Transversely polarized collinear and TMD observables within the Collins-Soper-Sterman formalism

Daniel Pitonyak

Penn State University-Berks, Reading, PA

supported by
TMD Topical Collaboration

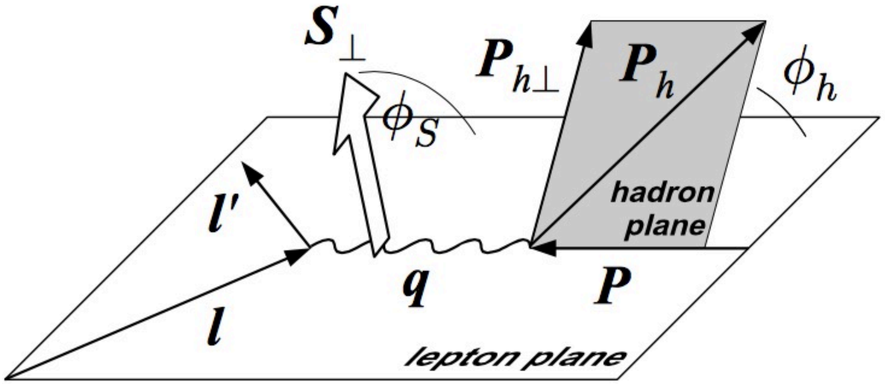
Jefferson Lab Theory Seminar
Newport News, VA
December 11, 2017

Outline

- Background
  - Transverse single-spin asymmetries (TSSAs)
  - TMD and collinear twist-3 (CT3) functions
- TMD and CT3 observables
  - Sivers and Collins effects
  - A_N in pp ➔ $\{\gamma\,,\pi\}\ X$
- Relations between TMD and CT3 functions
  - Using CSS operators
  - Physical interpretation using “naïve” operators
- Towards a global analysis of TMD and CT3 observables
- Summary

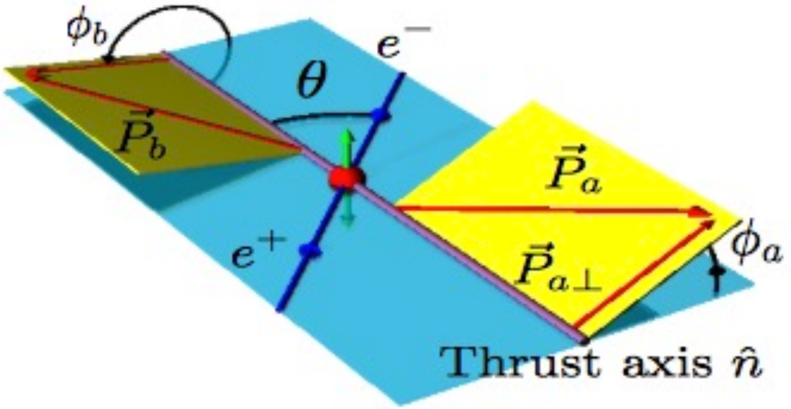
Background

$e\,N \rightarrow e'\,h\,X$



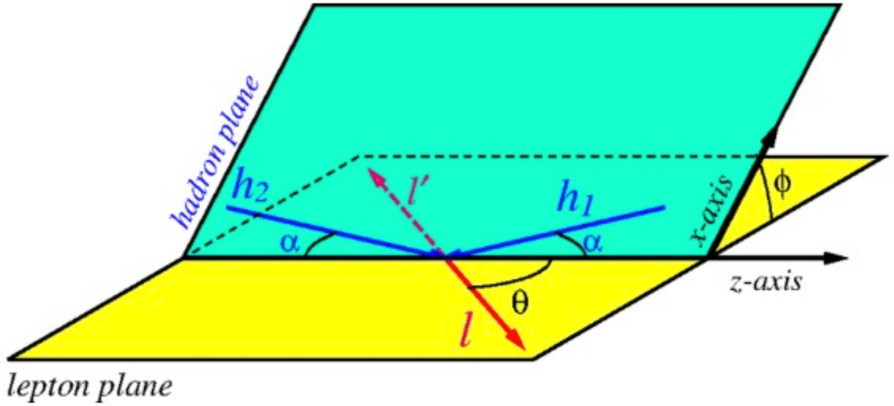
Sivers ~ *sin($\phi_h - \phi_s$)*, Collins ~ *sin($\phi_h + \phi_s$)*, …

$e^+e^- \rightarrow h_a h_b X$



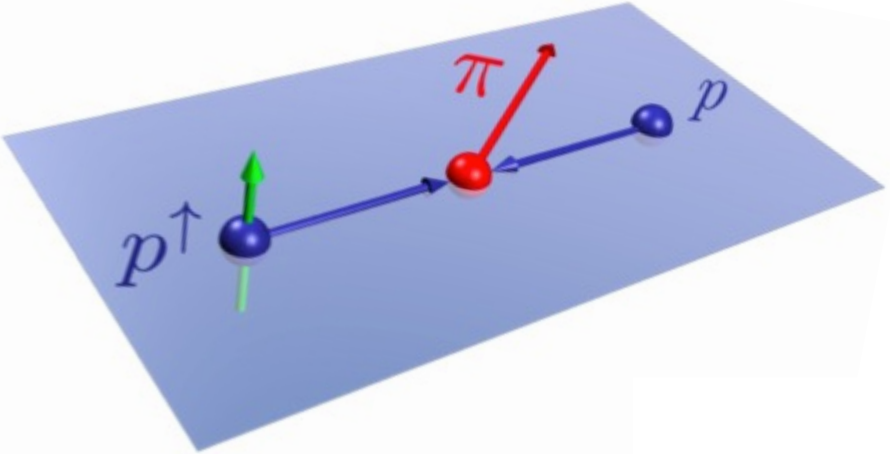
Collins ~ *cos($\phi_a + \phi_b$)*, …

$p^\uparrow\{p,\pi\} \rightarrow \{l^+l^-, W/Z\}X$



Sivers ~ *sin(ϕ_s)* (lepton pair) / Sivers ~ *cos($\phi_{W/Z}$)* (boson)

$p^\uparrow\{p,l\} \rightarrow \{\pi,\gamma\}X$



$A_N \sim d\sigma_L - d\sigma_R$

TMD PDFs (x, k_T)

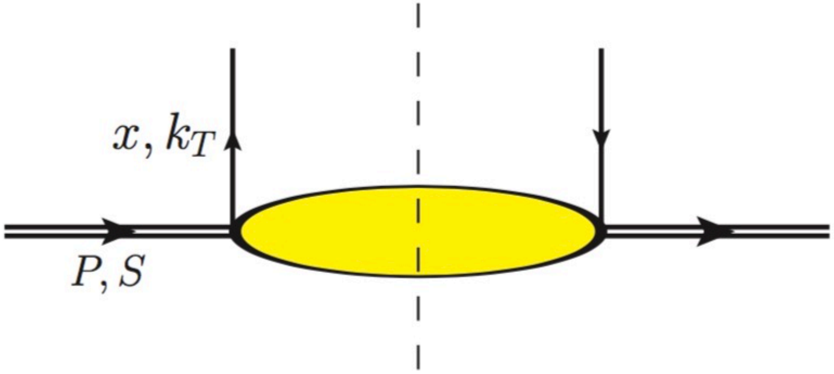
q pol. H pol.	U	L	T
U	f_1		$h_1^{\perp}$
L		g_{1L}	$h_{1L}^{\perp}$
T	$f_{1T}^{\perp}$	g_{1T}	h_{1T} $h_{1T}^{\perp}$

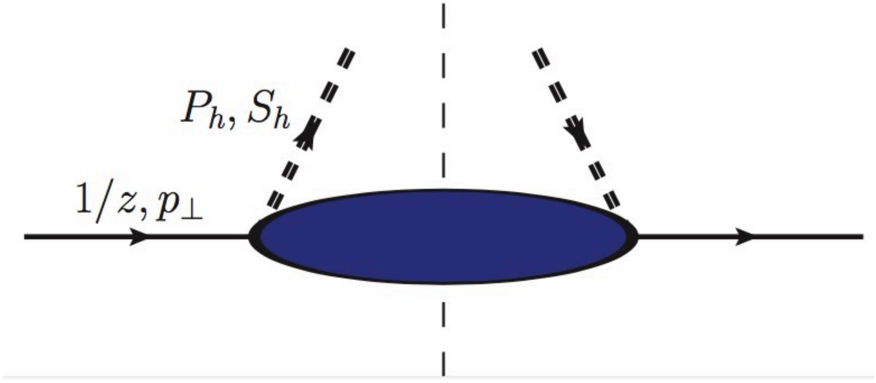
(Mulders, Tangerman (1996); Goeke, Metz, Schlegel (2005))

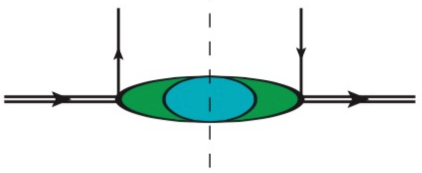
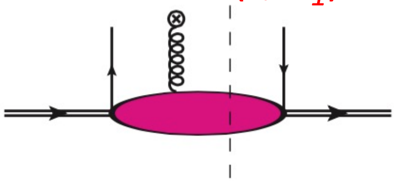
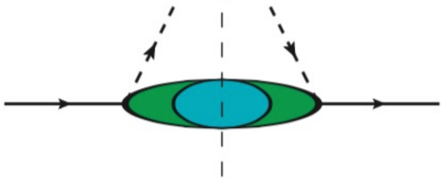
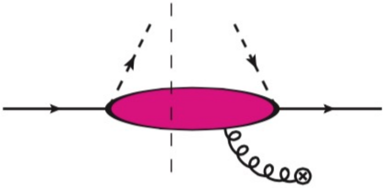
TMD FFs (z,$p_\perp$)

q pol. H pol.	U	L	T
U	D_1		$H_1^{\perp}$
L		G_{1L}	$H_{1L}^{\perp}$
T	$D_{1T}^{\perp}$	G_{1T}	H_{1T} $H_{1T}^{\perp}$

(Boer, Jakob, Mulders (1997))



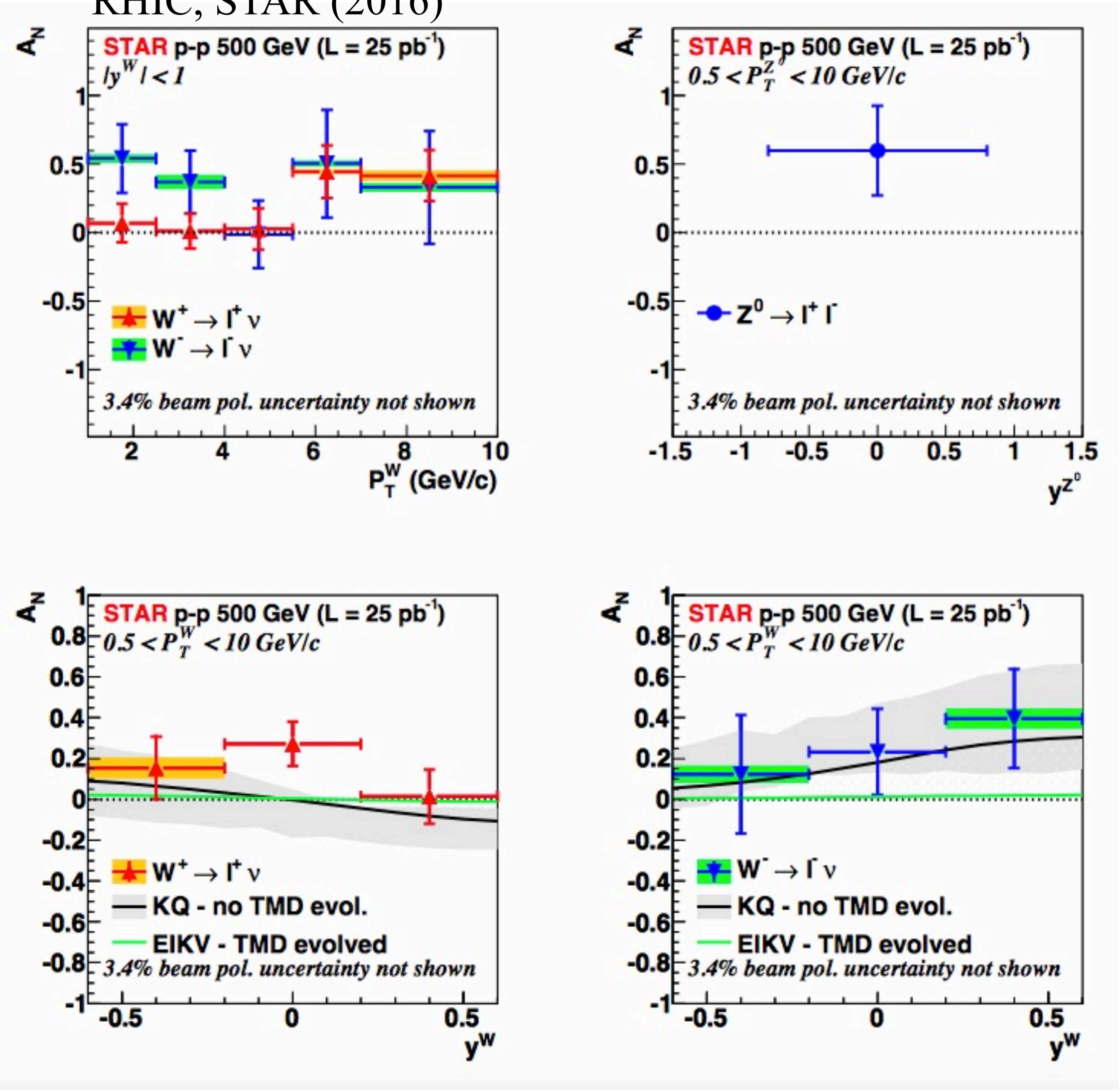


	CT3 PDF (x)		CT3 PDF (x, x_1)	CT3 FF (z)		CT3 FF (z, z_1)
Hadron Pol.						
	intrinsic	kinematical	dynamical	intrinsic	kinematical	dynamical
U	e	$h_1^{\perp(1)}$	H_{FU}	E, H	$H_1^{\perp(1)}$	$\hat{H}_{FU}^{\Re,\Im}$
L	h_L	$h_{1L}^{\perp(1)}$	H_{FL}	H_L, E_L	$H_{1L}^{\perp(1)}$	$\hat{H}_{FL}^{\Re,\Im}$
T	g_T	$f_{1T}^{\perp(1)}$, $g_{1T}^{\perp(1)}$	F_{FT}, G_{FT}	D_T, G_T	$D_{1T}^{\perp(1)}$, $G_{1T}^{\perp(1)}$	$\hat{D}_{FT}^{\Re,\Im}, \hat{G}_{FT}^{\Re,\Im}$

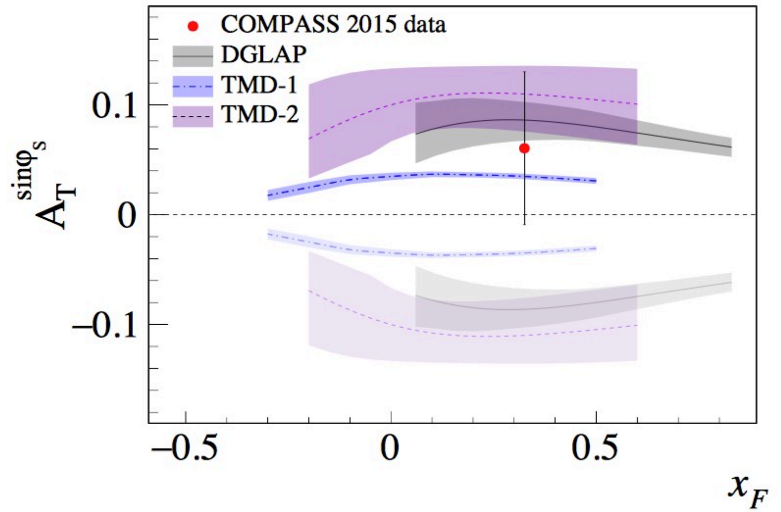
TMD and CT3 Observables

Drell-Yan Sivers effect

RHIC, STAR (2016)

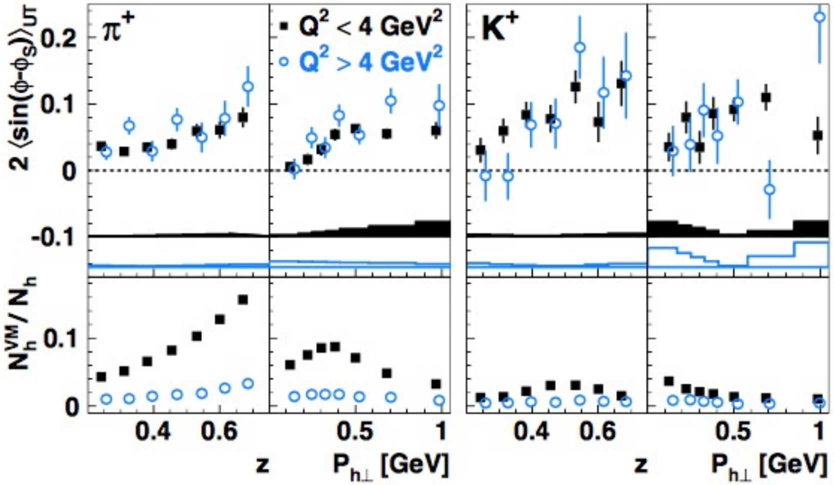


COMPASS (2017)

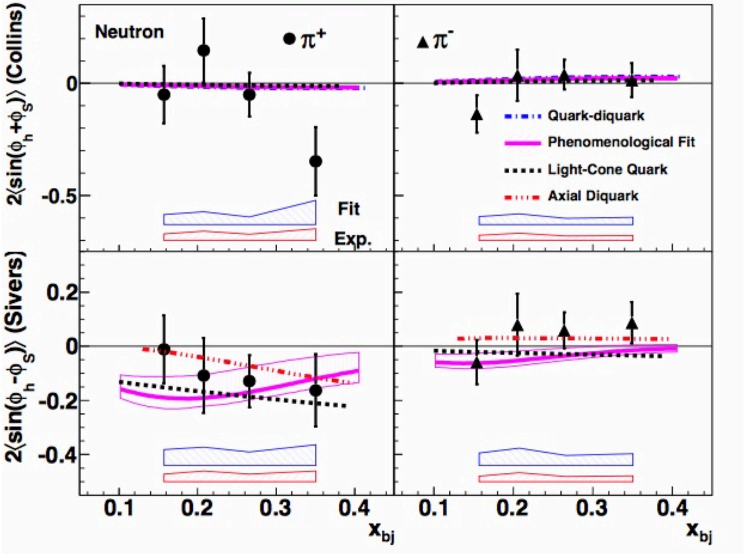


SIDIS Sivers effect (*sin($\phi_h - \phi_s$)*)

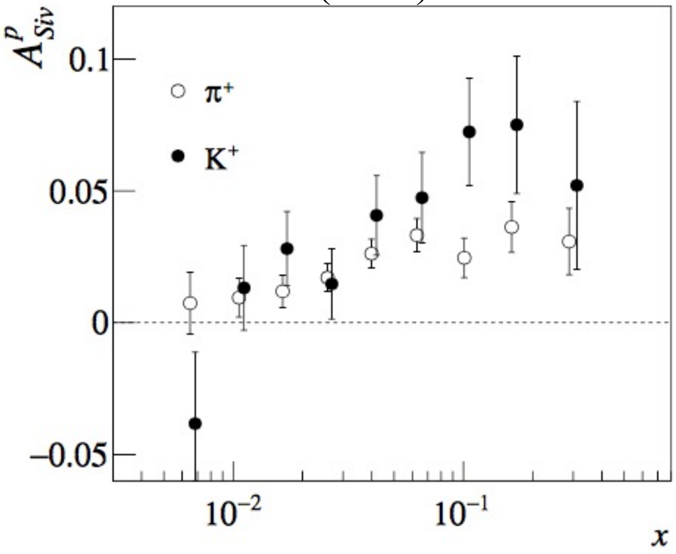
HERMES (2009)



JLab, Hall A (2011)

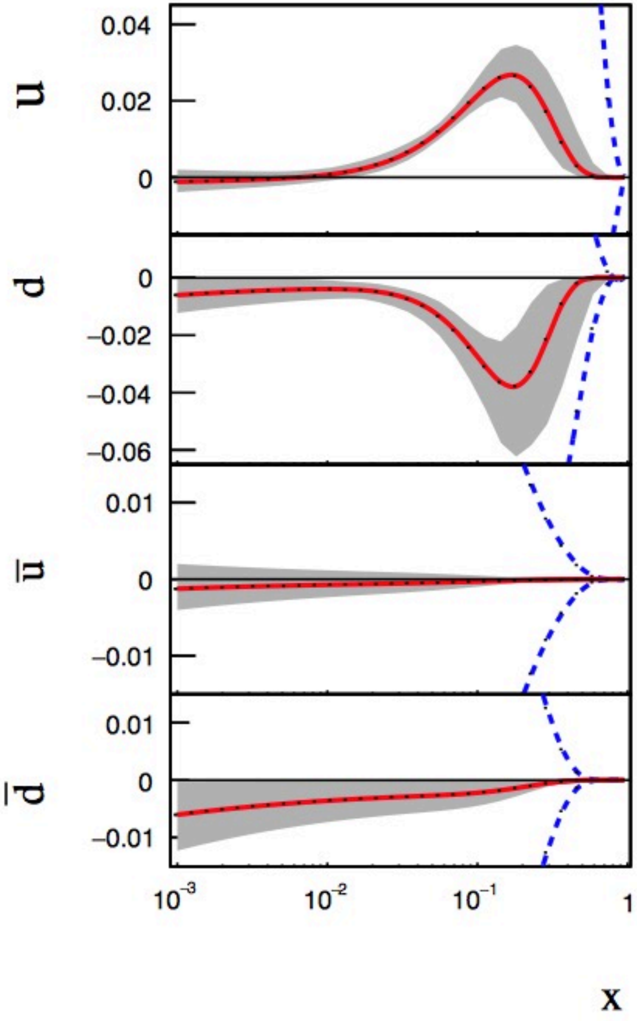


COMPASS (2015)

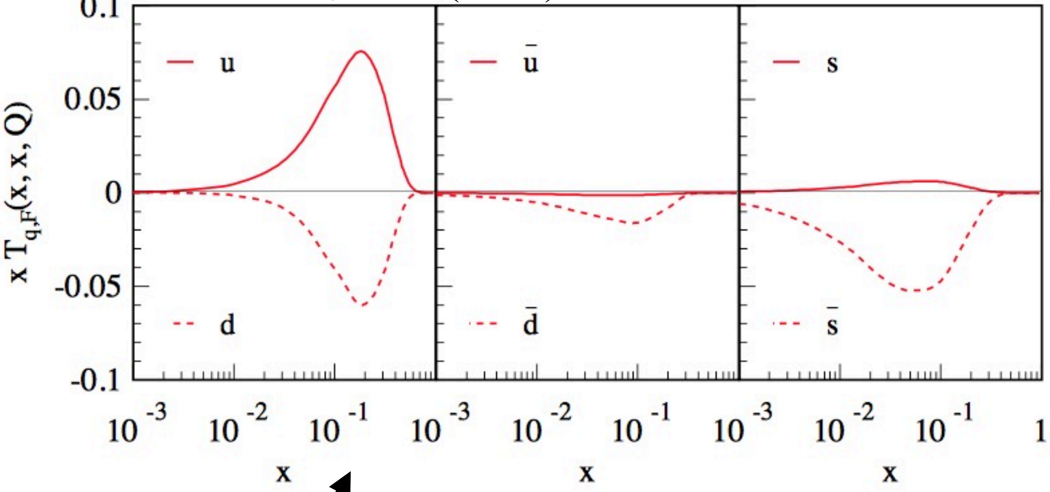


$$F_{UT}^{\sin(\phi_h-\phi_S)} = \mathcal{C}\left[-\frac{\hat{h}\cdot\vec{k}_T}{M}\boldsymbol{f_{1T}^{\perp}}D_1\right]$$

Anselmino, et al. (2017)



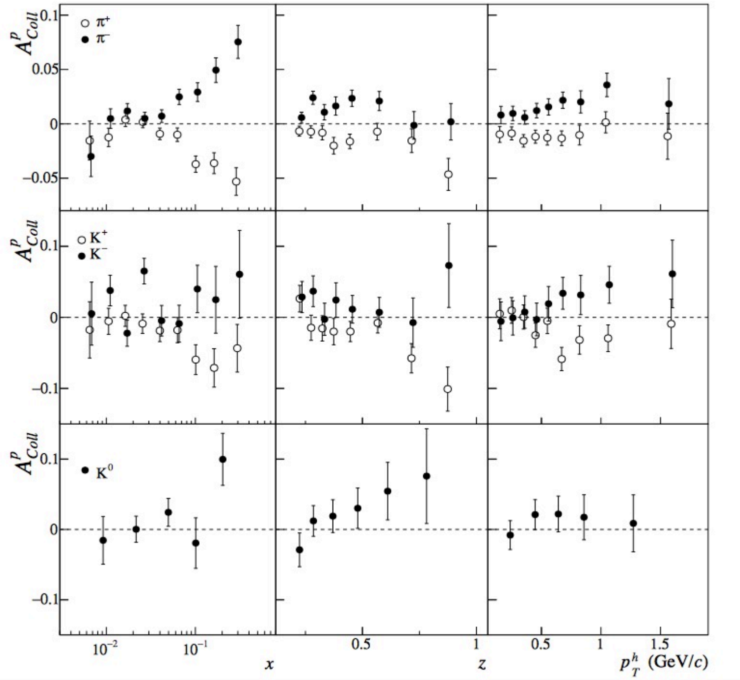
Echevarria, et al. (2014)



TMDs in CSS formalism

SIDIS Collins effect (*sin($\phi_h + \phi_s$)*)

COMPASS (2015)

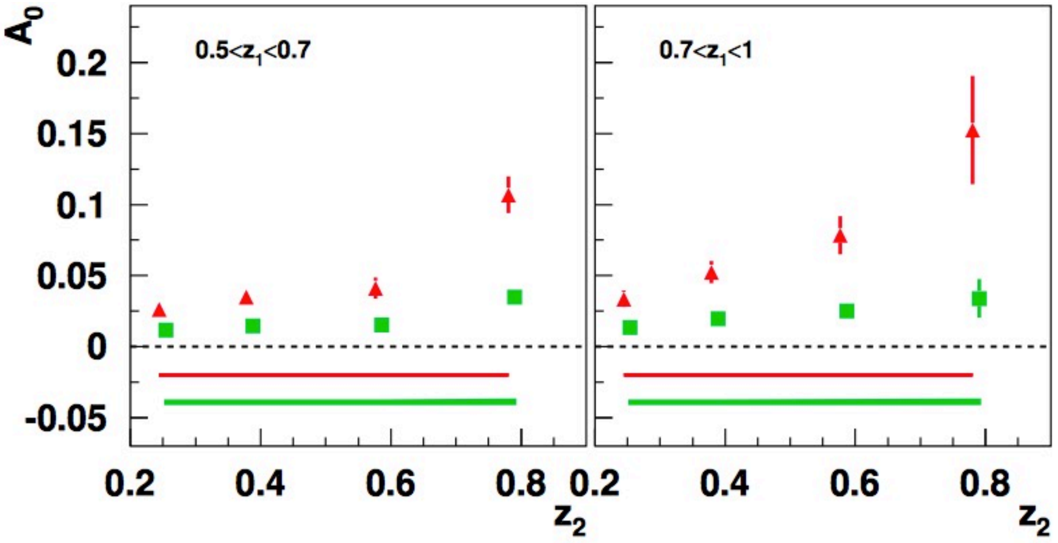


Also data from JLab Hall A (2011, 2014) and HERMES

$$F_{UT}^{\sin(\phi_h+\phi_S)} = \mathcal{C}\left[-\frac{\hat{h}\cdot\vec{p}_\perp}{M_h}\boldsymbol{h_1}\boldsymbol{H_1^\perp}\right]$$

e^+e^- Collins effect (*cos($2\phi_0$)*)

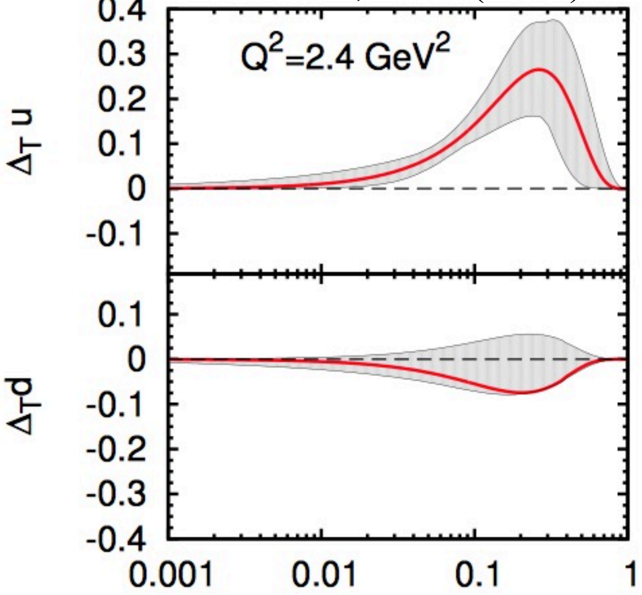
Belle (2008)

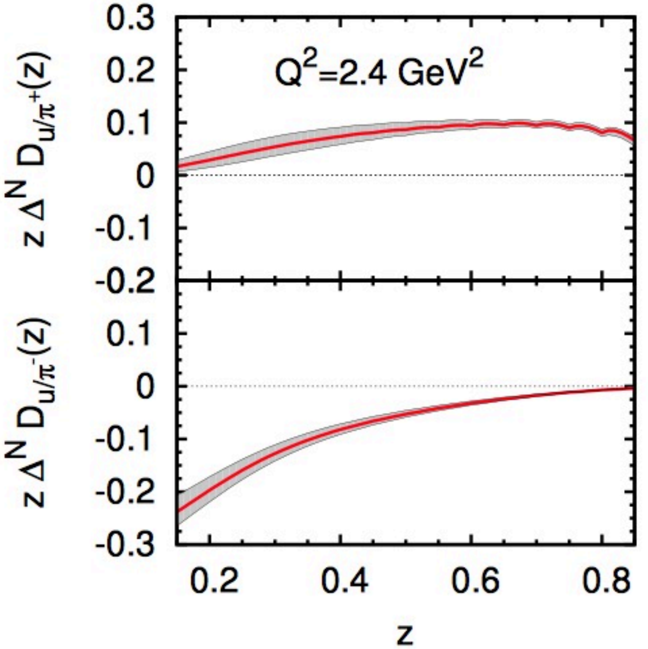


Also data from BaBar (2014) and BESIII (2016)

$$F_{UU}^{\cos(2\phi_0)} = \mathcal{C}\left[\frac{2\hat{h}\cdot\vec{p}_{a\perp}\,\hat{h}\cdot\vec{p}_{b\perp} - \vec{p}_{a\perp}\cdot\vec{p}_{b\perp}}{M_aM_b}\boldsymbol{H_1^\perp}\,\boldsymbol{\bar{H}_1^\perp}\right]$$

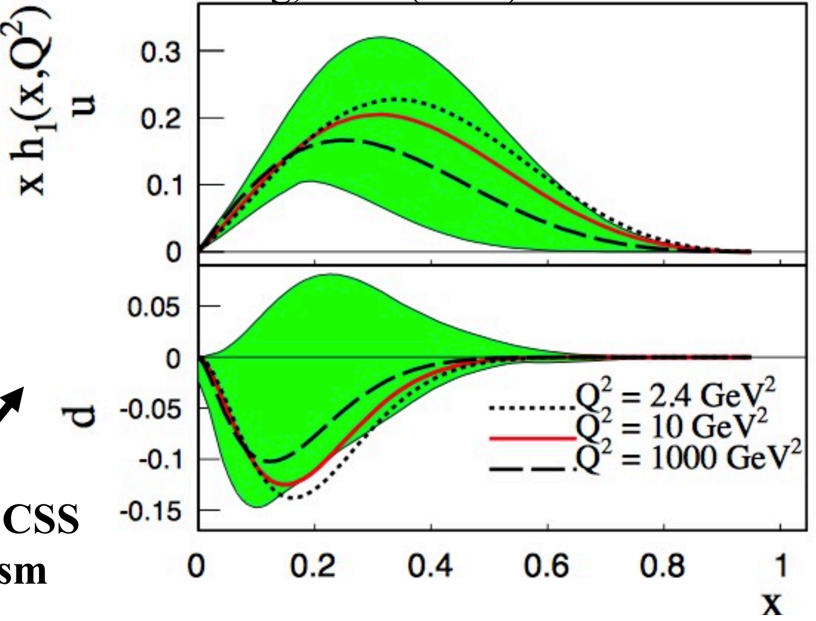
Anselmino, et al. (2015)

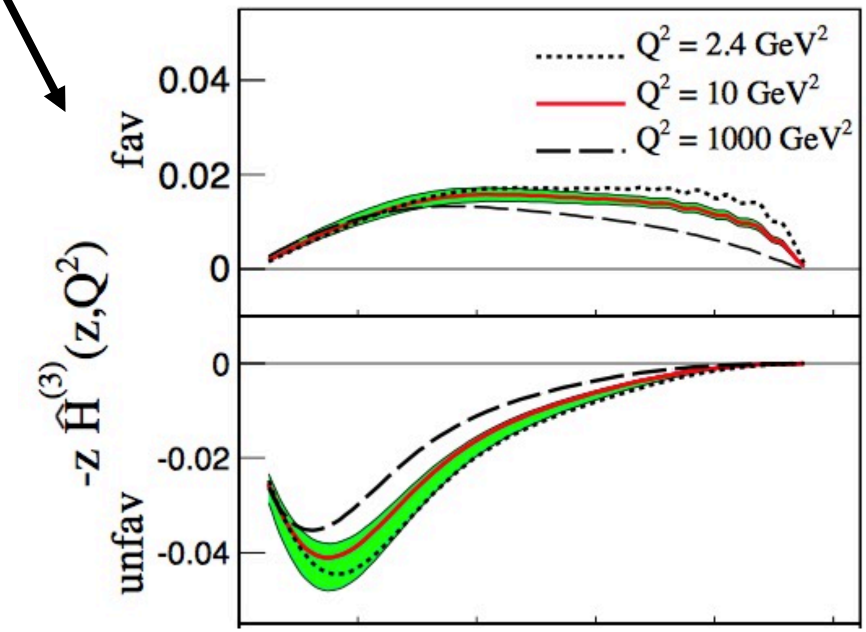


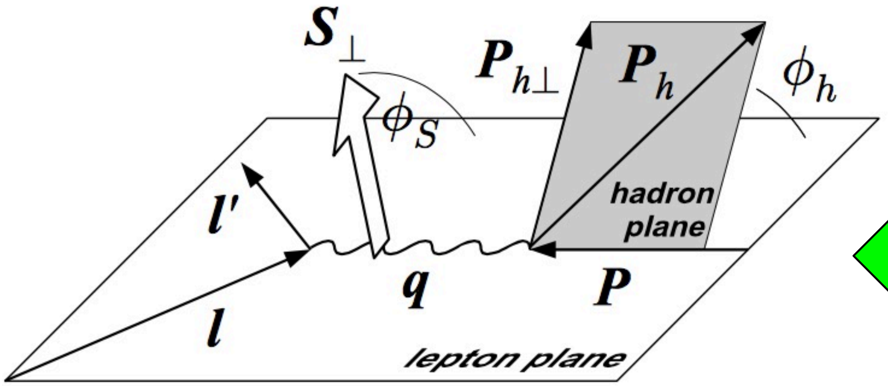


TMDs in CSS formalism

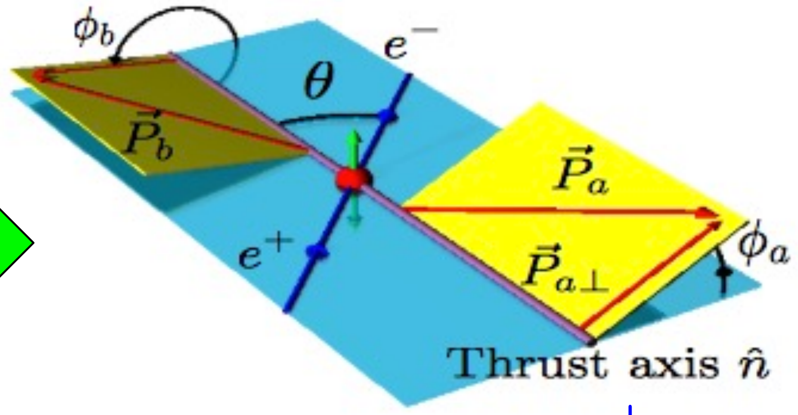
Kang, et al. (2016)



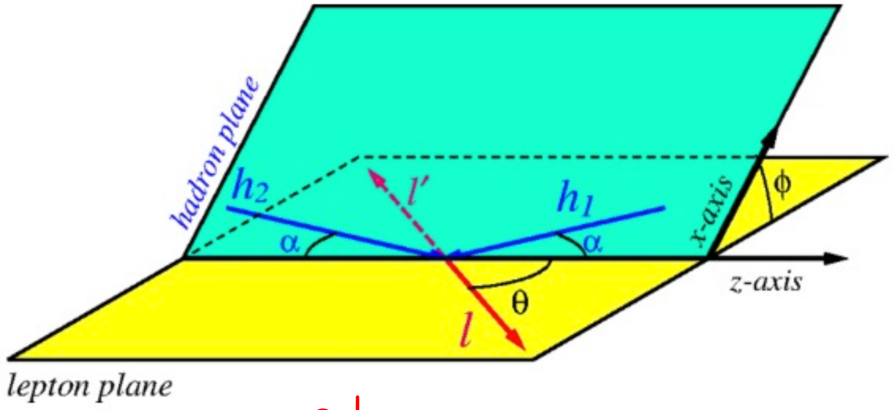




h_1, $f_{1T}^\perp$, $H_1^\perp$

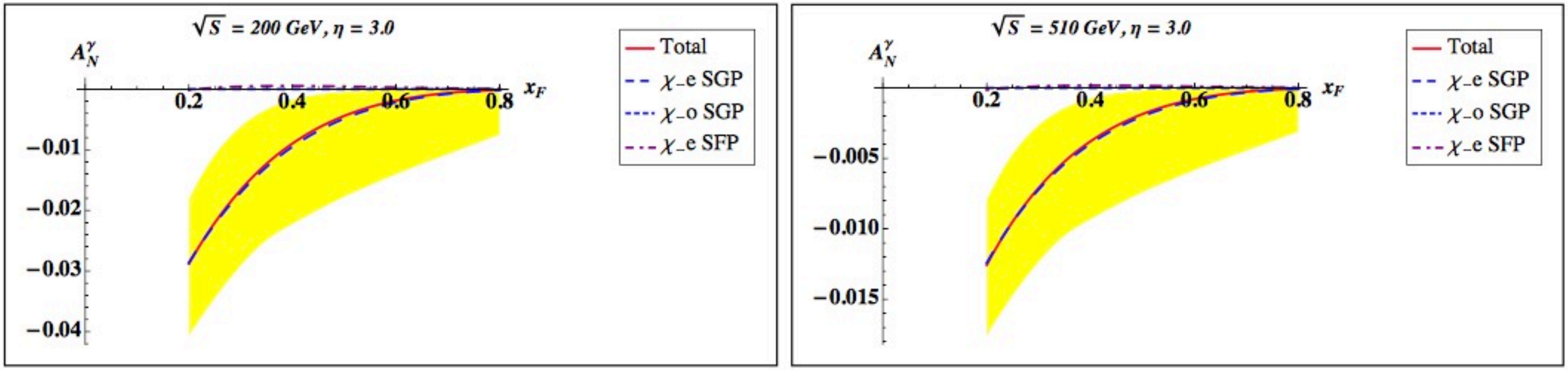


$H_1^\perp$



$f_{1T}^\perp$

A_N in pp -> γ X



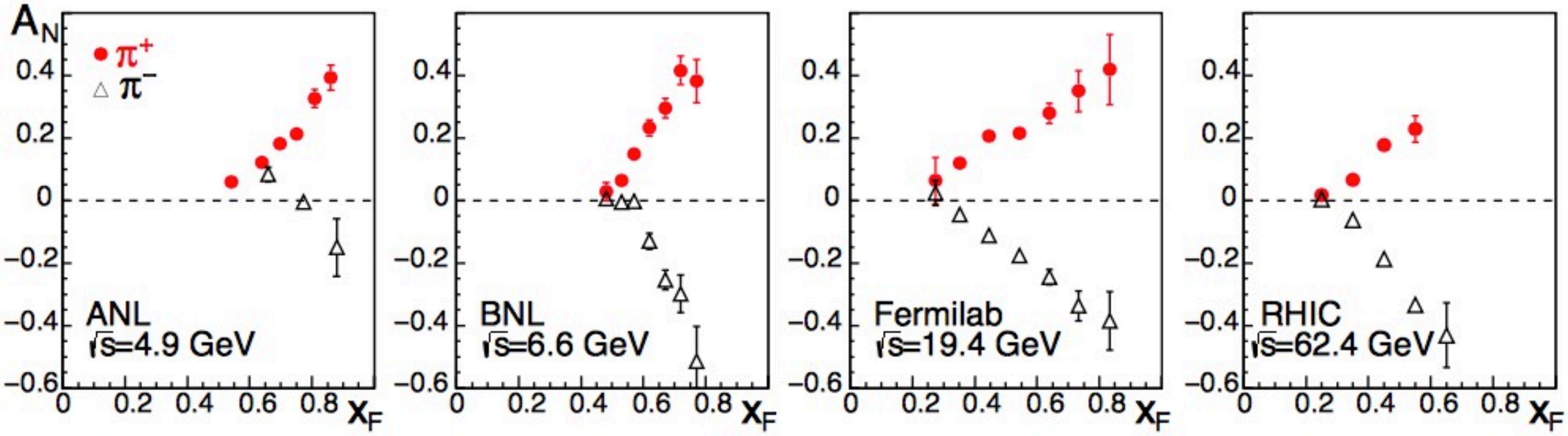
(Kanazawa, Koike, Metz, DP – PRD **91** (2015))
(See also Gamberg, Kang, Prokudin (2013))

Qiu-Sterman term is the main
cause of A_N in *pp* -> γ *X*

$$d\Delta\sigma^{\pi} \sim H \otimes f_1 \otimes \boldsymbol{F_{FT}(x,x)}$$

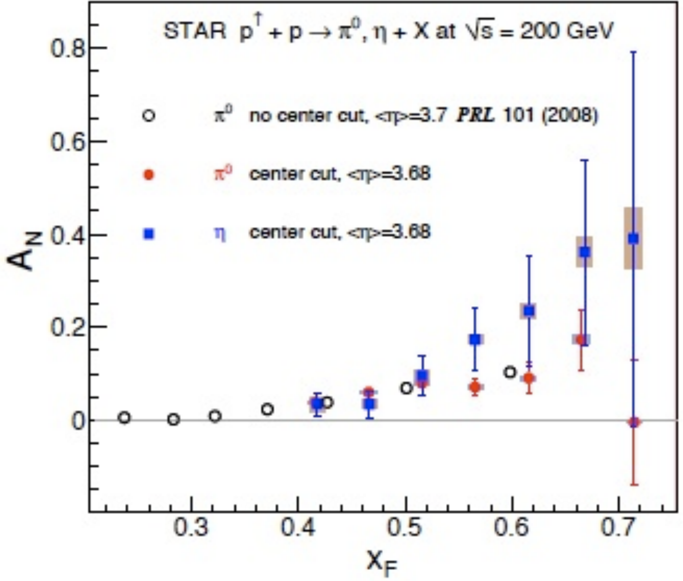
Qiu-Sterman function

A_N in *pp -> π X* – PUZZLE FOR 40+ YEARS!

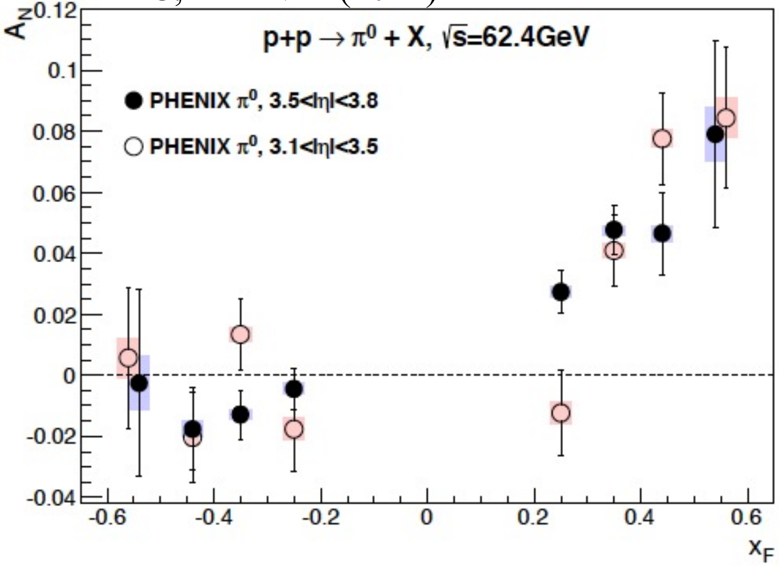


1976 →

RHIC, STAR (2012)



RHIC, PHENIX (2014)



$$d\Delta\sigma^{\pi} \sim H \otimes f_1 \otimes \boldsymbol{F_{FT}(x,x)}$$

$$E_\ell \frac{d^3\Delta\sigma(\vec{s}_T)}{d^3\ell} = \frac{\alpha_s^2}{S}\sum_{a,b,c}\int_{z_{\min}}^{1}\frac{dz}{z^2}D_{c\to h}(z)\int_{x'_{\min}}^{1}\frac{dx'}{x'}\frac{1}{x'S+T/z}\phi_{b/B}(x')$$
$$\times\sqrt{4\pi\alpha_s}\left(\frac{\epsilon^{\ell s_T n\bar{n}}}{z\hat{u}}\right)\frac{1}{x}\left[T_{a,F}(x,x)-x\left(\frac{d}{dx}T_{a,F}(x,x)\right)\right]H_{ab\to c}(\hat{s},\hat{t},\hat{u})$$

$$\boxed{F_{FT} \sim T_F}$$

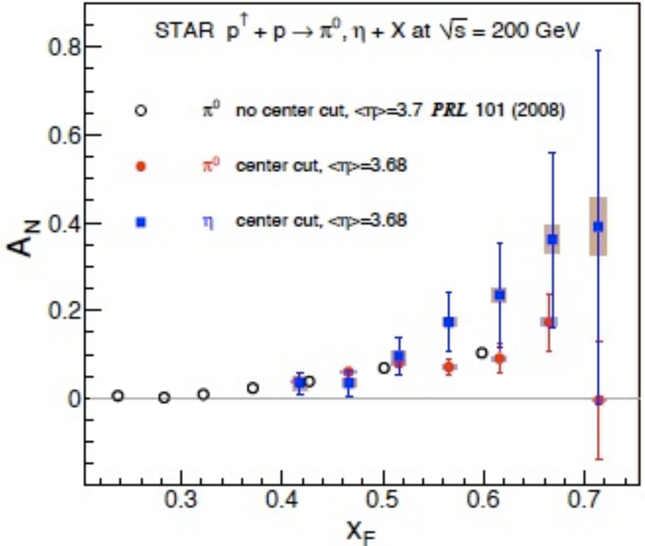
(Qiu and Sterman (1999), Kouvaris, et al. (2006))

For many years the Qiu-Sterman/Sivers-type contribution was thought to be the dominant source of TSSAs in $p^{\uparrow}p \longrightarrow \pi\ X$

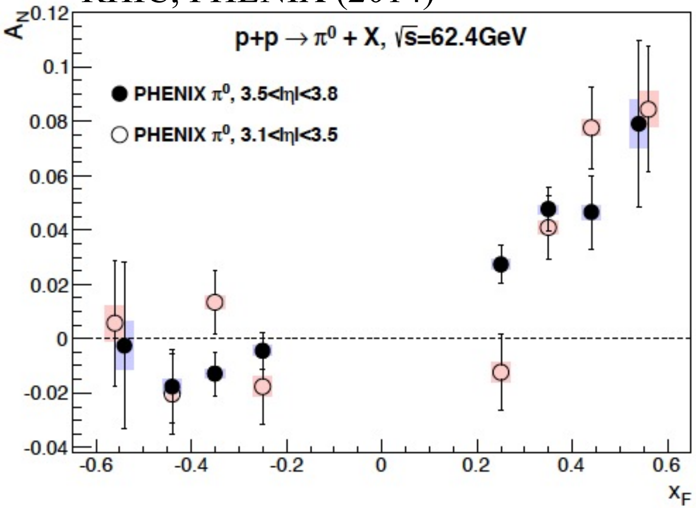
$$\boxed{p^{\uparrow}p \to h\,X}$$

$$\boxed{\ell N^{\uparrow} \to \ell'\,h\,X}$$

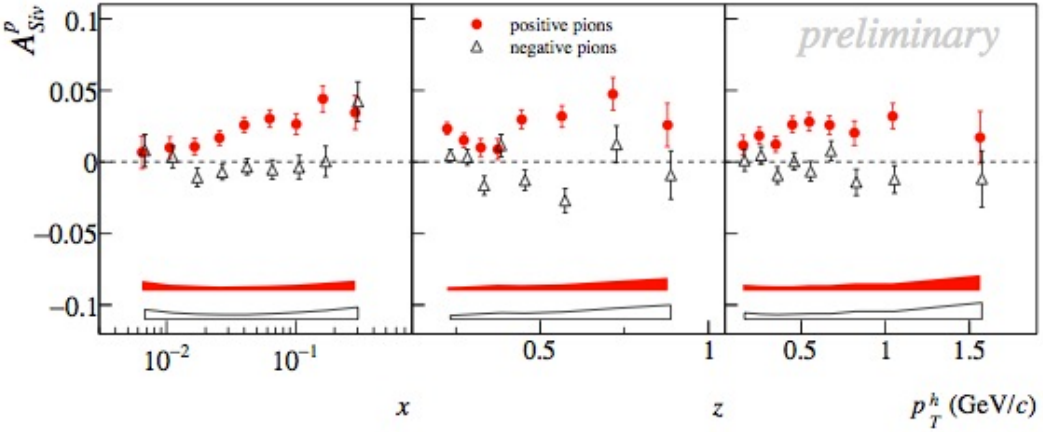
RHIC, STAR (2012)



RHIC, PHENIX (2014)



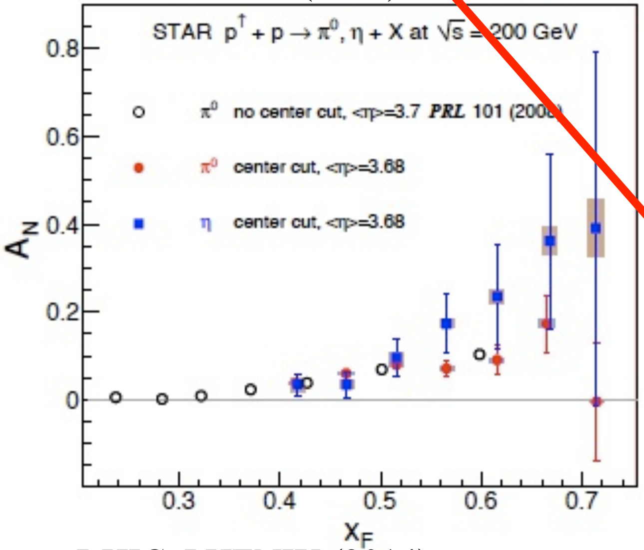
CERN, COMPASS (2013)



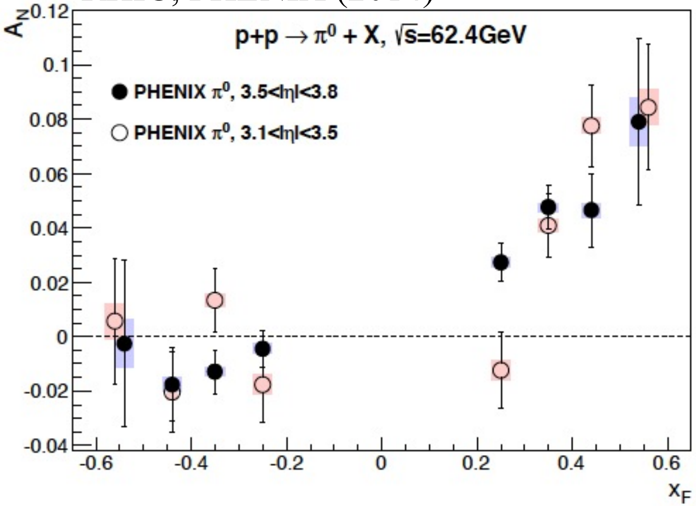
$$\pi\, F_{FT}(x,x) = f_{1T}^{\perp(1)}(x)$$

$$\boxed{p^{\uparrow} p \to h\, X}$$

RHIC, STAR (2012)

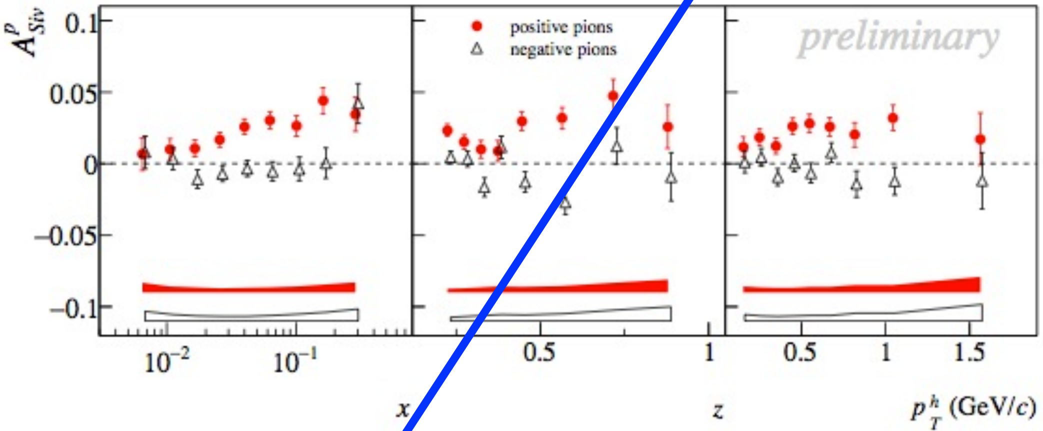


RHIC, PHENIX (2014)



$$\boxed{\ell N^{\uparrow} \to \ell'\, h\, X}$$

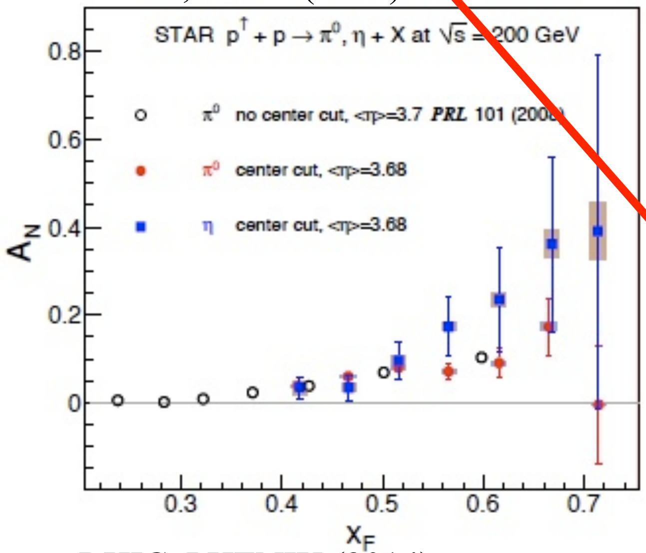
CERN, COMPASS (2013)



$$\pi\, F_{FT}(x,x) = f_{1T}^{\perp(1)}(x)$$

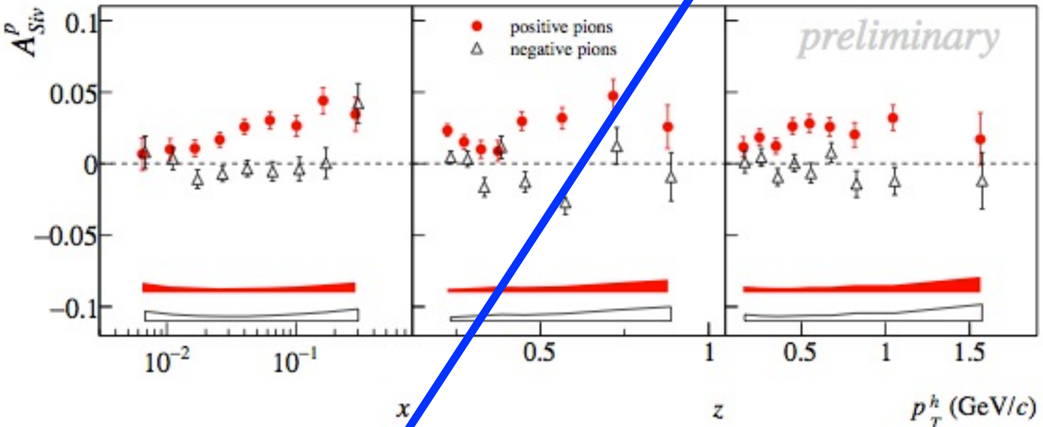
$$p^{\uparrow}p \to h\,X$$

RHIC, STAR (2012)



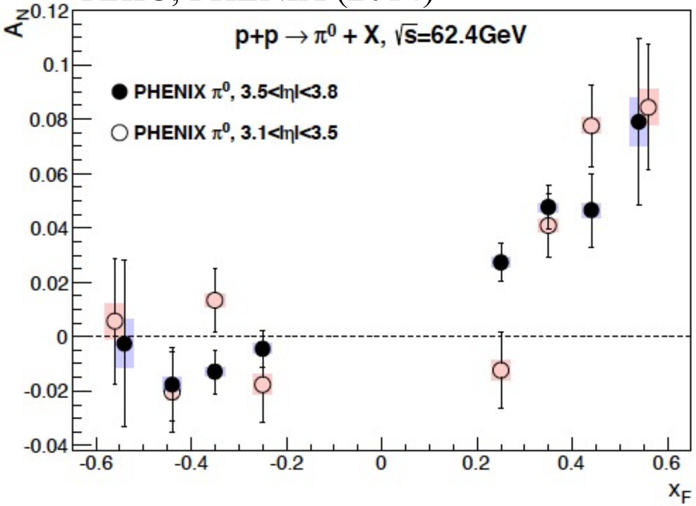
$$\ell N^{\uparrow} \to \ell'\,h\,X$$

CERN, COMPASS (2013)

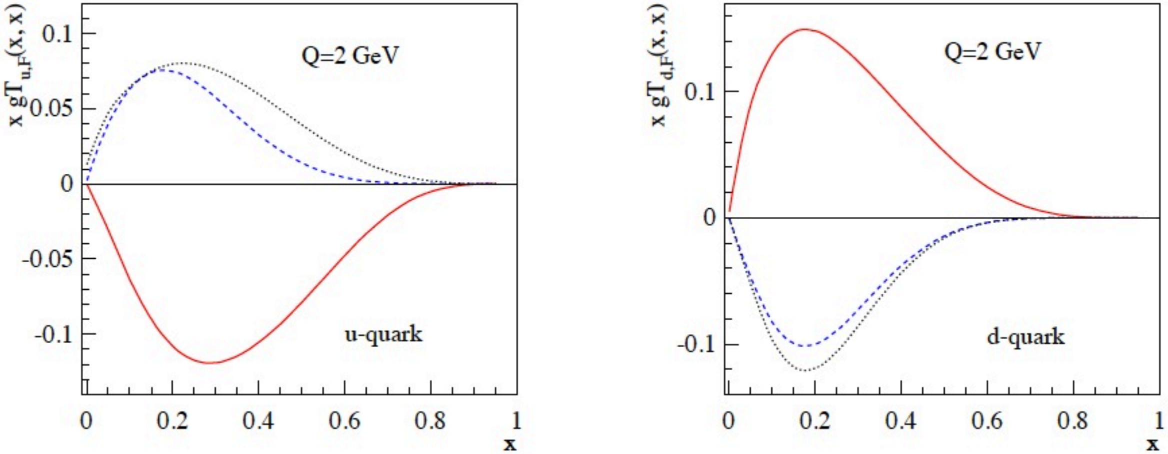


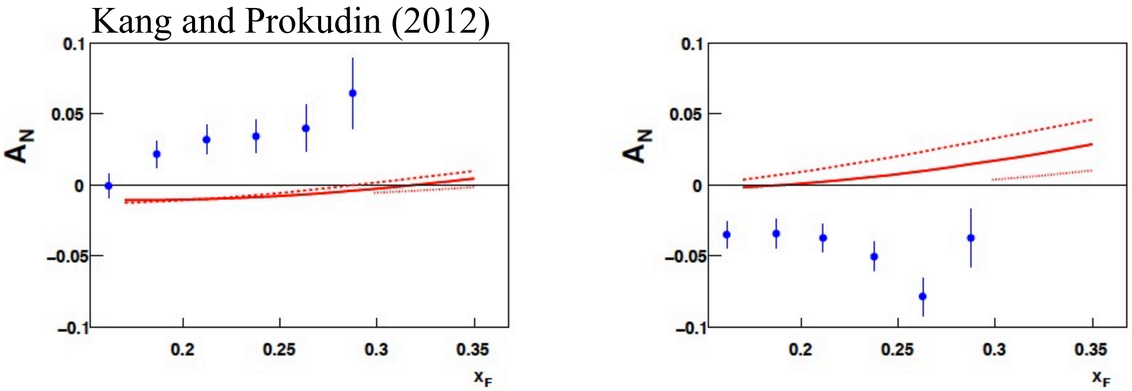
$$\pi\, F_{FT}(x,x) = f_{1T}^{\perp(1)}(x)$$

RHIC, PHENIX (2014)



“sign mismatch” (Kang, Qiu, Vogelsang, Yuan (2011))

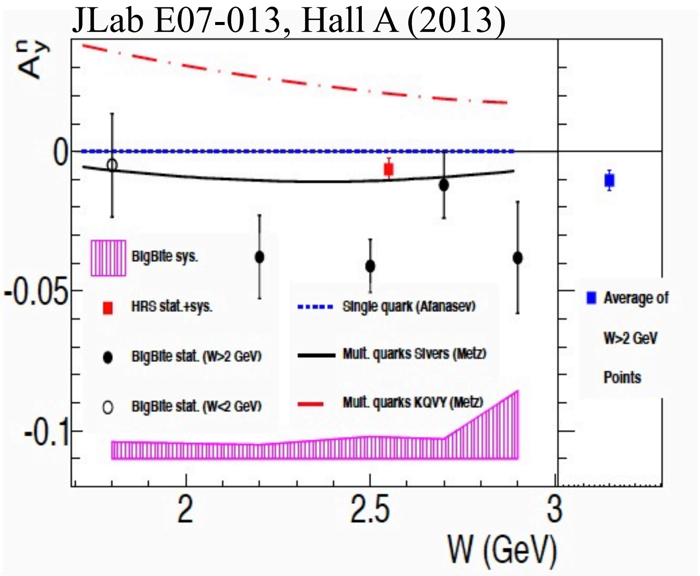




Kang and Prokudin (2012)

Proton-proton data from BRAHMS for π^+ (left) and π^- (right)

Nodes in Sivers cannot resolve issue



JLab E07-013, Hall A (2013)

Neutron TSSA in inclusive DIS

Metz, DP, Schäfer, Schlegel, Vogelsang, Zhou - PRD **86** (2012)

Sivers input agrees reasonably well with the JLab data ➡ FIRST INDICATION on the PROCESS DEPENDENENCE of the Sivers function (see also Gamberg, Kang, Prokudin (2013))

KQVY input gives the wrong sign ➡ Qiu-Sterman function cannot be the main cause of the large TSSAs seen in pion production from *pp* collisions

$$\cancel{d\Delta\sigma^{\pi} \sim H \otimes f_1 \otimes \boldsymbol{F_{FT}(x,x)}}$$

$$\cancel{d\Delta\sigma^{\pi} \sim H \otimes f_1 \otimes \boldsymbol{F_{FT}(x,x)}}$$

$$d\Delta\sigma^{\pi} \sim \boldsymbol{h_1} \otimes S \otimes \left(\boldsymbol{H_1^{\perp(1)}}, \boldsymbol{H}, \int \frac{dz_1}{z_1^2} \frac{\boldsymbol{\hat{H}_{FU}^{\Im}}}{(1/z - 1/z_1)^2} \right)$$

$$\begin{aligned} E_h \frac{d\Delta\sigma^{Frag}(S_T)}{d^3\vec{P}_h} = &- \frac{4\alpha_s^2 M_h}{S}\, \epsilon^{P'PP_hS_T} \sum_i \sum_{a,b,c} \int_0^1 \frac{dz}{z^3} \int_0^1 dx' \int_0^1 dx\ \delta(\hat{s}+\hat{t}+\hat{u}) \frac{1}{\hat{s}\,(-x'\hat{t}-x\hat{u})} \\ &\times\ h_1^a(x)\, f_1^b(x') \left\{ \left[H_1^{\perp(1),c}(z) - z\frac{dH_1^{\perp(1),c}(z)}{dz} \right] S_{H_1^\perp}^i + \frac{1}{z} H^c(z)\, S_H^i \right. \\ &\qquad \left. + \frac{2}{z} \int_z^\infty \frac{dz_1}{z_1^2} \frac{1}{\left(\frac{1}{z}-\frac{1}{z_1}\right)^2} \hat{H}_{FU}^{c,\Im}(z,z_1)\, S_{\hat{H}_{FU}}^i \right\} \end{aligned}$$

(Metz and DP - PLB **723** (2013))

$$d\Delta\sigma^{\pi} \sim \boldsymbol{h_1} \otimes S \otimes \left(\boldsymbol{H_1^{\perp(1)}}, \boldsymbol{H}, \int \frac{dz_1}{z_1^2} \frac{\boldsymbol{\hat{H}_{FU}^{\Im}}}{(1/z - 1/z_1)^2} \right)$$

$$H^q(z) \;=\; -2z\, H_1^{\perp(1),q}(z) + \boxed{2z \int_z^{\infty} \frac{dz_1}{z_1^2} \, \frac{1}{\frac{1}{z} - \frac{1}{z_1}} \, \hat{H}_{FU}^{q,\Im}(z, z_1)}$$

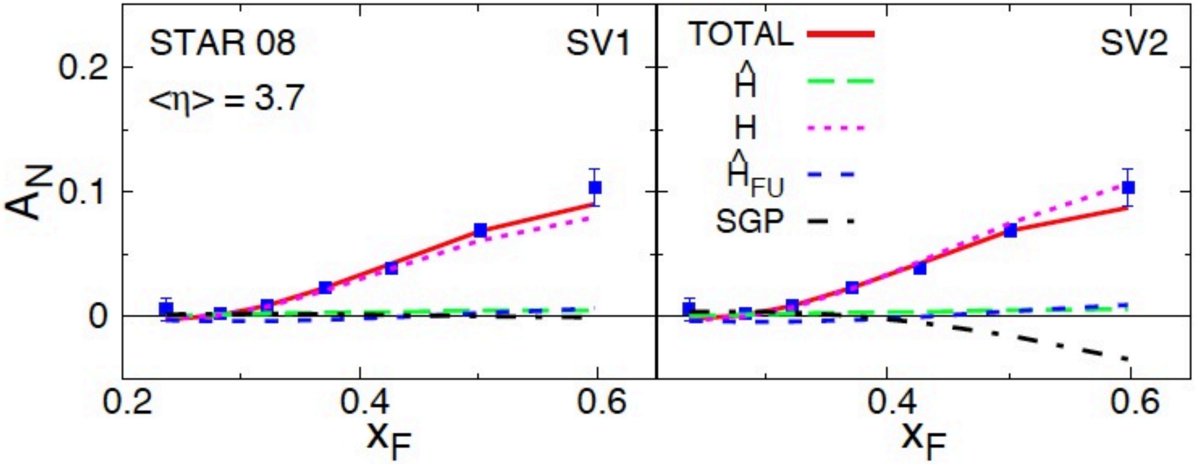
QCD e.o.m.
relation
(EOMR)

$$\longrightarrow \equiv \tilde{H}^q(z)$$

$$d\Delta\sigma^{\pi} \sim \boldsymbol{h_1} \otimes S \otimes \left(\boldsymbol{H_1^{\perp(1)}}, \boldsymbol{H}, \int \frac{dz_1}{z_1^2} \frac{\boldsymbol{\hat{H}_{FU}^{\Im}}}{(1/z - 1/z_1)^2} \right)$$

$$d\Delta\sigma^{\pi} \sim \boldsymbol{h_1} \otimes \hat{S} \otimes \left(\boldsymbol{H_1^{\perp(1)}}, \boldsymbol{\tilde{H}}, \int \frac{dz_1}{z_1^2} \frac{\boldsymbol{\hat{H}_{FU}^{\Im}}}{(1/z - 1/z_1)^2} \right)$$

Also included the Qiu-Sterman term $\pi\, \boldsymbol{F_{FT}(x,x)} = \boldsymbol{f_{1T}^{\perp(1)}(x)}$



(Kanazawa, Koike, Metz, DP, PRD **89**(RC) (2014))

Fragmentation term is the main cause of A_N in *pp* -> *π X*

$$d\Delta\sigma^{\pi} \sim \boldsymbol{h_1} \otimes \hat{S} \otimes \left(\boldsymbol{H_1^{\perp(1)}}, \boldsymbol{\tilde{H}}, \int \frac{dz_1}{z_1^2} \frac{\boldsymbol{\hat{H}_{FU}^{\Im}}}{(1/z - 1/z_1)^2} \right)$$

$$\frac{H^q(z)}{z} = -\left(1 - z\frac{d}{dz}\right) H_1^{\perp(1),q}(z) - \frac{2}{z}\int_z^{\infty} \frac{dz_1}{z_1^2} \frac{\hat{H}_{FU}^{q,\Im}(z,z_1)}{(1/z - 1/z_1)^2}$$

Lorentz invariance relation (LIR)

(Kanazawa, Koike, Metz, DP, Schlegel, PRD **93** (2016))

$$d\Delta\sigma^{\pi} \sim \boldsymbol{h_1} \otimes \hat{S} \otimes \left(\boldsymbol{H_1^{\perp(1)}}, \boldsymbol{\tilde{H}}, \int \frac{dz_1}{z_1^2} \frac{\boldsymbol{\hat{H}_{FU}^{\Im}}}{(1/z - 1/z_1)^2} \right)$$

⬇

$$d\Delta\sigma^{\pi} \sim \boldsymbol{h_1} \otimes \tilde{S} \otimes \left(\boldsymbol{H_1^{\perp(1)}}, \boldsymbol{\tilde{H}} \right)$$

$$E_h \frac{d\Delta\sigma^{Frag}(S_T)}{d^3\vec{P}_h} = -\frac{4\alpha_s^2 M_h}{S}\, \epsilon^{P'PP_hS_T} \sum_i \sum_{a,b,c} \int_0^1 \frac{dz}{z^3} \int_0^1 dx' \int_0^1 dx\; \delta(\hat{s}+\hat{t}+\hat{u}) \frac{1}{\hat{s}}$$

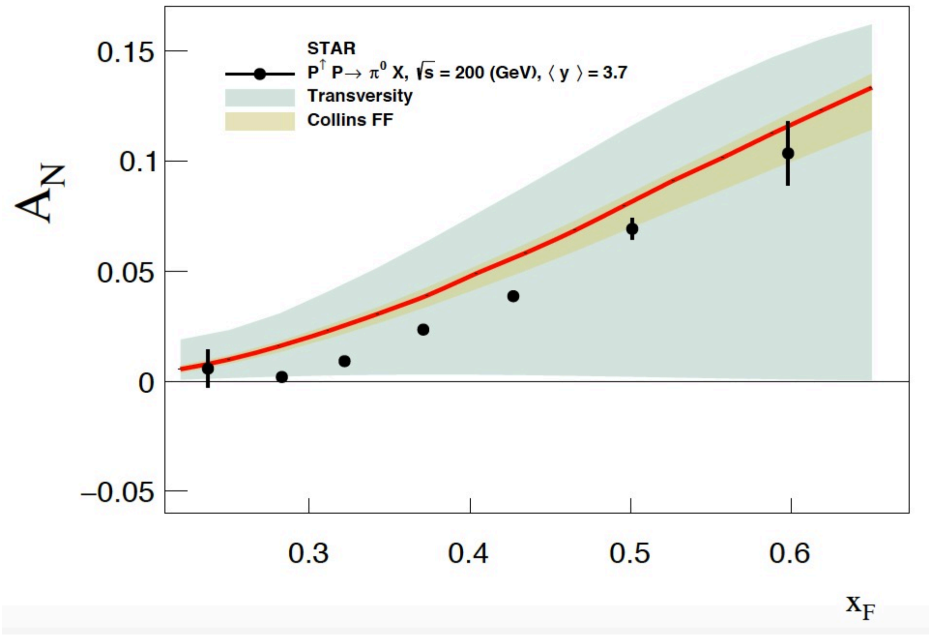
$$\times\; h_1^a(x)\, f_1^b(x') \left\{ \left[H_1^{\perp(1),c}(z) - z\frac{dH_1^{\perp(1),c}(z)}{dz} \right] \tilde{S}^i_{H_1^\perp} + \left[-2H_1^{\perp(1),c}(z) + \frac{1}{z}\tilde{H}^c(z) \right] \tilde{S}^i_H \right\}$$

where $\tilde{S}^i_{H_1^\perp} \equiv \dfrac{S^i_{H_1^\perp} - S^i_{H_{FU}}}{-x'\hat{t} - x\hat{u}}$ and $\tilde{S}^i_H \equiv \dfrac{S^i_H - S^i_{H_{FU}}}{-x'\hat{t} - x\hat{u}}$

(Gamberg, Kang, DP, Prokudin, PLB **770** (2017))

$$d\Delta\sigma^{\pi} \sim \boldsymbol{h_1} \otimes \hat{S} \otimes \left(\boldsymbol{H_1^{\perp(1)}}, \boldsymbol{\tilde{H}}, \int \frac{dz_1}{z_1^2} \frac{\boldsymbol{\hat{H}_{FU}^{\Im}}}{(1/z - 1/z_1)^2} \right)$$

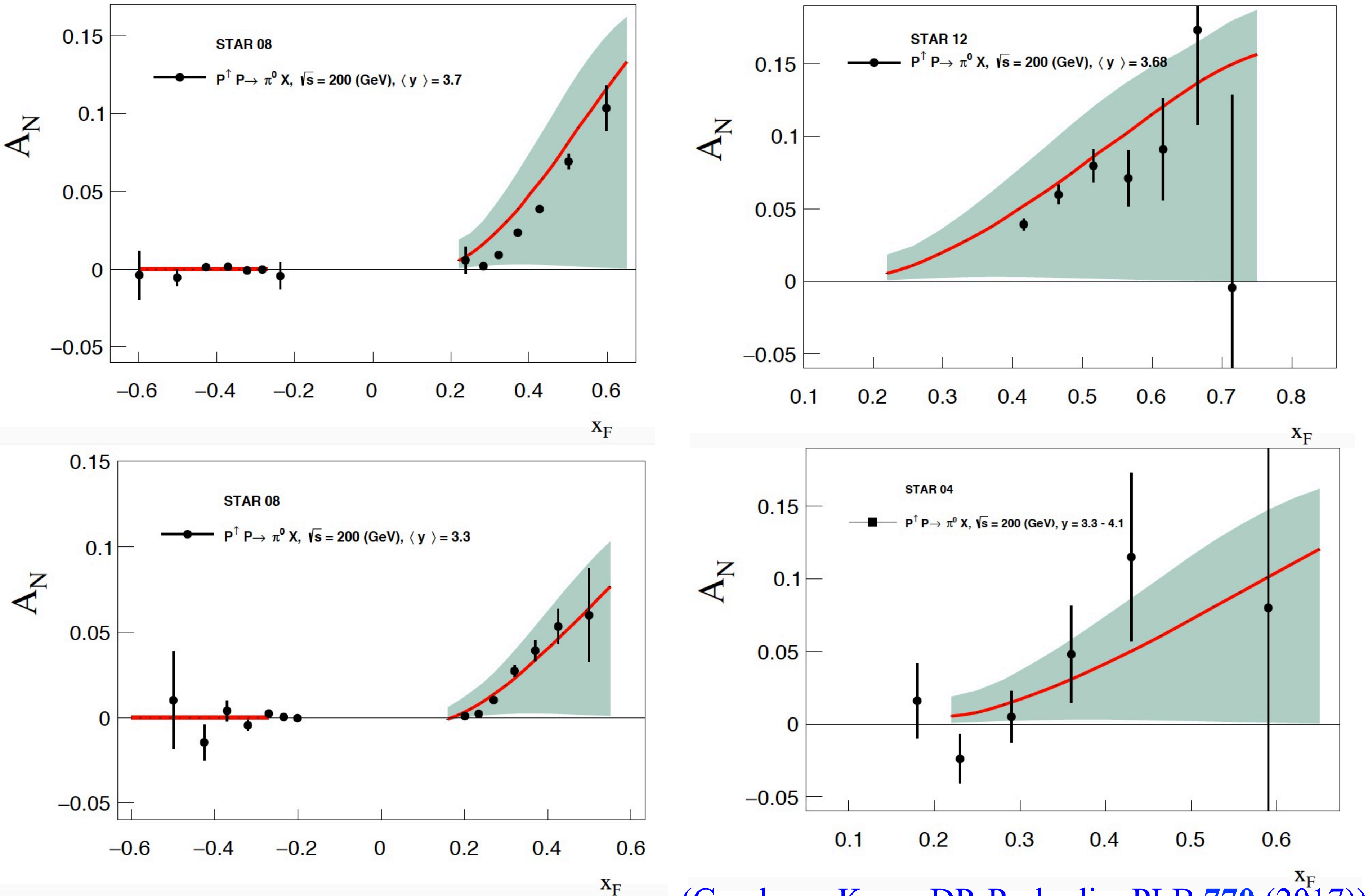
$$d\Delta\sigma^{\pi} \sim \boxed{\boldsymbol{h_1} \otimes \tilde{S} \otimes \Big(\boldsymbol{H_1^{\perp(1)}}}, \boldsymbol{\tilde{H}} \Big)$$



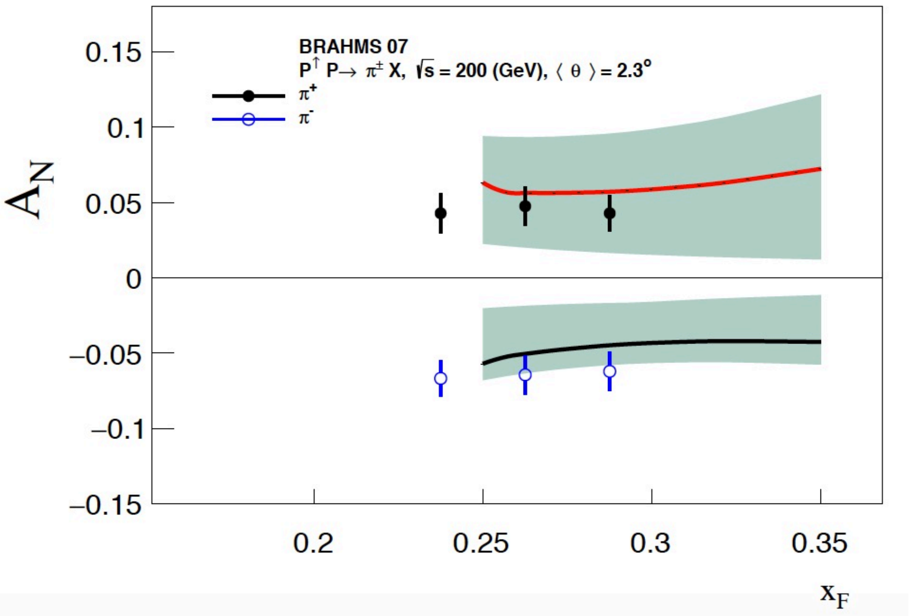
Fragmentation term is the main cause of A_N in *pp -> π X*

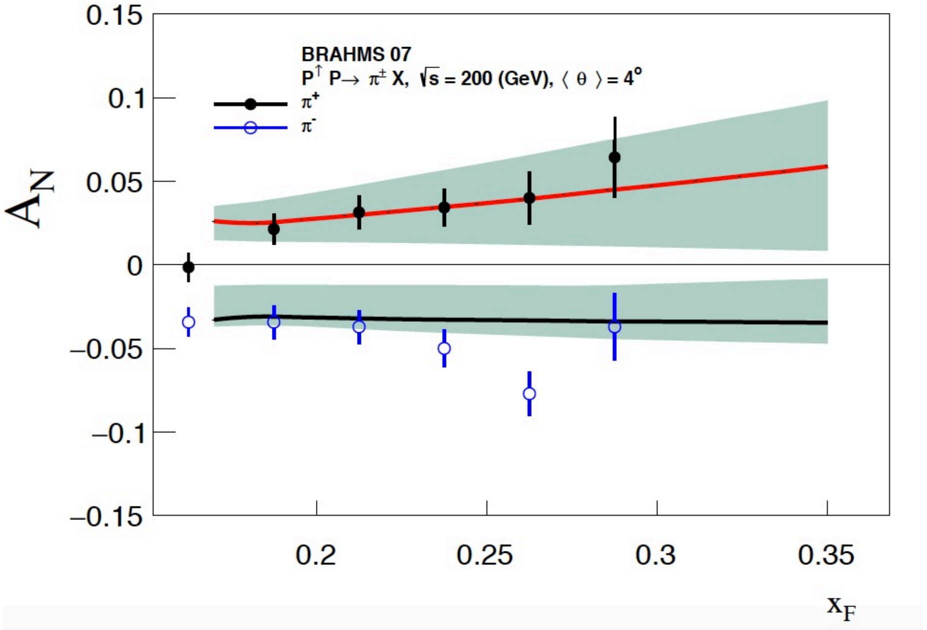
The A_N data from RHIC can be used along with measurements from SoLID to constrain ***transversity at large x***

(Gamberg, Kang, DP, Prokudin, PLB **770** (2017))

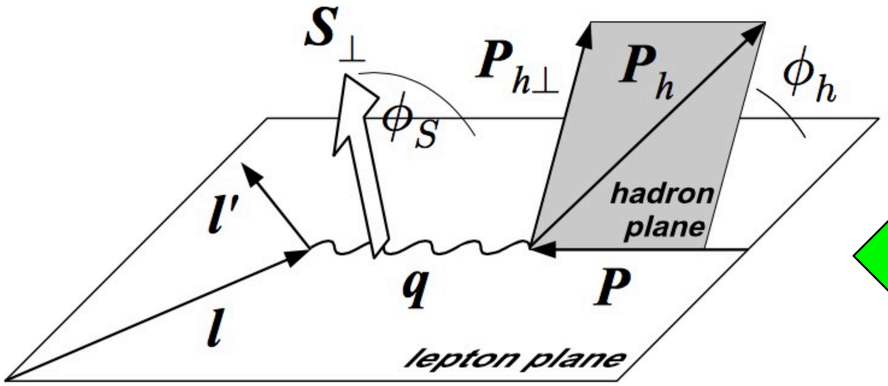


(Gamberg, Kang, DP, Prokudin, PLB **770** (2017))

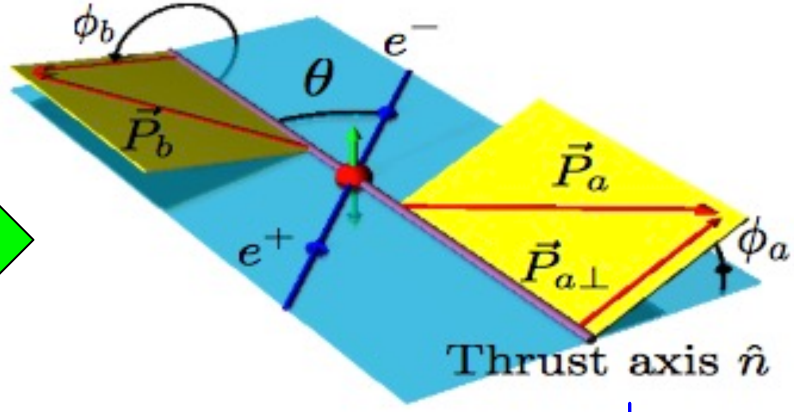




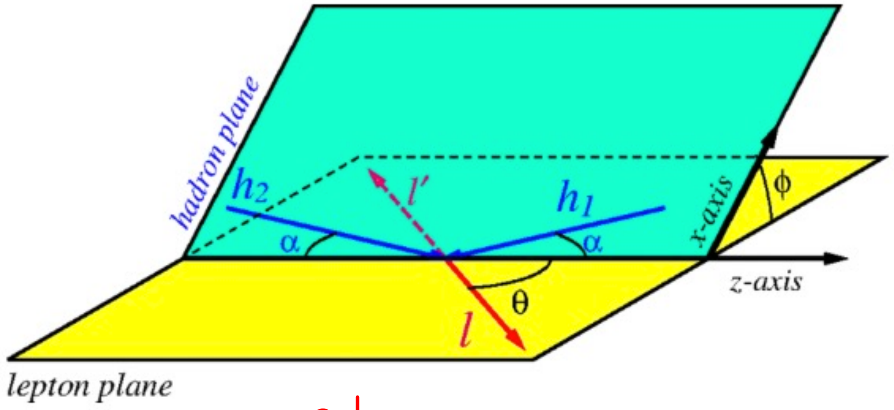
(Gamberg, Kang, DP, Prokudin, PLB **770** (2017))



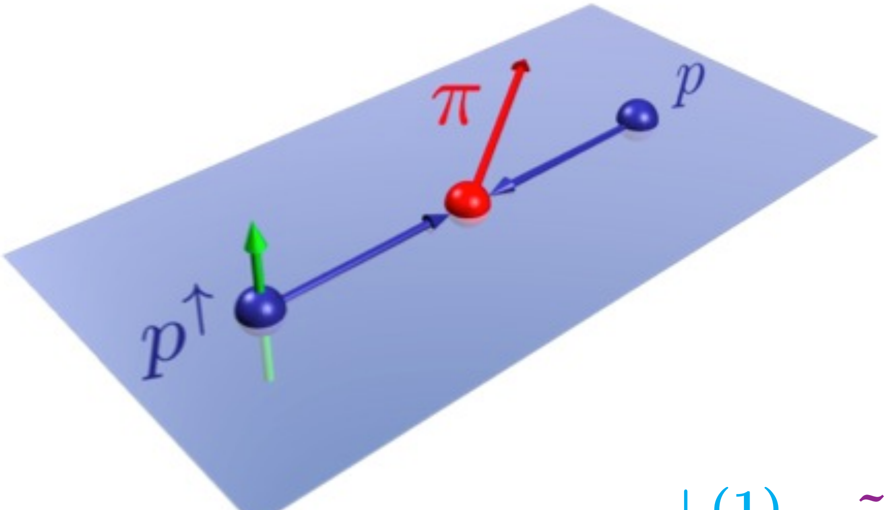
h_1, $f_{1T}^\perp$, $H_1^\perp$



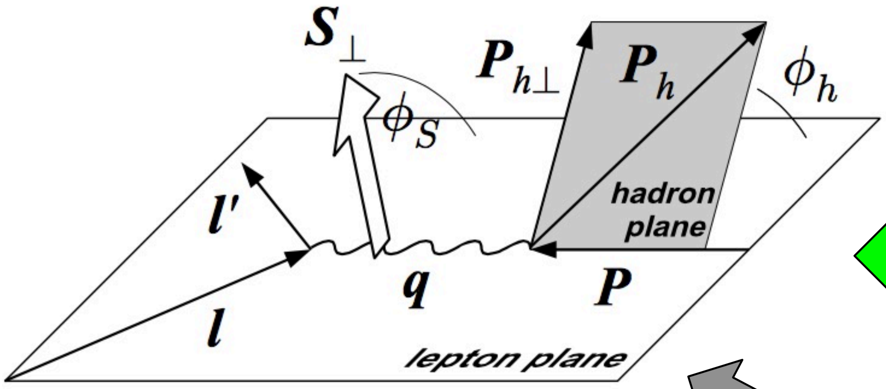
$H_1^\perp$



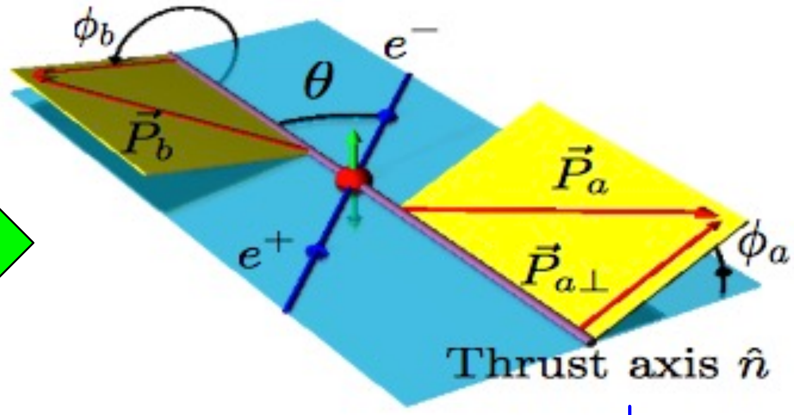
$f_{1T}^\perp$



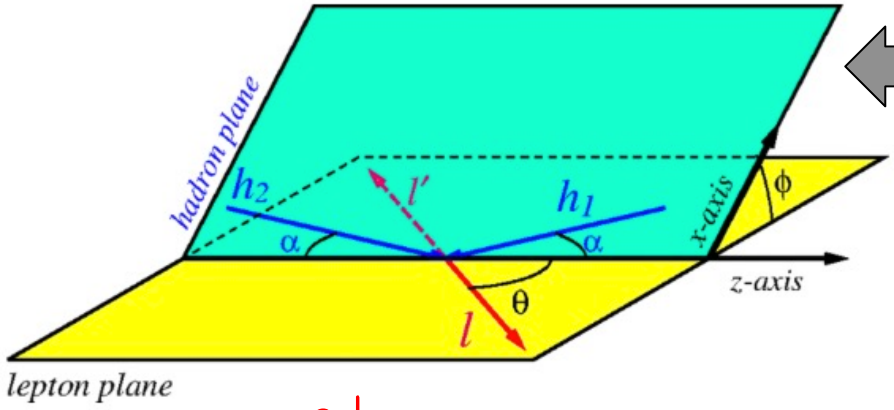
h_1, F_{FT}, $H_1^{\perp(1)}$, $\tilde{H}$



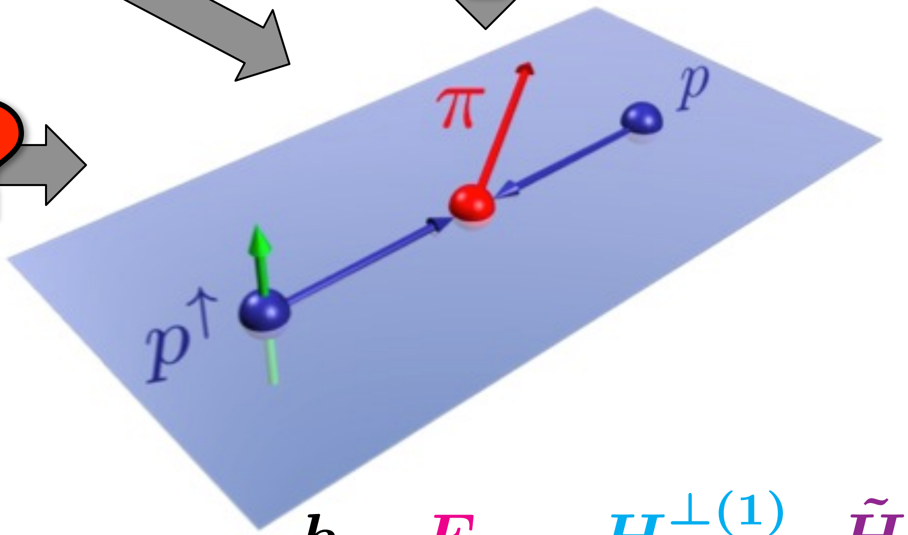
$h_1,\ f_{1T}^{\perp},\ H_1^{\perp}$



$H_1^{\perp}$



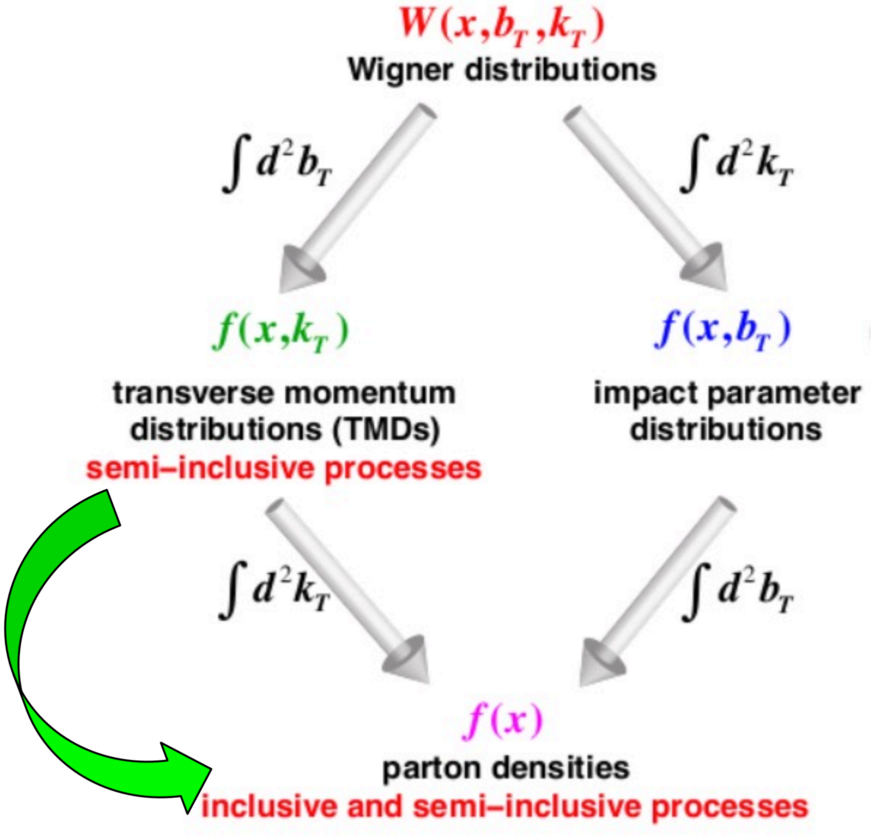
$f_{1T}^{\perp}$



$h_1,\ F_{FT},\ H_1^{\perp(1)},\ \tilde{H}$

Relations between TMD and CT3 Functions

Figure from EIC Whitepaper



$W(x, b_T, k_T)$

Wigner distributions

$\int d^2 b_T$ $\quad$ $\int d^2 k_T$

$f(x, k_T)$

transverse momentum distributions (TMDs)

semi–inclusive processes

$f(x, b_T)$

impact parameter distributions

$\int d^2 k_T$ $\quad$ $\int d^2 b_T$

$f(x)$

parton densities

inclusive and semi–inclusive processes

One naively expects that we can obtain collinear functions by integrating TMDs over k_T

“Original CSS” (Collins, Soper, Sterman (1985); Ji, Ma, Yuan (2005); Collins (2011); ...)

Takes into account “complications” of QCD (e.g., parton re-scattering and gluon radiation)

"Original CSS" (Collins, Soper, Sterman (1985); Ji, Ma, Yuan (2005); Collins (2011); ...)

"b-space" correlator

$$\tilde{\Phi}^{[\gamma^+]}(x,\vec{b}_T;Q^2,\mu_Q) = \boldsymbol{\tilde{f}_1(x,b_T;Q^2,\mu_Q)} - iM\epsilon^{ij}b_T^i S_T^j \left[\boldsymbol{-\frac{1}{M^2}\frac{1}{b_T}\frac{\partial}{\partial b_T}\tilde{f}_{1T}^{\perp}(x,b_T;Q^2,\mu_Q)} \right]$$

Boer, Gamberg, Musch, Prokudin (2011)

$$\equiv \boldsymbol{\tilde{f}_{1T}^{\perp(1)}(x,b_T;Q^2,\mu_Q)}$$

“Original CSS” (Collins, Soper, Sterman (1985); Ji, Ma, Yuan (2005); Collins (2011); ...)

“b-space” correlator

$$\tilde{\Phi}^{[\gamma^+]}(x,\vec{b}_T;Q^2,\mu_Q) = \boldsymbol{\tilde{f}_1(x,b_T;Q^2,\mu_Q)} - iM\epsilon^{ij}b_T^i S_T^j\left[\boldsymbol{-\frac{1}{M^2}\frac{1}{b_T}\frac{\partial}{\partial b_T}\tilde{f}_{1T}^{\perp}(x,b_T;Q^2,\mu_Q)}\right]$$

Boer, Gamberg, Musch, Prokudin (2011)

$$\equiv \boldsymbol{\tilde{f}_{1T}^{\perp(1)}(x,b_T;Q^2,\mu_Q)}$$

$$\boldsymbol{\tilde{f}_1(x,b_T;Q^2,\mu_Q)} \sim \left(\tilde{C}^{f_1}(x/\hat{x},b_*(b_T);\mu_{b_*}^2,\mu_{b_*},\alpha_s(\mu_{b_*}))\otimes \boldsymbol{f_1(\hat{x};\mu_{b_*})}\right)$$

Collins (2011); ...

$$\times\ \exp\left[-S_{pert}(b_*(b_T);\mu_{b_*},Q,\mu_Q) - S_{NP}^{f_1}(b_T,Q)\right]$$

“Original CSS” (Collins, Soper, Sterman (1985); Ji, Ma, Yuan (2005); Collins (2011); ...)

“b-space” correlator

$$\tilde{\Phi}^{[\gamma^+]}(x,\vec{b}_T;Q^2,\mu_Q) = \boldsymbol{\tilde{f}_1(x,b_T;Q^2,\mu_Q)} - iM\epsilon^{ij}b_T^i S_T^j\left[-\frac{\mathbf{1}}{\boldsymbol{M^2}}\frac{\mathbf{1}}{\boldsymbol{b_T}}\frac{\boldsymbol{\partial}}{\boldsymbol{\partial b_T}}\boldsymbol{\tilde{f}_{1T}^{\perp}(x,b_T;Q^2,\mu_Q)}\right]$$

Boer, Gamberg, Musch, Prokudin (2011)

$$\equiv \boldsymbol{\tilde{f}_{1T}^{\perp(1)}(x,b_T;Q^2,\mu_Q)}$$

$$\boldsymbol{\tilde{f}_1(x,b_T;Q^2,\mu_Q)} \sim \left(\tilde{C}^{f_1}(x/\hat{x},b_*(b_T);\mu_{b_*}^2,\mu_{b_*},\alpha_s(\mu_{b_*}))\otimes \boldsymbol{f_1(\hat{x};\mu_{b_*})}\right)$$
$$\times\ \exp\left[-S_{pert}(b_*(b_T);\mu_{b_*},Q,\mu_Q) - S_{NP}^{f_1}(b_T,Q)\right]$$

Collins (2011); ...

$$\boldsymbol{\tilde{f}_{1T}^{\perp(1)}(x,b_T;Q^2,\mu_Q)} \sim \left(\tilde{C}^{f_{1T}^{\perp}}(\hat{x}_1,\hat{x}_2,b_*(b_T);\mu_{b_*}^2,\mu_{b_*},\alpha_s(\mu_{b_*}))\otimes \boldsymbol{F_{FT}(\hat{x}_1,\hat{x}_2;\mu_{b_*})}\right)$$
$$\times\ \exp\left[-S_{pert}(b_*(b_T);\mu_{b_*},Q,\mu_Q) - S_{NP}^{f_{1T}^{\perp}}(b_T,Q)\right]$$

Aybat, Collins, Qiu, Rogers (2012); Echevarria, Idilbi, Kang, Vitev (2014); ...

“Original CSS” (Collins, Soper, Sterman (1985); Ji, Ma, Yuan (2005); Collins (2011); ...)

$$\boldsymbol{\tilde{f}_1(x, b_T; Q^2, \mu_Q)} \sim \left(\tilde{C}^{f_1}(x/\hat{x}, b_*(b_T); \mu_{b_*}^2, \mu_{b_*}, \alpha_s(\mu_{b_*})) \otimes \boldsymbol{f_1(\hat{x}; \mu_{b_*})}\right)$$

$$\times\ \exp\left[-S_{pert}(b_*(b_T); \mu_{b_*}, Q, \mu_Q) - S_{NP}^{f_1}(b_T, Q)\right]$$

<ins>perturbative Sudakov factor</ins>

$$\underbrace{-\ln(Q/\mu_{b_*})\tilde{K}(b_*, \mu_{b_*}) - \int_{\mu_{b_*}}^{\mu_Q} \frac{d\mu'}{\mu'}\left[\gamma(\alpha_s(\mu'); 1) - \gamma_K(\alpha_s(\mu')) \ln(Q/\mu')\right]}_{\text{same for unpol. and pol.}}$$

<ins>non-perturbative Sudakov factor</ins>

$$g_{f_1}(x, b_T) + g_K(b_T) \ln(Q/Q_0)$$

different for each TMD

universal

"Original CSS" (Collins, Soper, Sterman (1985); Ji, Ma, Yuan (2005); Collins (2011); ...)

$$\boldsymbol{\tilde{f}_1(x, b_T; Q^2, \mu_Q)} \sim \left(\tilde{C}^{f_1}(x/\hat{x}, b_*(b_T); \mu_{b_*}^2, \mu_{b_*}, \alpha_s(\mu_{b_*})) \otimes \boldsymbol{f_1(\hat{x}; \mu_{b_*})}\right)$$

$$\times \exp\left[-S_{pert}(b_*(b_T); \mu_{b_*}, Q, \mu_Q) - S_{NP}^{f_1}(b_T, Q)\right]$$

perturbative Sudakov factor

$$\underbrace{-\ln(Q/\mu_{b_*})\tilde{K}(b_*, \mu_{b_*}) - \int_{\mu_{b_*}}^{\mu_Q} \frac{d\mu'}{\mu'}\left[\gamma(\alpha_s(\mu'); 1) - \gamma_K(\alpha_s(\mu'))\ln(Q/\mu')\right]}_{\text{same for unpol. and pol.}}$$

non-perturbative Sudakov factor

$$g_{f_1}(x, b_T) + g_K(b_T)\ln(Q/Q_0)$$

different for each TMD

universal

$$b_*(b_T) \equiv \sqrt{\frac{b_T^2}{1 + b_T^2/b_{\max}^2}} \qquad \mu_{b_*} = C_1/b_*(b_T)$$

"Original CSS" (Collins, Soper, Sterman (1985); Ji, Ma, Yuan (2005); Collins (2011); ...)

$$\boldsymbol{\tilde{f}_1(x, b_T; Q^2, \mu_Q)} \quad \sim \quad \left(\tilde{C}^{f_1}(x/\hat{x}, b_*(b_T); \mu_{b_*}^2, \mu_{b_*}, \alpha_s(\mu_{b_*})) \otimes \boldsymbol{f_1(\hat{x}; \mu_{b_*})}\right)$$

$$\times \ \exp\left[-S_{pert}(b_*(b_T); \mu_{b_*}, Q, \mu_Q) - S_{NP}^{f_1}(b_T, Q)\right]$$

perturbative Sudakov factor

$$\underbrace{-\ln(Q/\mu_{b_*})\tilde{K}(b_*, \mu_{b_*}) - \int_{\mu_{b_*}}^{\mu_Q} \frac{d\mu'}{\mu'}\left[\gamma(\alpha_s(\mu'); 1) - \gamma_K(\alpha_s(\mu'))\ln(Q/\mu')\right]}_{\text{same for unpol. and pol.}}$$

non-perturbative Sudakov factor

$$g_{f_1}(x, b_T) + g_K(b_T)\ln(Q/Q_0)$$

g_{f_1}: different for each TMD

g_K: universal

$$b_*(b_T) \equiv \sqrt{\frac{b_T^2}{1 + b_T^2/b_{\max}^2}} \qquad \mu_{b_*} = C_1/b_*(b_T)$$

Note: $\boldsymbol{b_*(0) = 0 \text{ and } (\mu_{b_*})_{b_* \to 0} = \infty}$ ➡ problematic large logarithms in S_{pert}

(Bozzi, Catani, de Florian, Grazzini (2006); Collins, Gamberg, Prokudin, Rogers, Sato, Wang (2016))

"Original CSS" (Collins, Soper, Sterman (1985); Ji, Ma, Yuan (2005); Collins (2011); ...)

$$\tilde{f}_1(x, b_T; Q^2, \mu_Q) \sim \left(\tilde{C}^{f_1}(x/\hat{x}, b_*(b_T); \mu_{b_*}^2, \mu_{b_*}, \alpha_s(\mu_{b_*})) \otimes f_1(\hat{x}; \mu_{b_*})\right) \times \exp\left[-S_{pert}(b_*(b_T); \mu_{b_*}, Q, \mu_Q) - S_{NP}^{f_1}(b_T, Q)\right]$$

<u>perturbative Sudakov factor</u>

$$\underbrace{-\ln(Q/\mu_{b_*})\tilde{K}(b_*, \mu_{b_*}) - \int_{\mu_{b_*}}^{\mu_Q} \frac{d\mu'}{\mu'}\left[\gamma(\alpha_s(\mu'); 1) - \gamma_K(\alpha_s(\mu'))\ln(Q/\mu')\right]}_{\text{same for unpol. and pol.}}$$

<u>non-perturbative Sudakov factor</u>

$$g_{f_1}(x, b_T) + g_K(b_T)\ln(Q/Q_0)$$

g_{f_1}: different for each TMD

g_K: universal

$$b_*(b_T) \equiv \sqrt{\frac{b_T^2}{1 + b_T^2/b_{\max}^2}} \qquad \mu_{b_*} = C_1/b_*(b_T)$$

<u>**Note**</u>: $b_*(0) = 0$ and $(\mu_{b_*})_{b_* \to 0} = \infty$ ➡ problematic large logarithms in S_{pert}

(Bozzi, Catani, de Florian, Grazzini (2006); Collins, Gamberg, Prokudin, Rogers, Sato, Wang (2016))

“Original CSS” (Collins, Soper, Sterman (1985); Ji, Ma, Yuan (2005); Collins (2011); ...)

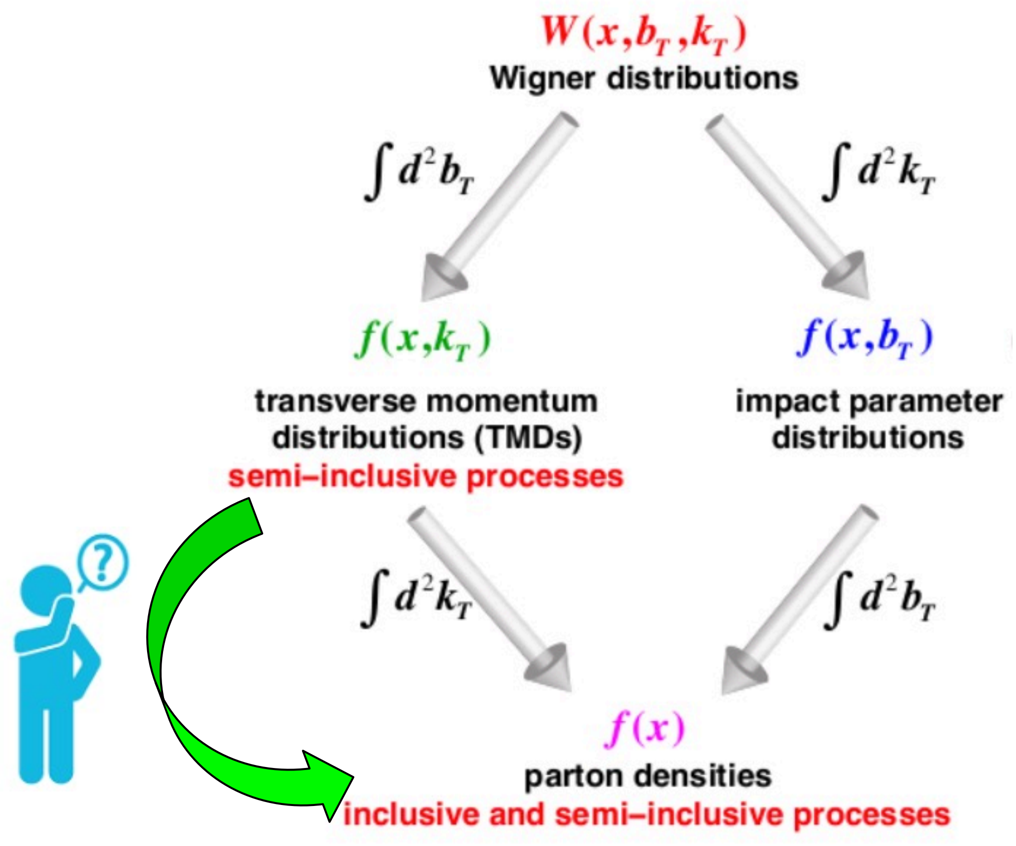
$$\int d^2k_T\, f_1(x, k_T; Q^2, \mu_Q) = \boldsymbol{\tilde{f}_1(x, b_T \to 0; Q^2, \mu_Q) = 0\,!}$$

(Collins, Gamberg, Prokudin, Rogers, Sato, Wang (2016))

$$\int d^2k_T\, \frac{k_T^2}{2M^2}\, f_{1T}^{\perp}(x, k_T; Q^2, \mu_Q) \equiv f_{1T}^{\perp(1)}(x; Q^2, \mu_Q) = \boldsymbol{\tilde{f}_{1T}^{\perp(1)}(x, b_T \to 0; Q^2, \mu_Q) = 0\,!}$$

(Gamberg, Metz, DP, Prokudin, to appear soon)

Figure from EIC Whitepaper



"Original CSS" (Collins, Soper, Sterman (1985); Ji, Ma, Yuan (2005); Collins (2011); ...)

$$\int d^2k_T\, f_1(x, k_T; Q^2, \mu_Q) = \boldsymbol{\tilde{f}_1(x, b_T \to 0; Q^2, \mu_Q)} = \mathbf{0}\,!$$

(Collins, Gamberg, Prokudin, Rogers, Sato, Wang (2016))

$$\int d^2k_T\, \frac{k_T^2}{2M^2}\, f_{1T}^{\perp}(x, k_T; Q^2, \mu_Q) \equiv f_{1T}^{\perp(1)}(x; Q^2, \mu_Q) = \boldsymbol{\tilde{f}_{1T}^{\perp(1)}(x, b_T \to 0; Q^2, \mu_Q)} = \mathbf{0}\,!$$

(Gamberg, Metz, DP, Prokudin, to appear soon)

TMDs lose their physical interpretation in the "Original CSS" formalism!

“Original CSS” (Collins, Soper, Sterman (1985); Ji, Ma, Yuan (2005); Collins (2011); ...)

$$\int d^2k_T\, f_1(x,k_T;Q^2,\mu_Q) = \boldsymbol{\tilde{f}_1(x,b_T\to 0;Q^2,\mu_Q)} = \mathbf{0\,!}$$

(Collins, Gamberg, Prokudin, Rogers, Sato, Wang (2016))

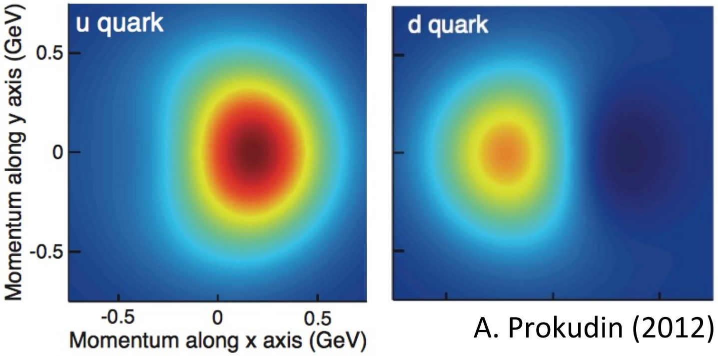
$$\int d^2k_T\, \frac{k_T^2}{2M^2}\, f_{1T}^{\perp}(x,k_T;Q^2,\mu_Q) \equiv f_{1T}^{\perp(1)}(x;Q^2,\mu_Q) = \boldsymbol{\tilde{f}_{1T}^{\perp(1)}(x,b_T\to 0;Q^2,\mu_Q)} = \mathbf{0\,!}$$

(Gamberg, Metz, DP, Prokudin, to appear soon)

TMDs lose their physical interpretation in the “Original CSS” formalism!

$$\langle k_T^i(x)\rangle_{UT} = \int d^2k_T\, k_T^i\left(-\frac{\vec{k}_T\times\vec{S}_T}{M}f_{1T}^{\perp}(x,k_T)\right)$$

avg. TM of unpolarized quarks in a transversely polarized spin-1/2 target



A. Prokudin (2012)

"Original CSS" (Collins, Soper, Sterman (1985); Ji, Ma, Yuan (2005); Collins (2011); ...)

$$\int d^2k_T\, f_1(x, k_T; Q^2, \mu_Q) = \boldsymbol{\tilde{f}_1(x, b_T \to 0; Q^2, \mu_Q) = 0\,!}$$

(Collins, Gamberg, Prokudin, Rogers, Sato, Wang (2016))

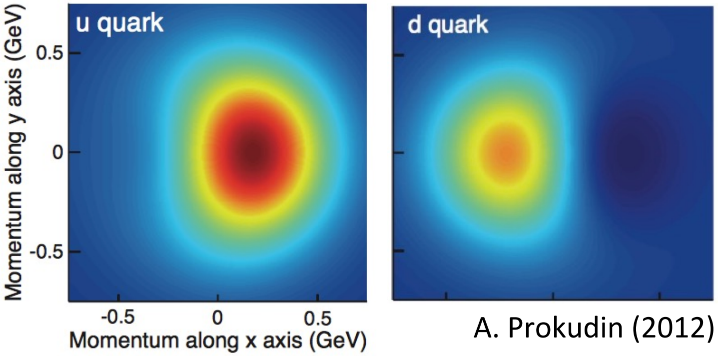
$$\int d^2k_T\, \frac{k_T^2}{2M^2} \boxed{f_{1T}^{\perp}(x, k_T; Q^2, \mu_Q)} \equiv f_{1T}^{\perp(1)}(x; Q^2, \mu_Q) = \boldsymbol{\tilde{f}_{1T}^{\perp(1)}(x, b_T \to 0; Q^2, \mu_Q) = 0\,!}$$

(Gamberg, Metz, DP, Prokudin, to appear soon)

TMDs lose their physical interpretation in the "Original CSS" formalism!

$$\langle k_T^i(x) \rangle_{UT} = \int d^2k_T\, k_T^i \left(-\frac{\vec{k}_T \times \vec{S}_T}{M} f_{1T}^{\perp}(x, k_T) \right)$$

avg. TM of unpolarized quarks in a transversely polarized spin-1/2 target



A. Prokudin (2012)

"Improved CSS" (Unpolarized) (Collins, Gamberg, Prokudin, Rogers, Sato, Wang (2016))*

Place a lower cut-off on b_T: $b_T \to b_c(b_T)$ where $b_c(b_T) = \sqrt{b_T^2 + b_0^2/(C_5 Q)^2}$

➡ $\mu_{b_*} \to \bar{\mu} \equiv \dfrac{C_1}{b_*(b_c(b_T))}$ so μ_{b_*} is cut off at $\mu_c \approx \dfrac{C_1 C_5 Q}{b_0}$

*Other modifications are discussed in this reference that attempt to improve the agreement of the CSS W+Y formulation with the differential cross section over all transverse momentum regions.

"Improved CSS" (Unpolarized) (Collins, Gamberg, Prokudin, Rogers, Sato, Wang (2016))

Place a lower cut-off on b_T: $b_T \to b_c(b_T)$ where $b_c(b_T) = \sqrt{b_T^2 + b_0^2/(C_5 Q)^2}$

➡ $\mu_{b_*} \to \bar{\mu} \equiv \dfrac{C_1}{b_*(b_c(b_T))}$ so μ_{b_*} is cut off at $\mu_c \approx \dfrac{C_1 C_5 Q}{b_0}$

$$\boldsymbol{\tilde{f}_1(x, b_c(b_T); Q^2, \mu_Q)} \quad \sim \quad \left(\tilde{C}^{f_1}(x/\hat{x}, b_*(b_c(b_T)); \bar{\mu}^2, \bar{\mu}, \alpha_s(\bar{\mu})) \otimes \boldsymbol{f_1(\hat{x}; \bar{\mu})}\right)$$

$$\times \; \exp\left[-S_{pert}(b_*(b_c(b_T)); \bar{\mu}, Q, \mu_Q) - S_{NP}^{f_1}(b_c(b_T), Q)\right]$$

"Improved CSS" (Unpolarized) (Collins, Gamberg, Prokudin, Rogers, Sato, Wang (2016))

Place a lower cut-off on b_T: $b_T \to b_c(b_T)$ where $b_c(b_T) = \sqrt{b_T^2 + b_0^2/(C_5 Q)^2}$

➡ $\mu_{b_*} \to \bar{\mu} \equiv \dfrac{C_1}{b_*(b_c(b_T))}$ so μ_{b_*} is cut off at $\mu_c \approx \dfrac{C_1 C_5 Q}{b_0}$

$$\boldsymbol{\tilde{f}_1(x, b_c(b_T); Q^2, \mu_Q)} \sim \left(\tilde{C}^{f_1}(x/\hat{x}, b_*(b_c(b_T)); \bar{\mu}^2, \bar{\mu}, \alpha_s(\bar{\mu})) \otimes \boldsymbol{f_1(\hat{x}; \bar{\mu})}\right)$$
$$\times \exp\left[-S_{pert}(b_*(b_c(b_T)); \bar{\mu}, Q, \mu_Q) - S_{NP}^{f_1}(b_c(b_T), Q)\right]$$

"Improved CSS" (Polarized) (Gamberg, Metz, DP, Prokudin, to appear soon)

$$\tilde{\Phi}^{[\gamma^+]}(x, \vec{b}_T; Q^2, \mu_Q) = \boldsymbol{\tilde{f}_1(x, b_T; Q^2, \mu_Q)} - iM\epsilon^{ij} b_T^i S_T^j \boldsymbol{\tilde{f}_{1T}^{\perp(1)}(x, b_T; Q^2, \mu_Q)}$$

"Improved CSS" (Unpolarized) (Collins, Gamberg, Prokudin, Rogers, Sato, Wang (2016))

Place a lower cut-off on b_T: $b_T \to b_c(b_T)$ where $b_c(b_T) = \sqrt{b_T^2 + b_0^2/(C_5 Q)^2}$

➡ $\mu_{b_*} \to \bar{\mu} \equiv \dfrac{C_1}{b_*(b_c(b_T))}$ so μ_{b_*} is cut off at $\mu_c \approx \dfrac{C_1 C_5 Q}{b_0}$

$$\tilde{\boldsymbol{f}}_1(\boldsymbol{x}, \boldsymbol{b_c(b_T)}; \boldsymbol{Q^2}, \boldsymbol{\mu_Q}) \sim \left(\tilde{C}^{f_1}(x/\hat{x}, b_*(b_c(b_T)); \bar{\mu}^2, \bar{\mu}, \alpha_s(\bar{\mu})) \otimes \boldsymbol{f_1(\hat{x}; \bar{\mu})}\right)$$
$$\times \exp\left[-S_{pert}(b_*(b_c(b_T)); \bar{\mu}, Q, \mu_Q) - S_{NP}^{f_1}(b_c(b_T), Q)\right]$$

"Improved CSS" (Polarized) (Gamberg, Metz, DP, Prokudin, to appear soon)

$$\tilde{\Phi}^{[\gamma^+]}(x, \vec{b}_T; Q^2, \mu_Q) = \tilde{\boldsymbol{f}}_1(\boldsymbol{x}, \boldsymbol{b_T}; \boldsymbol{Q^2}, \boldsymbol{\mu_Q}) - iM\epsilon^{ij} b_T^i S_T^j \tilde{\boldsymbol{f}}_{\boldsymbol{1T}}^{\perp(1)}(\boldsymbol{x}, \boldsymbol{b_T}; \boldsymbol{Q^2}, \boldsymbol{\mu_Q})$$

b_T -> $b_c(b_T)$

NO b_T -> $b_c(b_T)$ replacement –
kinematic factor NOT associated
with the scale evolution

b_T -> $b_c(b_T)$

“Improved CSS” (Unpolarized) (Collins, Gamberg, Prokudin, Rogers, Sato, Wang (2016))

Place a lower cut-off on b_T: $b_T \to b_c(b_T)$ where $b_c(b_T) = \sqrt{b_T^2 + b_0^2/(C_5 Q)^2}$

➡ $\mu_{b_*} \to \bar{\mu} \equiv \dfrac{C_1}{b_*(b_c(b_T))}$ so μ_{b_*} is cut off at $\mu_c \approx \dfrac{C_1 C_5 Q}{b_0}$

$$\boldsymbol{\tilde{f}_1(x, b_c(b_T); Q^2, \mu_Q)} \sim \left(\tilde{C}^{f_1}(x/\hat{x}, b_*(b_c(b_T)); \bar{\mu}^2, \bar{\mu}, \alpha_s(\bar{\mu})) \otimes \boldsymbol{f_1(\hat{x}; \bar{\mu})}\right) \times \exp\left[-S_{pert}(b_*(b_c(b_T)); \bar{\mu}, Q, \mu_Q) - S_{NP}^{f_1}(b_c(b_T), Q)\right]$$

“Improved CSS” (Polarized) (Gamberg, Metz, DP, Prokudin, to appear soon)

$$\tilde{\Phi}^{[\gamma^+]}(x, \vec{b}_T, b_c(b_T); Q^2, \mu_Q) = \boldsymbol{\tilde{f}_1(x, b_c(b_T); Q^2, \mu_Q)} - iM\epsilon^{ij} b_T^i S_T^j \boldsymbol{\tilde{f}_{1T}^{\perp(1)}(x, b_c(b_T); Q^2, \mu_Q)}$$

$$\boldsymbol{\tilde{f}_{1T}^{\perp(1)}(x, b_c(b_T); Q^2, \mu_Q)} \sim \left(\tilde{C}^{f_{1T}^{\perp}}(\hat{x}_1, \hat{x}_2, b_*(b_c(b_T)); \bar{\mu}^2, \bar{\mu}, \alpha_s(\bar{\mu})) \otimes \boldsymbol{F_{FT}(\hat{x}_1, \hat{x}_2; \bar{\mu})}\right) \times \exp\left[-S_{pert}(b_*(b_c(b_T)); \bar{\mu}, Q, \mu_Q) - S_{NP}^{f_{1T}^{\perp}}(b_c(b_T), Q)\right]$$

Analogous modification for fragmentation functions...

$$\boldsymbol{\tilde{D}_1(z, b_c(b_T); Q^2, \mu_Q)} \sim \left(\tilde{C}^{D_1}(z/\hat{z}, b_*(b_c(b_T)); \bar{\mu}^2, \bar{\mu}, \alpha_s(\bar{\mu})) \otimes \boldsymbol{D_1(\hat{z}; \bar{\mu})}\right) \times \exp\left[-S_{pert}(b_*(b_c(b_T)); \bar{\mu}, Q, \mu_Q) - S_{NP}^{D_1}(b_c(b_T), Q)\right]$$

$$\boldsymbol{\tilde{H}_1^{\perp(1)}(z, b_c(b_T); Q^2, \mu_Q)} \sim \left(\tilde{C}^{H_1^\perp}(z/\hat{z}, b_*(b_c(b_T)); \bar{\mu}^2, \bar{\mu}, \alpha_s(\bar{\mu})) \otimes \boldsymbol{H_1^{\perp(1)}(\hat{z}; \bar{\mu})}\right) \times \exp\left[-S_{pert}(b_*(b_c(b_T)); \bar{\mu}, Q, \mu_Q) - S_{NP}^{H_1^\perp}(b_c(b_T), Q)\right]$$

We then *define* the momentum-space functions...

$$\boldsymbol{f_1(x, k_T; Q^2, \mu_Q; C_5)} \equiv \int \frac{db_T}{2\pi}\, b_T J_0(k_T b_T)\, \boldsymbol{\tilde{f}_1(x, b_c(b_T); Q^2, \mu_Q)}$$

$$\boldsymbol{D_1(z, p_T; Q^2, \mu_Q; C_5)} \equiv \int \frac{db_T}{2\pi}\, b_T J_0(p_T b_T)\, \boldsymbol{\tilde{D}_1(z, b_c(b_T); Q^2, \mu_Q)}$$

$$\frac{\vec{k}_T^2}{2M^2}\, \boldsymbol{f_{1T}^{\perp}(x, k_T; Q^2, \mu_Q; C_5)} \equiv k_T \int \frac{db_T}{4\pi} b_T^2 J_1(k_T b_T)\, \boldsymbol{\tilde{f}_{1T}^{\perp(1)}(x, b_c(b_T); Q^2, \mu_Q)}$$

$$\frac{\vec{p}_T^2}{2z^2 M_h^2}\, \boldsymbol{H_1^{\perp}(z, p_T; Q^2, \mu_Q; C_5)} \equiv p_T \int \frac{db_T}{4\pi} b_T^2\, J_1(p_T\, b_T)\, \boldsymbol{\tilde{H}_1^{\perp(1)}(z, b_c(b_T); Q^2, \mu_Q)}$$

(Gamberg, Metz, DP, Prokudin, to appear soon)

which leads to...

$$\int d^2\vec{k}_T\, \boldsymbol{f_1(x, k_T; Q^2, \mu_Q; C_5)} = \boldsymbol{\tilde{f}_1(x, b_c(0); Q^2, \mu_Q)} = \boldsymbol{f_1(x; \mu_c)} + O(\alpha_s(Q)) + O((m/Q)^p)$$

$$\int d^2\vec{p}_T\, \boldsymbol{D_1(z, p_T; Q^2, \mu_Q; C_5)} = \boldsymbol{\tilde{D}_1(z, b_c(0); Q^2, \mu_Q)} = \boldsymbol{D_1(z; \mu_c)} + O(\alpha_s(Q)) + O((m/Q)^p)$$

$$\int d^2\vec{k}_T\, \frac{\vec{k}_T^2}{2M^2}\, \boldsymbol{f_{1T}^{\perp}(x, k_T; Q^2, \mu_Q; C_5)} = \boldsymbol{\tilde{f}_{1T}^{\perp(1)}(x, b_c(0); Q^2, \mu_Q)} = \pi\, \boldsymbol{F_{FT}(x, x; \mu_c)} + O(\alpha_s(Q)) + O((m/Q)$$

$$\int d^2\vec{p}_T\, \frac{\vec{p}_T^2}{2z^2M_h^2} \boldsymbol{H_1^{\perp}(z, p_T; Q^2, \mu_Q; C_5)} = \boldsymbol{\tilde{H}_1^{\perp(1)}(z, b_c(0); Q^2, \mu_Q)} = \boldsymbol{H_1^{\perp(1)}(z; \mu_c)} + O(\alpha_s(Q)) + O((m/Q)^{p''})$$

At LO in the "Improved CSS" formalism we recover the relations one expects from the "naïve" operator definitions of the functions

(Gamberg, Metz, DP, Prokudin, to appear soon)

which leads to...

$$\int d^2\vec{k}_T\, \boldsymbol{f_1(x, k_T; Q^2, \mu_Q; C_5)} = \boldsymbol{\tilde{f}_1(x, b_c(0); Q^2, \mu_Q)} = \boldsymbol{f_1(x; \mu_c)} + O(\alpha_s(Q)) + O((m/Q)^p)$$

$$\int d^2\vec{p}_T\, \boldsymbol{D_1(z, p_T; Q^2, \mu_Q; C_5)} = \boldsymbol{\tilde{D}_1(z, b_c(0); Q^2, \mu_Q)} = \boldsymbol{D_1(z; \mu_c)} + O(\alpha_s(Q)) + O((m/Q)^p)$$

$$\int d^2\vec{k}_T\, \frac{\vec{k}_T^2}{2M^2}\, \boldsymbol{f_{1T}^{\perp}(x, k_T; Q^2, \mu_Q; C_5)} = \boldsymbol{\tilde{f}_{1T}^{\perp(1)}(x, b_c(0); Q^2, \mu_Q)} = \pi\, \boldsymbol{F_{FT}(x, x; \mu_c)} + O(\alpha_s(Q)) + O((m/Q)$$

$$\int d^2\vec{p}_T\, \frac{\vec{p}_T^{\,2}}{2z^2M_h^2} \boldsymbol{H_1^{\perp}(z, p_T; Q^2, \mu_Q; C_5)} = \boldsymbol{\tilde{H}_1^{\perp(1)}(z, b_c(0); Q^2, \mu_Q)} = \boldsymbol{H_1^{\perp(1)}(z; \mu_c)} + O(\alpha_s(Q)) + O((m/Q)^{p''})$$

At LO in the “Improved CSS” formalism we recover the relations one expects from the “naïve” operator definitions of the functions

The “Improved CSS” formalism (approximately) restores the physical interpretation of TMDs!

(Gamberg, Metz, DP, Prokudin, to appear soon)

$$\int d^2\vec{k}_T \, \frac{\vec{k}_T^2}{2M^2} \, \boldsymbol{f_{1T}^{\perp}(x, k_T; Q^2, \mu_Q; C_5)} = \pi \, \boldsymbol{F_{FT}(x, x; \mu_c)} + O(\alpha_s(Q)) + O((m/Q)^{p'})$$

(Gamberg, Metz, DP, Prokudin, to appear soon)

$$\int d^2\vec{k}_T \, \frac{\vec{k}_T^2}{2M^2}\, f_{1T}^{\perp}(x, k_T; Q^2, \mu_Q; C_5) = \pi\, F_{FT}(x,x;\mu_c) + O(\alpha_s(Q)) + O((m/Q)^{p'})$$

$$\langle k_T^i(x;\mu)\rangle_{UT}$$

$$= \frac{1}{2}\int d^2k_T\, k_T^i \int \frac{db^-}{2\pi}\int\frac{d^2b_T}{(2\pi)^2}\, e^{ixP^+b^-} e^{-i\vec{k}_T\cdot\vec{b}_T}\langle P,S|\bar{\psi}(0)\gamma^+\mathcal{W}_{\mathrm{DIS}}(0;b)\psi(b)|P,S\rangle\Big|_{b^+=0}$$

$$= \frac{1}{2}\int\frac{db^-dy^-}{4\pi}\, e^{ixP^+b^-}\langle P,S|\bar{\psi}(0)\gamma^+\mathcal{W}(0;y^-)gF^{+i}(y^-)\,\mathcal{W}(y^-;b^-)\psi(b^-)|P,S\rangle$$

$$= -\pi\, M\epsilon^{ij}S_T^j\, F_{FT}(x,x;\mu)$$

(Boer, Mulders, Teryaev (1998); Burkardt (2004); Meissner, Metz, Goeke (2007))

(Gamberg, Metz, DP, Prokudin, to appear soon)

$$\int d^2\vec{k}_T \, \frac{\vec{k}_T^2}{2M^2} \, f_{1T}^{\perp}(x, k_T; Q^2, \mu_Q; C_5) = \pi \, F_{FT}(x, x; \mu_c) + O(\alpha_s(Q)) + O((m/Q)^{p'})$$

"Naïve" TMD operator – UV renormalization at LO and soft factor at LO, Wilson lines on the lightcone

$$\langle k_T^i(x;\mu)\rangle_{UT} = \frac{1}{2}\int d^2k_T \, k_T^i \boxed{\int \frac{db^-}{2\pi}\int \frac{d^2b_T}{(2\pi)^2} e^{ixP^+b^-} e^{-i\vec{k}_T\cdot\vec{b}_T} \langle P,S|\bar{\psi}(0)\gamma^+\mathcal{W}_{\rm DIS}(0;b)\psi(b)|P,S\rangle\Big|_{b^+=0}}$$

$$= \frac{1}{2}\int \frac{db^-dy^-}{4\pi} e^{ixP^+b^-}\langle P,S|\bar{\psi}(0)\gamma^+\mathcal{W}(0;y^-)gF^{+i}(y^-)\,\mathcal{W}(y^-;b^-)\psi(b^-)|P,S\rangle$$

$$= -\pi\, M\epsilon^{ij}S_T^j\, F_{FT}(x,x;\mu)$$

(Gamberg, Metz, DP, Prokudin, to appear soon)

$$\int d^2\vec{k}_T \, \frac{\vec{k}_T^2}{2M^2} \, f_{1T}^{\perp}(x,k_T;Q^2,\mu_Q;C_5) = \pi \, F_{FT}(x,x;\mu_c) + O(\alpha_s(Q)) + O((m/Q)^{p'})$$

This is *NOT* the operator that defines TMDs in CSS

“Naïve” TMD operator – UV renormalization at LO and soft factor at LO, Wilson lines on the lightcone

$\langle k_T^i(x;\mu)\rangle_{UT}$

$$= \frac{1}{2}\int d^2k_T\, k_T^i \boxed{\int \frac{db^-}{2\pi}\int \frac{d^2b_T}{(2\pi)^2} e^{ixP^+b^-} e^{-i\vec{k}_T\cdot\vec{b}_T} \langle P,S|\bar{\psi}(0)\gamma^+\mathcal{W}_{\rm DIS}(0;b)\psi(b)|P,S\rangle\Big|_{b^+=0}}$$

$$= \frac{1}{2}\int \frac{db^-dy^-}{4\pi}\, e^{ixP^+b^-} \langle P,S|\bar{\psi}(0)\gamma^+\mathcal{W}(0;y^-)gF^{+i}(y^-)\,\mathcal{W}(y^-;b^-)\psi(b^-)|P,S\rangle$$

$$= -\pi\, M\epsilon^{ij}S_T^j\, F_{FT}(x,x;\mu)$$

(Gamberg, Metz, DP, Prokudin, to appear soon)

$$\int d^2\vec{k}_T\,\frac{\vec{k}_T^2}{2M^2}\,f_{1T}^{\perp}(x,k_T;Q^2,\mu_Q;C_5) = \pi\,F_{FT}(x,x;\mu_c) + O(\alpha_s(Q)) + O((m/Q)^{p'})$$

$$\langle k_T^i(x;\mu)\rangle_{UT}$$

$$= \frac{1}{2}\int d^2k_T\,k_T^i\int\frac{db^-}{2\pi}\int\frac{d^2b_T}{(2\pi)^2}e^{ixP^+b^-}e^{-i\vec{k}_T\cdot\vec{b}_T}\langle P,S|\bar{\psi}(0)\gamma^+\mathcal{W}_{\mathrm{DIS}}(0;b)\psi(b)|P,S\rangle\Big|_{b^+=0}$$

$$= \frac{1}{2}\boxed{\int\frac{db^-dy^-}{4\pi}\,e^{ixP^+b^-}\langle P,S|\bar{\psi}(0)\gamma^+\mathcal{W}(0;y^-)gF^{+i}(y^-)\,\mathcal{W}(y^-;b^-)\psi(b^-)|P,S\rangle}$$

$$= -\pi\,M\epsilon^{ij}S_T^j\,F_{FT}(x,x;\mu)$$

“Naïve” collinear operator – LO term of the UV renormalized correlator, Wilson lines on the lightcone

(Gamberg, Metz, DP, Prokudin, to appear soon)

$$\int d^2\vec{k}_T\,\frac{\vec{k}_T^{\,2}}{2M^2}\,f_{1T}^{\perp}(x,k_T;Q^2,\mu_Q;C_5)=\pi\,F_{FT}(x,x;\mu_c)+O(\alpha_s(Q))+O((m/Q)^{p'})$$

This *IS* the operator for the Qiu-Sterman function that enters the OPE within CSS

$$\langle k_T^i(x;\mu)\rangle_{UT}$$

$$=\frac{1}{2}\int d^2k_T\,k_T^i\int\frac{db^-}{2\pi}\int\frac{d^2b_T}{(2\pi)^2}\,e^{ixP^+}\,e^{-i\vec{k}_T\cdot\vec{b}_T}\langle P,S|\bar{\psi}(0)\gamma^+\mathcal{W}_{\rm DIS}(0;b)\psi(b)|P,S\rangle\Big|_{b^+=0}$$

$$=\frac{1}{2}\boxed{\int\frac{db^-dy^-}{4\pi}\,e^{ixP^+b^-}\langle P,S|\bar{\psi}(0)\gamma^+\mathcal{W}(0;y^-)gF^{+i}(y^-)\,\mathcal{W}(y^-;b^-)\psi(b^-)|P,S\rangle}$$

$$=-\pi\,M\epsilon^{ij}S_T^j\,F_{FT}(x,x;\mu)$$

“Naïve” collinear operator – LO term of the UV renormalized correlator, Wilson lines on the lightcone

(Gamberg, Metz, DP, Prokudin, to appear soon)

$$\int d^2\vec{k}_T\,\frac{\vec{k}_T^2}{2M^2}\,f_{1T}^{\perp}(x,k_T;Q^2,\mu_Q;C_5)=\pi\,\boxed{F_{FT}(x,x;\mu_c)}+O(\alpha_s(Q))+O((m/Q)^{p'})$$

avg. TM of unpolarized quarks in a transversely polarized spin-1/2 target

$$\langle k_T^i(x;\mu)\rangle_{UT}$$

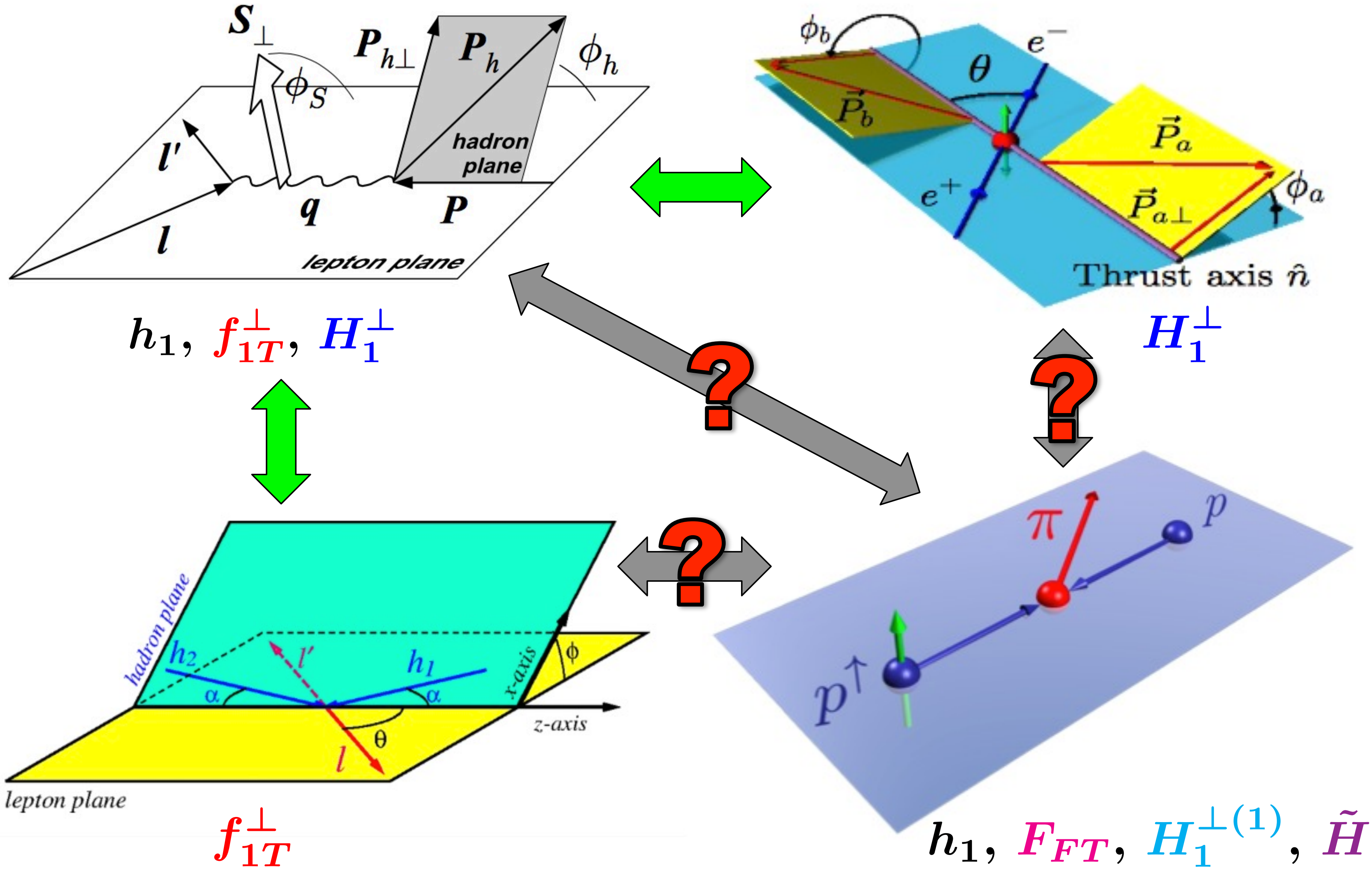
$$=\frac{1}{2}\int d^2k_T\,k_T^i\int\frac{db^-}{2\pi}\int\frac{d^2b_T}{(2\pi)^2}\,e^{ixP^+b^-}e^{-i\vec{k}_T\cdot\vec{b}_T}\,\langle P,S|\bar{\psi}(0)\gamma^+\mathcal{W}_{\rm DIS}(0;b)\psi(b)|P,S\rangle\Big|_{b^+=0}$$

$$=\frac{1}{2}\int\frac{db^-dy^-}{4\pi}\,e^{ixP^+b^-}\,\langle P,S|\bar{\psi}(0)\gamma^+\mathcal{W}(0;y^-)gF^{+i}(y^-)\,\mathcal{W}(y^-;b^-)\psi(b^-)|P,S\rangle$$

$$=-\pi\,M\epsilon^{ij}S_T^j\,F_{FT}(x,x;\mu)$$

The Qiu-Sterman function can fundamentally be understood as an avg. TM, and the first *k_T*-moment of the Sivers function (using "Improved CSS") retains this interpretation at LO

(Gamberg, Metz, DP, Prokudin, to appear soon)



$h_1,\ f_{1T}^{\perp},\ H_1^{\perp}$

$H_1^{\perp}$

$f_{1T}^{\perp}$

$h_1,\ F_{FT},\ H_1^{\perp(1)},\ \tilde{H}$

Recall the current phenomenology of TMD observables...

$$\tilde{f}_{1T}^{\perp(1)}(x, b_T; Q^2, \mu_Q) \quad \sim \quad \boxed{F_{FT}(x, x; \mu_{b_*})} \exp\left[-S_{pert}(b_*(b_T); \mu_{b_*}, Q, \mu_Q) - S_{NP}^{f_{1T}^\perp}(b_T, Q)\right]$$

$$\swarrow$$

$$\boxed{g_{f_{1T}^\perp}(x, b_T)} + g_K(b_T)\ln(Q/Q_0)$$

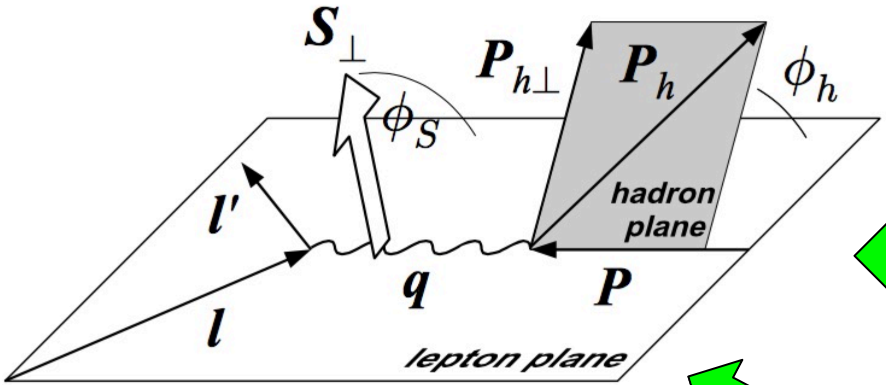
$$\tilde{H}_1^{\perp(1)}(z, b_T; Q^2, \mu_Q) \quad \sim \quad \boxed{H_1^{\perp(1)}(z; \mu_{b_*})} \exp\left[-S_{pert}(b_*(b_T); \mu_{b_*}, Q, \mu_Q) - S_{NP}^{H_1^\perp}(b_T, Q)\right]$$

$$\swarrow$$

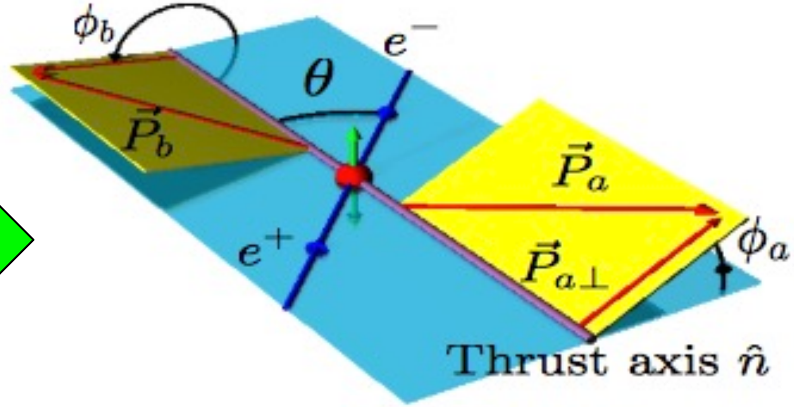
$$\boxed{g_{H_1^\perp}(z, b_T)} + g_K(b_T)\ln(Q/Q_0)$$

The **CT3 functions** (along with the NP *g*-functions) are what get extracted in analyses of TSSAs in ***TMD processes*** that use CSS evolution!

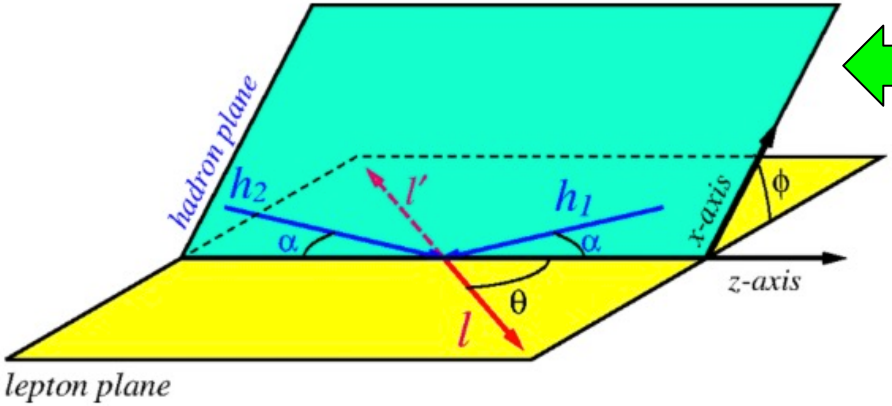
(Echevarria, Idilbi, Kang, Vitev (2014); Kang, Prokudin, Sun, Yuan (2016))



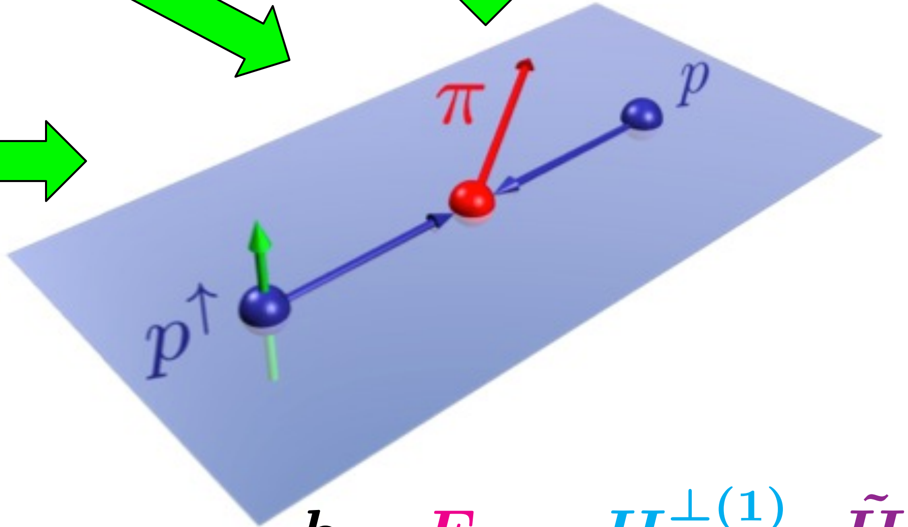
h_1, F_{FT}, $H_1^{\perp(1)}$



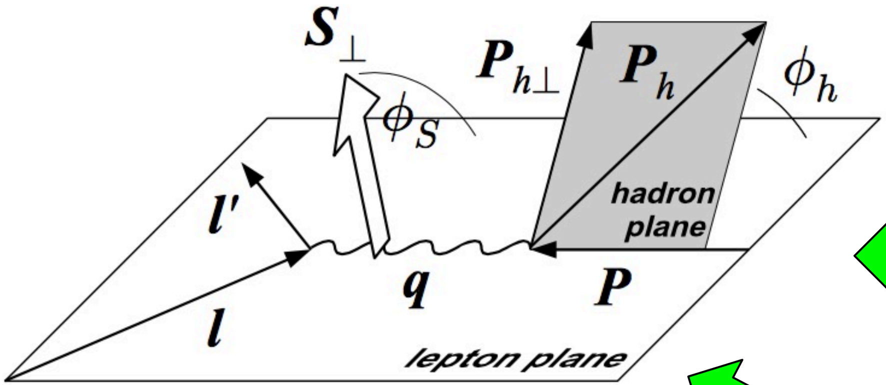
$H_1^{\perp(1)}$



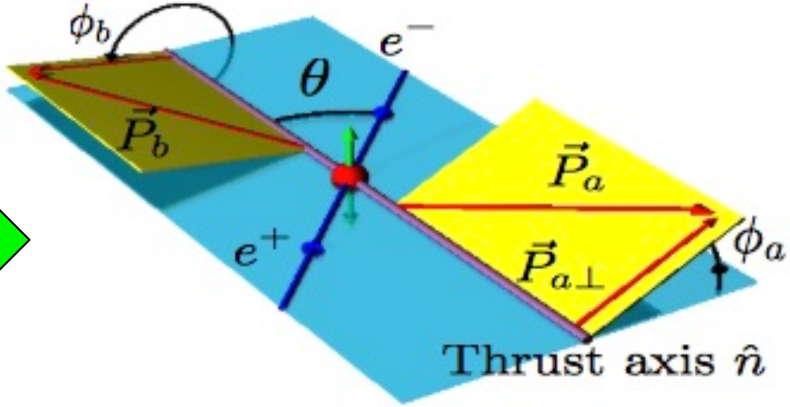
F_{FT}



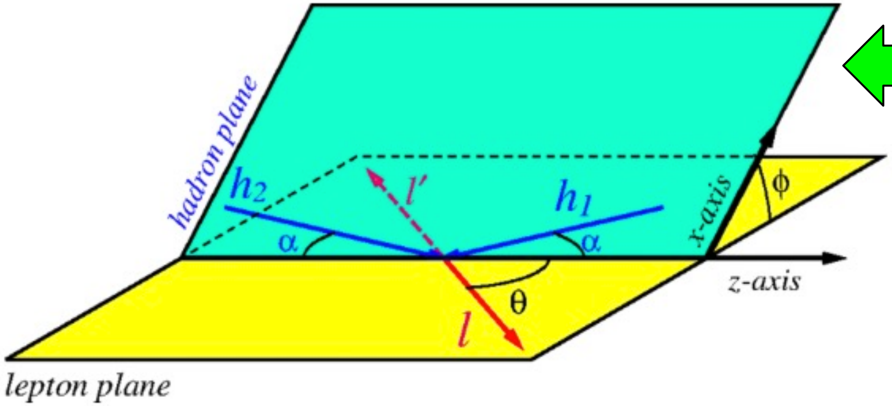
h_1, F_{FT}, $H_1^{\perp(1)}$, $\tilde{H}$



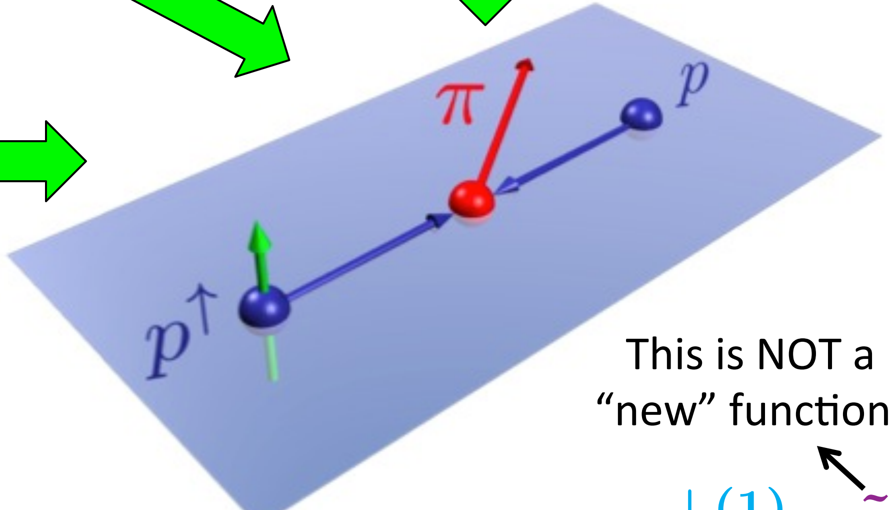
$h_1, F_{FT}, H_1^{\perp(1)}$



$H_1^{\perp(1)}$



F_{FT}



This is NOT a "new" function!

$h_1, F_{FT}, H_1^{\perp(1)}, \tilde{H}$

$A_{UT}^{\sin\phi_S}$ in SIDIS integrated over P_T (Mulders, Tangerman (1996); Bacchetta, et al. (2007))

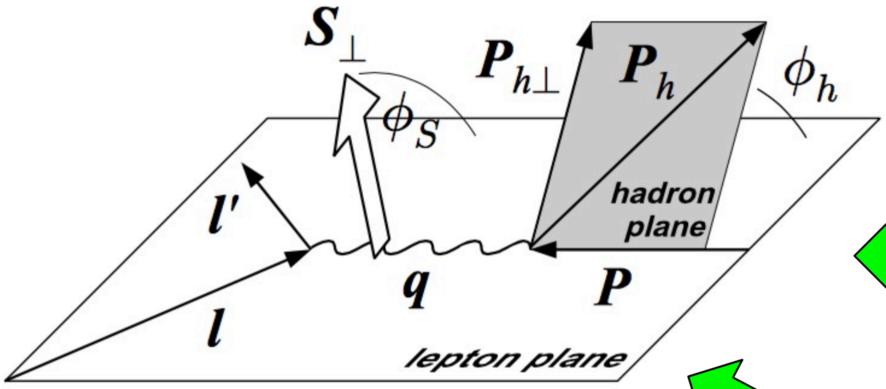
$$F_{UT}^{\sin\phi_S} \propto \sum_a e_a^2 \, \frac{2M_h}{Q} \, h_1^a(x) \, \frac{\tilde{\boldsymbol{H}}^a(\boldsymbol{z})}{z}$$

$A_{UT}^{\sin\phi_S}$ in $e^+e^- \rightarrow h_1 h_2 X$ integrated over q_T (Boer, Jakob, Mulders (1997))

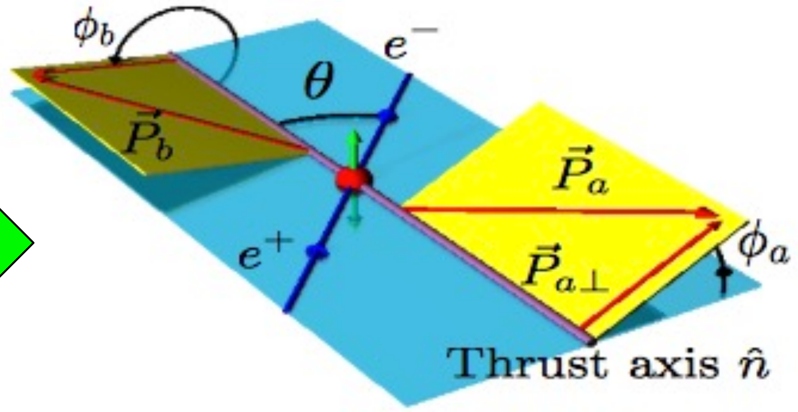
$$F_{UT}^{\sin\phi_S} \propto \sum_{a,\bar{a}} e_a^2 \left(\frac{2M_2}{Q} \, D_1^a(z_1) \frac{D_T^{\bar{a}}(z_2)}{z_2} + \frac{2M_1}{Q} \, \frac{\tilde{\boldsymbol{H}}^a(\boldsymbol{z}_1)}{z_1} H_1^{\bar{a}}(z_2) \right)$$

And also the TMD version of these (and other) observables (but with many more terms)

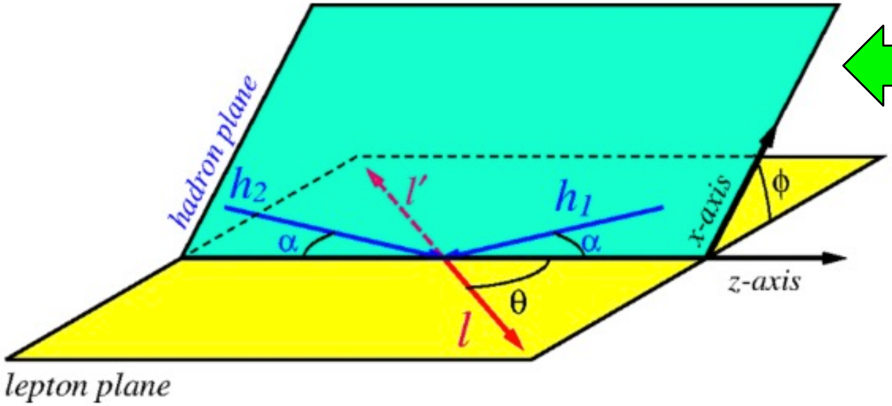
Note: data from COMPASS, HERMES, and Belle show nonzero effects for the unintegrated version of the above asymmetries



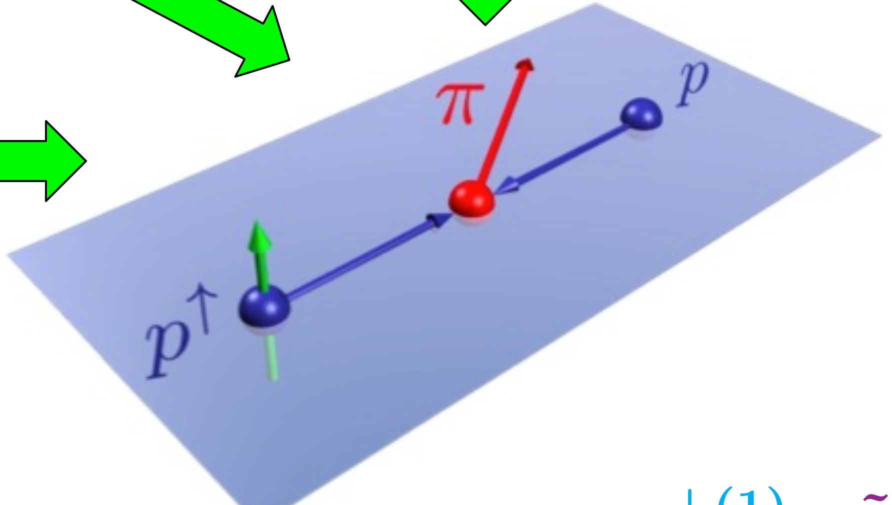
h_1, F_{FT}, $H_1^{\perp(1)}$, $\tilde{H}$



$H_1^{\perp(1)}$, $\tilde{H}$



F_{FT}



h_1, F_{FT}, $H_1^{\perp(1)}$, $\tilde{H}$

Towards a Global Analysis of TMD and CT3 Observables

$$H^q(z) \;=\; -2z\,H_1^{\perp(1),q}(z) + 2z\int_z^\infty \frac{dz_1}{z_1^2}\,\frac{1}{\frac{1}{z}-\frac{1}{z_1}}\,\hat{H}_{FU}^{q,\Im}(z,z_1)$$

QCD e.o.m. relation (EOMR)

$$\frac{H^q(z)}{z} = -\left(1 - z\frac{d}{dz}\right)H_1^{\perp(1),q}(z) - \frac{2}{z}\int_z^\infty \frac{dz_1}{z_1^2}\,\frac{\hat{H}_{FU}^{q,\Im}(z,z_1)}{(1/z-1/z_1)^2}$$

Lorentz invariance relation (LIR)

(Kanazawa, Koike, Metz, DP, Schlegel, PRD **93** (2016))

$$H^q(z) = -2z\,H_1^{\perp(1),q}(z) + 2z\int_z^\infty \frac{dz_1}{z_1^2}\,\frac{1}{\frac{1}{z}-\frac{1}{z_1}}\,\hat{H}_{FU}^{q,\Im}(z,z_1)$$

QCD e.o.m. relation (EOMR)

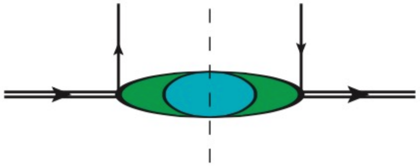
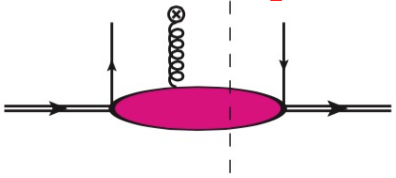
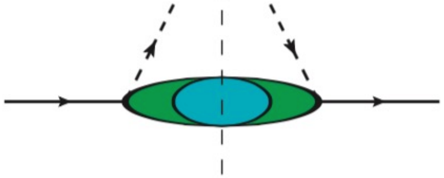
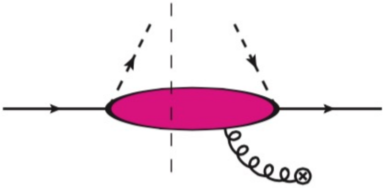
$$\frac{H^q(z)}{z} = -\left(1 - z\frac{d}{dz}\right)H_1^{\perp(1),q}(z) - \frac{2}{z}\int_z^\infty \frac{dz_1}{z_1^2}\,\frac{\hat{H}_{FU}^{q,\Im}(z,z_1)}{(1/z-1/z_1)^2}$$

Lorentz invariance relation (LIR)

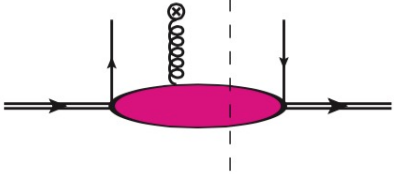
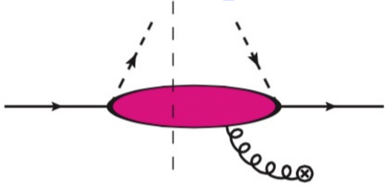
$$H(z) = \int_z^1 dz_1 \int_{z_1}^\infty \frac{dz_2}{z_2^2}\,2\left[\frac{\left(2(\frac{2}{z_1}-\frac{1}{z_2}) + \frac{1}{z_1}\left(\frac{1}{z_1}-\frac{1}{z_2}\right)\delta\left(\frac{1}{z_1}-\frac{1}{z}\right)\right)}{\left(\frac{1}{z_1}-\frac{1}{z_2}\right)^2}\,\hat{H}_{FU}^{\Im}(z_1,z_2)\right]$$

$$H_1^{\perp(1)}(z) = -\frac{2}{z}\int_z^1 dz_1 \int_{z_1}^\infty \frac{dz_2}{z_2^2}\,\frac{\left(\frac{2}{z_1}-\frac{1}{z_2}\right)}{\left(\frac{1}{z_1}-\frac{1}{z_2}\right)^2}\,\hat{H}_{FU}^{\Im}(z_1,z_2)$$

(Kanazawa, Koike, Metz, DP, Schlegel, PRD **93** (2016))

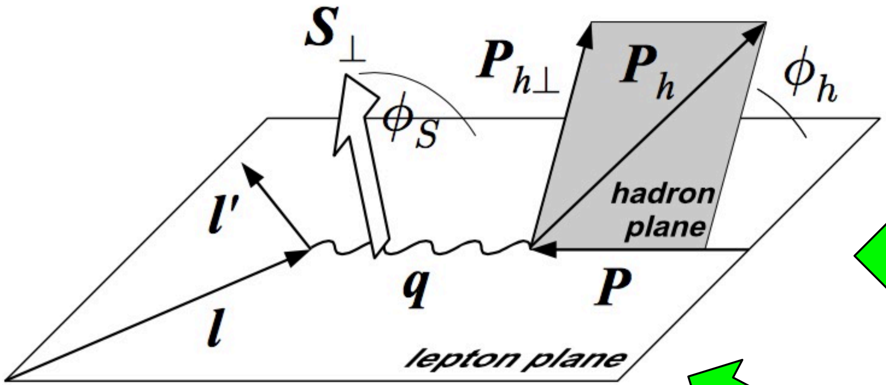
Hadron Pol.	PDF (x) 		PDF (x, x_1) 	FF (z) 		FF (z, z_1) 
	intrinsic	kinematical	dynamical	intrinsic	kinematical	dynamical
U	$\cancel{e}$	$\cancel{h_1^{\perp(1)}}$	H_{FU}	$\cancel{E}, \cancel{H}$	$\cancel{H_1^{\perp(1)}}$	$\hat{H}_{FU}^{\Re,\Im}$
L	$\cancel{h_L}$	$\cancel{h_{1L}^{\perp(1)}}$	H_{FL}	$\cancel{E_L}, \cancel{H_L}$	$\cancel{H_{1L}^{\perp(1)}}$	$\hat{H}_{FL}^{\Re,\Im}$
T	$\cancel{g_T}$	$\cancel{f_{1T}^{\perp(1)}}$, $\cancel{g_{1T}^{\perp(1)}}$	F_{FT}, G_{FT}	$\cancel{D_T}, \cancel{G_T}$	$\cancel{D_{1T}^{\perp(1)}}$, $\cancel{G_{1T}^{\perp(1)}}$	$\hat{D}_{FT}^{\Re,\Im}, \hat{G}_{FT}^{\Re,\Im}$

(Kanazawa, Koike, Metz, DP, Schlegel, PRD **93** (2016))

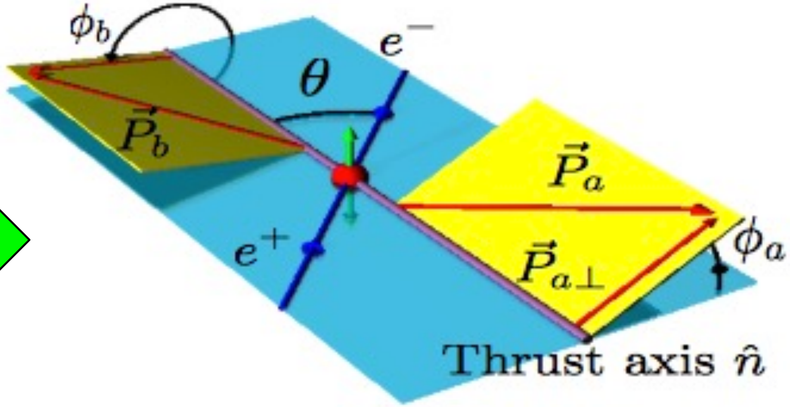
Hadron Pol.	PDF (x, x_1) 	FF (z, z_1) 
U	dynamical H_{FU}	dynamical $\hat{H}_{FU}^{\Re,\Im}$
L	H_{FL}	$\hat{H}_{FL}^{\Re,\Im}$
T	F_{FT}, G_{FT}	$\hat{D}_{FT}^{\Re,\Im}, \hat{G}_{FT}^{\Re,\Im}$

***ALL** transverse spin observables* are driven by *multi-parton correlations*

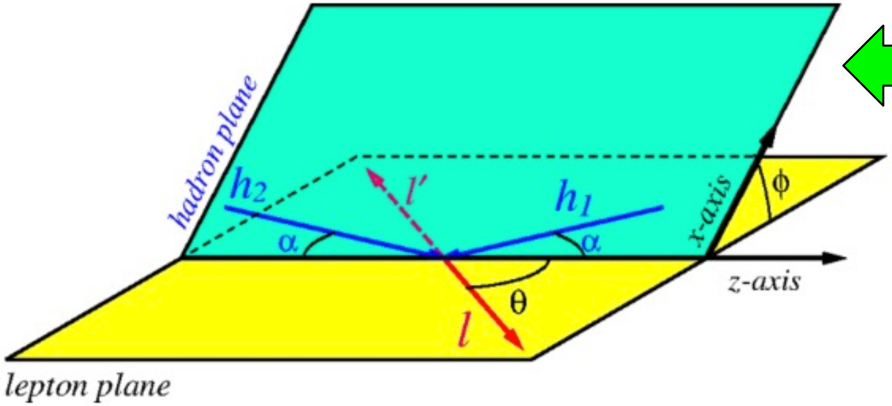
(Kanazawa, Koike, Metz, DP, Schlegel, PRD **93** (2016))



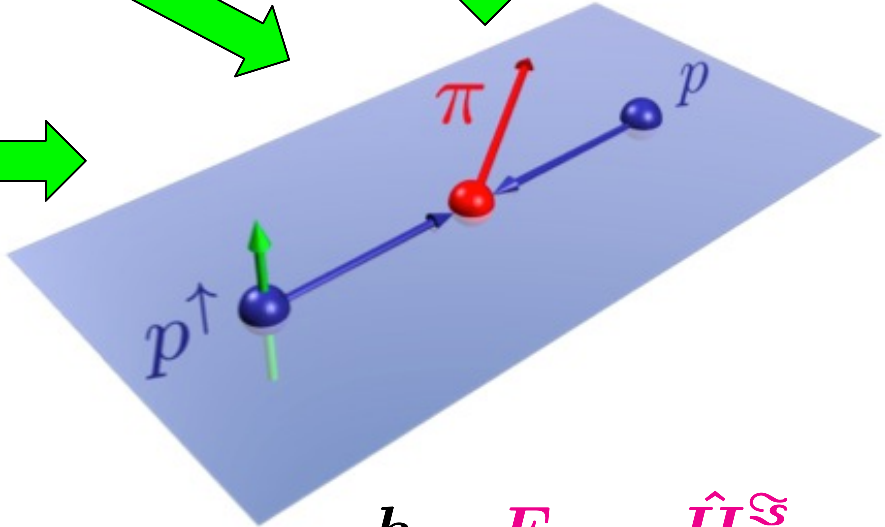
h_1, F_{FT}, $\hat{H}_{FU}^{\sphericalangle}$



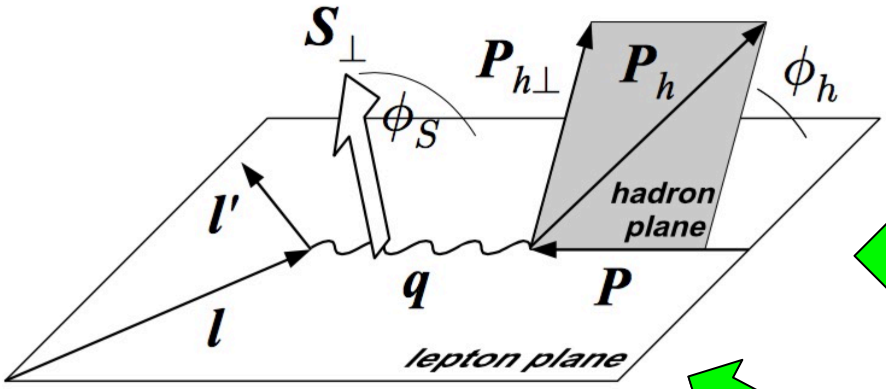
$\hat{H}_{FU}^{\sphericalangle}$



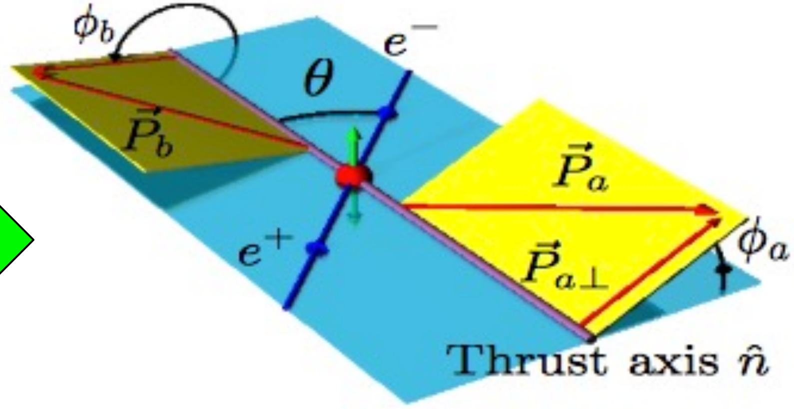
F_{FT}



h_1, F_{FT}, $\hat{H}_{FU}^{\sphericalangle}$

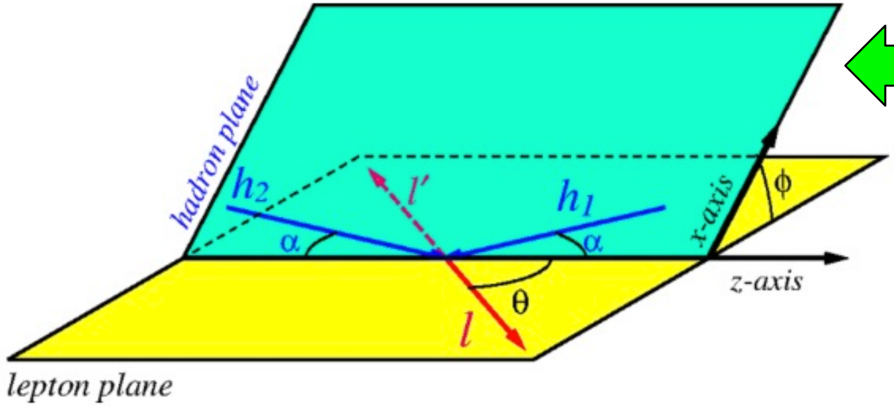


h_1, F_{FT}, $H_1^{\perp(1)}$, $\tilde{H}$

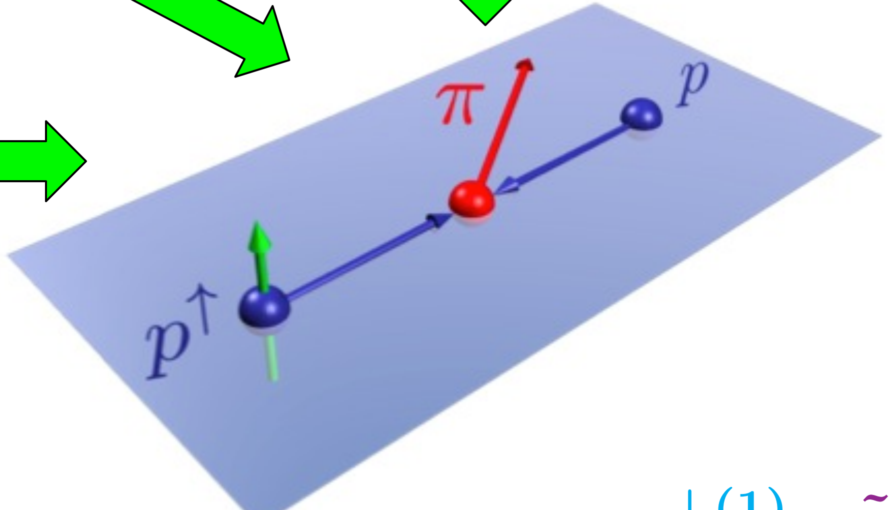


$H_1^{\perp(1)}$, $\tilde{H}$

Need to perform a “global” analysis!



F_{FT}



h_1, F_{FT}, $H_1^{\perp(1)}$, $\tilde{H}$

Summary

- TSSAs have been studied in both TMD processes (SIDIS, e^+e^-, DY) and collinear processes (A_N in *pp* & *lp* collisions).

- The current TMD formalism using "Improved CSS" (iCSS) allows one to rigorously connect these two different types of observables. We have extended the original work on the unpolarized cross section to now include polarization.

- With the iCSS formalism, we are able at LO to restore the physical interpretation of (integrated) TMDs.

- (LIRs + EOMRs + iCSS) = ***ALL*** transverse spin observables are driven by 3-parton (dynamical) functions.

- A global analysis of TMD *AND* collinear twist-3 transverse-spin observables is now possible.