Gluonic Structure of Hadrons and Nuclei

William Detmold, MIT

JLab, Oct 30th, 2017

Cover image from EIC whitepaper arXiv::1212.1701

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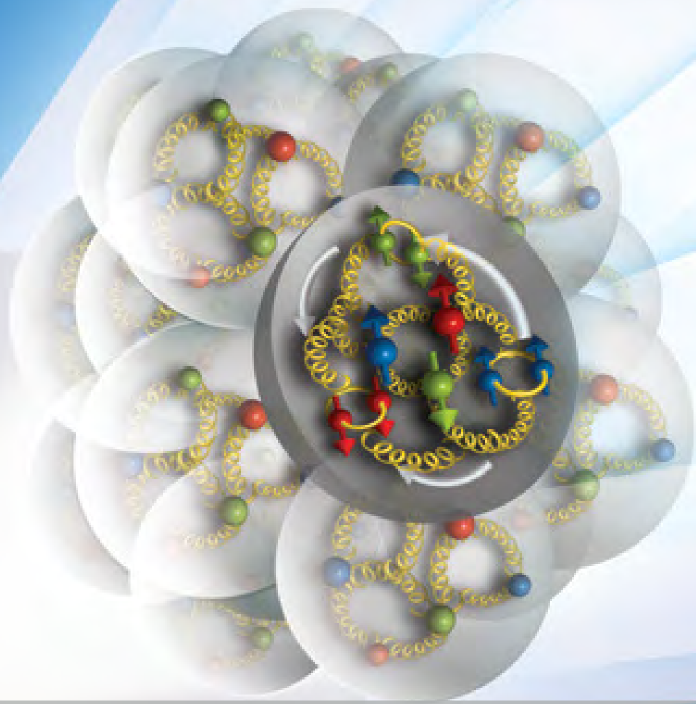

Work primarily with
Phiala Shanahan

William Detmold, MIT

JLab, Oct 30th, 2017

- The past 60+ years have provided detailed view of the quark structure of nucleons
- Gluonic structure relatively unexplored
- Electron-Ion Collider
  - High priority in 2015 long range plan
  - Over-arching goal: "Understanding the glue that binds us all"
- What can LQCD do to help?

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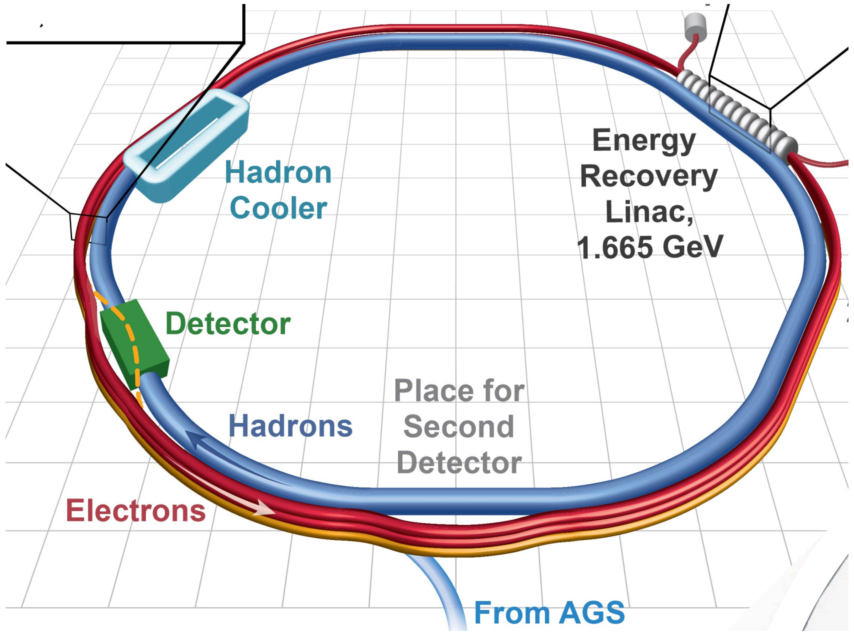

Gluonic structure

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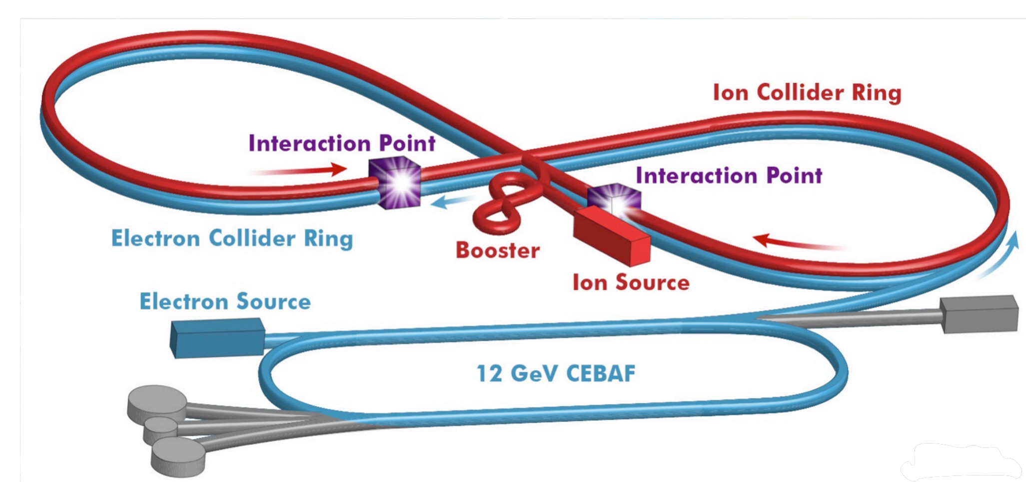
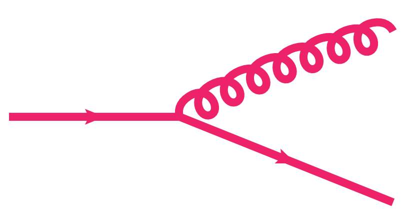
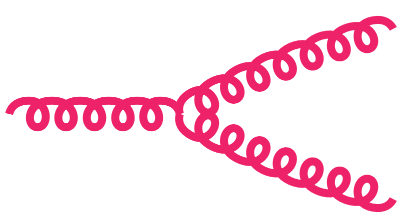


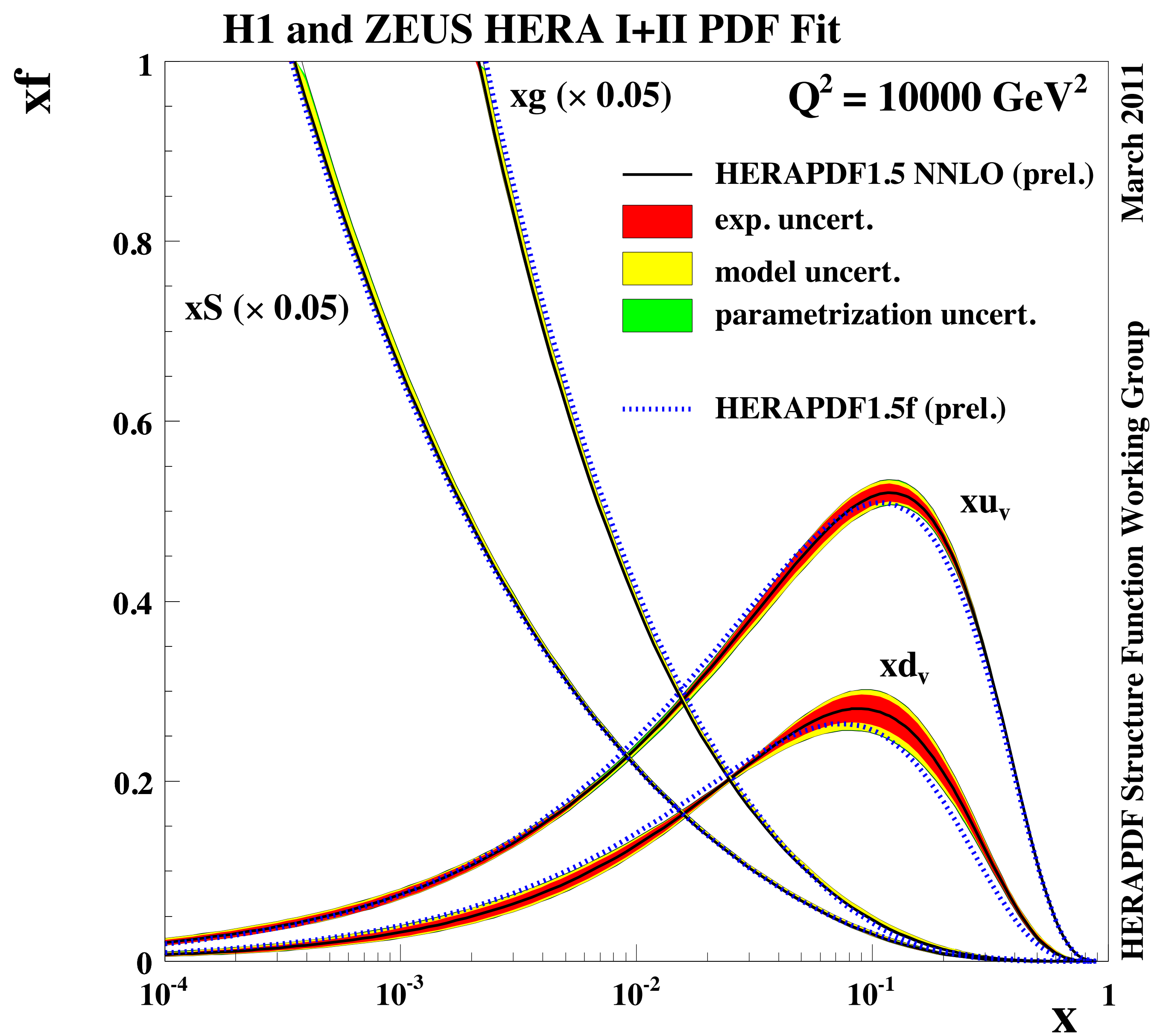
Gluonic structure

- Unpolarised gluon PDF g(x)
  - extracted from scaling violations in DIS,…
  - dominant at small Bjorken x
  - sharp rise due to QCD evolution





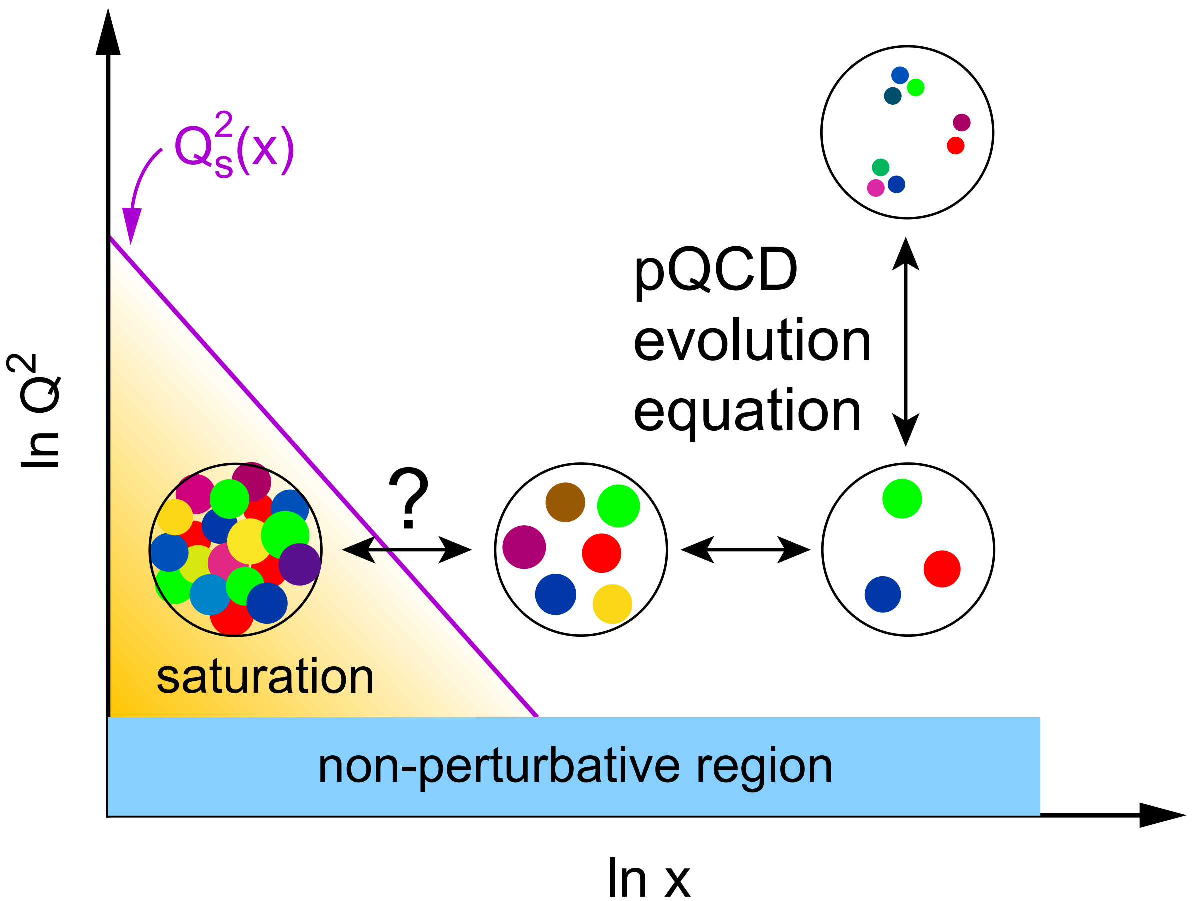
- Important input for LHC



Gluon saturation?

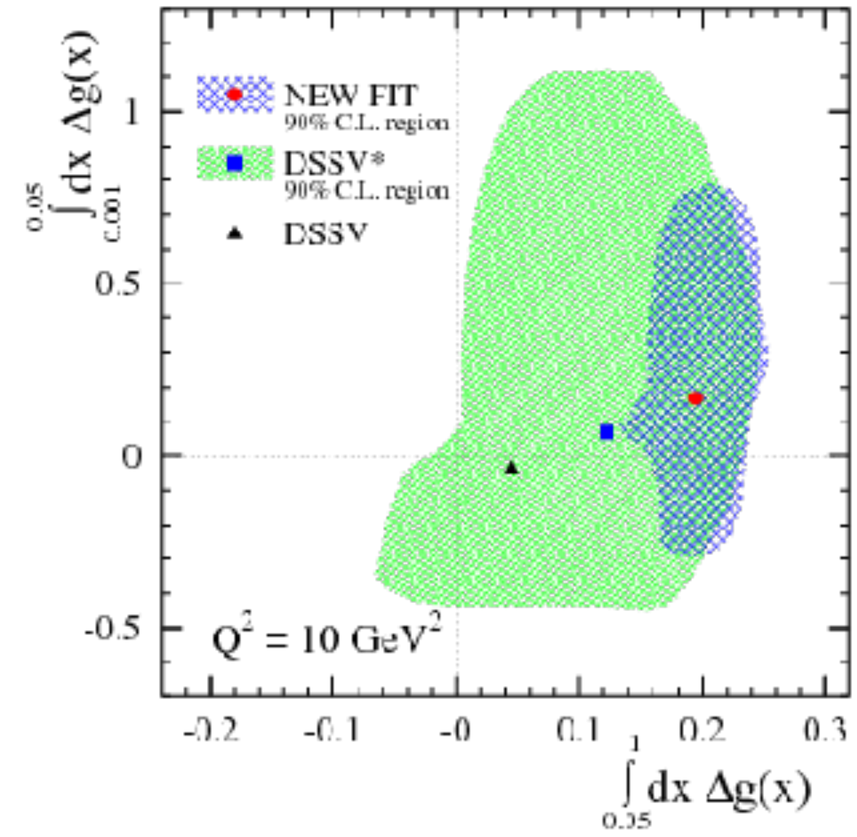
- Small x behaviour uncertain
  - Large gluon density makes recombination important
    [Balitsky-Kovchegov, JIMWLK]

  - "Colour glass condensate"??
- Nuclear environment to enhance saturation
- Key motivation for EIC

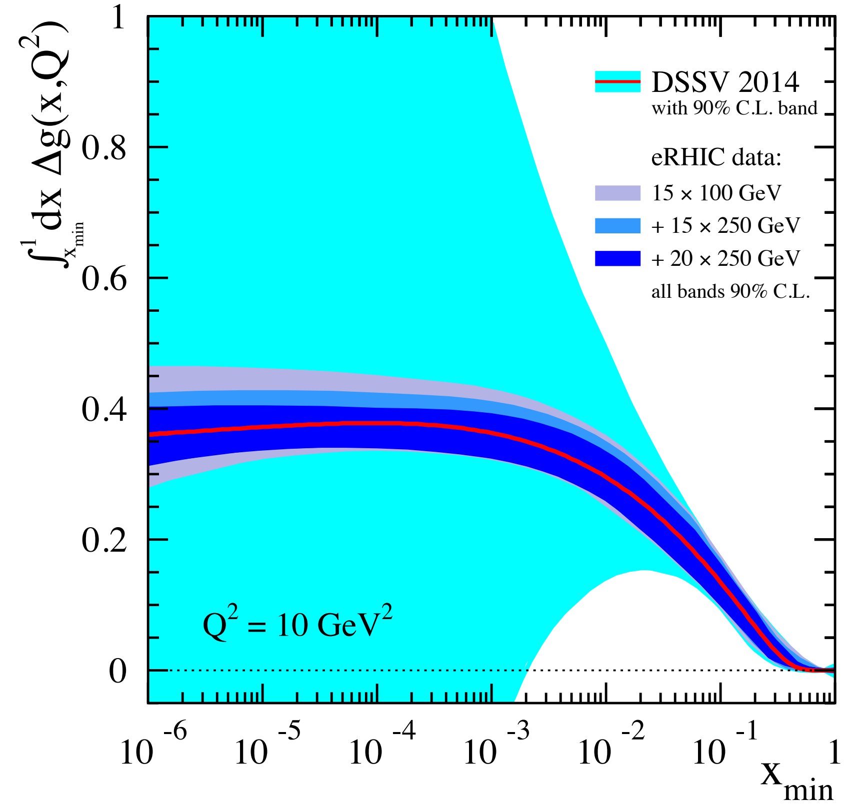


Gluon angular momentum

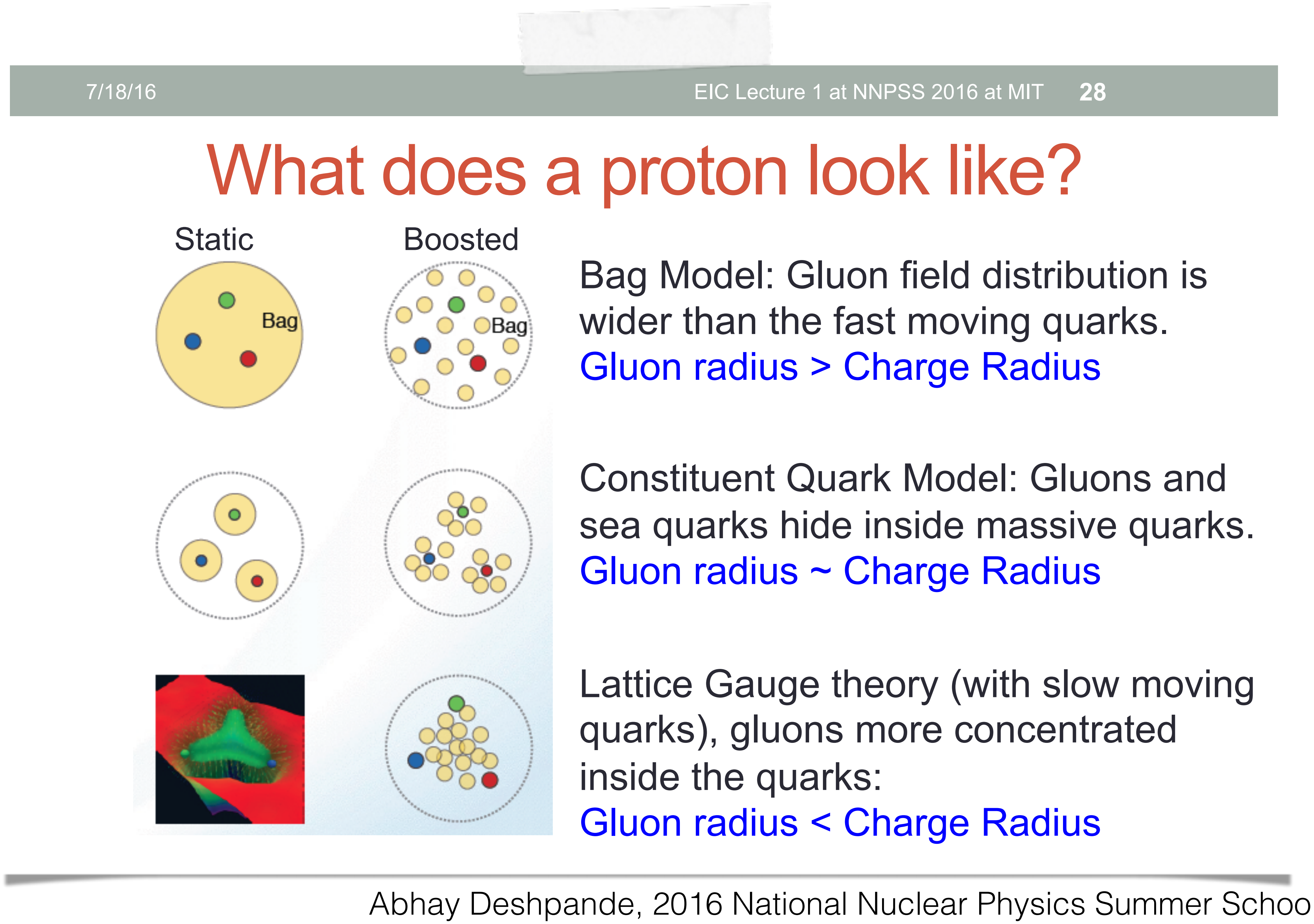
- Gluon helicity much less well constrained
  - Major focus of RHIC-spin program
  - Asymmetries in polarised *pp→ πX, DX, BX, jets*
- Orbital angular momentum of gluons even less understood
  - Gluon TMDs
- Further major motivation for EIC



de Florian et. al, Phys.Rev.Lett. 113, 012001 (2014)



Gluonic Radius

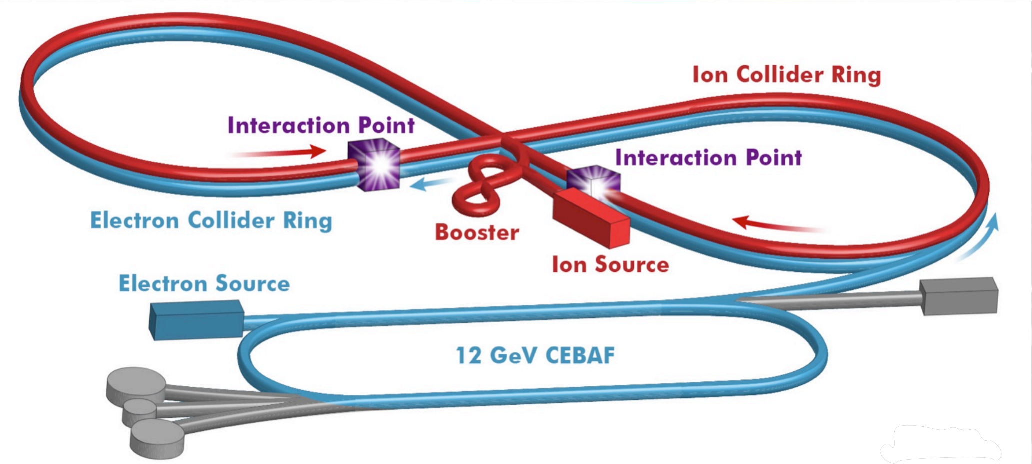


- A natural question

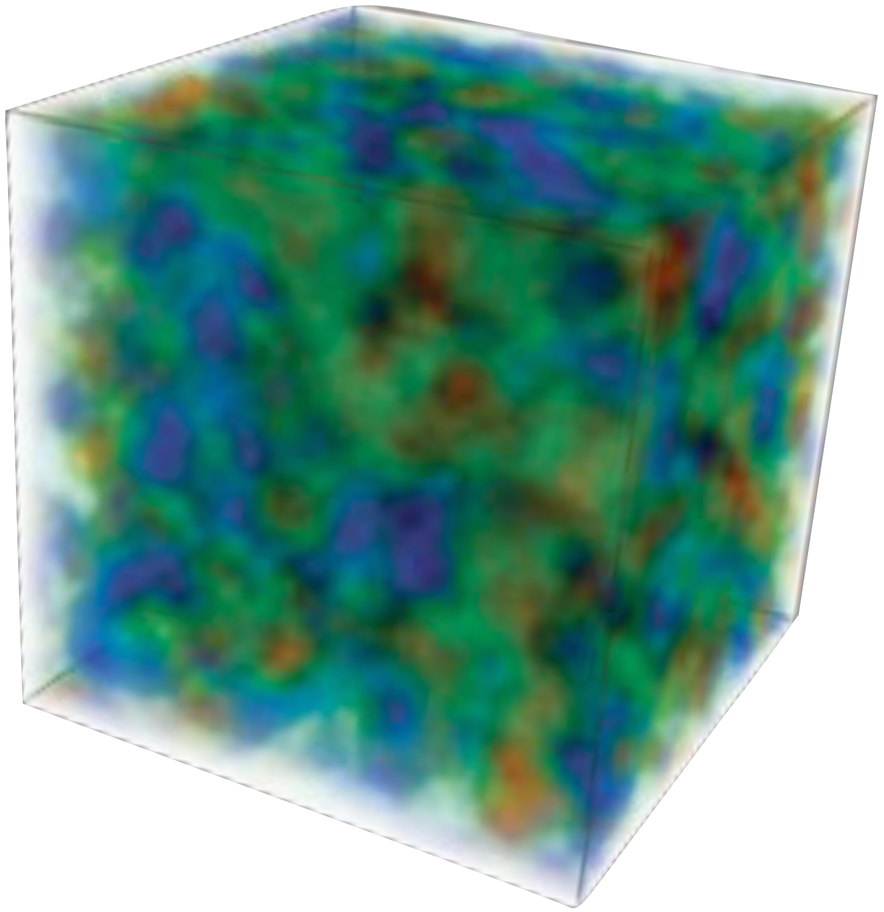
Gluonic Radius

- However not so simple to answer
  - Experimentally challenging
    - DIS probes are EW so sensitivity to gluons is poor
    - Other processes less clean: heavy flavour production, …
  - The proton is a quantum system
    - Quarks and gluons mix via evolution
    - Nonsinglet quantities uniquely quarky
    - Double helicity flip uniquely gluonic

Lattice QCD input



- EIC is a precision gluon structure machine
  - Timescale is >2025
- What can lattice QCD do?
  - Gluonic observables are challenging - signal to noise
  - Few calculations so far
    - Gluon momentum fraction
      [Meyer&Negele; Gockeler et al., Alexandru et al.]
    - Gluon angular momentum
      [Liu et al., Yang et al, Alexandru et al.]



Gluon helicty/spin

- Two common decompositions of the proton spin:
- Ji (1996)

quark orbital angular momentum

$$J_N = \sum_{q=u,d,s,c\ldots} \left(\frac{1}{2}\Delta\Sigma_q + L_q\right) + J_g$$

quark helicity

gluon spin

- Jaffe-Manohar (1990)

$$J_N = \sum_{q=u,d,s,c\ldots} \left(\frac{1}{2}\Delta\Sigma_q + \mathcal{L}_q\right) + \Delta g + \mathcal{L}_g$$

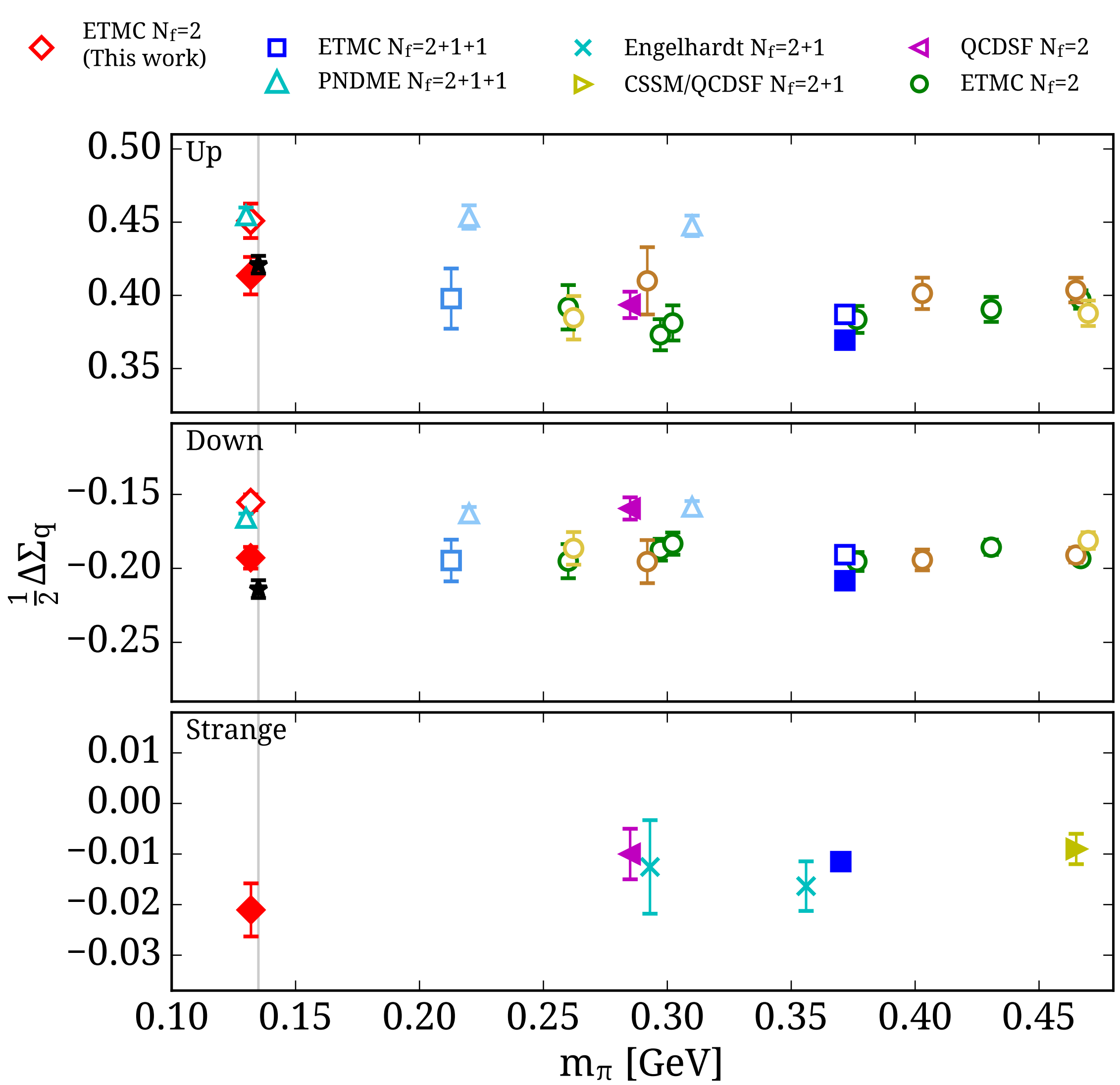
gluon helicity

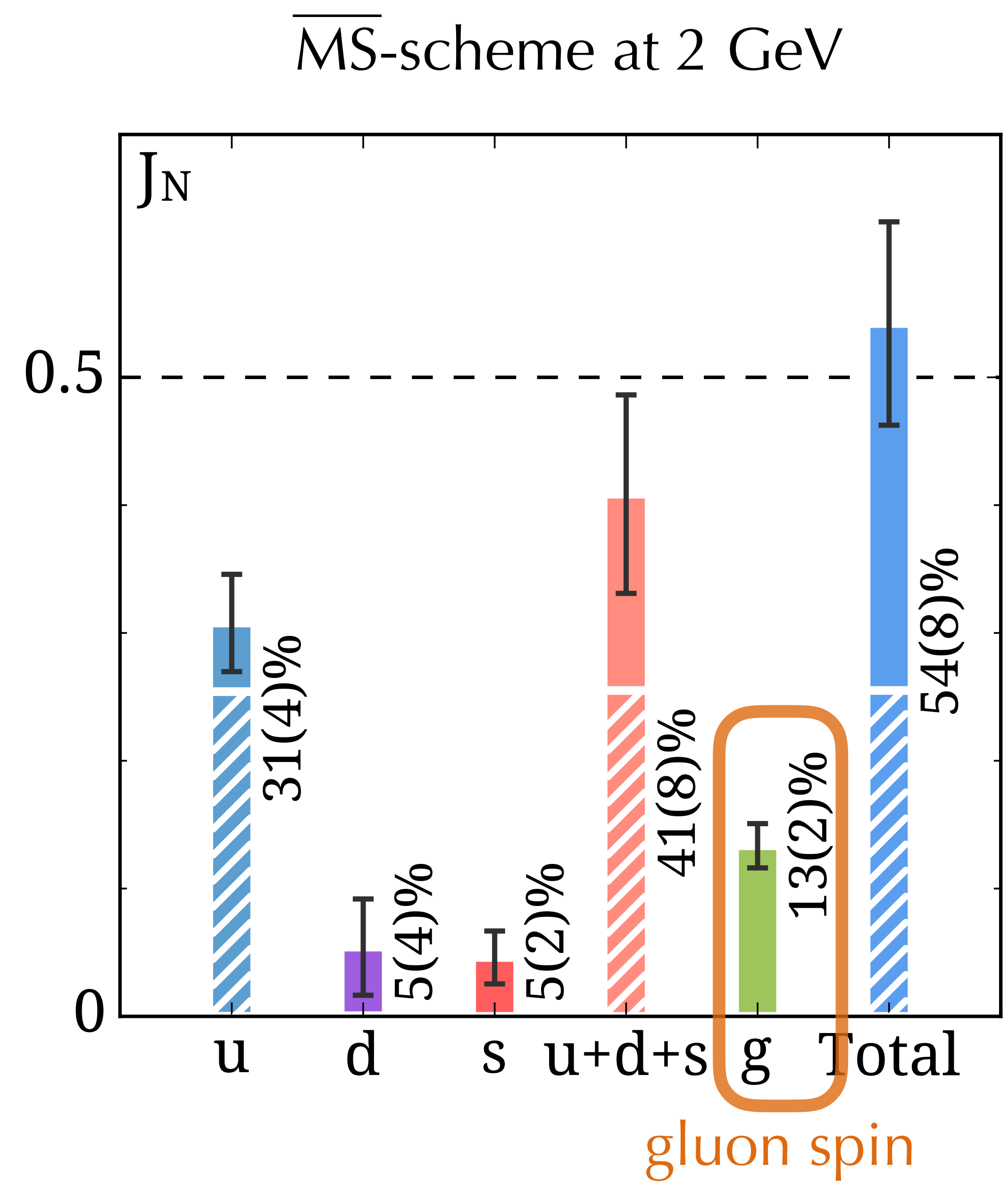
- Interpolation between decompositions [M. Engelhardt, PRD 95 094505 (2017)]

Spin decomposition of nucleon

C. Alexandrou et al., arXiv:1706.02973

- Physical pion mass
- All terms calculated directly



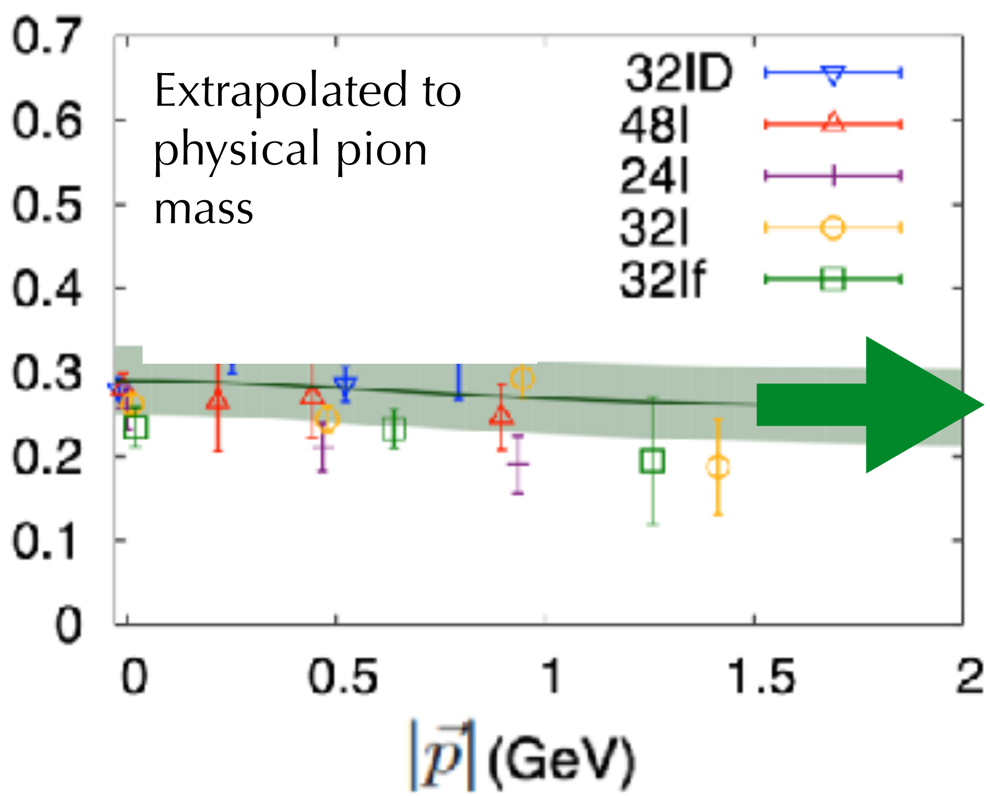


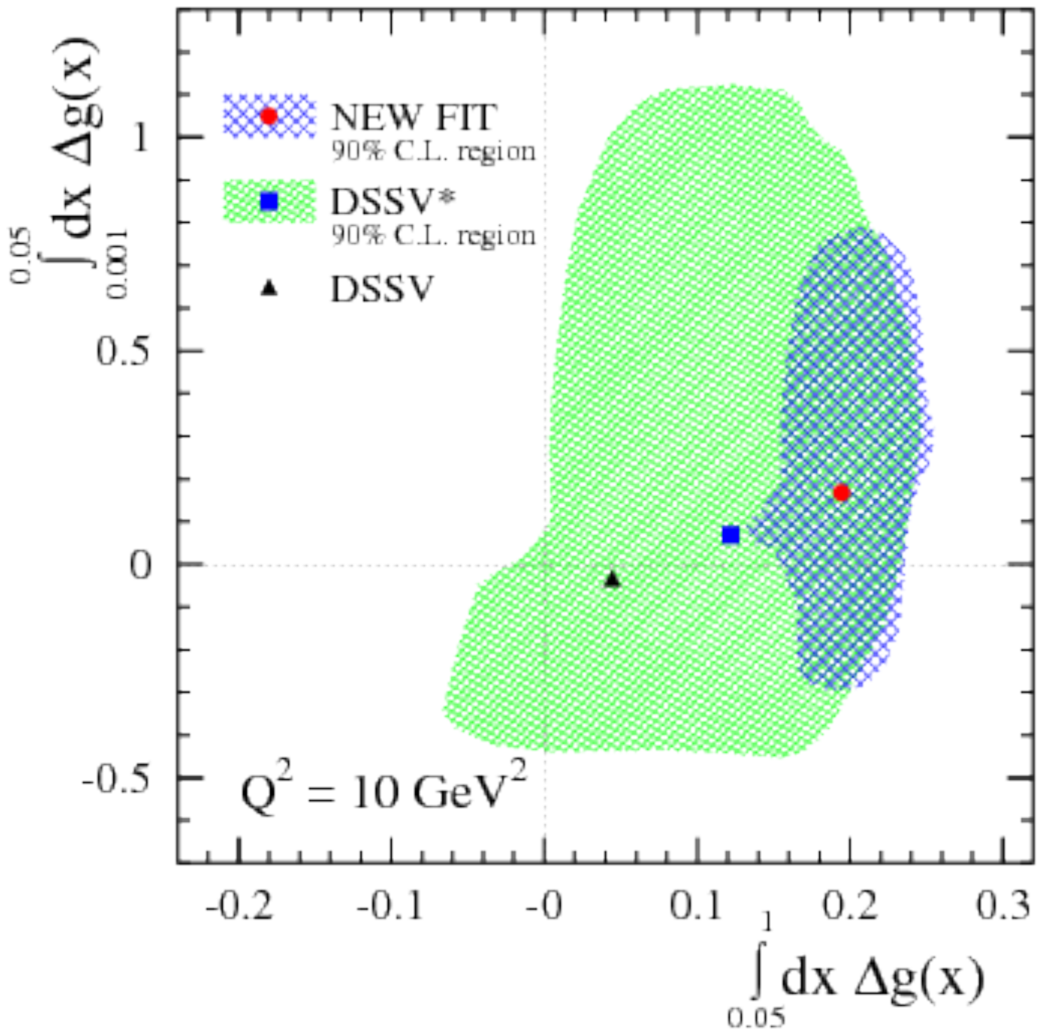
Hashed: connected
Solid: disconnected

Spin decomposition of nucleon

Y.-B. Yang et al., PRL 118, 102001 (2017)

- Gluon helicity: not directly calculable
- Match to calculable ME in infinite momentum frame limit using large momentum effective theory [Ji et al.]





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Volume 223, number 2 PHYSICS LETTERS B 8 June 1989

NUCLEAR GLUONOMETRY ☆

R.L. JAFFE and Aneesh MANOHAR
Center for Theoretical Physics, Laboratory for Nuclear Science and Department of Physics, Massachusetts Institute of Technology, Cambridge, MA 02139, USA

Received 24 March 1989

We identify a new leading twist structure function in QCD which can be measured in deep elastic scattering from polarized targets (such as nuclei) with spin $\geqslant 1$. The structure function measures a gluon distribution in the target and vanishes for a bound state of protons and neutrons, thereby providing a clear signature for exotic gluonic components in the target.

1. Introduction

The physical photon has four structure functions [1,2]. Three are familiar: F_1^γ, $F_L^\gamma = F_2^\gamma - 2xF_1^\gamma$ and g_1^γ. The fourth, called F_3^γ by Ahmed and Ross [2], corresponds to the imaginary part of the double helicity flip Compton amplitude, $A_{+-,-+}$ in the notation of ref. [3]. The other three are proportional to helicity conserving Compton amplitudes, $\binom{F_1}{g_1} \propto (A_{++,++} \pm A_{-+,-+})$, $F_L \propto A_{0+,0+}$. In parton models both F_L^γ and F_3^γ would be expected to vanish in the Bjorken limit since massless quarks do not couple to longitudinal photons, nor flip the photon helicity by two units. In QCD both F_L^γ and F_3^γ get contributions from the box graph [1,2,4] which persist in the scaling limit because the short-distance behavior of the box graph violates parton model assumptions.

Witten [5] pointed out that these contributions to F_L^γ are associated with towers of *photon* operators which appear in the operator product expansion (OPE) of two electromagnetic currents. Their coefficient functions have been calculated from the box graph. Recently, one of us [6] identified the tower of photon operators which contribute to F_3^γ. By analogy it is evident that there must be a tower of *gluon* operators in QCD, with coefficient functions of order $\alpha_s(Q^2)$ obtained from the box graph, which generate a double helicity flip Compton amplitude on a hadronic target. These operators belong to different representations of the Lorentz group than the other operators which appear in the OPE and therefore do not mix under renormalization with quark operators and the other gluon operators. These operators have vanishing matrix elements in any state with spin less than one and appear to have been overlooked in all QCD analyses in the past. We name the hadronic structure function associated with this tower of operators $\Delta(x, Q^2)$ (to avoid confusion with the parity-violating structure function $F_3(x, Q^2)$ of neutrino scattering). $\Delta(x, Q^2)$ can be measured by scattering an *unpolarized* electron beam from a target *aligned* ((that is, polarized either along or against) perpendicular to the beam. [Actually any direction not exactly parallel to the beam will do, but perpendicular is best.] The only targets with $J \geqslant 1$ are nuclei. $\Delta(x, Q^2)$ vanishes identically for a nucleus made up of protons, neutrons and pions regardless of Fermi motion or binding corrections in the approximation in which the nucleons or pions scatter independently. It is therefore an unambiguous probe of the gluonic components of the nuclear wavefunction which cannot be identified with individual nucleons or pions.

If the scattering cross section is measured as a function of the usual variables, $x = Q^2/2\nu$, $y = \nu/ME$ and the azimuthal angle ϕ between the plane formed by the beam and the alignment axis and the plane formed by the beam and the scattered electron (fig. 1), then in the scaling limit (Q^2, $\nu \to \infty$, x fixed),

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- **WANTED**: well defined purely gluonic observables
  - Gluonic transversity

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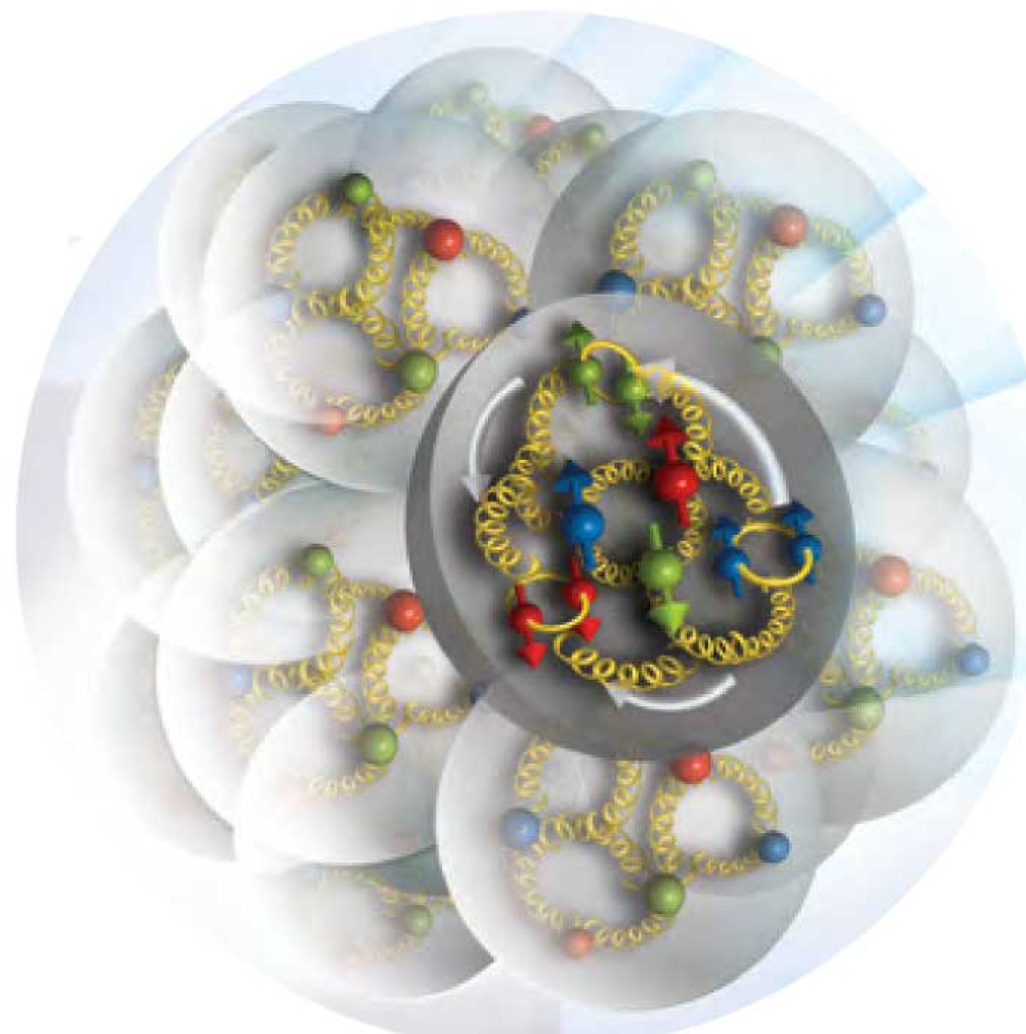

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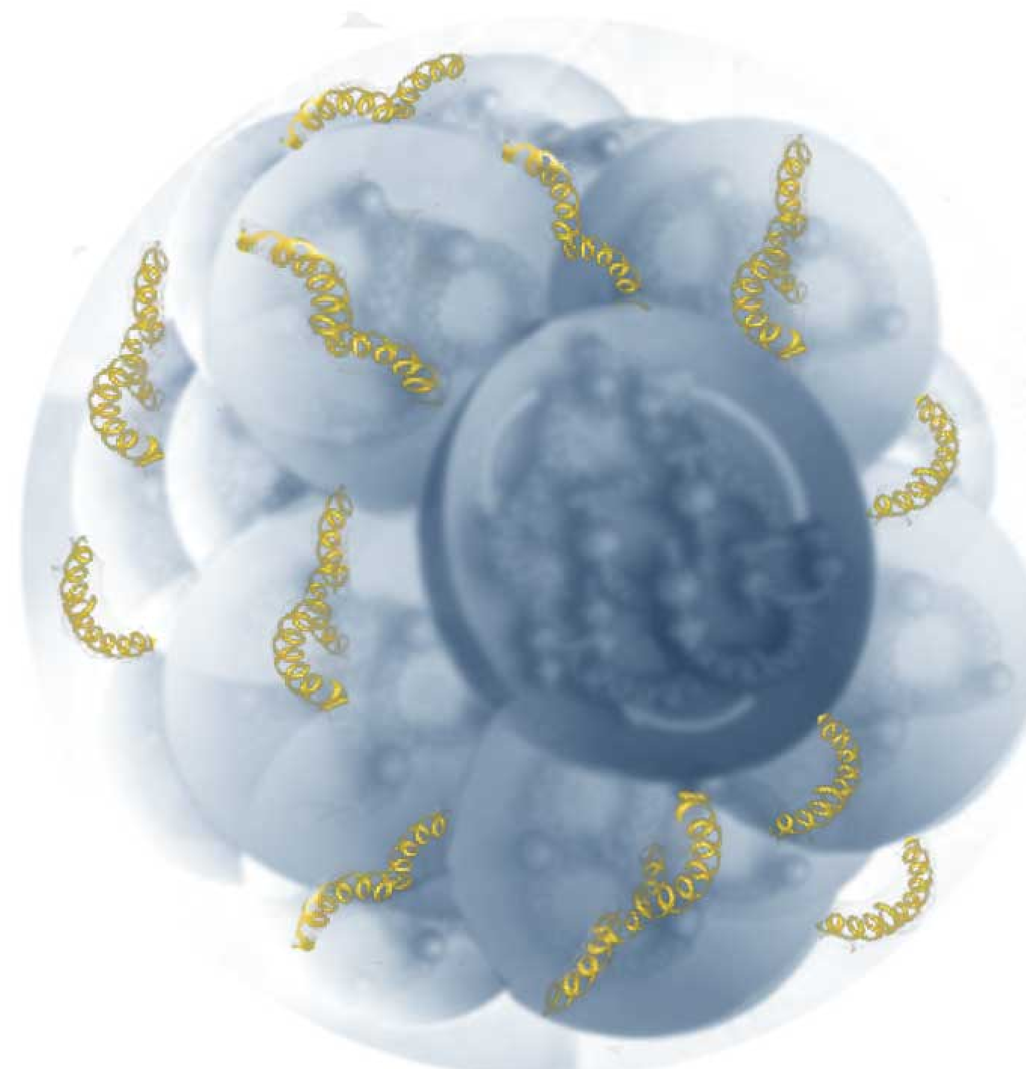
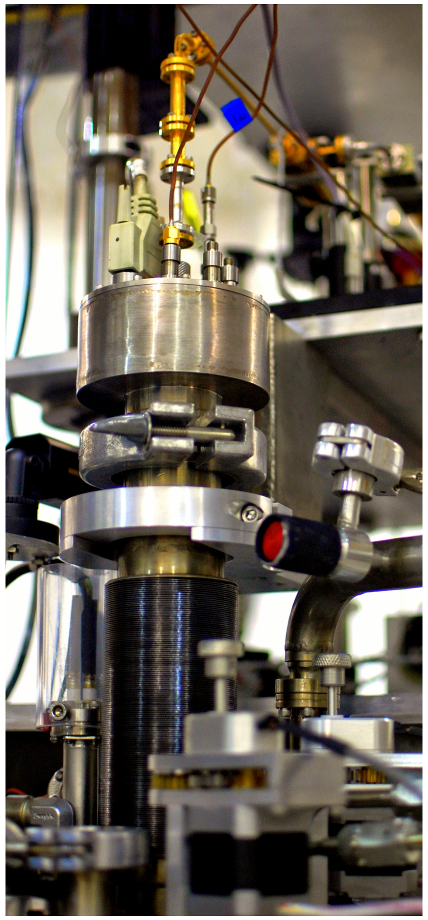


Double Helicity Flip Gluon Structure

- Targets with J≥1 have an additional leading twist gluon parton distribution $\Delta(x,Q^2)$: double helicity flip [Jaffe & Manohar 1989]
  - Unambiguously gluonic: no analogous quark PDF at twist-2
  - Vanishes in nucleon: nonzero value in nucleus probes nuclear effects directly
  - Experimentally measurable
    - NH_3: JLab LoI 2015 [PI: James Maxwell]
    - Polarised nuclei at EIC under serious consideration [R. Milner]
  - Moments calculable in LQCD

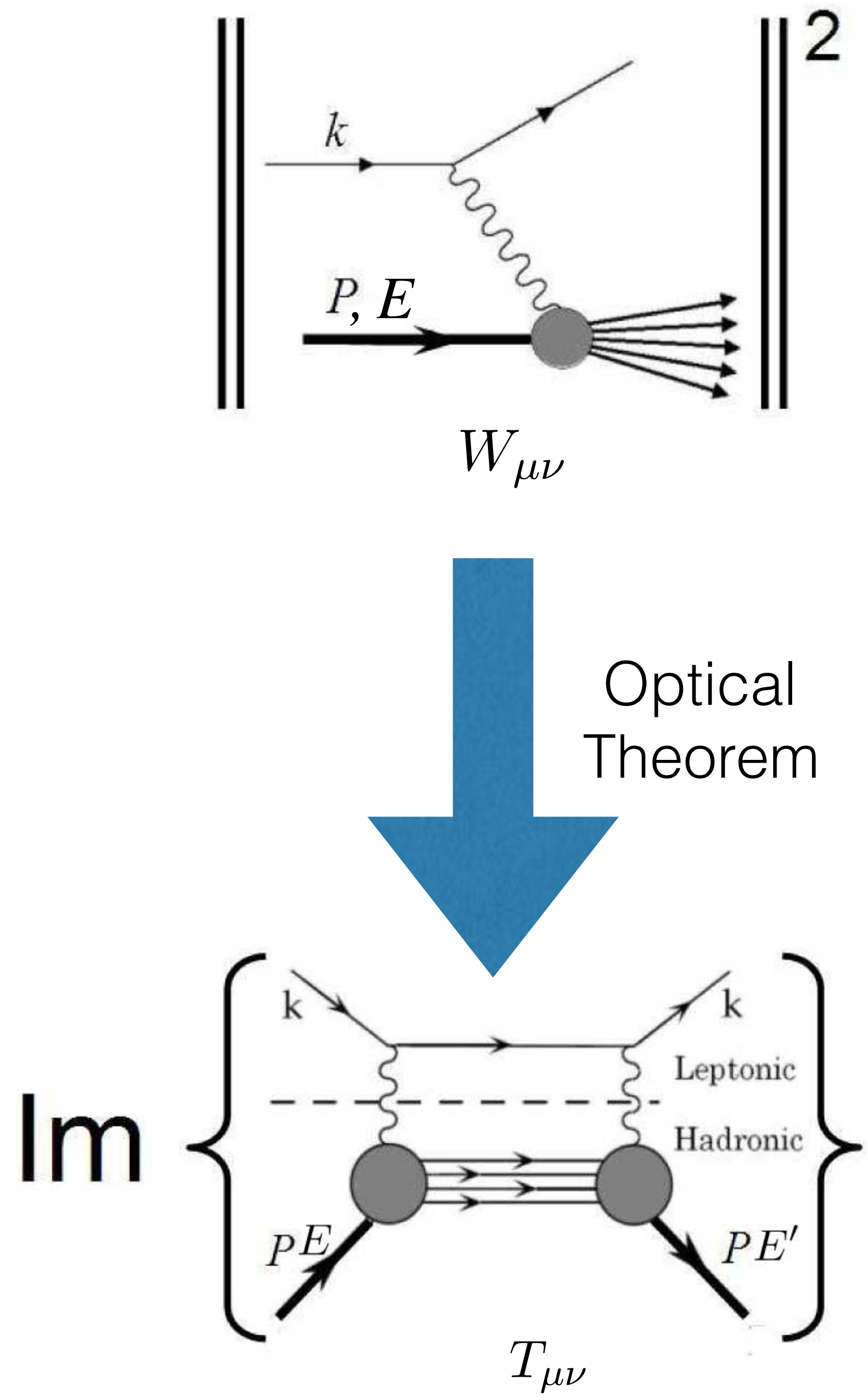


- Deep inelastic scattering on J=1 target
  [Hoodbhoy, Jaffe, Manohar 1989]

$$W_{\mu\nu}(p,q,E',E) = \frac{1}{4\pi}\int d^4x\, e^{iq\cdot x}\langle p',E'|[j_\mu(x),j_\nu(0)]|p,E\rangle$$

$$\begin{aligned} W_{\mu\nu}^{\lambda_f\lambda_i} &= -F_1\hat{g}_{\mu\nu} + \frac{F_2}{M\nu}\hat{p}_\mu\hat{p}_\nu - b_1 r_{\mu\nu} \\ &+ \frac{1}{6}b_2(s_{\mu\nu}+t_{\mu\nu}+u_{\mu\nu}) + \frac{1}{2}b_3(s_{\mu\nu}-u_{\mu\nu}) + \frac{1}{2}b_4(s_{\mu\nu}-t_{\mu\nu}) \\ &+ \frac{ig_1}{\nu}\epsilon_{\mu\nu\lambda\sigma}q^\lambda s^\sigma + \frac{ig_2}{M\nu^2}\epsilon_{\mu\nu\lambda\sigma}q^\lambda(p\cdot q s^\sigma - s\cdot q p^\sigma) + W_{\mu\nu}^{\Delta=2} \end{aligned}$$

- where $\{s,t,u\}_{\mu\nu} = \{s,t,u\}_{\mu\nu}(E,E',p,q)$
- Contains double helicity flip
  [Jaffe, Manohar 1989]



Double Helicity Flip Gluon Structure

- Double helicity flip structure function

$$W_{\mu\nu}^{\Delta=2} = \frac{1}{2}\Bigg\{ \Bigg[\left(E'^*_\mu - \frac{q\cdot E'^*}{\kappa\nu}\left(p_\mu - \frac{M^2}{\nu}q_\mu\right)\right)\left(E_\nu - \frac{q\cdot E}{\kappa\nu}\left(p_\nu - \frac{M^2}{\nu}q_\nu\right)\right) + (\mu \leftrightarrow \nu)\Bigg] - \left[g_{\mu\nu} - \frac{q_\mu q_\nu}{q^2} + \frac{q^2}{\kappa\nu^2}\left(p_\mu - \frac{\nu}{q^2}q_\mu\right)\left(p_\nu - \frac{\nu}{q^2}q_\nu\right)\right]\left[E'^*\cdot E + \frac{M^2}{\kappa\nu^2} q\cdot E'^*\, q\cdot E\right]\Bigg\}\Delta(x,Q^2)$$

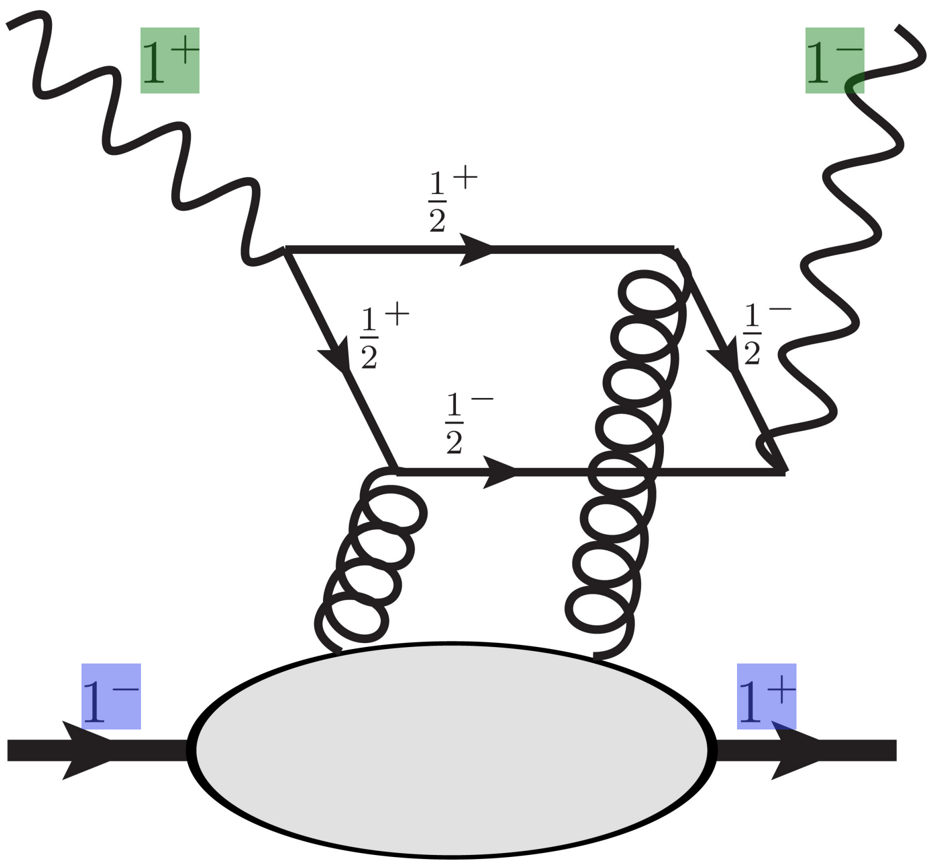
- Express in helicity amplitude basis

$$W_{\mu\nu}(p,q,E,E') = E'^{*\alpha}E^\beta W_{\mu\nu,\alpha\beta}(p,q)$$

$$W_{\mu\nu,\alpha\beta}(p,q) = \sum_{hH,h'H'} P(hH,h'H')_{\mu\nu,\alpha\beta} A_{hH,h'H'}(p,q)$$

- Changes both photon and target helicity by 2 units

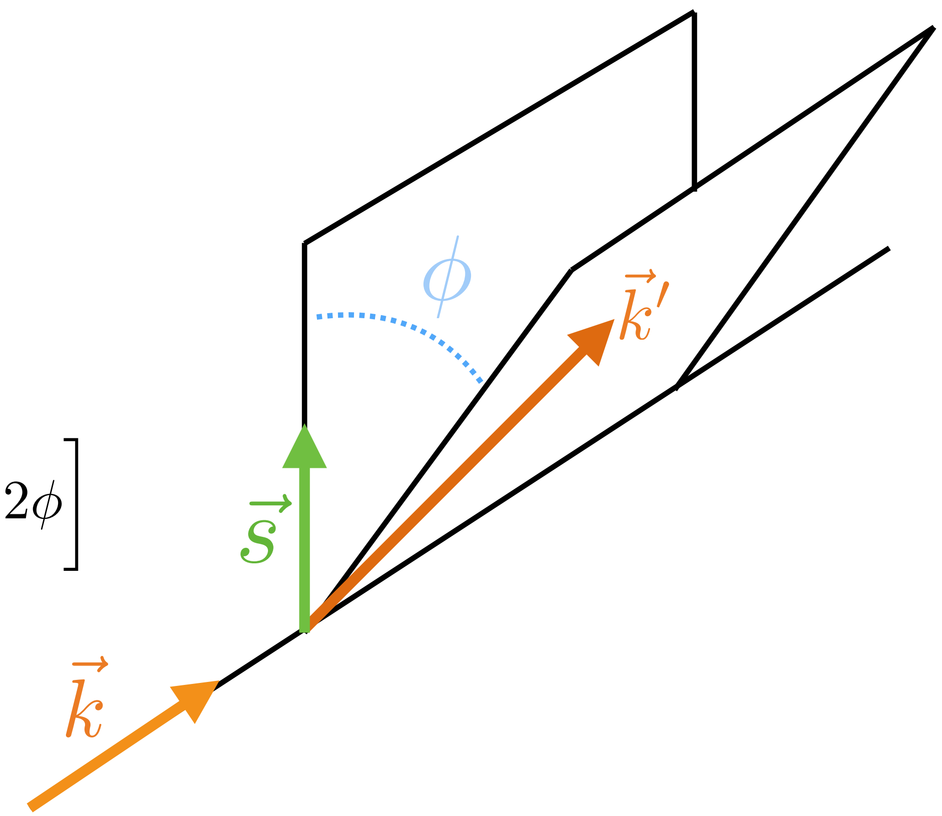
$$\Delta(x,Q^2) = A_{+-,-+}$$



Double Helicity Flip Gluon Structure

- Measurable in unpolarised electron DIS on transversely polarised target as azimuthal variation

$$\lim_{Q^2\to\infty} \frac{d\sigma}{dx\,dy\,d\phi} = \frac{e^4 ME}{4\pi^2 Q^4}\Big[xy^2 F_1(x,Q^2) + (1-y)F_2(x,Q^2) - \frac{x(1-y)}{2}\Delta(x,Q^2)\cos 2\phi\Big]$$



- Parton model interpretation

$$\Delta(x,Q^2) = -\frac{\alpha_s(Q^2)}{2\pi}\mathrm{Tr}\mathcal{Q}^2\, x^2 \int_x^1 \frac{dy}{y^3}\left[g_{\hat{x}}(y,Q^2) - g_{\hat{y}}(x,Q^2)\right]$$

where $g_{\hat{x},\hat{y}}(x,\mu^2)$ is probability of finding a gluon with momentum fraction *x* linearly polarised in x,y direction

Double Helicity Flip Gluon Structure

- Moments

$$\int_0^1 dx\ x^{n-1}\Delta(x,Q^2) = \frac{\alpha_s(Q^2)}{3\pi(n+2)} A_n(Q^2) \qquad n = 2, 4, \ldots$$

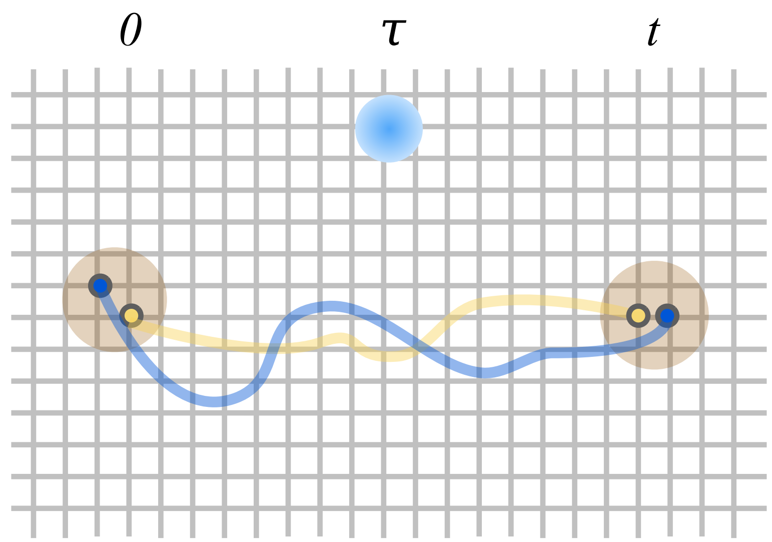
- Determined by matrix elements of local gluonic operators

$$\begin{aligned}\langle p, E'|\mathcal{S}[G_{\mu\mu_1} \overleftrightarrow{D}_{\mu_3} \ldots \overleftrightarrow{D}_{\mu_n} G_{\nu\mu_2}]|p, E\rangle \\ = (-2i)^{n-2}\mathcal{S}[\{(p_\mu E'^*_{\mu_1} - p_{\mu_1}E'^*_{\mu})(p_\nu E'^*_{\mu_2} - p_{\mu_2}E'^*_{\nu}) \\ +(\mu \leftrightarrow \nu)\}p_{\mu_3}\ldots p_{\mu_n}]A_n(Q^2)\end{aligned}$$

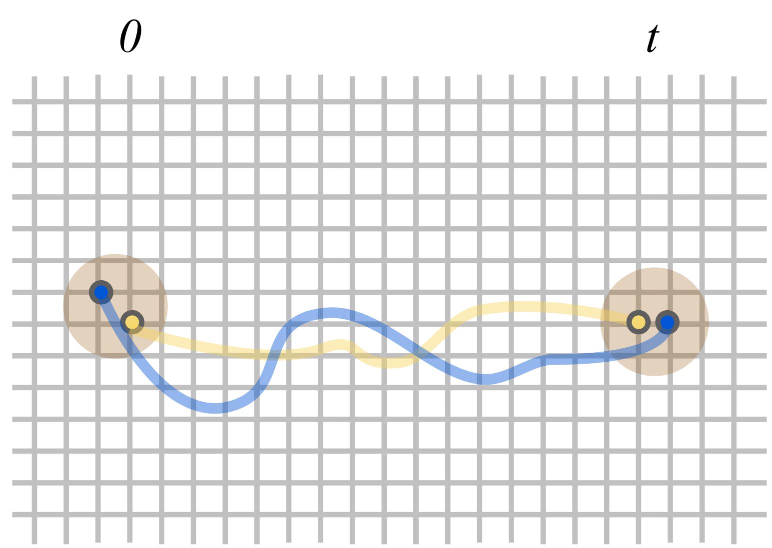
  - Symmetrised and trace subtracted in $\mu_1 \ldots \mu_n$
  - Local operators suitable for calculation in lattice QCD

- Extract matrix element from ratio of correlators

$C_3(t,\tau) =$



$C_2(t) =$



$$\frac{C_3(t,\tau)}{C_2(t)} \propto A_2, \qquad 0 \ll \tau \ll t$$

Hypercubic group

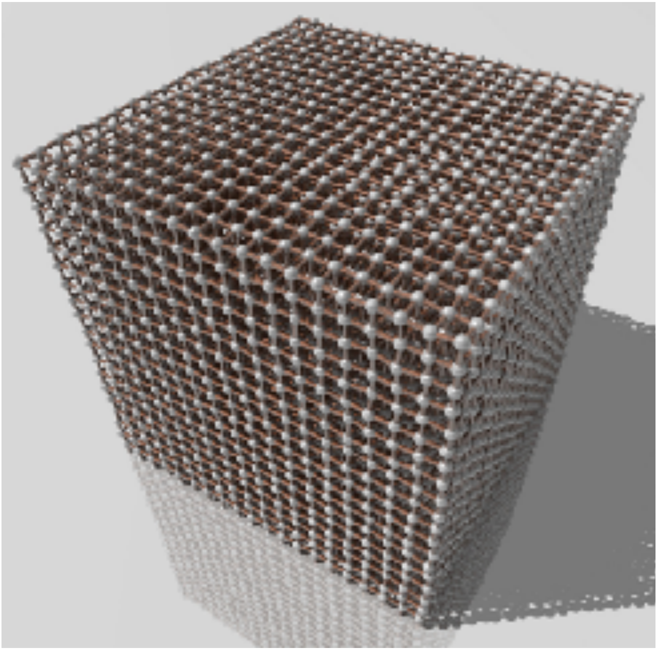
- Lattice symmetries significantly reduced from O(4) by discretisation and boundary conditions

- H(4): finite group of rotations by π/2 and reflections

$$H(4) = \{(a,\pi)|a \in \mathbb{Z}_2^4,\, \pi \in S_4\}$$

- 20 irreducible representations

$$4 \cdot \mathbf{1} \oplus 2 \cdot \mathbf{2} \oplus 4 \cdot \mathbf{3} \oplus 4 \cdot \mathbf{4} \oplus 4 \cdot \mathbf{6} \oplus 2 \cdot \mathbf{8}$$



Nice Example

- Continuum operator $\mathcal{O}_{\mu\nu} = \bar{q}\gamma_{\{\mu}D_{\nu\}}q$ belongs to

$$\left(\tfrac{1}{2},\tfrac{1}{2}\right) \otimes \left(\tfrac{1}{2},\tfrac{1}{2}\right) = (0,0) \oplus [(1,0) \oplus (0,1)] \oplus (1,1)$$

- Hypercubic decomposition

$$\mathbf{4}_1 \otimes \mathbf{4}_1 = \mathbf{1}_1 \oplus \mathbf{3}_1 \oplus \mathbf{6}_1 \oplus \mathbf{6}_3$$

- Lattice operators (symmetric traceless):

$$\mathcal{O}_{14} + \mathcal{O}_{41}, \qquad \mathcal{O}_{44} - \frac{1}{3}\left(\mathcal{O}_{11} + \mathcal{O}_{22} + \mathcal{O}_{33}\right)$$

- Have same continuum limit ($\mathbf{6}_3$ requires $\mathbf{p} \neq 0$)
- No operators of lower dimension 😎

Nasty example

- Continuum operator $\mathcal{O}_{\{\mu\nu\rho\}} = \overline{q}\gamma_{\{\mu}D_\nu D_{\rho\}}q$ lives in

$$\left(\tfrac{1}{2},\tfrac{1}{2}\right)\otimes\left(\tfrac{1}{2},\tfrac{1}{2}\right)\otimes\left(\tfrac{1}{2},\tfrac{1}{2}\right) = 4\cdot\left(\tfrac{1}{2},\tfrac{1}{2}\right)\oplus 2\cdot\left(\tfrac{3}{2},\tfrac{1}{2}\right)\oplus 2\cdot\left(\tfrac{1}{2},\tfrac{3}{2}\right)\oplus\left(\tfrac{3}{2},\tfrac{3}{2}\right)$$

- Hypercubic decomposition

$$\mathbf{4}_1\otimes\mathbf{4}_1\otimes\mathbf{4}_1 = 4\cdot\mathbf{4}_1\oplus\mathbf{4}_2\oplus\mathbf{4}_4\oplus 3\cdot\mathbf{8}_1\oplus 2\cdot\mathbf{8}_2$$

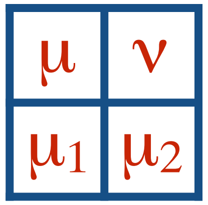
- Lattice operators:

$$\mathcal{O}_{111},\qquad \mathcal{O}_{\{123\}},\qquad \mathcal{O}_{\{441\}} - \tfrac{1}{2}\left(\mathcal{O}_{\{221\}} + \mathcal{O}_{\{331\}}\right)$$

- Same continuum limit but $\mathcal{O}_{111}$ mixes with $\overline{q}\gamma_1 q \in \mathbf{4_1}$ and the coefficient absorbs the missing dimensions 😩
  - Always the case for all n > 4 quark operators

n=2 operator

- Focus on n=2 operator $\mathcal{O}_{\mu\nu\mu_1\mu_2} = S\left[G_{\mu\mu_1} G_{\nu\mu_2}\right]$



- Construct in the clean H(4) irrupts

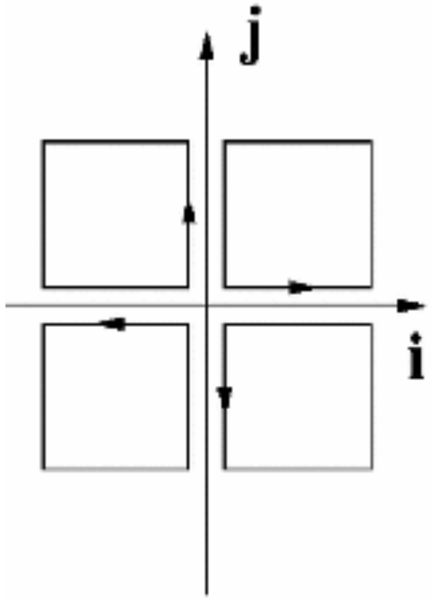
$$4\ \tau_1^{(1)} \oplus 3\tau_1^{(2)} \oplus 7\tau_1^{(3)} \oplus 10\tau_1^{(6)} \oplus \tau_2^{(1)} \oplus 2\tau_2^{(2)} \oplus 3\tau_2^{(3)} \oplus 6\tau_2^{(6)} \oplus 3\tau_3^{(3)} \oplus 10\tau_3^{(6)} \oplus \tau_4^{(1)} \oplus 3\tau_4^{(3)} \oplus 6\tau_4^{(6)}$$

  - No mixing until subheading twist

- Build from clover field strength tensor

$$G_{\mu\nu}(x) = \frac{1}{4}\frac{1}{2}\left(P_{\mu\nu}(x) - P^\dagger_{\mu\nu}(x)\right)$$

$$\begin{aligned} P_{\mu\nu}(x) =& U_\mu(x)U_\nu(x+\mu)U^\dagger_\mu(x+\nu)U^\dagger_\nu(x) \\ &+ U_\nu(x)U^\dagger_\mu(x-\mu+\nu)U^\dagger_\nu(x-\mu)U_\mu(x-\mu) \\ &+ U^\dagger_\mu(x-\mu)U^\dagger_\nu(x-\mu-\nu)U_\mu(x-\mu-\nu)U_\nu(x-\nu) \\ &+ U^\dagger_\nu(x-\nu)U_\mu(x-\nu)U_\nu(x-\nu+\mu)U^\dagger_\mu(x). \end{aligned}$$



- Focus in bare operator and ignore renormalisation

$$\mathcal{O}^{(E)}_{m,n} = Z_2^m \mathcal{O}^{\text{latt.}}_{m,n}$$

n=2 operator

- Focus on n=2 operator $\mathcal{O}_{\mu\nu\mu_1\mu_2} = S\left[G_{\mu\mu_1} G_{\nu\mu_2}\right]$

μ	ν
μ_1	μ_2

- Construct in the clean H(4) irrupts

$$4\,\tau_1^{(1)} \oplus 3\tau_1^{(2)} \oplus 7\tau_1^{(3)} \oplus 10\tau_1^{(6)} \oplus \tau_2^{(1)} \oplus 2\tau_2^{(2)} \oplus 3\tau_2^{(3)} \oplus 6\tau_2^{(6)} \oplus 3\tau_3^{(3)} \oplus 10\tau_3^{(6)} \oplus \tau_4^{(1)} \oplus 3\tau_4^{(3)} \oplus 6\tau_4^{(6)}$$

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- Build from clover field strength tensor

$$G_{\mu\nu}(x) = \frac{1}{4}\frac{1}{2}\left(P_{\mu\nu}(x) - P^\dagger_{\mu\nu}(x)\right)$$

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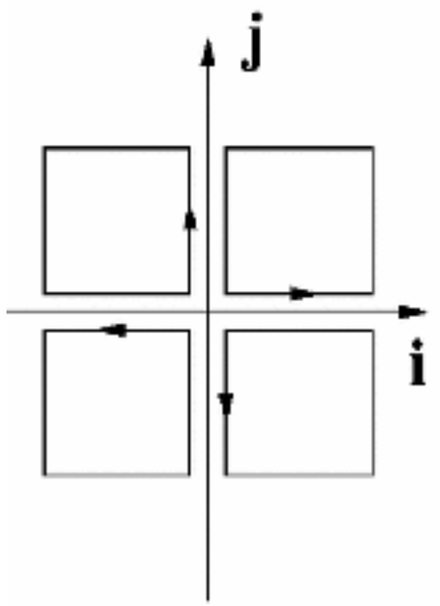


- Focus in bare operator and ignore renormalisation

$$\mathcal{O}^{(E)}_{m,n} = Z_2^m \mathcal{O}^{\text{latt.}}_{m,n}$$

Double Helicity Flip Gluon Structure

- First LQCD calculation [WD & P Shanahan PRD 94 (2016), 014507]
- First moment in φ meson (simplest spin-1 system, nuclei eventually)
- Lattice details: clover fermions, Lüscher-Weisz gauge action

L/a	T/a	β	am_l	am_s
24	64	6.1	-0.2800	-0.2450
a (fm)	L (fm)	T (fm)	m_π (MeV)	m_K (MeV)
0.1167(16)	2.801(29)	7.469(77)	450(5)	596(6)
m_ϕ (MeV)	$m_\pi L$	$m_\pi T$	N_{cfg}	N_{src}
1040(3)	6.390	17.04	1042	10^5

- Many systematics not addressed!: a→0, L→∞, m_{phys}
- Extremely high statistics: O(100,000) measurements

Double Helicity Flip Gluon Structure

- More specifically (including off-forward case

$$C^{\rm 2pt}_{jk}(\vec{p},t) = \left\langle \eta_k(\vec{p},t)\eta^\dagger_j(\vec{p},0)\right\rangle$$
$$= |Z_\phi(\vec{p})|^2\left(e^{-\mathcal{E}t}+e^{-\mathcal{E}(T-t)}\right)\sum_\lambda \epsilon_k(\vec{p},\lambda)\epsilon^*_j(\vec{p},\lambda)$$

$$C^{\rm 3pt}_{jk}(\vec{p},\vec{p}\,',t,\tau,\mathcal{O}) \equiv \left\langle \eta_k(\vec{p},t)\,\mathcal{O}(\vec{p}\,'-\vec{p},\tau)\,\eta^\dagger_j(\vec{p}\,',0)\right\rangle - \left\langle \eta_k(\vec{p},t)\,\eta^\dagger_j(\vec{p}\,',0)\right\rangle\left\langle \mathcal{O}(\vec{p}\,'-\vec{p},\tau)\right\rangle$$
$$= Z^\dagger_\phi(\vec{p})Z_\phi(\vec{p}\,')e^{-\mathcal{E}t}\sum_{\lambda\lambda'}\epsilon_k(\vec{p},\lambda)\epsilon^*_j(\vec{p}\,',\lambda')\langle\vec{p},\lambda|\mathcal{O}|\vec{p}\,',\lambda'\rangle$$

$$R_{jk}(\vec{p},\vec{p}\,',t,\tau,\mathcal{O}) = \frac{C^{\rm 3pt}_{jk}(\vec{p},\vec{p}\,',t,\tau,\mathcal{O})}{C^{\rm 2pt}_{kk}(\vec{p}\,',t)}\sqrt{\frac{C^{\rm 2pt}_{jj}(\vec{p},t-\tau)C^{\rm 2pt}_{kk}(\vec{p}\,',t)C^{\rm 2pt}_{kk}(\vec{p}\,',\tau)}{C^{\rm 2pt}_{kk}(\vec{p}\,',t-\tau)C^{\rm 2pt}_{jj}(\vec{p},t)C^{\rm 2pt}_{jj}(\vec{p},\tau)}}$$

- Use appropriate combinations of polarisations
- Study for boost momenta up to (1,1,1)
- Examine all elements of each lattice irrep

$$\epsilon^\mu(\vec{p},\lambda) = \left(\frac{\vec{p}\cdot\vec{e}_\lambda}{m},\vec{e}_\lambda+\frac{\vec{p}\cdot\vec{e}_\lambda}{m(m+E)}\vec{p}\right)$$

$$\vec{e}_\pm = \mp\frac{m}{\sqrt{2}}(0,1,\pm i),$$
$$\vec{e}_0 = m(1,0,0).$$

Double Helicity Flip Gluon Structure

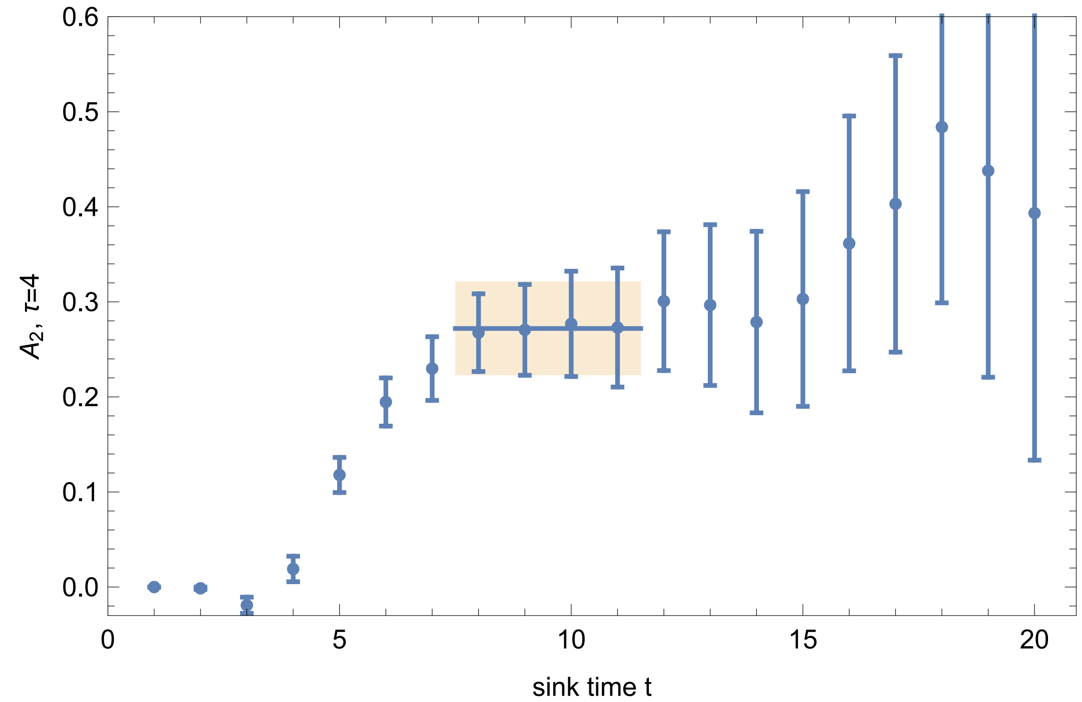
- Form many combinations of polarisations boosts etc
- Example: p=(0,0,0)

$$
\begin{array}{c} \\ \rho_0 \\ \rho_+ \\ \rho_- \end{array}
\begin{array}{c}
\begin{array}{ccc} \rho_0 & \rho_+ & \rho_- \end{array} \\
\begin{pmatrix}
\frac{2m^2 A_2}{\sqrt{3}} & 0 & 0 \\
0 & -\frac{m^2 A_2}{\sqrt{3}} & 0 \\
0 & 0 & -\frac{m^2 A_2}{\sqrt{3}}
\end{pmatrix}
\end{array}
$$

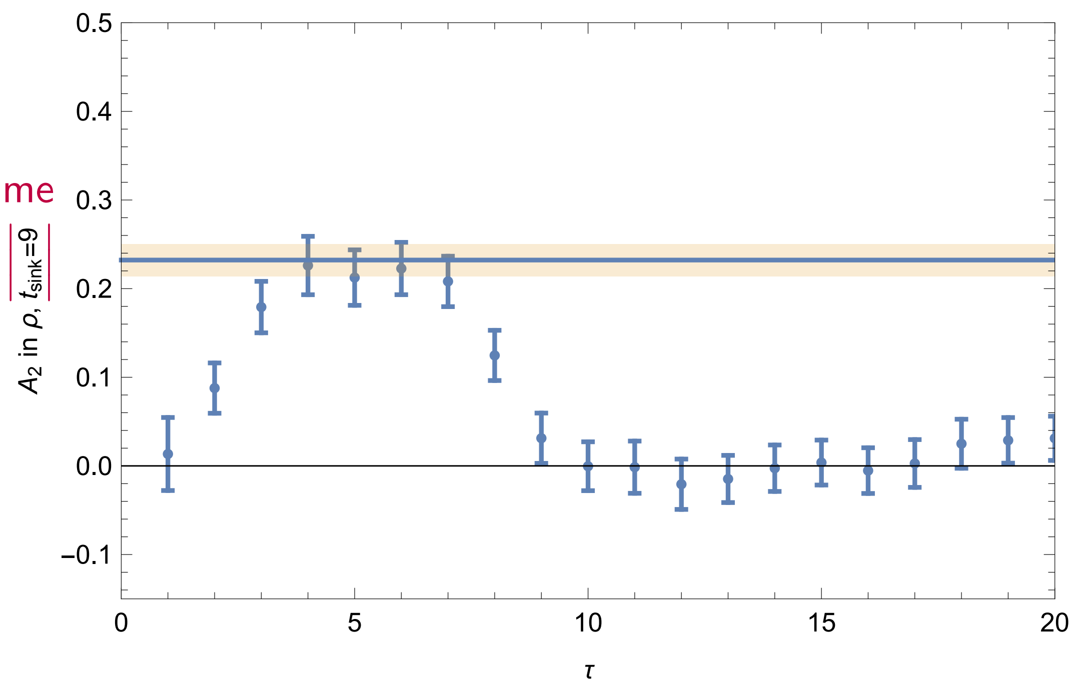
- Example p=p(1,1,1)

$$
\begin{array}{c} \\ \rho_0 \\ \rho_+ \\ \rho_- \end{array}
\begin{array}{c}
\begin{array}{ccc} \rho_0 & \rho_+ & \rho_- \end{array} \\
\begin{pmatrix}
\frac{2\left(m^3+\sqrt{m^2+3p^2}m^2+4p^2m+2p^2\sqrt{m^2+3p^2}\right)A_2}{\sqrt{3}\left(m+\sqrt{m^2+3p^2}\right)} & \frac{(1-i)p^2\left(m+2\sqrt{m^2+3p^2}\right)A_2}{\sqrt{6}\left(m+\sqrt{m^2+3p^2}\right)} & -\frac{(1+i)p^2\left(m+2\sqrt{m^2+3p^2}\right)A_2}{\sqrt{6}\left(m+\sqrt{m^2+3p^2}\right)} \\
\frac{(1+i)p^2\left(m+2\sqrt{m^2+3p^2}\right)A_2}{\sqrt{6}\left(m+\sqrt{m^2+3p^2}\right)} & -\frac{\left(m^3+\sqrt{m^2+3p^2}m^2+4p^2m+2p^2\sqrt{m^2+3p^2}\right)A_2}{\sqrt{3}\left(m+\sqrt{m^2+3p^2}\right)} & -\frac{2ip^2\left(m+2\sqrt{m^2+3p^2}\right)A_2}{\sqrt{3}\left(m+\sqrt{m^2+3p^2}\right)} \\
-\frac{(1-i)p^2\left(m+2\sqrt{m^2+3p^2}\right)A_2}{\sqrt{6}\left(m+\sqrt{m^2+3p^2}\right)} & \frac{2ip^2\left(m+2\sqrt{m^2+3p^2}\right)A_2}{\sqrt{3}\left(m+\sqrt{m^2+3p^2}\right)} & -\frac{\left(m^3+\sqrt{m^2+3p^2}m^2+4p^2m+2p^2\sqrt{m^2+3p^2}\right)A_2}{\sqrt{3}\left(m+\sqrt{m^2+3p^2}\right)}
\end{pmatrix}
\end{array}
$$

Double Helicity Flip Gluon Structure

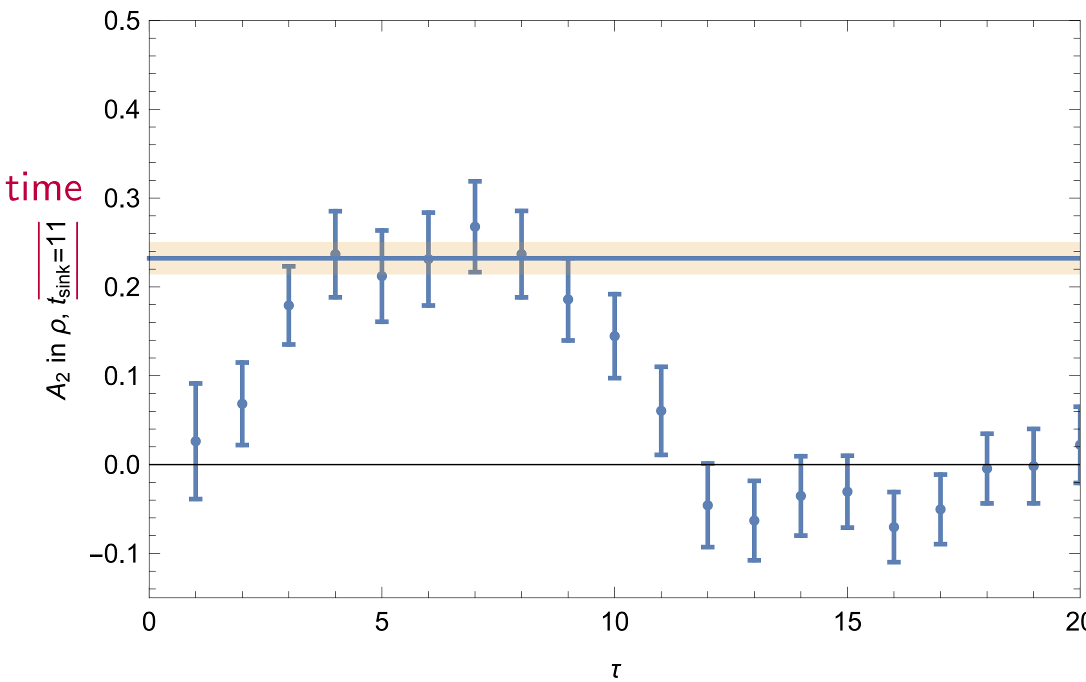


sink time



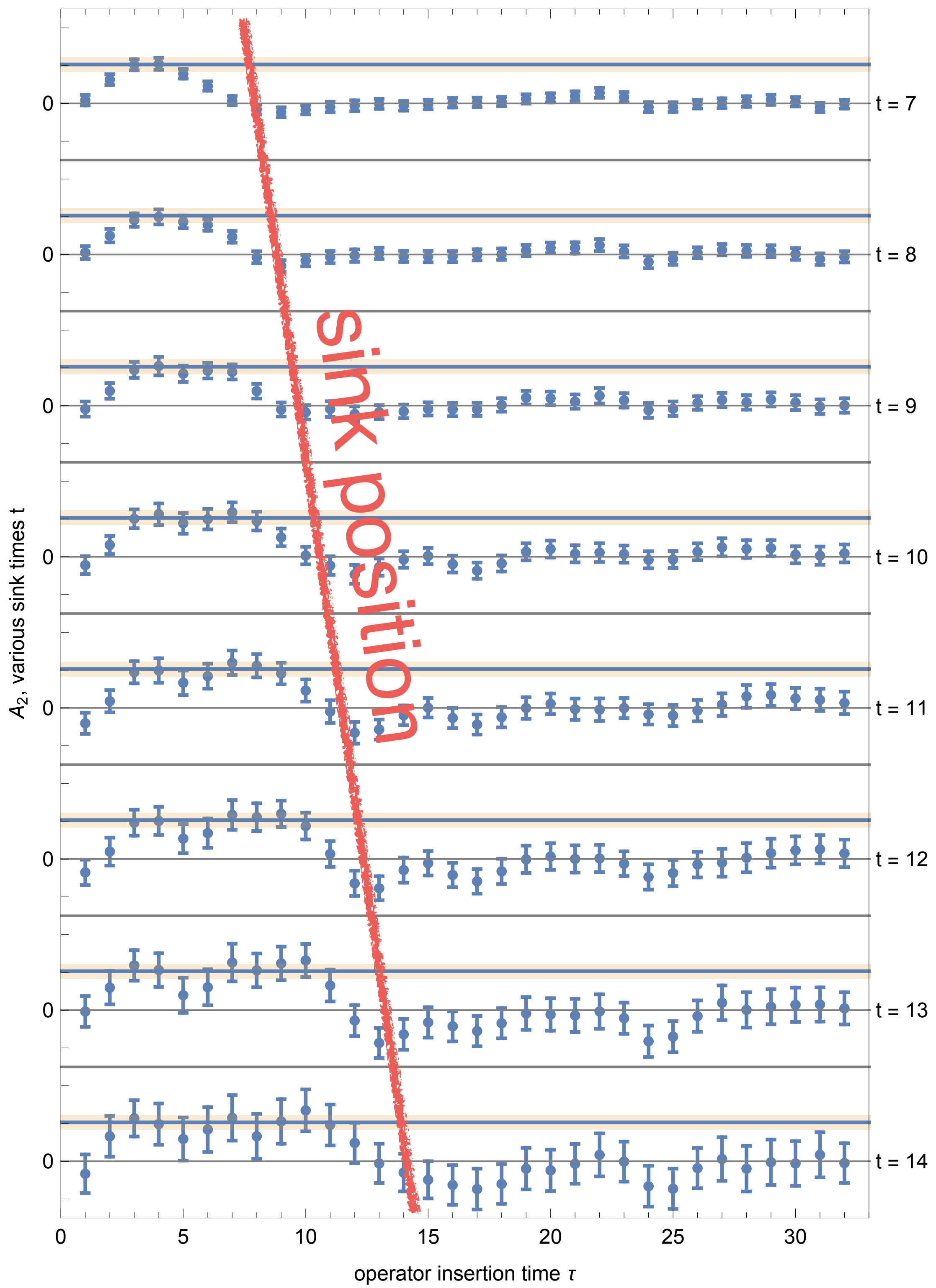
operator insertion time

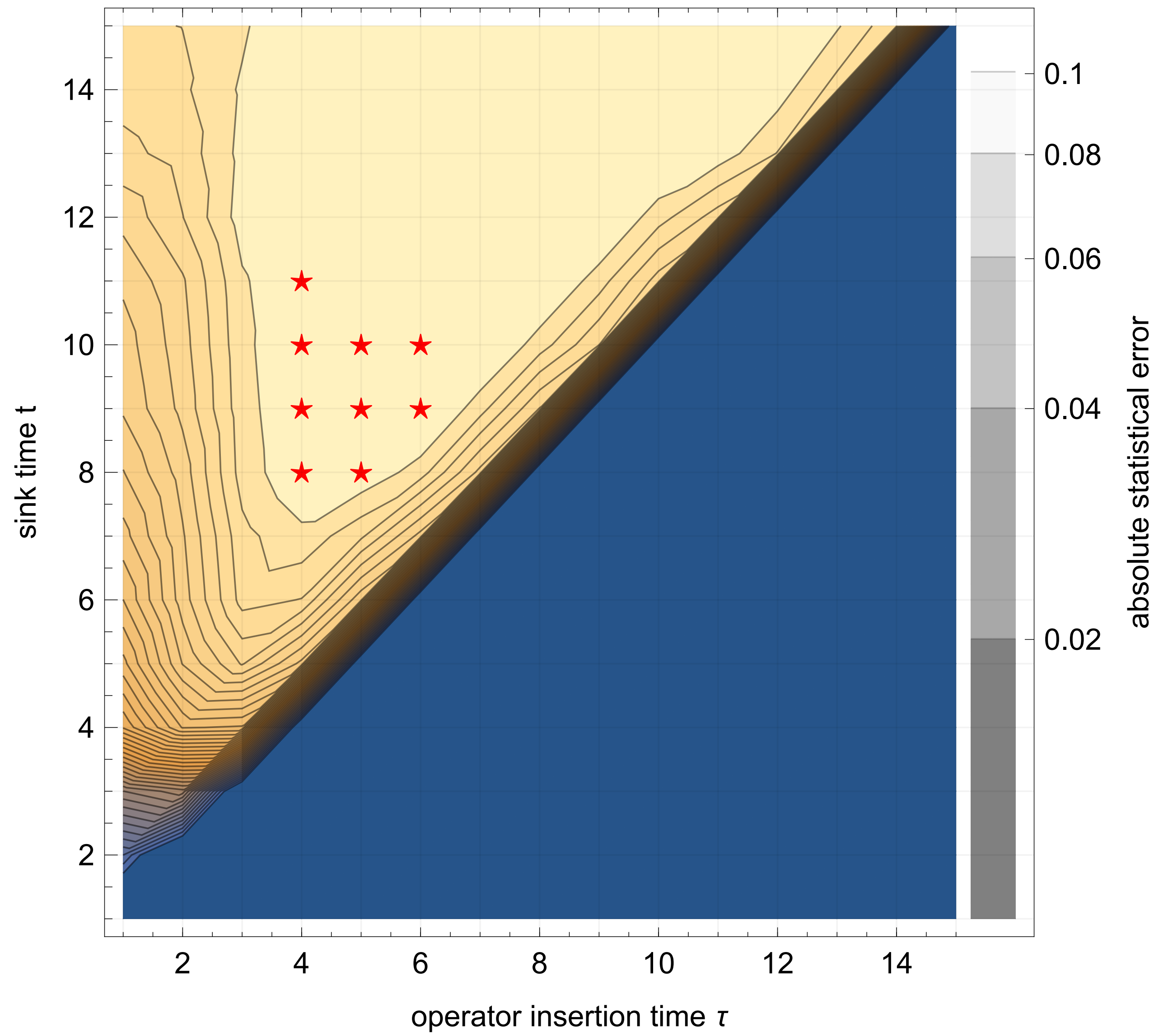
sink time



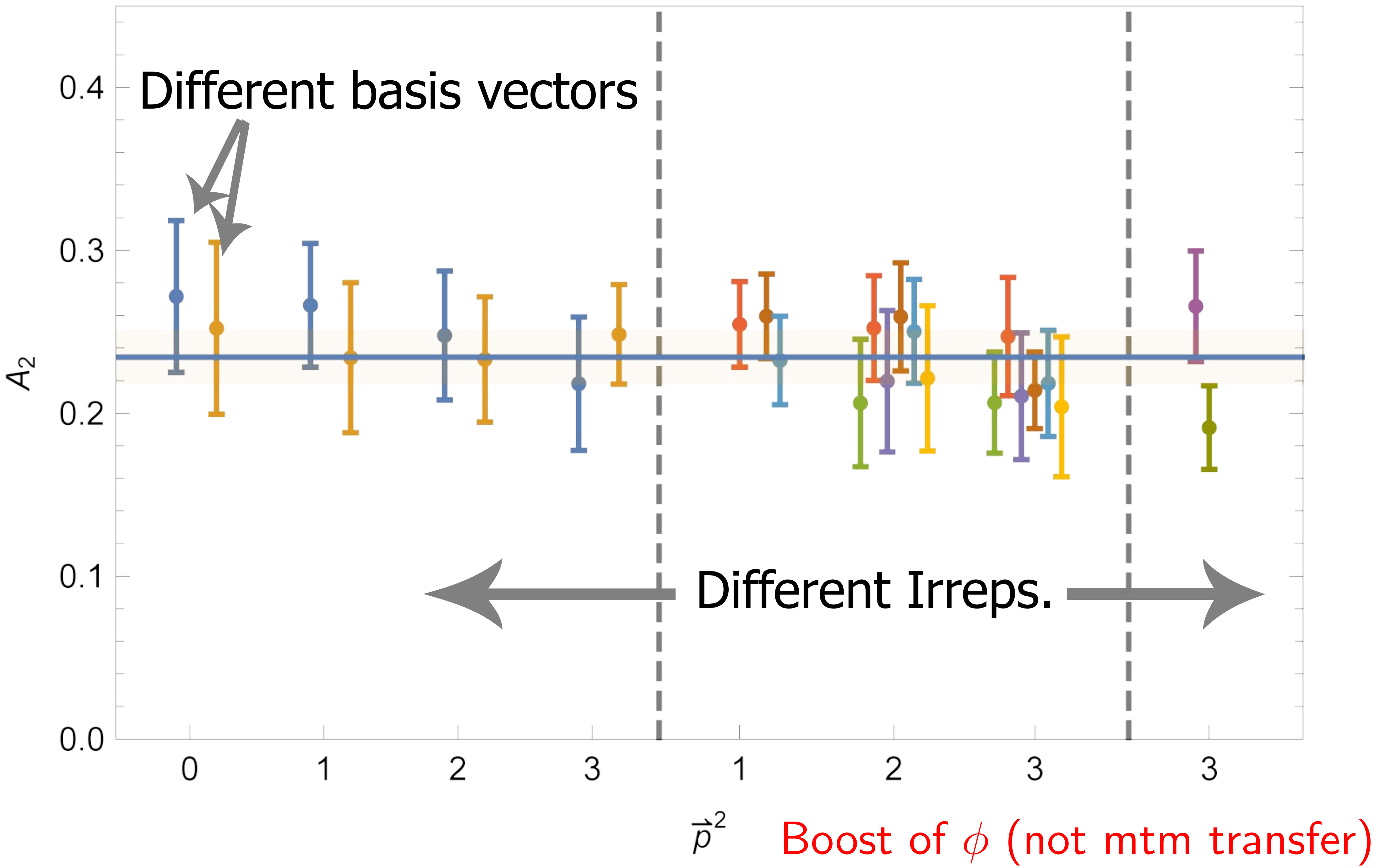
operator insertion time

Double Helicity Flip Gluon Structure





Double Helicity Flip Gluon Structure



Gluonic Soffer bound

- Soffer bound on quark transversity

$$|\delta q(x)| \leq \frac{1}{2}(q(x) + \Delta q(x))$$

- Moment space

$$\langle x^2 \rangle_{\delta q} \leq \frac{1}{2}(\langle x^2 \rangle_q + \langle x^2 \rangle_{\Delta q})$$

  - Saturated at ~80% from LQCD [Diehl et al. 2005 @ heavy quark mass]

- Gluonic analogue

Transversity

Spin-independent

$$|A_2| \leq \frac{1}{24}(5B_{2,1} - 6B_{2,2})$$

Spin-dependent → 0

Unpolarised gluon structure

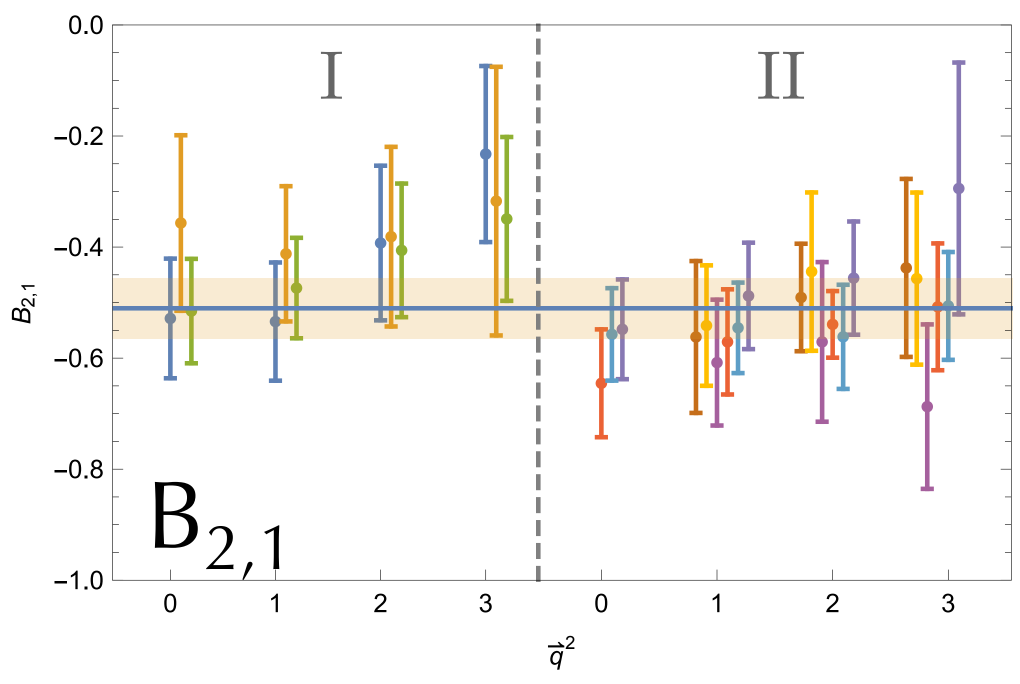
- Spin-independent gluon operator:

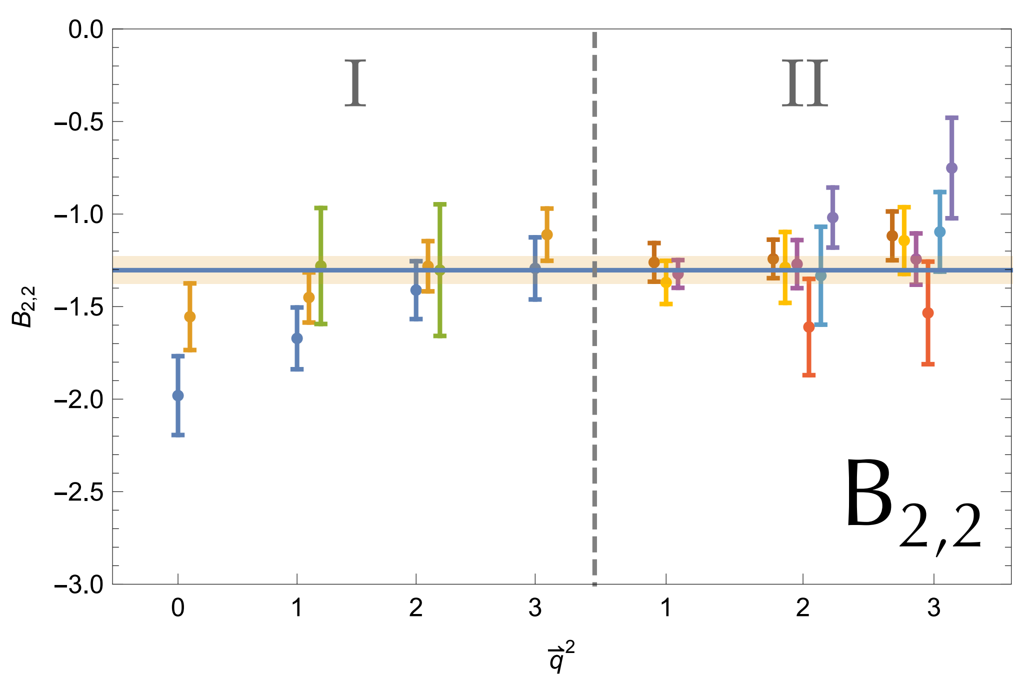
$$\overline{\mathcal{O}}_{\mu_1\ldots\mu_n} = S\left[G_{\mu_1\alpha}\overleftrightarrow{D}_{\mu_3}\ldots\overleftrightarrow{D}_{\mu_n}G_{\mu_2}{}^{\alpha}\right]$$

- Matrix elements at n=2 define lowest moment of structure functions

$$\langle pE'|\overline{\mathcal{O}}_{\mu_1\mu_2}|pE\rangle = S\left[M^2 E'^*_{\mu_1}E_{\mu_2}\right]\boxed{B_{2,1}(\mu^2)} + S\left[(E\cdot E'^*)p_{\mu_1}p_{\mu_2}\right]\boxed{B_{2,2}(\mu^2)}$$

Two reduced matrix elements





- Analysis as in transversity case

- Mixing with quark ops. neglected, pQCD calcs. shown that it is small: Alexandrou 1611.06901

Gluonic Soffer bound

- Gluonic bound satisfied similarly

$$|A_2| \leq \frac{1}{24}(5B_{2,1} - 6B_{2,2})$$

Transversity

Spin-independent

Spin-dependent → 0

$$|0.24| \leq \frac{1}{24}\left[5(-0.5) - 6(-1.4)\right] = 0.24$$

- CAUTION: bare matrix elements!!
- All for φ meson: next step is deuteron!!

Gluonic radii

- Radii defined by slope of FFs vs Q^2
- Matrix elements of the spin-independent gluon operator
- Off-forward matrix elements are complicated:

$$
\begin{aligned}
&\left\langle p'E' \left| S\left[G_{\mu\alpha} i\overleftrightarrow{D}_{\mu_1}\dots i\overleftrightarrow{D}_{\mu_n} G_\nu{}^\alpha\right]\right| pE\right\rangle \\
&\quad = \sum_{\substack{m\ \text{even}\\ m=0}}^{n} \Bigg\{ B_{1,m}^{(n+2)}(\Delta^2) M^2 S\left[E_\mu E'^*_\nu \Delta_{\mu_1}\dots\Delta_{\mu_m} P_{\mu_{m+1}}\dots P_{\mu_n}\right] \\
&\qquad + B_{2,m}^{(n+2)}(\Delta^2) S\left[(E\cdot E'^*) P_\mu P_\nu \Delta_{\mu_1}\dots\Delta_{\mu_m} P_{\mu_{m+1}}\dots P_{\mu_n}\right] \\
&\qquad + B_{3,m}^{(n+2)}(\Delta^2) S\left[(E\cdot E'^*) \Delta_\mu \Delta_\nu \Delta_{\mu_1}\dots\Delta_{\mu_m} P_{\mu_{m+1}}\dots P_{\mu_n}\right] \\
&\qquad + B_{4,m}^{(n+2)}(\Delta^2) S\left[\left((E'^*\cdot P) E_\mu P_\nu + (E\cdot P) E'^*_\mu P_\nu\right) \Delta_{\mu_1}\dots\Delta_{\mu_m} P_{\mu_{m+1}}\dots P_{\mu_n}\right] \\
&\qquad + B_{5,m}^{(n+2)}(\Delta^2) S\left[\left((E'^*\cdot P) E_\mu \Delta_\nu - (E\cdot P) E'^*_\mu \Delta_\nu\right) \Delta_{\mu_1}\dots\Delta_{\mu_m} P_{\mu_{m+1}}\dots P_{\mu_n}\right] \\
&\qquad + \frac{B_{6,m}^{(n+2)}(\Delta^2)}{M^2} S\left[(E\cdot P)(E'^*\cdot P) P_\mu P_\nu \Delta_{\mu_1}\dots\Delta_{\mu_m} P_{\mu_{m+1}}\dots P_{\mu_n}\right] \\
&\qquad + \frac{B_{7,m}^{(n+2)}(\Delta^2)}{M^2} S\left[(E\cdot P)(E'^*\cdot P) \Delta_\mu \Delta_\nu \Delta_{\mu_1}\dots\Delta_{\mu_m} P_{\mu_{m+1}}\dots P_{\mu_n}\right] \Bigg\}.
\end{aligned}
$$

Gluonic radii

- Radii defined by slope of FFs vs Q^2
- Matrix elements of the spin-independent gluon operator
- Off-forward matrix elements are complicated:

$$\left\langle p'E' \left| S\left[G_{\mu\alpha} i\overleftrightarrow{D}_{\mu_1} \ldots i\overleftrightarrow{D}_{\mu_n} G_\nu{}^\alpha\right]\right| pE\right\rangle$$

$$= \sum_{\substack{m \text{ even} \\ m=0}}^{n} \Big\{ B_{1,m}^{(n+2)}(\Delta^2) M^2 S\left[E_\mu E'^*_\nu \Delta_{\mu_1} \ldots \Delta_{\mu_m} P_{\mu_{m+1}} \ldots P_{\mu_n}\right]$$

$$+ B_{2,m}^{(n+2)}(\Delta^2) S\left[(E \cdot E'^*) \ldots\right]$$

$$+ B_{3,m}^{(n+2)}(\Delta^2) S\left[(E \cdot E \ldots\right]$$

$$+ B_{4,m}^{(n+2)}(\Delta^2) S\left[((E'^* \ldots \ldots P_{\mu_n}\right]$$

$$+ B_{5,m}^{(n+2)}(\Delta^2) S\left[((E \ldots \ldots P_{\mu_n}\right]$$

$$+ \frac{B_{6,m}^{(n+2)}(\Delta^2)}{M^2} S\left[(E \cdot P)(E'^* \cdot P) P_\mu P_\nu \Delta_{\mu_1} \ldots \Delta_{\mu_m} P_{\mu_{m+1}} \ldots P_{\mu_n}\right]$$

$$+ \frac{B_{7,m}^{(n+2)}(\Delta^2)}{M^2} S\left[(E \cdot P)(E'^* \cdot P) \Delta_\mu \Delta_\nu \Delta_{\mu_1} \ldots \Delta_{\mu_m} P_{\mu_{m+1}} \ldots P_{\mu_n}\right] \Big\}.$$

Many gluonic radii:
Defined by slope of each form factor at Q^2=t=0

Gluonic radii

- Matrix elements of the gluon transversity operator
- Similarly complicated:

$$
\begin{aligned}
&\left\langle p'E' \middle| S\left[G_{\mu\mu_1}\overleftrightarrow{D}_{\mu_3}\dots\overleftrightarrow{D}_{\mu_n}G_{\nu\mu_2}\right]\middle| pE\right\rangle \\
&= \sum_{\substack{m\ \mathrm{odd}\\ m=3}}^{n}\Big\{\boxed{A^{(n)}_{1,m-3}(t,\mu^2)}S\left[(P_\mu E_{\mu_1}-E_\mu P_{\mu_1})(P_\nu E'^*_{\mu_2}-E'^*_\nu P_{\mu_2})\Delta_{\mu_3}\dots\Delta_{\mu_{m-1}}P_{\mu_m}\dots P_{\mu_n}\right] \\
&\quad+\boxed{A^{(n)}_{2,m-3}(t,\mu^2)}S\left[(\Delta_\mu E_{\mu_1}-E_\mu\Delta_{\mu_1})(\Delta_\nu E'^*_{\mu_2}-E'^*_\nu\Delta_{\mu_2})\Delta_{\mu_3}\dots\Delta_{\mu_{m-1}}P_{\mu_m}\dots P_{\mu_n}\right] \\
&\quad+\boxed{A^{(n)}_{3,m-3}(t,\mu^2)}S\Big[\big((\Delta_\mu E_{\mu_1}-E_\mu\Delta_{\mu_1})(P_\nu E'^*_{\mu_2}-E'^*_\nu P_{\mu_2})-(\Delta_\mu E'^*_{\mu_1}-E'^*_\mu\Delta_{\mu_1})(P_\nu E_{\mu_2}-E_\nu P_{\mu_2})\big) \\
&\qquad\times\Delta_{\mu_3}\dots\Delta_{\mu_{m-1}}P_{\mu_m}\dots P_{\mu_n}\Big] \\
&\quad+\boxed{A^{(n)}_{4,m-3}(t,\mu^2)}S\left[(E_\mu E'^*_{\mu_1}-E_{\mu_1}E'^*_\mu)(P_\nu\Delta_{\mu_2}-P_{\mu_2}\Delta_\nu)\Delta_{\mu_3}\dots\Delta_{\mu_{m-1}}P_{\mu_m}\dots P_{\mu_n}\right] \\
&\quad+\frac{\boxed{A^{(n)}_{5,m-3}(t,\mu^2)}}{M^2}S\Big[\big((E\cdot P)(P_\mu\Delta_{\mu_1}-\Delta_\mu P_{\mu_1})(\Delta_\nu E'^*_{\mu_2}-E'^*_\nu\Delta_{\mu_2}) \\
&\qquad+(E'^*\cdot P)(P_\mu\Delta_{\mu_1}-\Delta_\mu P_{\mu_1})(\Delta_\nu E_{\mu_2}-E_\nu\Delta_{\mu_2})\big)\Delta_{\mu_3}\dots\Delta_{\mu_{m-1}}P_{\mu_m}\dots P_{\mu_n}\Big] \\
&\quad+\frac{\boxed{A^{(n)}_{6,m-3}(t,\mu^2)}}{M^2}S\Big[\big((E\cdot P)(P_\mu\Delta_{\mu_1}-\Delta_\mu P_{\mu_1})(P_\nu E'^*_{\mu_2}-E'^*_\nu P_{\mu_2}) \\
&\qquad-(E'^*\cdot P)\,(P_\mu\Delta_{\mu_1}-\Delta_\mu P_{\mu_1})(P_\nu E_{\mu_2}-E_\nu P_{\mu_2})\big)\Delta_{\mu_3}\dots\Delta_{\mu_{m-1}}P_{\mu_m}\dots P_{\mu_n}\Big] \\
&\quad+\frac{\boxed{A^{(n)}_{7,m-3}(t,\mu^2)}}{M^2}(E'^*\cdot E)S\left[(P_\mu\Delta_{\mu_1}-\Delta_\mu P_{\mu_1})(P_\nu\Delta_{\mu_2}-\Delta_\nu P_{\mu_2})\Delta_{\mu_3}\dots\Delta_{\mu_{m-1}}P_{\mu_m}\dots P_{\mu_n}\right] \\
&\quad+\frac{\boxed{A^{(n)}_{8,m-3}(t,\mu^2)}}{M^4}(E\cdot P)(E'^*\cdot P)S\left[(P_\mu\Delta_{\mu_1}-\Delta_\mu P_{\mu_1})(P_\nu\Delta_{\mu_2}-\Delta_\nu P_{\mu_2})\Delta_{\mu_3}\dots\Delta_{\mu_{m-1}}P_{\mu_m}\dots P_{\mu_n}\right]\Big\}
\end{aligned}
$$

Gluonic radii

- Complicated over and under-determined systems of equations (different polarisation and boosts at same momentum transfer)
- Some GFFs suppressed by orders of magnitude
- Some GFFs related by symmetries at some momenta

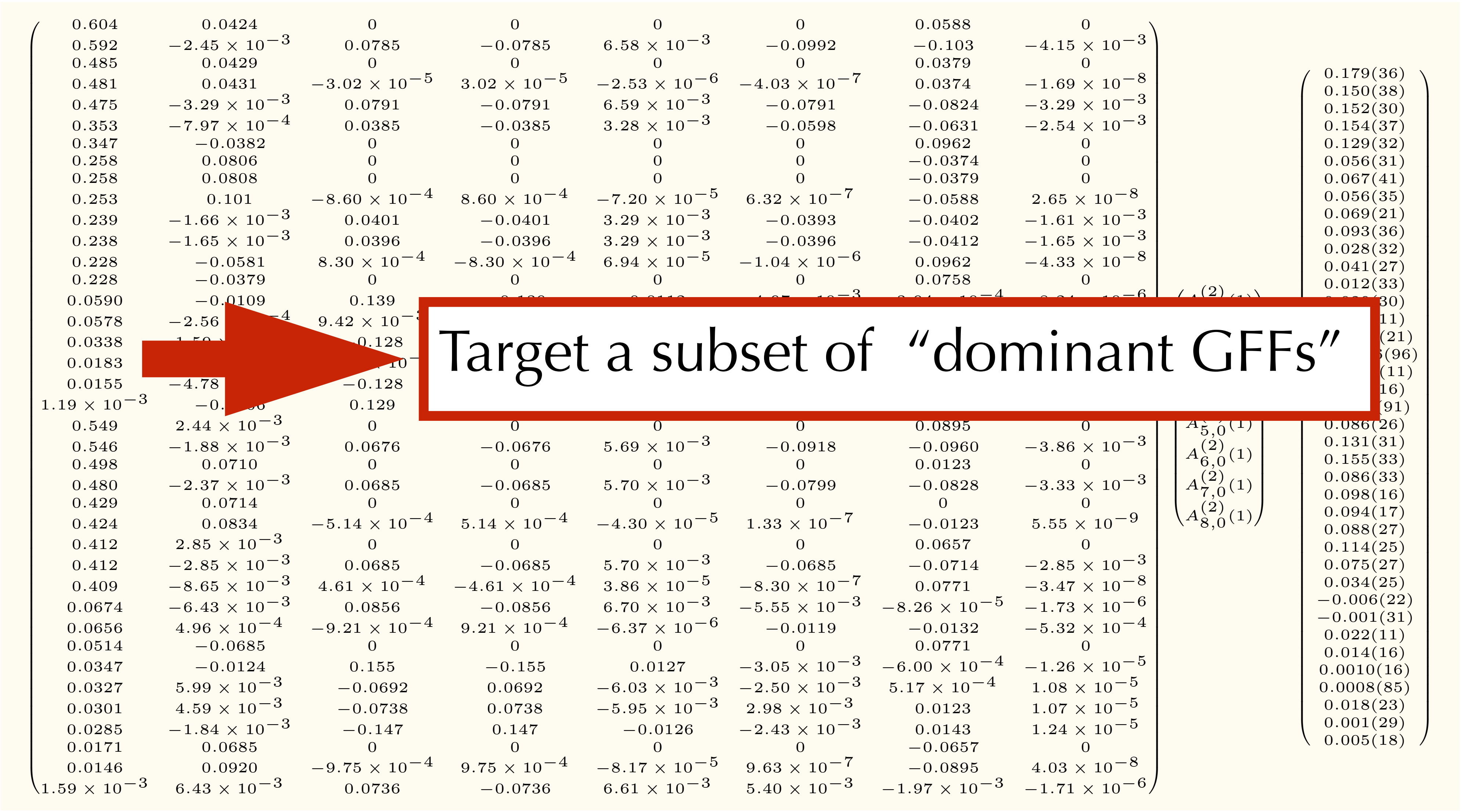
Simplest example:
Transversity GFFs
One basis (2 vectors)
|p|=1 (lattice units)

$$
\begin{pmatrix}
0.604 & 0.0424 & 0 & 0 & 0 & 0 & 0.0588 & 0 \\
0.592 & -2.45\times10^{-3} & 0.0785 & -0.0785 & 6.58\times10^{-3} & -0.0992 & -0.103 & -4.15\times10^{-3} \\
0.485 & 0.0429 & 0 & 0 & 0 & 0 & 0.0379 & 0 \\
0.481 & 0.0431 & -3.02\times10^{-5} & 3.02\times10^{-5} & -2.53\times10^{-6} & -4.03\times10^{-7} & 0.0374 & -1.69\times10^{-8} \\
0.475 & -3.29\times10^{-3} & 0.0791 & -0.0791 & 6.59\times10^{-3} & -0.0791 & -0.0824 & -3.29\times10^{-3} \\
0.353 & -7.97\times10^{-4} & 0.0385 & -0.0385 & 3.28\times10^{-3} & -0.0598 & -0.0631 & -2.54\times10^{-3} \\
0.347 & -0.0382 & 0 & 0 & 0 & 0 & 0.0962 & 0 \\
0.258 & 0.0806 & 0 & 0 & 0 & 0 & -0.0374 & 0 \\
0.258 & 0.0808 & 0 & 0 & 0 & 0 & -0.0379 & 0 \\
0.253 & 0.101 & -8.60\times10^{-4} & 8.60\times10^{-4} & -7.20\times10^{-5} & 6.32\times10^{-7} & -0.0588 & 2.65\times10^{-8} \\
0.239 & -1.66\times10^{-3} & 0.0401 & -0.0401 & 3.29\times10^{-3} & -0.0393 & -0.0402 & -1.61\times10^{-3} \\
0.238 & -1.65\times10^{-3} & 0.0396 & -0.0396 & 3.29\times10^{-3} & -0.0396 & -0.0412 & -1.65\times10^{-3} \\
0.228 & -0.0581 & 8.30\times10^{-4} & -8.30\times10^{-4} & 6.94\times10^{-5} & -1.04\times10^{-6} & 0.0962 & -4.33\times10^{-8} \\
0.228 & -0.0379 & 0 & 0 & 0 & 0 & 0.0758 & 0 \\
0.0590 & -0.0109 & 0.139 & -0.139 & 0.0112 & -4.97\times10^{-3} & -3.94\times10^{-4} & -8.24\times10^{-6} \\
0.0578 & -2.56\times10^{-4} & 9.42\times10^{-3} & -9.42\times10^{-3} & 3.89\times10^{-4} & -4.65\times10^{-3} & 2.51\times10^{-4} & 5.25\times10^{-6} \\
0.0338 & 1.59\times10^{-3} & -0.128 & 0.128 & -0.0107 & 3.18\times10^{-4} & 0.0154 & 1.33\times10^{-5} \\
0.0183 & 6.36\times10^{-3} & -1.29\times10^{-4} & 1.29\times10^{-4} & 3.84\times10^{-4} & 4.84\times10^{-3} & 5.99\times10^{-3} & 5.18\times10^{-6} \\
0.0155 & -4.78\times10^{-3} & -0.128 & 0.128 & -0.0111 & -4.52\times10^{-3} & 9.41\times10^{-3} & 8.14\times10^{-6} \\
1.19\times10^{-3} & -0.0106 & 0.129 & -0.129 & 0.0108 & -3.22\times10^{-4} & -6.45\times10^{-4} & -1.35\times10^{-5} \\
0.549 & 2.44\times10^{-3} & 0 & 0 & 0 & 0 & 0.0895 & 0 \\
0.546 & -1.88\times10^{-3} & 0.0676 & -0.0676 & 5.69\times10^{-3} & -0.0918 & -0.0960 & -3.86\times10^{-3} \\
0.498 & 0.0710 & 0 & 0 & 0 & 0 & 0.0123 & 0 \\
0.480 & -2.37\times10^{-3} & 0.0685 & -0.0685 & 5.70\times10^{-3} & -0.0799 & -0.0828 & -3.33\times10^{-3} \\
0.429 & 0.0714 & 0 & 0 & 0 & 0 & 0 & 0 \\
0.424 & 0.0834 & -5.14\times10^{-4} & 5.14\times10^{-4} & -4.30\times10^{-5} & 1.33\times10^{-7} & -0.0123 & 5.55\times10^{-9} \\
0.412 & 2.85\times10^{-3} & 0 & 0 & 0 & 0 & 0.0657 & 0 \\
0.412 & -2.85\times10^{-3} & 0.0685 & -0.0685 & 5.70\times10^{-3} & -0.0685 & -0.0714 & -2.85\times10^{-3} \\
0.409 & -8.65\times10^{-3} & 4.61\times10^{-4} & -4.61\times10^{-4} & 3.86\times10^{-5} & -8.30\times10^{-7} & 0.0771 & -3.47\times10^{-8} \\
0.0674 & -6.43\times10^{-3} & 0.0856 & -0.0856 & 6.70\times10^{-3} & -5.55\times10^{-3} & -8.26\times10^{-5} & -1.73\times10^{-6} \\
0.0656 & 4.96\times10^{-4} & -9.21\times10^{-4} & 9.21\times10^{-4} & -6.37\times10^{-6} & -0.0119 & -0.0132 & -5.32\times10^{-4} \\
0.0514 & -0.0685 & 0 & 0 & 0 & 0 & 0.0771 & 0 \\
0.0347 & -0.0124 & 0.155 & -0.155 & 0.0127 & -3.05\times10^{-3} & -6.00\times10^{-4} & -1.26\times10^{-5} \\
0.0327 & 5.99\times10^{-3} & -0.0692 & 0.0692 & -6.03\times10^{-3} & -2.50\times10^{-3} & 5.17\times10^{-4} & 1.08\times10^{-5} \\
0.0301 & 4.59\times10^{-3} & -0.0738 & 0.0738 & -5.95\times10^{-3} & 2.98\times10^{-3} & 0.0123 & 1.07\times10^{-5} \\
0.0285 & -1.84\times10^{-3} & -0.147 & 0.147 & -0.0126 & -2.43\times10^{-3} & 0.0143 & 1.24\times10^{-5} \\
0.0171 & 0.0685 & 0 & 0 & 0 & 0 & -0.0657 & 0 \\
0.0146 & 0.0920 & -9.75\times10^{-4} & 9.75\times10^{-4} & -8.17\times10^{-5} & 9.63\times10^{-7} & -0.0895 & 4.03\times10^{-8} \\
1.59\times10^{-3} & 6.43\times10^{-3} & 0.0736 & -0.0736 & 6.61\times10^{-3} & 5.40\times10^{-3} & -1.97\times10^{-3} & -1.71\times10^{-6}
\end{pmatrix}
\begin{pmatrix}
A^{(2)}_{1,0}(1) \\ A^{(2)}_{2,0}(1) \\ A^{(2)}_{3,0}(1) \\ A^{(2)}_{4,0}(1) \\ A^{(2)}_{5,0}(1) \\ A^{(2)}_{6,0}(1) \\ A^{(2)}_{7,0}(1) \\ A^{(2)}_{8,0}(1)
\end{pmatrix}
=
\begin{pmatrix}
0.179(36) \\ 0.150(38) \\ 0.152(30) \\ 0.154(37) \\ 0.129(32) \\ 0.056(31) \\ 0.067(41) \\ 0.056(35) \\ 0.069(21) \\ 0.093(36) \\ 0.028(32) \\ 0.041(27) \\ 0.012(33) \\ 0.029(30) \\ 0.024(11) \\ -0.005(21) \\ -0.0056(96) \\ -0.002(11) \\ 0.009(16) \\ 0.0162(91) \\ 0.086(26) \\ 0.131(31) \\ 0.155(33) \\ 0.086(33) \\ 0.098(16) \\ 0.094(17) \\ 0.088(27) \\ 0.114(25) \\ 0.075(27) \\ 0.034(25) \\ -0.006(22) \\ -0.001(31) \\ 0.022(11) \\ 0.014(16) \\ 0.0010(16) \\ 0.0008(85) \\ 0.018(23) \\ 0.001(29) \\ 0.005(18)
\end{pmatrix}
$$

Gluonic radii

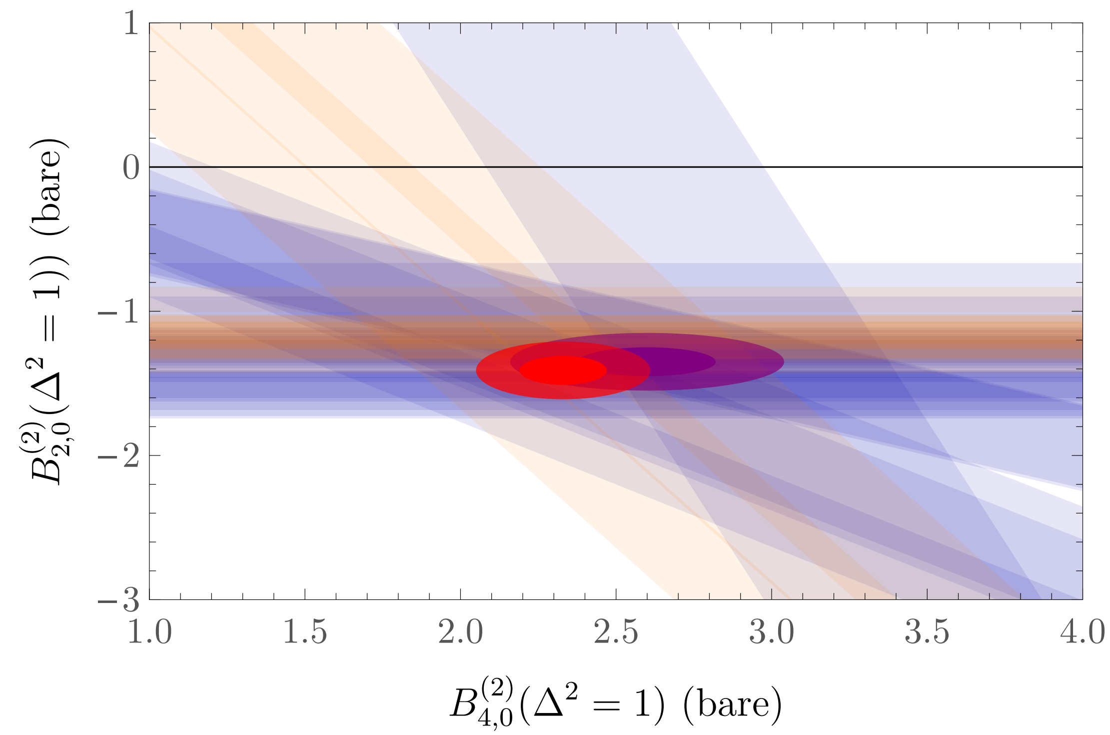
- Complicated over and under-determined systems of equations (different polarisation and boosts at same momentum transfer)
- Some GFFs suppressed by orders of magnitude
- Some GFFs related by symmetries at some momenta

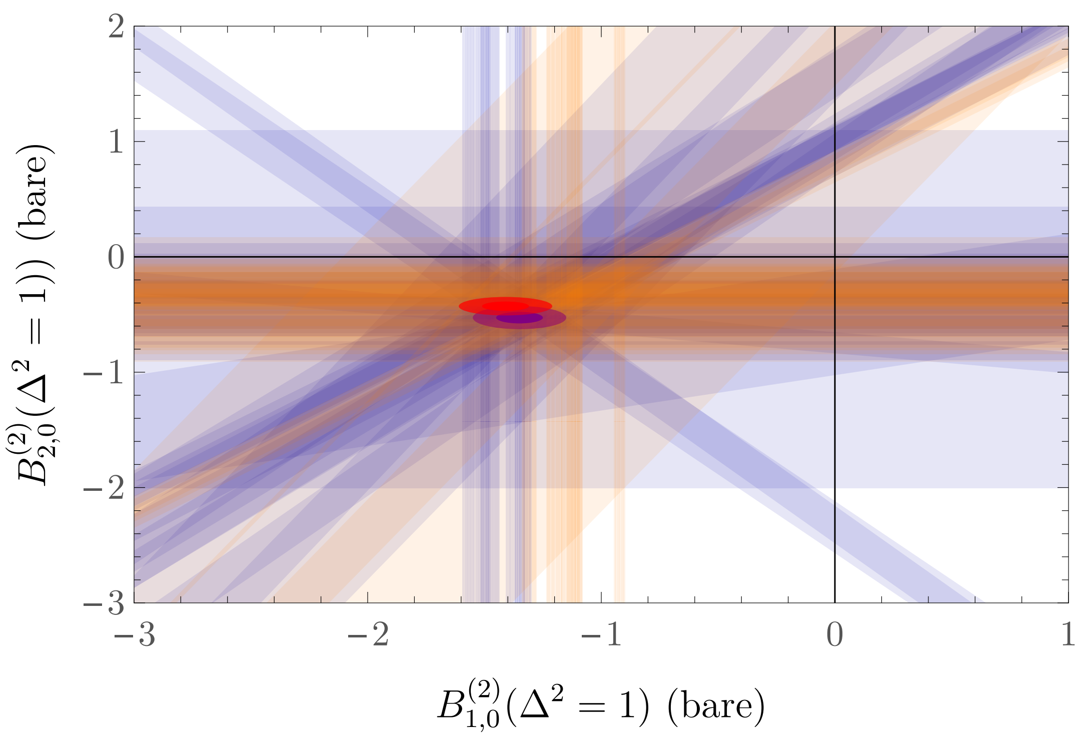
Simplest example:
Transversity GFFs
One basis (2 vectors)
|p|=1 (lattice units)

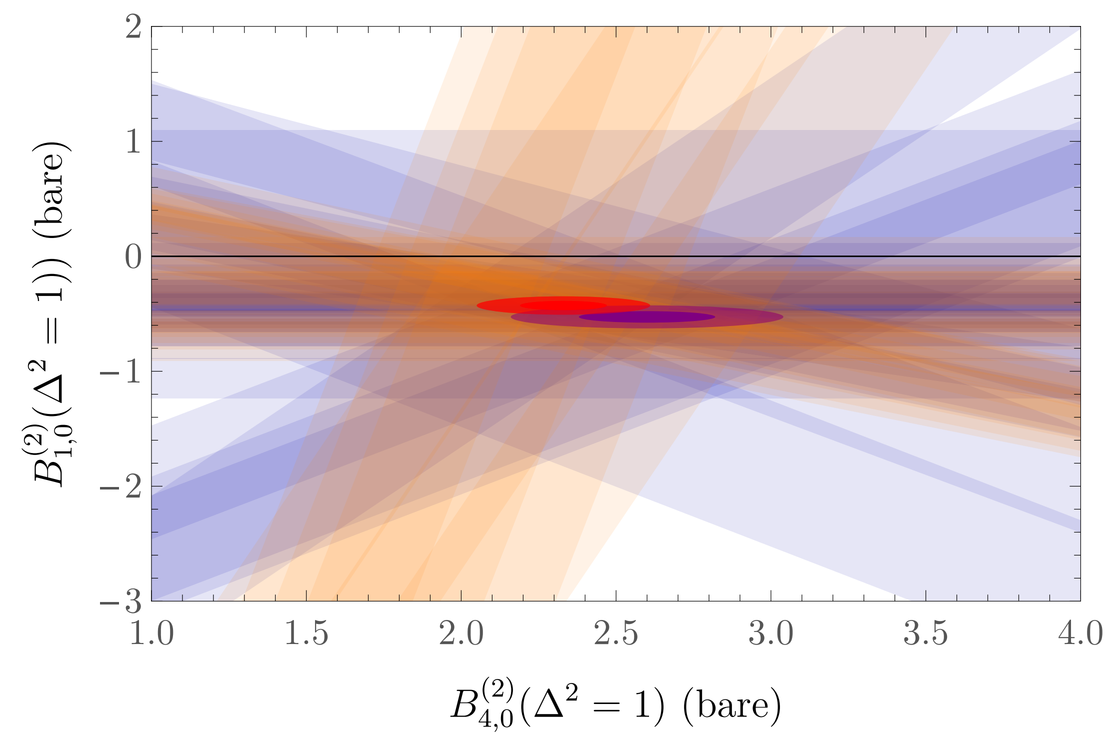


Gluonic radii

- Example: Spin-indep GFFs, lowest non-zero momentum transfer
- 3 dominant GFFs, others set to 0±10
- Only tightly-constrained bands shown in each projection.



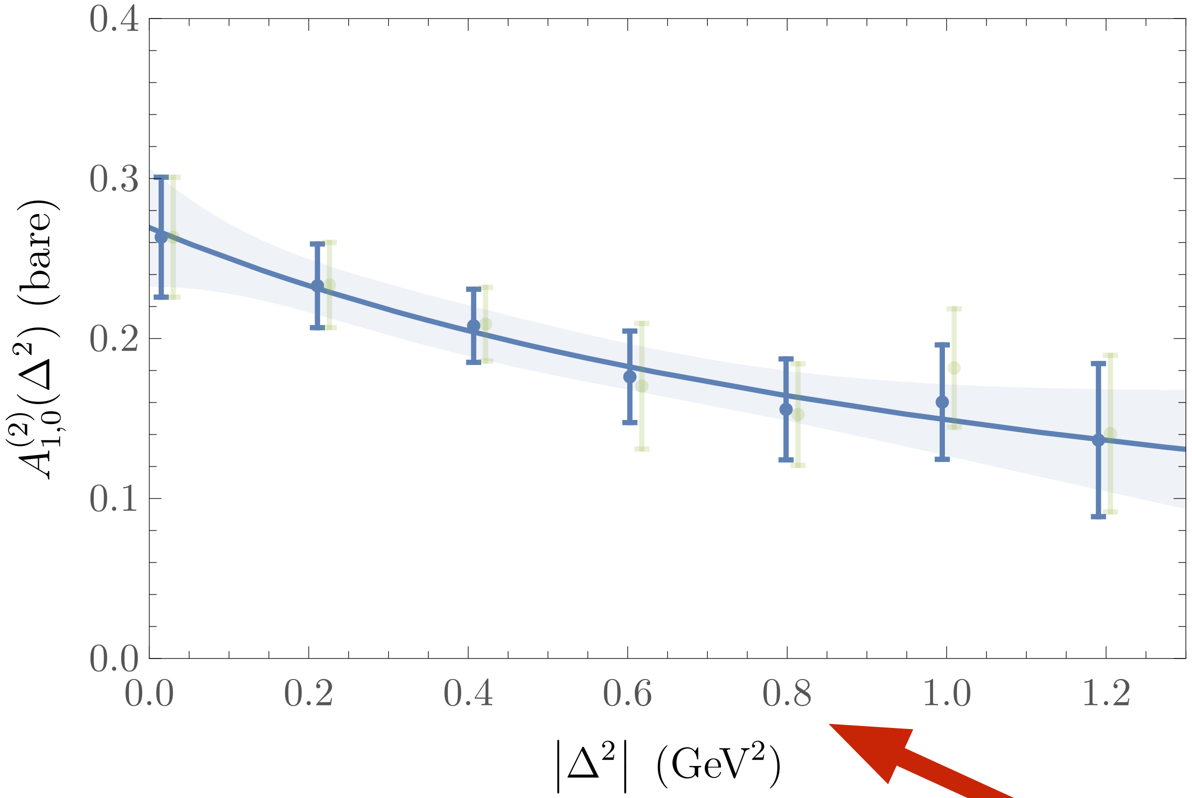


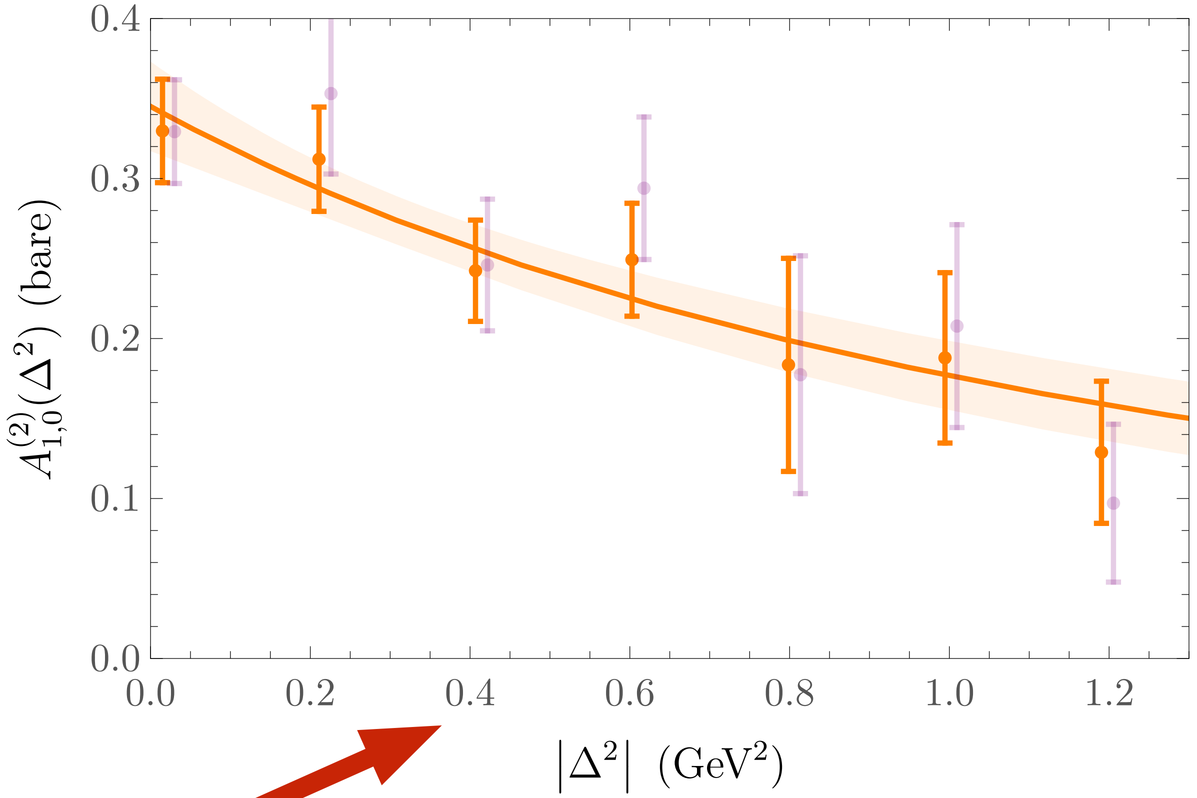


Gluonic radii

WD, D. Pefkou, P. Shanahan PRD 95 (2017), 114515

One GFF can be resolved for all momenta



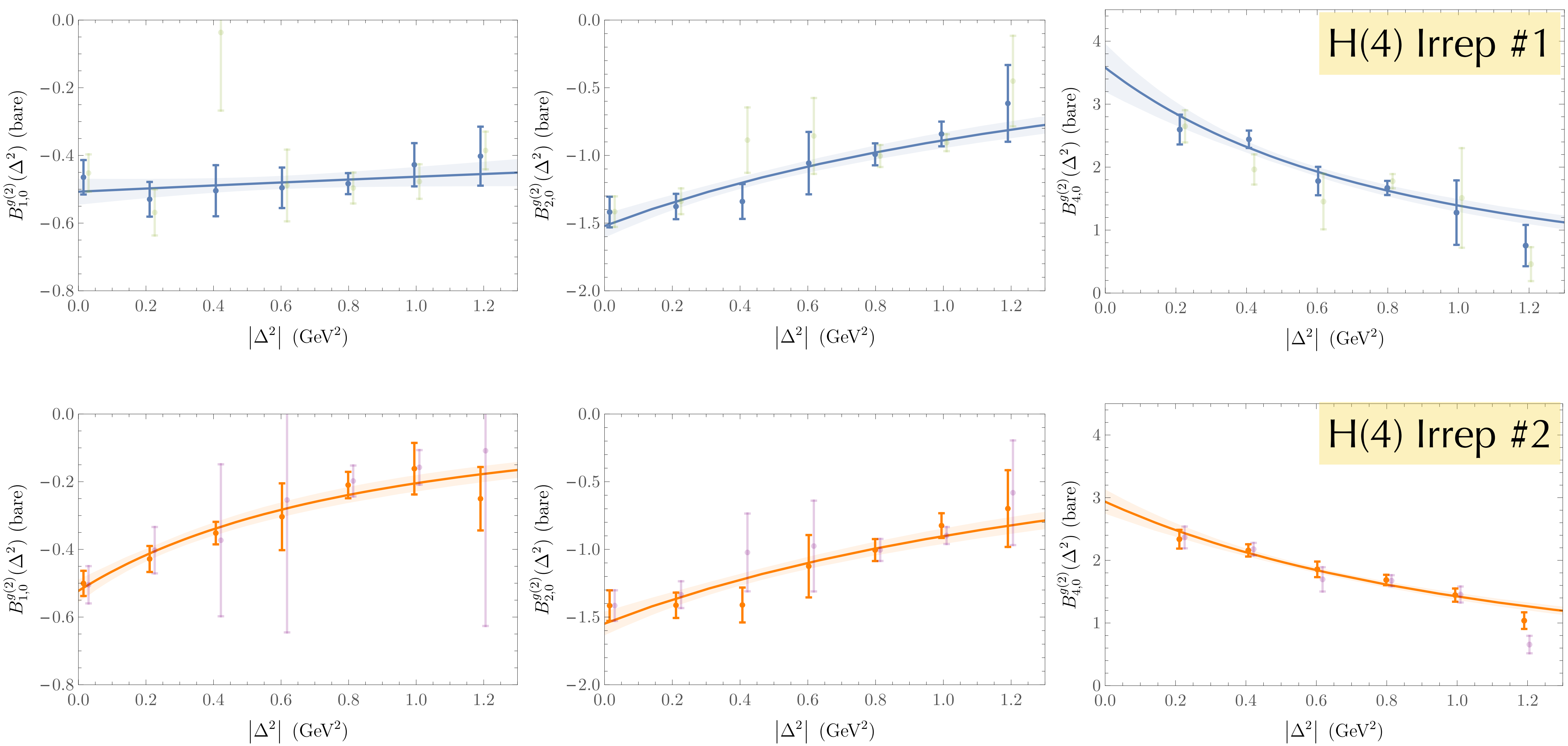


Different H(4) irreps

Unpolarised gluon GFFs

WD, D. Pefkou, P. Shanahan PRD 95 (2017), 114515

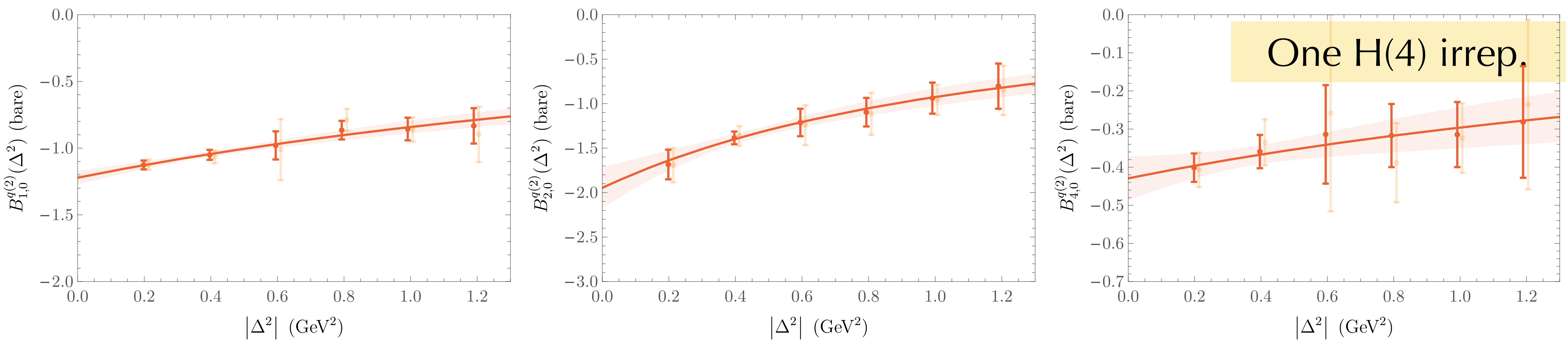
- Three GFFs can be resolved for all momenta



Unpolarised quark GFFs

WD, D. Pefkou, P. Shanahan PRD 95 (2017), 114515

- Three quark GFFs can be resolved for all momenta
- GFF decomposition has precisely the same structure as in the spin-independent gluon case

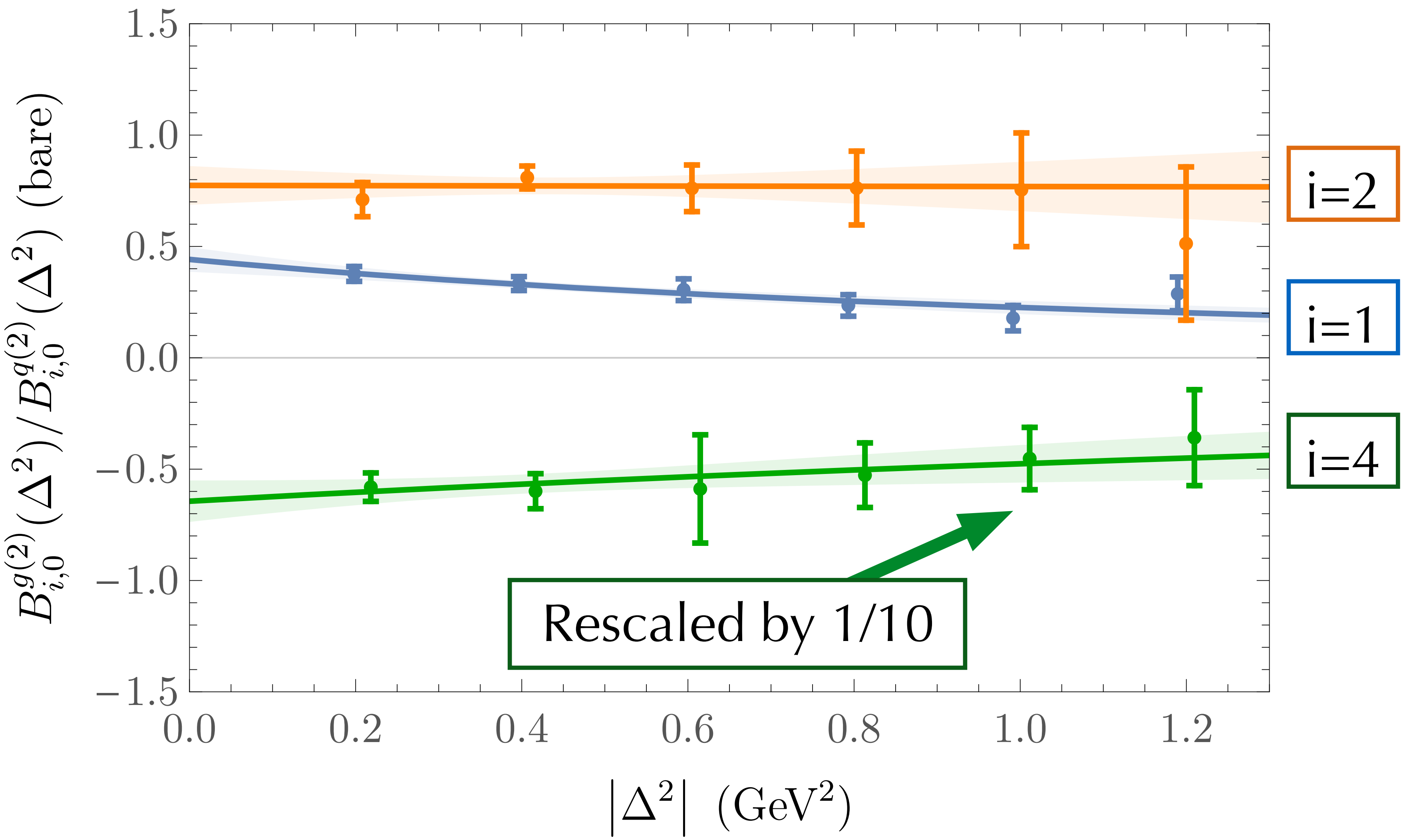


Same three GFFs that are resolved in the gluon case

Gluonic radii

WD, D. Pefkou, P. Shanahan PRD 95 (2017), 114515

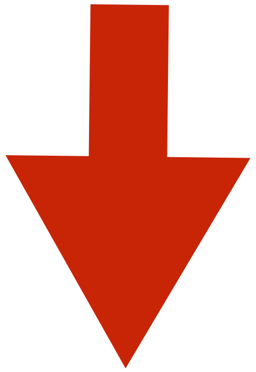
- Ratio of gluon to quark unpolarised GFFs



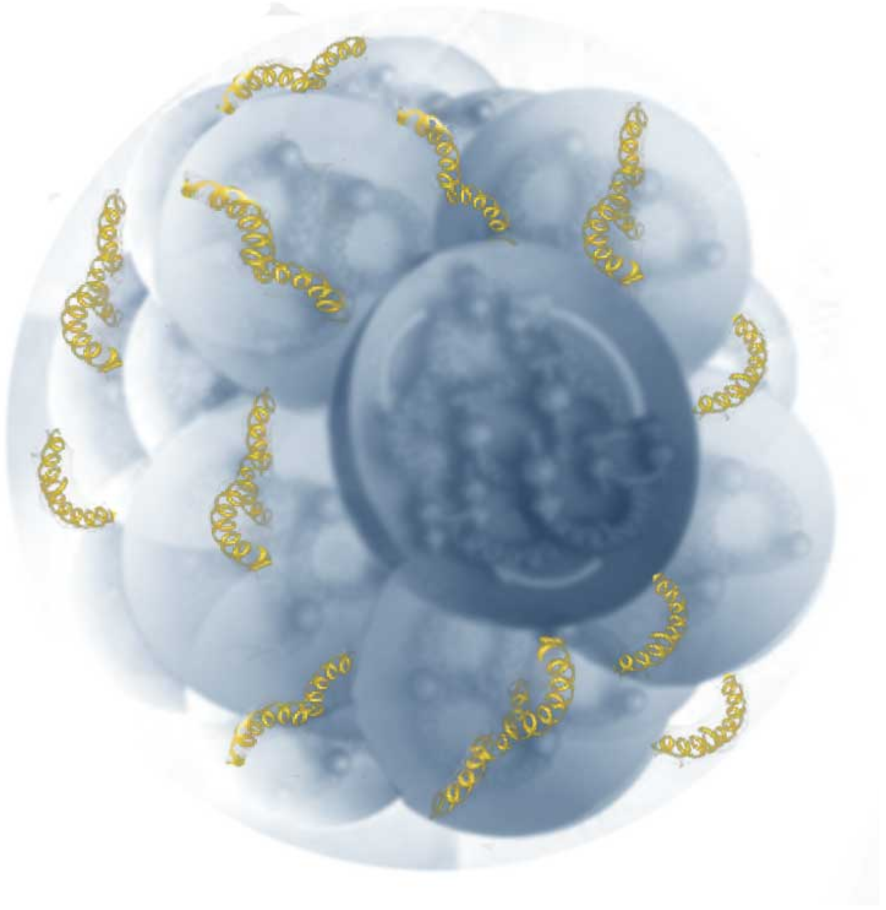
- Gluon vs quark radius is a non-trivial question
  More complicated than intuitive pictures

Gluon structure of nuclei

- First investigations: ϕ meson
  simplest spin-1 system (has fwd limit



- Phenomenologically relevant:
  nucleon, nuclei

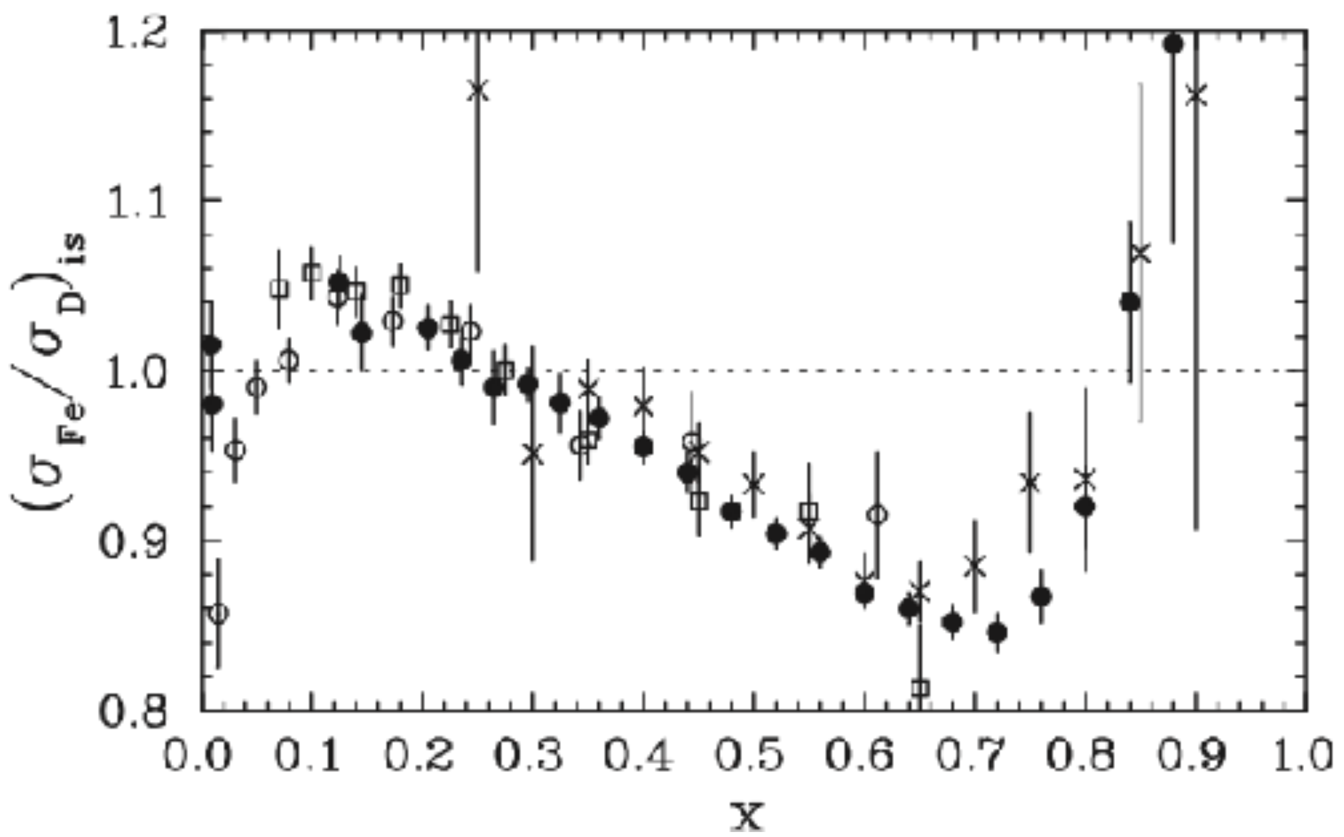


Gluon structure of nuclei

- Nuclear modification of proton structure (EMC effect)

$$\mathrm{F}_2(x, Q^2) = \sum_{\mathrm{q=u,d,s..}} x z_{\mathrm{q}}^2 \left[\mathrm{q}(x, Q^2) + \bar{\mathrm{q}}(x, Q^2) \right]$$

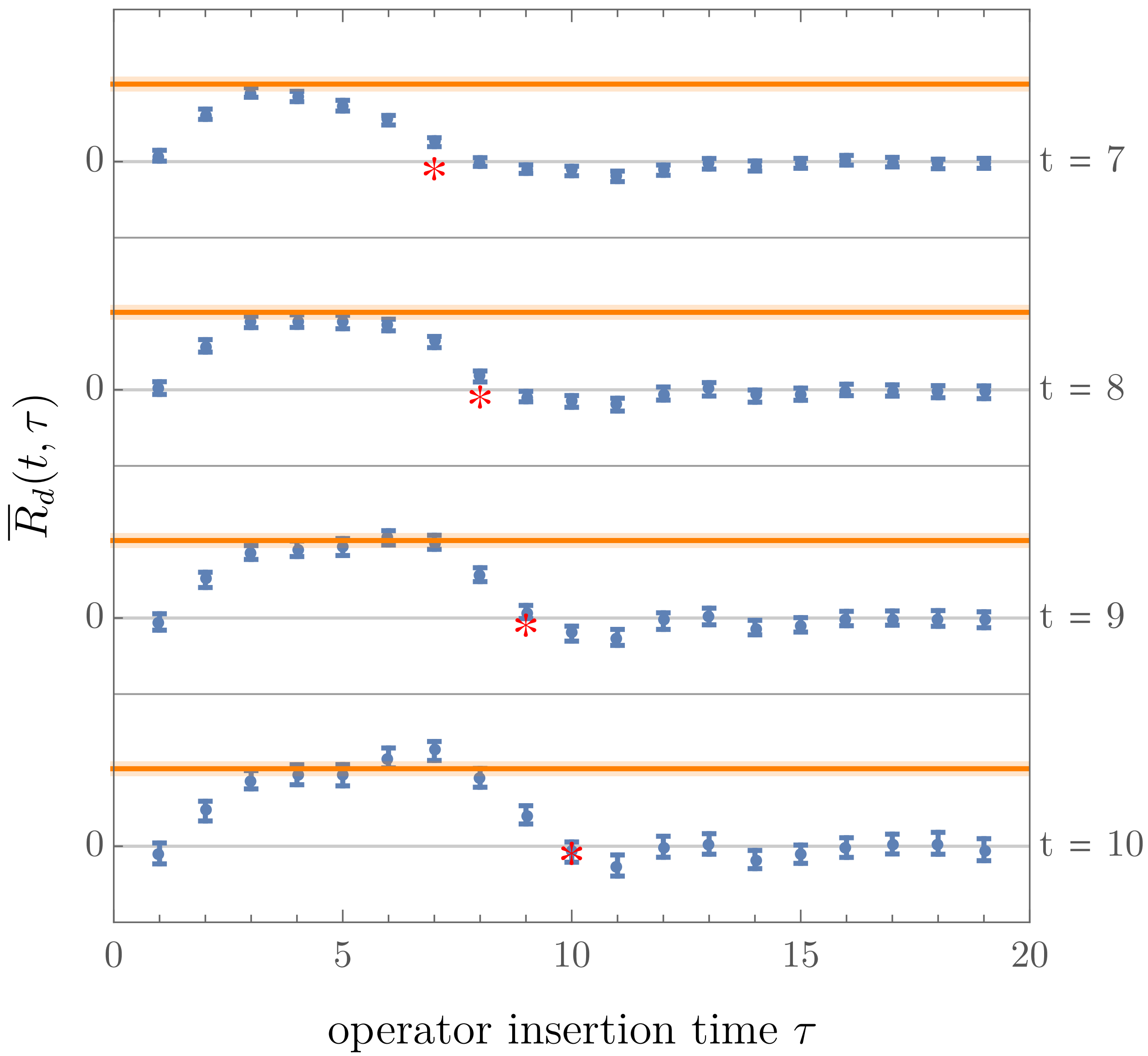
- Important to understand from QCD: nuclear targets essential many current experiments (DUNE, …)
- Look for gluon analogue of EMC effect
    - Target for EIC discovery

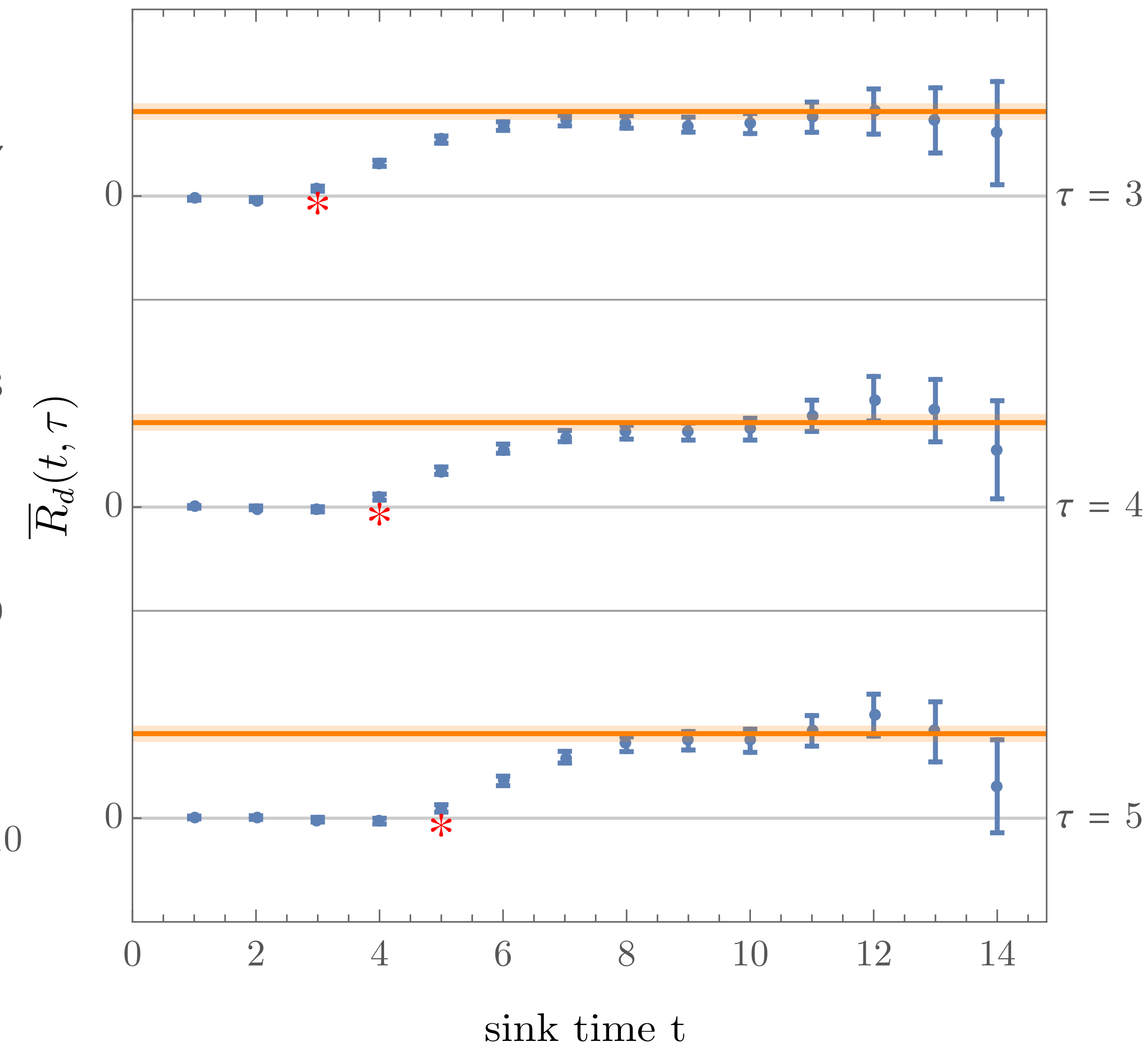


Gluon structure of nuclei

NPLQCD, arXiv:1709.00395

- Clean signals for spin-independent gluon operator in deuteron

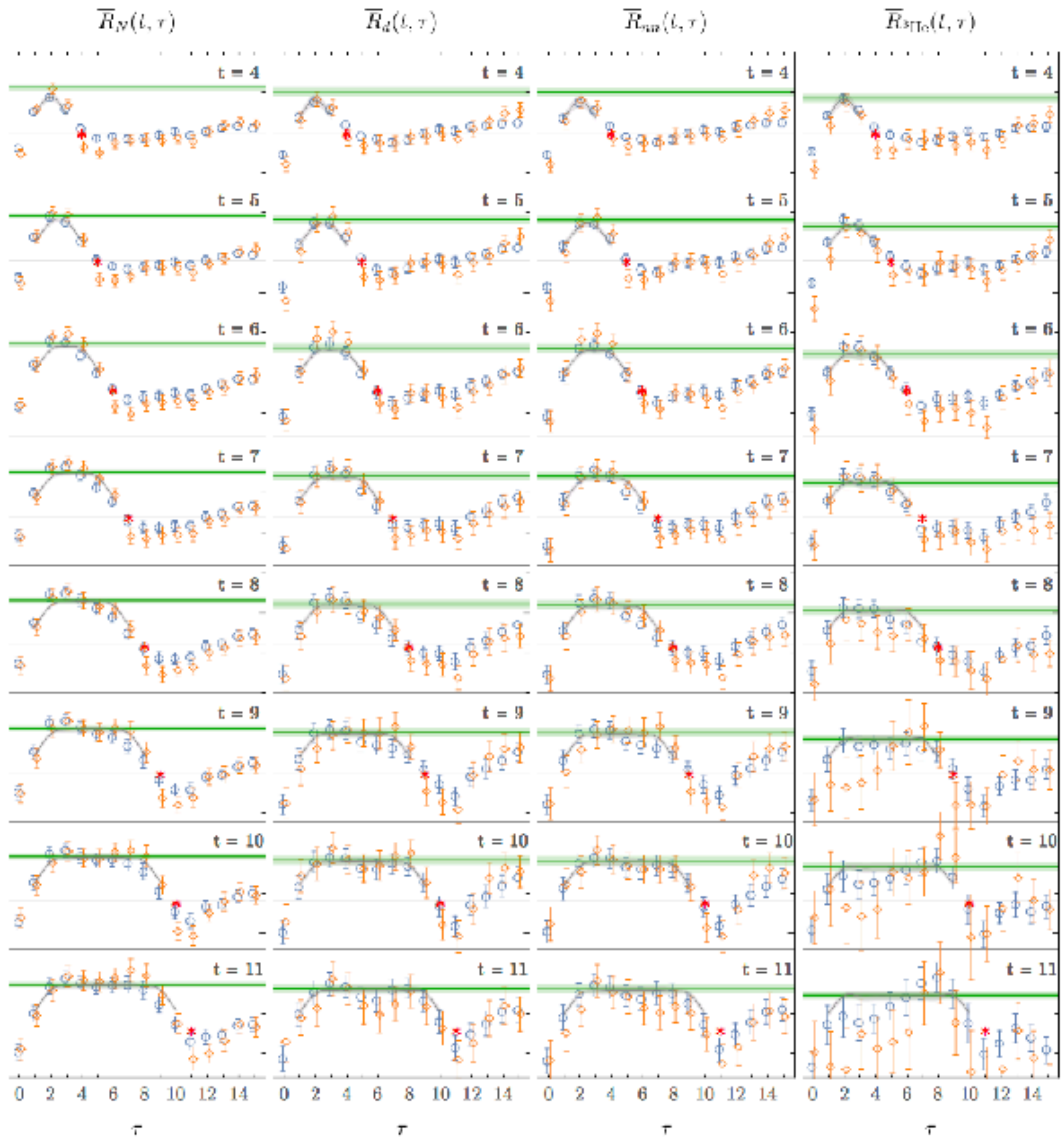




Gluon structure of nuclei

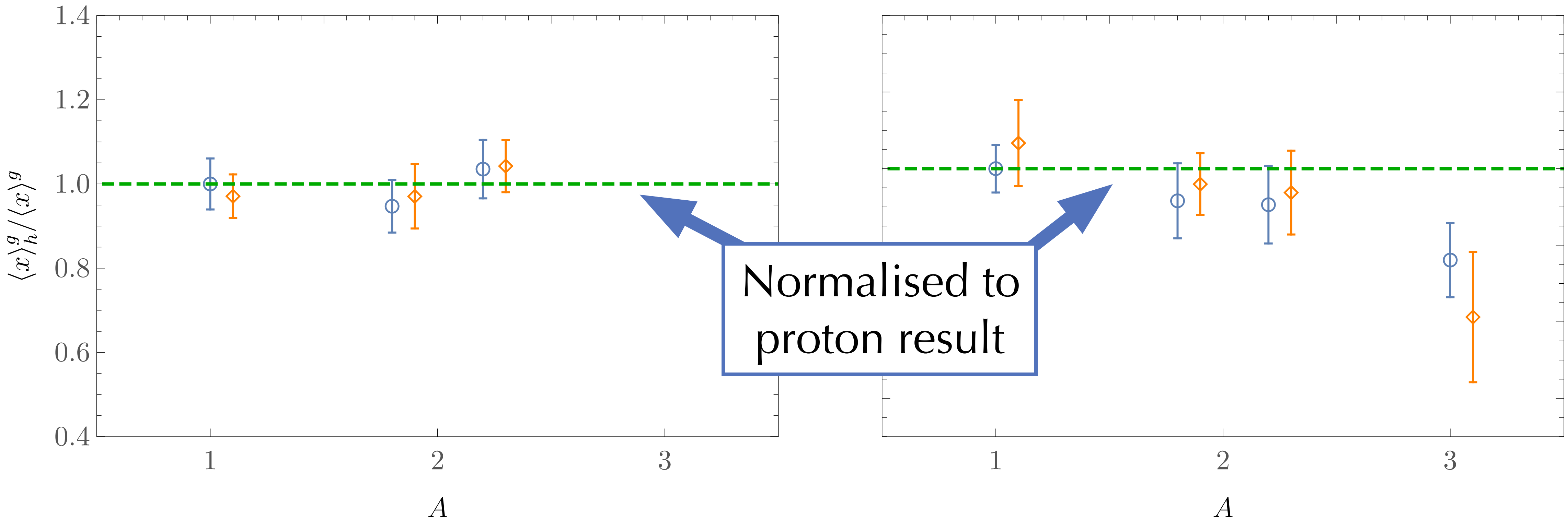
NPLQCD, arXiv:1709.00395

- Signals for spin-independent gluon operator
- Deuteron, Di-neutron, ^{3}He
- One/two state fits to extract moment



NPLQCD, arXiv:1709.00395

- Matrix elements of the Spin-independent gluon operator in nucleon and light nuclei
- Present statistics: can't distinguish from no-EMC effect scenario
  - Constraint from momentum sum-rule
- Small additional uncertainty from mixing with quark operators



m_π ~450 MeV

m_π ~800 MeV

Gluonic transversity

- Double helicity flip structure function $\Delta(x,Q^2)$
- Hadrons: gluonic transversity (parton model interpretation)

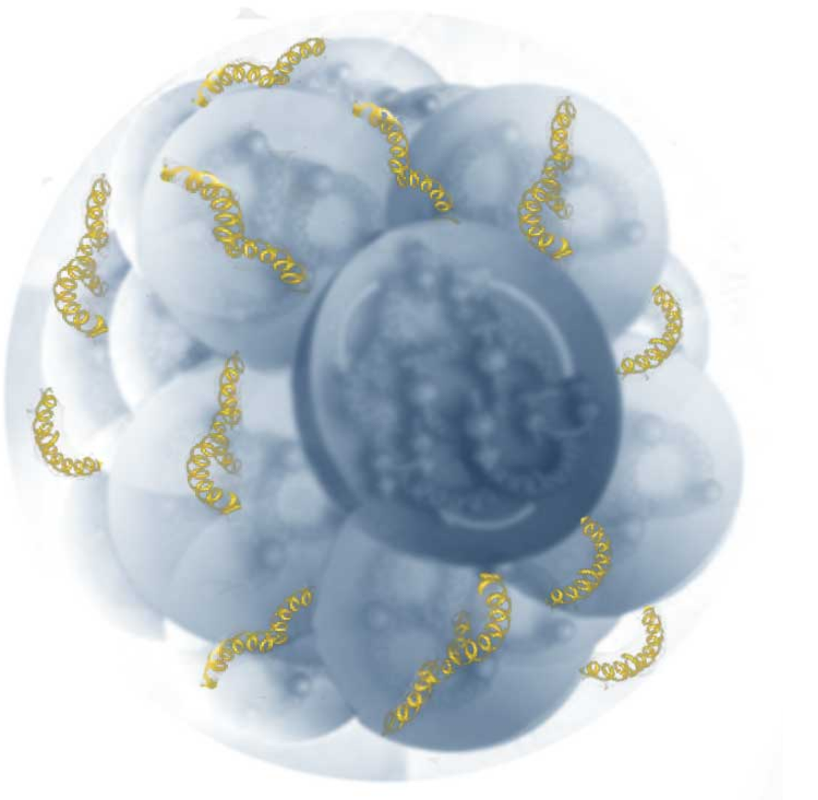
$$\Delta(x,Q^2) = -\frac{\alpha_s(Q^2)}{2\pi}\mathrm{Tr}\mathcal{Q}^2\, x^2 \int_x^1 \frac{dy}{y^3}\left[g_{\hat{x}}(y,Q^2) - g_{\hat{y}}(x,Q^2)\right]$$

$g_{\hat{x},\hat{y}}(y,Q^2)$: probability of finding a gluon with momentum fraction y linearly polarised in $\hat{x}$, $\hat{y}$ direction

- Nuclei: Exotic Glue
  - Gluons not associated with individual nucleons in nucleus

$$\langle p|\mathcal{O}|p\rangle = 0$$

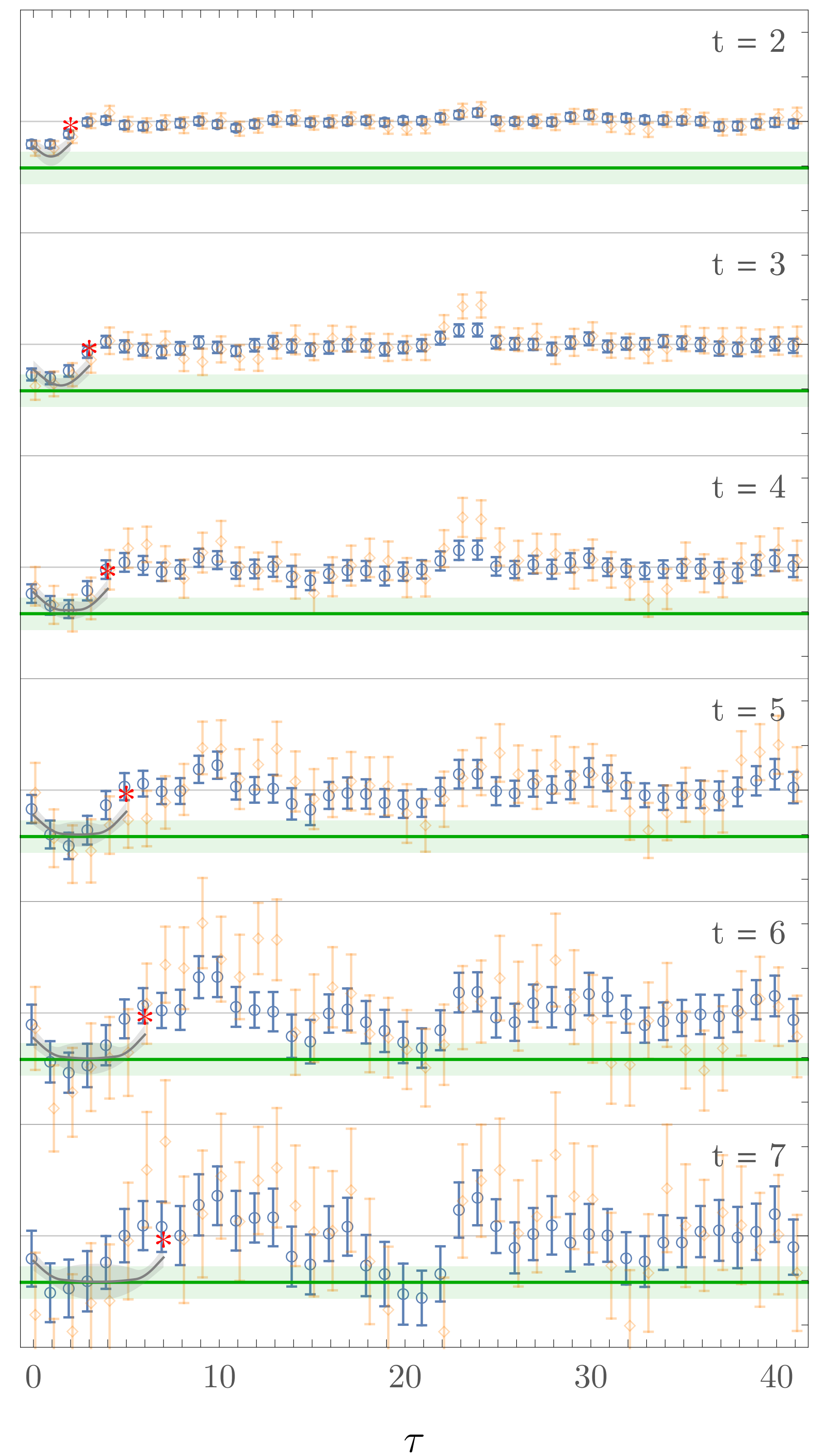
$$\langle N,Z|\mathcal{O}|N,Z\rangle \neq 0$$



NPLQCD, arXiv:1709.00395

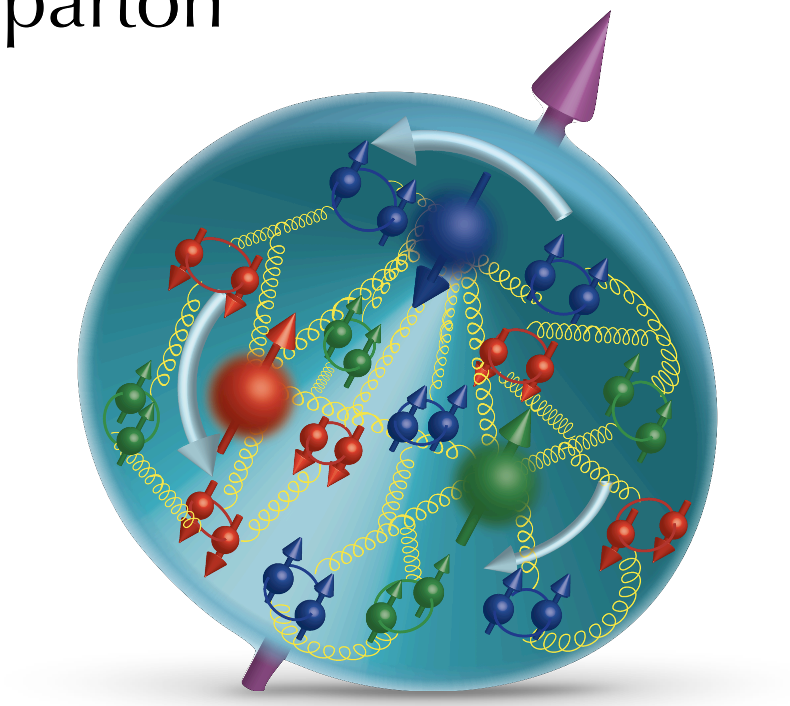
- First moment of gluon transversity in the deuteron
- First evidence for non-nucleonic gluon structure
  - m_π ~800 MeV
  - Fit systematics large
  - Calculations feasible
- Magnitude as expected from large-N_c (relative to unpol)

Ratio of 3pt and 2pt functions



Gluons, gluons, gluons….

- EIC will dramatically alter our knowledge of the gluonic structure of nucleons and nuclei
  - Eventually have a complete 3D picture of parton structure (PDFs, GPDs, TMDs)
  - $\Delta(x,Q^2)$ has an interesting role
    - Purely gluonic
    - Non-nucleonic



- Lattice calculations of gluon structure are progressing and will be a strong motivator for these experiments
- Address similarities and differences in distributions of quark and gluons in hadrons and nuclei