isoscalar mesons in QCD

Jozef Dudek

WILLIAM & MARY
CHARTERED 1693

Jefferson Lab

arXiv:1708.06667 [hep-lat]

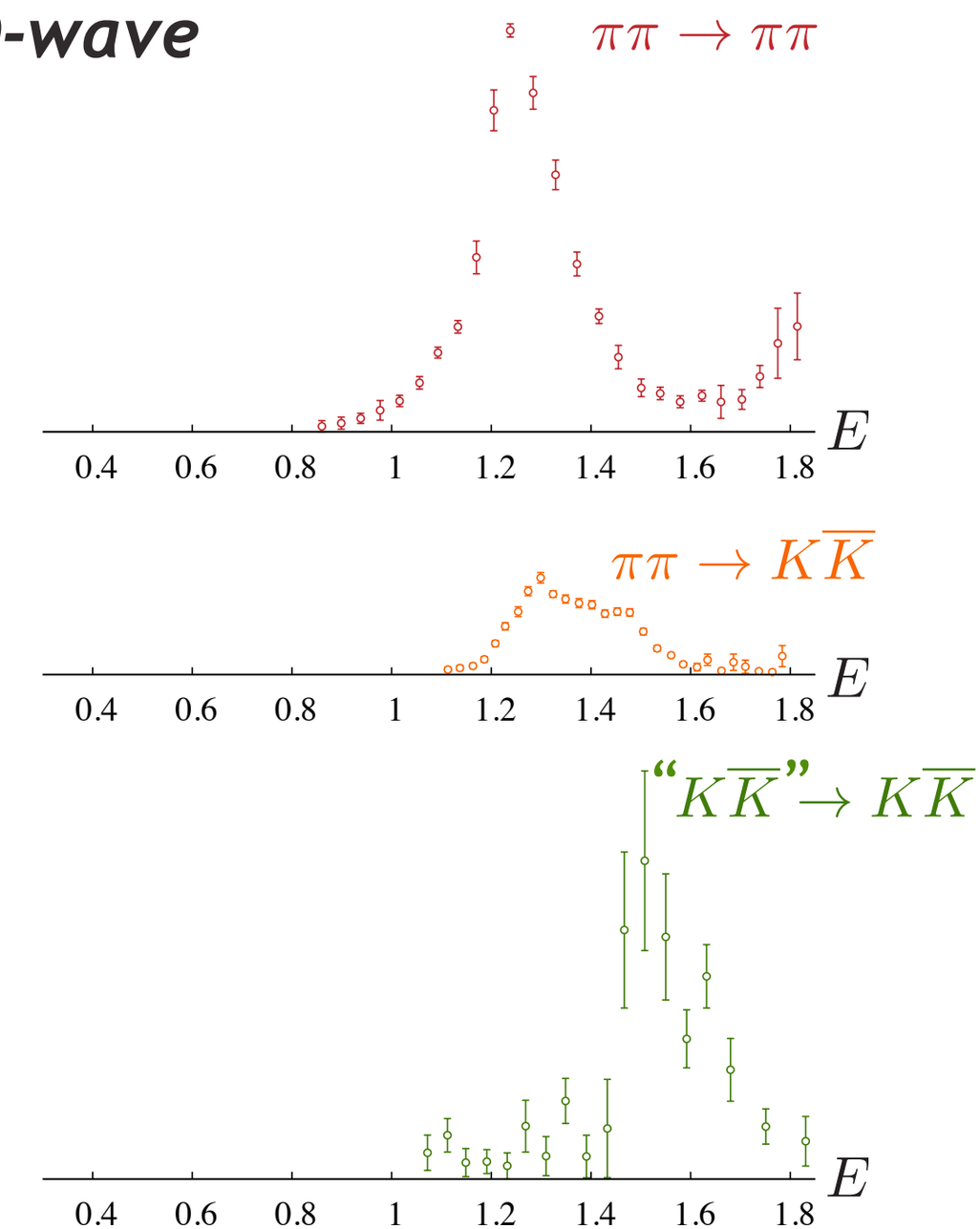
with Raul, Robert, David Wilson (Trinity)

hadspec

isoscalar meson resonances — tensors

coupled $\pi\pi$, $K\bar{K}$, $\eta\eta$ … scattering

J^P=2$^+$
D-wave



bump at ~1250 in $\pi\pi$
bump at ~1500 in $K\bar{K}$

two resonances ?

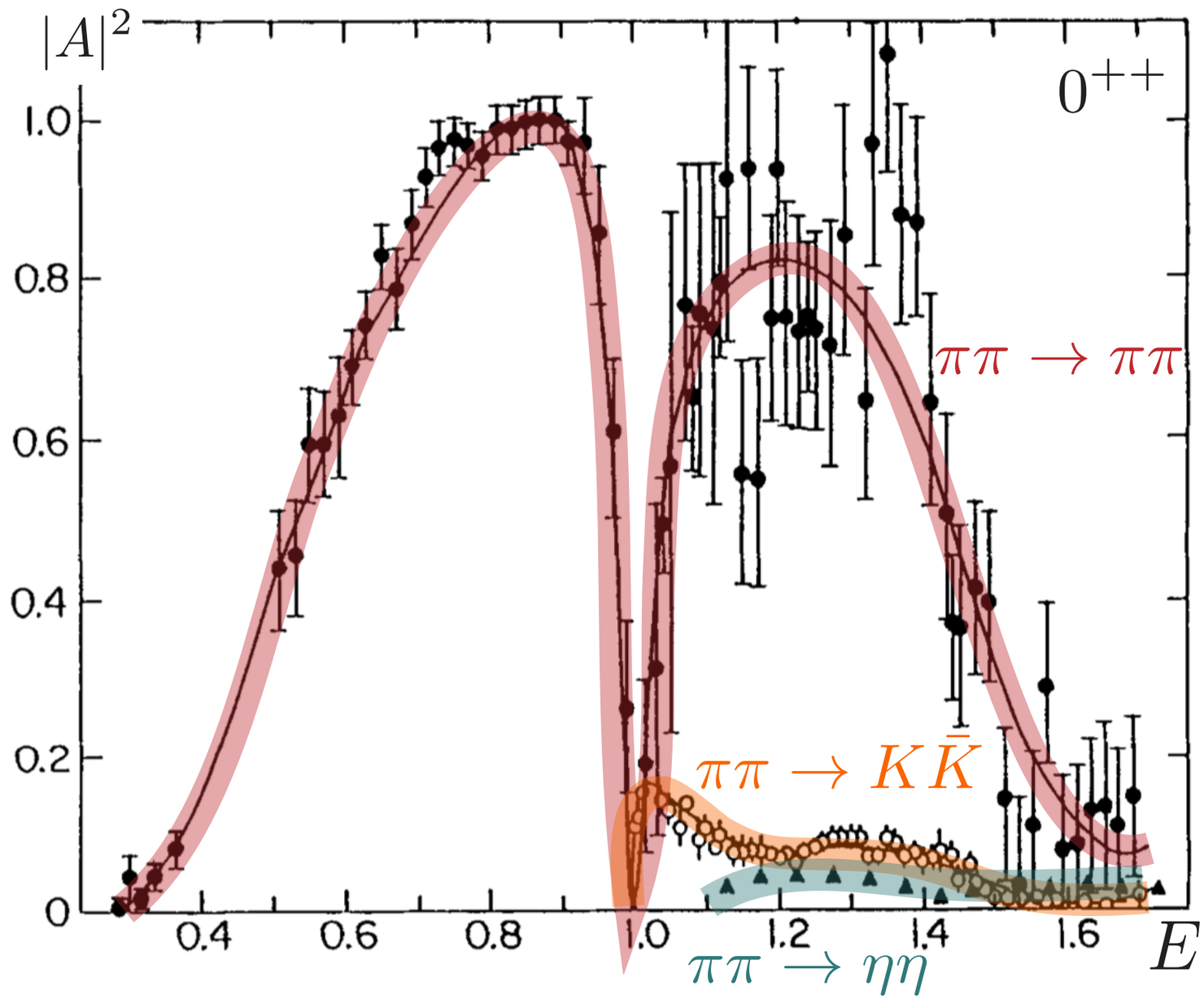
$q\bar{q}$ quark model

$$f_2(1270) \sim (u\bar{u} + d\bar{d})\left[{}^3P_2\right]$$
$$f_2'(1525) \sim (s\bar{s})\left[{}^3P_2\right]$$

isoscalar meson resonances — scalars

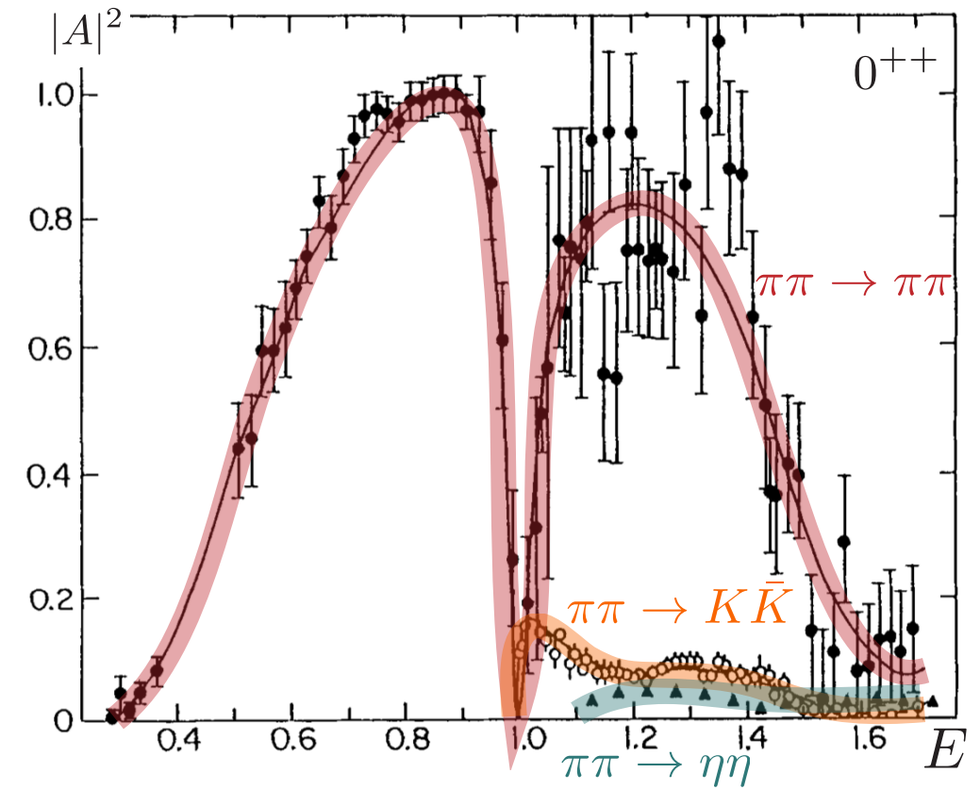
coupled $\pi\pi$, $K\bar{K}$, $\eta\eta$ … scattering

J^P=0⁺
S-wave

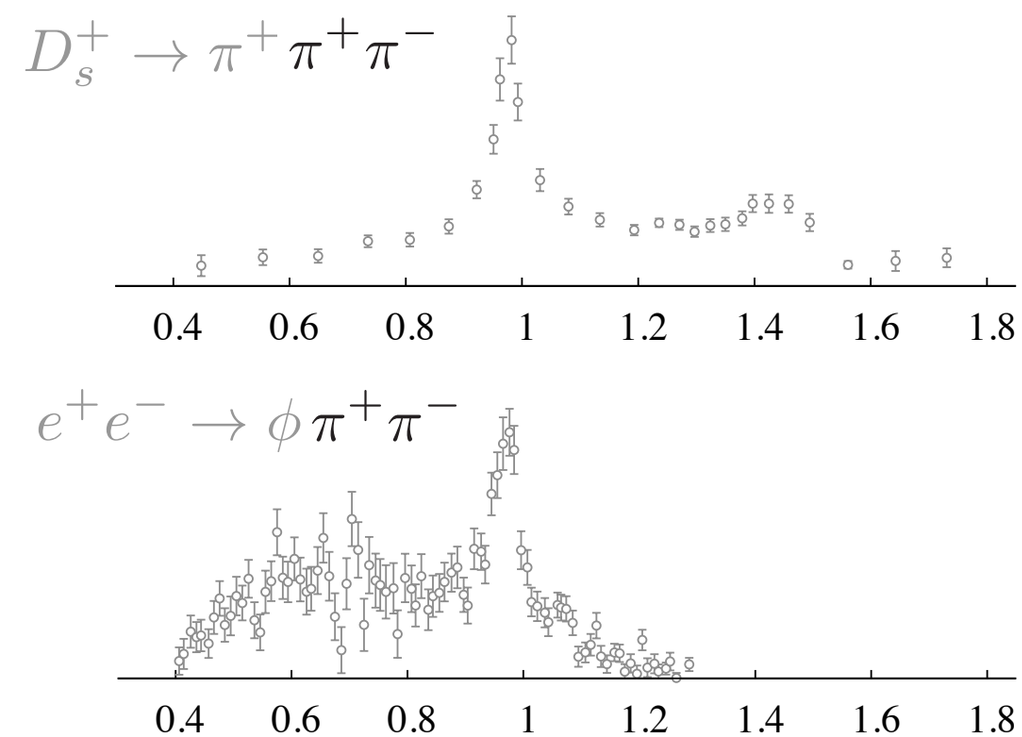


resonance content … ?

isoscalar meson resonances — scalars



in some processes
the **dip** is a **peak**

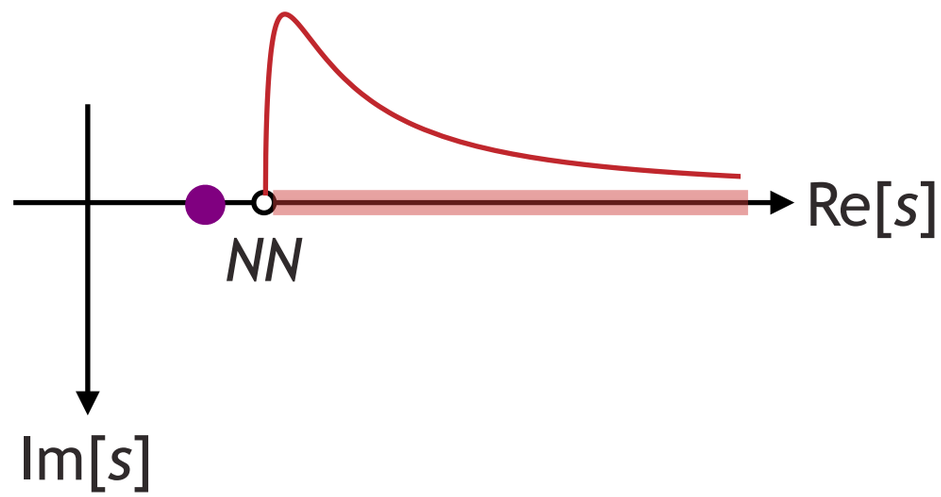


resonance content ?

a rigorous definition — pole singularity in a partial-wave amplitude $t_{ij}^{(\ell)}(s) \sim \frac{c_i\, c_j}{s_0 - s}$

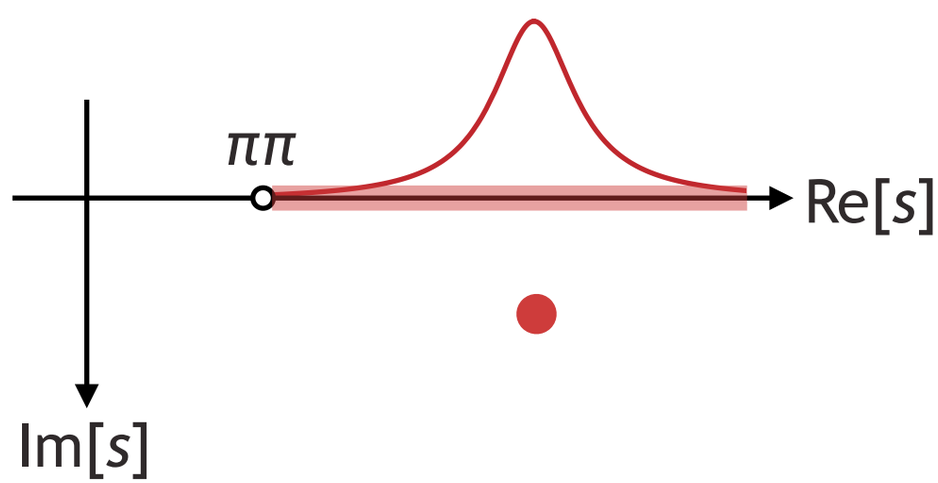
— bound state: $s_0 = M^2$

e.g. deuteron

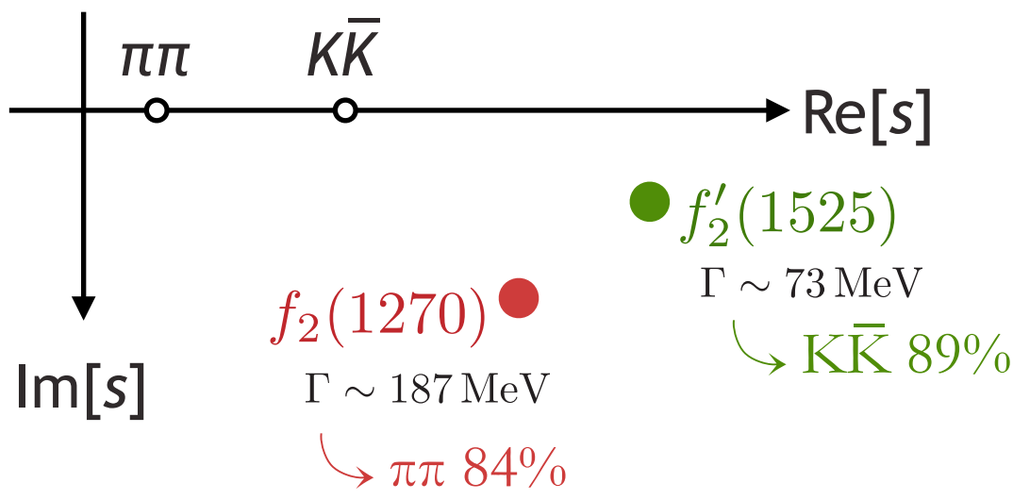


— resonance: $\sqrt{s_0} = M - i\frac{1}{2}\Gamma$

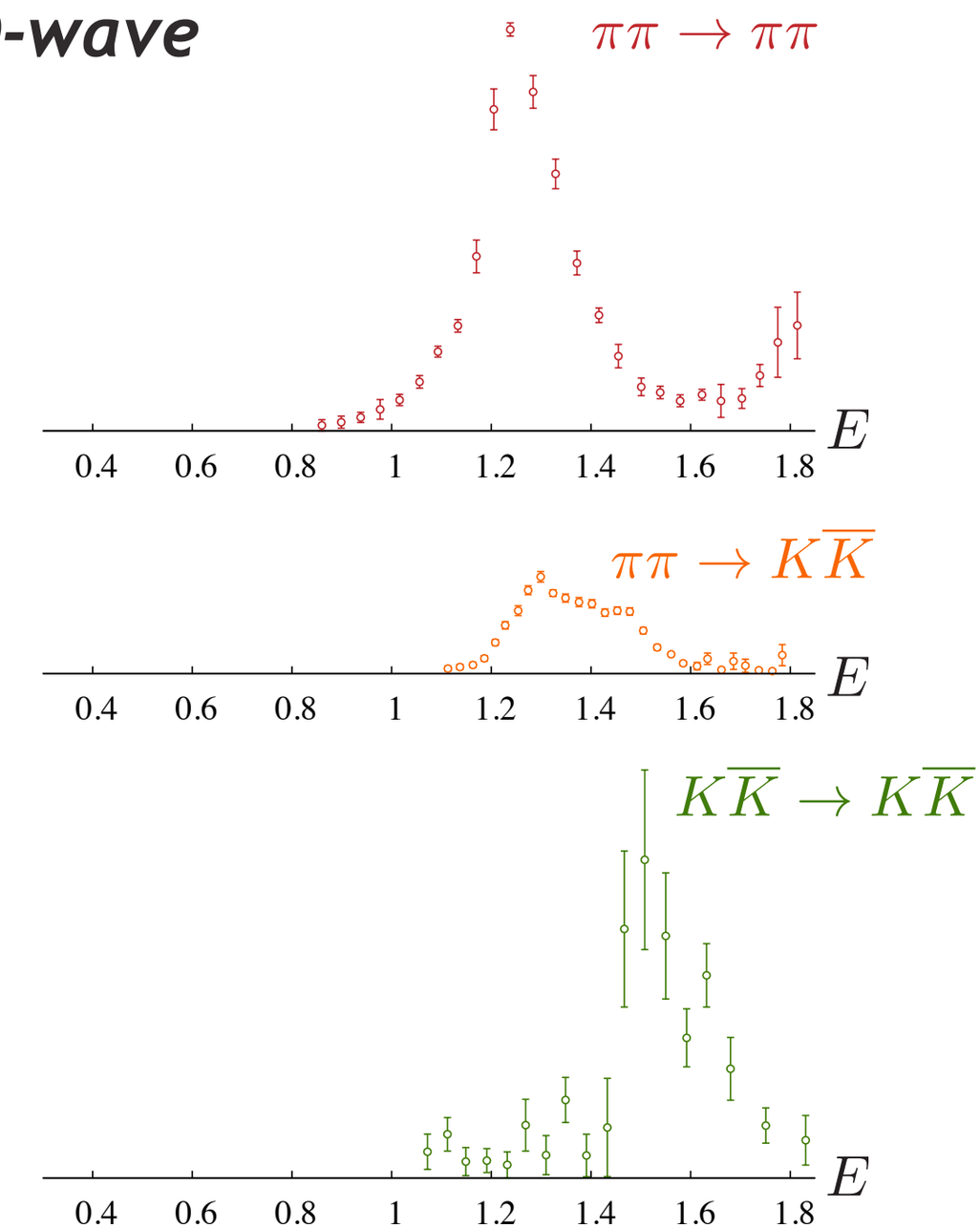
e.g. ρ meson



f_2 resonances



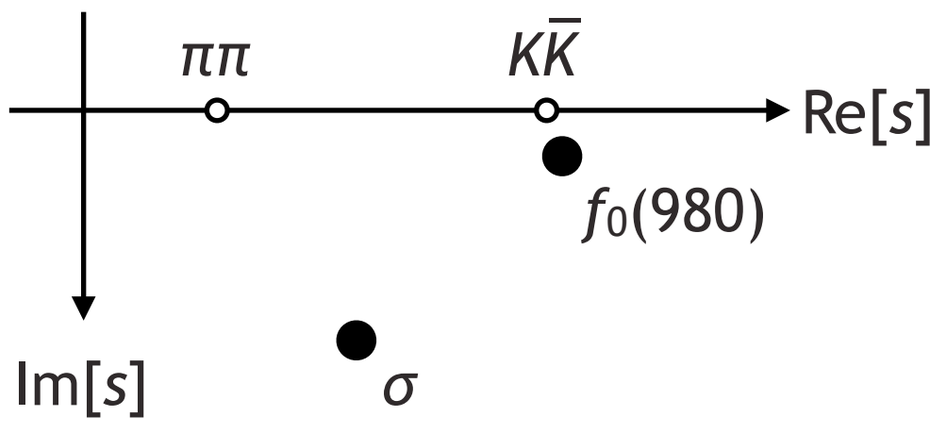
$J^P=2^+$
D-wave

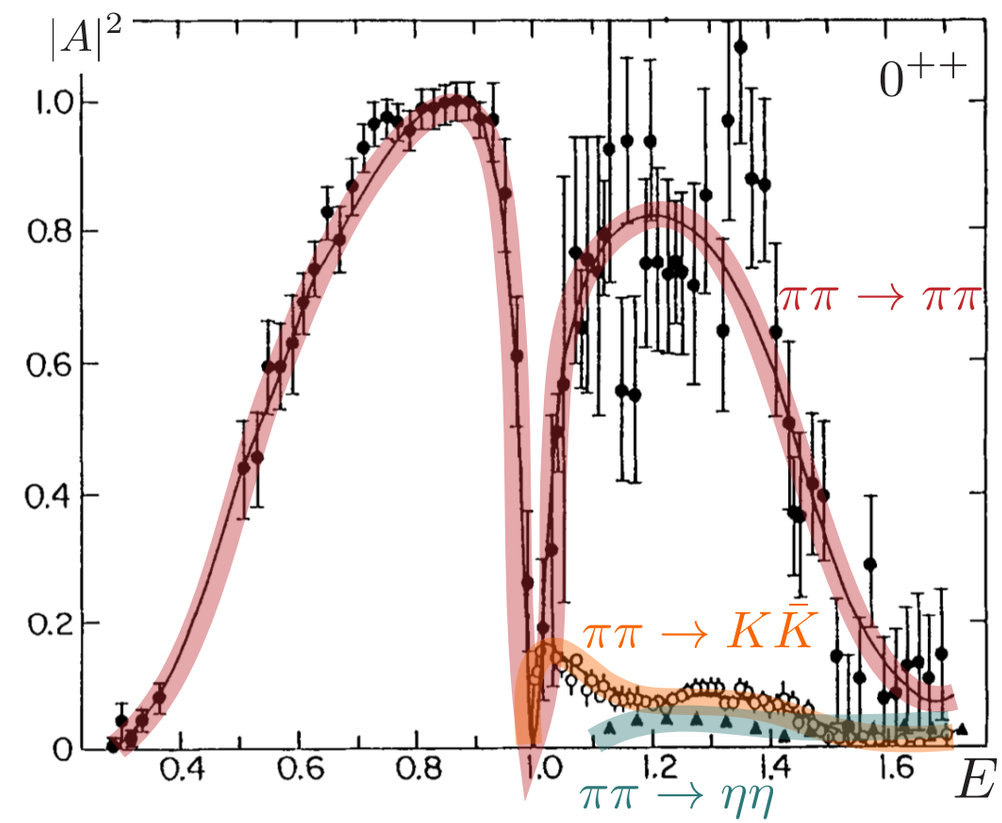


branching fractions
from the pole residues

$$t_{ij}^{(\ell)}(s) \sim \frac{c_i\, c_j}{s_0 - s}$$

f_0 resonances ?





$f_0(980)$ large coupling to $K\bar{K}$

other scalar mesons

broad, light $\sigma(600)$ in $\pi\pi$ $I=0$

broad, light $\kappa(800)$ in πK $I=½$, $S=1$

narrow $f_0(980)$ in $\pi\pi$ at $K\bar{K}$ threshold $I=0$

narrow $a_0(980)$ in $\pi\eta$ at $K\bar{K}$ threshold $I=1$

… a flavor nonet ?

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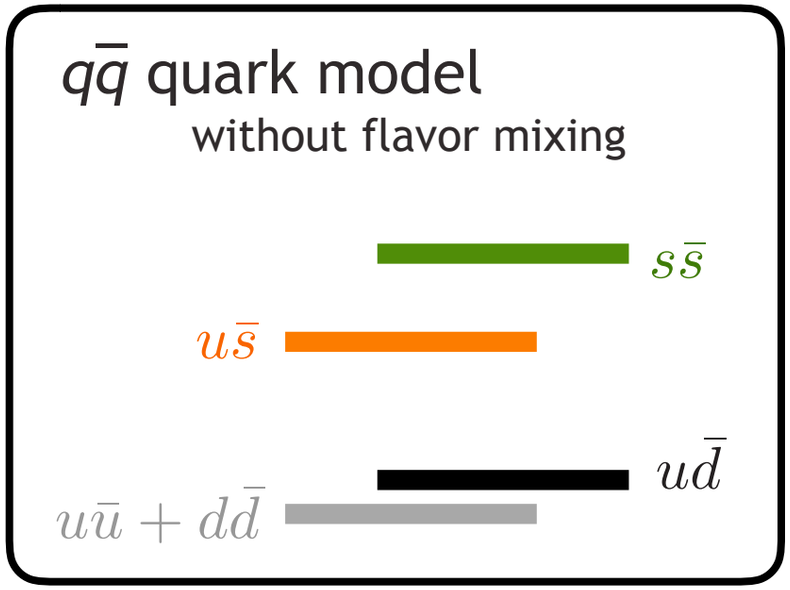

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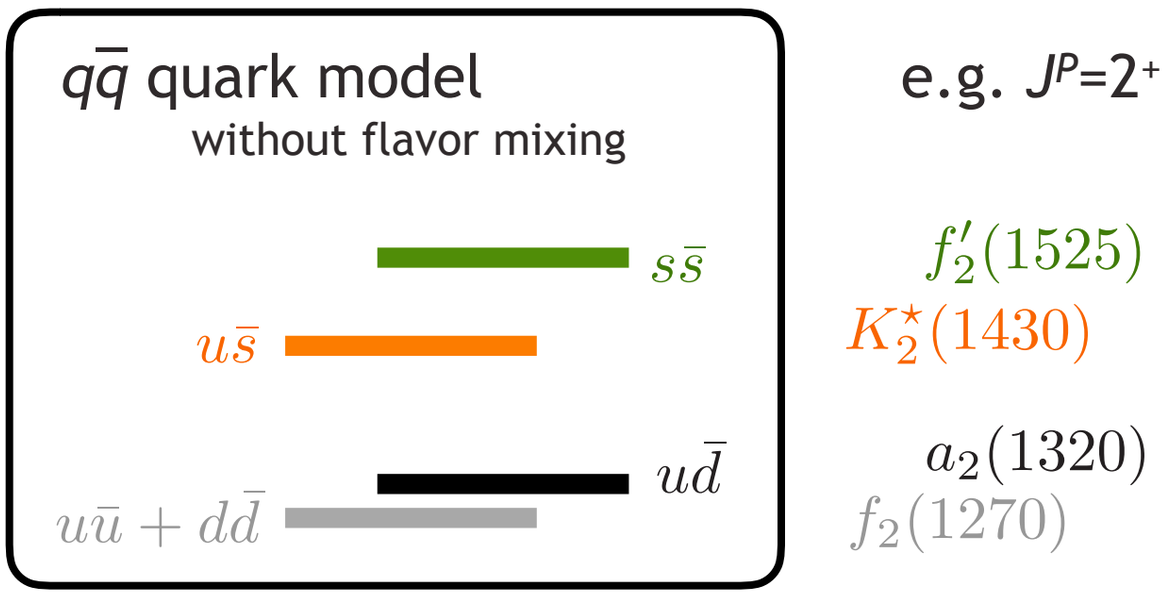

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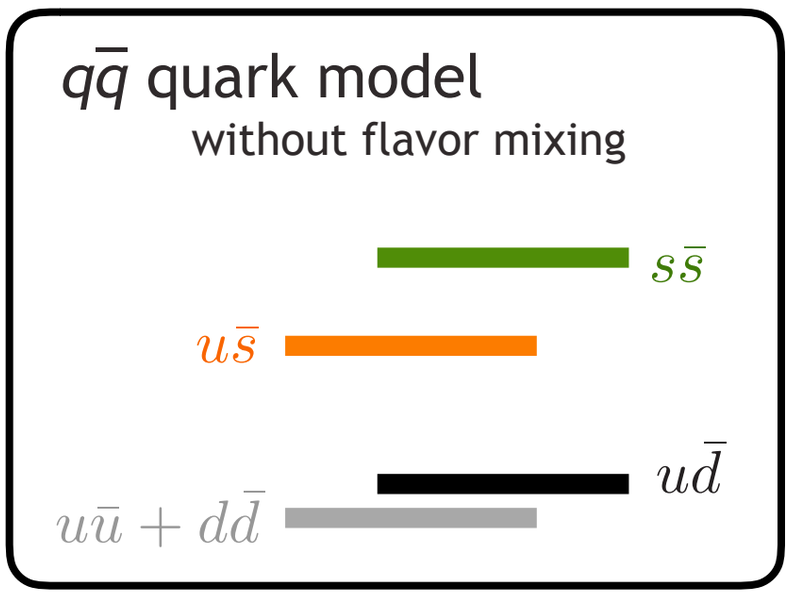

e.g. J^P=2⁺

$f_2'(1525)$

$K_2^\star(1430)$

$a_2(1320)$

$f_2(1270)$

but not J^P=0⁺

$a_0(980)$

$f_0(980)$

$\kappa(800)$

$\sigma(600)$

?

other scalar mesons

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broad, light $\kappa(800)$ in πK

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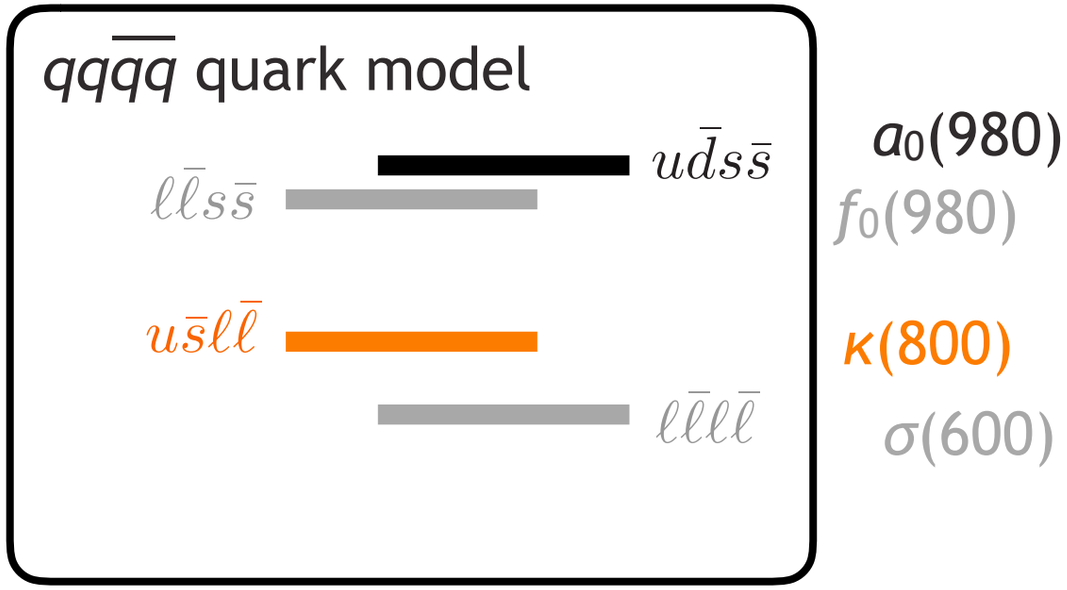

✔ hidden strangeness makes the f_0, a_0 heavier

✘ why so light, differing widths, proximity to $K\bar{K}$ accident ?

scalar mesons

broad, light $\sigma(600)$ in $\pi\pi$

broad, light $\kappa(800)$ in πK

?

narrow $f_0(980)$ in $\pi\pi$ at $K\bar{K}$ threshold

narrow $a_0(980)$ in $\pi\eta$ at $K\bar{K}$ threshold

} $K\bar{K}$ molecules ?

or some other explanation we've not thought of yet ?

… can these questions be explored in QCD ?

lattice QCD

you've heard this a million times …

in this case:

→ **discrete cubic grid** (*probably irrelevant*)

→ **larger quark mass** (*helpful — makes pions heavier*)

→ **finite spatial volume** (*vital — tool for scattering*)

compute two-point correlation functions $C_{ij}(t) = \langle 0 | \mathcal{O}_i(t)\, \mathcal{O}_j(0) | 0 \rangle$

to determine the discrete spectrum:

$$C_{ij}(t) = \sum_{\mathfrak{n}} e^{-E_{\mathfrak{n}} t} \langle 0 | \mathcal{O}_i | \mathfrak{n} \rangle \langle \mathfrak{n} | \mathcal{O}_j | 0 \rangle$$

lattice QCD

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operators

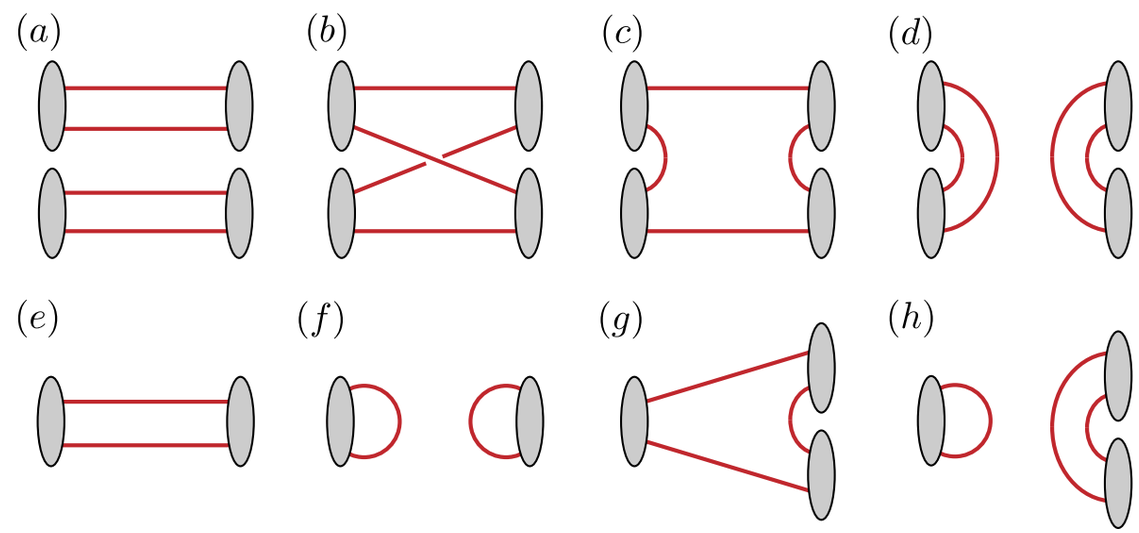
'single-meson'

$$\sum_{\mathbf{x}} e^{i\mathbf{p}\cdot\mathbf{x}}\, \overline{\psi} \Gamma \overleftrightarrow{D} \cdots \overleftrightarrow{D} \psi(\mathbf{x}, t)$$

'meson-meson'

$$\sum_{\hat{\mathbf{p}}_1, \hat{\mathbf{p}}_2} C(\mathbf{p}_1, \mathbf{p}_2; \mathbf{p})\, M_1(\mathbf{p}_1) M_2(\mathbf{p}_2)$$

Wick contractions



choice of meson-meson operators guided by non-interacting spectrum …

$$E_{\text{n.i.}} = \sqrt{m_1^2 + \left(\tfrac{2\pi}{L}\right)^2 \mathbf{p}_1^2} + \sqrt{m_2^2 + \left(\tfrac{2\pi}{L}\right)^2 \mathbf{p}_2^2}$$

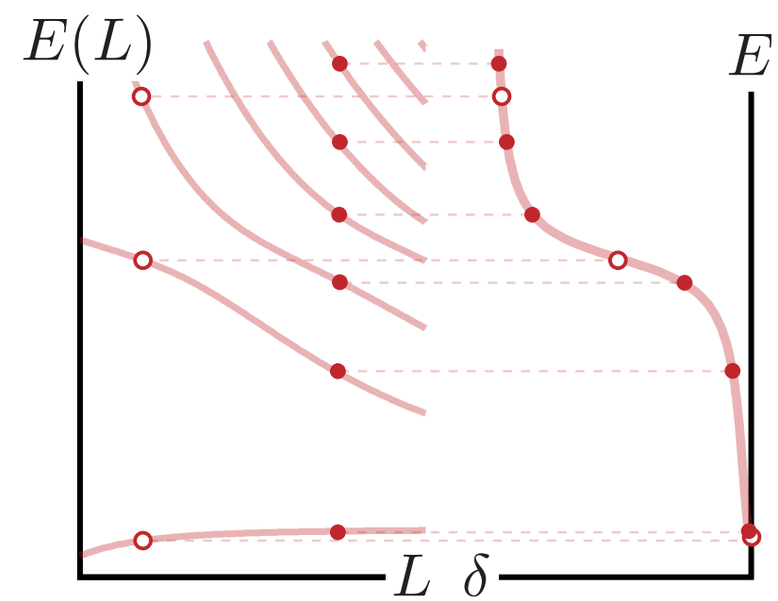
finite-volume formalism

$$\det\left[\mathbf{1} + i\boldsymbol{\rho}\cdot\mathbf{t}\cdot(\mathbf{1} + i\boldsymbol{\mathcal{M}})\right] = 0$$

$\boldsymbol{\rho}(E)$ phase-space

$\mathbf{t}(E)$ scattering matrix

$\boldsymbol{\mathcal{M}}(E, L)$ finite-volume function



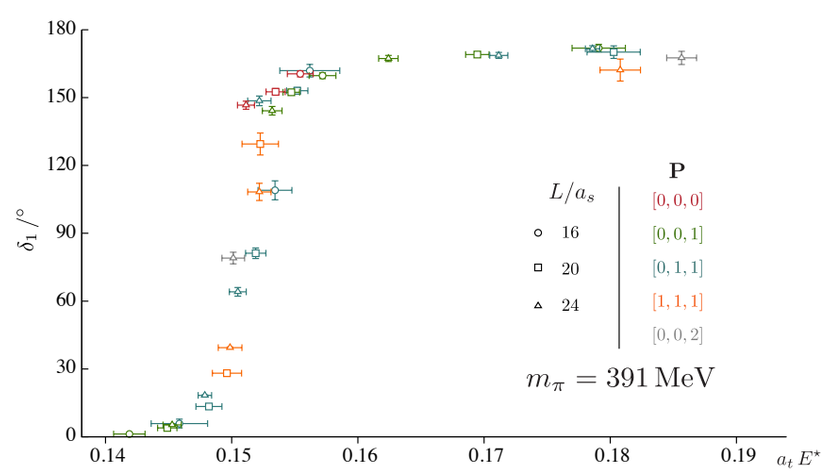
in the elastic case
(for one partial wave)

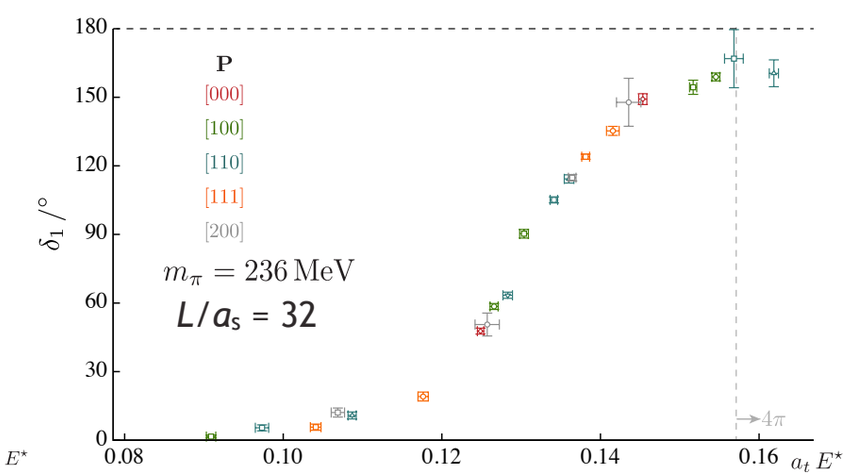
$$t = \frac{1}{\rho} e^{i\delta} \sin\delta$$

$$\cot\delta(E) = \mathcal{M}(E, L)$$

actually more information
in moving-frame spectra

I=1, J^P=1$^-$
$\pi\pi$ elastic





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$\mathbf{t}(E)$ scattering matrix

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can also be applied in **coupled-channel case** more challenging

$\mathbf{t}(E) \to E_{\mathfrak{n}}(L)$ (plug into eqn. above & solve)

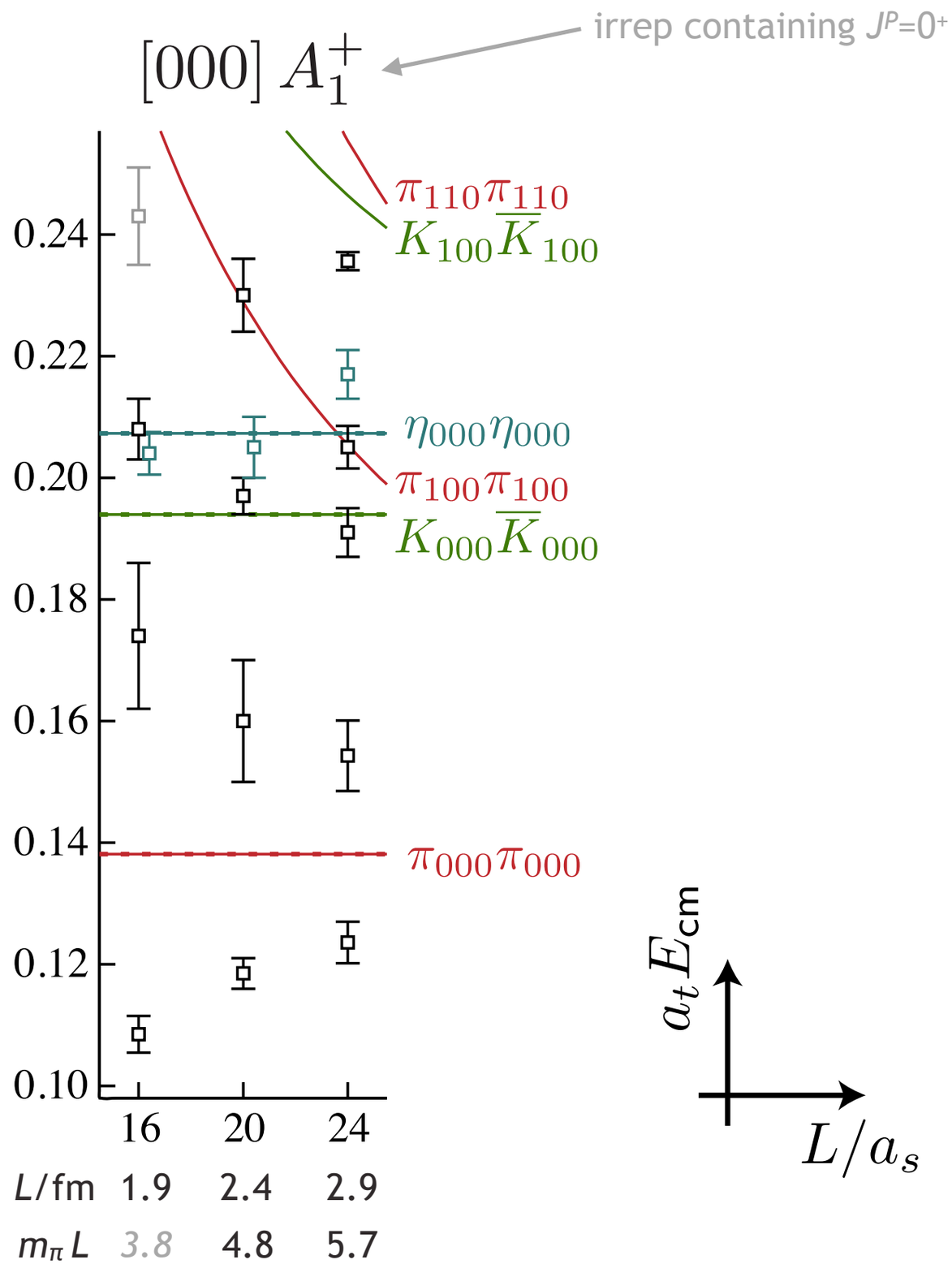
$E_{\mathfrak{n}}(L) \not\to \mathbf{t}(E_{\mathfrak{n}})$ (multiple unknowns in the scattering matrix)

do a global fit to the spectrum with an amplitude parameterization …

(guarantee unitarity by using a *K*-matrix)

finite-volume spectrum — *I*=0, *G*=+

$m_\pi \sim 391\,\mathrm{MeV}$



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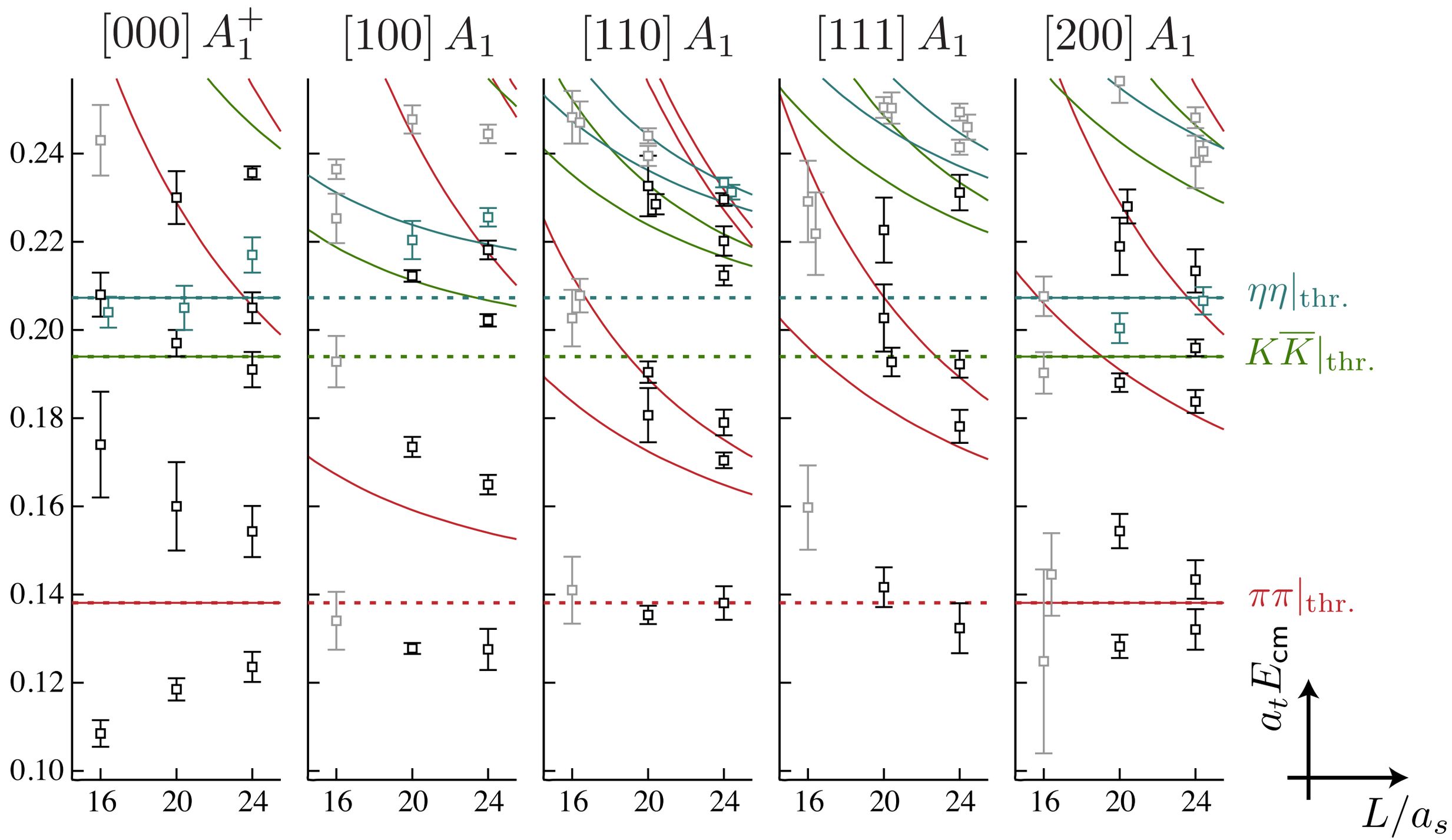
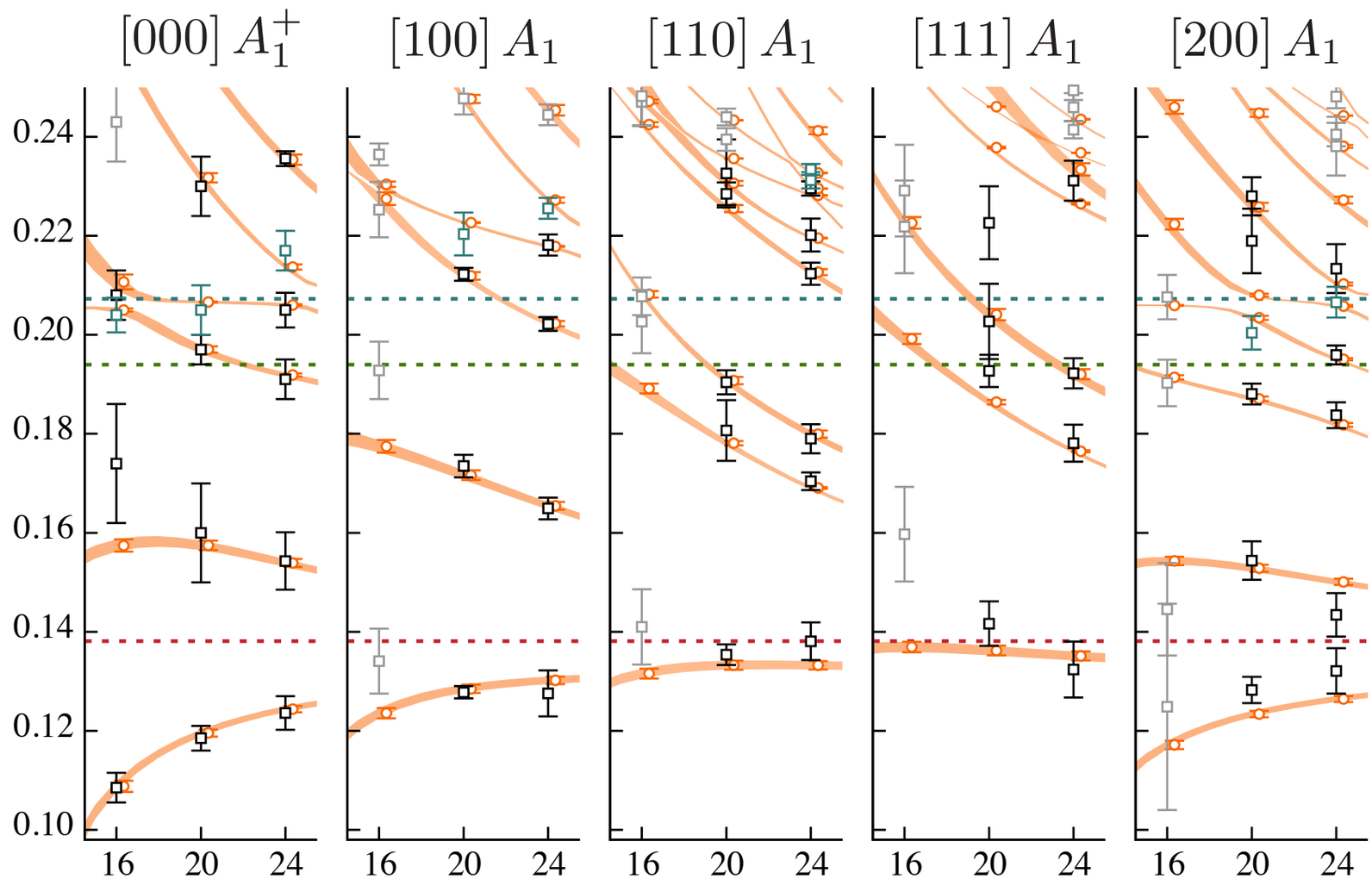


a *K*-matrix amplitude description

$$\mathbf{K}^{-1} = \begin{array}{c} \begin{array}{ccc} \pi\pi & K\overline{K} & \eta\eta \end{array} \\ \begin{pmatrix} a + b\,s & c + d\,s & e \\ c + d\,s & f & g \\ e & g & h \end{pmatrix} \end{array}$$

+ Chew-Mandelstam phase-space



χ^2/N_{dof} = 44.0/(57−8) = 0.90

too conservative systematic error estimation ?

a *K*-matrix amplitude description

$m_\pi \sim 391\,\mathrm{MeV}$

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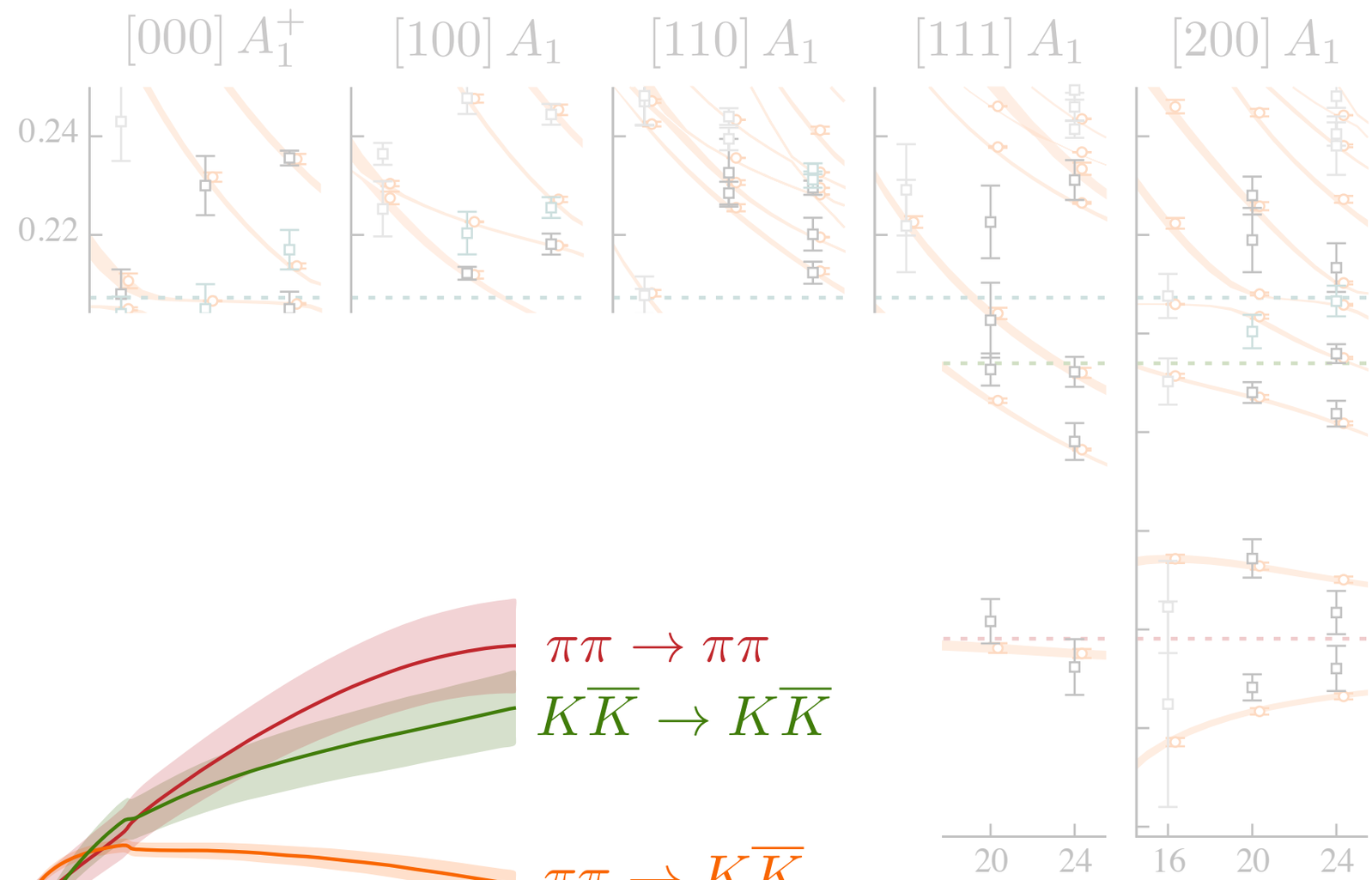

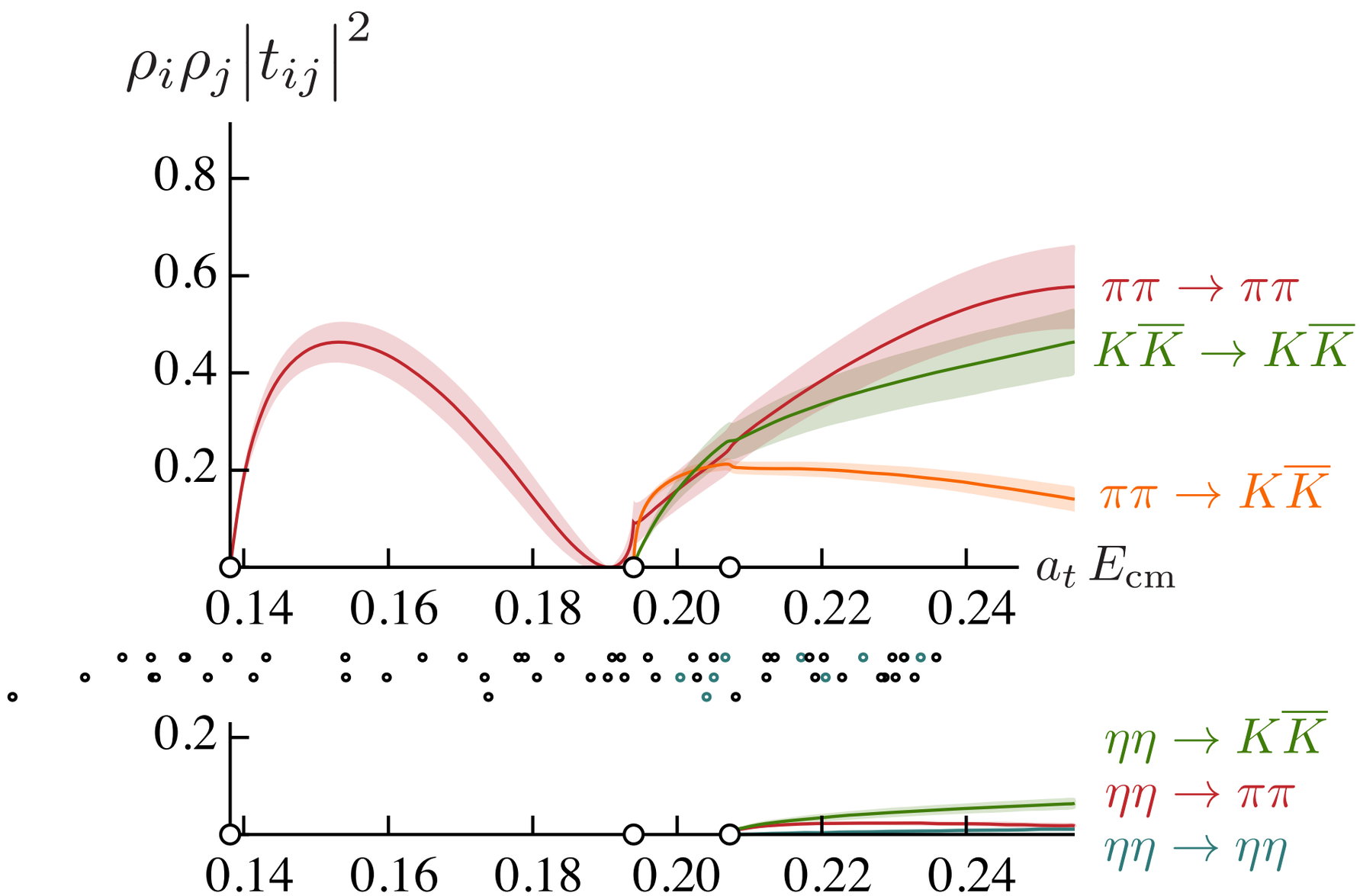

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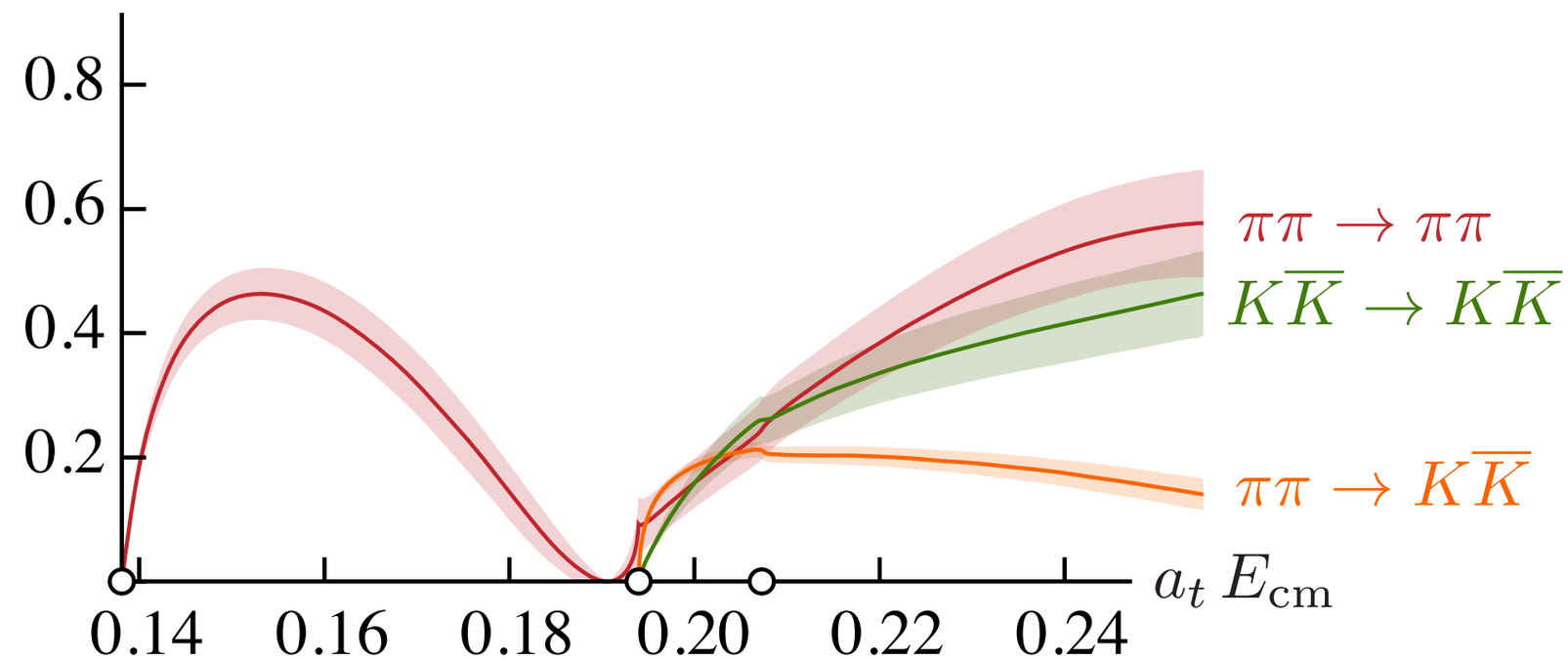
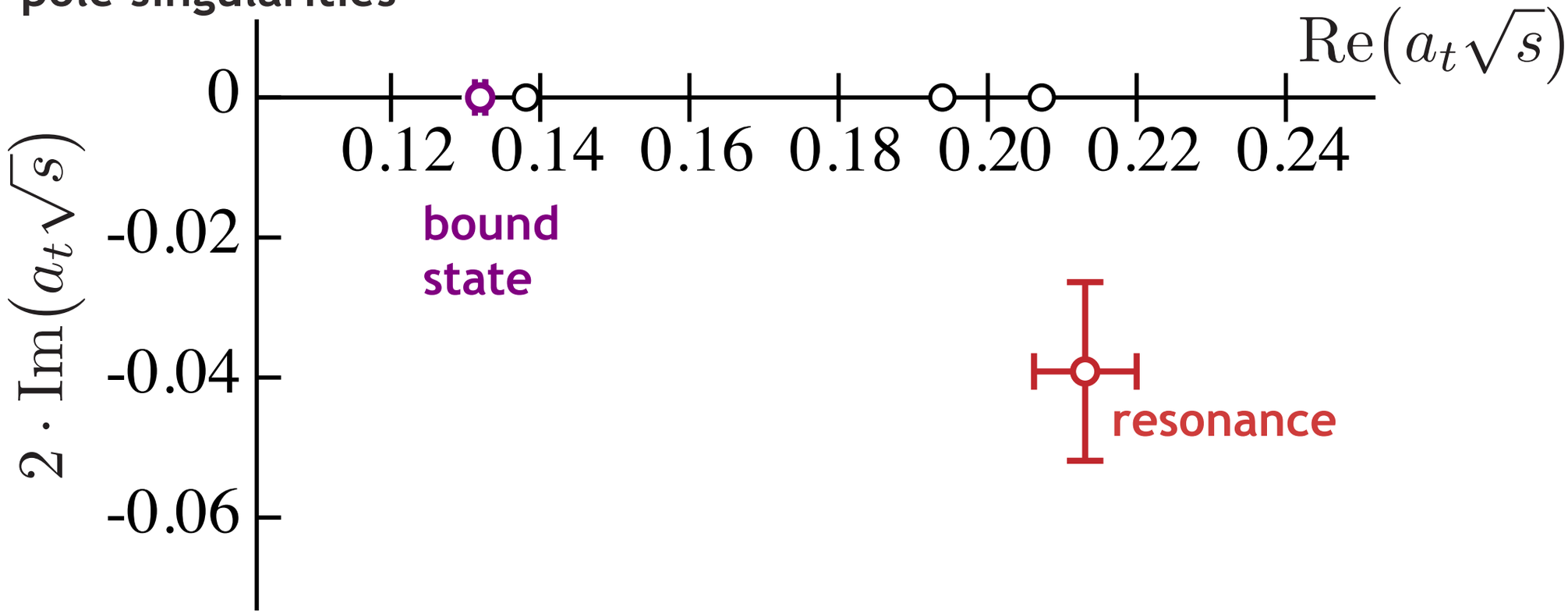
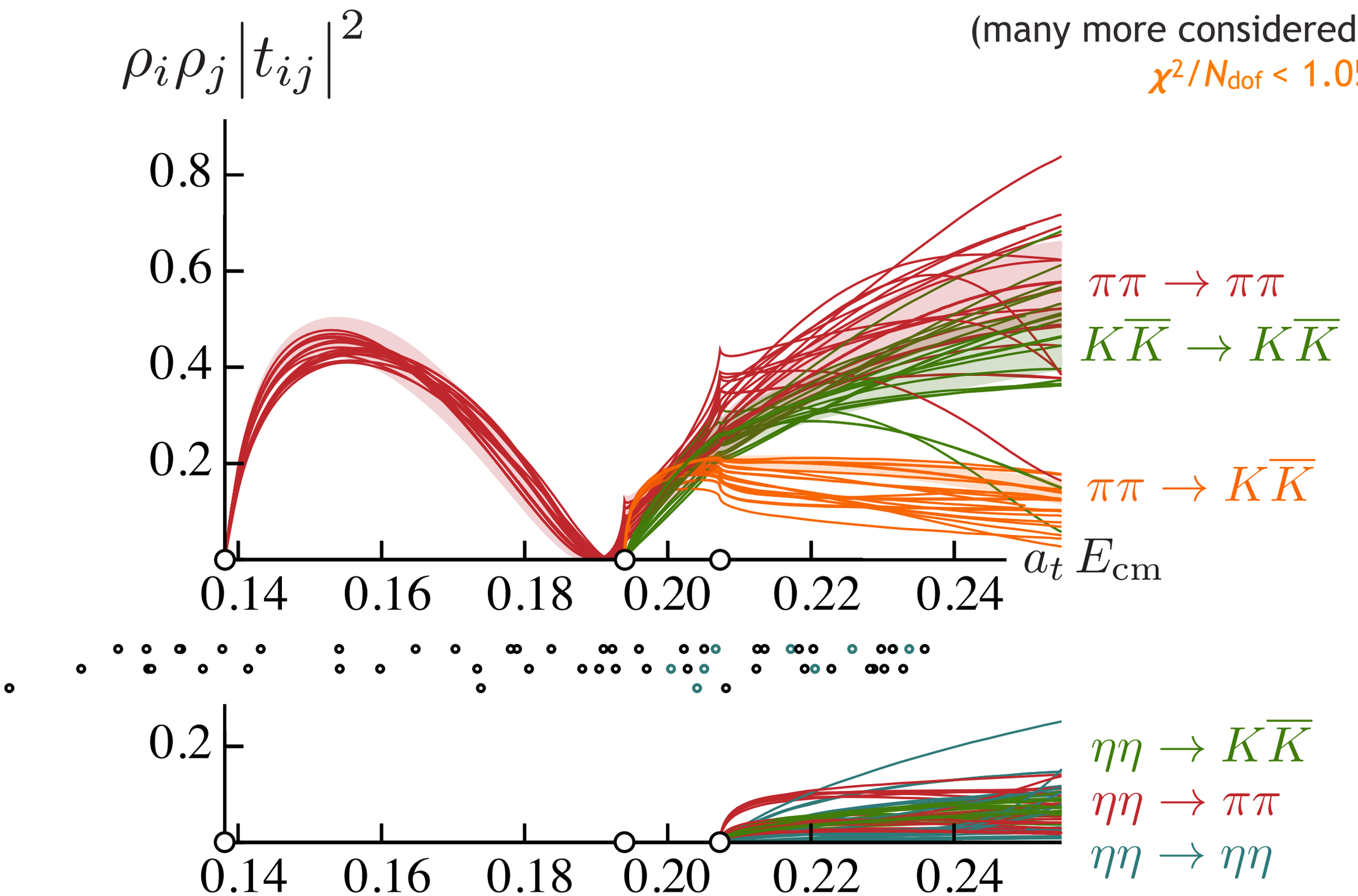


pole singularities



varying the amplitude parameterization $m_\pi \sim 391\,\mathrm{MeV}$

20 variations
(many more considered)
$\chi^2/N_{\mathrm{dof}} < 1.05$



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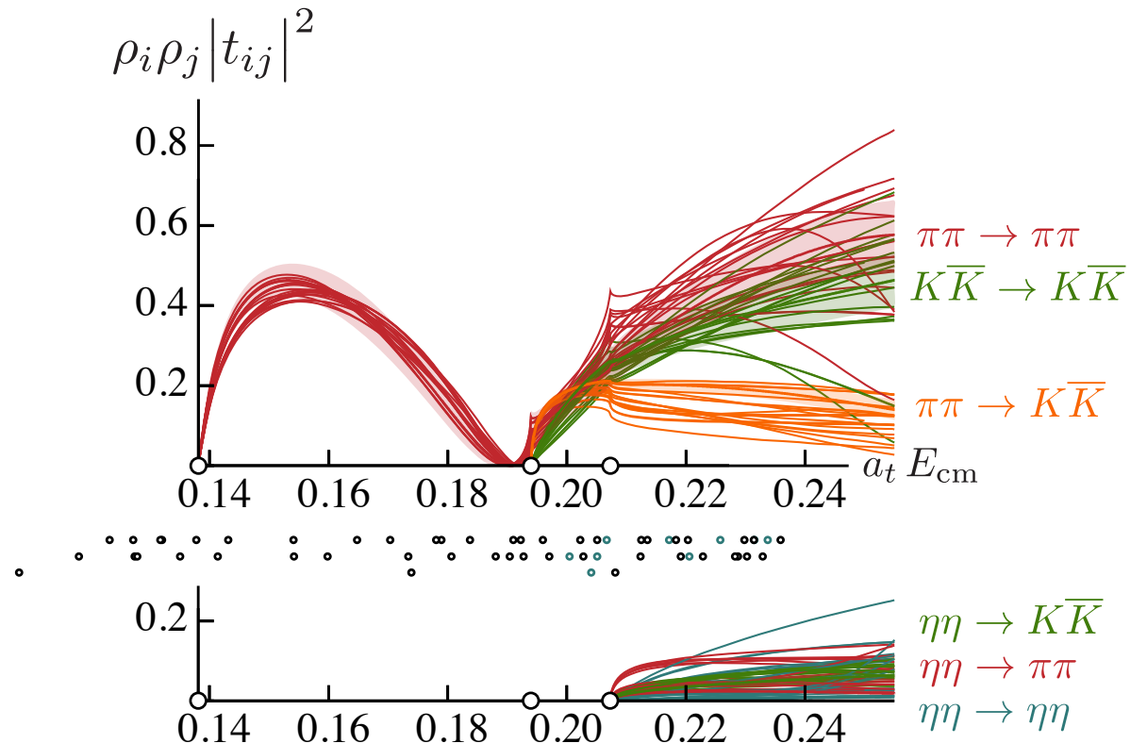
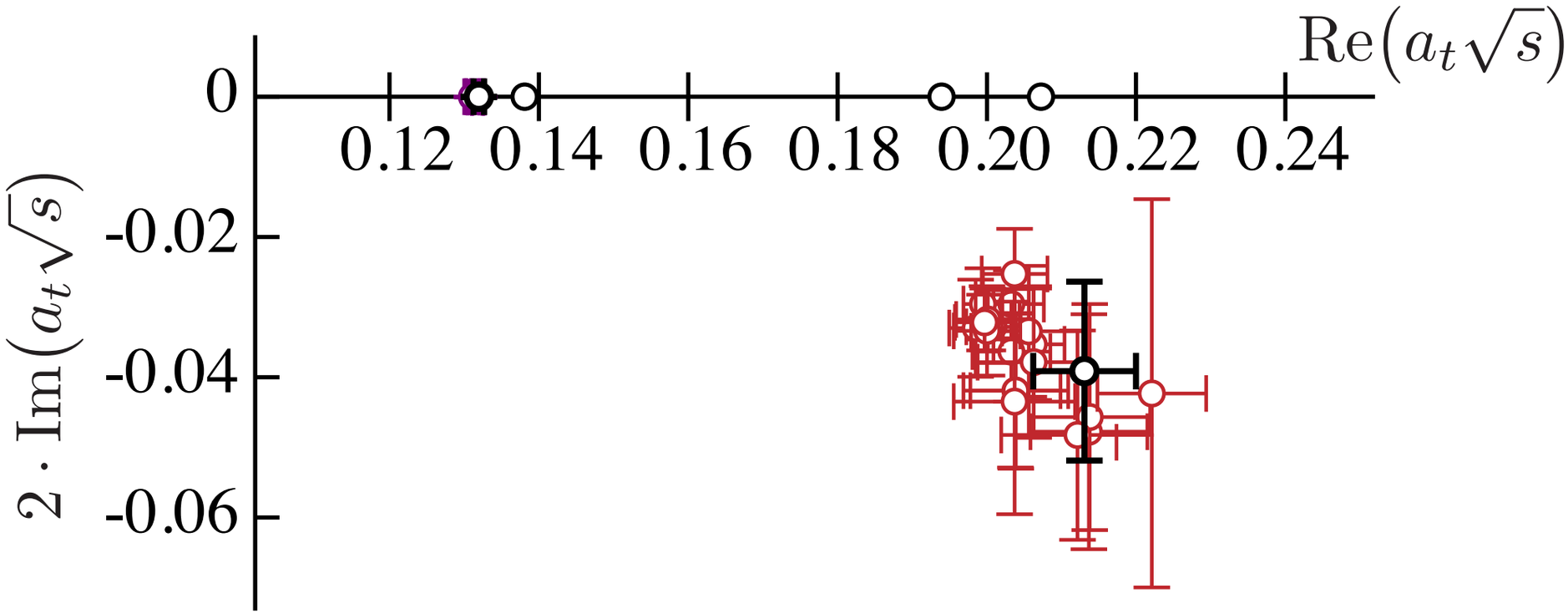
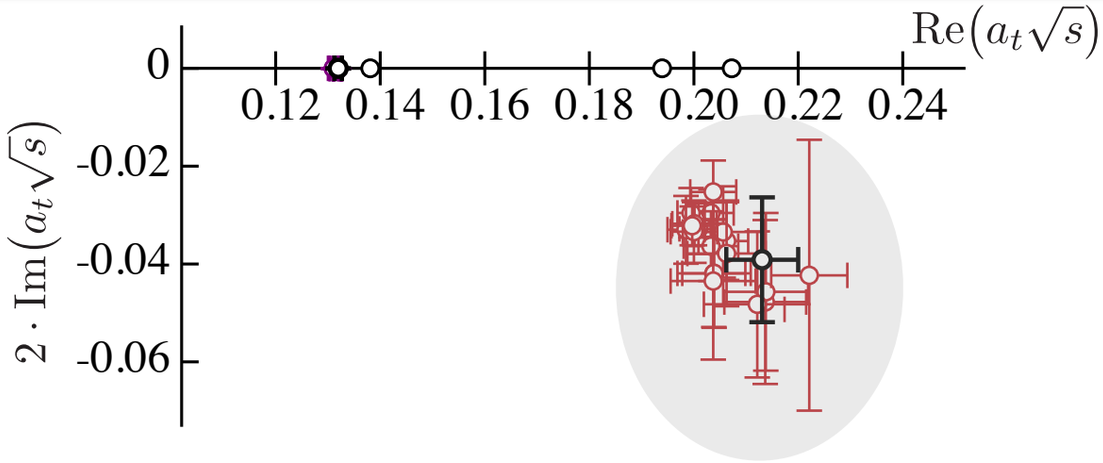


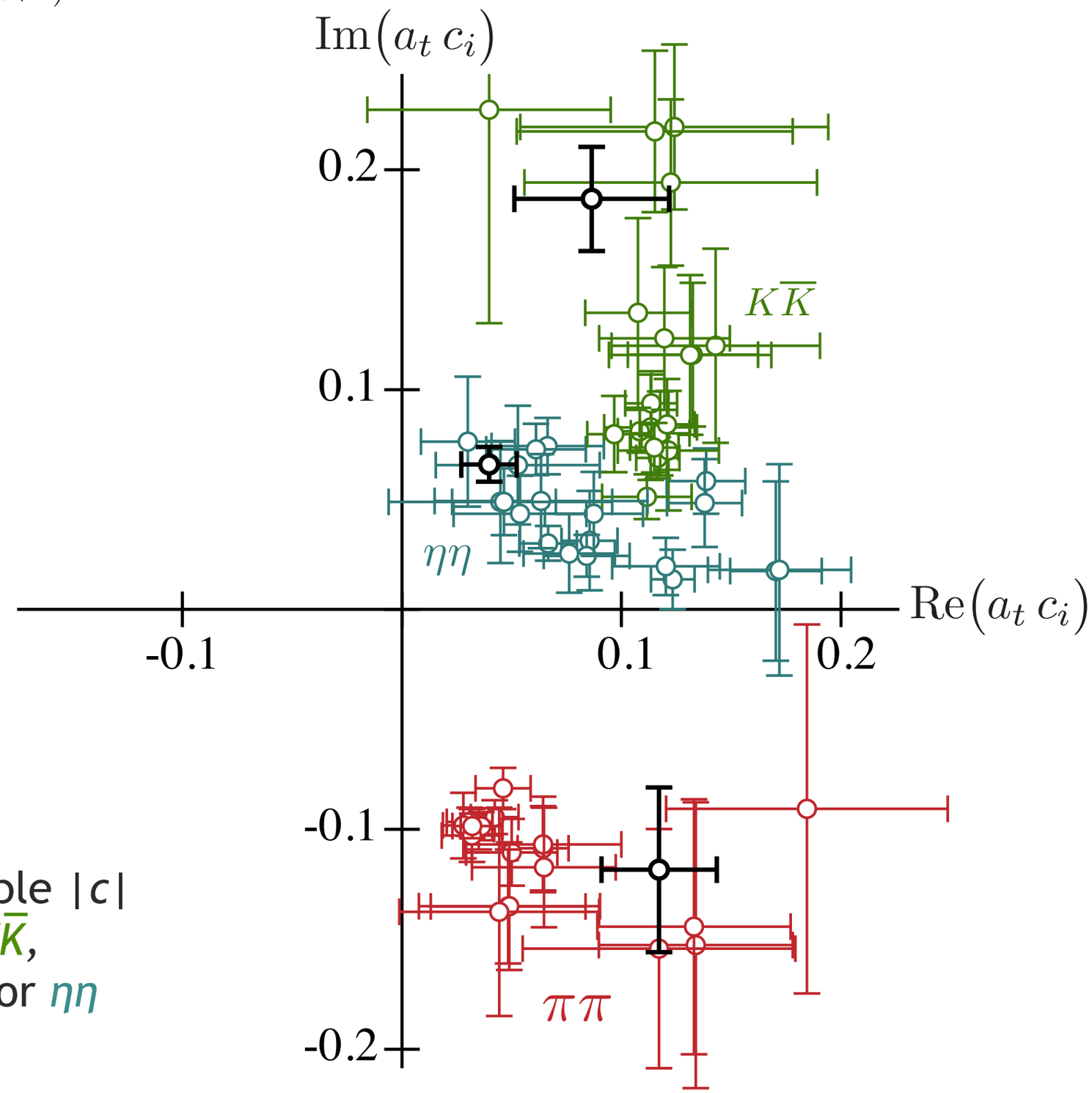


couplings from pole residue

$m_\pi \sim 391\,\mathrm{MeV}$



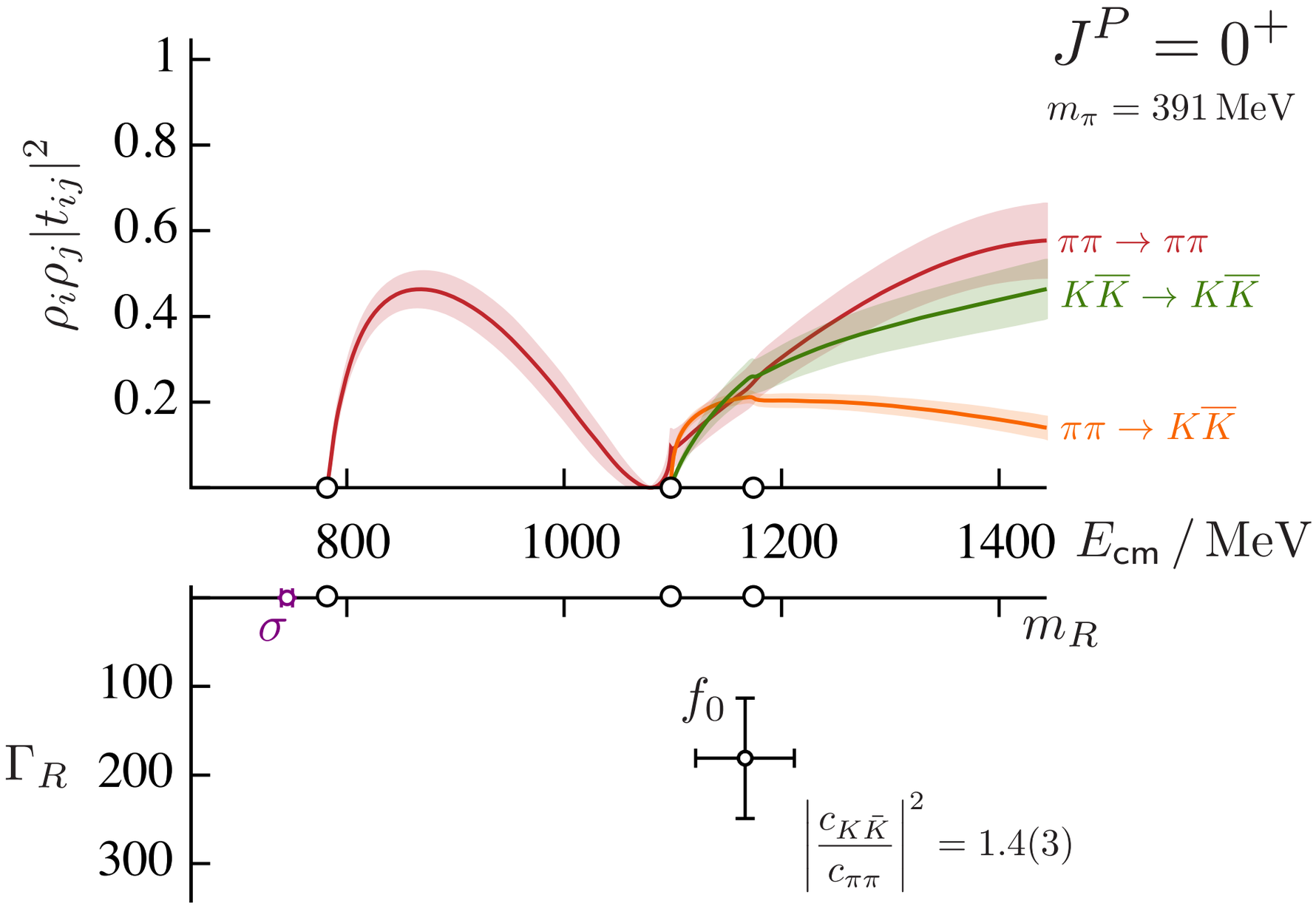
$$t_{ij}^{(\ell)}(s) \sim \frac{c_i\, c_j}{s_0 - s}$$

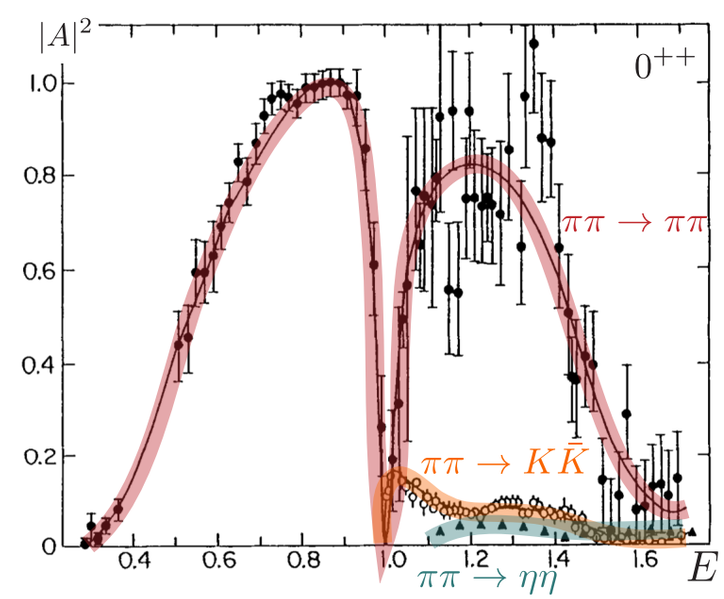


comparable |c|
for $\pi\pi$, $K\overline{K}$,
smaller for $\eta\eta$

S-wave summary

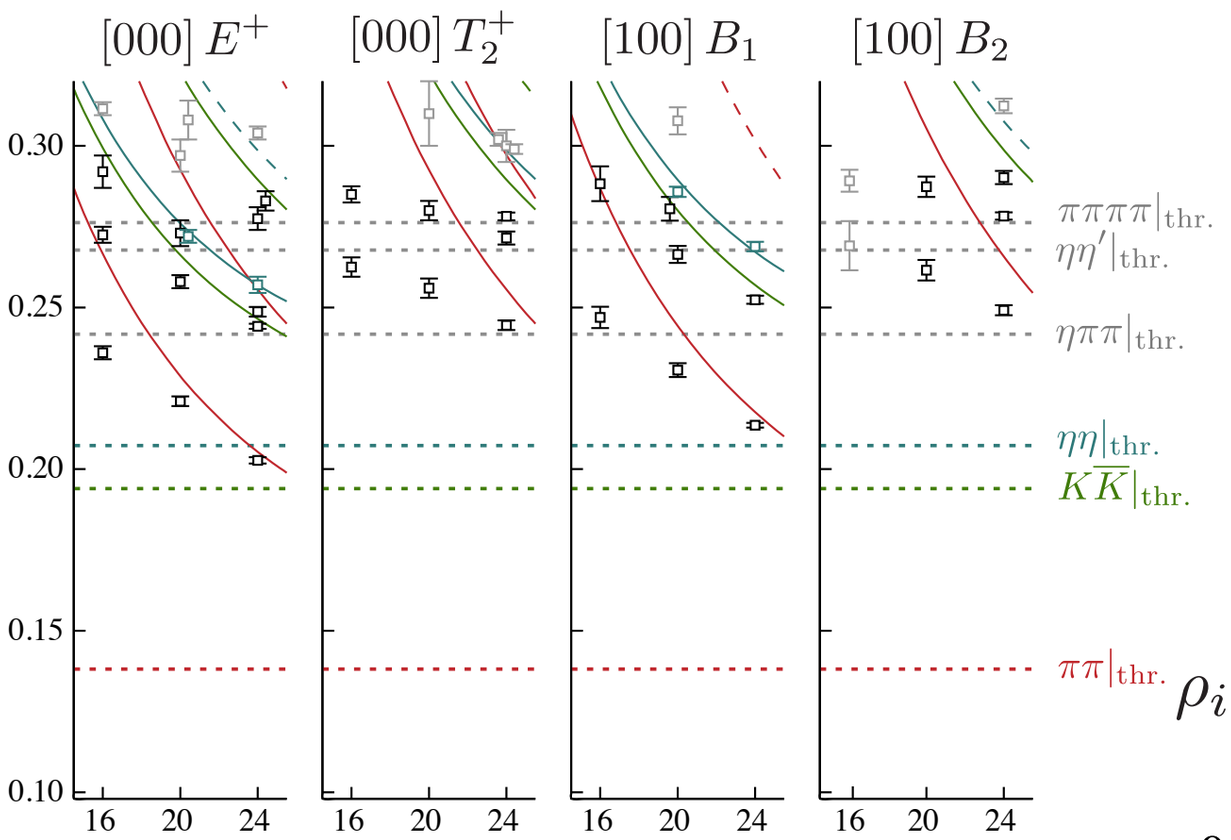
$m_\pi \sim 391\,\mathrm{MeV}$

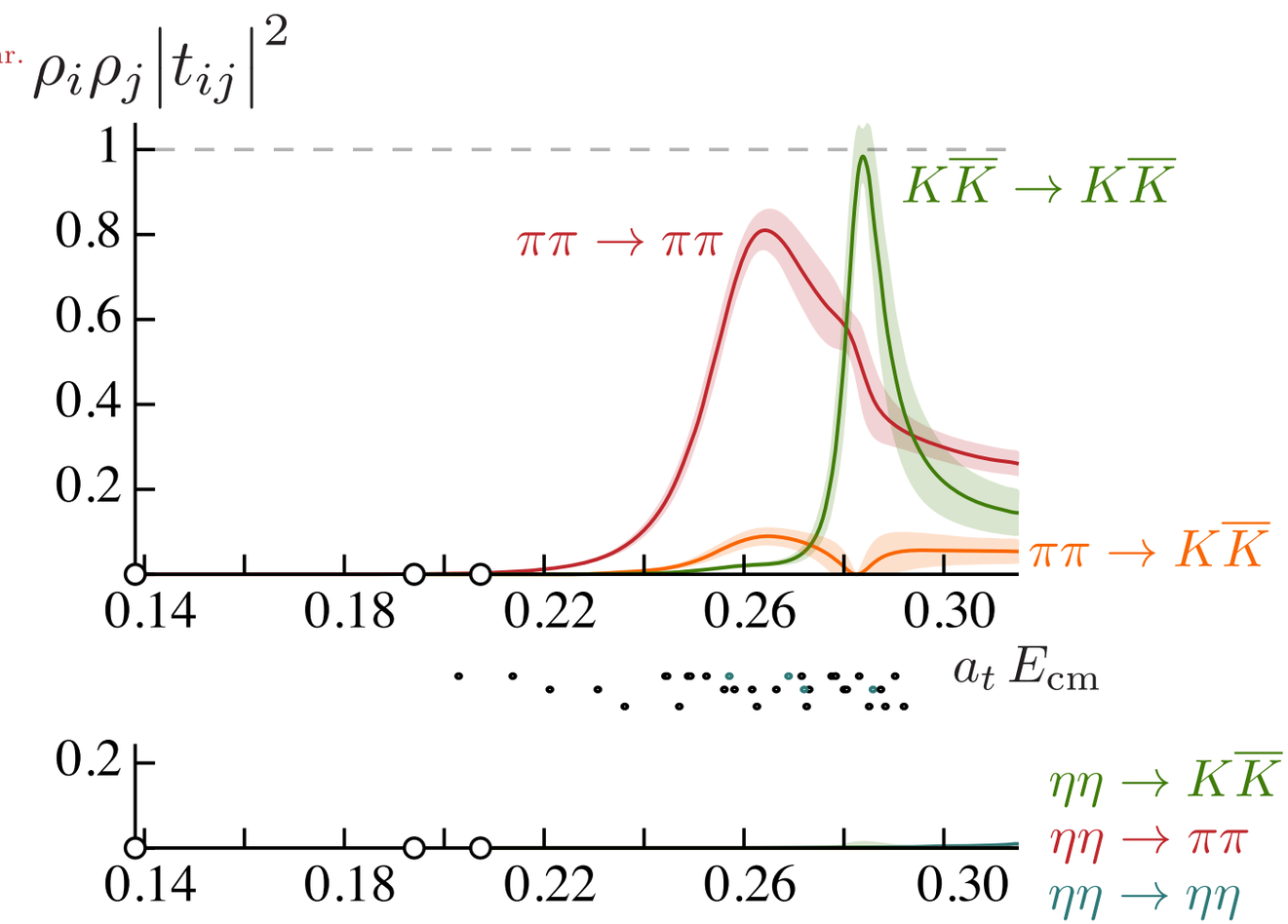




D-wave

$m_\pi \sim 391\,\mathrm{MeV}$

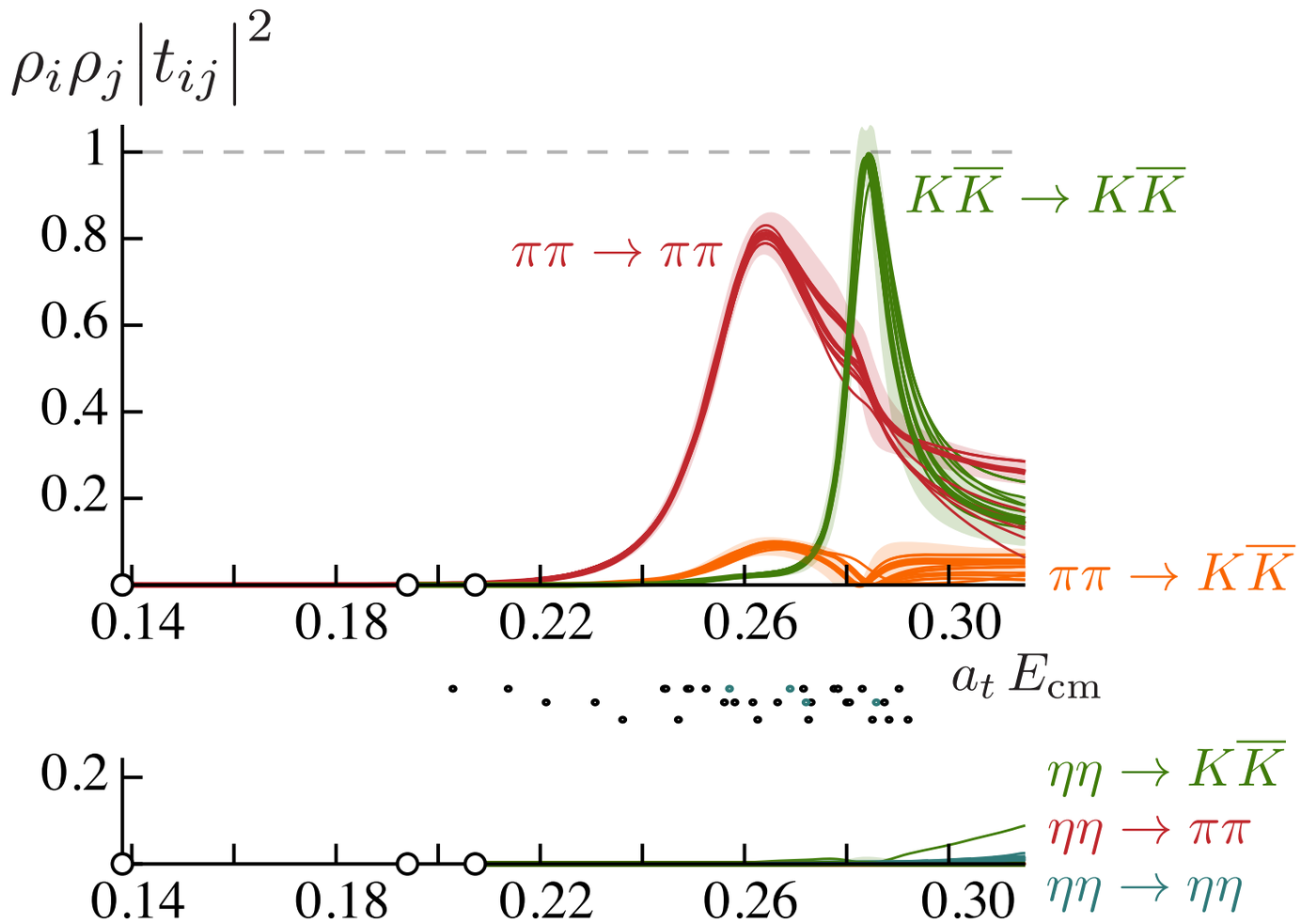




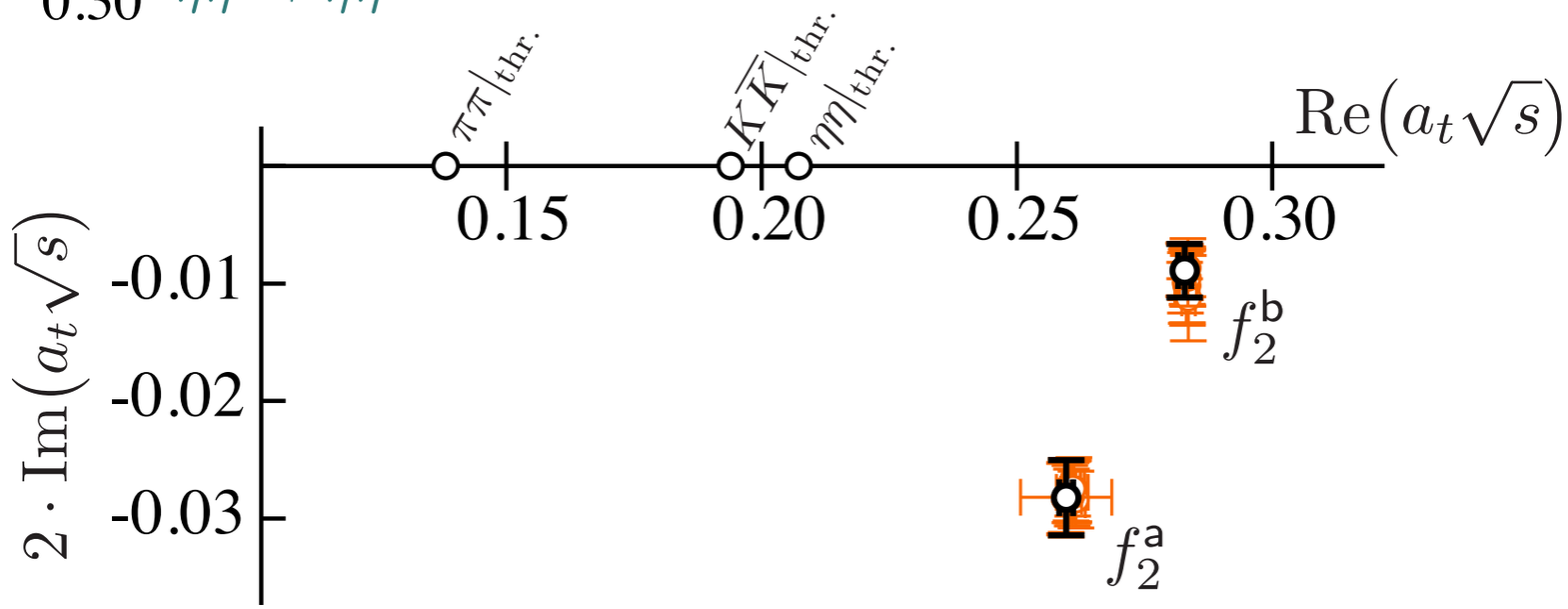
$\chi^2/N_{\mathrm{dof}} = 28.9/(34-9) = 1.15$

D-wave — amplitude variations

$m_\pi \sim 391\,\mathrm{MeV}$

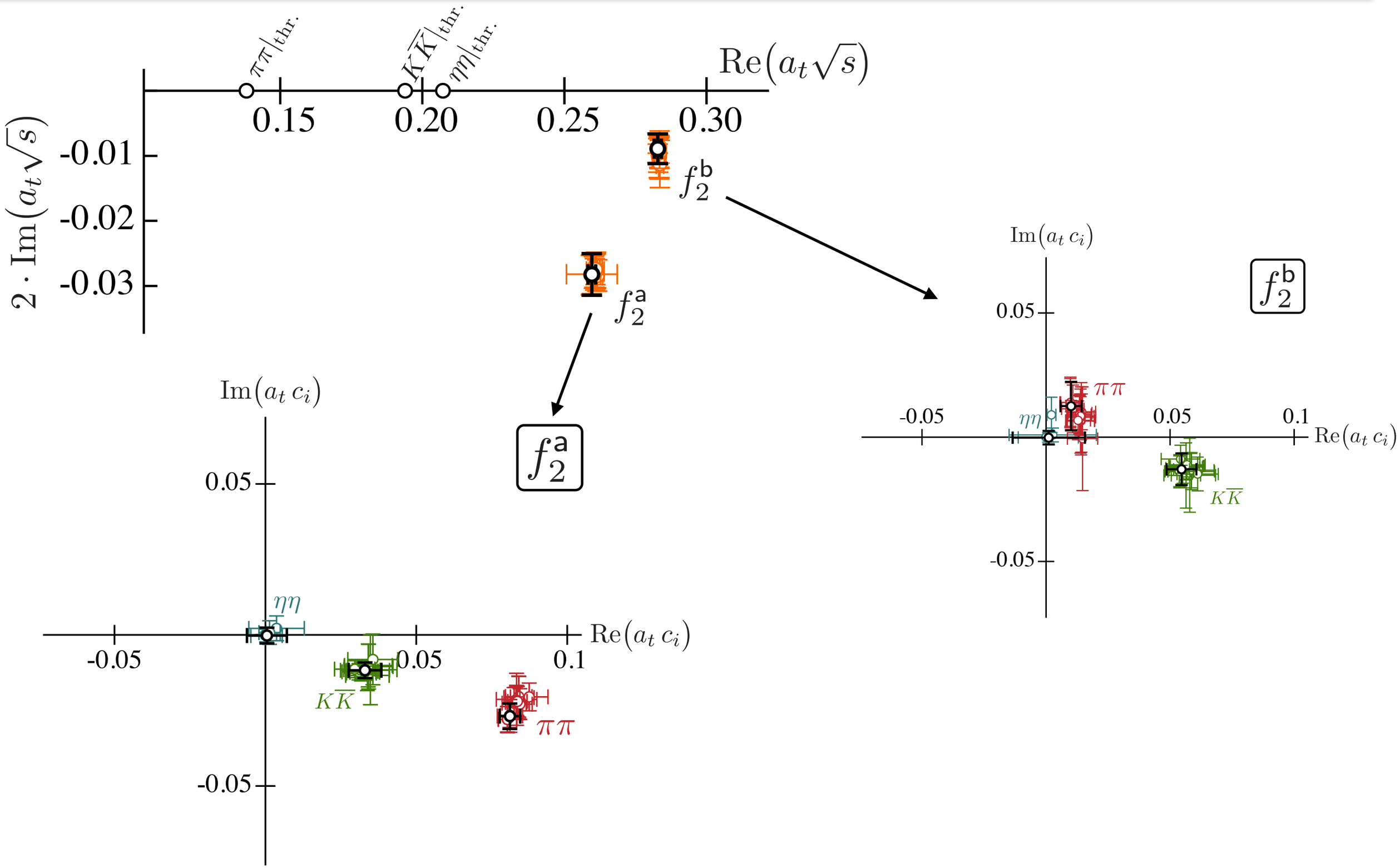


$\chi^2/N_{\mathrm{dof}} < 1.2$



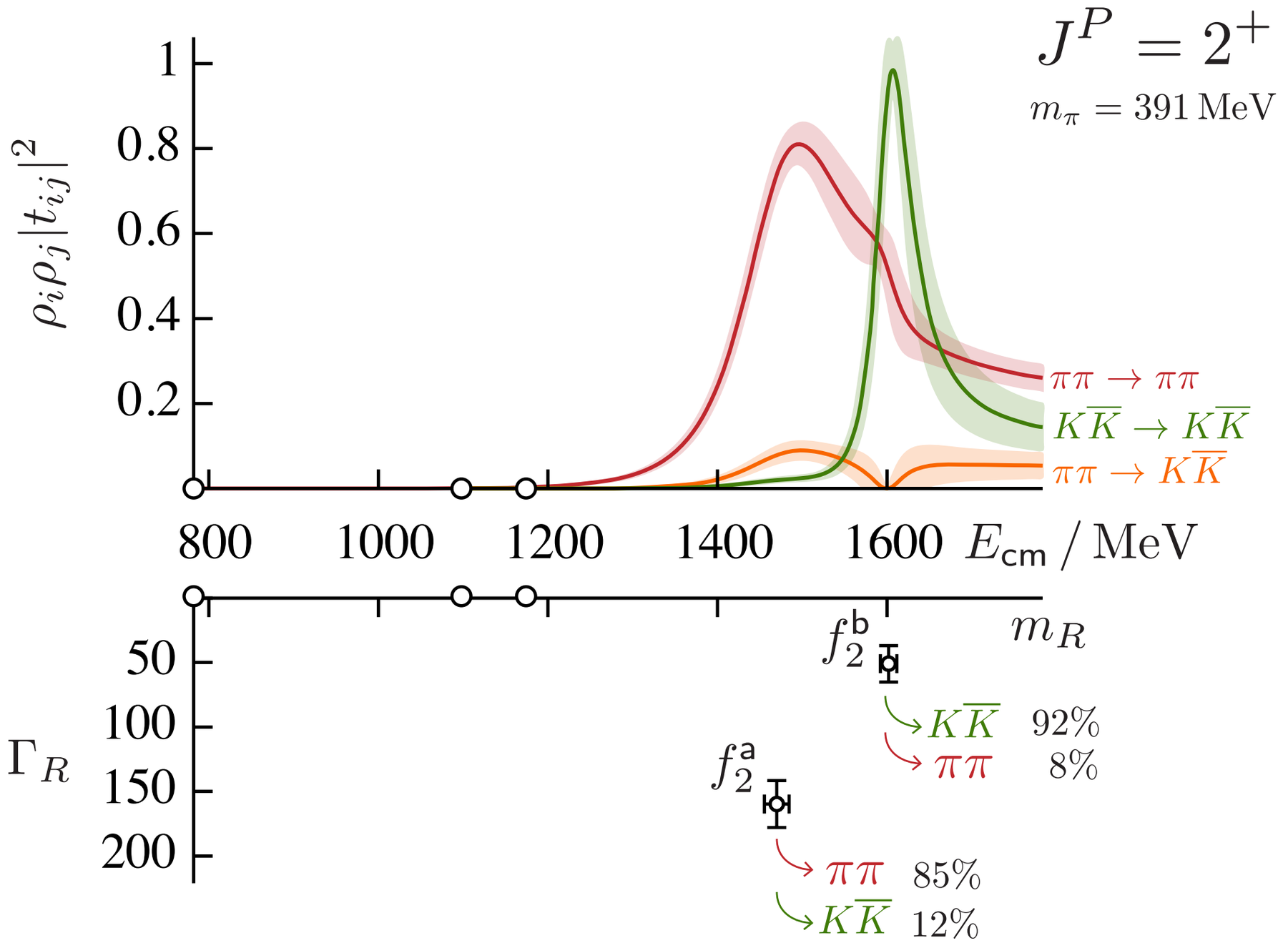
D-wave — resonance couplings

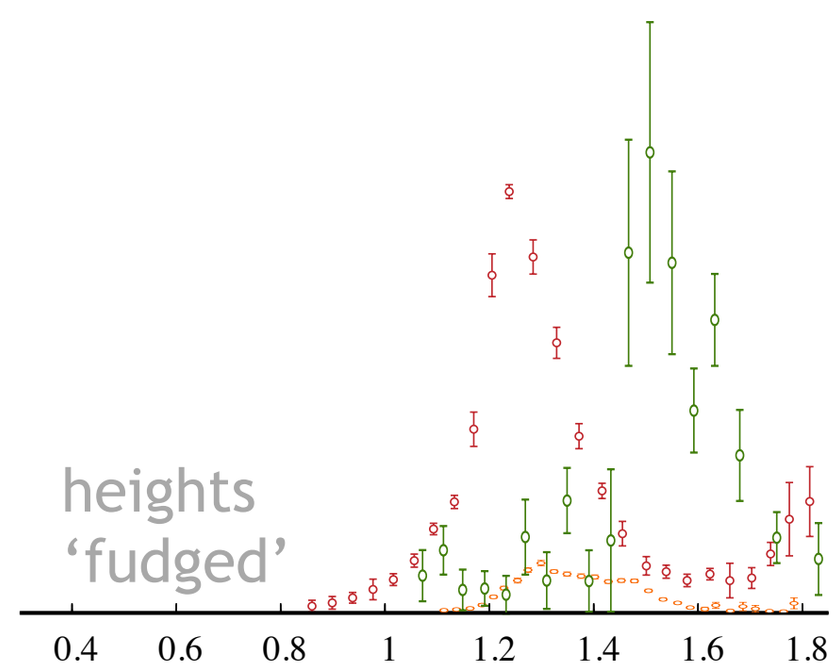
$m_\pi \sim 391\,\mathrm{MeV}$



D-wave summary

$m_\pi \sim 391\,\mathrm{MeV}$





other scalar mesons ?

$m_\pi \sim 391\,\mathrm{MeV}$

previously calculated coupled $\pi\eta, K\bar{K}$ I=1 scattering

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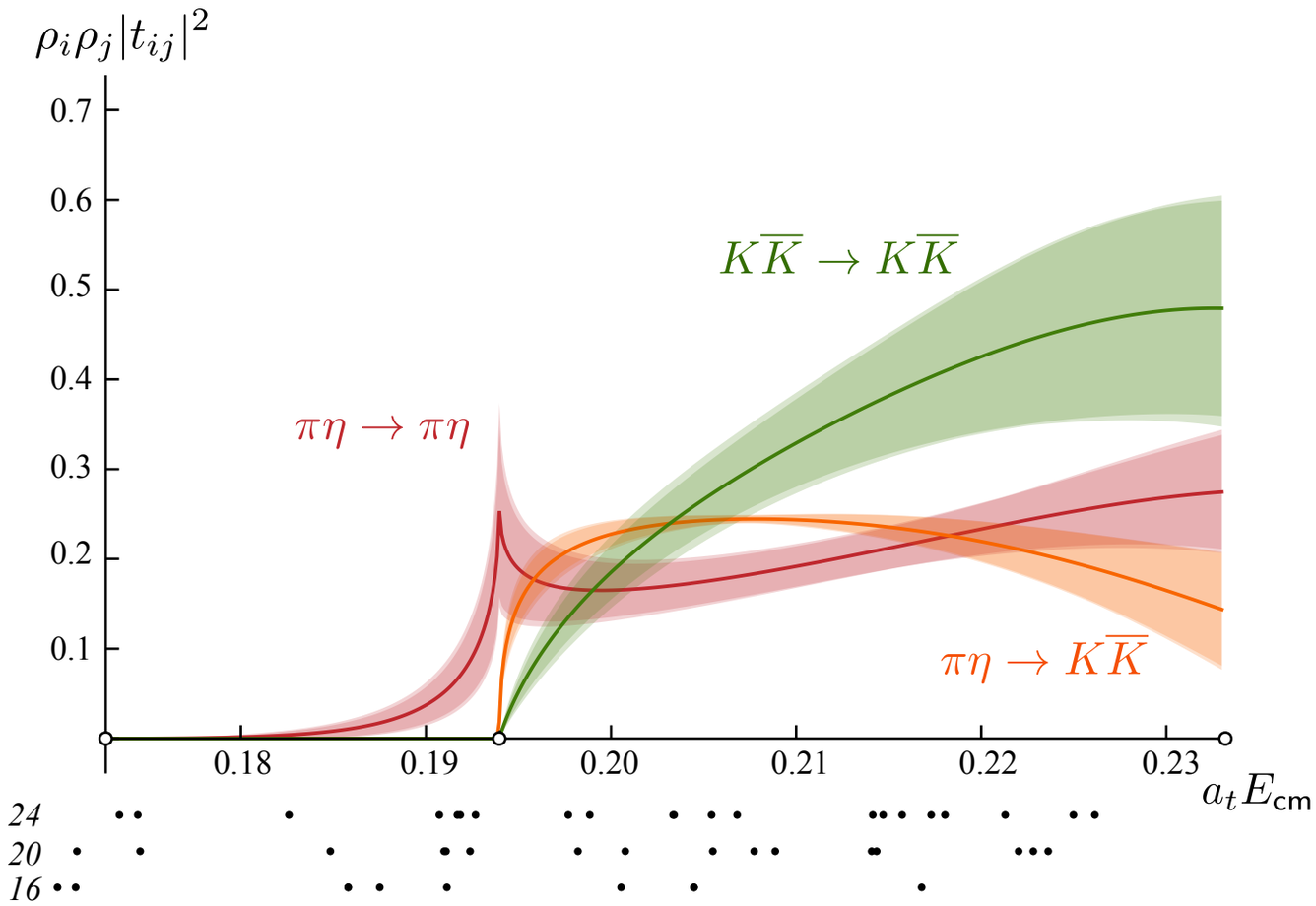
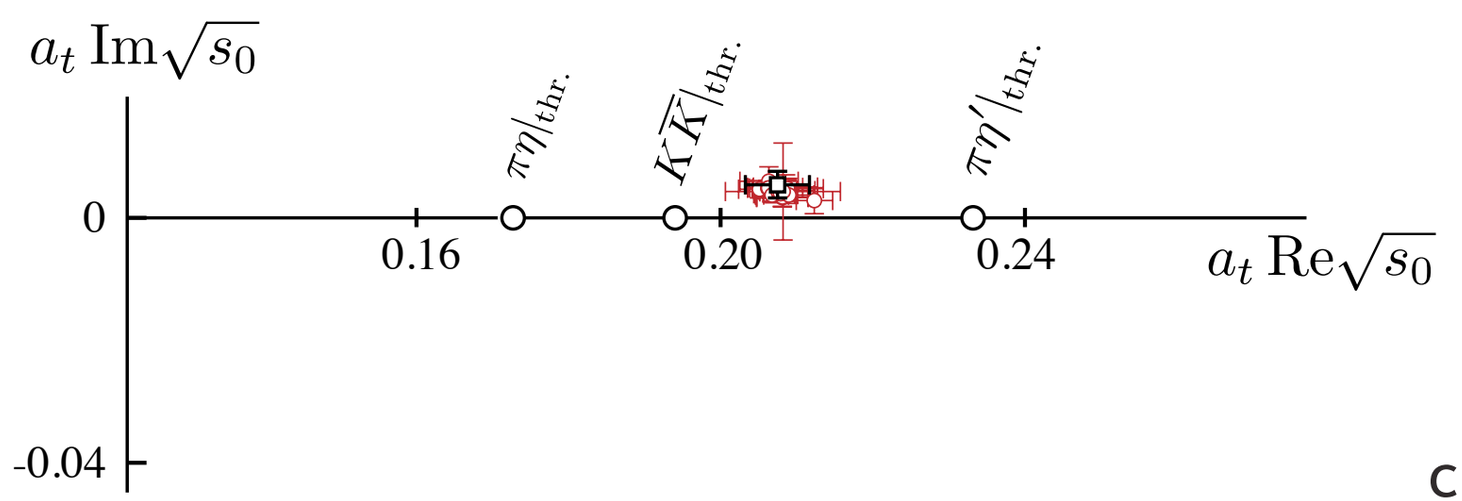


sharp asymmetric peak at $K\bar{K}$ threshold



c.f. $a_0(980)$

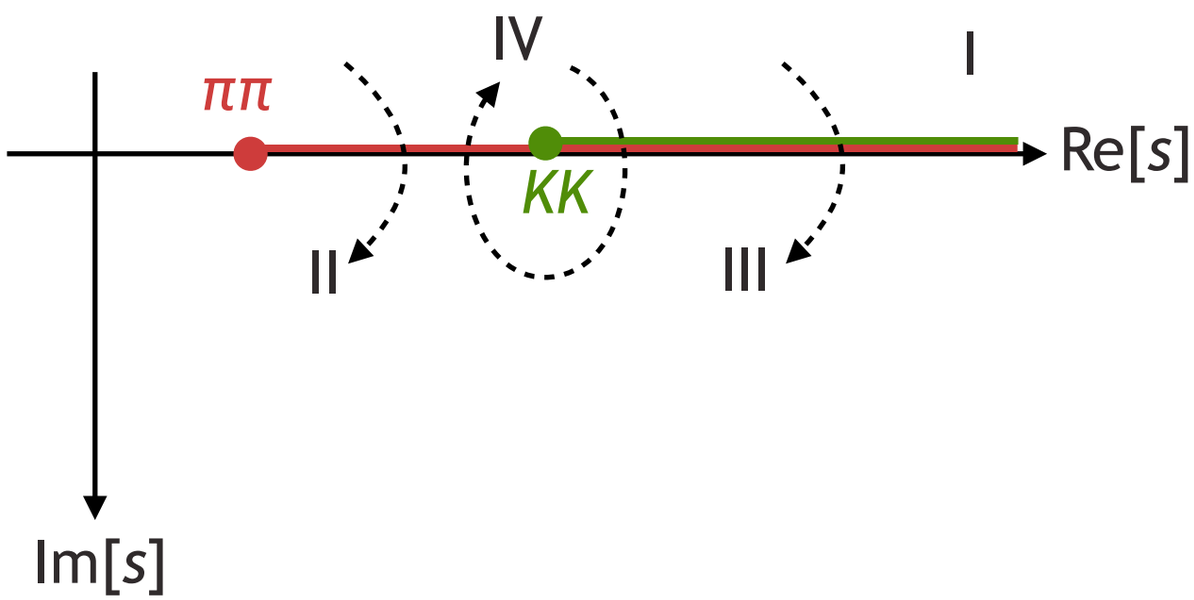
sheets ?

complex *s*-plane actually multi-sheeted

unitarity $\mathrm{Im}\big[t_{ij}(s)\big] = -\delta_{ij}\,\rho_i(s)$

$$\rho_i(s) = \sqrt{1 - \frac{4m_i^2}{s}}$$

square-root branch-point at each threshold



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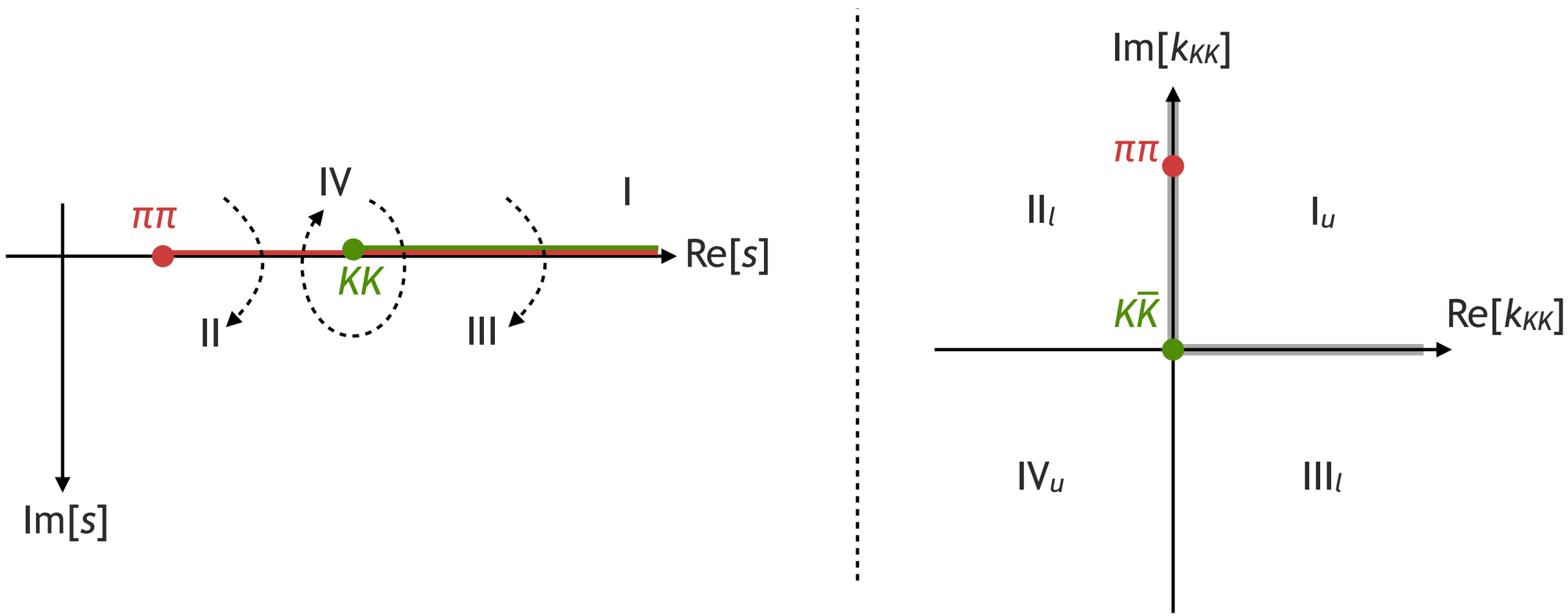


sheets ?

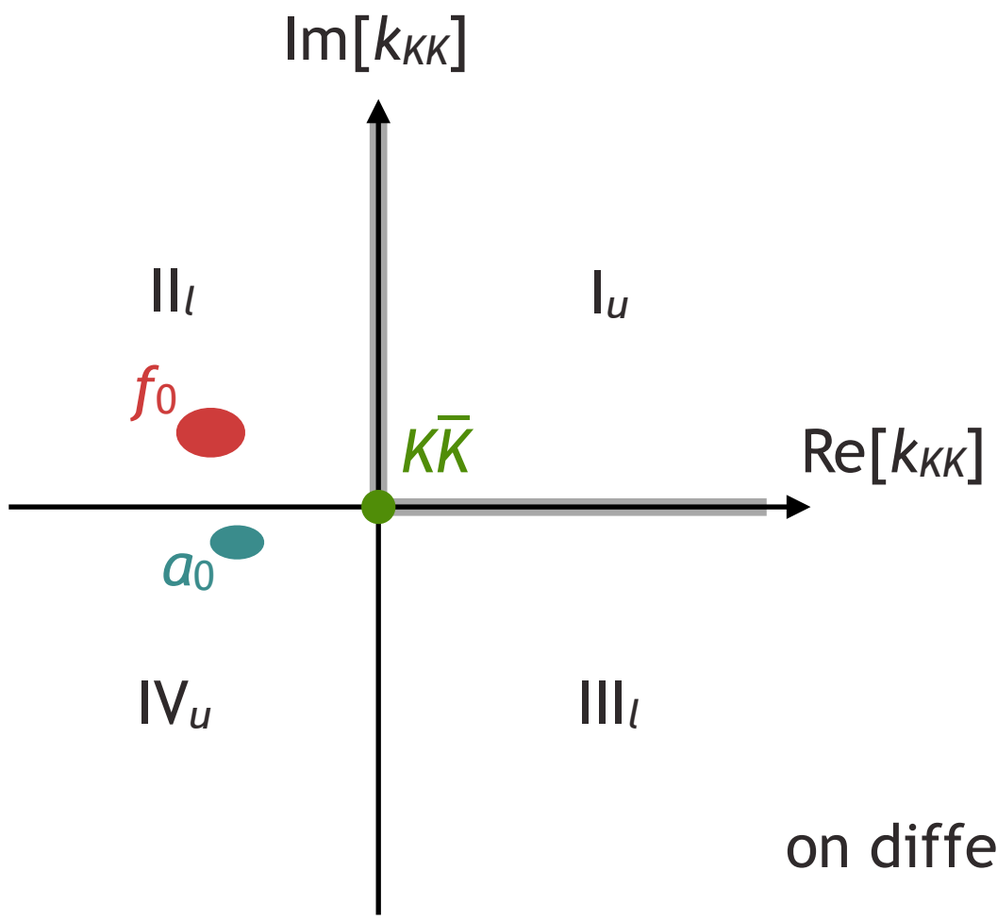
$m_\pi \sim 391\,\mathrm{MeV}$

$m_R(f_0) = 1166(45)\,\mathrm{MeV}, \quad \Gamma_R(f_0) = 181(68)\,\mathrm{MeV},$

$m_R(a_0) = 1177(27)\,\mathrm{MeV}, \quad \Gamma_R(a_0) = 49(33)\,\mathrm{MeV}.$

$$\left|c(a_0 \to K\overline{K})\right| \approx \left|c(f_0 \to K\overline{K})\right| \quad \sim 850\,\mathrm{MeV}$$
$$\left|c(a_0 \to \pi\eta)\right| \approx \left|c(f_0 \to \pi\pi)\right| \quad \sim 700\,\mathrm{MeV}.$$

look very similar (in mass and couplings), but …



on different sheets ?

‘explaining’ the sheet distribution

e.g. Flatté form $D(s) = m_0^2 - s - i g_1^2 \rho_1(s) - i g_2^2 \rho_2(s)$

has poles

$$\sqrt{s_0} \approx m_0 \pm \frac{i}{2}\frac{g_2^2 \rho_2}{m_0}\left[\left(\frac{g_1}{g_2}\right)^2 \frac{\rho_1}{\rho_2} - 1\right] \quad \text{on sheet II, if} \quad \left(\frac{g_1}{g_2}\right)^2 \frac{\rho_1}{\rho_2} > 1, \text{ or,}$$

$$\sqrt{s_0} \approx m_0 \pm \frac{i}{2}\frac{g_2^2 \rho_2}{m_0}\left[1 - \left(\frac{g_1}{g_2}\right)^2 \frac{\rho_1}{\rho_2}\right] \quad \text{on sheet IV, if} \quad \left(\frac{g_1}{g_2}\right)^2 \frac{\rho_1}{\rho_2} < 1, \text{ and,}$$

$$\sqrt{s_0} \approx m_0 \pm \frac{i}{2}\frac{g_2^2 \rho_2}{m_0}\left[1 + \left(\frac{g_1}{g_2}\right)^2 \frac{\rho_1}{\rho_2}\right] \quad \text{on sheet III, in all cases,}$$

$m_R(f_0) = 1166(45)\,\text{MeV}, \quad \Gamma_R(f_0) = 181(68)\,\text{MeV},$

$m_R(a_0) = 1177(27)\,\text{MeV}, \quad \Gamma_R(a_0) = 49(33)\,\text{MeV}.$

$$\left|c(a_0 \to K\overline{K})\right| \approx \left|c(f_0 \to K\overline{K})\right| \quad \sim 850\,\text{MeV}$$

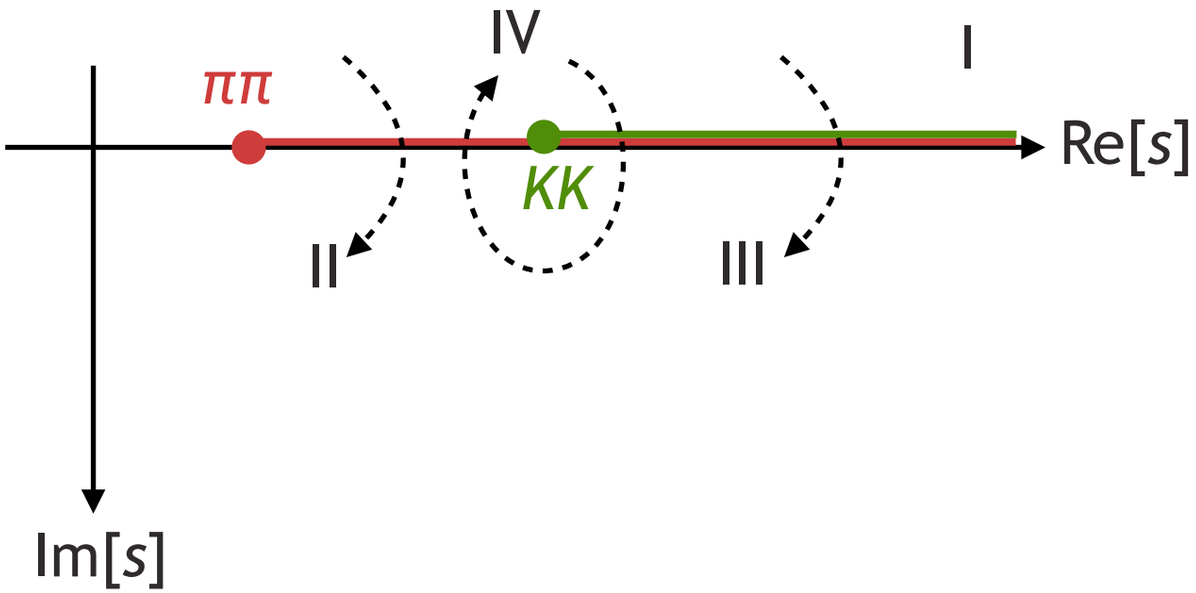
$$\left|c(a_0 \to \pi\eta)\right| \approx \left|c(f_0 \to \pi\pi)\right| \quad \sim 700\,\text{MeV}.$$

but larger phase-space for $\pi\pi$ than $\pi\eta$

interpreting the sheet distribution ?

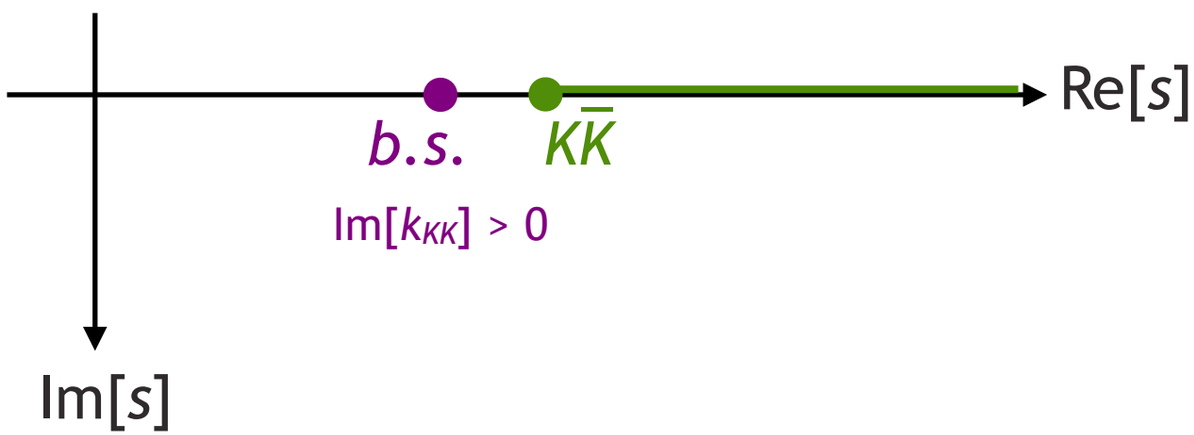
a pole on **only** sheet II or sheet IV ⇒ ‘molecular resonance’ ?

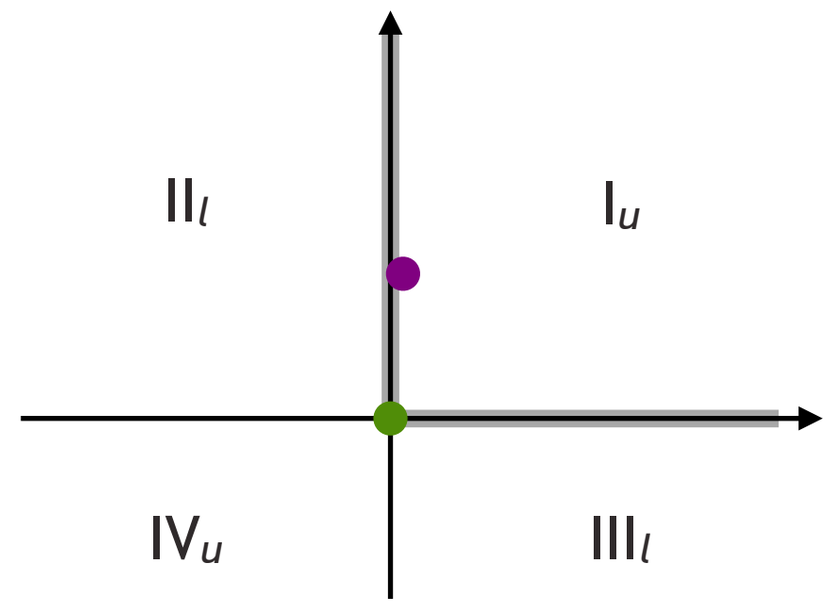
explained to me by Adam,
i’m still trying to understand …



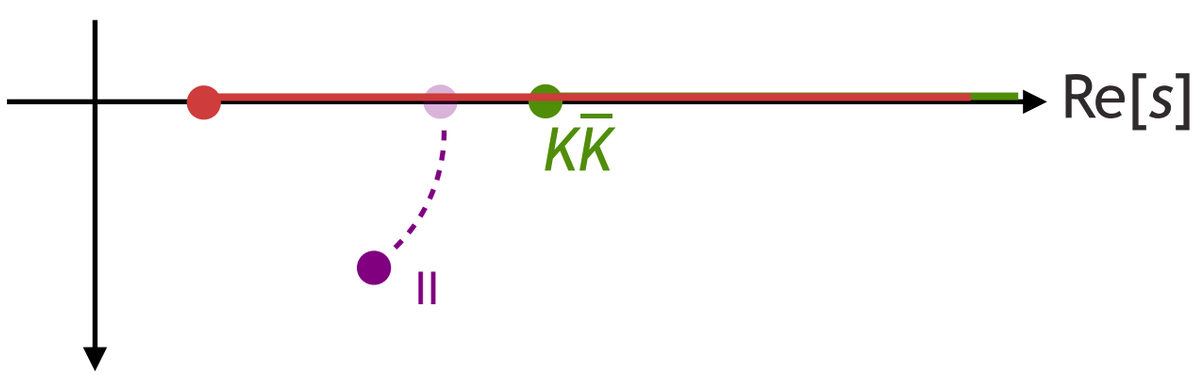
interpreting the sheet distribution ?

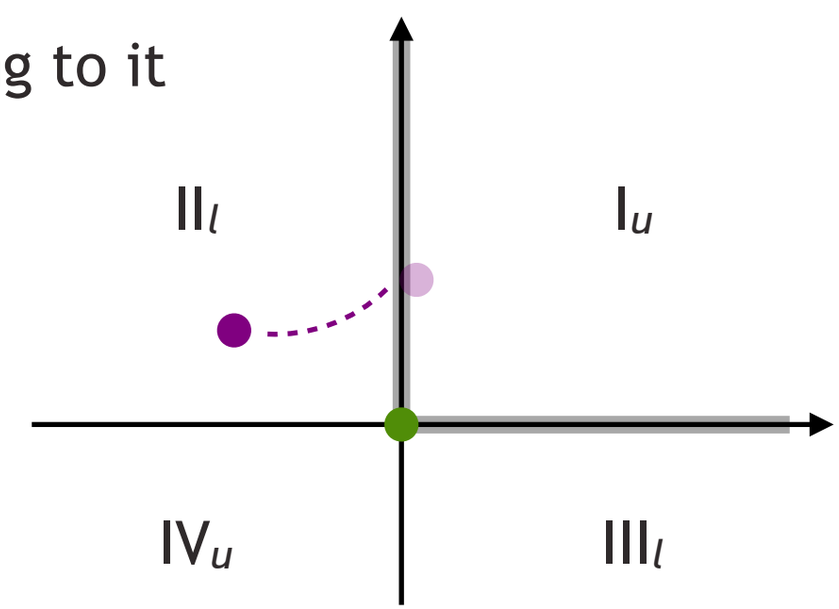
imagine no lower channel
and binding dynamics in $K\bar{K}$





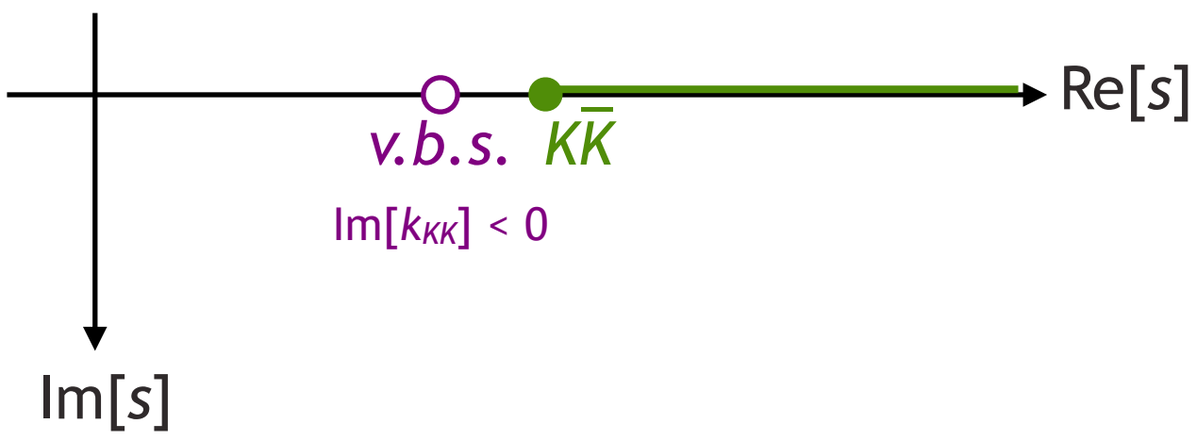
now 'turn on' the lower channel and allow a coupling to it

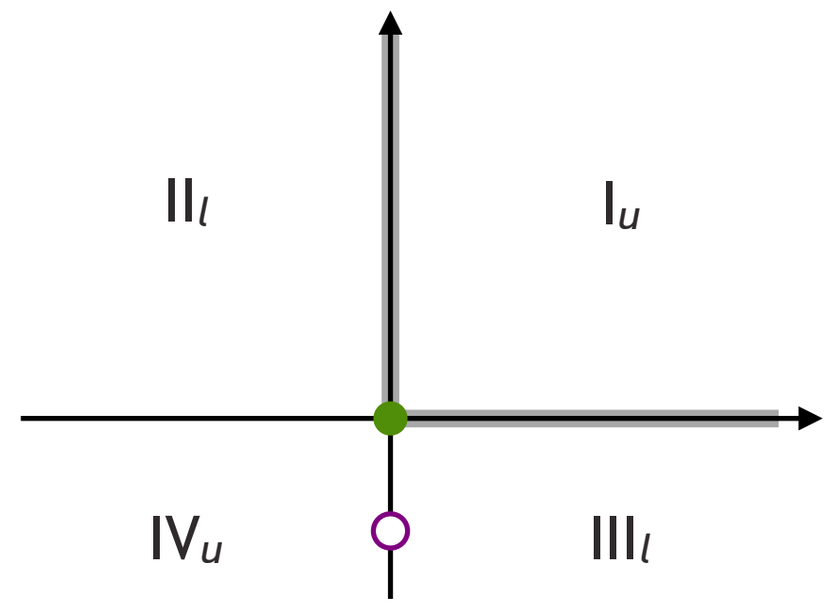




interpreting the sheet distribution ?

imagine no lower channel
and weaker attraction in $K\bar{K}$





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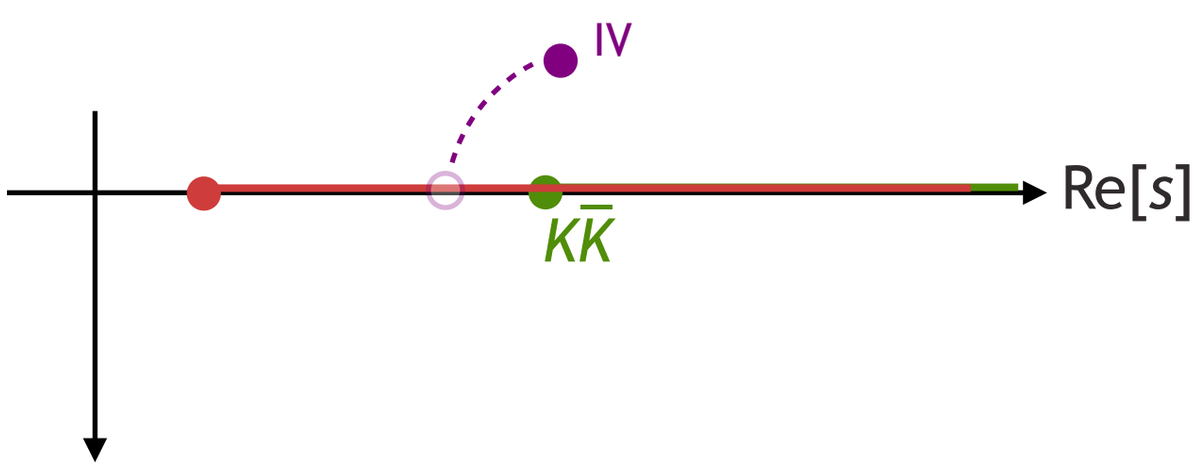

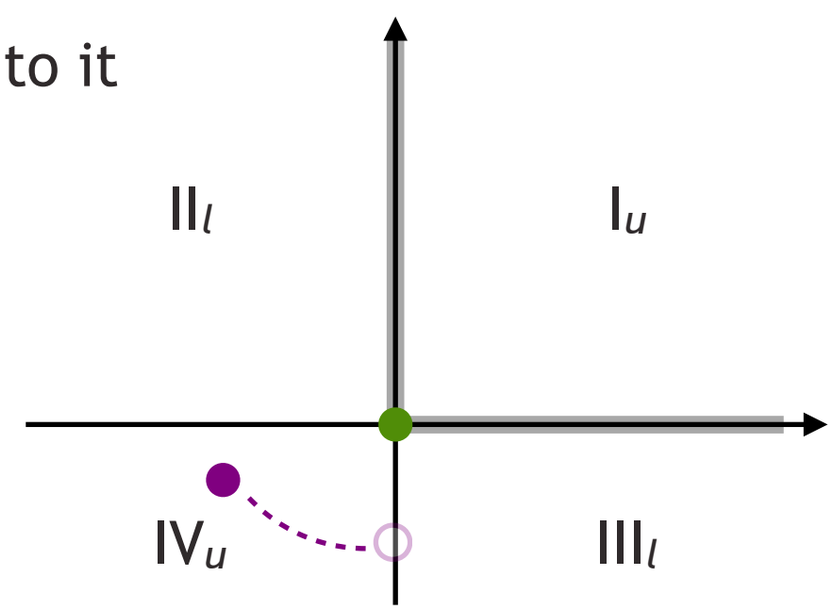

interpreting the sheet distribution ?

on the other hand …

an ‘ordinary’ resonance is expected to have ‘mirror’ poles:

e.g. Flatté form

$$D(s) = m_0^2 - s - i g_1^2 \rho_1(s) - i g_2^2 \rho_2(s)$$

has poles

$$\sqrt{s_0} \approx m_0 \pm \frac{i}{2}\frac{g_2^2 \rho_2}{m_0}\left[\left(\frac{g_1}{g_2}\right)^2 \frac{\rho_1}{\rho_2} - 1\right] \quad \text{on sheet II, if } \left(\frac{g_1}{g_2}\right)^2 \frac{\rho_1}{\rho_2} > 1, \text{ or,}$$

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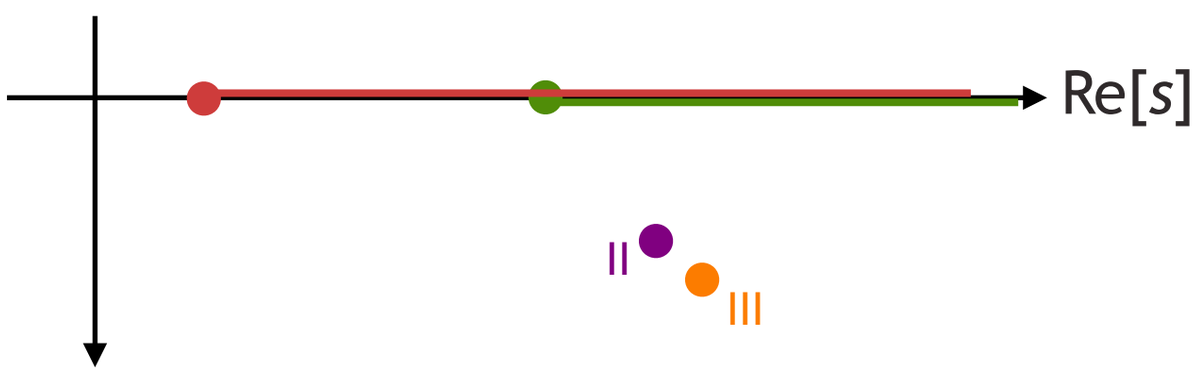
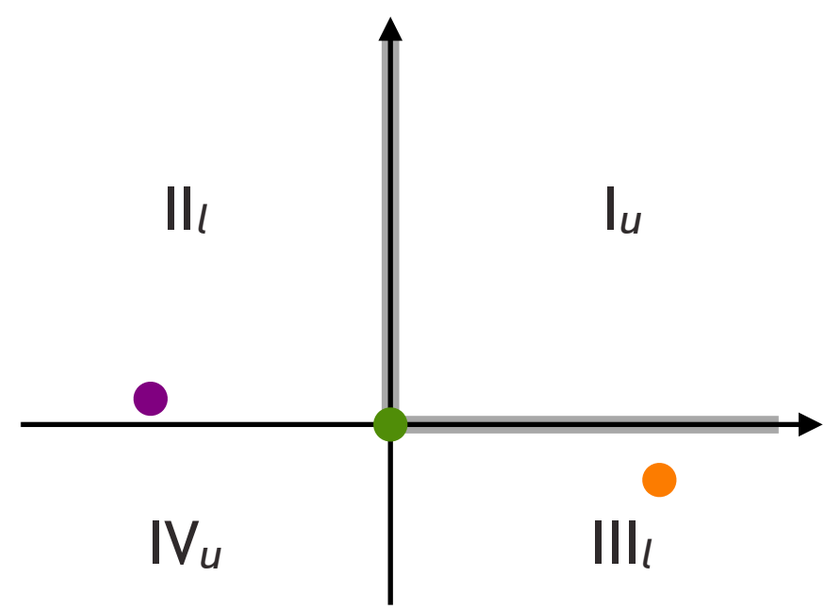
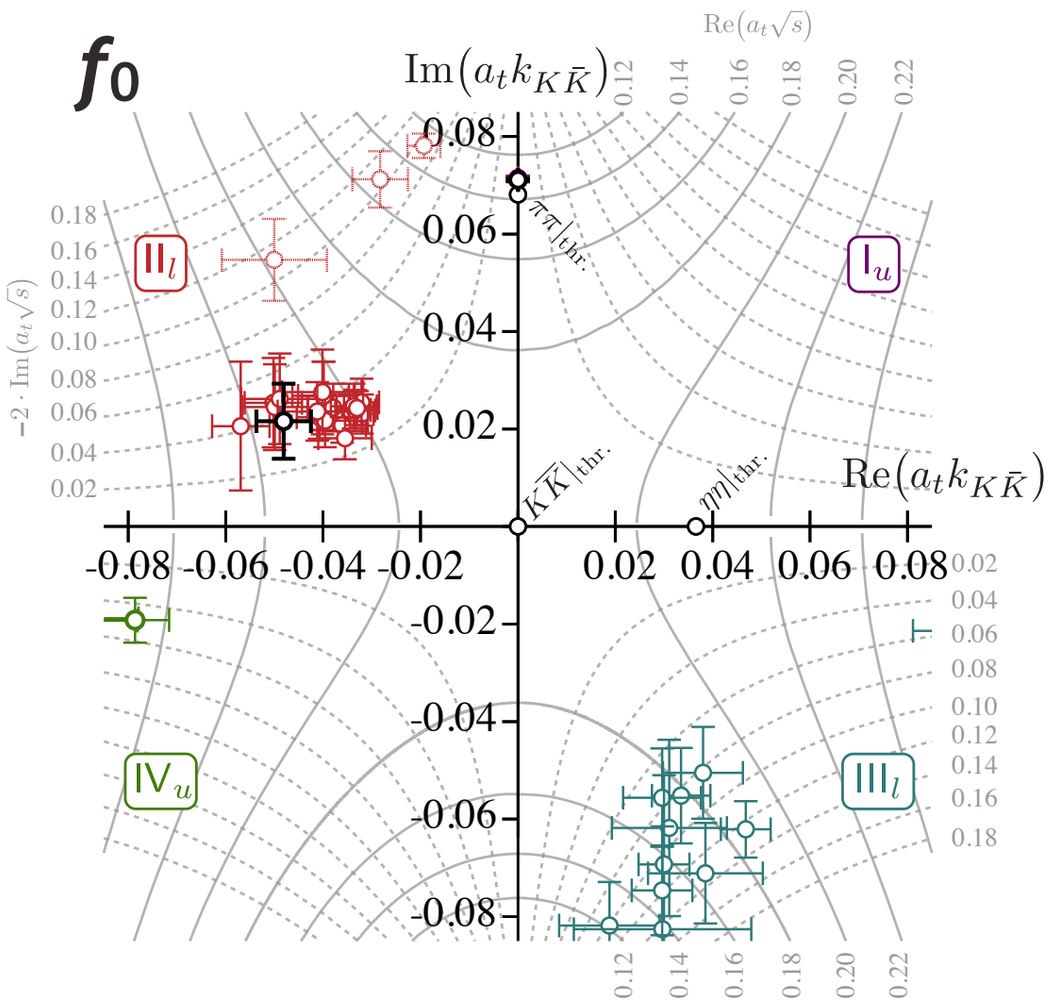
$$\sqrt{s_0} \approx m_0 \pm \frac{i}{2}\frac{g_2^2 \rho_2}{m_0}\left[1 + \left(\frac{g_1}{g_2}\right)^2 \frac{\rho_1}{\rho_2}\right] \quad \text{on sheet III, in all cases,}$$

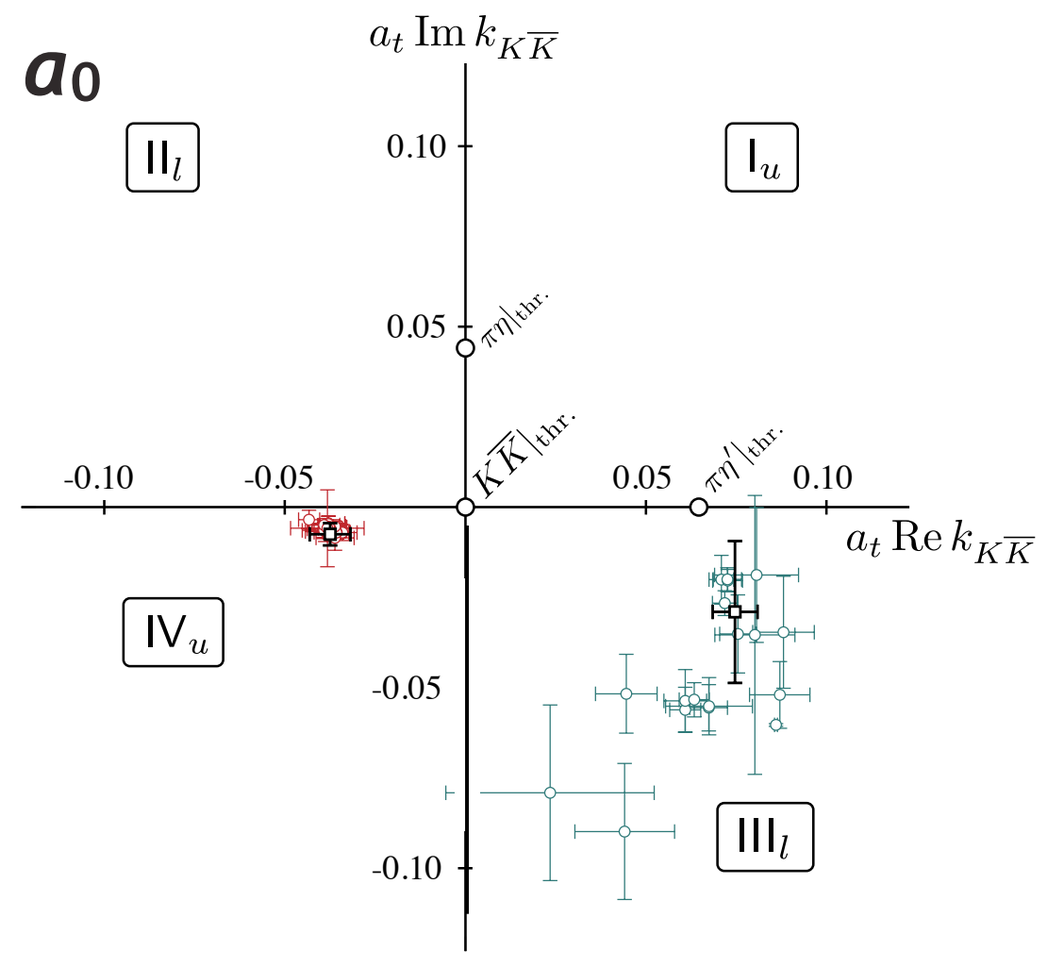




poles on other sheets in the lattice calc ?

$m_\pi \sim 391\,\mathrm{MeV}$





parameterization dependent
distant poles on sheet III

looks more like
one pole ⇒ 'molecular resonance' ?

summary — isoscalar mesons

first exploration of the lightest few isoscalar resonances in first-principles QCD

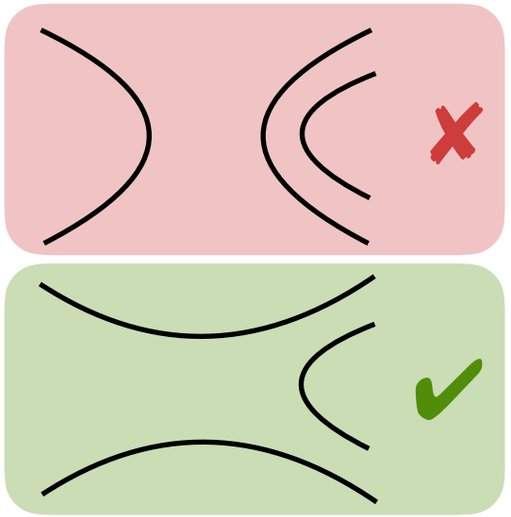
with unphysically heavy u,d quarks

tensors look 'ordinary'

$f_2^{\mathsf{a}} \sim f_2(1270) \sim u\bar{u} + d\bar{d}$ $\rightarrow \pi\pi$ dominantly

$f_2^{\mathsf{b}} \sim f_2'(1525) \sim s\bar{s}$ $\rightarrow K\overline{K}$ dominantly

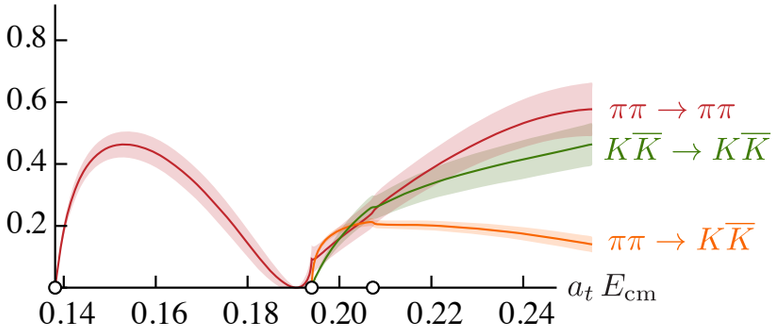
supports 'OZI' phenomenology



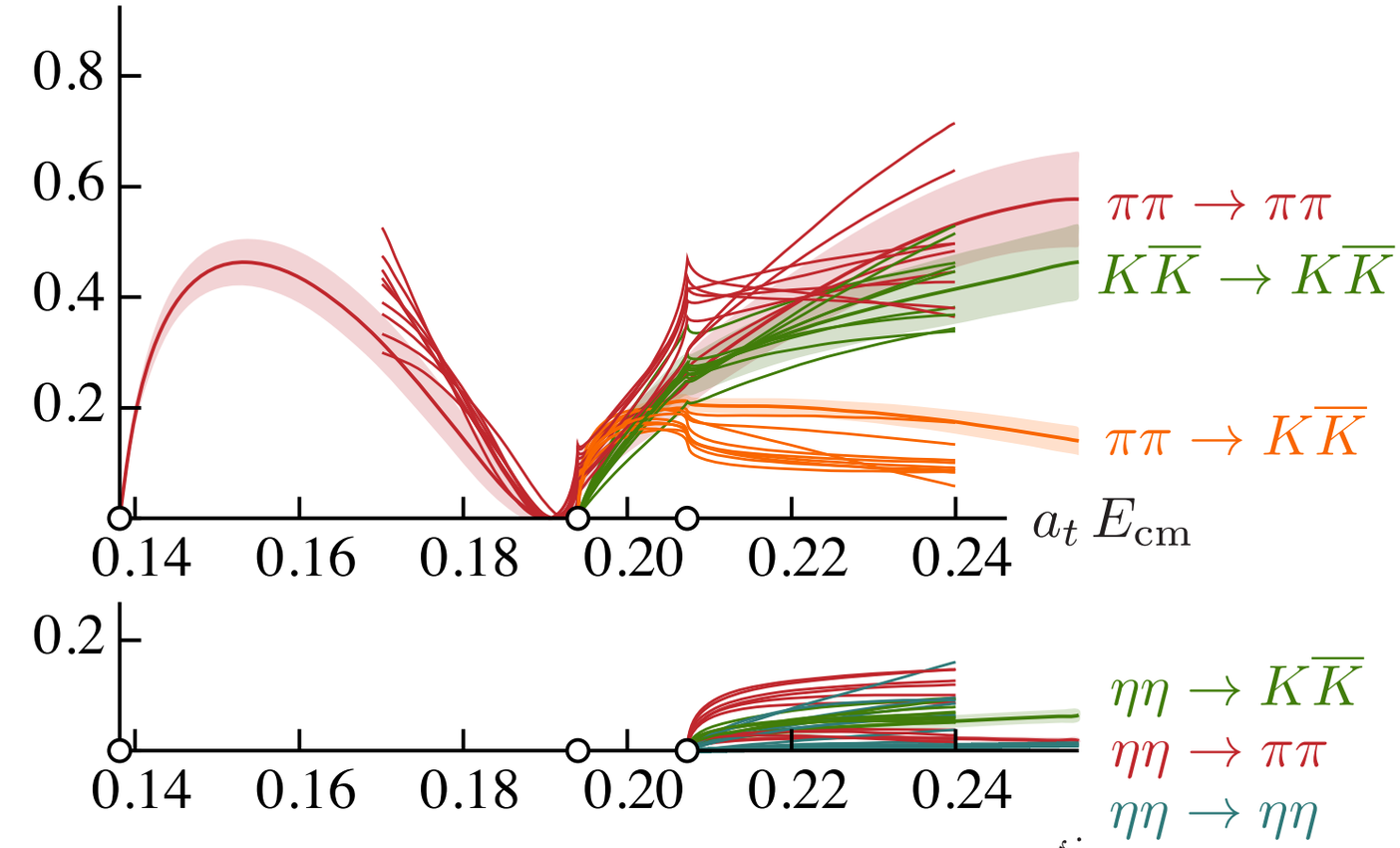
scalars are much more interesting

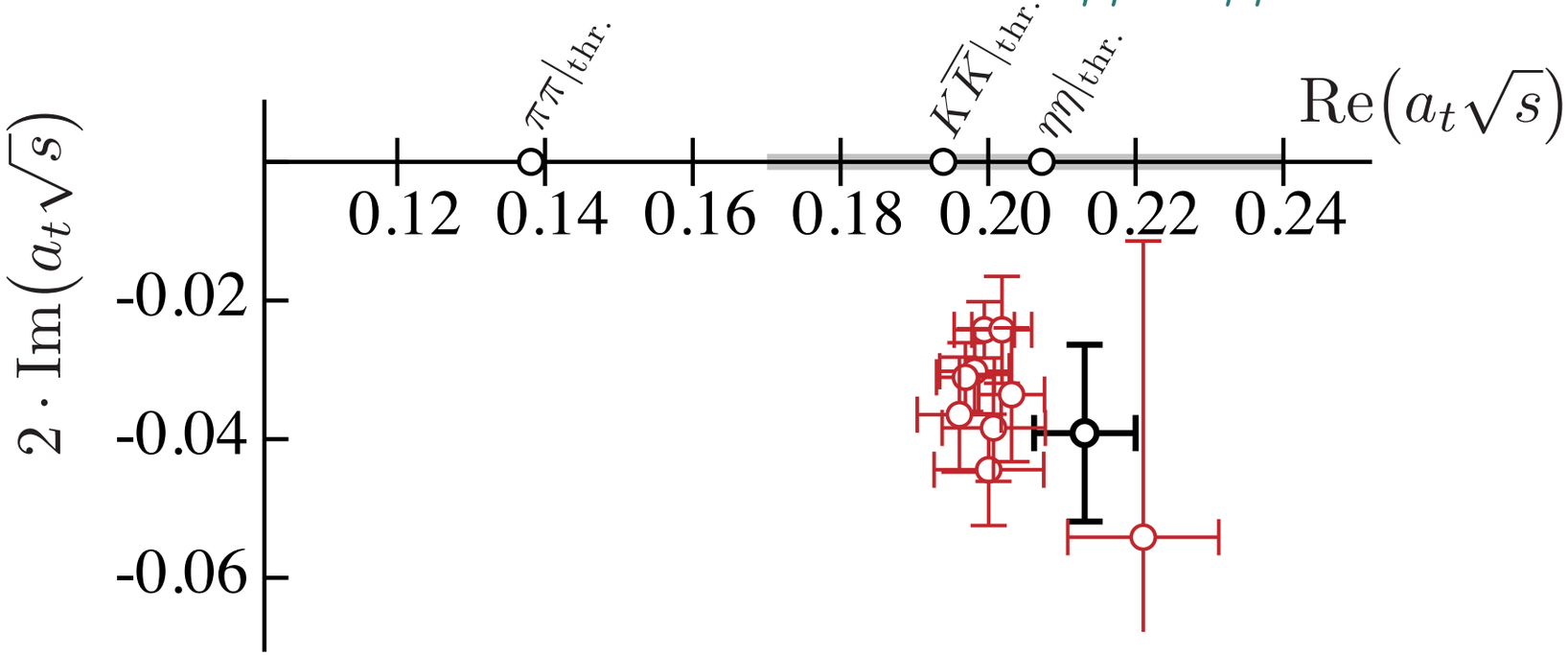
σ as stable bound-state below $\pi\pi$ threshold

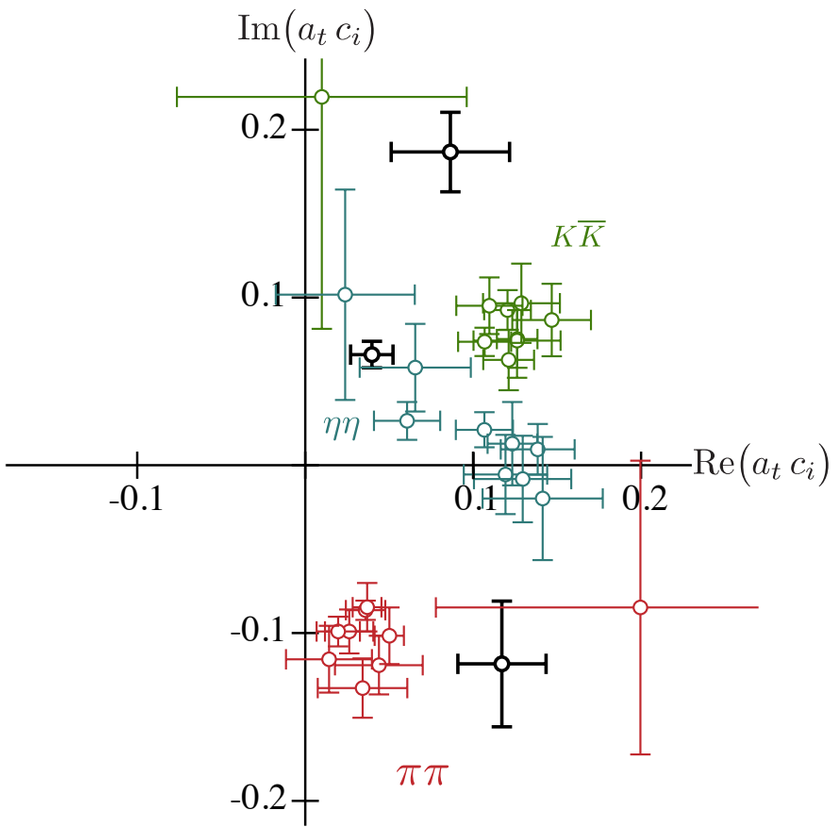
f_0 resonance close to $K\overline{K}$ threshold



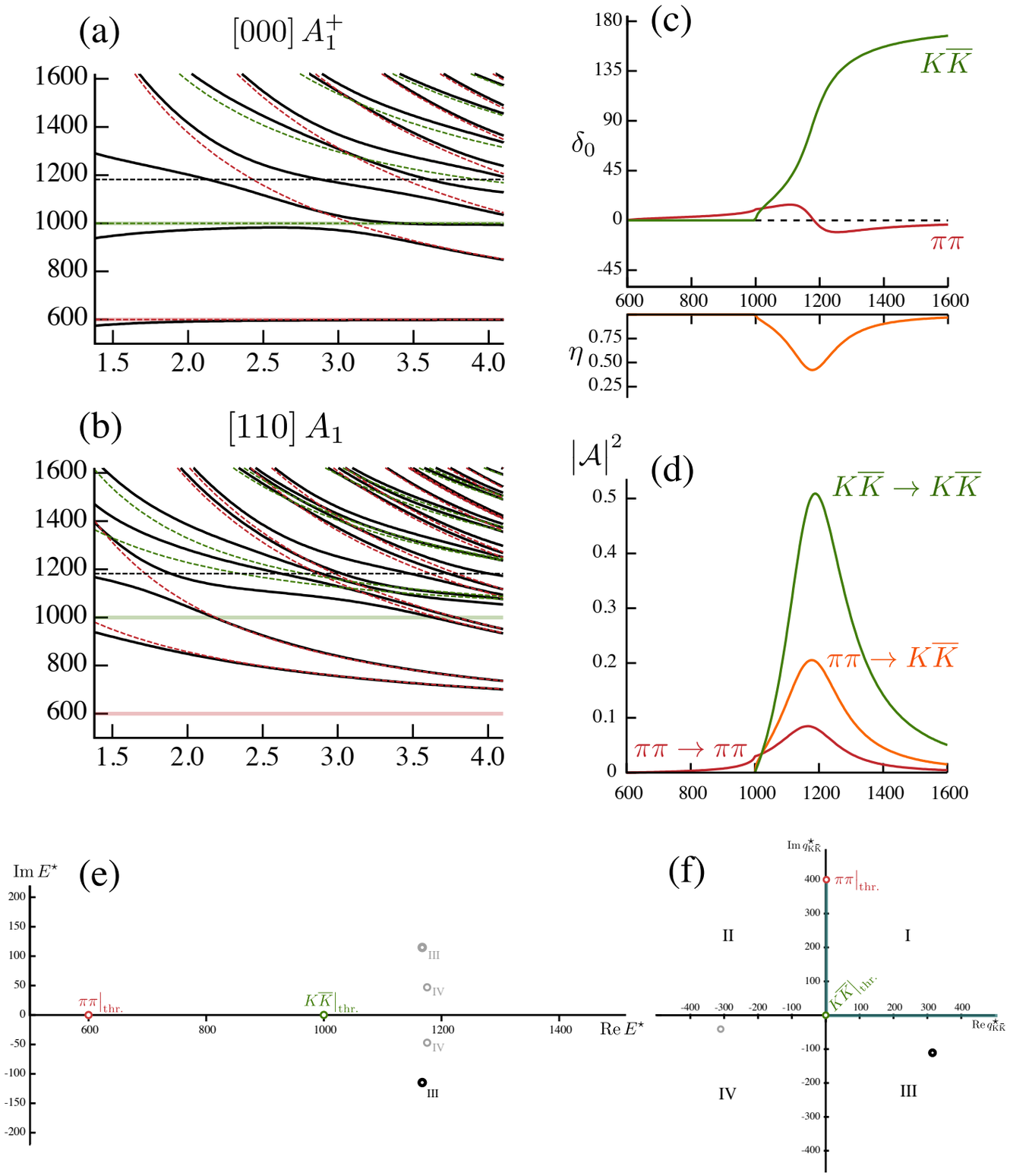
$K\overline{K}$ threshold region





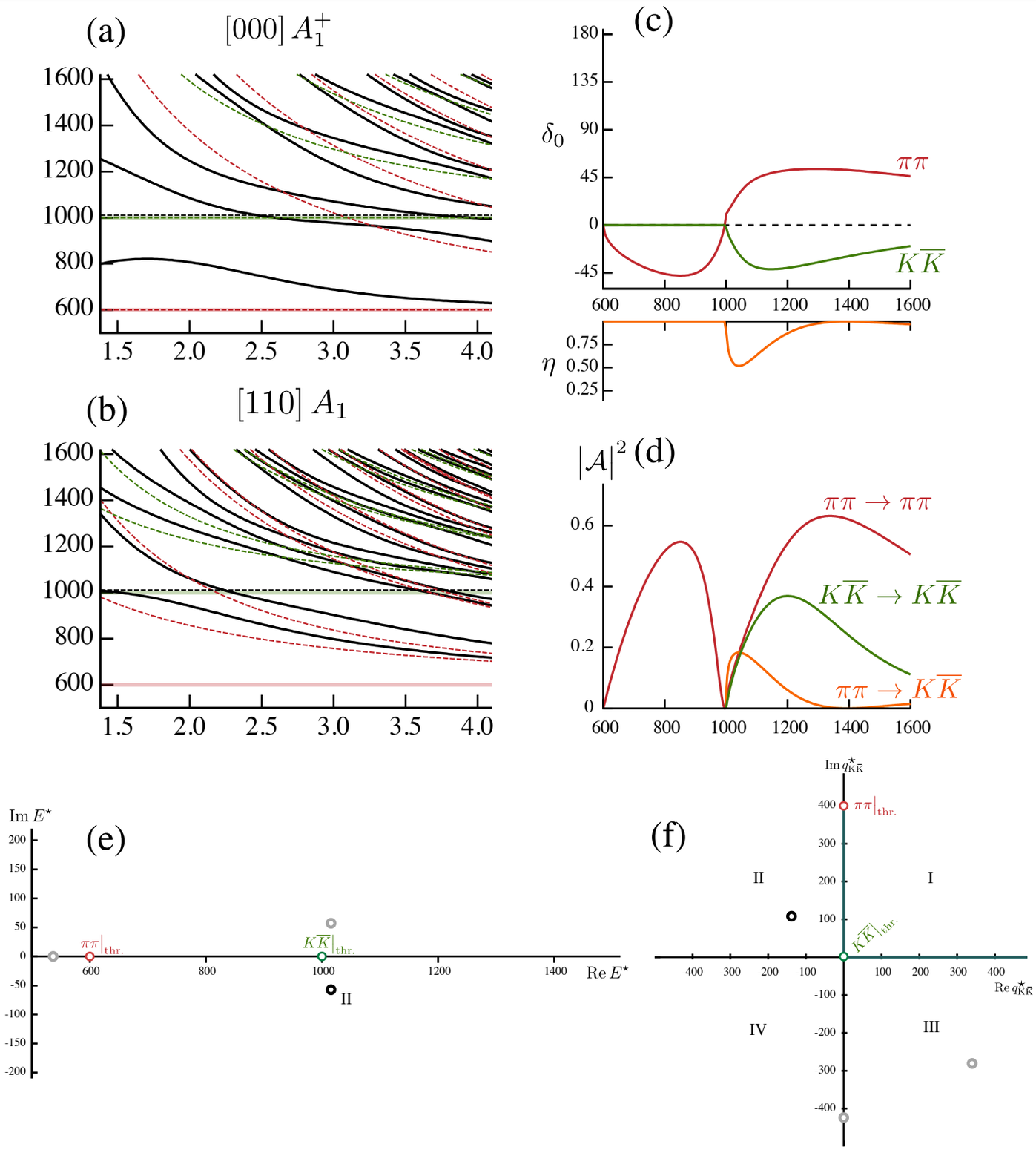


finite-volume Flatté



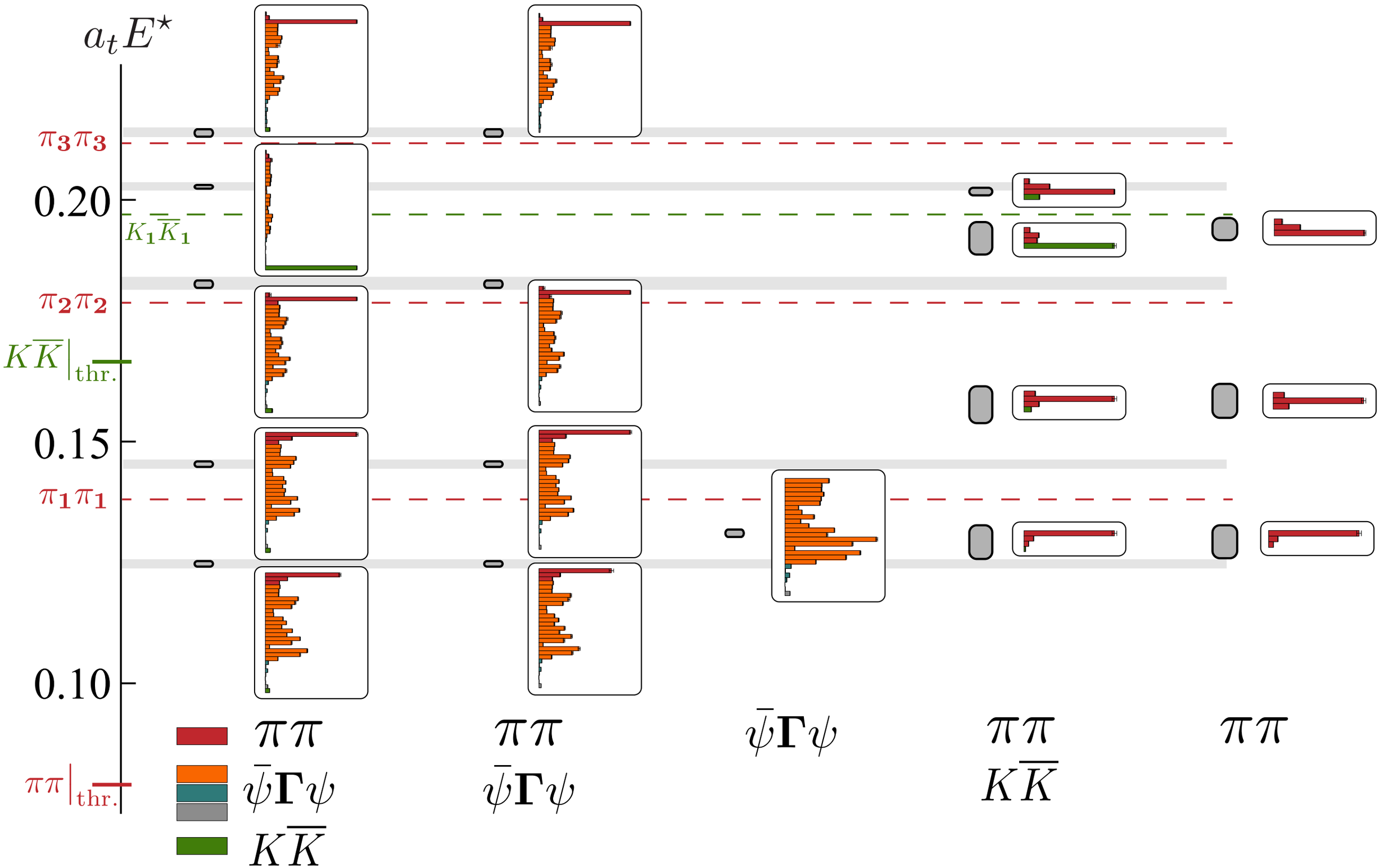
modelling an
'ordinary resonance'

finite-volume ‘dip-like’

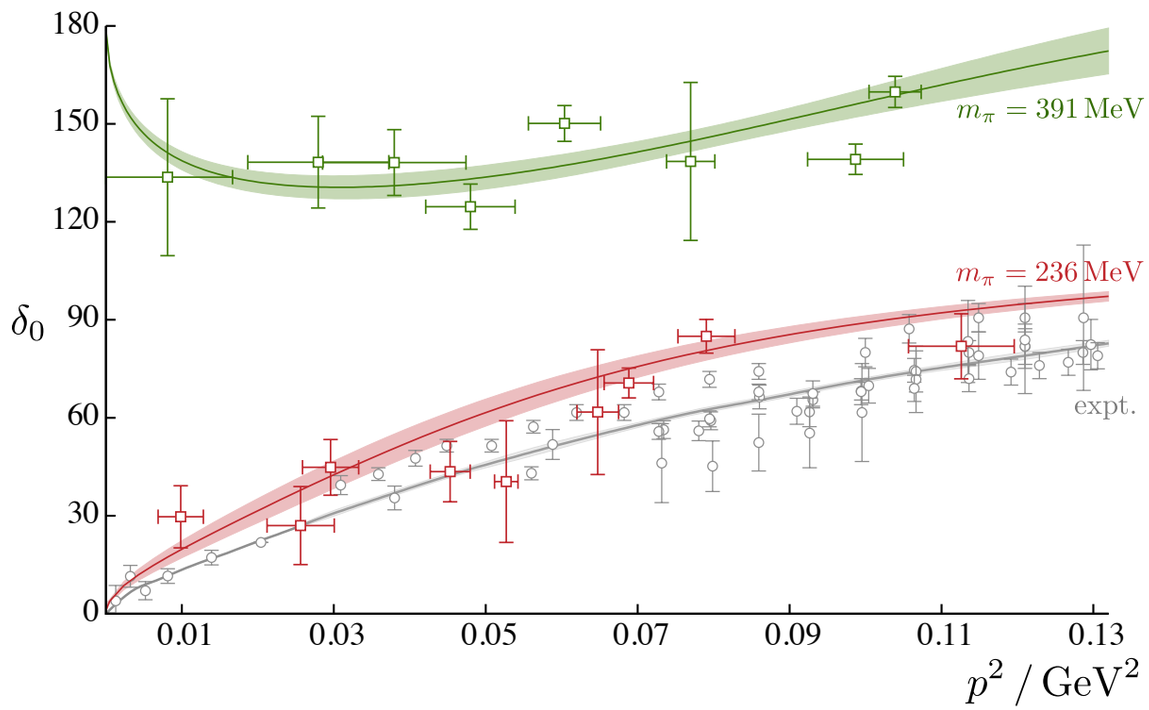


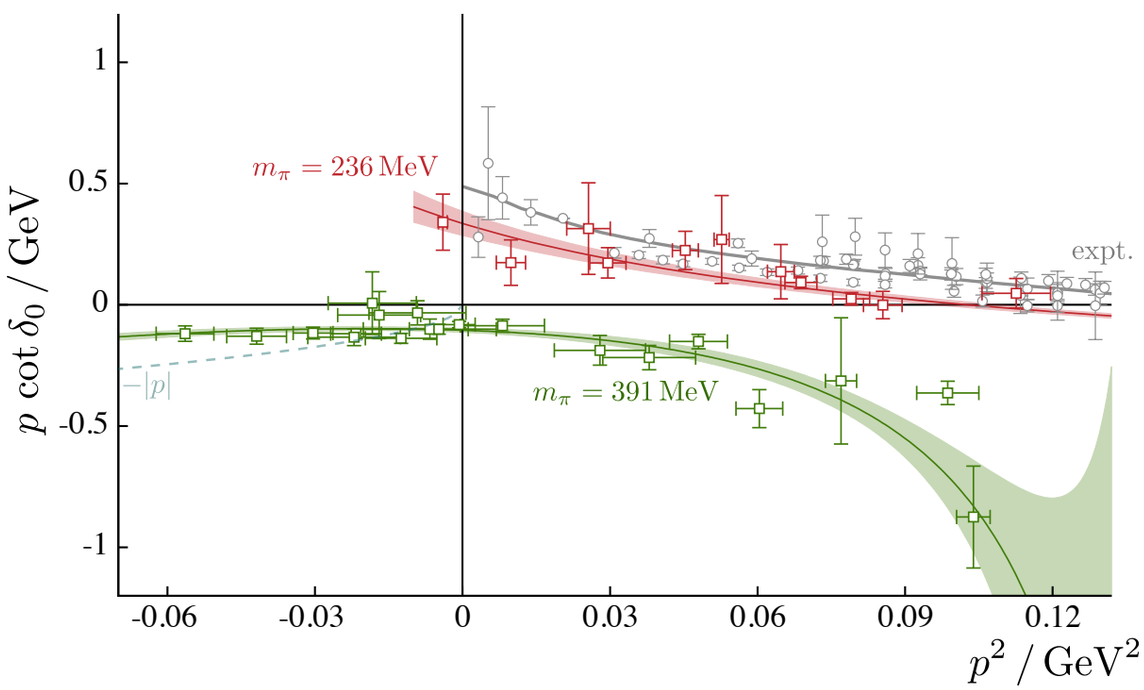
‘cartoon’ of $f_0(980)$?

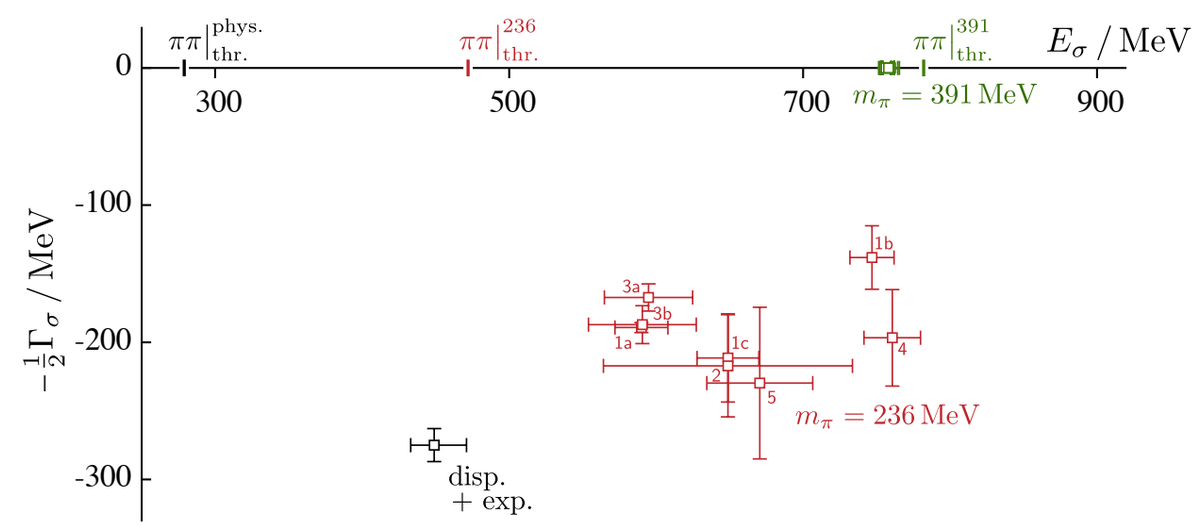
meson-meson ops are vital



quark mass evolution of σ

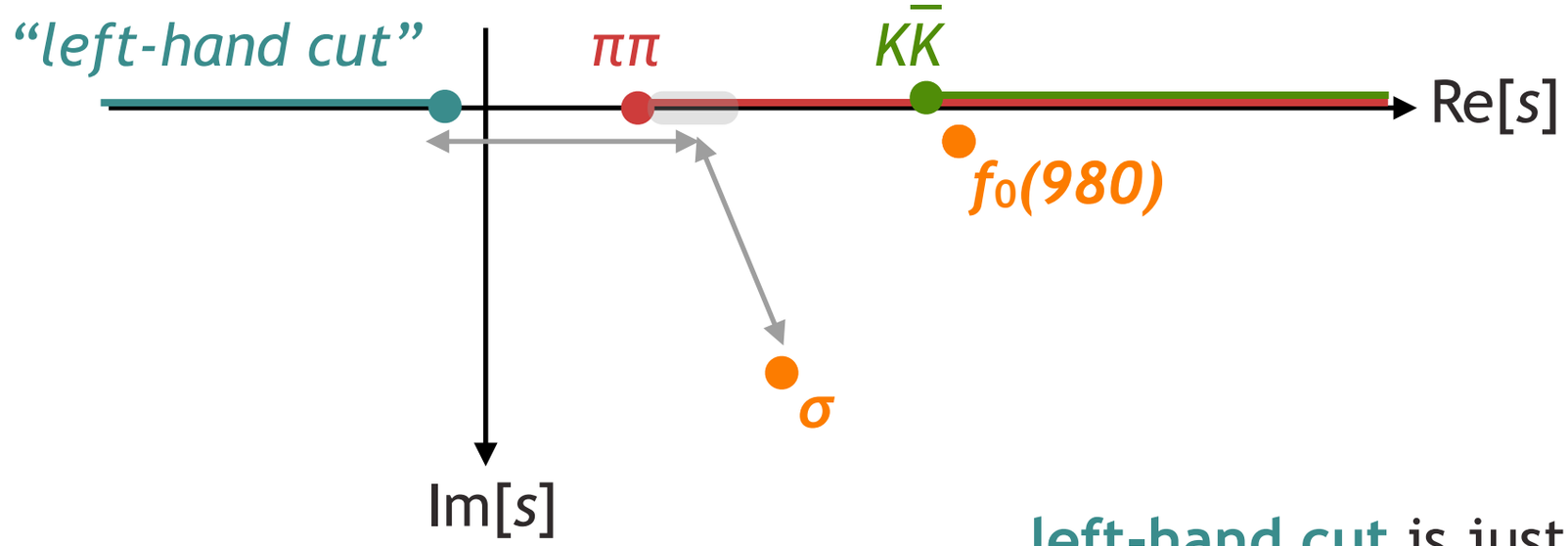






why is this case different ?

the real energy axis behaviour isn't just dominated by a single (nearby) pole

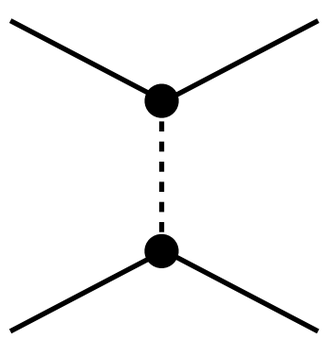


left-hand cut is just as close as the σ pole

but our amplitude parameterizations don't include it !

left-hand cut related to crossing symmetry

e.g.



$$t(s,t) \sim \frac{1}{t - M^2}$$

$$t_\ell(s) \sim \int d\cos\theta \, P_\ell(\cos\theta) \, t\big(s, t(\cos\theta)\big)$$

$$t_\ell(s) \sim \log\left(s - (4m_\pi^2 - M^2)\right)$$

branch point at $s = -M^2 + 4m_\pi^2$

σ pole with changing quark mass

- unitarized $SU(3)_F$ chiral perturbation theory

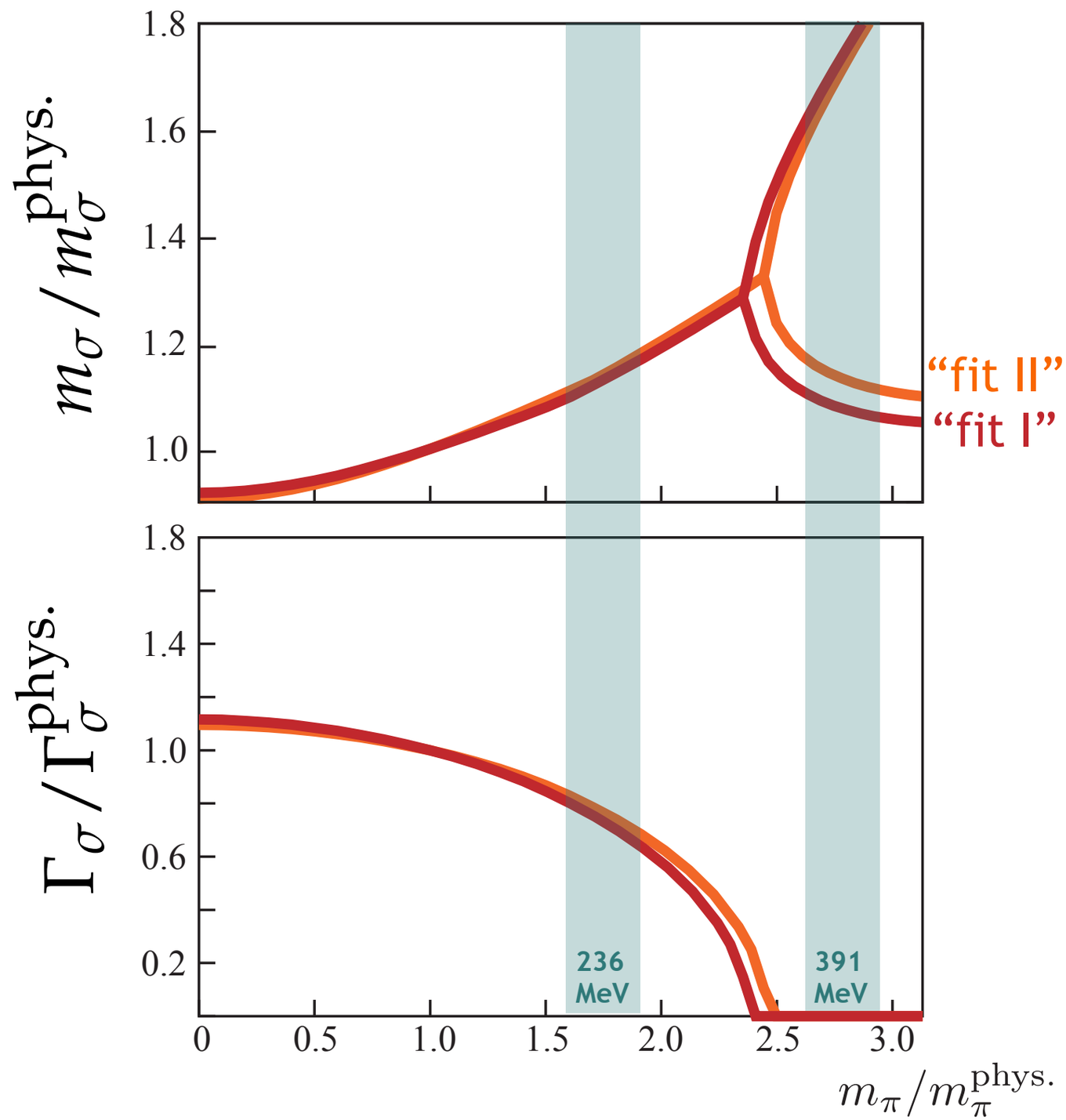
  - resonance poles become virtual bound states somewhere near $m_\pi \sim 2.5\ m_\pi^{phys}$

  - lowest mass v.b.s becomes a bound state somewhere slightly above $m_\pi \sim 3.0\ m_\pi^{phys}$

    "the exact m_π value when this happens is not very reliable"

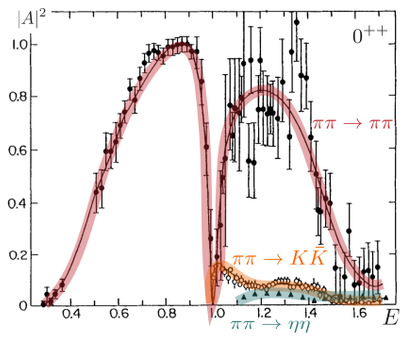
Nebreda & Pelaez PRD81 054035 (2010)

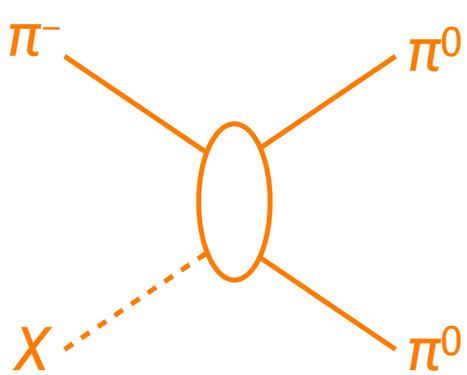
$$\sqrt{s_0} = m + \frac{i}{2}\Gamma$$



$f_0(980)$ dip — peak

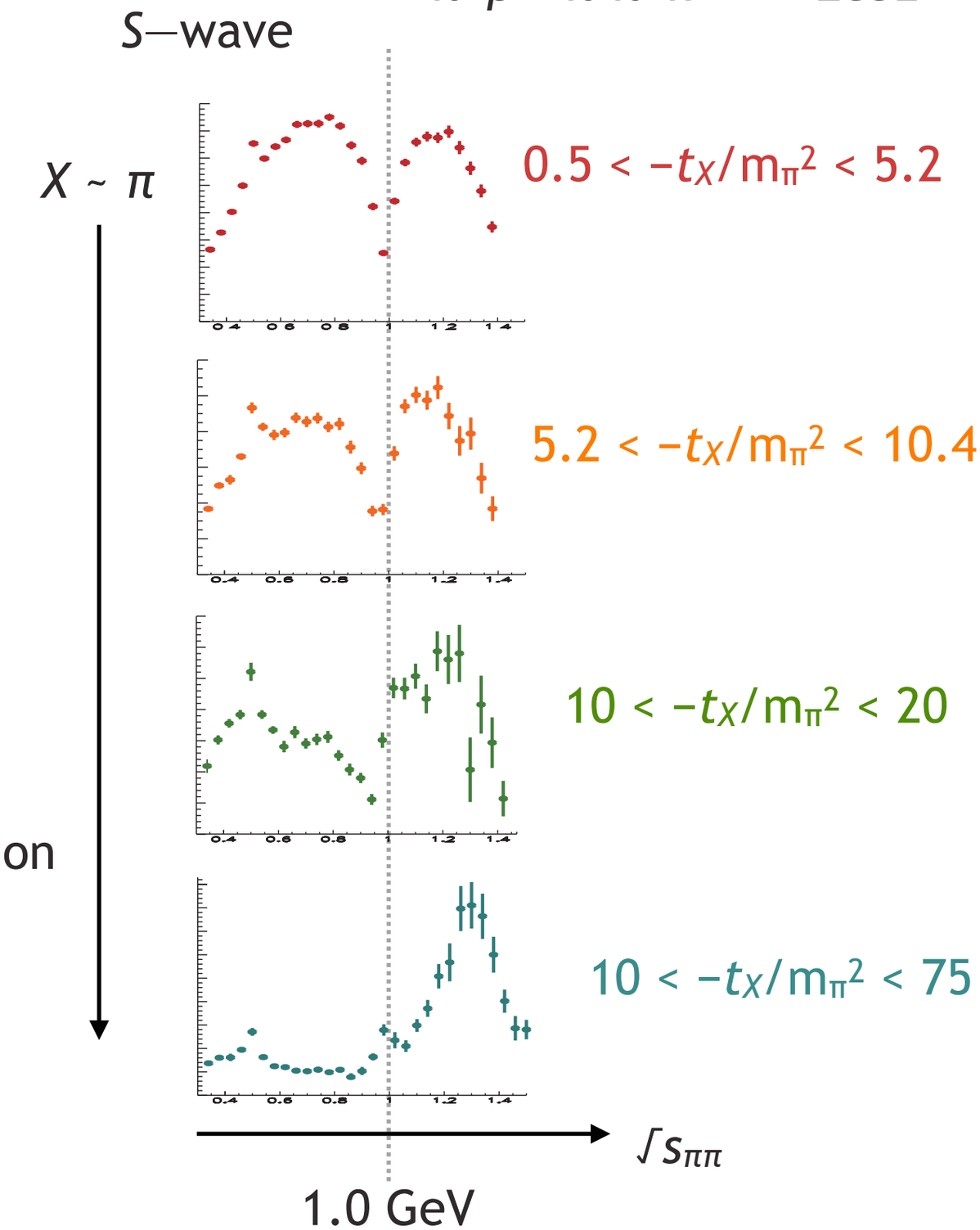
can even look different in 'same' process



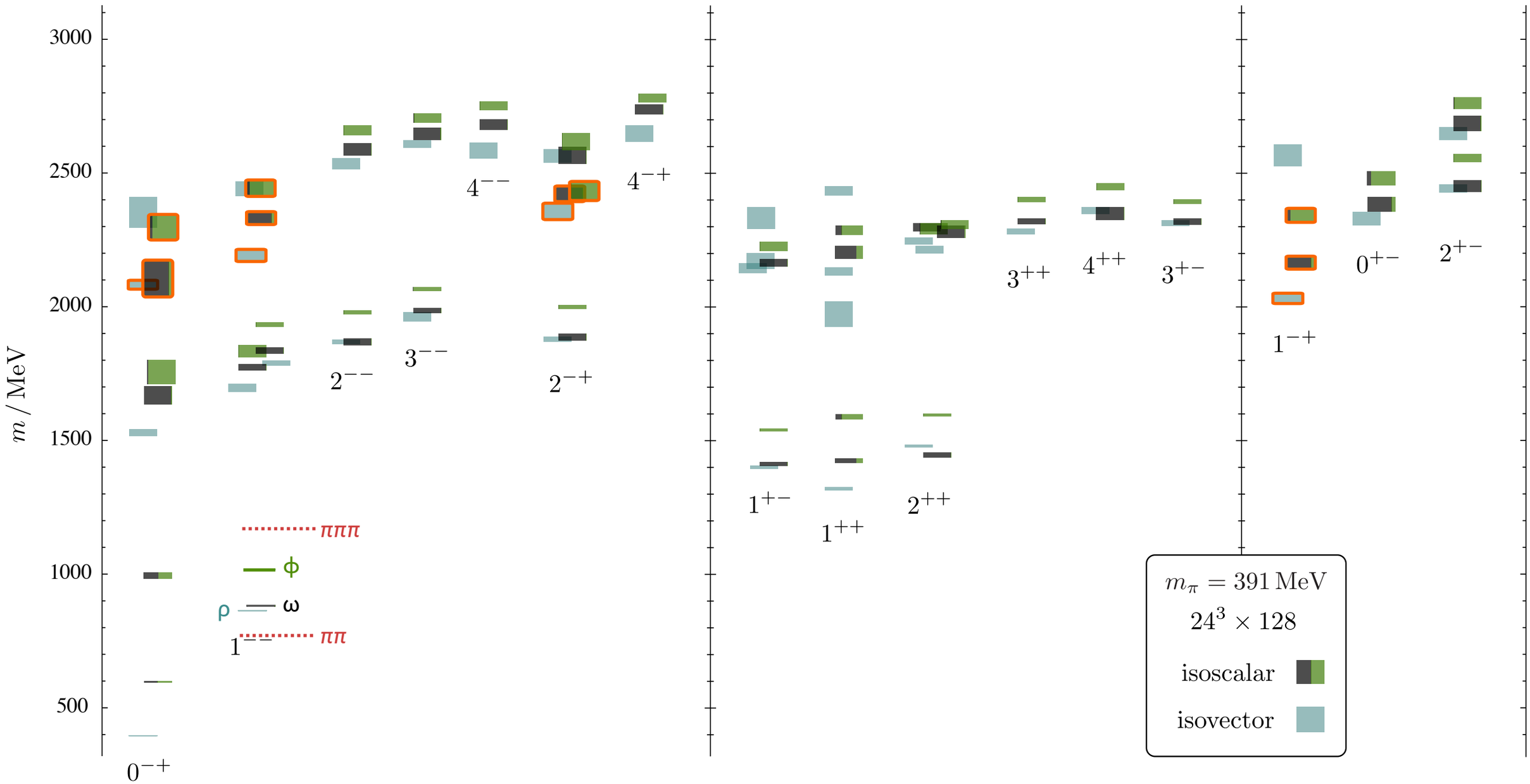


other exchanges
modify the production

$\pi^- p \to \pi^0\pi^0 n$ *E852*



‘local’ operator spectrum



tetraquark operators

generic local diquark operator

$$\delta_{RF}^{J[\Gamma]} = \langle \mathbf{3}r_a; \mathbf{3}r_b | Rr \rangle \langle F_a f_a; F_b f_b | F f \rangle \, q^T_{r_a f_a} (C\Gamma) \, q_{r_b f_b}$$

color reps. $R = \bar{\mathbf{3}}, \mathbf{6}$

spins $J^P = 0^\pm, 1^\pm$

no assumptions made at this point about good/bad diquarks

generic local tetraquark operator

$$\mathcal{T}^{J[\Gamma_1\Gamma_2]}_{\mathbf{1}[R_1R_2]\,F[F_1F_2]} = \langle J_1 m_1; J_2 m_2 | J m \rangle \langle R_1 r_1; R_2 r_2 | \mathbf{1} \rangle \langle F_1 f_1; F_2 f_2 | F f \rangle \, \delta^{J_1[\Gamma_1]}_{R_1F_1} \, \bar{\delta}^{J_2[\Gamma_2]}_{R_2F_2}$$

(+ C/G-parity symmetrisation ...)

spins $J \leq 2$

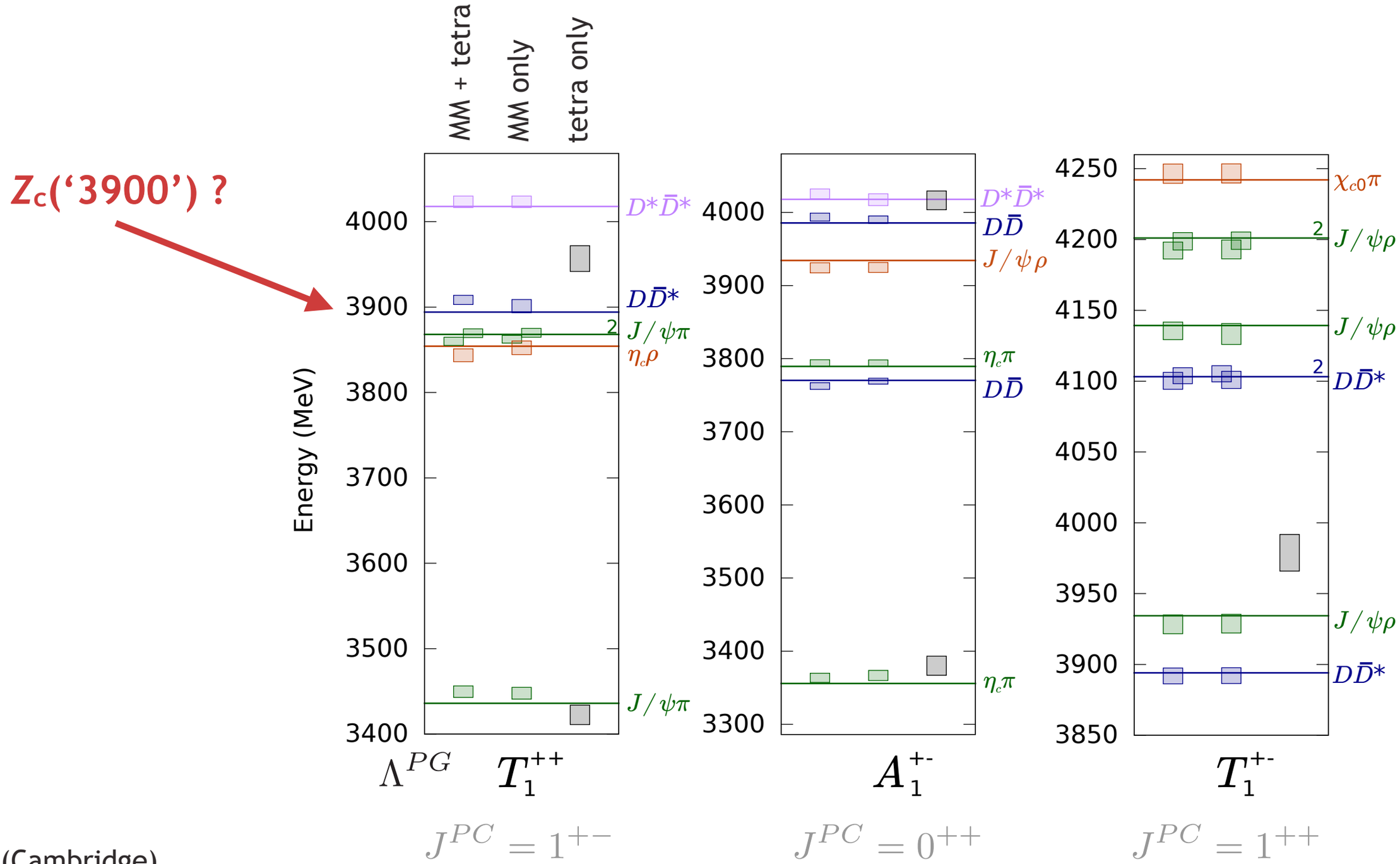
smeared quark fields, but otherwise **local**,
certainly not sampling the whole lattice volume

(diquark construction just makes fermion antisymmetry manifest)

tetraquark operators — hidden charm *I=1*

$m_\pi \sim 391\,\mathrm{MeV}$

all ‘expected’ meson-meson operators + several tetraquark operators



Gavin Cheung (Cambridge)
arXiv:1709.01417 [hep-lat]