Precision light and heavy meson physics from Lattice QCD

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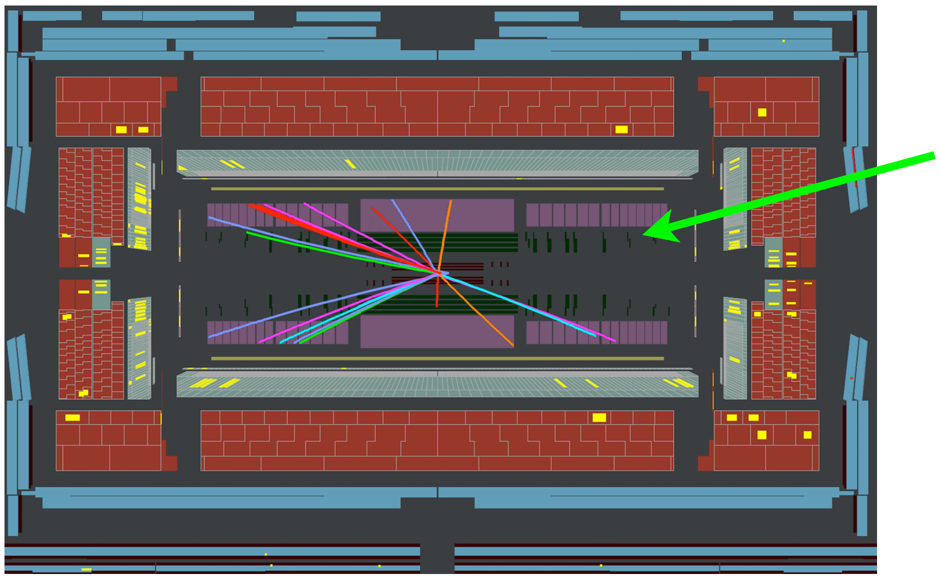
QCD is a key part of the Standard Model but quark confinement is a complication/interesting feature.

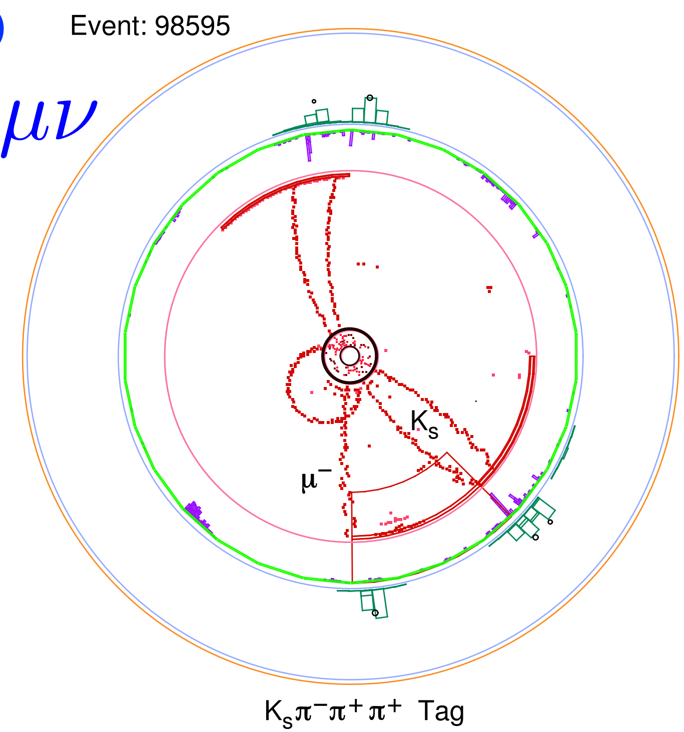


ATLAS@LHC

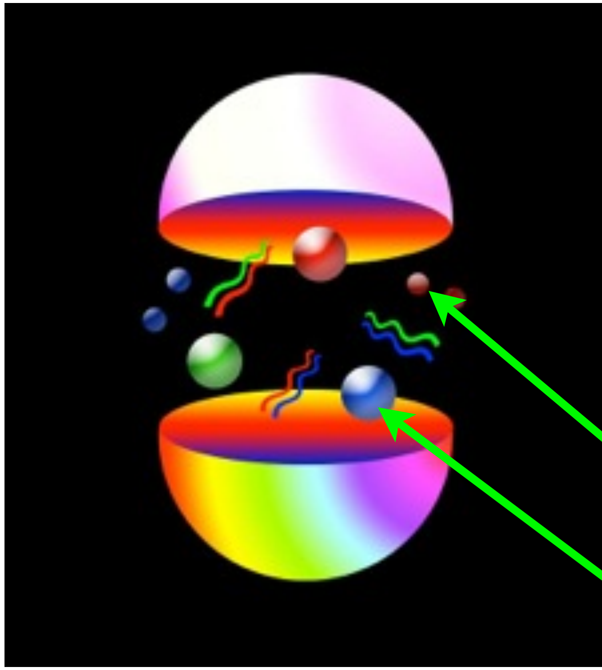
Connecting observed hadron properties to those of quarks requires full nonperturbative treatment of Quantum Chromodynamics.

CLEO

$D \to \mu\nu$



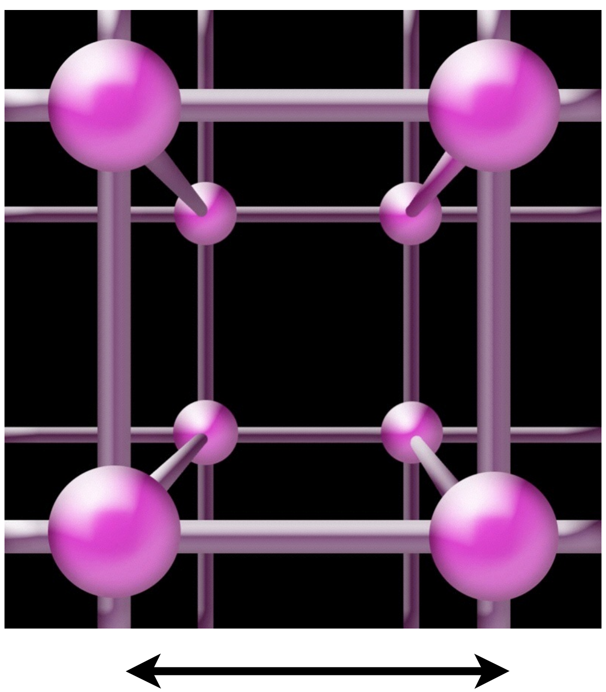
Some properties of hadrons can be very accurately measured and are calculable in lattice QCD - can test SM and determine parameters very accurately (1%).



Lattice QCD: fields defined on 4-d discrete space-(Euclidean) time. Lagrangian parameters: $\alpha_s, m_q a$

1) Generate sets of gluon fields for Monte Carlo integrn of Path Integral (inc effect of u, d, s, (c) sea quarks)

2) Calculate valence quark propagators and combine for “hadron correlators” . Fit for hadron masses and amplitudes



a

• Determine a to convert results in lattice units to physical units. Fix m_q from hadron mass

numerically extremely challenging

• cost increases as $a \to 0, m_{u/d} \to$ phys and with statistics, volume.

Darwin@Cambridge, part of UK's £15m HPC facility for theoretical particle physics and astronomy

DiRAC

www.dirac.ac.uk

Allows us to calculate quark propagators rapidly and store them for flexible re-use.

State-of-the-art commodity cluster: 9600 Intel Sandybridge cores, infiniband interconnect, fast switch and 2 Pbytes storage

Quark formalisms

Many ways to discretise Dirac Lagrangian onto lattice.
All should give same answers.

Issues are: Discretisation errors at power a^n
Numerical speed of matrix inversion
Chiral symmetry
Quark doubling

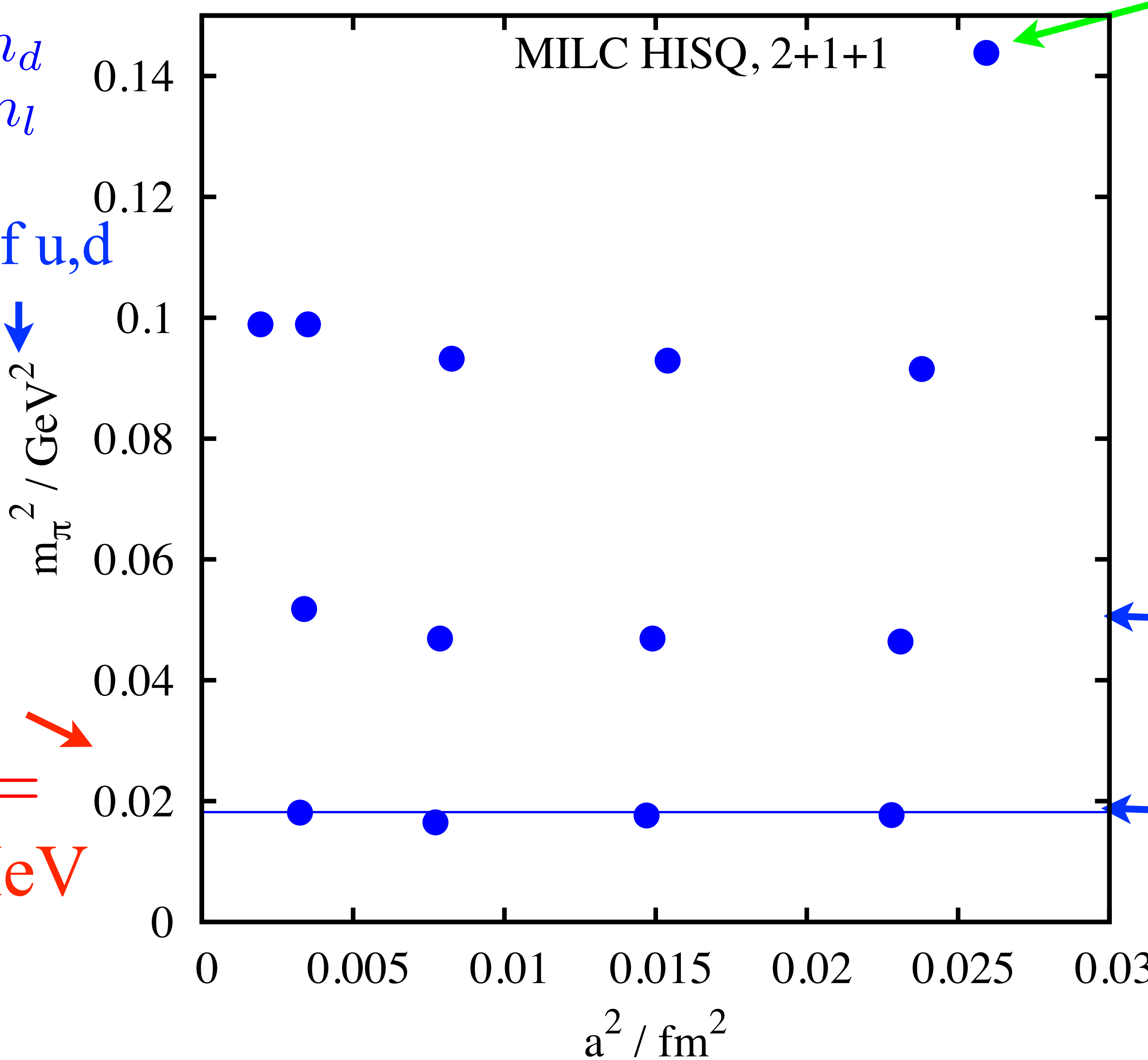
We use Highly Improved Staggered Quarks (HISQ) for u, d, s and c. Also (with extrapolation) for b.
Disc. errors $\alpha_s^2 a^2, a^4$. Numerically fast. Chiral symm.
Some complications from doublers ('tastes').

For b quarks, also use NRQCD. Fast for b quarks at moderate a, disc. errors now $\alpha_s^2 a^2$. Current normln perturbative, through α_s

Example parameters for ‘2nd generation’ calculations now being done with staggered quarks.

$m_u = m_d = m_l$

mass of u,d quarks



real world $m_{\pi^0} =$ 135 MeV

“2nd generation” lattices inc. c quarks in sea

HISQ = Highly improved staggered quarks - very accurate discretisation

E.Follana, et al, HPQCD, hep-lat/0610092.

$m_{u,d} \approx m_s/10$

$m_{u,d} \approx m_s/27$

Volume: $m_\pi L > 3$

other formalisms: clover (Fermilab), domain-wall, twisted-mass …

Hadron correlation functions ('2point functions') give masses and decay constants.

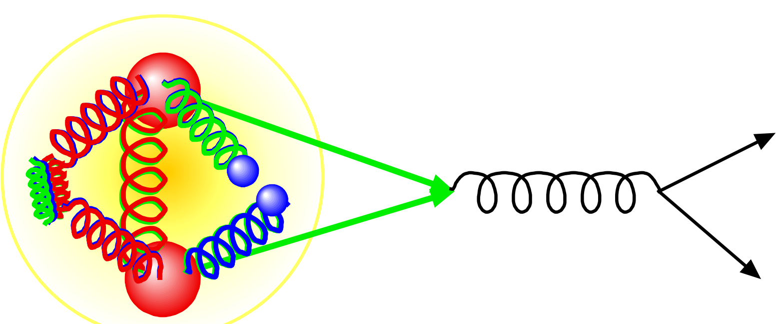
$$\langle 0|H^\dagger(T)H(0)|0\rangle = \sum_n A_n e^{-m_n T} \xrightarrow{\text{large } T} A_0 e^{-m_0 T}$$

H  H

masses of all hadrons with quantum numbers of H

$$A_n = \frac{|\langle 0|H|n\rangle|^2}{2m_n} = \frac{f_n^2 m_n}{2}$$

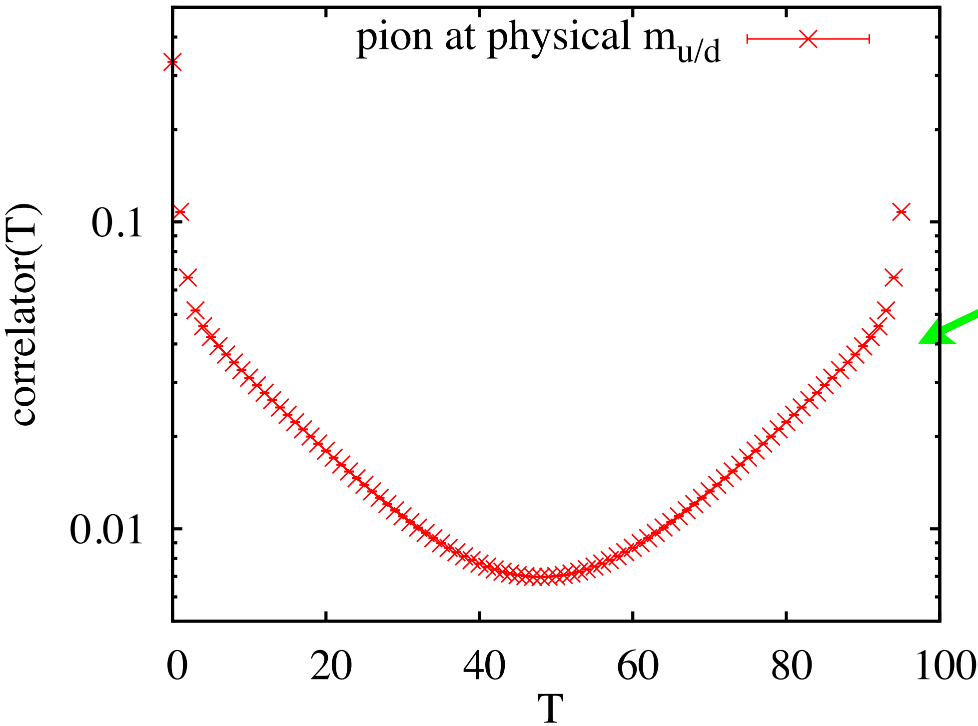
decay constant parameterises amplitude to annihilate - a property of the meson calculable in QCD. Can often relate to experimental decay rate.



1% accurate experimental info. for f and m for many mesons! Need accurate determination from lattice QCD to match

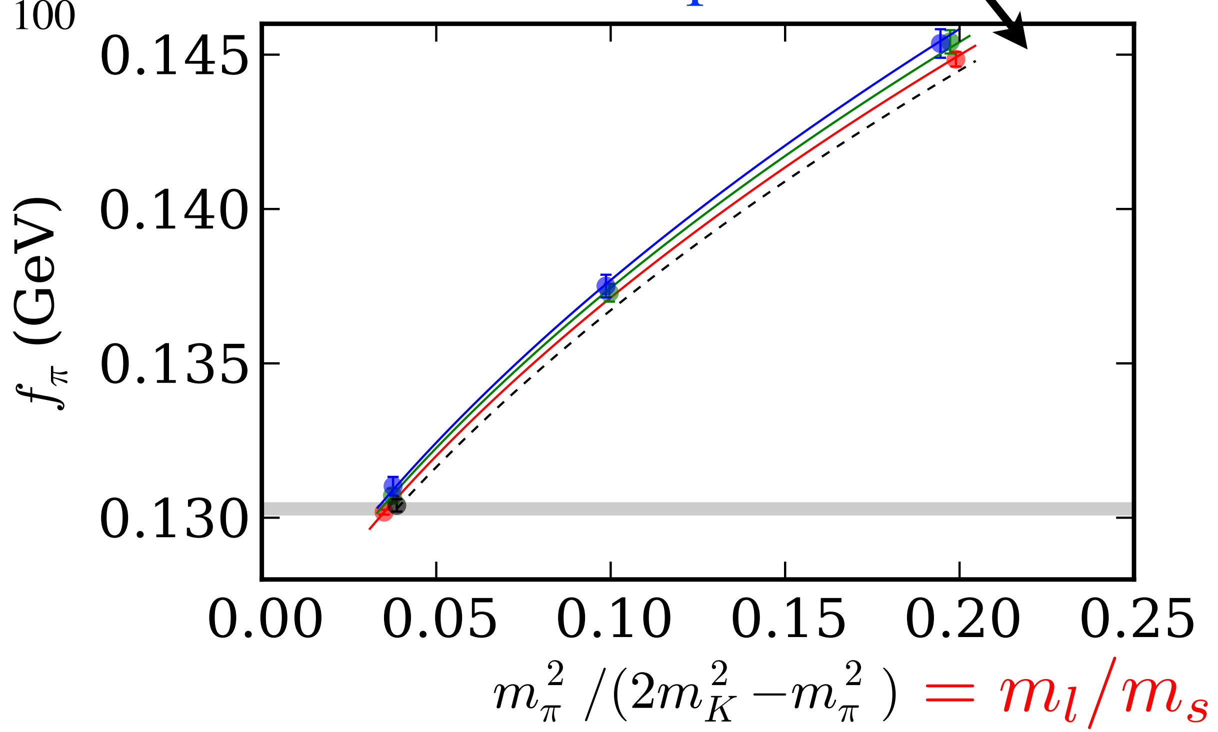
Example (state-of-the-art) calculation

R. Dowdall et al, HPQCD, 1303.1670.



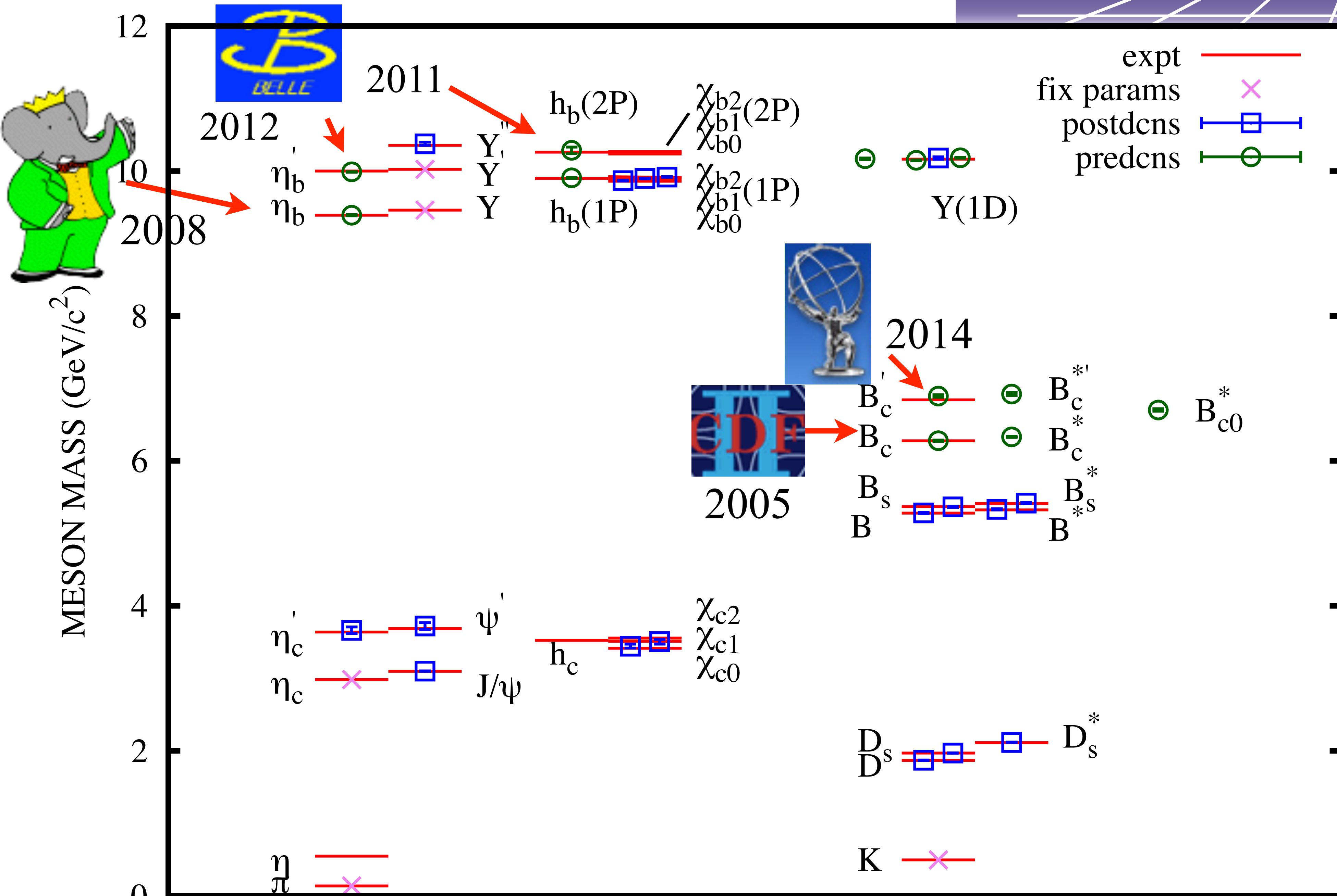
Extract meson mass and amplitude=decay constant from correlator for multiple lattice spacings and $m_{u/d}$. Very high statistics (16,000 samples of the correlator)

Extrapolation in a



Convert decay constant to GeV units using w_0 to fix relative lattice spacing. Very small discretisation errors. PCAC reln means no renormln factors needed.

The gold-plated meson spectrum

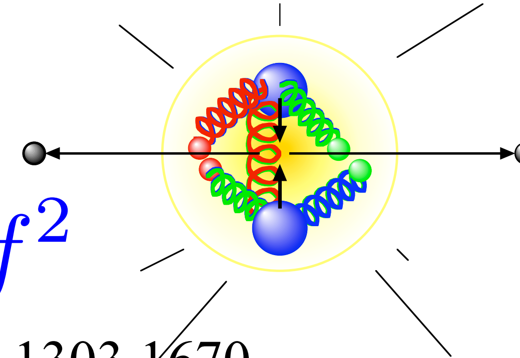


Uncties at few MeV level : Future: inc QED and $m_u \neq m_d$

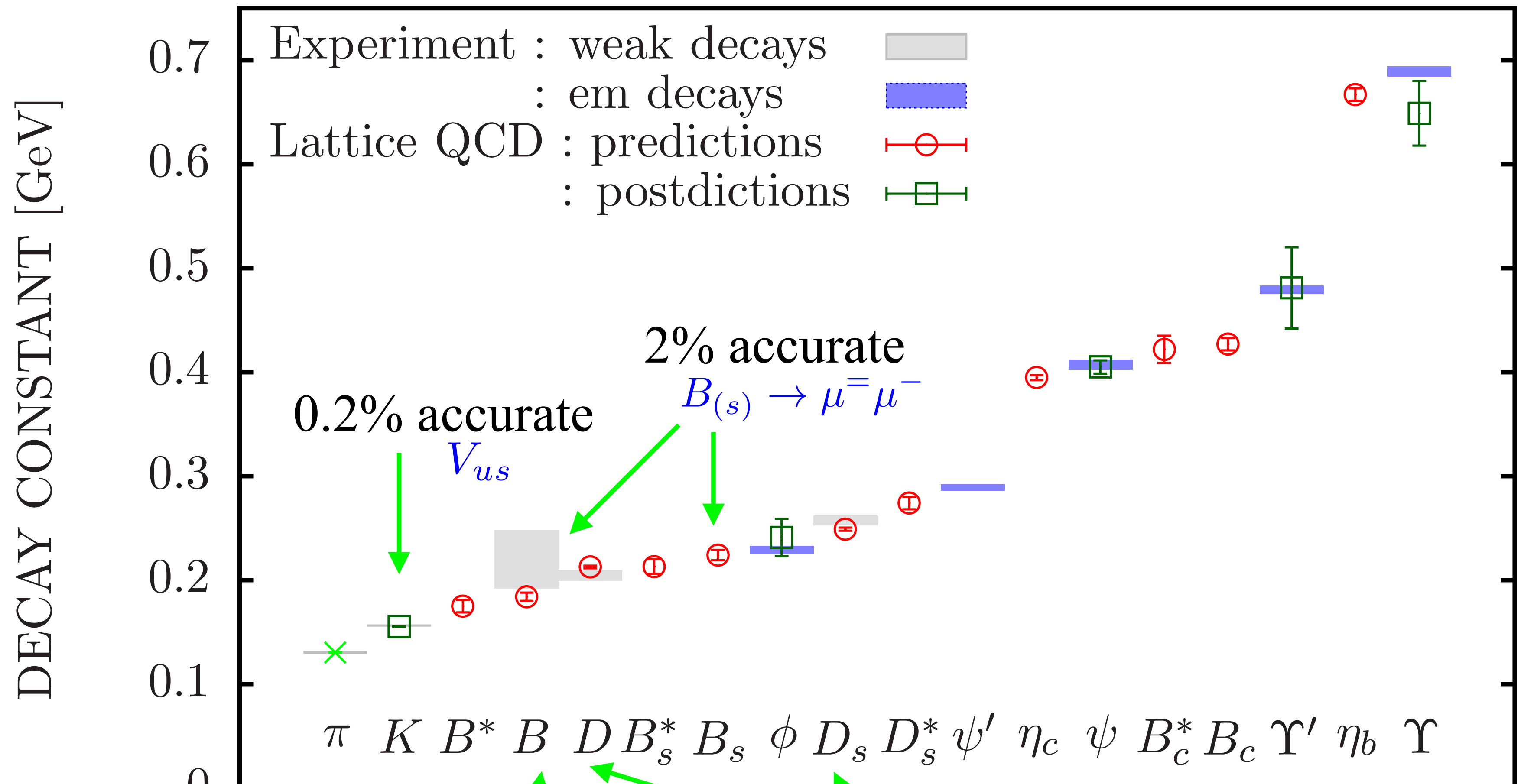
Meson decay constants

Parameterises hadronic information needed for annihilation rate to W or photon:

$$\Gamma \propto f^2$$



HPQCD 1503.05762, 1408.5768, 1302.2644, 1303.1670



decay constants of vector mesons now being pinned down

2012

BELLE $B \to \tau\nu$ V_{ub}

0.5% accuracy from lattice QCD now : **FNAL/MILC** 1407.3772

BES will improve expt. V_{cd} V_{cs}

Vector meson decay constants HPQCD 1312.5264, 1503.05762

We calculate:

$$f_{D_s^*}/f_{D_s} = 1.10(2)$$

allows us to fix total width

$$\Gamma(D_s^* \to D_s\gamma) = 0.066(26)\text{keV}$$

may be possible to see this?

$$\text{Br}(D_s^* \to \ell\nu) = 3.4(1.4) \times 10^{-5}$$

We also find:

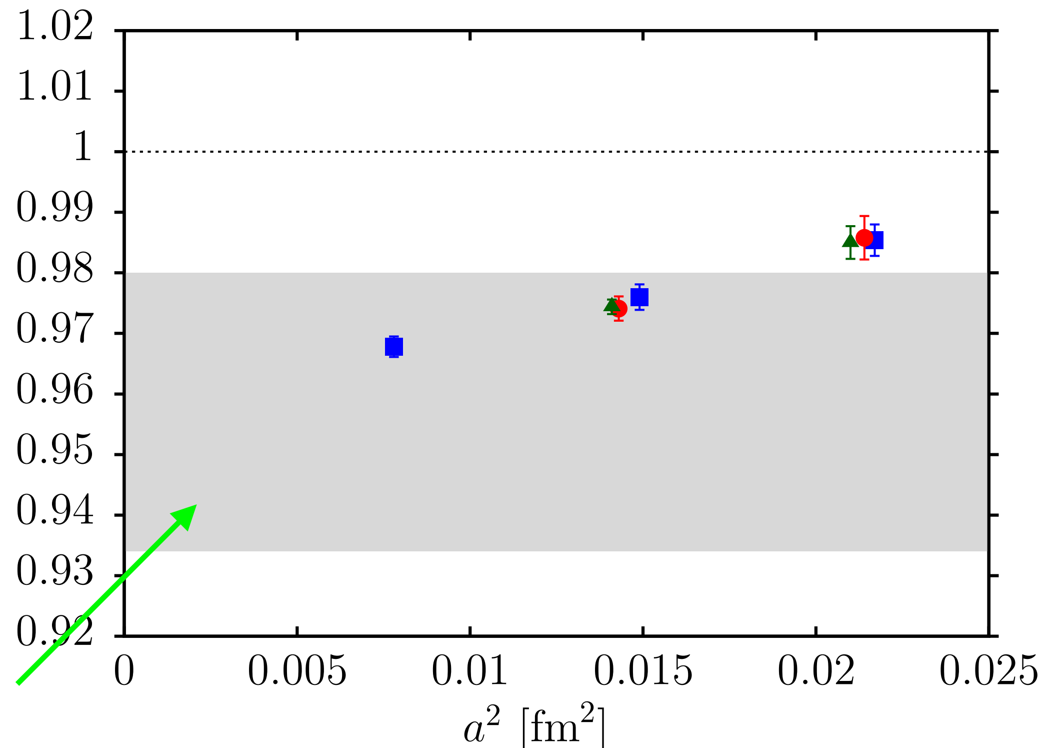
$$f_{B_s^*}/f_{B_s} = 0.953(23)$$

$$< 1$$

can test HQET

R_s

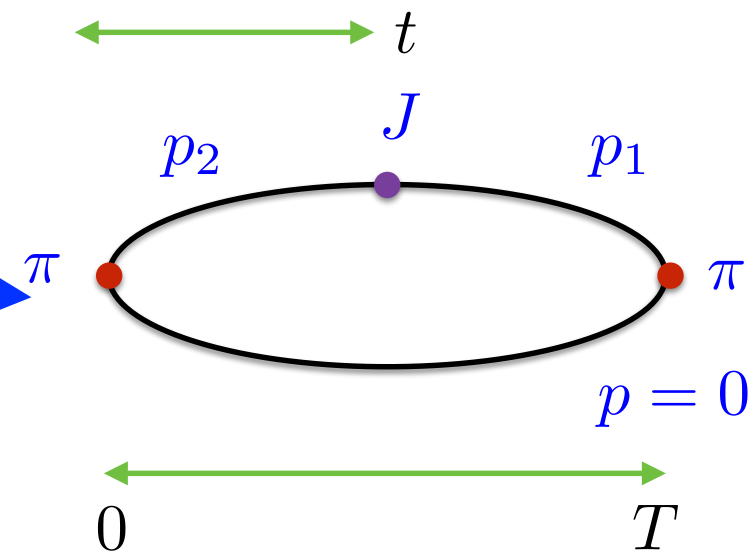
$$\frac{f_{B_s^*}\sqrt{M_{B_s^*}}}{f_{B_s}\sqrt{M_{B_s}}}$$



pert. matching uncty dominates

Further probes of hadron structure: form factors as functions of q^2

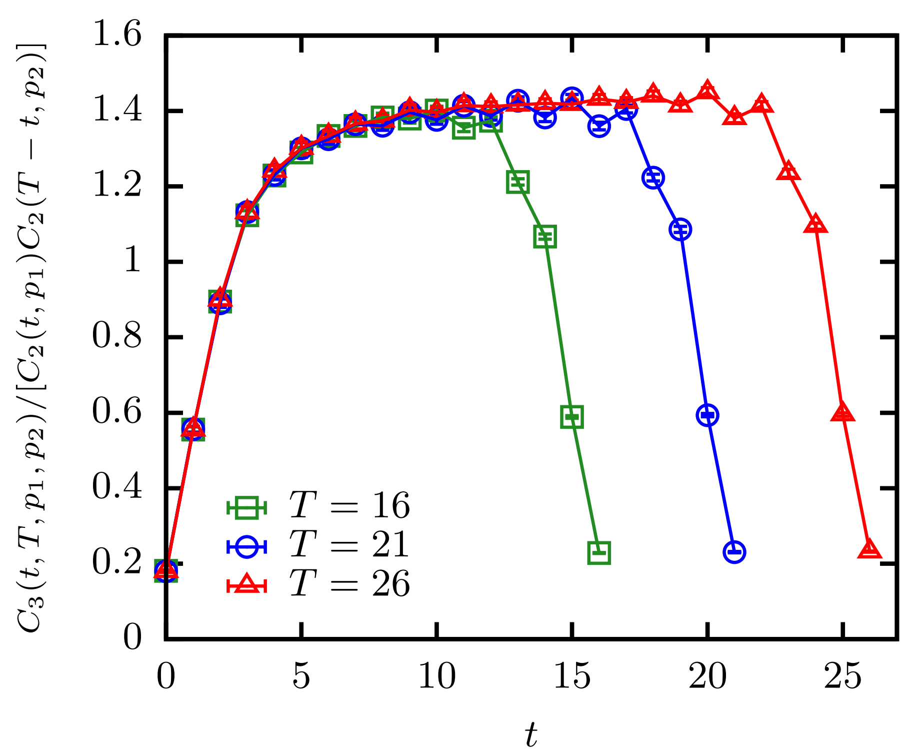
Needs '3-point' functions
e.g. for pion to pion transition
via vector current J



Need to calculate correlators for multiple T values and 0<t<T and fit as a function of t, T simultaneously with 2pt.

$$C_{3pt} = \sum_{i,j} b_i J_{ij} b_j e^{-E_i t} e^{-E_j (T-t)}$$

$$\langle \pi | V_\mu | \pi \rangle / (2Z\sqrt{E_i E_j})$$

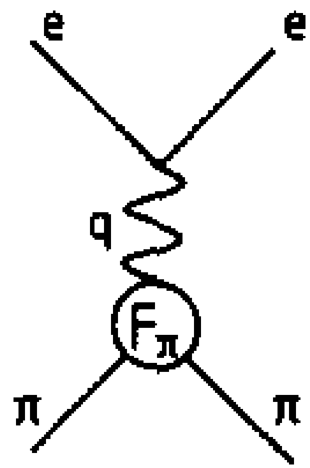


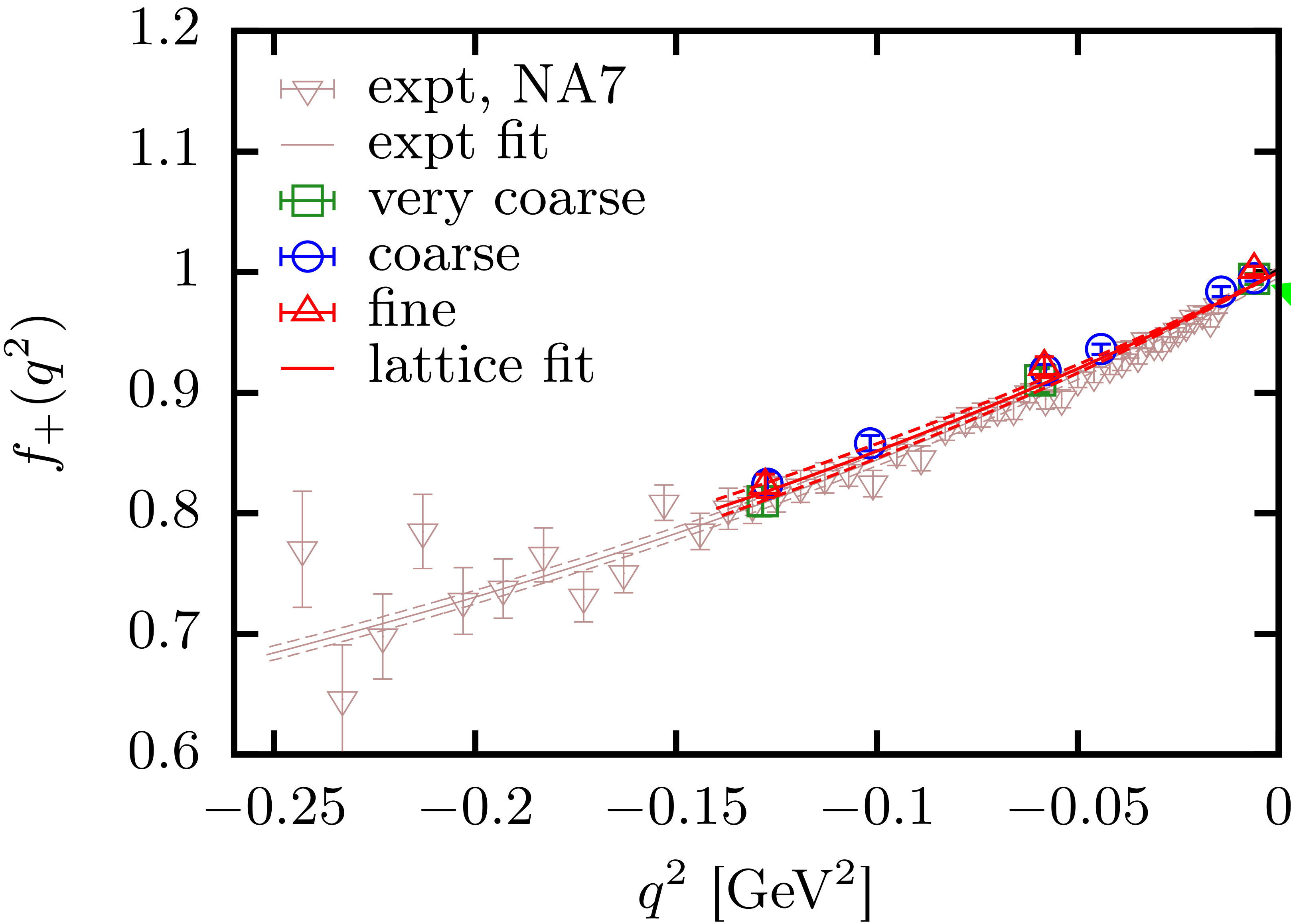
Normln of J must be fixed, e.g. here $f_+(0) = 1$ from charge cons.

π electromagnetic form factor at small q^2

$\pi - e$ scattering probes π electric charge distn

$$\langle \pi(p_1)|V_\mu|\pi(p_2)\rangle = f_+(q^2)(p_1+p_2)_\mu$$

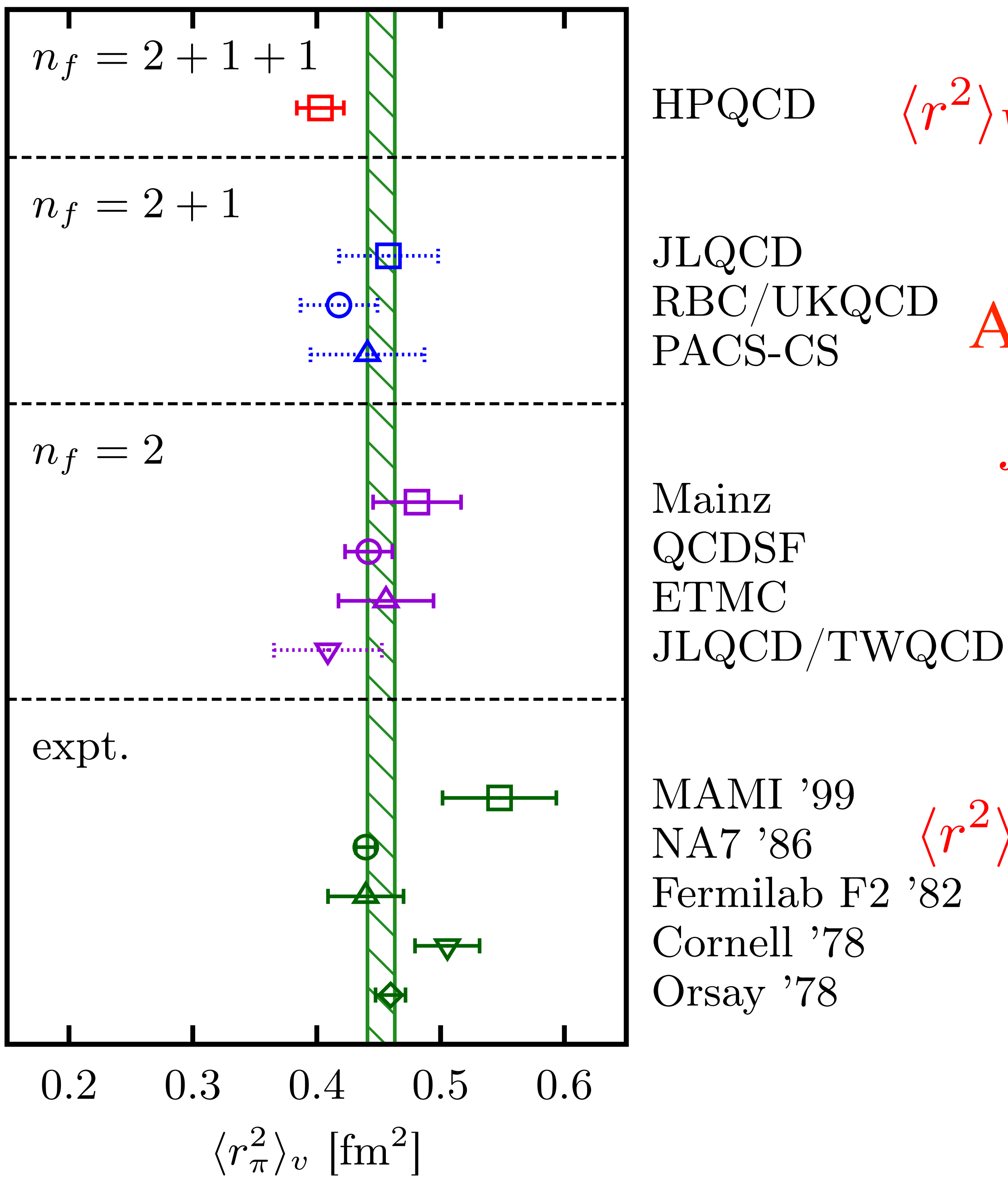




Working at **physical** u/d quark masses on HISQ 2+1+1 configs, lattice QCD raw results on top of experiment

J. Koponen et al, HPQCD, 1511.07382

J. Koponen et al, HPQCD, 1511.07382



$$\langle r^2 \rangle_{V,\mathrm{lat}} = 0.403(18)(6)\mathrm{fm}^2$$

At small q^2 can fit pole:

$$f_+(q^2) = \frac{1}{1 - q^2/M^2}$$

$$M^2 = 6/\langle r^2 \rangle \approx M_\rho^2$$

$$\langle r^2 \rangle_{V,\mathrm{NA7}} = 0.431(10)\mathrm{fm}^2$$

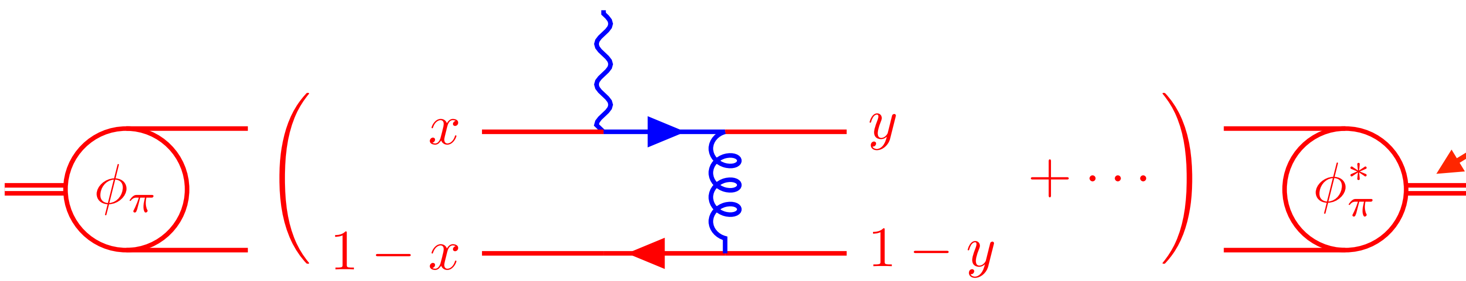
Meson form factors at high (space-like) q^2

To ‘understand’ form factor want to map it experimentally *and* theoretically up to high q^2 where perturbative QCD becomes valid. But where is this point?

EXPT: JLAB E12-06-101

Perturbative QCD: High q^2 photon must be accompanied by high momentum gluon exchange Lepage, Brodsky, PRD22(1980)2157

$$Q^2 = -q^2 \to \infty$$



coeffs of expansion of distn amplitude in Gegenbauer polyn., run to 0 with inc. Q^2

$$f_+(q^2) = F_\pi(Q^2) = \frac{8\pi f_\pi^2 \alpha_s(Q/2)}{Q^2} \left| 1 + \sum_{n=2}^{\infty} a_n^\pi(Q/2) \right|^2$$

Enter lattice QCD …

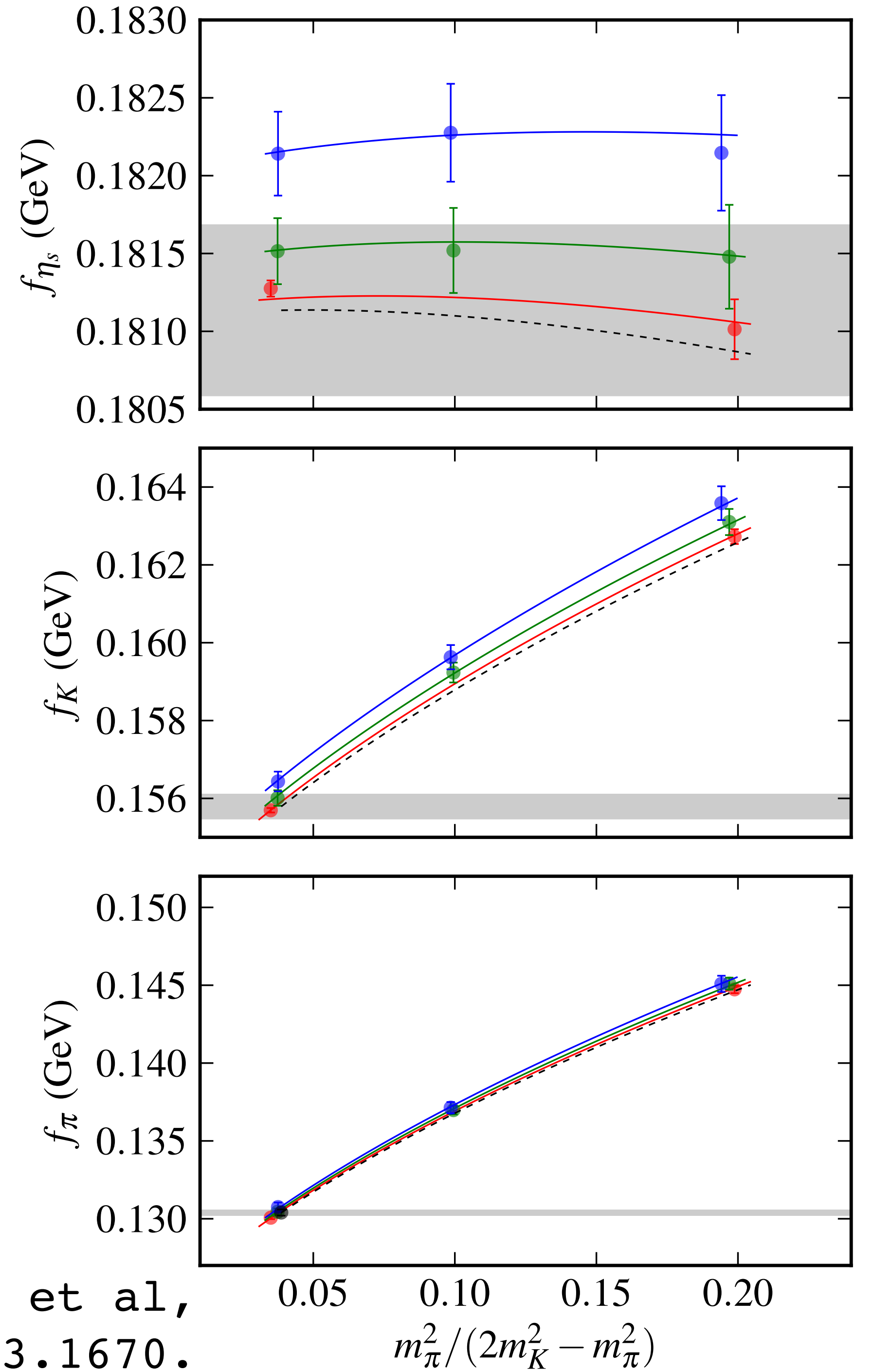
J. Koponen et al, HPQCD, 1701.04250.
See also: Bonnet et al, 0411028.

Need a formalism with small discretisation errors that is numerically fast. *HISQ*

Instead of π use η_s pseudoscalar made of s quarks for added speed.

η_s prevented from mixing with light states on lattice so not physical, but properties can be mapped out. $M_{\eta_s} = 688.5(2.2)\ \mathrm{MeV}$

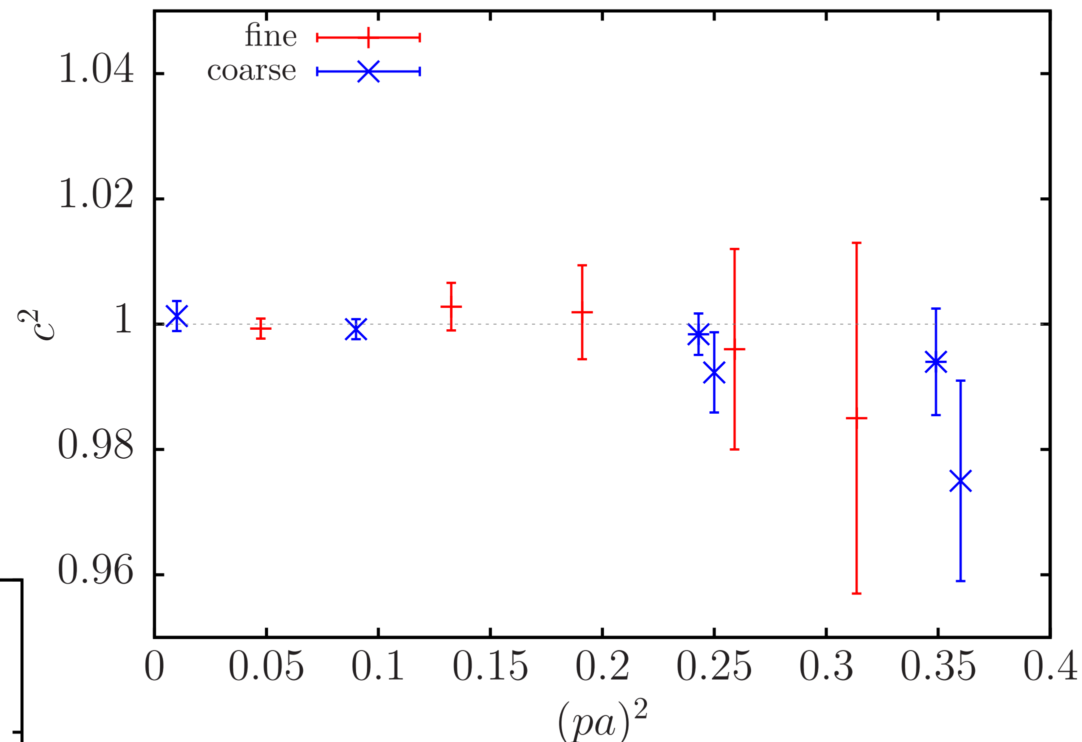
$f_{\eta_s} = 181.14(55)\ \mathrm{MeV}$

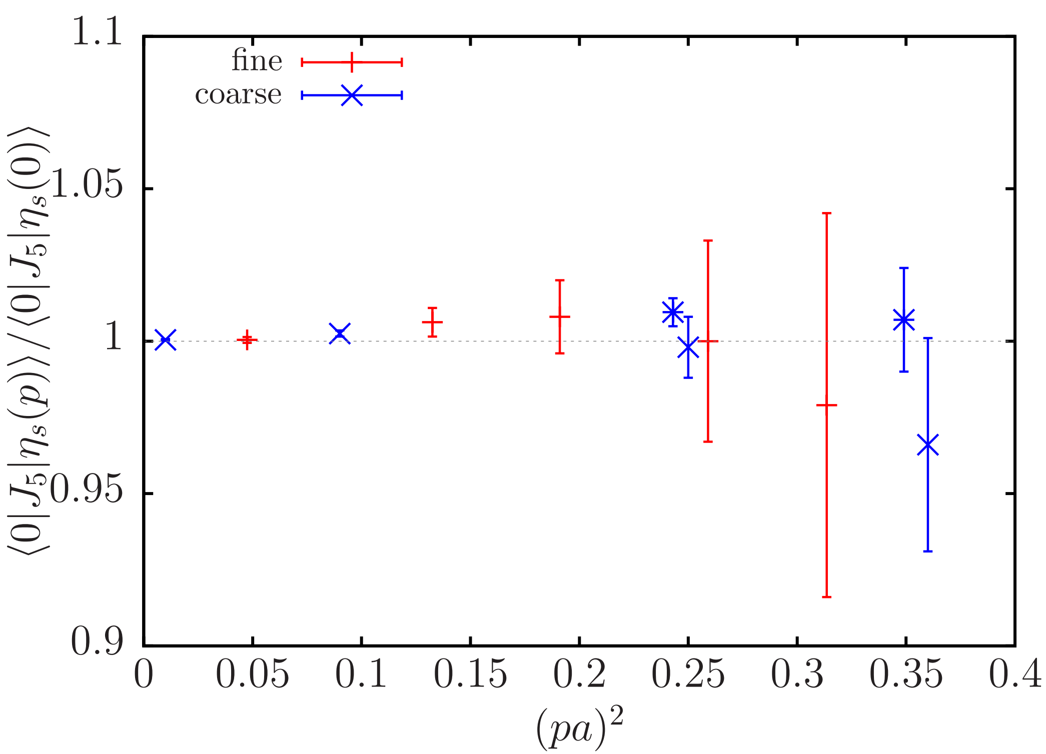


R. Dowdall et al, HPQCD, 1303.1670.

Work at two values of lattice spacing (0.12fm and 0.09fm) and two values of light sea quark mass ($m_s/5$ and $m_s/10$)

Use 'twisted boundary conditions' to insert momentum and test discretisation errors as a function of (pa)





For 3pt functions use Breit frame $\vec{p}_i = -\vec{p}_f$

Maximises Q^2 for a given (pa)

J. Koponen et al, HPQCD, 1701.04250.

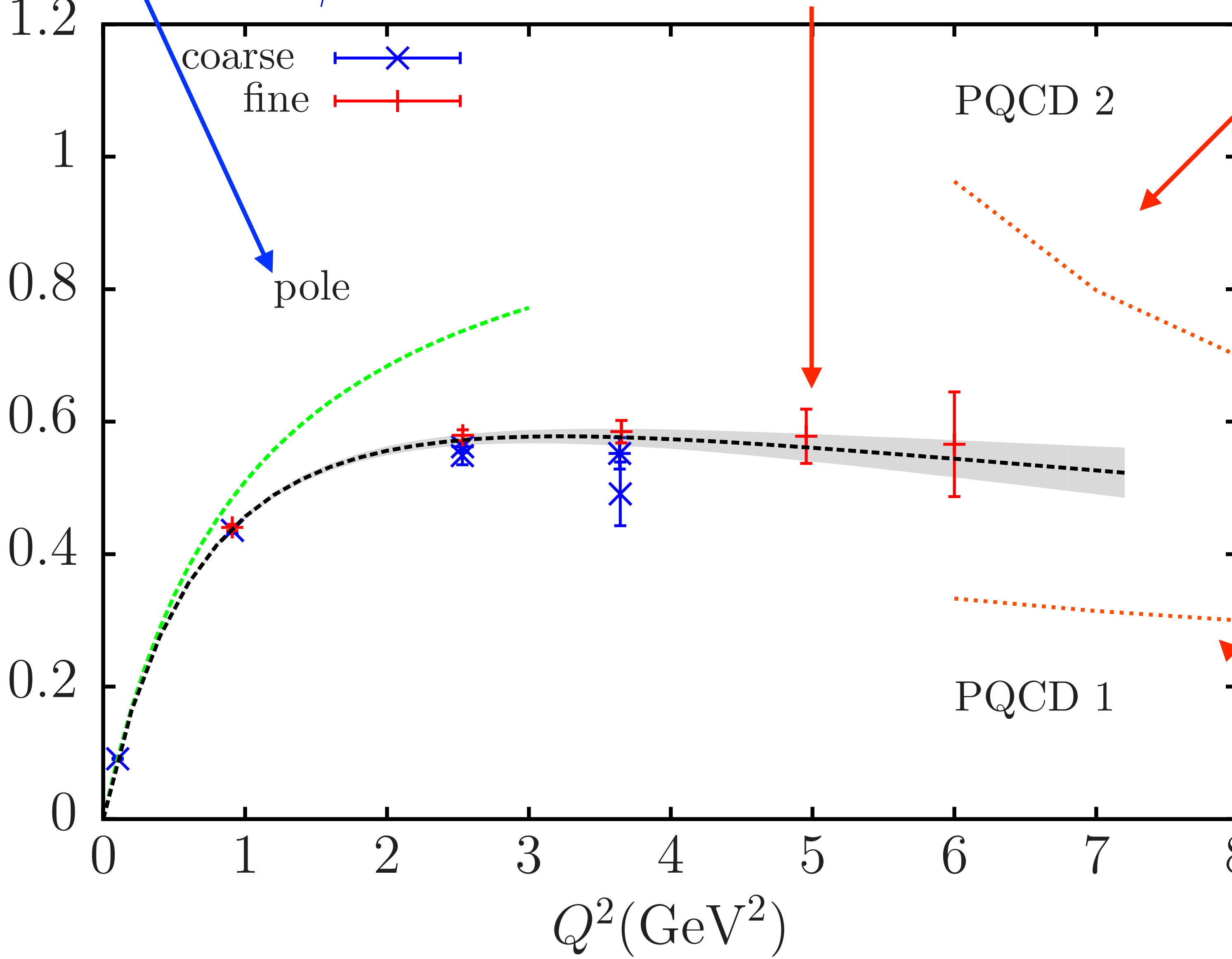
Results - Can reach Q^2 of 6 GeV^2 for (pa)<0.6

Disc. and sea quark mass effects very small

$$F = \frac{1}{1 + Q^2/M_\phi^2}$$

physical point limit of 'z-expansion' fit

pert. QCD inc. $\phi_\pi(2\mathrm{GeV}) = [x(1-x)]^{0.52}$



Braun et al, 1503.03656

asymp. pert. QCD

J. Koponen et al, HPQCD, 1701.04250.

z-expansion

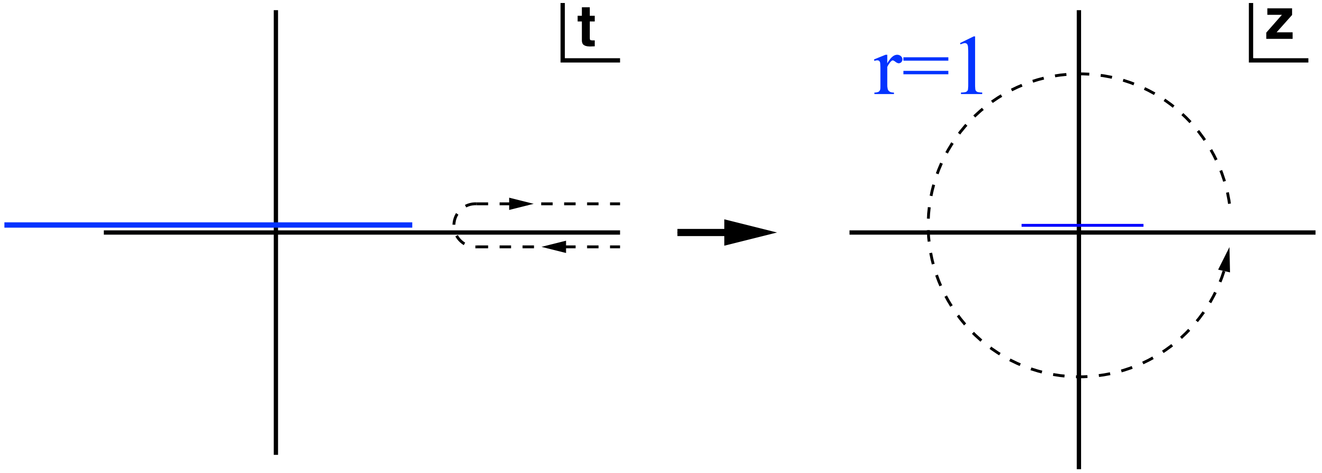
$$z(t, t_{cut}) = \frac{\sqrt{t_{cut} - t} - \sqrt{t_{cut}}}{\sqrt{t_{cut} - t} + \sqrt{t_{cut}}}$$

$$t = q^2$$

$$t_{cut} = 4M_K^2$$

Maps t region into

-1 < z < 1

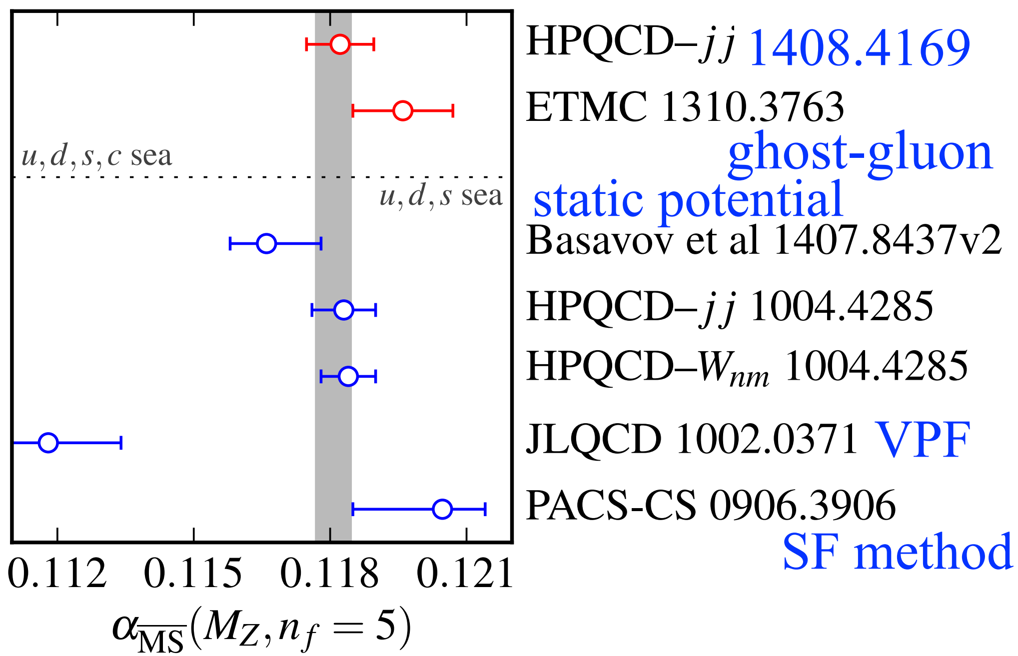


Fit:

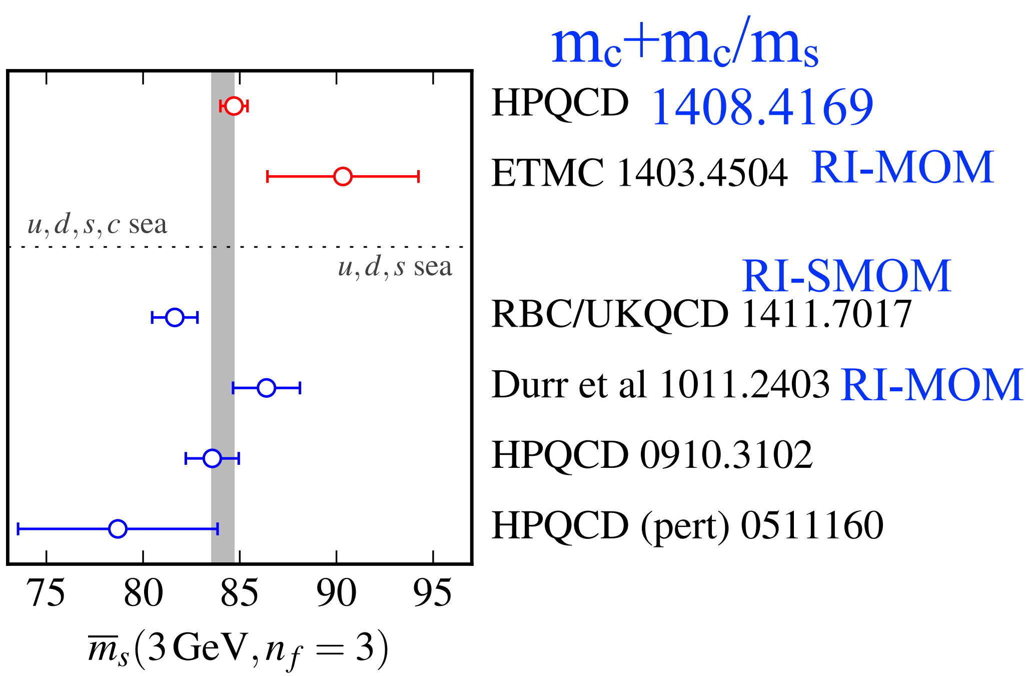
$$(1 + Q^2/M_\phi^2)F = 1 + \sum_i z^i A_i \left[1 + B_i(a\Lambda)^i + C_i(a\Lambda)^4 + D_i \frac{\delta m_{sea}}{10}\right]$$

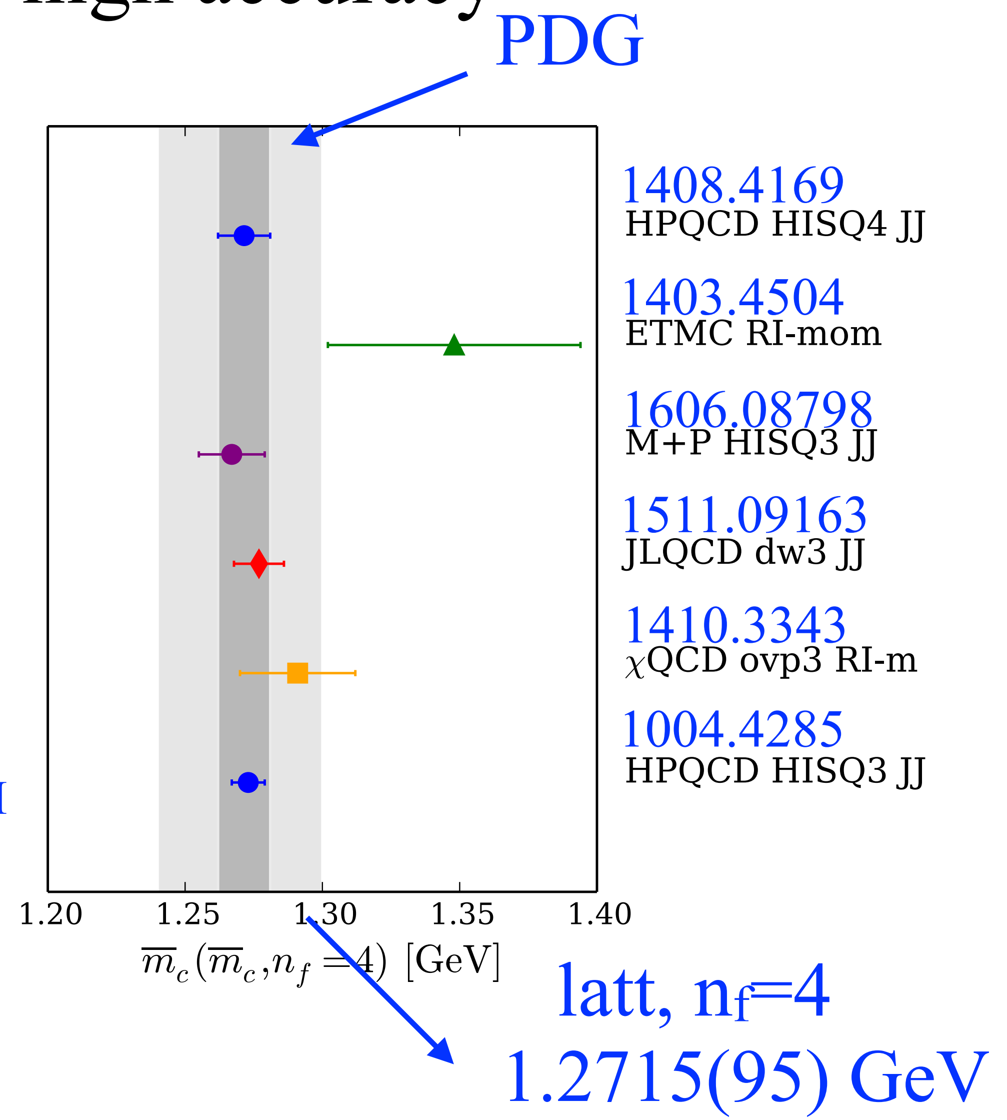
Further tests of pert. QCD underway: scaling with decay constant as change mass and ff of helicity-nonconserving scalar current.

Quark masses and strong coupling constant

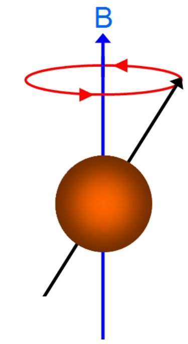


Multiple lattice QCD methods now that agree to high accuracy





Muon anomalous magnetic moment



$$\vec{\mu} = g\frac{e}{2m}\vec{S} \qquad a_\mu = \frac{g-2}{2}$$

Measure using polarised muons circulating in E and B fields. At a momentum where $\beta \times E$ terms cancel, difference between precession and cyclotron frequencies:

$$\omega_a = -\frac{e}{m}a_\mu B$$

BNL result:

$$a_\mu^{expt} = 11659208.9(6.3) \times 10^{-10}$$

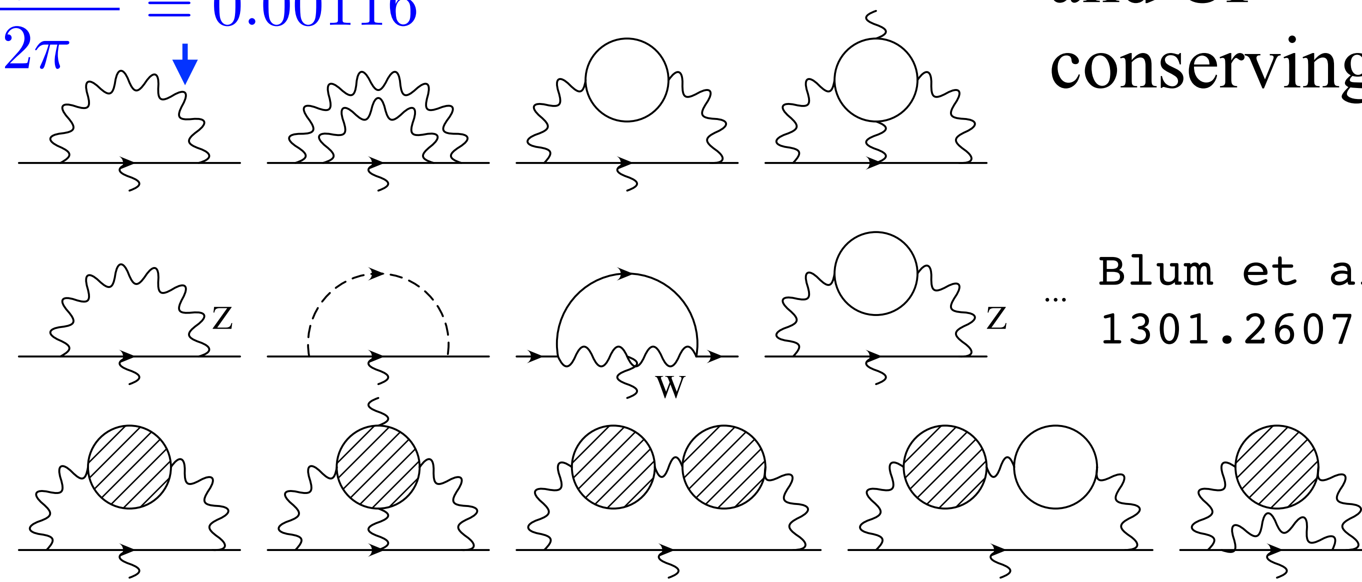
E989 (FNAL) will reduce exptl uncty to 1.6, starting 2017

Standard Model theory expectations

Contributions from QED, EW and QCD interactions. QED dominates. QCD contribs start at α_{QED}^2

$$\frac{\alpha_{QED}}{2\pi} = 0.00116$$

flavour and CP conserving



…

Blum et al, 1301.2607

LO Hadronic vacuum polarisation (HVP) dominates uncertainty in SM result

$$a_\mu^{QED} = 11658471.885(4) \times 10^{-10}$$

$$a_\mu^{EW} = 15.4(2) \times 10^{-10}$$

$$a_\mu^{E821} = 11659208.9(6.3) \times 10^{-10}$$

Hadronic (and other) contributions = EXPT - QED - EW

$$a_\mu^{expt} - a_\mu^{QED} - a_\mu^{EW} = 721.7(6.3) \times 10^{-10}$$

$$= a_\mu^{HVP} + a_\mu^{HOHVP} + a_\mu^{HLBL} + a_\mu^{new\,physics}$$

Focus on lowest order hadronic vacuum polarisation,
so assume:

$$a_\mu^{HLbL} = 10.5(2.6) \times 10^{-10}$$

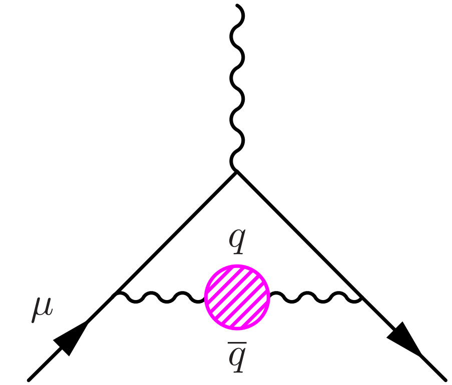
$$a_\mu^{HOHVP} = -8.85(9) \times 10^{-10}$$

NLO+NNLO

Kurz et al,
1403.6400

$$a_\mu^{HVP,no\,new\,physics} = 719.8(6.8) \times 10^{-10}$$

Lattice calculation of HVP,LO



Analytically continue to Euclidean q².

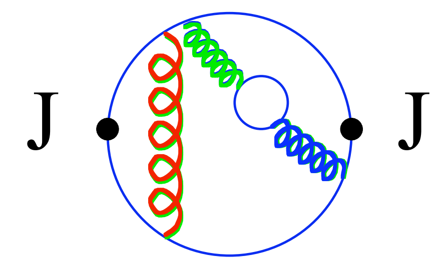
$$a_\mu^{HVP,i} = \frac{\alpha}{\pi}\int_0^\infty dq^2 f(q^2)(4\pi\alpha e_i^2)\hat{\Pi}_i(q^2)$$

Blum, hep-lat/ 0212018

connected contribution for flavour i

f(q²) divergent function with scale set by m_μ

$$\hat{\Pi}(q^2) = \Pi(q^2) - \Pi(0)$$



HPQCD method: time-moments of vector JJ correlators give expansion around q²=0

$$G_n \equiv \sum_{t,\vec{x}} t^n Z_V^2 \langle J^j(\vec{x},t)J^j(0)\rangle \qquad \hat{\Pi}(q^2) = \sum_{k=1}^{\infty} q^k \Pi_k$$

$$\Pi_k = (-1)^{k+1}\frac{G_{2k+2}}{(2k+2)!}$$

Chakraborty et al, HPQCD, 1403.1778

replace with [2,2] Padé

HPQCD 1403.1778, 1408.5768

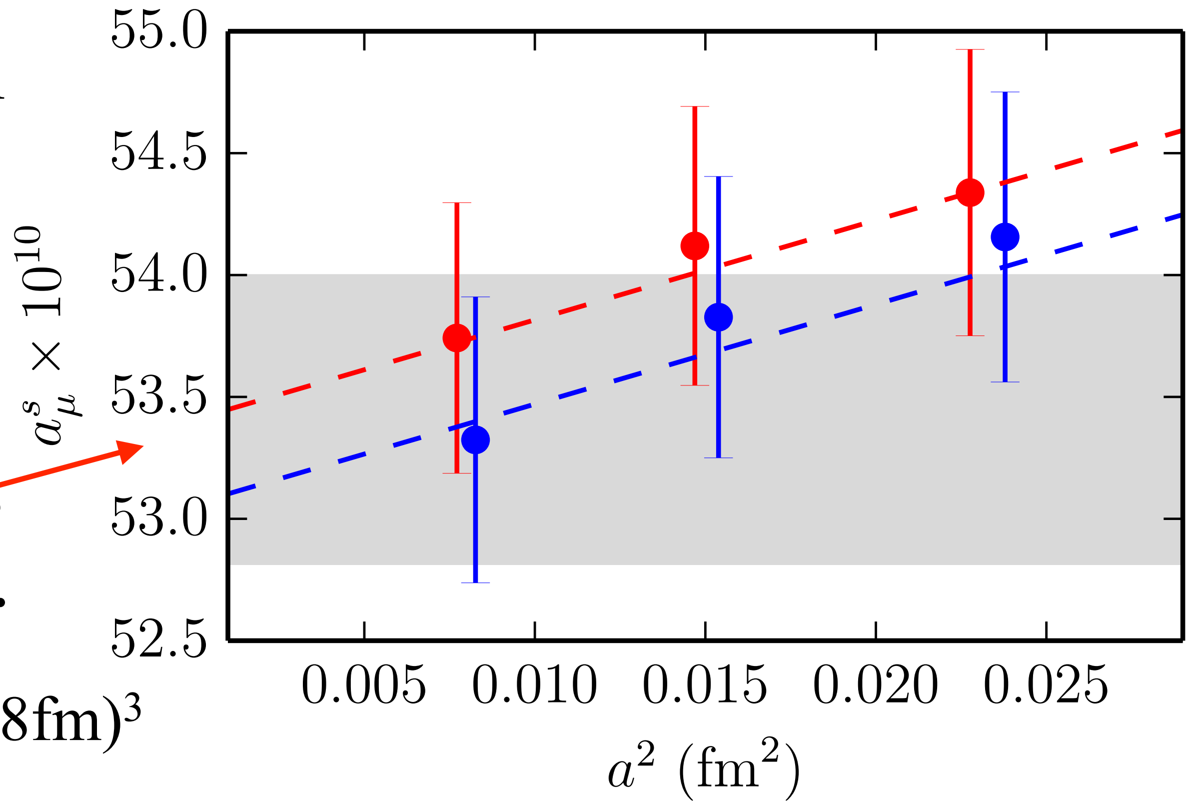
Test on STRANGE contribution

HISQ valence quarks on MILC 2+1+1 HISQ configs. Local J_V - nonpert. Z_V.

multiple a (fixed by w_0), m_l (inc. phys.), volumes.

up to $(5.8\mathrm{fm})^3$

Tune s from η_s



	a_μ^s
Uncertainty in lattice spacing (w_0, r_1):	1.0%
Uncertainty in Z_V:	0.4%
Monte Carlo statistics:	0.1%
$a^2 \to 0$ extrapolation:	0.1%
QED corrections:	0.1%
Quark mass tuning:	0.0%
Finite lattice volume:	< 0.1%
Padé approximants:	< 0.1%
Total:	1.1%

$$a_\mu^{HVP,s} = 53.41(59) \times 10^{-10}$$

Also

$$a_\mu^{HVP,c} = 14.4(4) \times 10^{-10}$$

$$a_\mu^{HVP,b} = 0.27(4) \times 10^{-10}$$

NRQCD

UP/DOWN contribution $m_u = m_d$

Chakraborty et al, HPQCD, 1601.03071

Much noisier and sensitive to u/d mass. Use

$$G(t) = \begin{cases} G_{\mathrm{data}}(t) & \text{for } t \le t^* \\ G_{\mathrm{fit}}(t) & \text{for } t > t^* \end{cases}$$

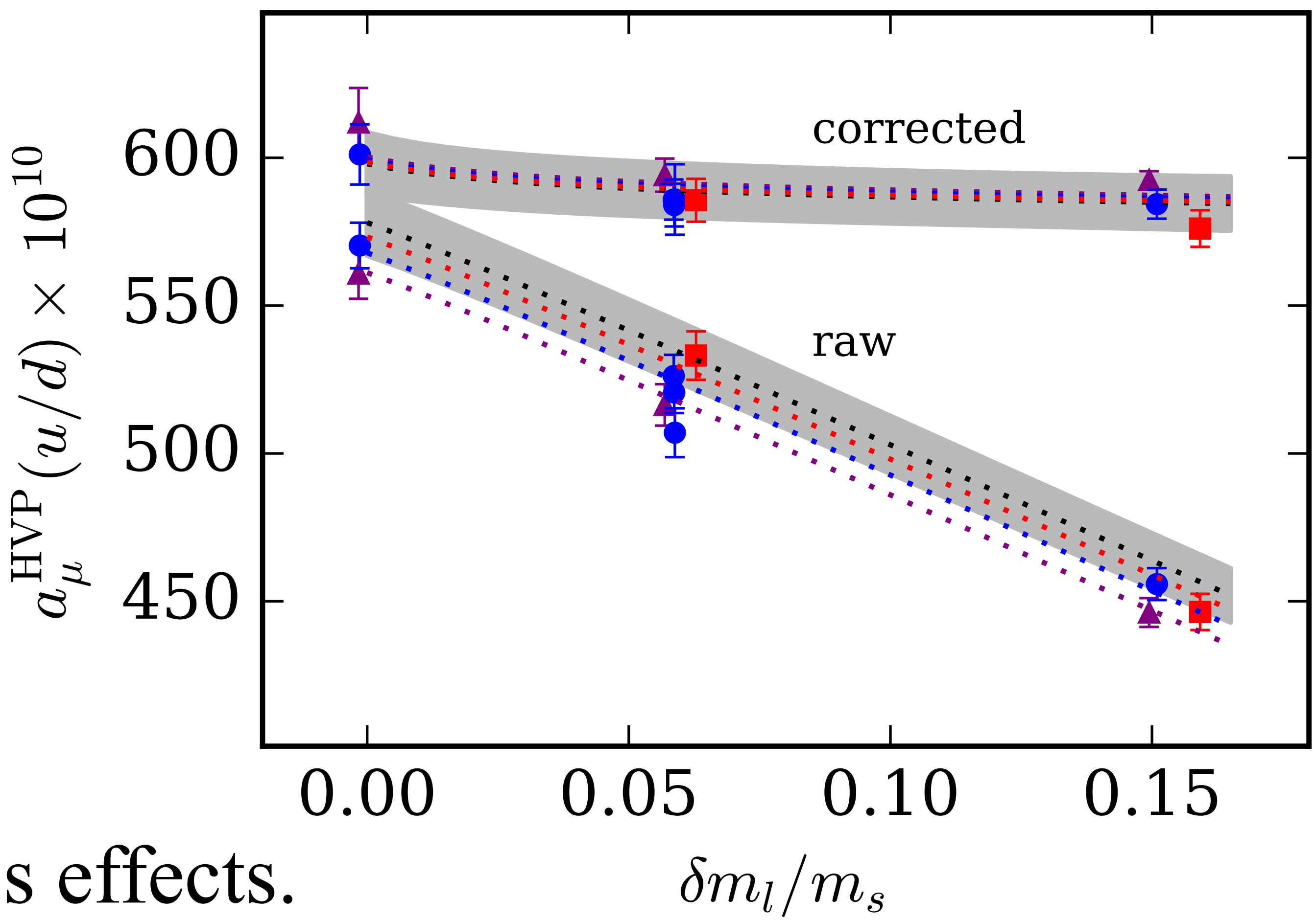
from Monte Carlo

from multi-exponential fit

$t^* = 1.5\mathrm{fm} = 6/m_\rho$ so 70% of result from G_{data}

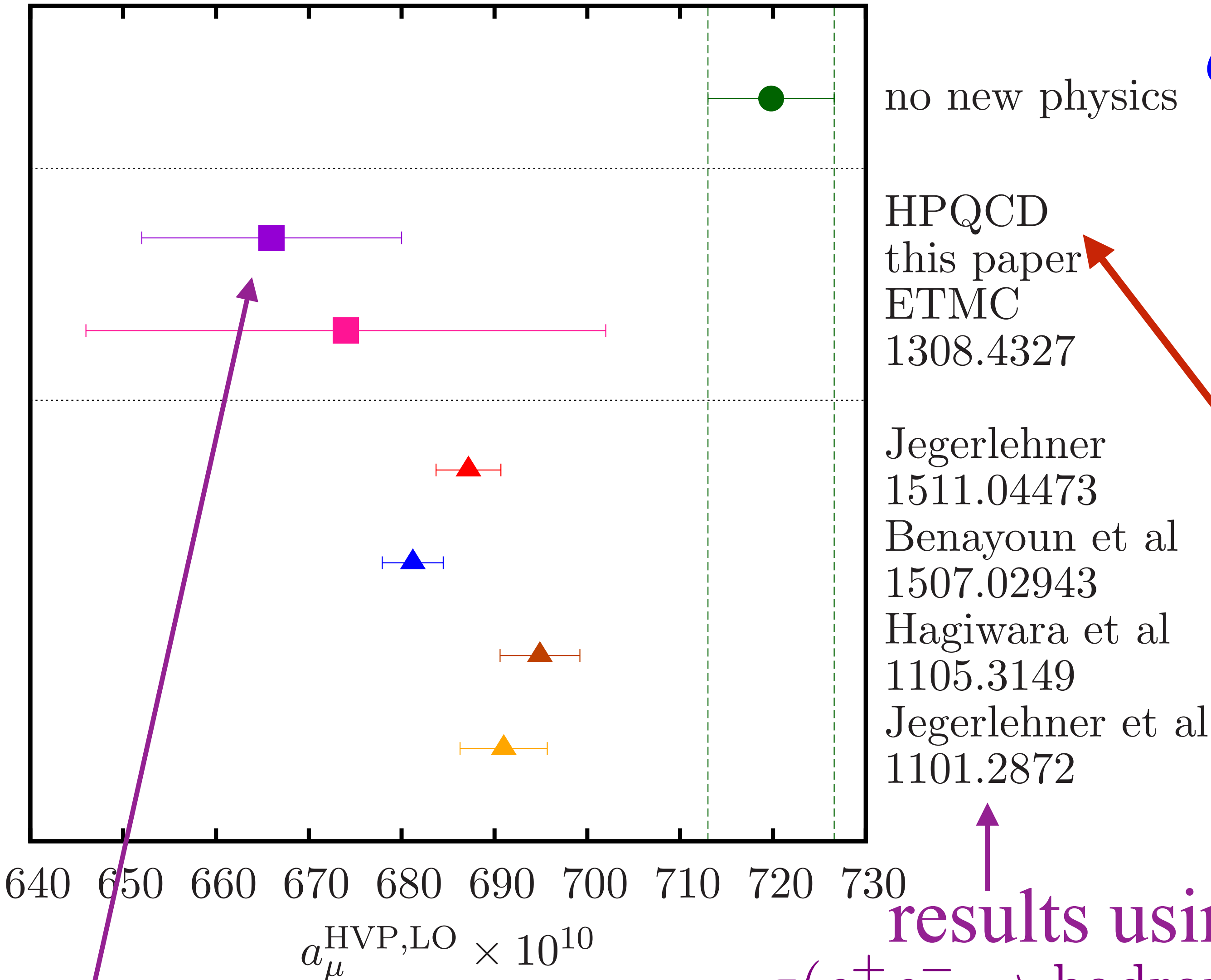
Must correct for finite vol. effects in $\pi\pi$ contribn using scalar QED (7%)

Rescale Π_j by $(m_\rho^{latt}/m_\rho^{expt})^{2j}$ to reduce u/d mass effects.



Combining numbers for a total

Chakraborty et al, HPQCD, 1601.03071



$a_\mu^{\rm HVP,LO} \times 10^{-10}$

598(11) u/d

53.4(6) s

14.4(4) c

0.27(4) b

Total 666(6)(12)

add syst from disc. diags (1.5%) in quad

results using $\sigma(e^+e^- \to \text{hadrons})$

3.5σ discrepancy with no new physics

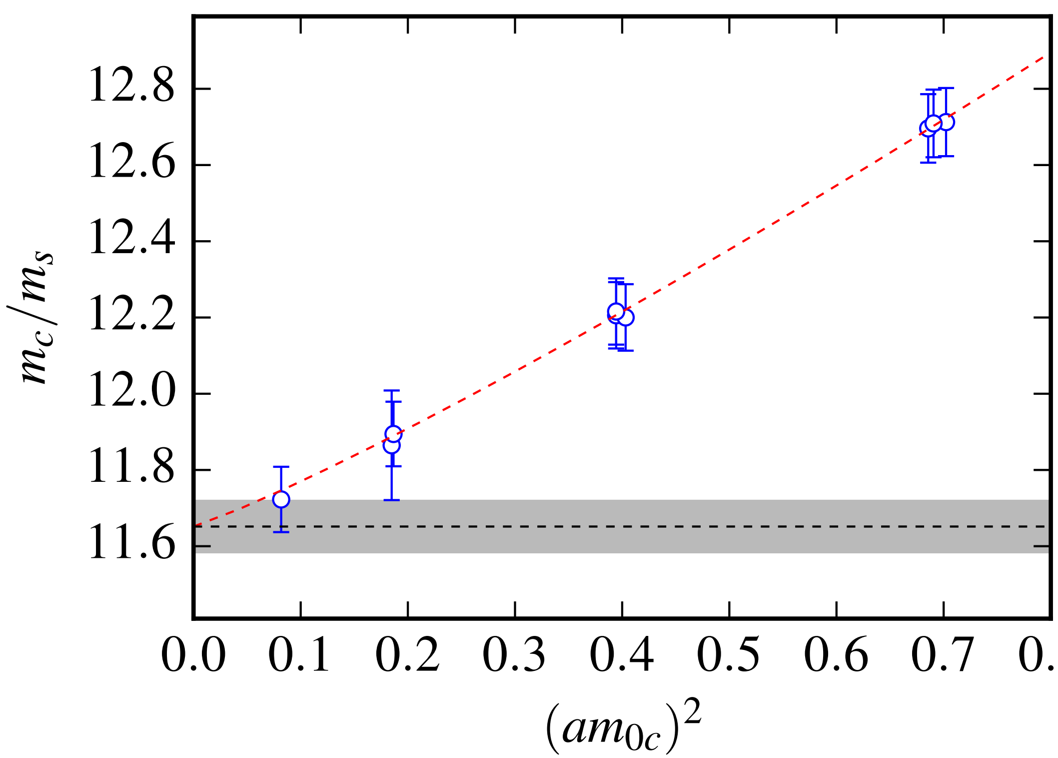
Conclusion

- Lattice QCD calculations now on '2nd generation' gluon configs with charm in the sea and $m_{u,d}$ at physical value (so no extrapoln).
- Gold-plated hadron masses and decay constants provide stringent tests of QCD/SM.
- NEW calculation of light meson electromagnetic form factor up to $Q^2 = 6\ \mathrm{GeV}^2$ shows behaviour inconsistent with asymptotic pert. QCD. Further calc. underway …
- Accurate quark mass determinations being tested with multiple methods
- sub-1% uncertainties on lattice QCD calculations for HVP contribution to a_μ are within sight.

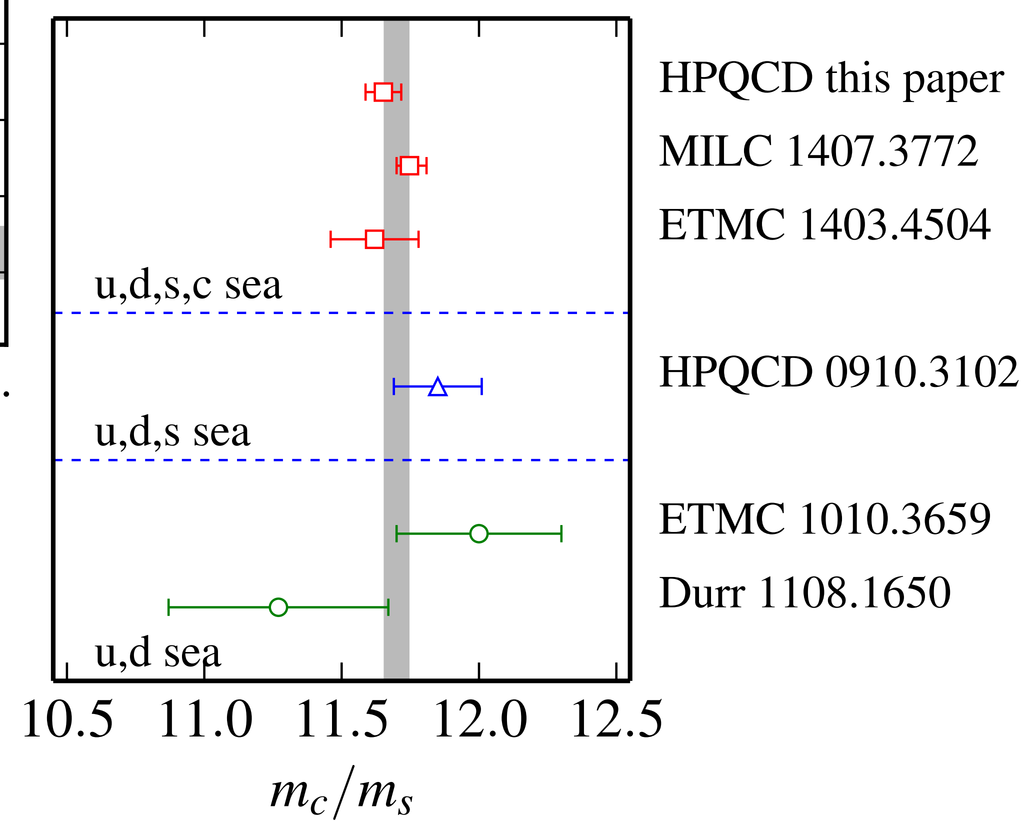
Spares

m_c/m_s

Mass ratio direct from lattice QCD using same formalism for both quarks (HISQ). Not possible to connect heavy and light masses without lattice.



Good consistency between different lattice actions

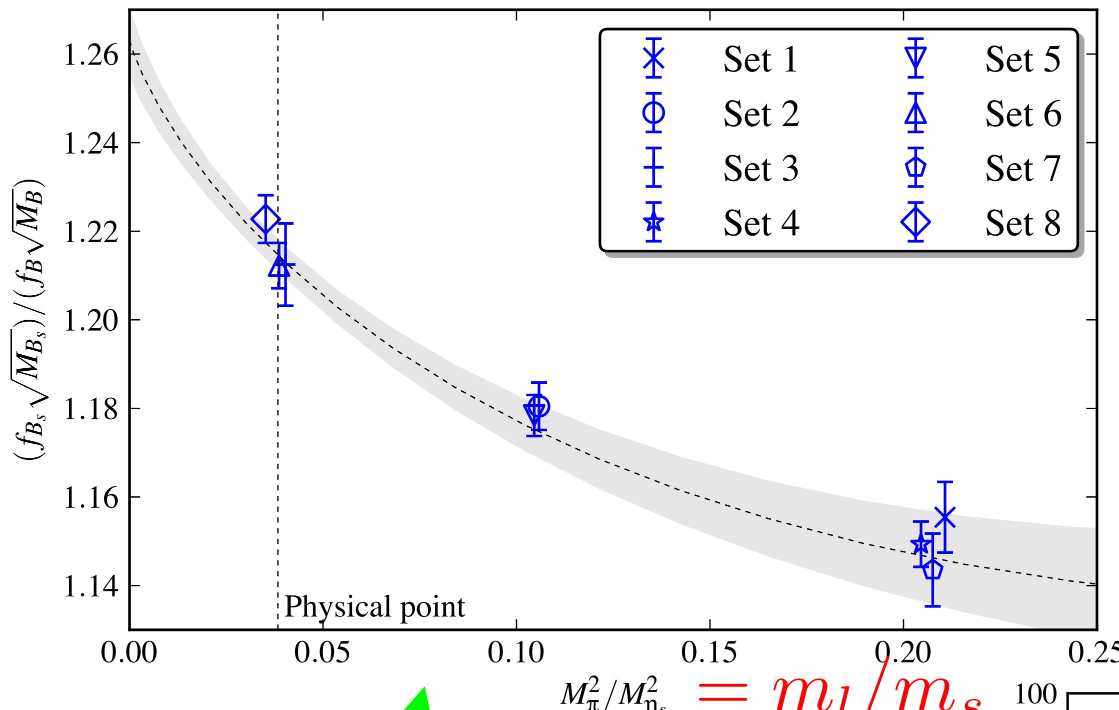


$$\frac{m_c}{m_s} = 11.652(65)$$

HPQCD, 1408.4169

B meson decay constants: HPQCD results from NRQCD b and physical u/d quarks

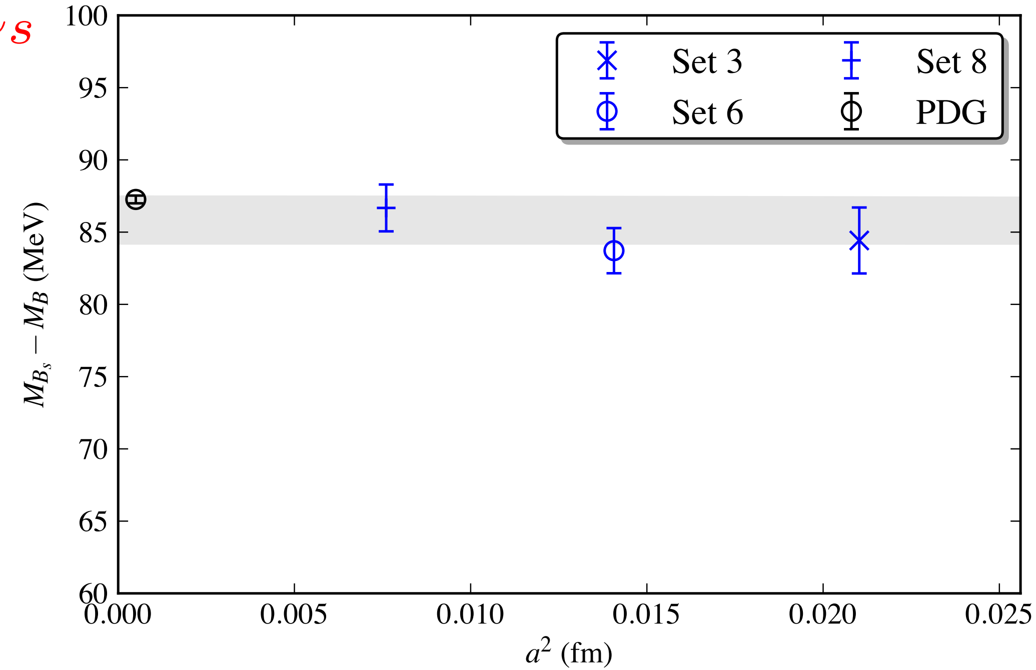
HPQCD: R Dowdall et al, 1302.2644.



$= m_l/m_s$

Use 2+1+1 configs with u/d down to physical values + improved NRQCD

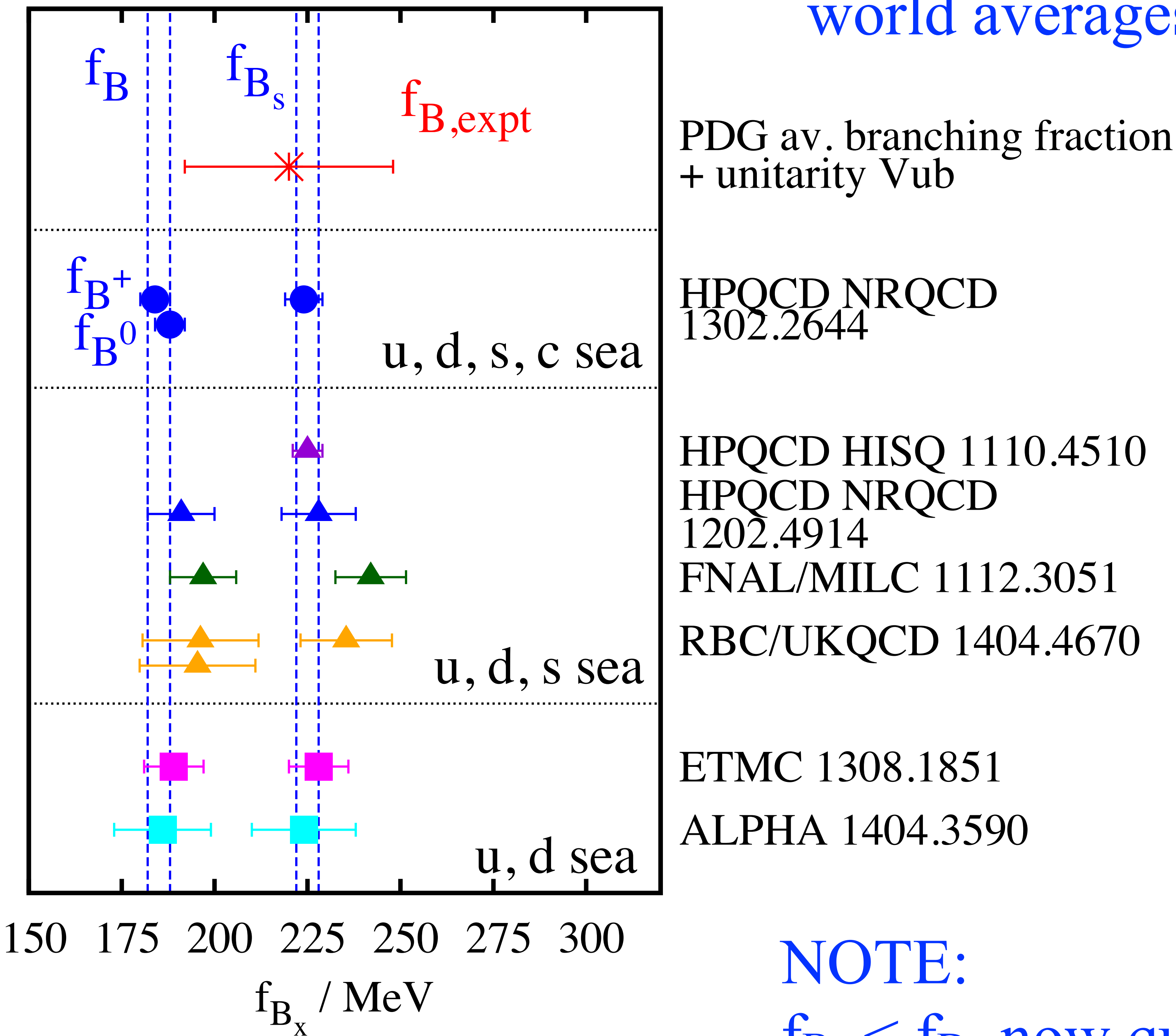
meson mass difference correct to 2%



B_s to B decay constant ratio accurate to 0.6% - since Z factors cancel. Separate decay constants to 2%

185(3) 225(3) averages

B, B_s decay constant world averages



Different approaches to b quark being used and to normalising weak current

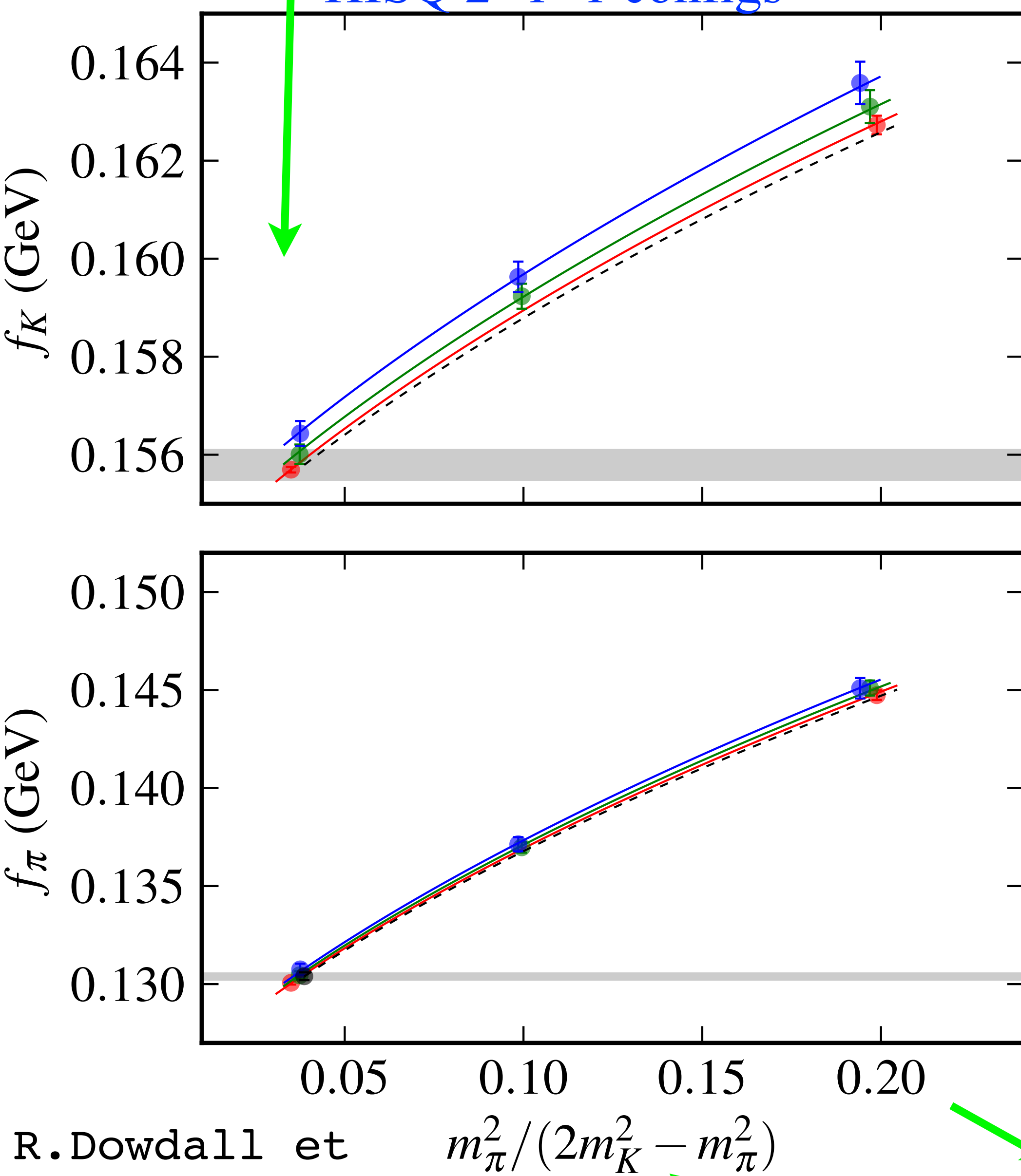
NOTE:

$f_{Bs} < f_{Ds}$ now quite clear

Constraining new physics with lattice QCD

* results at physical u/d quark masses*

HISQ 2+1+1 configs

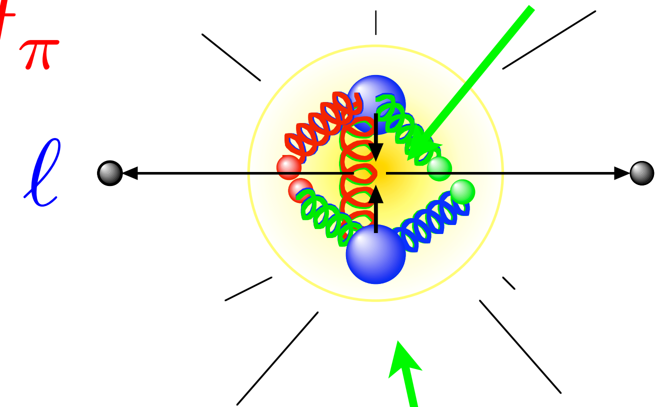


$= m_{u,d}/m_s$

R.Dowdall et al, HPQCD: 1303.1670

f_K/f_π

V_{us}/V_{ud}



Annihilation of K/π to W allows CKM element determination given decay constants from lattice QCD

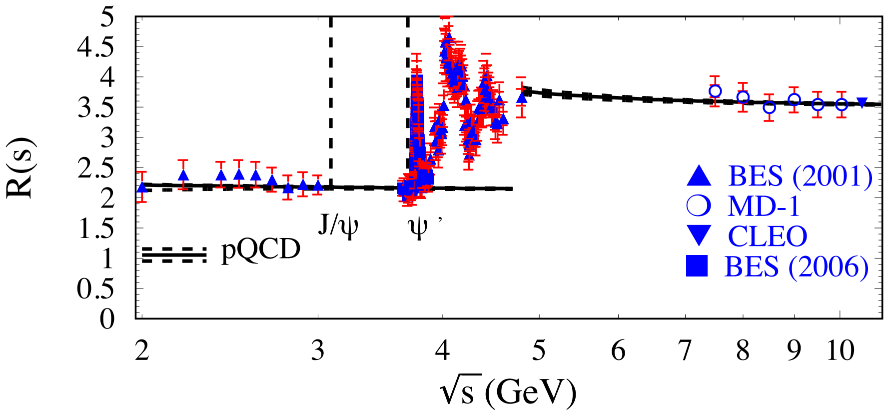
expt for $\dfrac{\Gamma(K^+ \to \ell\nu)}{\Gamma(\pi^+ \to \ell\nu)}$

$$\frac{|V_{us}|f_{K^+}}{|V_{ud}|f_{\pi^+}} = 0.27598(35)_{\mathrm{Br}(K^+)}(25)_{EM}$$

$\dfrac{f_{K^+}}{f_{\pi^+}}$ from lattice gives CKM

Charm contribution to

$$R = \sigma(e^+e^- \to \text{hadrons})/\sigma_{pt}$$

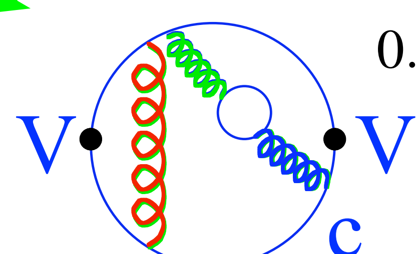


Subtract u,d,s with pert. th. to isolate charm contribution.
Calculate inverse-s-moment:

$$\mathcal{M}_n \equiv \int \frac{ds}{s^{n+1}} R_c(s)$$

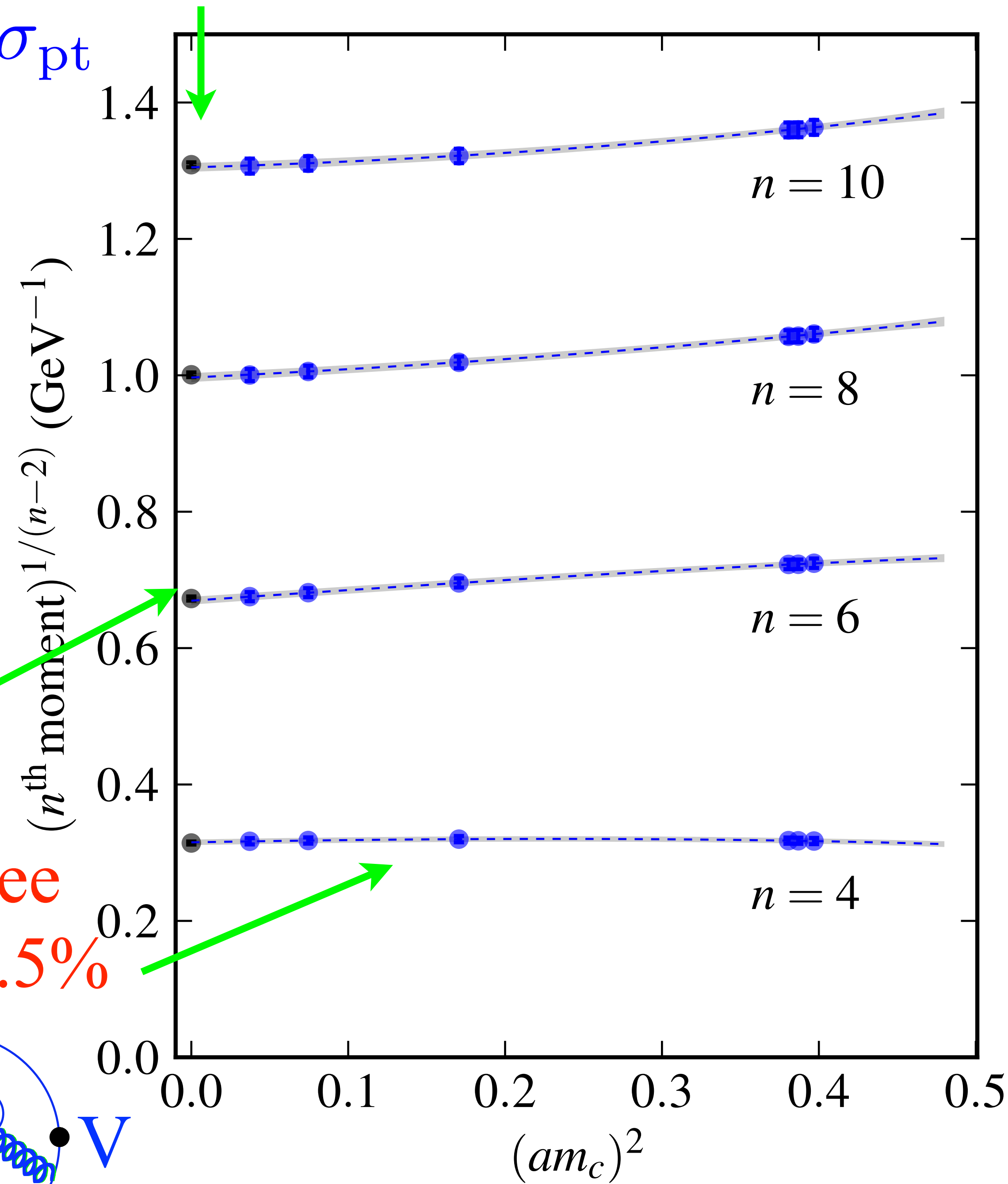
Lattice calcln:

$$M_n = \sum_t t^n G(t)$$



Agree to 1.5%

'expt' from J. Kuhn et al, hep-ph/0702103



G. Donald et al, HPQCD,1208.2855

Error budget for HVP,LO calculation

TABLE III: Error budget for the connected contributions to the muon anomaly a_μ from vacuum polarization of u/d quarks.

	$a_\mu^{\mathrm{HVP,LO}}(u/d)$
QED corrections:	$1.0\,\%$
Isospin breaking corrections:	$1.0\,\%$
Staggered pions, finite volume:	$0.7\,\%$
Noise reduction (t^*):	$0.5\,\%$
Valence m_ℓ extrapolation:	$0.4\,\%$
Monte Carlo statistics:	$0.4\,\%$
Padé approximants:	$0.4\,\%$
$a^2 \to 0$ extrapolation:	$0.3\,\%$
Z_V uncertainty:	$0.4\,\%$
Correlator fits:	$0.2\,\%$
Tuning sea-quark masses:	$0.2\,\%$
Lattice spacing uncertainty:	$< 0.05\,\%$
Total:	$1.9\,\%$