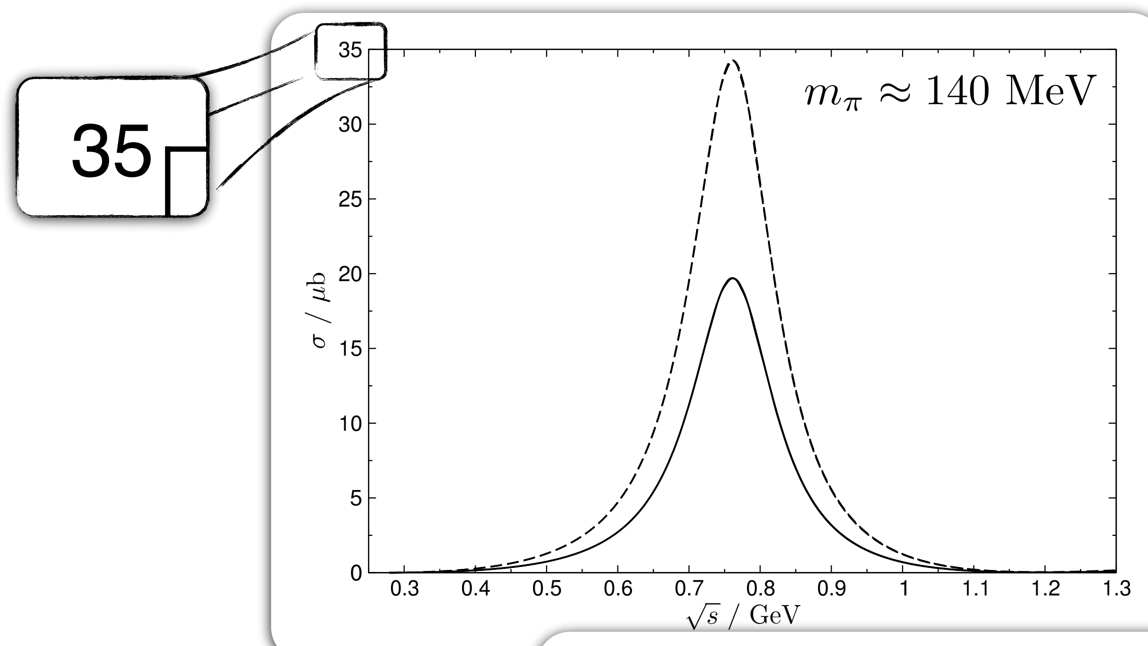
contractions:



$\pi\gamma$-to-$\pi\pi$ cross section



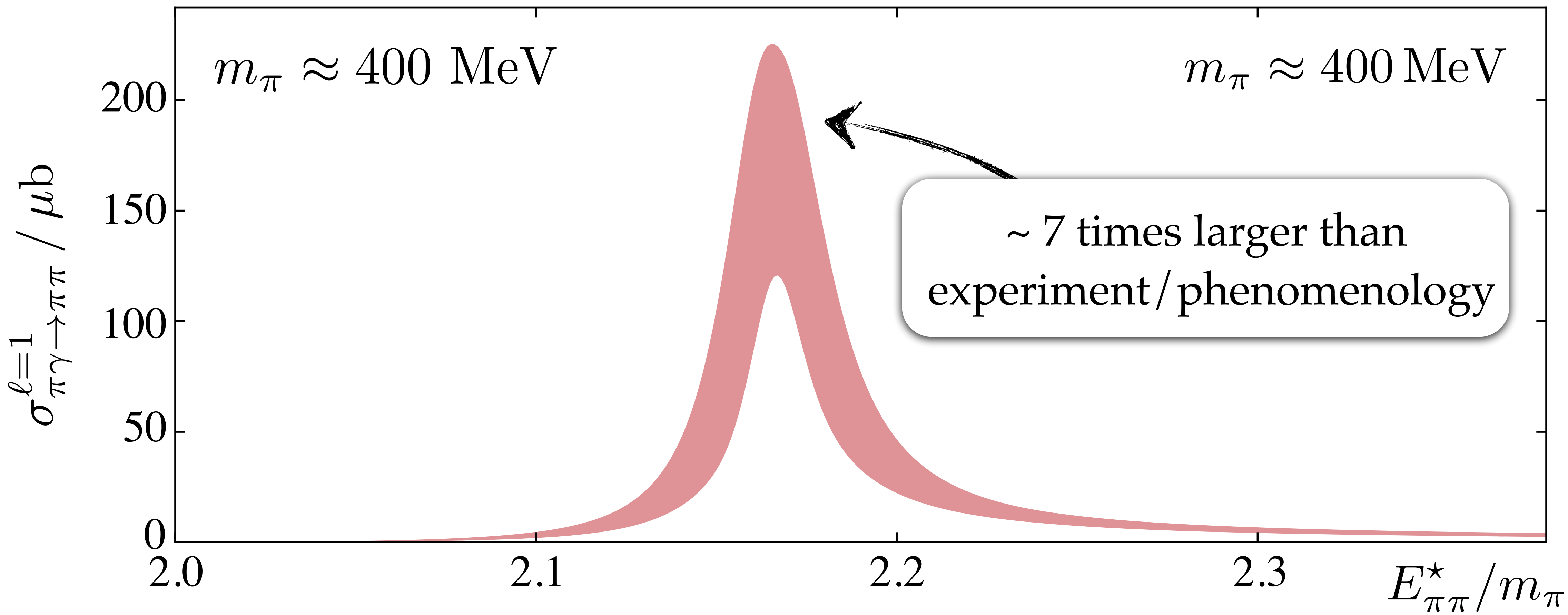
~ 7 times larger than experiment / phenomenology



Hoferichter, Kubis, & Sakkas (2012)

non trivial quark-mass dependence!

$\pi\gamma$-to-$\pi\pi$ cross section

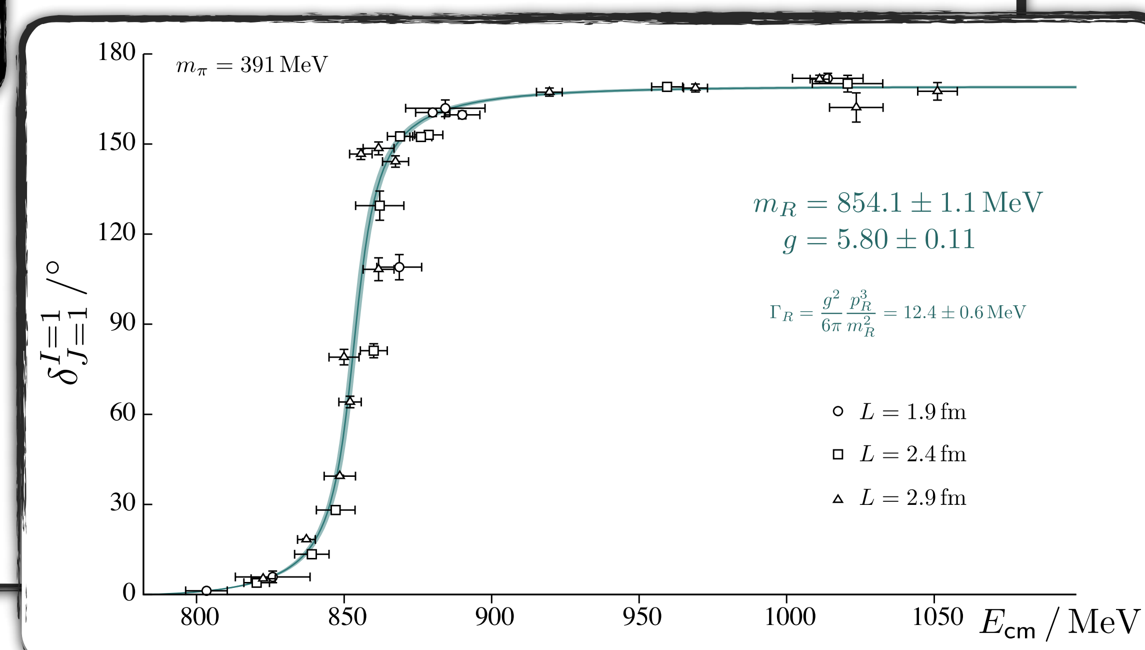


~ 7 times larger than experiment / phenomenology

$$\lim_{E^\star_{\pi\pi}\to m_\rho} \sigma(\pi^+\gamma \to \pi^+\pi^0) \propto \frac{q^\star_{\pi\gamma}\, F^2_{\pi\rho}(m_\rho, 0)}{m_\pi^2} \times \frac{1}{\Gamma_1(m_\rho)}$$

0.60 x (physical)

12 x (physical)



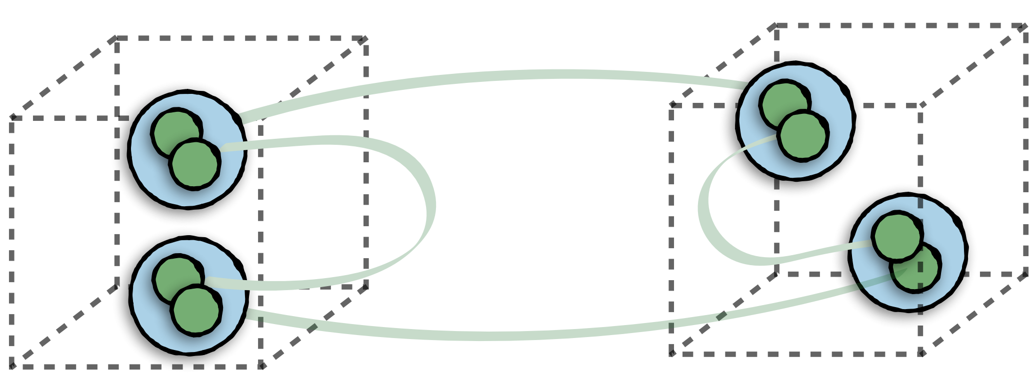
On determining correlation function using small basis of operators

Extracting the spectrum

Two-point correlation functions:

$$
\begin{aligned}
C^{2pt.}_{ab}(t,\mathbf{P}) &\equiv \langle 0|\mathcal{O}_b(t,\mathbf{P})\mathcal{O}^\dagger_a(0,-\mathbf{P})|0\rangle \\
&= \sum_n \langle 0|\mathcal{O}_b(t,\mathbf{P})|n,L\rangle\langle n,L|\mathcal{O}^\dagger_a(0,-\mathbf{P})|0\rangle \\
&= \sum_n \langle 0|e^{t\hat{H}_{QCD}}\mathcal{O}_b(0,\mathbf{P})e^{-t\hat{H}_{QCD}}|n,L\rangle\langle n,L|\mathcal{O}^\dagger_a(0,-\mathbf{P})|0\rangle \\
&= \sum_n Z_{b,n}Z^\dagger_{a,n}e^{-E_n t}
\end{aligned}
$$

spectrum



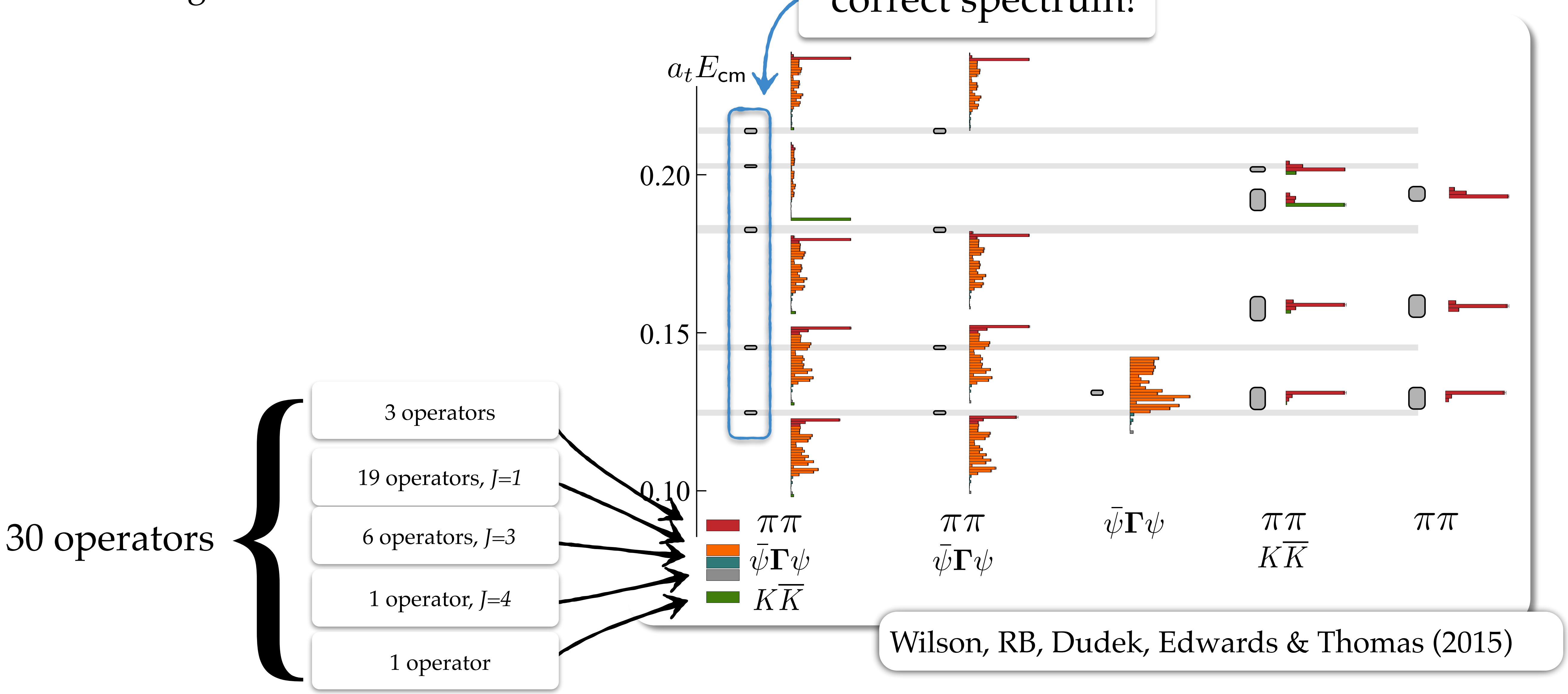
time=0 time=t

Extracting the spectrum

Two-point correlation functions:

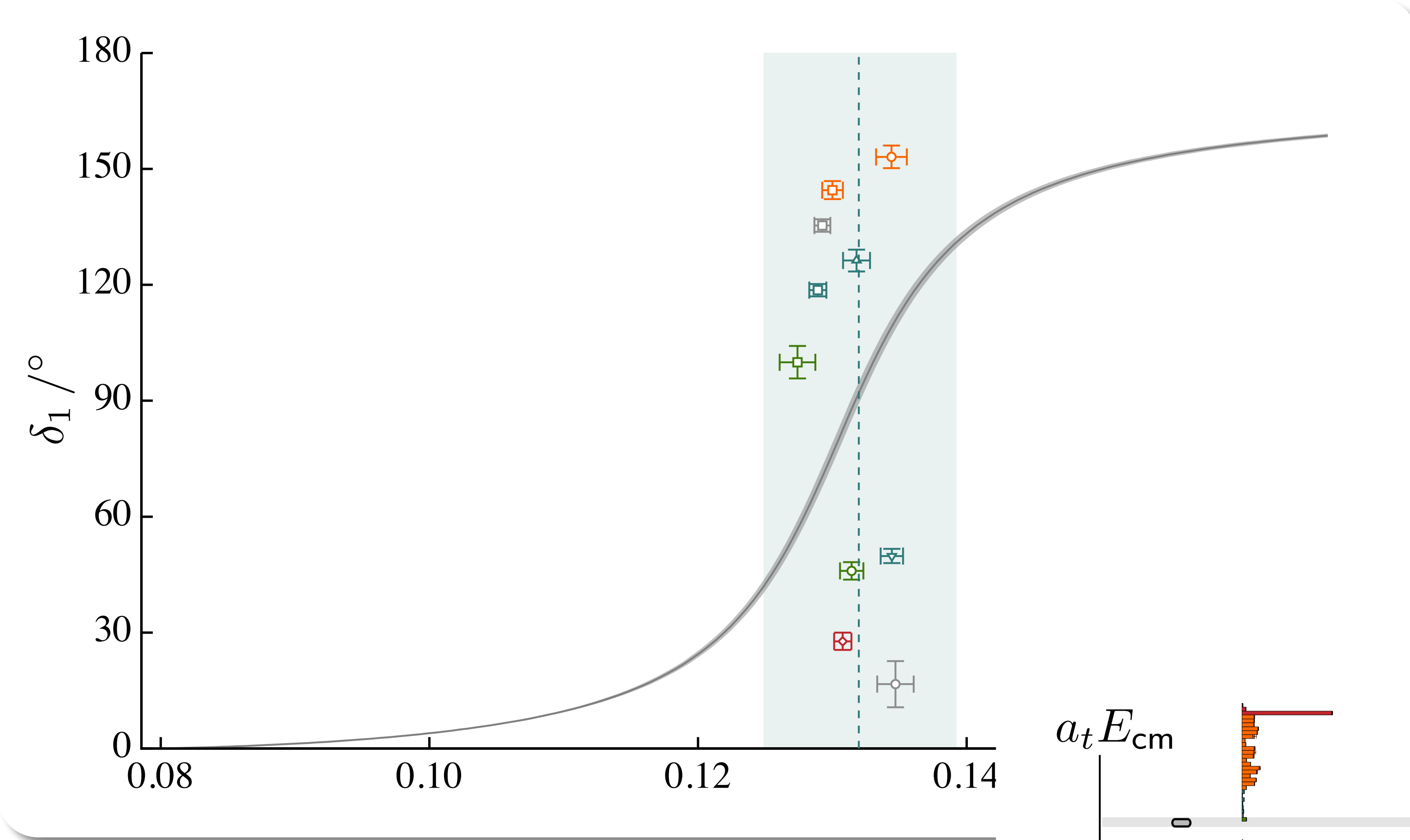
$$C_{ab}^{2pt.}(t,\mathbf{P}) \equiv \langle 0|\mathcal{O}_b(t,\mathbf{P})\mathcal{O}_a^\dagger(0,-\mathbf{P})|0\rangle = \sum_n Z_{b,n} Z_{a,n}^\dagger e^{-E_n t}$$

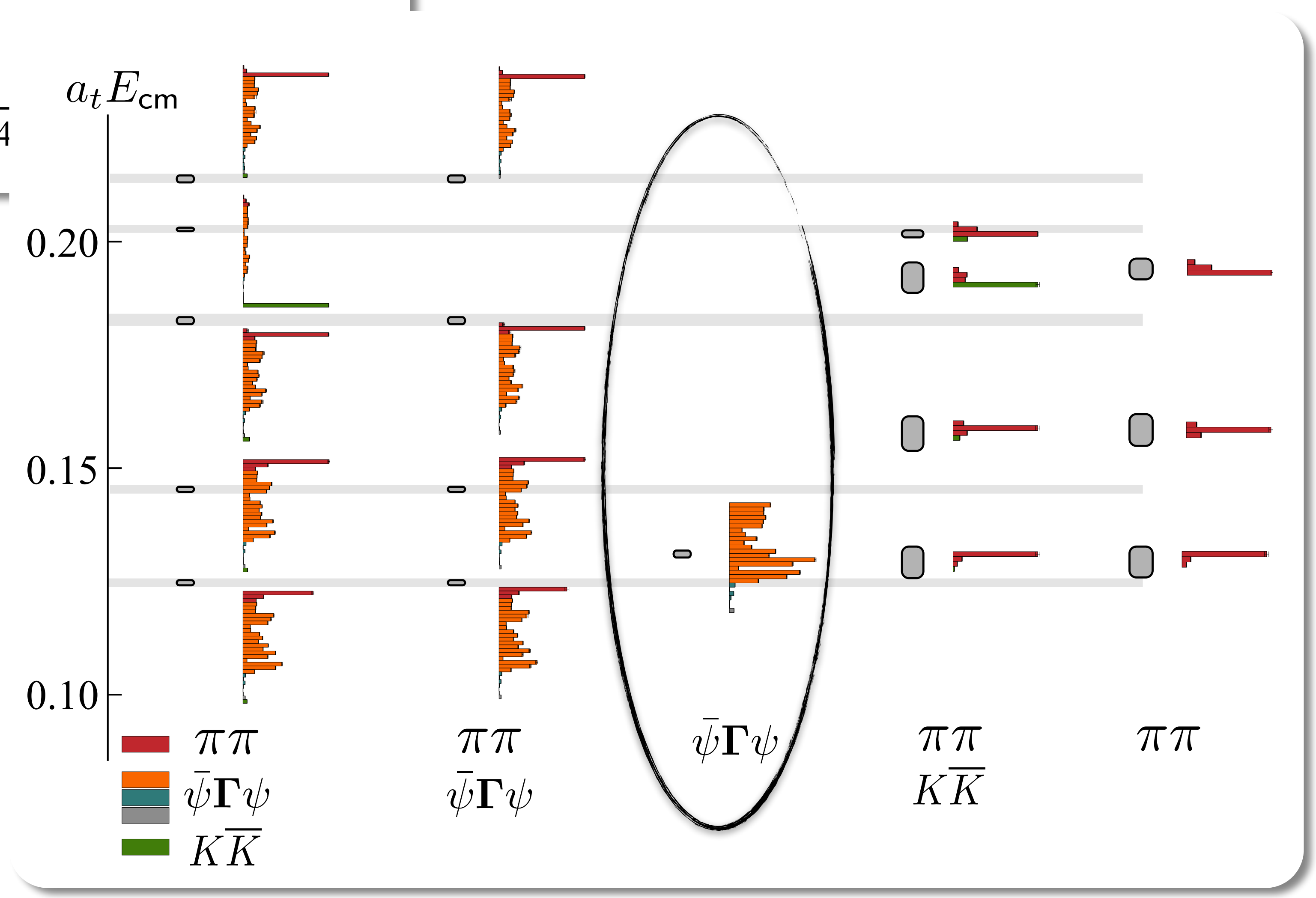
- Use a large basis of operators with the same quantum numbers
- Diagonalize correlation function



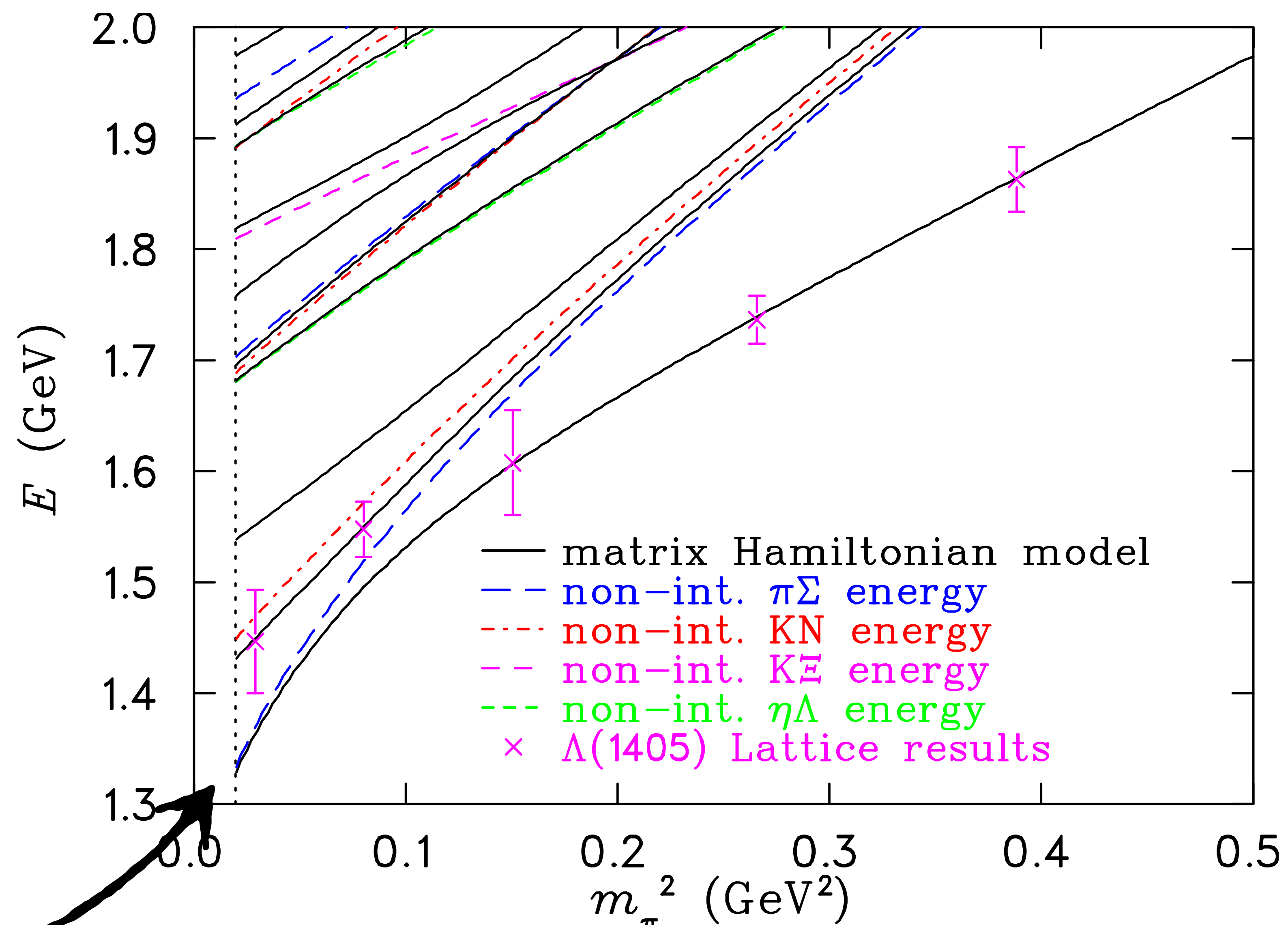
Wilson, RB, Dudek, Edwards & Thomas (2015)

The incorrect answer





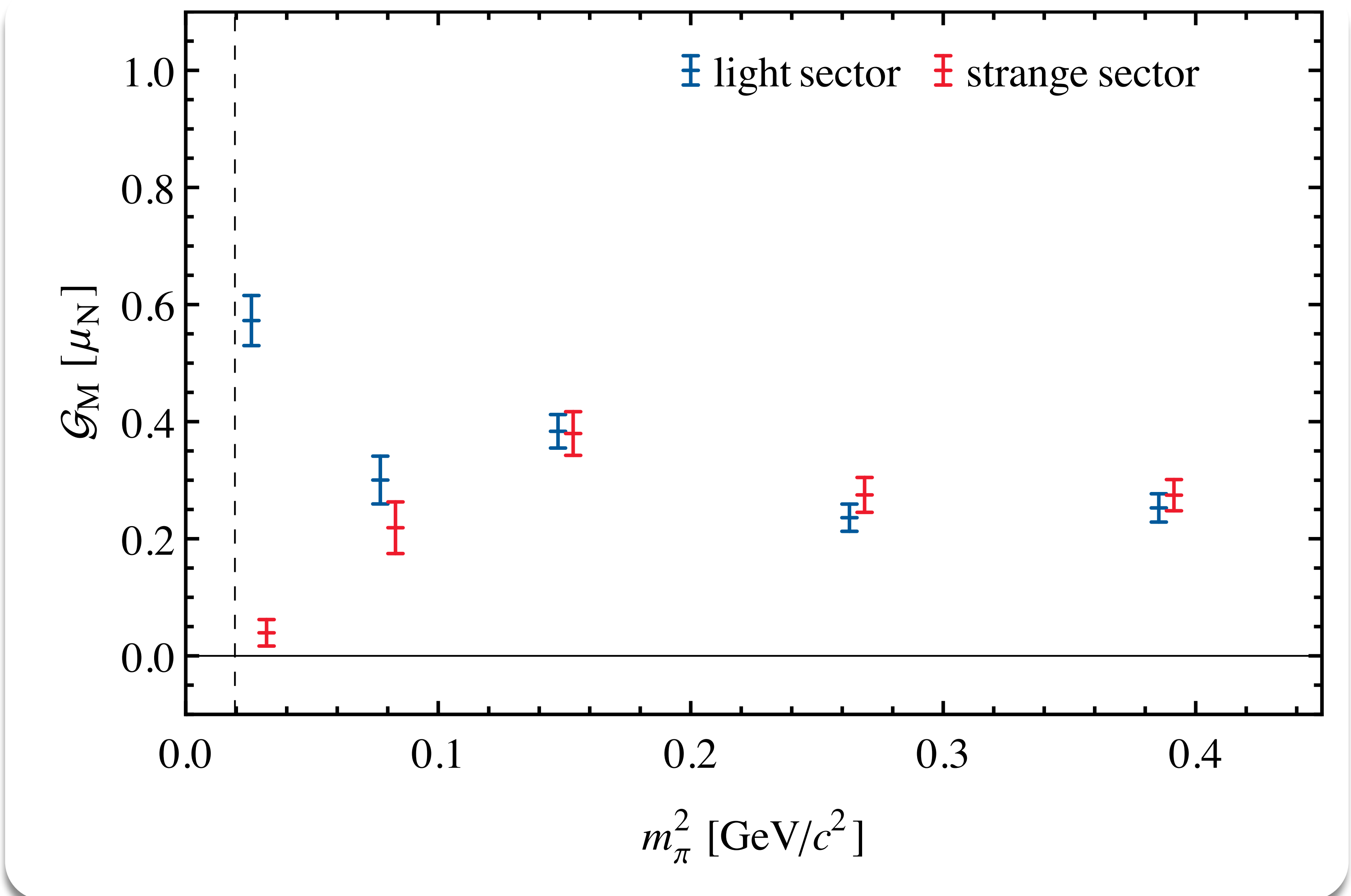
On the $\Lambda(1405)$ spectrum



- Noisy spectrum
- missing ground state
- found only one pole?
- claim resonance is a KN molecule, but did not use KN operators?
- **where's the width of the Λ(1405)?**

Hall, Kamleh, Leinweber, Menadue, Owen, Thomas & Young (2014)

On the $\Lambda(1405)$ form factor



- claim resonance is a KN molecule, but did not use KN operators?
- claim: finite volume matrix element of QED current equal infinite volume form factor
- only true for stable states
- Ignore coupling with $\Sigma\pi$ in analysis of form factor
- for unstable states, the matrix element is **not proportional** to infinite volume form factor

Hall, Kamleh, Leinweber, Menadue, Owen, Thomas & Young (2014)

Two-body maitre elements

$$\boxed{|\langle \mathbf{2}|\mathcal{J}|\mathbf{2}\rangle_L| = \frac{1}{\sqrt{L^3}}\sqrt{\mathrm{Tr}\left[\mathcal{R}\ \mathcal{W}_{L,\mathrm{df}}\ \mathcal{R}\ \mathcal{W}_{L,\mathrm{df}}\right]}}$$

$$\langle \mathbf{2}|\mathcal{J}|\mathbf{2}\rangle_L = \text{FV matrix element}$$

this was reported, everything else was not

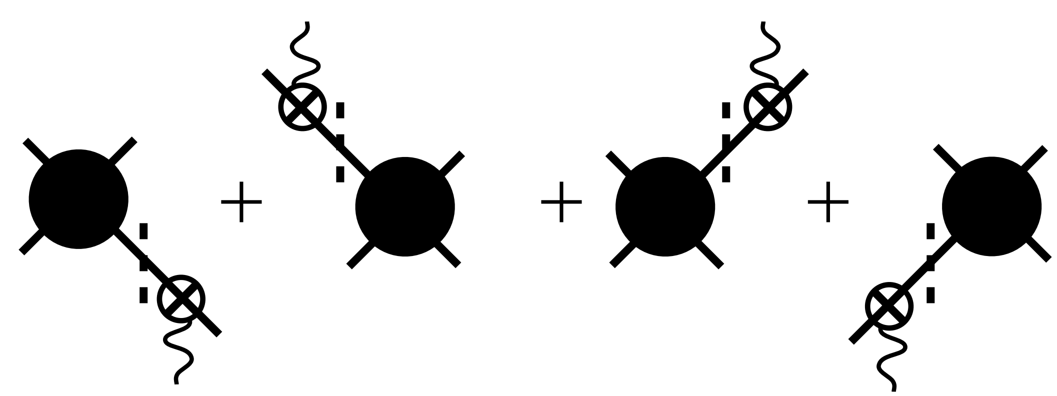
$$\mathcal{R} = \mathcal{R}(P, L, \mathcal{M}, \frac{\partial \mathcal{M}}{\partial E_{\mathbf{2}}})$$

$$\mathcal{W}_{L,\mathrm{df}} = \mathcal{W}_{\mathrm{df}} + \mathcal{M}\ [G(L)\cdot w]\ \mathcal{M}$$

$$G(L) = \text{FV function}$$

$$w = \text{single/stable particle form factor, e.g., } N, \pi, \overline{K}, \ldots$$

$$\mathcal{W} = \text{infinite volume } 2 + \mathcal{J} \to 2 \text{ amplitude}$$

$$i\mathcal{W} = i\mathcal{W}_{\mathrm{df}} + \Big[$$

$$\Big]$$

form factors are defined inside the residues of this amplitude

PRL **116,** 082004 (2016) PHYSICAL REVIEW LETTERS week ending 26 FEBRUARY 2016

Hamiltonian Effective Field Theory Study of the $N^*(1535)$ Resonance in Lattice QCD

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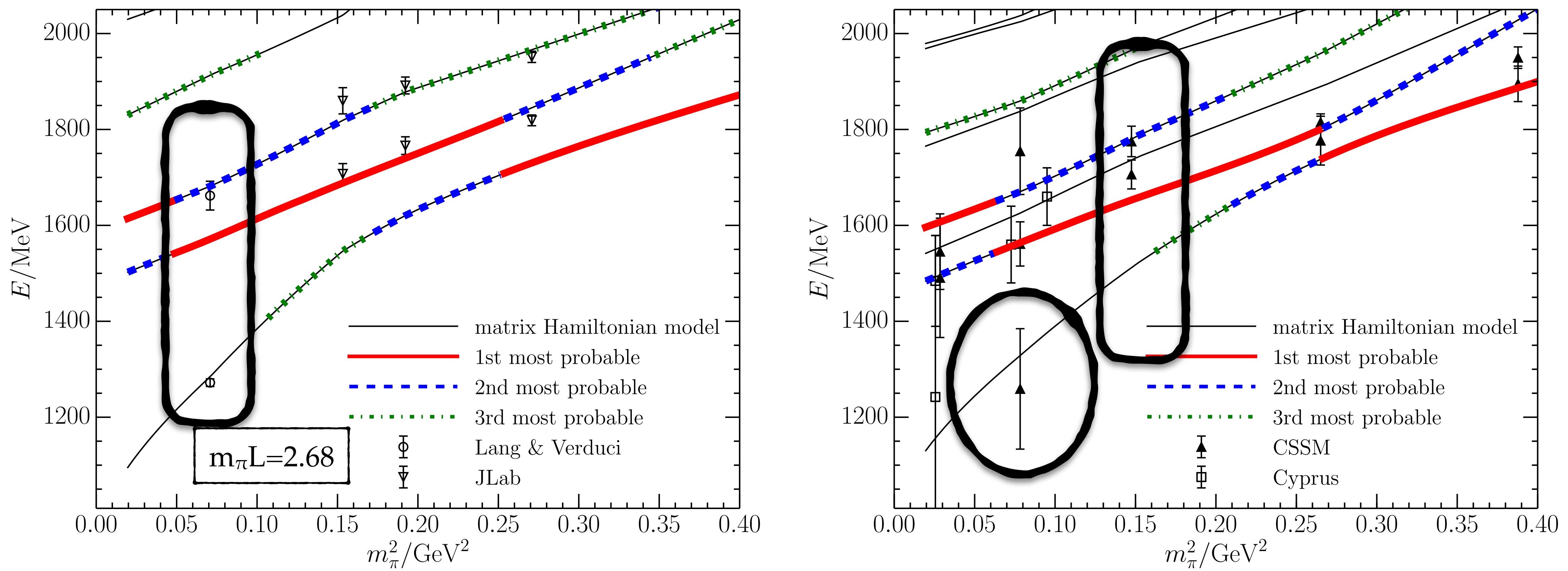
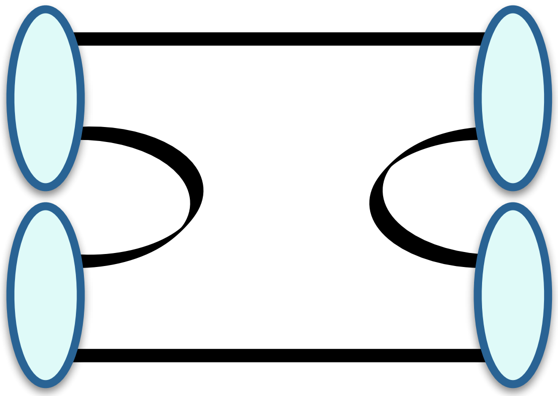
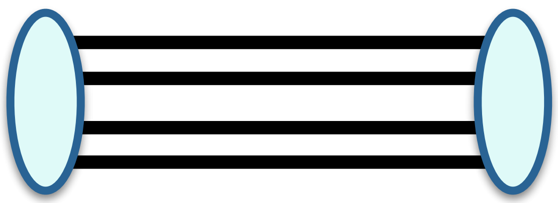
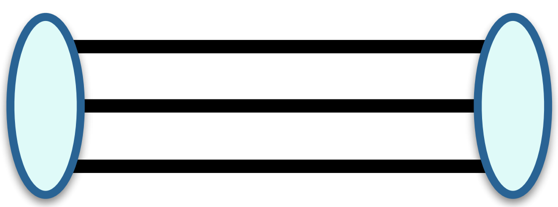
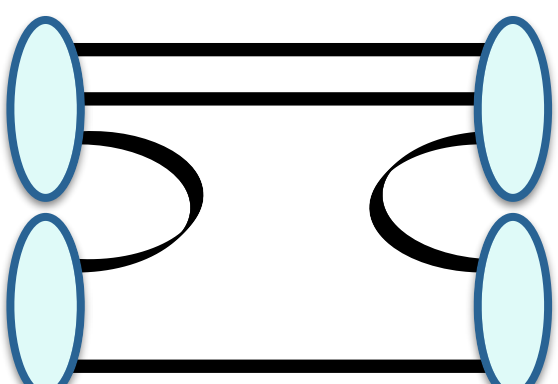
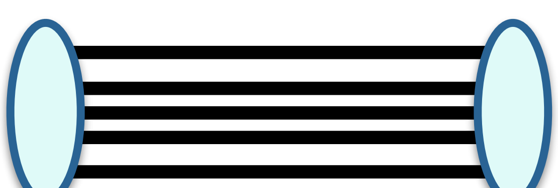


FIG. 2. The pion mass dependence of the $L \simeq 1.98$ fm (left) and $L \simeq 2.90$ fm (right) finite-volume energy eigenstates. The different line types and colors indicate the strength of the bare basis state in the Hamiltonian model eigenvector.

- Noisy spectrum
- missing states
- all, except Lang & Verduci, ignore $N\pi$, $N\pi\pi$, $N\eta$ operators
- requires experimental input to go from χ^2/N_{dof}=4.6 to 1.7

Contraction cost

Estimates by Robert

			Cost $\propto$ Volume	assuming dense tensors
meson:		$\mathrm{tr}\,(MPMP)$	$\sim N^3$	"easy"
meson-meson:			$\sim 100 \times N^3$	affordable
tetraquark:			$\sim N^5$	sparse in spin, but not really affordable
baryon:			$\sim N^4$	affordable
baryon-meson:			$\sim 100 \times N^4$	to do a decent job, not really affordable
pentaquark:			$\sim N^6$	prohibitive