

Light-nuclei spectra from chiral dynamics

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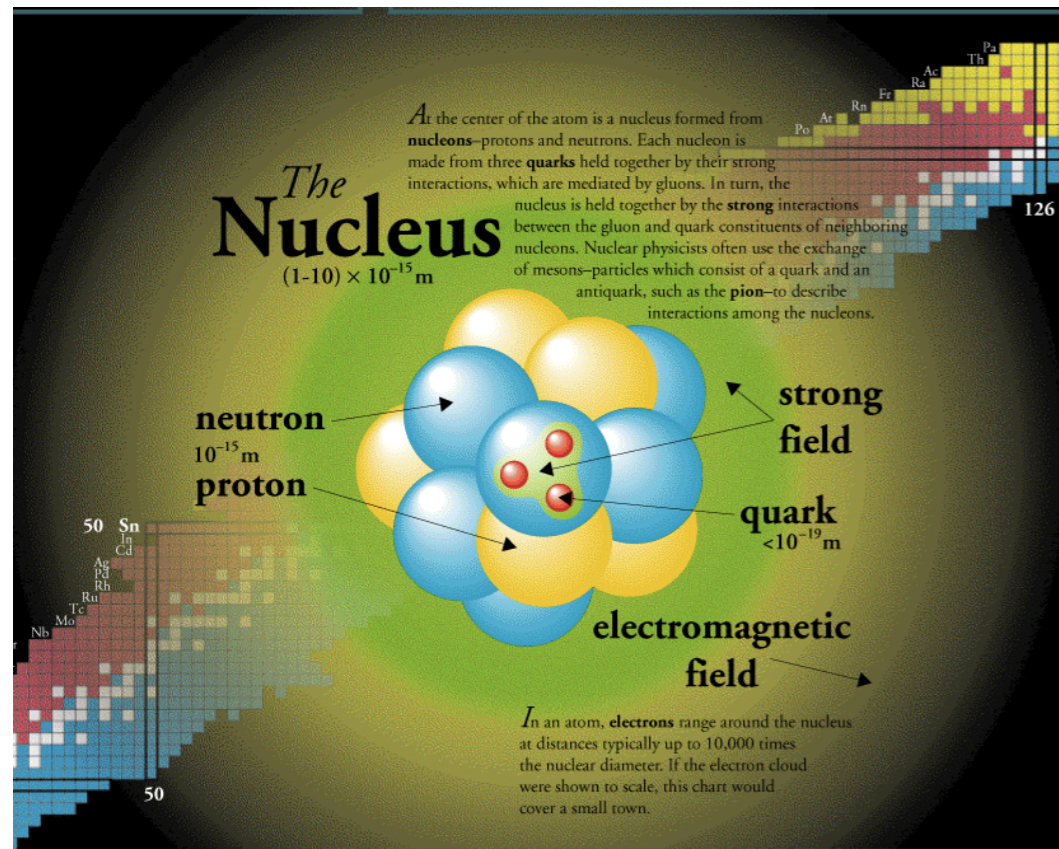


WORK NOT POSSIBLE WITHOUT EXTENSIVE COMPUTER RESOURCES

Argonne Laboratory Computing Resource Center (Blues, Bebob)

Argonne Leadership Computing Facility (Theta)

Nuclear Physics in a Nutshell:



- Atomic nuclei are complex quantum many-body systems of strongly interacting fermions (nucleons) displaying interesting properties: shell structure, pairing, deformation, strong clustering, etc
- QCD is the underlying theory of the strong interaction but nucleons are the relevant degree of freedom for the low-energy nuclear physics nuclear interactions

Nuclear forces: very complicated problem to derive in terms of quarks and gluons

- Historically research on the nuclear force has proceeded along different ways for example:

Phenomenological approach:

use the general form of a potential allowed by the symmetries (rotation, translation, isospin, etc);

potential terms are responsible to describe various phenomena remarked in NN interactions: for example the tensor term is important for the long-range part of potential and arises naturally from pion-exchange

In these potentials, the IR and SR parts are usually determined in a fully phenomenological way while for the LR part, an OPEP is often used

χ EFT approach:

pions, nucleons, Δ 's as degrees of freedom

construct their interactions consistently with the symmetries of the underlying theory, low-energy QCD

Atomic nuclei:

Collection of point-like particles whose dynamics is dictated by:

$$H = \sum_i \frac{\mathbf{p}_i^2}{2m} + \sum_{i < j} v_{ij} + \sum_{i < j < k} V_{ijk} + \dots$$

- v_{ij} : 2NF fitted on NN scattering data, deuteron BE, nn scattering length
- V_{ijk} : 3NF fitted on properties of light nuclei
- 2NF > 3NF > 4NF > ...
- Calculations of the properties of light nuclei based upon the bare nuclear forces shown that 3NFs are important

Models of nuclear interactions:

- phenomenological: Argonne (NN) + Urbana-Illinois (NNN)
- local, chiral effective field theory: χ EFT (NN+NNN+...)

Numerical methods:

- Ab initio approaches are aimed at solving the many-body Schrödinger equation associated with the nuclear Hamiltonian

$$H \Psi(\mathbf{R}; s_1, \dots, s_A; t_1, \dots, t_A) = E \Psi(\mathbf{R}; s_1, \dots, s_A; t_1, \dots, t_A)$$

- Our many-body method of choice is **Quantum Monte Carlo** (in particular **VMC** and **GFMC**) which allows for solving the nuclear Schrödinger equation with the required 1-2% accuracy level for both the ground- and the low-lying excited states of $A \leq 12$ nuclei

Nuclear Models: NN phenomenological potentials

- is a r-space potential controlled by ~ 4300 np and pp scattering data below 350 MeV of the Nijmegen database with a total $\chi^2 \cong 1$
- it is expressed as a sum of EM and OPE terms and phenomenological intermediate- and short-range parts

2NF Argonne V18 (AV18):

Wiringa, Stoks, Schiavilla
PRC 51, 38 (1995)

$$v_{18}(r_{12}) = v_{12}^{\gamma} + v_{12}^{\pi} + v_{12}^I + v_{12}^S = \sum_{p=1}^{18} v^p(r_{12}) O_{12}^p$$

Static part:

$$O_{12}^{l=1,\dots,6} = [\mathbf{1}, \boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2, S_{12}] \otimes [\mathbf{1}, \boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2]$$

Spin-orbit, (Spin-orbit)², L² part:

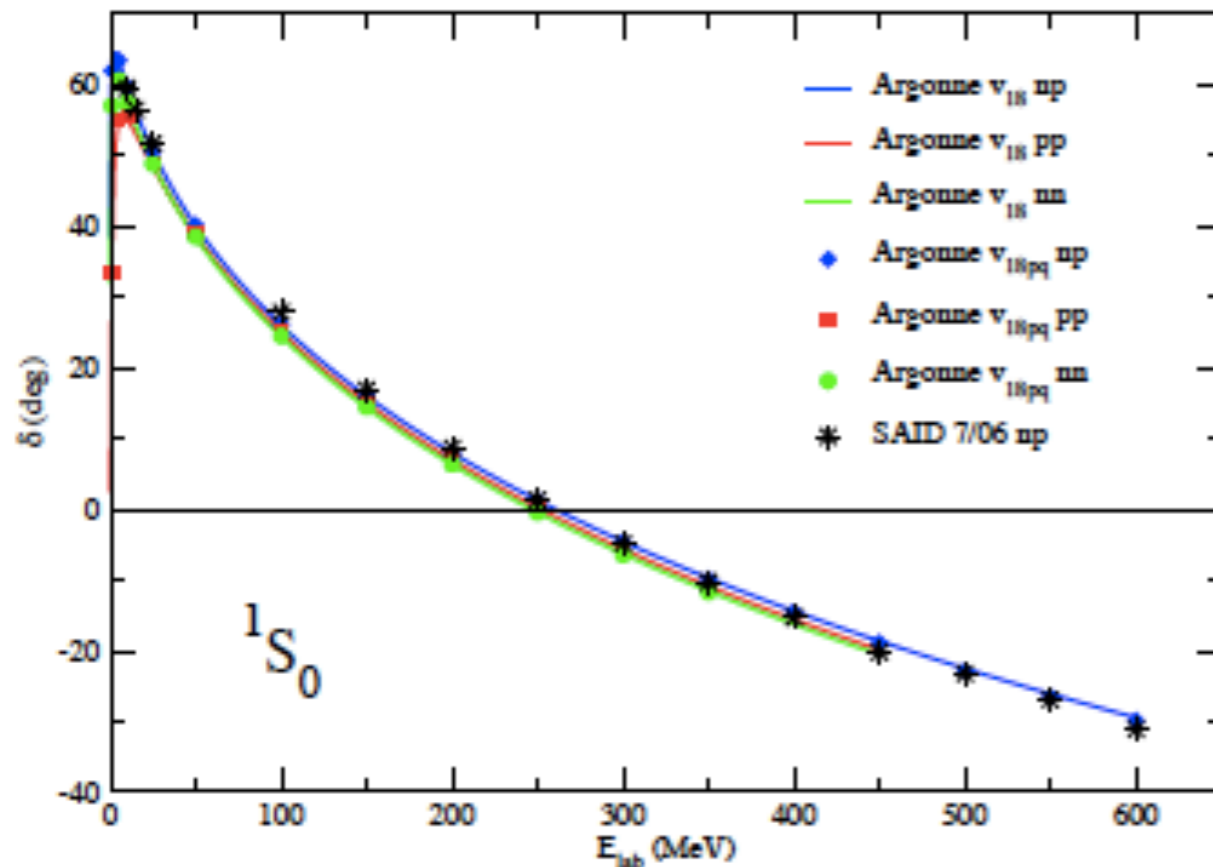
$$O_{12}^{l=7,\dots,14} = [\mathbf{L} \cdot \mathbf{S}, (\mathbf{L} \cdot \mathbf{S})^2] \otimes [\mathbf{1}, \boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2]$$

$$[\mathbf{L}^2, \mathbf{L}^2 \boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2] \otimes [\mathbf{1}, \boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2]$$

Charge-dependent part:

$$O_{12}^{l=15,\dots,18} = [\mathbf{1}, \boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2, S_{12}] \otimes T_{12}, \tau_1^z + \tau_2^z$$

$$S_{12} = 3 \boldsymbol{\sigma}_1 \cdot \hat{\mathbf{r}} \boldsymbol{\sigma}_2 \cdot \hat{\mathbf{r}} - \boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2 \quad T_{12} = 3 \tau_{1z} \tau_{2z} - \boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2$$



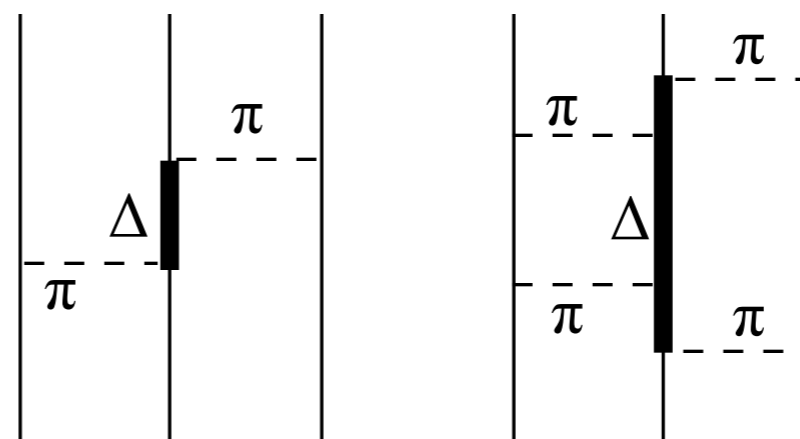
- The AV18 model has a total of 42 independent parameters. While the fit was made up to 350 MeV, the phase shifts are qualitatively good up to much larger energies ≥ 600 MeV

Nuclear Models: NNN phenomenological potentials

3NF Urbana-Illinois (UIX-IL7): • an Hamiltonian which only includes AV18 does not provide enough binding in the light nuclei

Urbana IX: J. Carlson et al. NP A401, 59 (1983)

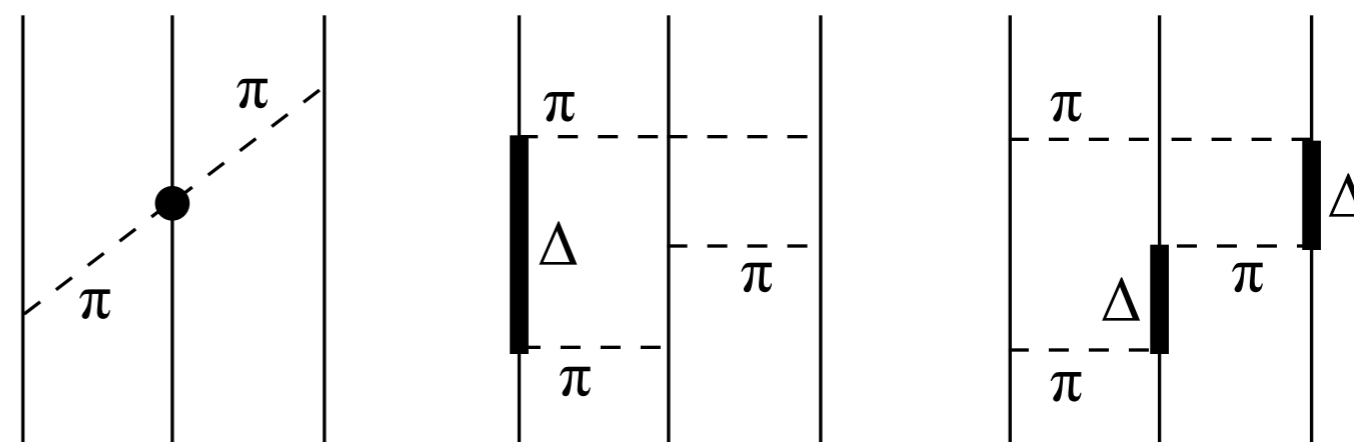
contains the attractive Fujita and Miyazawa two-pion exchange interaction and a phenomenological repulsive term; the two parameters were obtained by fitting the binding energy of ^3H and the density of nuclear matter in conjunction with the AV18



Good description for s-shell nuclei ($A=3,4$); inadequate description of the absolute binding energies and spin-orbit splitting of heavier nuclei

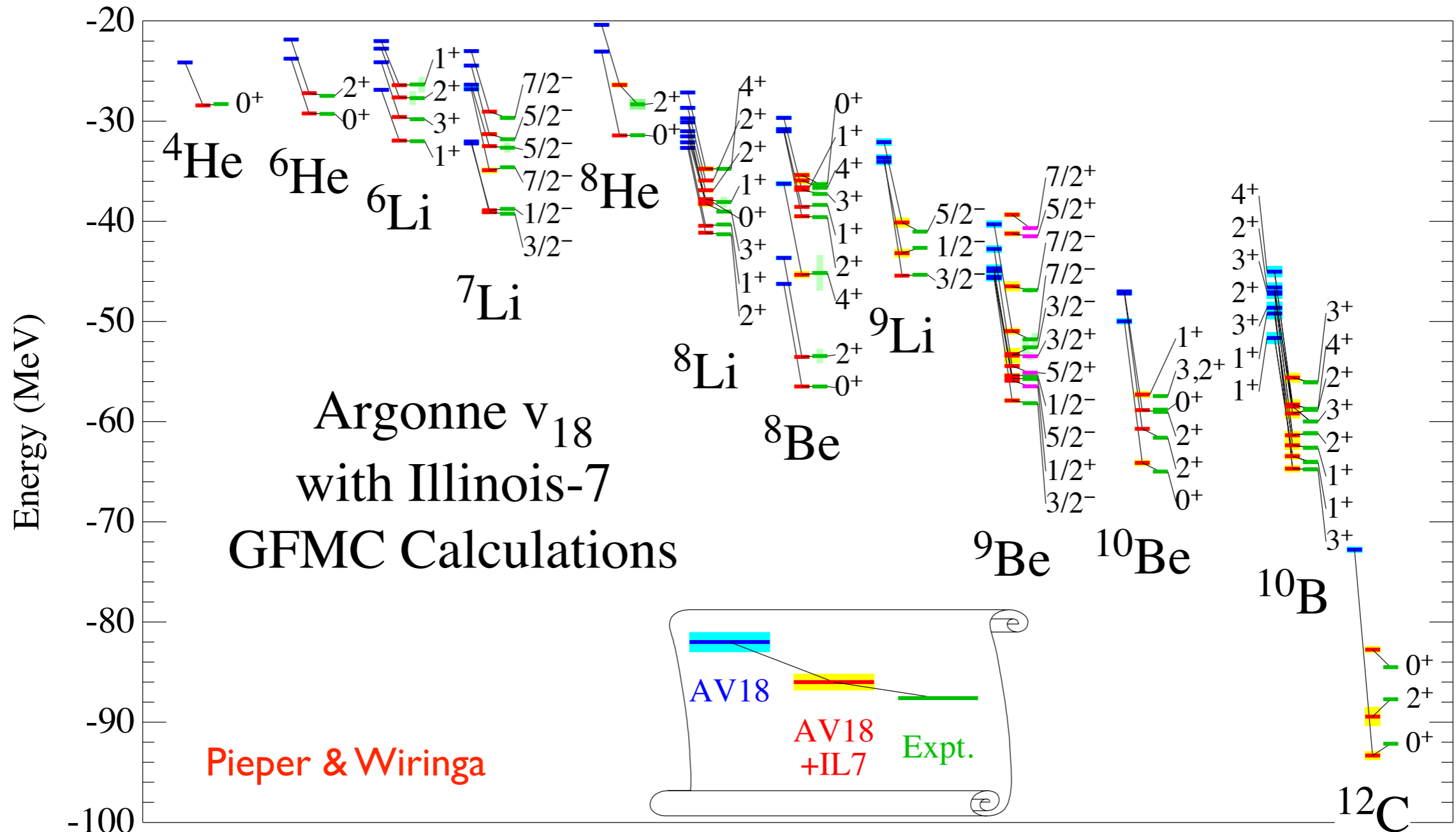
Illinois 7: S. Pieper et al. PRC 64, 014001 (2001)

also includes terms originating from three-pion rings containing one or two Δ s and the two-pion S-wave contribution; the four parameters were searched to obtain a best fit, in conjunction with the AV18, for energies of about 20 nuclear ground and low-lying excited states in $A \leq 10$ nuclei



Good description for light nuclei up to $A=12$; inadequate description of the neutron star matter equation of state

AV18/IL7: the structure of light nuclei



Pros: • Very good description of several nuclear binding energies using AV18+IL7 (GFMC ground-state: uncertainties within 1-2%)

Cons: • Phenomenological interactions are phenomenological, not clear how to improve their quality
 • They do not provide rigorous schemes to consistently derive 2N and 3N forces and compatible electroweak currents

The nuclear χ EFT approach:

S. Weinberg, Phys. Lett. **B251**, 288 (1990); Nucl. Phys. **B363**, 3 (1991); Phys. Lett **B295**, 114 (1992)

- In χ EFT, the symmetries of quantum chromodynamics (QCD), including its approximate chiral-symmetry, are employed to constrain the interactions of pions (π) among themselves, with baryons (N and Δ -isobars) or with external fields (such as electroweak)
- In particular, π 's couple to baryons by powers of its momentum Q , and the Lagrangian (\mathcal{L}_{eff}) can be expanded systematically in powers of Q/Λ (according to a power counting scheme); ($Q \ll \Lambda \approx 1$ GeV is the chiral-symmetry breaking scale and $Q \sim m_\pi$)

$$\mathcal{L}_{eff} = \mathcal{L}^{(0)} + \mathcal{L}^{(1)} + \mathcal{L}^{(2)} + \dots$$

- χ EFT allows for a perturbative treatment in terms of powers of a Q - as opposed to a coupling constant- expansion
- The unknown coefficients in this perturbative expansion are called LEC's and are fixed by comparison with experimental data
- The χ -expansion provides a practical calculation scheme to construct two- and many-body potential as well as external currents, capable (in principle) of systematic improvements

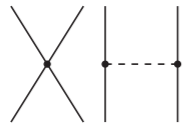
Chiral 2N

Δ -less

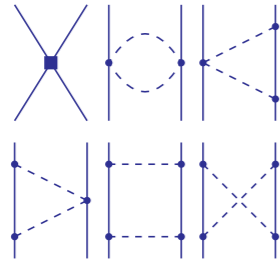
Additional in Δ -full

$$\Delta = m_\Delta - m_N \sim 300 \text{ MeV} \sim 2m_\pi$$

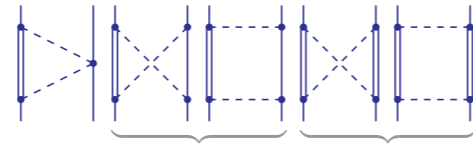
LO
 $(Q/\Lambda_\chi)^0$



NLO
 $(Q/\Lambda_\chi)^2$

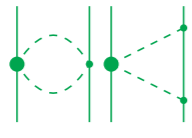


Kaiser et al.'97
Entem & Machleidt '02

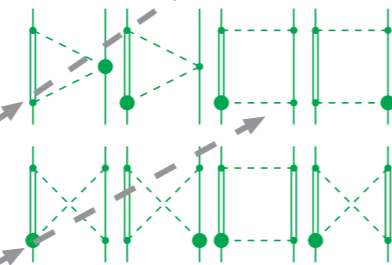


Ordóñez et al.'96; Kaiser et al.'98;
Krebs et al. '07

NNLO
 $(Q/\Lambda_\chi)^3$

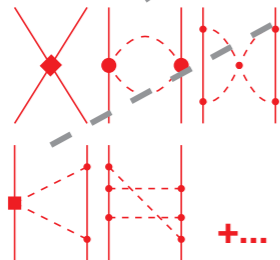


+...
Kaiser et al.'97

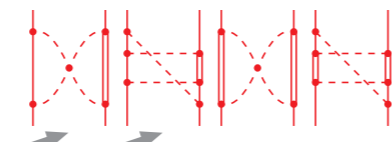


+...
Krebs et al.'07

N³LO
 $(Q/\Lambda_\chi)^4$

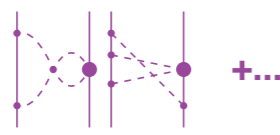


+...
Kaiser '00-'01-'02;
Entem & Machleidt '02



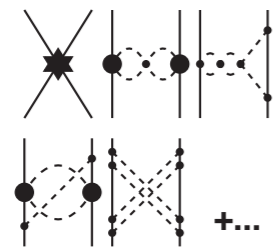
+...
Kaiser '15

N⁴LO
 $(Q/\Lambda_\chi)^5$



+...
Entem et al.'15, Epelbaum et al.'15

N⁵LO
 $(Q/\Lambda_\chi)^6$



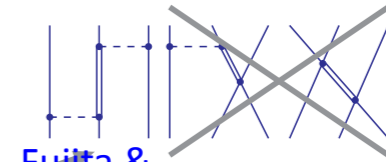
+...
Entem et al.'15

Chiral 3N

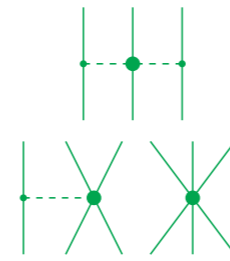
Δ -less

Additional in Δ -full

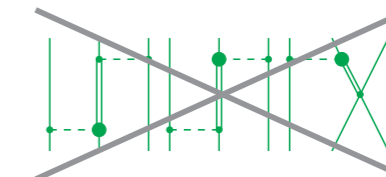
$$\Delta = m_\Delta - m_N \sim 300 \text{ MeV} \sim 2m_\pi$$



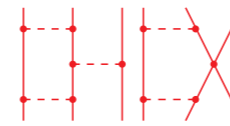
Fujita & Miyazawa '57
van Kolck'96;
Epelbaum'08



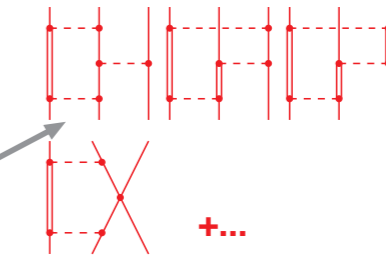
U. van Kolck '94; Epelbaum et al.'02;
Nogga et al.'05; Navratil et al.'07



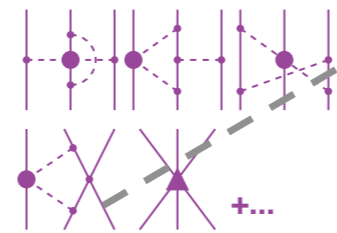
Epelbaum et al. '08



+...
Bernard et al. '08;
Ishikawa & Robilotta '07



+...
Epelbaum et al. '08



+...
Krebs et al. '12-'13;
Girlanda et al. '11

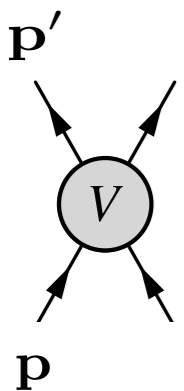
Entem & Machleidt '11
Machleidt & Sammarruca '16

Chiral 2N potentials: some recent developments

- First generation of chiral NN potential up to N3LO:
[Entem-Machleidt PRC 68, 041001 2003](#); [Epelbaum-Gloeckle-Meissner JNP A747, 362 2005](#)
- Optimized N2LO NN potential (π N LECs are tuned to NN peripheral scattering):
[Ekström et al. PRL 110, 192502 2013](#); [JPG 42, 034003 2015](#)
- N2LO potential: a simultaneous fit of NN and 3N forces to low NN data ($E_{\text{lab}}=35$ MeV), deuteron BE, BE and CR of hydrogen, helium, carbon and oxygen isotopes;
[Carlsson et al. PRC 91, 051301\(R\) 2015](#)
- New generation of chiral NN potentials up to N4LO: improved choice of the regulator, no SFR;
[Epelbaum et al. PRL 112, 102501 2014](#); [EPJ A51, 53 2015](#); [PRL 115, 122301 2015](#)
- Chiral 2π and 3π exchange up to N4LO and up to N5LO in NN peripheral scattering;
[Entem et al. PRC 91, 014002 2015](#); [PRC 92, 064001 2015](#), [arXiv:1703.05454 2017](#)
- The LENPIC collaboration [arXiv: 1705.01530v1](#)
-

Note: Many of the available versions of chiral potentials are formulated in momentum-space and are strongly nonlocal: $\Rightarrow \mathbf{p} \rightarrow -i\nabla$ hard to use in QMC methods

Nonlocalities due to contact interactions and to regulator functions



Nonlocal regulator

$$V_{\text{NN}}(\mathbf{p}, \mathbf{p}') \rightarrow \exp \left[- [(\mathbf{p}^2 + \mathbf{p}'^2)/\Lambda^2]^n \right] V_{\text{NN}}(\mathbf{p}, \mathbf{p}')$$

Local regulator

$$V_{\text{NN}}(\mathbf{p}, \mathbf{p}') \rightarrow \exp \left[- [(\mathbf{p}' - \mathbf{p})^2/\Lambda^2]^n \right] V_{\text{NN}}(\mathbf{p}, \mathbf{p}')$$

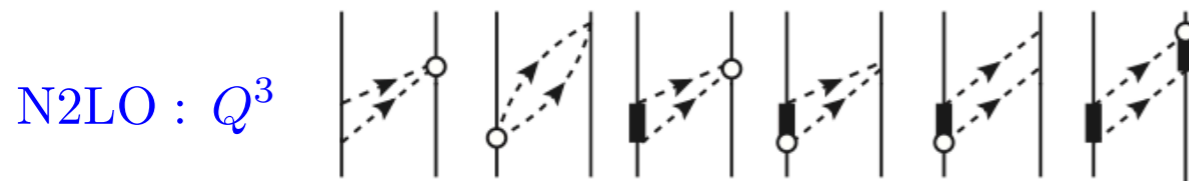
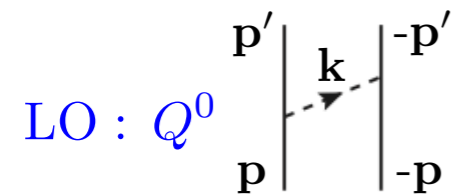
- Local NN potentials up to N2LO:
[Gezerlis et al. PRL 111, 032501 2013](#); [PRC 90, 054323 2014](#); [Lynn et al. PRL 113, 192501 2014](#)
- Minimally nonlocal/local NN potentials including N2LO Δ contributions;
[Piarulli et al. PRC 91, 024003 2015](#); [PRC 94, 054007 2016](#)

Local chiral NN potential:

The local chiral NN potential we designed can be written as: $v_{12} = v_{12}^{\text{EM}} + v_{12}^{\text{L}} + v_{12}^{\text{S}}$

v_{12}^{EM} : EM component including Coulomb, DF, VP and MM interactions

v_{12}^{L} : long-range component including

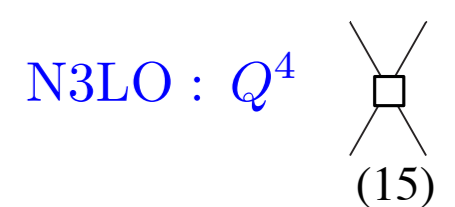
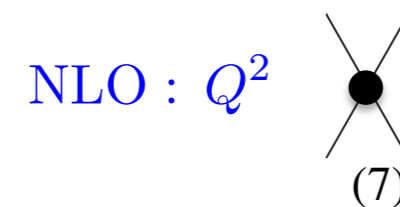
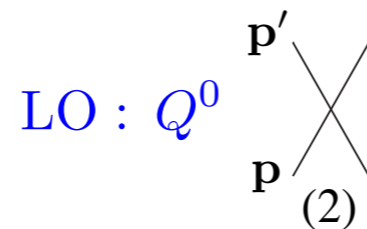


- dependence only on the momentum transfer $\mathbf{k}=\mathbf{p}'-\mathbf{p}$

- known LECs: $g_A, F_\pi, h_A = 3g_A/\sqrt{2}$

- unknown LECs: c_1, c_2, c_3, c_4 ($\mathcal{L}_{\pi N}^{(2)}$)
 $b_3 + b_8$ ($\mathcal{L}_{\pi N \Delta}^{(2)}$) taken from π -N scattering
 (Krebs et al. EPJ A32, 127 2007)

v_{12}^{S} : short-range component including



At NLO and N3LO strongly nonlocal contact terms: proportional to \mathbf{K}^2 and \mathbf{K}^4 where $\mathbf{K}=(\mathbf{p}'+\mathbf{p})/2$; we use Fierz rearrangements to remove these nonlocalities (see also Gezerlis et al. PRL 111, 032501 2013; PRC 90, 054323 2014)

Contact terms of type $\mathbf{k} \times \mathbf{K}$ or $k^2 \mathbf{K}^2$ still persist: they can not Fierz-transformed away

Coordinate-space v_{12}^L :

In coordinate-space the long-range part of the interaction can be written as

$$v_{12}^L = \left[\sum_{l=1}^6 v_L^l(r) O_{12}^l \right] + v_L^{\sigma T}(r) O_{12}^{\sigma T} + v_L^{tT}(r) O_{12}^{tT}$$

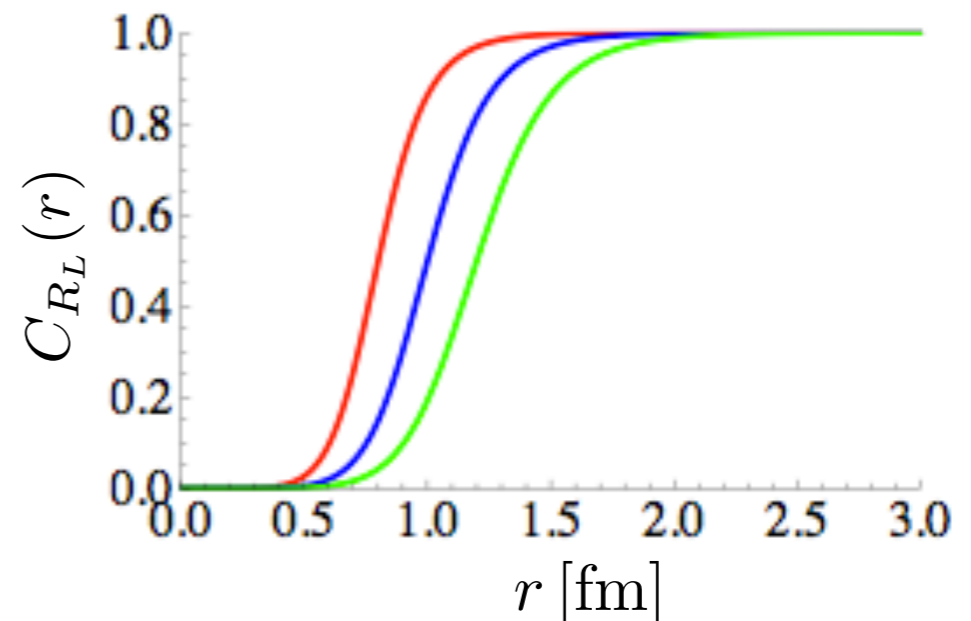
- Charge-independent part: $O_{12}^{l=1,\dots,6} = [\mathbf{1}, \boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2, S_{12}] \otimes [\mathbf{1}, \boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2]$
- Charge-dependent part: $O_{12}^{\sigma T} = \boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2 T_{12}$
 $O_{12}^{tT} = S_{12} T_{12}$ $T_{12} = 3\tau_{1z}\tau_{2z} - \boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2$

The radial functions have divergencies of type $1/r^n$, $1 \leq n \leq 6$; we need a regulator function which ensures that the short-distance part of the long-range potential at r smaller than R_L are smoothly cut off

- $C_{R_L}(r) = 1 - \frac{1}{(r/R_L)^6 e^{(r-R_L)/a_L} + 1}$

$$R_L = (0.8, 1.0, 1.2) \text{ fm}$$

$$a_L = R_L/2$$



Coordinate-space v_{12}^S :

In coordinate-space the short-range part of the interaction can be written as

$$v_{12}^S = \sum_{l=1}^{16} v_S^l(r) O_{12}^l$$

- Static part: $O_{12}^{l=1,\dots,6} = [\mathbf{1}, \boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2, S_{12}] \otimes [\mathbf{1}, \boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2]$
- Spin-orbit, (Spin-orbit)², L² part: $O_{12}^{l=7,\dots,11} = \mathbf{L} \cdot \mathbf{S}, \mathbf{L} \cdot \mathbf{S} \boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2, (\mathbf{L} \cdot \mathbf{S})^2, \mathbf{L}^2, \mathbf{L}^2 \boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2$
- Charge-dependent part: $O_{12}^{l=12,\dots,16} = T_{12}, (\tau_1^z + \tau_2^z), \boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2 T_{12}, S_{12} T_{12}, \mathbf{L} \cdot \mathbf{S} T_{12}$

In order to have a fully nonlocal interaction, we are neglecting \mathbf{p}^2 -terms of type

- $\{ v_S^p(r) + v_S^{p\sigma}(r) \boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2 + v_S^{pt}(r) S_{12} + v_S^{pt\tau}(r) S_{12} \boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2, \mathbf{p}^2 \}$

The FT of the single contact terms is carried out with a Gaussian regulator, depending only on the momentum transfer k , such that

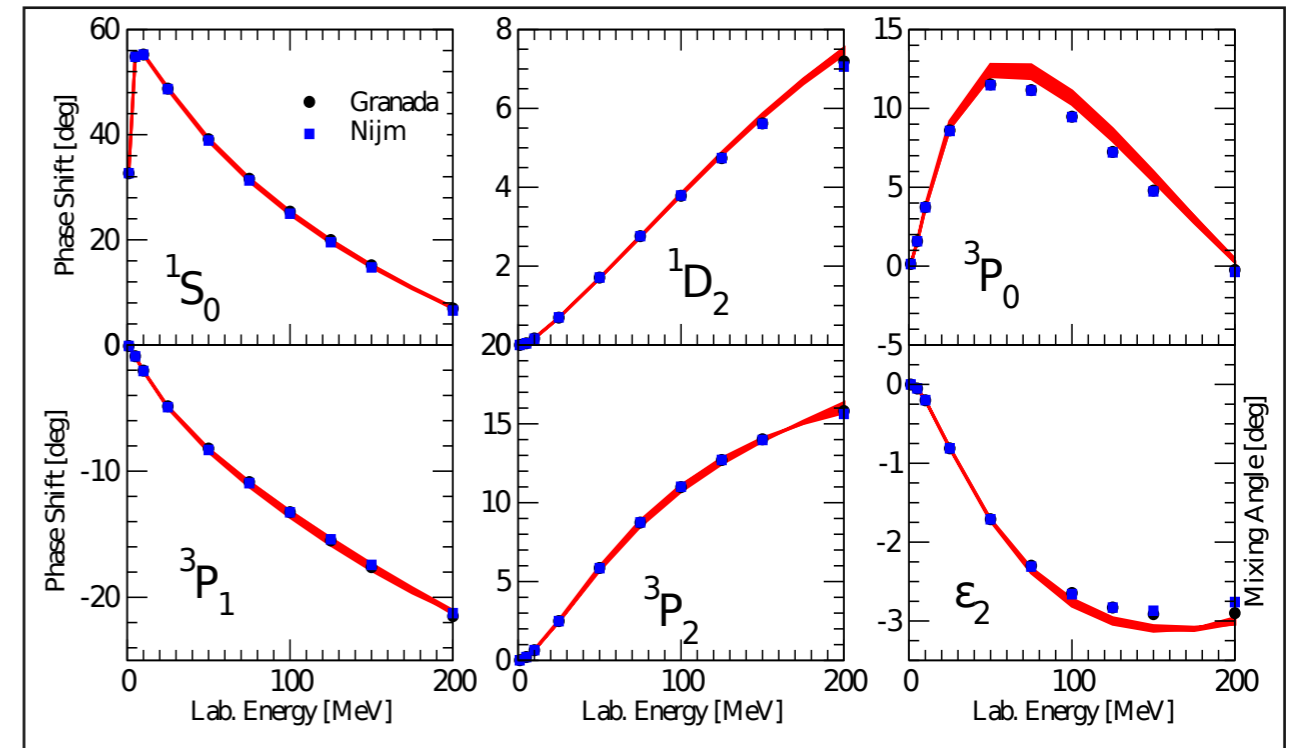
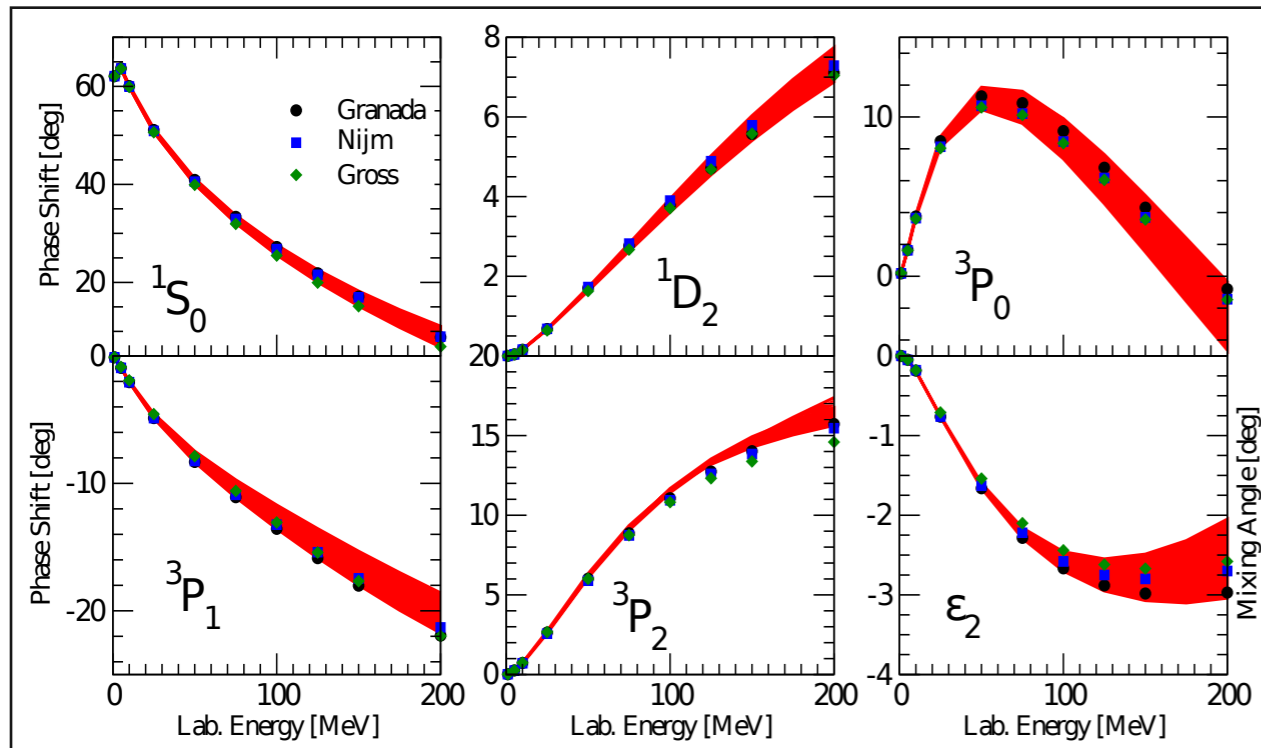
- $\tilde{C}_{R_S}(k) = e^{-R_S^2 k^2/4} \longrightarrow C_{R_S}(r) = \frac{1}{\pi^{3/2} R_S^3} e^{-(r/R_S)^2}$

In combination with $R_L = (0.8, 1.0, 1.2)$ fm we consider $R_S = (0.6, 0.7, 0.8)$ fm corresponding to typical momentum-space cutoff $\Lambda_S = 2/R_S$ (700, 600, 500) MeV

Fitting procedure:

The LECs fixed by fitting the pp and np Granada database up to two ranges of $E_{\text{lab}} = 125$ MeV and 200 MeV, the deuteron BE and the nn scattering length: we first fit the phase shifts then we refine the fit with a direct comparison with the database

To minimizing χ^2 the we use the Practical Optimization Using No Derivatives (for Squares), POUNDers (M. Kortelainen, PRC 82, 024313 2010)



model	order	R_L (fm)	R_S (fm)	E_{LAB} (MeV)	χ^2/datum
Model b	LO	1.0	0.7	125	59.88
Model b	NLO	1.0	0.7	125	2.18
Model b	N2LO	1.0	0.7	125	2.32
Model b	N3LO	1.0	0.7	125	1.07
Model a	N3LO	1.2	0.8	125	1.05
Model c	N3LO	0.8	0.6	125	1.11
Model \tilde{a}	N3LO	1.2	0.8	200	1.37
Model \tilde{b}	N3LO	1.0	0.7	200	1.37
Model \tilde{c}	N3LO	0.8	0.6	200	1.40

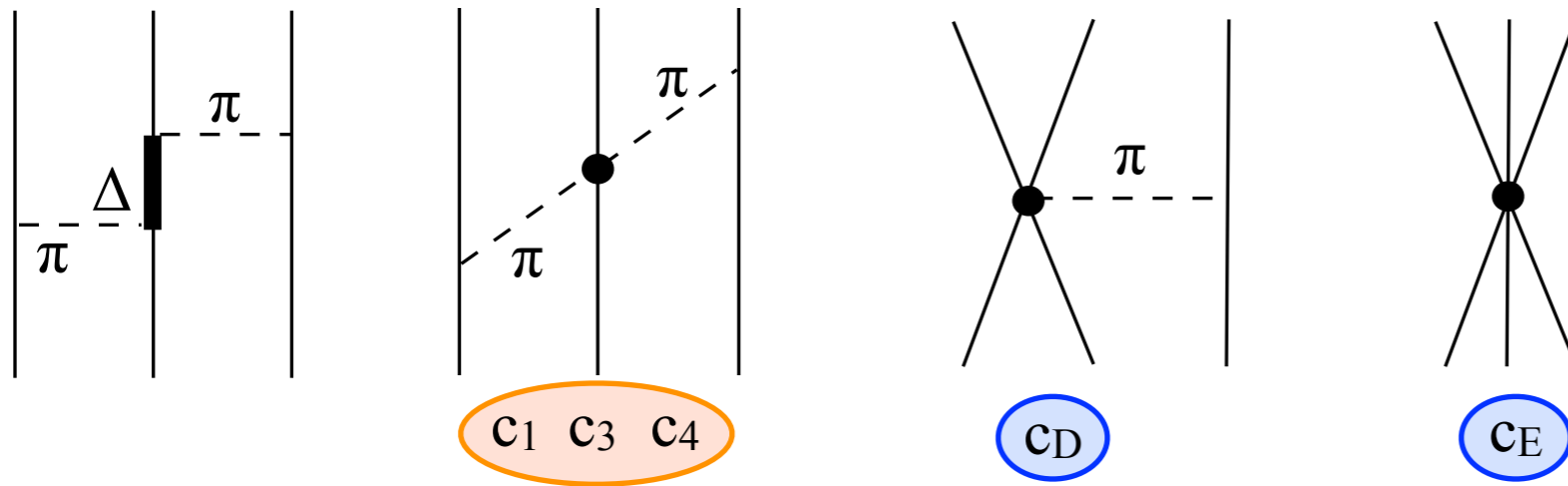
Model a : $(R_L, R_S) = (1.2, 0.8)$

Model b : $(R_L, R_S) = (1.0, 0.7)$

Model c : $(R_L, R_S) = (0.8, 0.6)$

Local chiral 3N potential:

Inclusion of 3N forces at N2LO:



same as NN interaction

need to be fixed

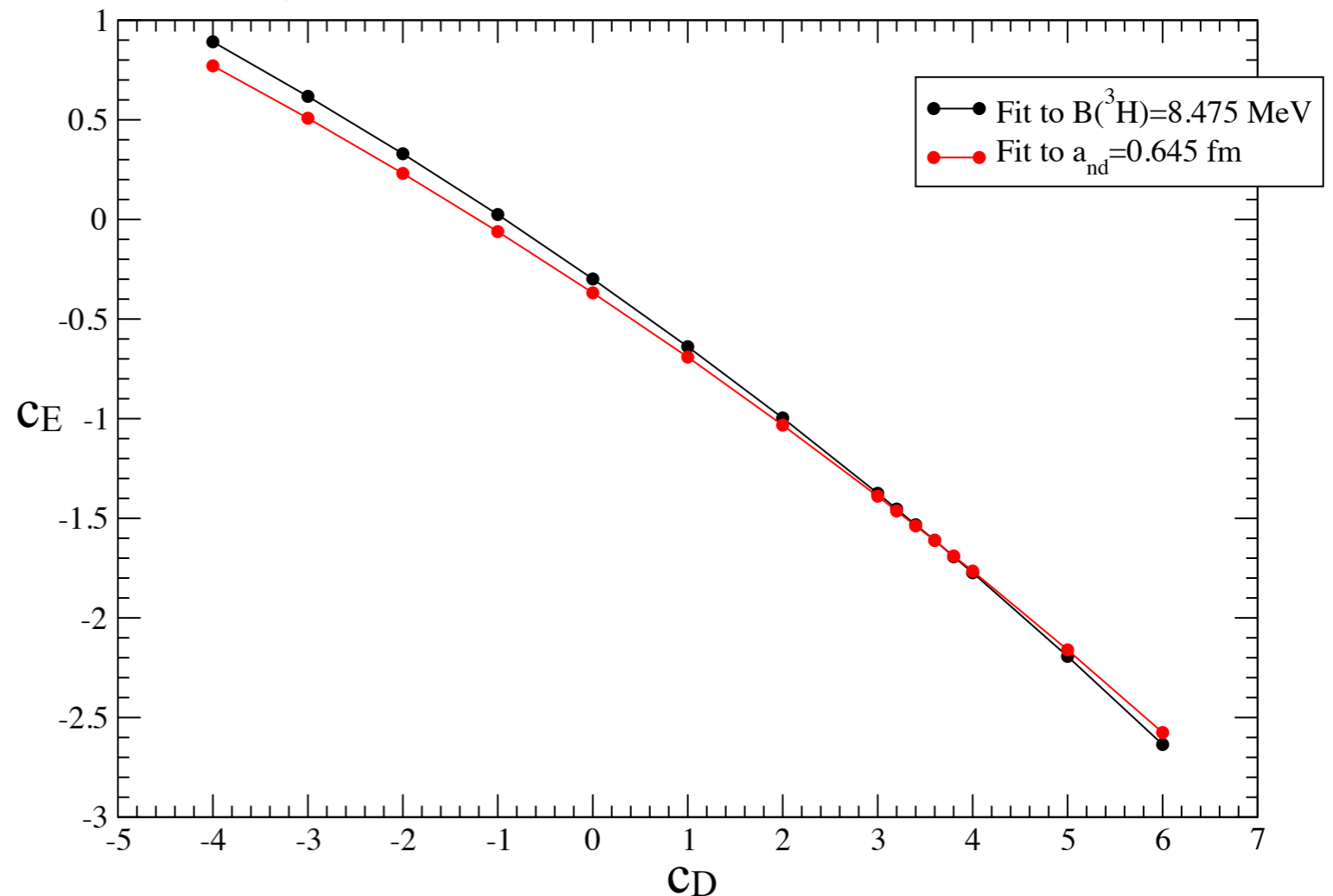
Note:
same regulator functions and
cutoff used in the NN interaction

Fit to:

- ${}^3\text{H}$ binding energy
- nd scattering length

Model	c_D	c_E
Ia	3.666	-1.638
Ib	-2.061	-0.982
IIa	1.278	-1.029
IIb	-4.480	-0.412

courtesy of Laura E. Marcucci (Universita' di Pisa)



Quantum Monte Carlo (QMC) methods:

- Encompasses a large family of computational methods whose common aim is the study of complex quantum systems
- Provide a reliable solution (or an accurate approximation) of the quantum many-body problem

VMC, GFMC: sampling in coordinate space



limited number of nucleons $A=12$

R.B. Wiringa, PRC **43**, 1585 (1991)

Carlson, *et al.*, Rev. Mod. Phys. **87**, 1067 (2015)

AFDMC:

sampling in coordinate space + spin-isospin coordinate



larger nuclei $A \sim 50$ & nuclear matter

Smith and Fantoni, Phys. Lett. B **446**, 99 (1999)

Carlson, *et al.*, Rev. Mod. Phys. **87**, 1067 (2015)

CVMC:

sampling in coordinate space + cluster expansion



closed shell nuclei (± 1): $A=40$

Pieper, *et al.*, Phys. Rev. C **46**, 1741 (1992)

Lonardonì, *et al.*, arXiv:1705.04337

QMC: Variational Monte Carlo (VMC)

R.B. Wiringa, PRC 43, 1585 (1991)

- Minimize the expectation value of H:

$$E_T = \frac{\langle \Psi_T | H | \Psi_T \rangle}{\langle \Psi_T | \Psi_T \rangle} \geq E_0$$

- Trial wave function (involves variational parameters):

$$|\Psi_T\rangle = \left[1 + \sum_{i<j<k} U_{ijk} \right] \left[S \prod_{i<j} (1 + U_{ij}) \right] |\Psi_J\rangle$$

$|\Psi_J\rangle = \left[\prod_{i<j} f_c(r_{ij}) \right] |\Phi(JMTT_z)\rangle$ (s-shell nuclei): Jastrow wave function, fully antisymmetric

$S \prod_{i<j}$: represents a symmetrized product

$U_{ij} = \sum_{p=2,6} u_p(r_{ij}) O_{ij}^p$: pair correlation operators

$U_{ijk} = \sum_x \epsilon_x V_{ijk}^x$: three-body correlation operators

$|\Psi_T\rangle$ are spin-isospin vectors in $3A$ dimension with $2^A \begin{pmatrix} A \\ Z \end{pmatrix}$

- The search in the parameter space is made using COBYLA (Constrained Optimization BY Linear Approximations) algorithm available in NLOpt library
- The typical number of variational parameters for s-shell nuclear wave functions is about two dozen for a two-body potential; four to six parameters are added if a three-body potential is included in the Hamiltonian

QMC: Green's function Monte Carlo (GFMC)

J. Carlson et al., Rev. Mod. Phys. **87**, 1067 (2015)

- The diffusion Monte Carlo (DMC) method such as GFMC, overcomes the limitation of VMC calculation by using a projection technique to enhance the true ground-state component of a starting trial wave function
- The method relies on the observation that the trial wave function can be expanded in the complete set of eigenstates of the Hamiltonian according to

$$|\Psi_T\rangle = \sum_n c_n |\Psi_n\rangle \quad H|\Psi_n\rangle = E_n |\Psi_n\rangle$$

which implies

$$\lim_{\tau \rightarrow \infty} |\Psi(\tau)\rangle = \lim_{\tau \rightarrow \infty} e^{-(H-E_0)\tau} |\Psi_T\rangle = c_0 |\Psi_0\rangle \quad |\Psi(\tau=0)\rangle = |\Psi_T\rangle$$

where τ is the imaginary time. The GFMC projects out lowest energy state from the best variational trial wave function

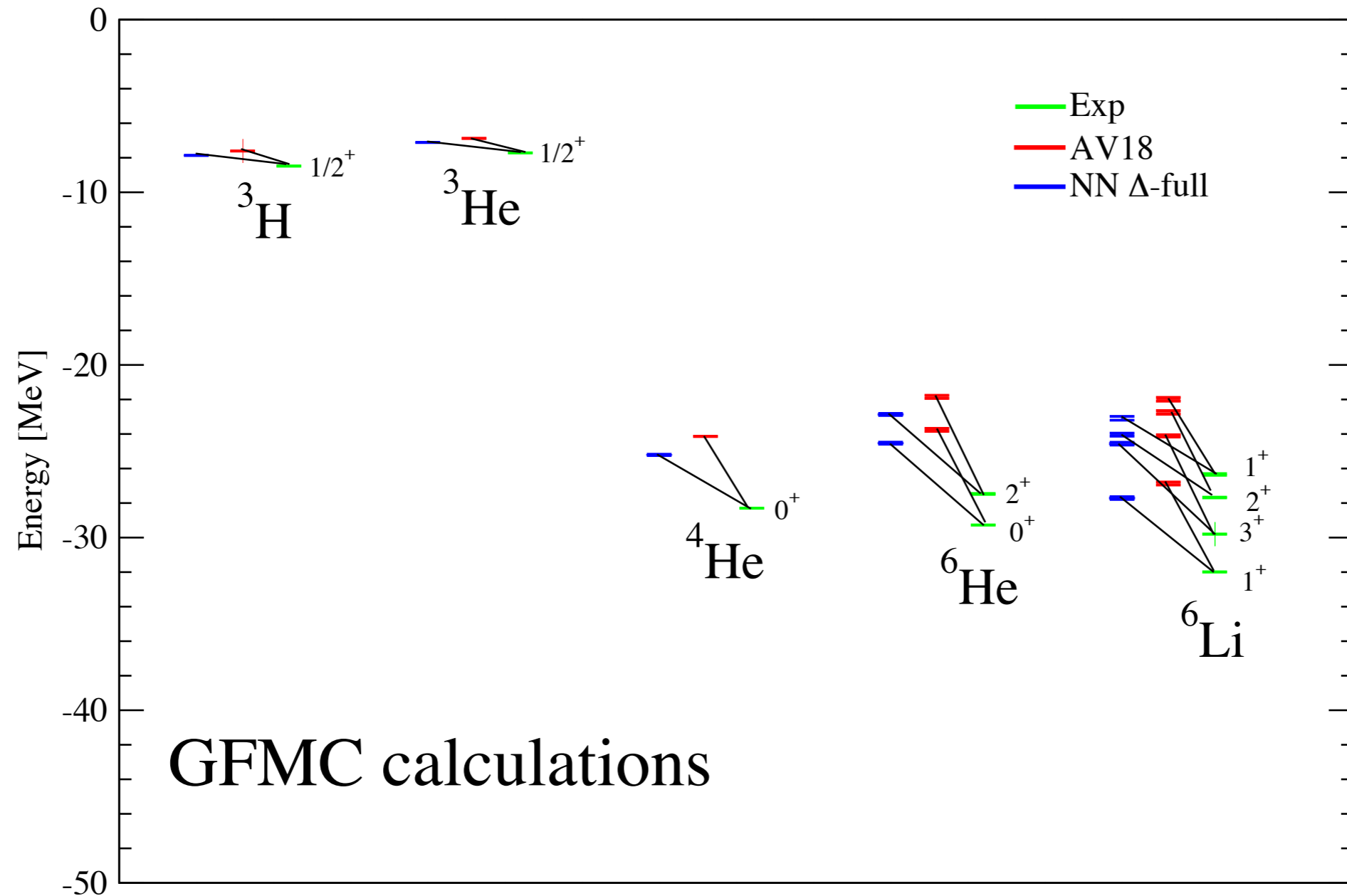
- The evaluation of $\Psi(\tau)$ is done stochastically in small time steps $\Delta\tau$ ($\tau = n \Delta\tau$) using a Green's function formulation

Propagator does not contain p^2 , L^2 , $(\mathbf{L} \cdot \mathbf{S})^2$

- it is carried out with a simplified version H' of the full Hamiltonian H ; H' is a projection of H on the first eight-operator preserving the potential in all S and P waves as well as the deuteron channel; the remaining terms are calculated perturbatively

QMC for $A \leq 6$ with only local NN interaction

- The $A \leq 6$ ground- and excited state energies with only local NN chiral interaction compared with the corresponding GFMC results obtained with AV18 and experimental values



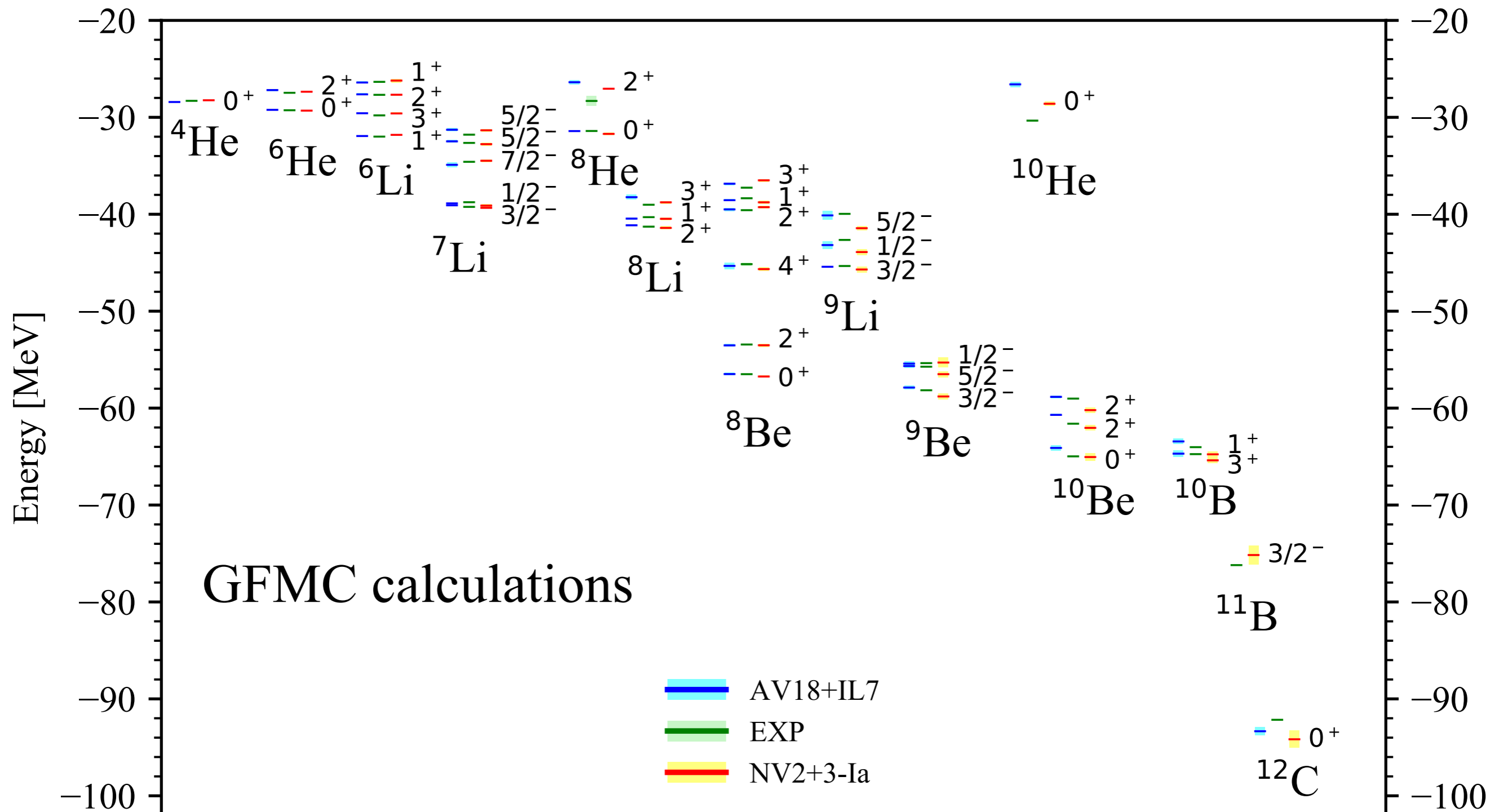
3N interactions are needed!!

Piarulli et al. PRC 94, 054007 20016

- For $A=3, 4$ benchmark with the HH calculations
- For $A=3, 4$, and 6 the energies differ by about 0.2-0.3, 1.0, and 0.5-1.3 MeV, respectively, from the corresponding ones obtained using the AV18

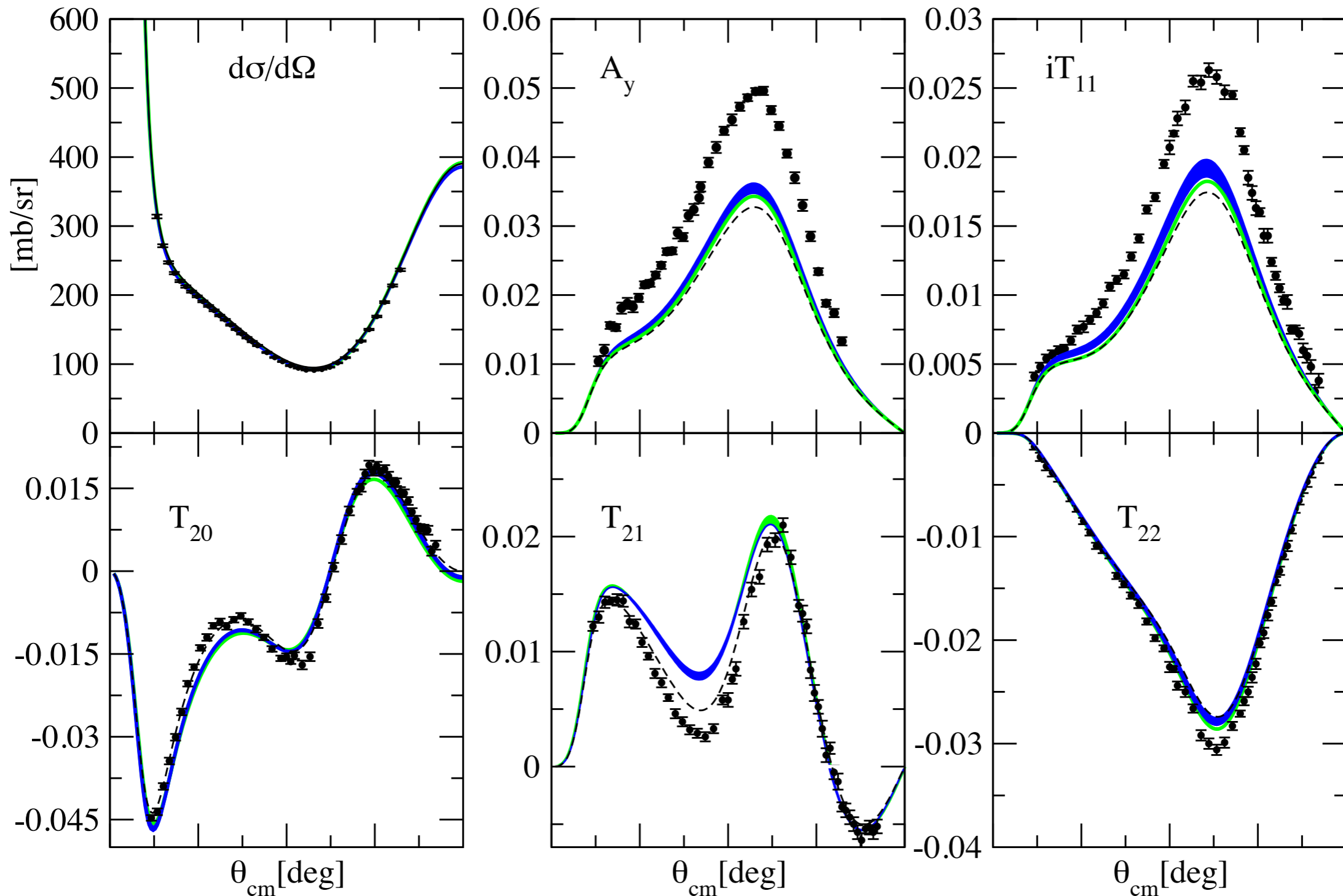
QMC for $A \leq 12$ with local NN+NNN interactions

- The $A \leq 12$ ground- and excited state energies with local NN+NNN chiral interaction, AV18+IL7, and the corresponding experimental values



- Compared to the AV18+UIX, these local chiral potentials give better description for absolute binding energies and spin-orbit splitting for p-shell nuclei

- Polarization observables in pd elastic scattering at 3 MeV, obtained in HH calculations with the NV2+3 models Ia-Ib (IIa-IIb), are shown by the green (blue) band. The black dashed line are results obtained with only the two-body interaction NV2-Ia



subleading contact terms in 3N interaction??? Additional 10 LECs

Conclusions:

- We have constructed a family of local NN potential with chiral TPE including Δ -isobar up to N2LO and contact interactions up to N3LO in the chiral expansion
- Three versions of this NN chiral potential for three different cutoff values have been developed with good fits to np and pp Granada database
- Corresponding local NNN chiral interaction up N2LO have been also developed; they involve two new LECs fixed by fitting the binding energy of ${}^3\text{H}$ and nd scattering length
- A subset of these local NN and NN+NNN chiral interactions have been used to in HH and QMC calculations of binding energies and rms proton radii for some nuclei with $A \leq 12$

Plans:

- Test other versions of NV2+3 with different energy fits and regulators and compare
- Studies of the effect of subleading 3N contact interactions in light nuclei
- Different strategies to fit 3NI
- Comprehensive treatment of radii, moments, electroweak transitions in VMC/GFMC including exchange currents