Electromagnetic interactions of light hadrons in covariant chiral perturbation theory

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Photon beams

Electromagnetic interactions provide clean probes of the hadrons’ inner structure

- Low photon energies (∼ 100 MeV): Compton scattering

- Slightly higher (≳ 140 MeV): pion photoproduction
Electromagnetic interactions provide **clean probes** of the hadrons’ inner structure

- Low photon energies (∼ 100 MeV): Compton scattering

- Even higher: start feeling resonance production
Non-perturbative QCD and chiral perturbation theory

\[ E_\gamma \approx \mathcal{O}(m_\pi) \Rightarrow \alpha_s = \mathcal{O}(1) \]

Perturbative QCD breaks down

\[ \implies \text{EFT: expansion around other parameters} \]
Non-perturbative QCD and chiral perturbation theory

\[ E_γ \approx \mathcal{O}(m_π) \Rightarrow \alpha_s = \mathcal{O}(1) \]

Perturbative QCD breaks down

\[ \Rightarrow \text{EFT: expansion around other parameters} \]

**Chiral perturbation theory:**

- Small masses, momenta \( \left( \frac{m_π}{1 \text{ GeV}}, \frac{p_\text{ext}}{1 \text{ GeV}} \ll 1 \right) \):
  combined expansion

- New degrees of freedom:
  quarks and gluons \( \Rightarrow \) mesons and baryons
Lagrangian ordered in terms of chiral parameters

Lowest-order **meson** Lagrangian $\sim p_{\text{ext}}^2, m_\pi^2$

$$\mathcal{L}^{(2)}_{\phi \phi \gamma} = \frac{F_0^2}{4} \text{Tr} \left( \nabla_\mu U \nabla^\mu U^\dagger + \chi_+ \right)$$

Lowest-order **baryon** Lagrangian $\sim p_{\text{ext}}$

$$\mathcal{L}^{(1)}_{\phi B \gamma} = \text{Tr} \left( \bar{B} (i \not{\partial} - m) B \right) + \frac{D}{2} \text{Tr} \left( \bar{B} \gamma^\mu \{ u_\mu, B \} \gamma_5 \right) + \frac{F}{2} \text{Tr} \left( \bar{B} \gamma^\mu [ u_\mu, B ] \gamma_5 \right)$$
Inclusion of the decuplet

The spin-3/2 states **couple strongly** to the spin-1/2 octet baryons


\[
L^{(1)}_{\Delta\phi B} = \frac{-i\sqrt{2}}{F_0 M_\Delta} \bar{B}^{ab_\epsilon cda} \gamma^{\mu\nu\lambda} (\partial_\mu \Delta_\nu)^{d_b e_c} (D_\lambda \phi)^{c e} + \text{H.c.}
\]

\[
L^{(2)}_{\Delta\gamma B} = -\frac{3ie g_M}{\sqrt{2}m(m + M_\Delta)} \bar{B}^{ab_\epsilon cda} Q^{c e} (\partial_\mu \Delta_\nu)^{d_b e_c} \tilde{F}^{\mu\nu} + \text{H.c.}
\]
Matching a diagram to a specific order

\[ O = 4L + \sum kV_k - 2N_\pi - N_N - N_\Delta \cdot \frac{1}{2} \]

- Propagators: meson \( \sim m_\pi^{-2} \), spin-1/2 baryon \( \sim p_{\text{ext}}^{-1} \)
- Spin-3/2 baryon: new scale \( \delta = M_\Delta - m_N \approx 0.3 \text{ GeV} > m_\pi \)
- \( (\delta/m_p)^2 \approx (m_\pi/m_p) \implies \) far from resonance mass: \( \frac{1}{2} \)

Compton scattering and polarizabilities

Hiller Blin, Gutsche, Ledwig and Lyubovitskij
Results

Compton scattering

Polarizabilities

- In EM field: hadrons deformed due to charged components
- Size of deformation: related to polarizabilities
- Experiment: Compton scattering off hadron targets
Compton scattering and $\gamma_0$

- Amplitude expansion **around low photon energy** $\omega$
  - $\mathcal{O}(\omega^0)$: total charge
  - $\mathcal{O}(\omega^1)$: anomalous magnetic moment
  - $\mathcal{O}(\omega^2)$: $\alpha_E$ and $\beta_M$
  - $\mathcal{O}(\omega^3)$: spin-dependent polarizabilities $\gamma_i$

- Forward spin polarizability $\gamma_0$
  - response to deformation relative to spin axis
  - photon scattering in extreme forward direction


\[
\gamma_0 \left[ \vec{\sigma} \cdot (\vec{\epsilon} \times \vec{\epsilon}^*) \right] = -\frac{i}{4\pi} \frac{\partial}{\partial \omega^2} \frac{\epsilon^\mu \mathcal{M}_{\mu\nu}^{SD} \epsilon^{*\nu}}{\omega} \bigg|_{\omega=0}
\]
Approaching the experimental results

- **Sum rule**: Gell-Mann et al., Phys. Rev. 95 (1954) 1612
  \[
  \gamma_0 = -\frac{1}{4\pi^2} \int_{\omega_0}^{\infty} d\omega \frac{\sigma_{3/2}(\omega) - \sigma_{1/2}(\omega)}{\omega^3}
  \]
  \(\sigma_{3/2}(\sigma_{1/2})\): photon and target helicities are (anti)parallel

  \(\gamma_0^P = [-1.01 \pm 0.08\text{(stat)} \pm 0.10\text{(syst)}] \cdot 10^{-4}\text{fm}^4\)

  \(\gamma_0^P = [-1.1 \pm 0.4] \cdot 10^{-4}\text{fm}^4\) and \(\gamma_0^n = [-0.3 \pm 0.2] \cdot 10^{-4}\text{fm}^4\)

- **First goal** is to reproduce these values theoretically
- Then extend the theoretical model to **predict** polarizabilities of not yet measured states \(\rightarrow\) **hyperons**
Hyperons

\[ SU(2) \rightarrow SU(3) \]

\[
\begin{pmatrix}
 p \\
 n
\end{pmatrix}
\rightarrow
\begin{pmatrix}
 \frac{1}{\sqrt{2}} \Sigma^0 + \frac{1}{\sqrt{6}} \Lambda \\
 \Sigma^- \\
 -\frac{1}{\sqrt{2}} \Sigma^0 + \frac{1}{\sqrt{6}} \Lambda
\end{pmatrix}
\begin{pmatrix}
 p \\
 n
\end{pmatrix}
\]

\[ \pi^\pm, \pi^0 \rightarrow \pi^\pm, \pi^0, K^\pm, K^0, \eta \]

\[ \Delta(1232) \rightarrow \Delta^\pm, \Delta^{++}, \Delta^0, \Sigma^{*\pm}, \Sigma^{*0}, \Xi^-, \Xi^*0, \Omega \]

- Hyperons: baryons with **strangeness** \( S \neq 0 \)
- Short lifetimes \( \rightarrow \) properties computed on the lattice
- Gives space for theoretical predictions
Compton scattering

Diagrams contributing to $\gamma_0$ up to $\mathcal{O}(p^{7/2})$
Renormalization

- Loop diagrams: **divergences** and **power counting breaking terms**

  \[ \frac{1}{\epsilon} = \frac{1}{4 - \text{dim}} \quad \text{and} \quad \text{e.g. terms } \propto p^2 \text{ at } \mathcal{O}(p^3) \]

- Fully analytical \(\Rightarrow\) match with **Lagrangian terms**

- **Low-energy constants** of these terms a priori unknown

- EOMS-renormalization prescription:


  - $\overline{MS}$ absorbs $L = \frac{2}{\epsilon} + \log(4\pi) - \gamma_E$ into LECs
  - Also subtracts PCBT by redefinition of LECs
  - Usually converges faster than other counting schemes (relativistic or not)
Results

Compton scattering

Renormalization

- \( \infty \) and PCBT: do not enter pieces \( \sim \omega^3 \) relevant for \( \gamma_0 \)

- Leading order for \( \gamma_0 \) \( \implies \) no unknown LECs

- Results independent of renormalization or unknown LECs

\[ \downarrow \]

pure predictions of ChPT
Results with different covariant ChPT models

\[ \gamma_0 \left[ 10^{-4}\text{fm}^{-4} \right] \]

Results

Compton scattering

Results for the hyperons

- Only HBChPT existed: incompatible with exp. (nucleons)
  Vijaya Kumar et al., Phys. Rev. D 84 (2011) 076007

- $g_M$ not well known

- We estimate it from electromagnetic decay width $\Gamma_{\Delta \to \gamma N}$

  \[ g_M = 3.16(16) \]

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<tr>
<th>$\gamma_0$ [fm$^{-4}$ $10^{-4}$]</th>
<th>$\Sigma^+$</th>
<th>$\Sigma^-$</th>
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Electromagnetic transition of **negatively charged** hyperons to spin-3/2 partners $SU(3)$ forbidden $\implies$ no uncertainty from $g_M$
Neutral pion photoproduction close to threshold

Hiller Blin, Ledwig and Vicente Vacas
Results

\( \gamma p \rightarrow p\pi^0 \)

Cross sections

\begin{align*}
\begin{array}{c|c}
\text{Reaction} & \text{Relative dipole moment} \\
\hline
\gamma p \rightarrow \pi^+ n & 1 \\
\gamma p \rightarrow \pi^0 p & -\frac{m_{\pi}}{m_N} \\
\gamma n \rightarrow \pi^- p & -\left(1 + \frac{m_{\pi}}{m_N}\right) \\
\gamma n \rightarrow \pi^0 n & 0 \\
\end{array}
\end{align*}

- Close to threshold: charged channels have much larger cross sections than neutral ones
- The charged channels are well described in low-order ChPT. The neutral channel is NOT Bernard et al. (1992) NPB
- Inclusion of the \( \Delta(1232) \) spin-3/2 resonance is essential Hemmert et al. (1997) PLB
Results

$\gamma p \rightarrow p \pi^0$

$\gamma p \rightarrow p \pi^0$ data

- Can be used to test the convergence of ChPT models
- Measured polarization observables:
  
  \[
  \frac{d\sigma}{d\Omega} \quad \text{and} \quad \Sigma = \frac{d\sigma_\perp - d\sigma_\parallel}{d\sigma_\perp + d\sigma_\parallel}
  \]
\( \mathcal{O}(p^4) \) ChPT (terms proportional to \( m^4_\pi \)): Hornidge et al., Phys. Rev. Lett. 111 (2013) 062004

▶ \( \mathcal{O}(p^4) \) relativistic ChPT
▶ \( \mathcal{O}(p^4) \) HBChPT
▶ Empirical fit

\( \chi^2_{\text{red}} \) vs. \( E_{\gamma}^{\text{max}} \) (MeV)
Putting all pieces together up to $O(p^3)$... and $\Delta(1232)$!
What could not be achieved without the $\Delta(1232)$ is now possible, **without the many new fitting constants of $O(p^4)$**
Results

**Fit of the low-energy constants**

<table>
<thead>
<tr>
<th>$g_0$</th>
<th>$\tilde{c}_{67}$</th>
<th>$\tilde{d}_{89} \cdot m_N^2$</th>
<th>$\tilde{d}_{168} \cdot m_N^2$</th>
<th>$h_A$</th>
<th>$g_M$</th>
<th>$g_E$</th>
<th>$\chi^2$/d.o.f.</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.05</td>
<td>2.45</td>
<td>1.67</td>
<td>-9.7</td>
<td>2.85</td>
<td>2.28</td>
<td>3.30</td>
<td>0.80</td>
</tr>
<tr>
<td>1.05</td>
<td>2.29</td>
<td>1.17</td>
<td>-10.4</td>
<td>2.85</td>
<td>2.90</td>
<td>3.53</td>
<td>0.96</td>
</tr>
</tbody>
</table>

- $g_0, \tilde{c}_{67} = c_6 + c_7$ *converge to the literature values*
  Ledwig et al. (2014) PRD

- $\tilde{d}_{89} = d_8 + d_9, g_E$ are of natural size

- $d_{18}$ *is very sensitive to higher-order input.* We fit the combination $\tilde{d}_{168} = 2d_{16} - d_{18}$

- $g_M, h_A$ prefer low values, but *literature value* gives good fit
Comparing theoretical curves with data

- **MAMI $\gamma p \rightarrow p\pi^0$ data**

- **$O(p^3)$ nucleonic model**
  Unable to reproduce the energy dependence

- **$O(p^{7/2})$ with $\Delta(1232)$**
  Good reproduction for all 800 data points

- **Only $\Delta(1232)$ contribution**
- **Only nucleonic contribution**
Results

\( \gamma p \rightarrow p\pi^0 \)

All data points for \( d\sigma/d\Omega \) and \( \Sigma \)

\begin{align*}
E_\gamma &= 203.91 \text{ MeV} \\
E_\gamma &= 201.57 \text{ MeV} \\
E_\gamma &= 199.16 \text{ MeV} \\
E_\gamma &= 196.75 \text{ MeV} \\
E_\gamma &= 194.35 \text{ MeV} \\
E_\gamma &= 191.94 \text{ MeV} \\
E_\gamma &= 189.57 \text{ MeV} \\
E_\gamma &= 187.16 \text{ MeV} \\
E_\gamma &= 184.79 \text{ MeV} \\
E_\gamma &= 182.4 \text{ MeV} \\
E_\gamma &= 180.02 \text{ MeV} \\
E_\gamma &= 177.66 \text{ MeV} \\
E_\gamma &= 175.22 \text{ MeV} \\
E_\gamma &= 172.91 \text{ MeV} \\
E_\gamma &= 170.53 \text{ MeV} \\
E_\gamma &= 168.16 \text{ MeV} \\
E_\gamma &= 165.78 \text{ MeV} \\
E_\gamma &= 163.44 \text{ MeV} \\
E_\gamma &= 161.08 \text{ MeV} \\
E_\gamma &= 158.72 \text{ MeV} \\
E_\gamma &= 156.38 \text{ MeV} \\
\end{align*}

\(~ 800 \text{ data points} ~\)
Summary and outlook

Summary I

Framework
- Electromagnetic probes of light baryons in $SU(3)$ ChPT
- Explicit inclusion of the $\Delta(1232)$ resonance
- Covariant renormalization scheme: EOMS

Hyperon polarizabilities
- We reproduce the nucleon $\gamma_0$ for the first time in $SU(3)$
- Predictive results for hyperon polarizabilities at $O(p^{7/2})$
- $\Sigma^-$ and $\Xi^-$ do not depend on uncertainties from LECs
Summary and outlook

Neutral pion photoproduction

- High-quality description of $\gamma p \rightarrow p\pi^0$ threshold data: **cross sections and polarization observables** match experimental data at $E_\gamma > 180$ MeV **for the first time**
- $O(p^{7/2})$ better than $\Delta$-less $O(p^4)$ HB or covariant ChPT
- Strong constraints of previously unknown LECs

Outlook

- **Compton scattering:**
  Magnetic moments, spin independent, photon virtuality, . . .
- **Pion photoproduction:**
  Cusp effect, charge production, photon virtuality, . . .
Additional material
Higher orders of the nucleonic Lagrangian

\[ \mathcal{L}_N = \bar{\Psi} \left\{ \frac{1}{8m} \left( c_6 f_{\mu\nu}^+ + c_7 \text{Tr} \left[ f_{\mu\nu}^+ \right] \right) \sigma^{\mu\nu} \right. \]

\[ + \frac{i}{2m} \varepsilon^{\mu\nu\alpha\beta} \left( d_8 \text{Tr} \left[ \tilde{f}_{\mu\nu}^+ u_\alpha \right] + d_9 \text{Tr} \left[ f_{\mu\nu}^+ \right] u_\alpha + \text{H.c.} \right) D_\beta \]

\[ + \frac{\gamma^\mu \gamma^5}{2} \left( d_{16} \text{Tr} \left[ \chi^+ \right] u_\mu + i d_{18} [D_\mu, \chi^-] \right) \right\} \Psi + \ldots \]
The explicit inclusion of $\Delta$ is essential to reproduce $M_{1^+}$
The contribution of $D$-waves needs to be taken into account in order to get the correct behavior for $E_0^+$. 

Schumann et al. (2015) PLB
Electron-scattering form factors

Matrix decomposition of the amplitude:

\[ \gamma^\mu F_1(Q^2) + \frac{i\sigma^{\mu\nu} q_\nu}{2m} F_2(Q^2) + \frac{i\sigma^{\mu\nu} \gamma_5 q_\nu}{2m} F_{EDM}(Q^2) \]

CP violating
Origin and consequences of CP violation:

- Standard model complex phases in quark mixing matrix: confirmed experimentally (electroweak), very small
- Not yet seen in the strong interaction sector, but possible
- Would lead to a permanent EDM of the neutron (nEDM)
- In the SM this EDM would only be of $\sim 10^{-32} e\ cm$
Postulated principles to explain matter-antimatter asymmetry:

- Baryon number conservation: violated
- Processes take place out of thermal equilibrium
- $C$ and $CP$ symmetries: violated

$CP$ violation from CKM matrix not enough for asymmetry

$\implies$ physics beyond the standard model?
Generating a neutron EDM

Precision EDM measurements: **neutral and stable** neutron

Current experimental upper limit for neutron EDM:

\[
\text{nEDM} < 2.9 \cdot 10^{-26} \text{ e cm}
\]

We explore one possibility to generate the nEDM from the theoretical perspective:

**Need:** \( CP \)-violating couplings of the \( \eta, \eta' \) to the nucleons
$CP$-violating $\eta, \eta' \rightarrow \pi\pi$ decays

We extract the $CP$-violating $\eta, \eta'$ coupling to the nucleon from the $CP$-violating $\eta, \eta'$ couplings to two pions:
Tight constraint on \( CP \)-violating decays

From the experiment:

\[
\frac{\Gamma(\eta \rightarrow \pi\pi)}{\Gamma_{\text{full}}^{\eta}} < \begin{cases} & 1.3 \times 10^{-5} \text{ for } \pi^+\pi^- \\ & 3.5 \times 10^{-4} \text{ for } \pi^0\pi^0 \\ \end{cases}
\]

\( \Rightarrow \) generates \( n\text{EDM}_{\text{theo}} < 10^{-18} \text{ e cm} \)

- \( n\text{EDM}_{\text{exp}} < 2.9 \cdot 10^{-26} \text{ e cm} \)

- \( CP \)-violating branching ratio should be reduced by further eight orders of magnitude

- Relevant for the rare \( \eta(\eta') \)-decay measurements by the GlueX and LHCb experiments
Contributions to form factors up to $O(p^3)$

We also include vector mesons explicitly.
Calculated all octet-baryon electromagnetic form factors

$$G_E(q^2) = F_1(q^2) + \frac{q^2}{4m_{B0}^2} F_2(q^2)$$

$$= 1 + q^2 \left\langle r_E^2 \right\rangle + \frac{q^4}{2} \frac{d^2}{(dq^2)^2} G_E(q^2) \bigg|_{q^2=0} + \mathcal{O}(q^6)$$
Calculated all octet-baryon electromagnetic form factors

\[ G_E(q^2) = F_1(q^2) + \frac{q^2}{4m_{B0}^2} F_2(q^2) \]  
\[ = 1 + q^2 \langle r_E^2 \rangle + \frac{q^4}{2} \frac{d^2}{(dq^2)^2} G_E(q^2) \bigg|_{q^2=0} + \mathcal{O}(q^6) \]  

Here: compare predictive higher-order piece with available proton data (polynomial fits)

Relevant for fits to constrain better \( r_E \): proton radius puzzle

 Might help to understand discrepancies between data from electron–proton scattering and muonic Lamb shift
**Electric form factor and charge radius**

**Experiment:** $11.7 - 16.6$ GeV$^{-4}$ Higinbotham et al., PRC 93 (2016) 055207

**SU(2) theory:** Ledwig et al., Phys. Rev. D 85 (2012) 034013 + our work

$$\frac{d^2}{(dq^2)^2} G_E(q^2)|_{q^2=0} \text{ [GeV}^{-4}]$$  

<table>
<thead>
<tr>
<th></th>
<th>Octet</th>
<th>Decuplet</th>
<th>Vector</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Our results $SU(2)$</td>
<td>4.9</td>
<td>0.6</td>
<td>12.0</td>
<td>17.5</td>
</tr>
<tr>
<td>Our results $SU(3)$</td>
<td>3.9</td>
<td>-0.1</td>
<td>10.7</td>
<td><strong>14.5</strong></td>
</tr>
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</table>

Another predictive quantity: peripheral transverse baryon electromagnetic densities
Dispersive representation of electromagnetic densities

\[ \rho_{1,2}(b) = \int_{4m^2}^{\infty} \frac{dt}{2\pi} K_0(\sqrt{tb}) \frac{\text{Im} F_{1,2}(t)}{\pi} \]

Bessel function \( K_0 \sim e^{-b\sqrt{t}} \) suppression at large \( t \)

\[ \implies \text{Distance } b \text{ is a filter of masses } \sqrt{t} \sim 1/b \]

**Two-pion cut:** low-mass states \( \rightarrow \) **peripheral transverse** \( \rho \)

Only these diagrams have **imaginary parts** at this cut
Spectral functions and densities

\[ \text{Im} F_1^\pi \]

- Octet
- Decuplet
- Total

\[ t (\text{GeV}^2) \]

\[ \rho_1^\pi (\text{fm}^{-2}) \]

- Octet
- Decuplet
- Total

\[ b (\text{fm}) \]

\[ \Lambda \text{ and } \Sigma^0: \text{ very compact charge/magnetic distributions} \]
Rosenbluth data and charge radius

\[ G_E \]

\[ Q^2 [\text{GeV}^2] \]

Rosenbluth data
linear, muonic hydrogen
linear, e^- scattering
chiral loops + linear e^- scattering
chiral loops, vector + linear e^- scattering
Octet charge densities

\[ \rho^V_1 \] \text{(vector region)}
\[ \rho^n_1 \] \text{(chiral region)}
\[ \rho^\Sigma_1^+ \]
\[ \rho^\Sigma_1^- \]

1. \[ 1 \times 10^{-7} \] to \[ 1 \times 10^{-2} \]
2. \[ 1 \times 10^{-6} \] to \[ 1 \times 10^{-1} \]
3. \[ 1 \times 10^{-5} \] to \[ 1 \times 10^{-1} \]
4. \[ 1 \times 10^{-4} \] to \[ 1 \times 10^{-1} \]
5. \[ 1 \times 10^{-3} \] to \[ 1 \times 10^{-1} \]
### Octet charge densities

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<th>VM SU(3) meson loops</th>
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<tr>
<td>ρ₁</td>
<td>Λ (fm⁻²)</td>
</tr>
<tr>
<td>0.000</td>
<td>0.002</td>
</tr>
<tr>
<td>0.004</td>
<td>0.006</td>
</tr>
<tr>
<td>0.008</td>
<td>0.010</td>
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### Graphical Representation

- **ρ₁ (fm⁻²)**: The graph shows the density of the first octet component as a function of the distance `b` in femtometers (fm). The curves represent contributions from different models: VM (green), SU(3) meson loops (blue), and the total (red).
- **ρ₁ Λ (fm⁻²)**: Similar to `ρ₁`, but for the lambda component.
- **ρ₁ Σ₀ (fm⁻²)**: Graph for the sigma-zero component.
- **ρ₁ Ξ⁻ (fm⁻²)**: Graph for the xi-negative component.
Octet magnetic densities

\[ \rho_{\Sigma} (\text{fm}^{-2}) \]

\[ \rho_{\Sigma} (\text{fm}^{-2}) \]

\[ \rho_{\Sigma} (\text{fm}^{-2}) \]

\[ \rho_{\Sigma} (\text{fm}^{-2}) \]
Octet magnetic densities

\[ \rho^\Lambda (\text{fm}^{-2}) \]

\[ \rho^\Sigma_0 (\text{fm}^{-2}) \]

\[ \rho^\Xi_0 (\text{fm}^{-2}) \]

\[ \rho^\Xi^- (\text{fm}^{-2}) \]

Graphs showing the magnetic densities for VM, SU(3), and Total for different values of \( b \) (fm).
Octet Im($F_1$)

Contributions to $\frac{\text{Im} F_1}{\pi} (\Lambda)$

Contributions to $\frac{\text{Im} F_1}{\pi} (\Sigma^0)$

Contributions to $\frac{\text{Im} F_1}{\pi} (\Xi^-)$
Octet $\text{Im}(F_2)$
Octet $\text{Im}(F_2)$

Contributions to $\text{Im}F_2^\Lambda(\pi)$

Contributions to $\text{Im}F_2^\Sigma_0(\pi)$

Contributions to $\text{Im}F_2^{\Xi_0}(\pi)$

Contributions to $\text{Im}F_2^{\Xi^-}(\pi)$