Status and future of event generation

the Lund perspective

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Outline

- Increase in perturbative precision
- Better non-perturbative modelling
- Heavy Ion collisions
- MCnet and Rivet
The structure of a proton collision
The hard/primary scattering
Immediate decay of unstable elementary particles
Radiation from particles before primary interaction
Radiation from produced particles
Additional sub-scatterings
... with accompanying radiation
Formation of **colour strings**
Fragmentation of strings into hadrons
Decay of unstable hadrons
Event Generator Philosophy

- As much theory as we can find
  - Add phenomenological models where there is no theory
  - Parameters correspond to physical properties of the model
  - Tune the parameters to data globally
  - Open software
  - It’s all about QCD
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The strong coupling is indeed strong. Leading order calculations is never enough.

Today we have automated programs that calculate observables to NLO precision.

Calculations exist for some observables no NNLO and even $N^3LO$

But that is not enough.
Perturbative QCD prediction for an observable

\[ \langle O \rangle = C_0 \rightarrow 1 \]
\[ + C_1 \alpha_s \rightarrow \alpha_s L^2 + \alpha_s L + \alpha_s \]
\[ + C_2 \alpha_s^2 \rightarrow \alpha_s^2 L^4 + \alpha_s^2 L^3 + \alpha_s^2 L^2 + \alpha_s^2 L + \alpha_s^2 \]
\[ \vdots \]
\[ + C_n \alpha_s^n \rightarrow \alpha_s^n L^{2n} + \alpha_s^n L^{2n-1} + \alpha_s^n L^{2n-2} + \alpha_s^n L^{2n-3} + \ldots \]

\[ \alpha_s \rightarrow \alpha_s(\mu_R) \] can be reabsorbed into \( C_i \) but residual dependence if series is cut off.

Any jet observable will have an additional resolution scale, \( \rho \), giving a dependence of \( C_i \) on the logarithm \( L = \log \frac{\mu_R}{\rho} \).

We clearly have a problem if \( L^2 \alpha_s \sim 1 \)

We need to resum and match
Perturbative QCD prediction for an observable

\[
\langle \mathcal{O} \rangle = C_0 \rightarrow 1 + C_1 \alpha_s \rightarrow \alpha_s L^2 + \alpha_s L + \alpha_s \\
+ C_2 \alpha_s^2 \rightarrow \alpha_s^2 L^4 + \alpha_s^2 L^3 + \alpha_s^2 L^2 + \alpha_s^2 L + \alpha_s^2 \\
+ \cdots \\
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\[
\langle O \rangle = C_0 + C_1 \alpha_s + C_2 \alpha_s^2 + \cdots + C_n \alpha_s^n \to \alpha_s L^2 + \alpha_s L + \alpha_s^2 L^2 + \alpha_s^2 L + \alpha_s^2 \quad \text{NNLO}
\]

\[
= 1 + \alpha_s L^2 + \alpha_s^2 L^3 + \alpha_s^2 L^2 + \alpha_s^2 L + \alpha_s^2 L^3 + \alpha_s^2 L^2 + \alpha_s^2 L + \cdots
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\vdots \\
\text{LL}
\]

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\vdots \\
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\vdots \\
\text{LL} \quad \text{NLL}
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We need to resum and match
Resummation

Can be done analytically for a given observable.

But we want to generate a whole event so that every observable is resummed.

The logarithmic divergencies comes from soft and collinear divergencies in the splitting kernels:

$$dP(k^2_\perp, z) \sim \frac{\alpha_s}{2\pi} \frac{dz}{z} \frac{dk^2_\perp}{k^2_\perp}$$
Parton Showers

When we generate a multi-parton event, we repeatedly use the splitting functions to generate more and more partons.

We regularize the divergencies by imposing an ordering. We typically require the hardest emission to be done first.

\[ dP(k^2, z) \rightarrow dP(k^2, z)\Delta(k^2, k^2_{\text{max}}) \]

where \( \Delta(k^2, k^2_{\text{max}}) \) is the Sudakov form factor, or no-emission probability.

\[ \Delta(k^2, k^2_{\text{max}}) = \exp \left( - \int_{k^2}^{k^2_{\text{max}}} dq^2 \int dz P(q^2, z) \right) \]
Expanding out the Sudakov in $\alpha_s$ gives us exactly the LL resummation for exclusive jet-cross sections.

Most showers also give NLL resummation modulo $O(1/N_c^2)$.

Parton Shower $\sim 1/$Jet Algorithm.
Multiplying an inclusive cross section with $\Delta(k_\perp^2, k_{\max}^2)$ gives the exclusive cross section for having no additional jets above a resolution scale, $k_\perp^2$.

We get the resummed exclusive cross sections for all parton multiplicities.

In addition, the parton shower is inherently unitary, so when summing everything together we get the inclusive cross section.
Matching and Merging — The Basic Idea

A fixed-order ME-generator gives the first few orders in $\alpha_s$ exactly.

The parton shower gives approximate (N)LL terms to all orders in $\alpha_s$ through the Sudakov form factors.

- Take a parton shower and correct the first few terms in the resummation with (N)LO ME.
- Take events generated with (N)LO ME with subtracted Parton Shower terms. Add parton shower.
- Take events samples generated with (N)LO ME, reweight and combine with Parton showers:
Tree-level Merging

Has been around the whole millennium: CKKW(-L), MLM, . . .
Combines samples of tree-level (LO) ME-generated events for different jet multiplicities. Reweights with proper Sudakov form factors (or approximations thereof).
Needs a merging scales to separate ME and shower region and avoid double counting. Only observables involving jets above that scale will be correct to LO.
Typically the merging scale dependence is beyond the precision of the shower: $\sim \mathcal{O}(L^3 \alpha_s^2) \frac{1}{N_c} + \mathcal{O}(L^2 \alpha_s^2)$. 
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NLO

The anatomy of NLO calculations.

\[ \langle O \rangle = \int d\phi_n (B_n + V_n) O_n(\phi_n) + \int d\phi_{n+1} B_{n+1} O_{n+1}(\phi_{n+1}). \]

Not practical, since \( V_n \) and \( B_{n+1} \) are separately divergent, although their sum is finite.

The standard subtraction method:

\[ \langle O \rangle = \int d\phi_n \left( B_n + V_n + \sum_p \int d\psi_{n,p}^{(a)} S_{n,p}^{(a)} \right) O_n(\phi_n) \]

\[ + \int d\phi_{n+1} \left( B_{n+1} O_{n+1}(\phi_{n+1}) - \sum_p S_{n,p}^{(a)} O_n\left(\frac{\phi_{n+1}}{\psi_{n,p}^{(a)}}\right) \right). \]
MC@NLO (Frixione et al.)

The subtraction terms must contain all divergencies of the real-emission matrix element. A parton shower splitting kernel does exactly that.

Generating two samples, one according to \( B_n + V_n + \int S_n^{PS} \), and one according to \( B_{n+1} - S_n^{PS} \), and just add the parton shower from which \( S_n \) is calculated.

POWHEG (Nason et al.)

Calculate \( \bar{B}_n = B_n + V_n + \int B_{n+1} \) and generate \( n \)-parton states according to that.

Generate a first emission according to \( B_{n+1}/B_n \), and then add any parton shower for subsequent emissions.
Really NLO?

Do NLO-generators always give NLO-predictions?

For simple Born-level processes such as $Z^0$-production, all inclusive $Z^0$ observables will be correct to NLO.

- $y_Z$
- $y_e$
- $p_{\perp e}$

But note that for $p_{\perp e} > m_Z/2$ the prediction is only leading order!
Multi-leg Matching

We need to be able to combine several NLO calculations and add (parton shower) resummation in order to get reliable predictions.

- No double (under) counting.
  - No parton shower emissions which are already included in (tree-level) ME states.
  - No terms in the PS no-emission resummation which are already in the NLO
- Dependence of any merging scale must not destroy NLO accuracy.
  - The NLO 0-jet cross section must not change too much when adding NLO 1-jet.
  - Dependence on logarithms of the merging scale should be less than $L^3 \alpha_s^2$ in order for predictions to be stable for small scales.
State of the art

- **SHERPA-MEPS@NLO**: (Höche et al.) CKKW-based using a merging scale. Any jet multiplicity possible if NLO is available. Residual dependence: $L^3 \alpha_s^2 / N_C^2$ — can’t take merging scale too low.

- **POWHEG-MiNLO**: (Hamilton et al.) No merging scale. 0 + 1-jet to NLO. NNLO possible?

- **PYTHIA8-UNLOPS**: (Prestel et al.) CKKW-L-based, with merging scale, but can be taken arbitrarily low. Lots of negative weights. Possible to go to NNLO? (also Herwig7)

- **FxFx**: (Frederix et al.) MLM-like merging procedure. Uncertain dependency on the merging scale. Any number of jets.

Also: GENEVA, VINCIA, ...
The future

- Proper NLL parton showers \( (\alpha_s \sim 1/N_C^2) \)
  DIRE (Hösche, Prestel), DEDUCTOR (Nagy, Soper),
  VINCIA (Skands)
- Electro-weak effects \( (\alpha_s^2 \sim \alpha_{EM}) \)
  PYTHIA8 (Sjöstrand, Prestel)
- PS-based NNLO subtraction terms
  DIRE? (Hösche, Prestel), Sherpa?
Phenomenological modelling

- PDFs: DGLAP-based fitted to data
- Particle Decays: measured and modelled
- Hadronization: Non-perturbative modelling
- Multi-parton interactions: (Non-)perturbative modelling
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**String Hadronisation**

What do we know about non-perturbative QCD?

- At small distances we have a **Coulomb-like** asymptotically free theory
- At larger distances we have a **linear** confining potential

For large distances, the field lines are compressed to vortex lines like the magnetic field in a superconductor

1+1-dimensional object $\sim$ a massless relativistic string
As a $q\bar{q}$-pair moves apart, they are slowed down and more and more energy is stored in the string.

If the energy is small, the $q\bar{q}$-pair will eventually stop and move together again. We get a “YoYo”-state which we interpret as a meson.

If high enough energy, the string will break as the energy in the string is large enough to create a new $q\bar{q}$-pair.

The energy in the string is given by the string tension

$$\kappa = \left| \frac{dE}{dz} \right| = \left| \frac{dE}{dt} \right| = \left| \frac{dp_z}{dz} \right| = \left| \frac{dp_z}{dt} \right| \sim 1\text{GeV/fm}$$
The quarks obtain a mass and a transverse momentum in the breakup through a tunneling mechanism (à la Schwinger)

\[ \mathcal{P} \propto e^{-\frac{\pi m_q^2}{\kappa}} = e^{-\frac{\pi m_{q_\perp}^2}{\kappa}} e^{-\frac{\pi p_{\perp}^2}{\kappa}} \]

Gives a natural suppression of heavy quarks

\[ d\bar{d} : u\bar{u} : s\bar{s} : c\bar{c} \approx 1 : 1 : 0.3 : 10^{-11} \]
For $1+1$ dimension and one flavour we have the probability for producing $n$ mesons

$$
\mathcal{P} \propto \left\{ \prod_{1}^{n} N d^{2} p_{i} \delta(p_{i}^{2} - m^{2}) \right\} \delta^{(2)}(\sum p_{i} - P_{tot}) \exp(-bA).
$$

- $bA$ is the imaginary part of the action for a relativistic string.
- $A/\kappa^2$ is the space-time area covered by the string before the break-ups.
The break-ups starts in the middle and spreads outward, but they are causally disconnected.

So we should be able to start anywhere. In particular we could start from either end and go inwards.
Requiring left-right symmetry we obtain a unique *fragmentation function* for a hadron taking a fraction $z$ of the energy of a string end in a breakup

\[ p(z) = N \frac{(1 - z)^a}{z} e^{-b m^2_\perp/z} \]

The Lund symmetric fragmentation function.

$N$ and $a$ are free parameters, related to each other by the normalization $\int f(z) dz \equiv 1$. 
$b$ is in principle given by $b \propto \sum_f \exp(-\pi m_f^2/\kappa)$ but is treated as a free parameter.

In a one-flavour world with only neutral pions, life is easy.

For more flavours we are guided by the quark-masses, but these are not well determined in a non-perturbative setting (pole mass? constituent mass?). Hence $b$ is treated as a free parameter to tune, rather than estimated from the quark masses.

Also the probability of producing a $s\bar{s}$-pair in a break-up is a free parameter.
... and then there are baryons!

Baryons can be produced by having $qq - \bar{q}\bar{q}$-breakups (diquarks behaves like an anti-colour)

A naive model would have very strong correlations between baryons and anti-baryons, which is not compatible with data.

Instead PYTHIA8 implements the so-called Popcorn model.

\[
\begin{array}{c}
\bullet \quad \bar{r} \\
\hline
\text{r} \\
\end{array}
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The Lund String model is non-perturbative, but it requires high energy.

We can chop off hadrons from either end randomly, but in the end we are left with a tiny string piece which we don’t know how to treat.

So as soon as we go below a couple of GeV in string mass we simply force the decay into two hadrons.
Gluons complicates the picture somewhat. They can be interpreted as a “kinks” on the string carrying energy and momentum
(The gluon carries twice the charge, $N_C/C_F \to 2$ for $N_C \to \infty$)

A bit tricky to go around the gluon corners, but we get a consistent picture of the energy–momentum structure of an event with no extra parameters (!)
Basics of Multiple Interactions

Starting Point:

\[ \frac{d\sigma_H}{dk_{\perp}^2} = \sum_{ij} \int dx_1 dx_2 f_i(x_1, \mu_F^2) f_j(x_2, \mu_F^2) \frac{d\hat{\sigma}_{Hij}}{dk_{\perp}^2} \]

The perturbative QCD $2 \rightarrow 2$ cross section is divergent. \( \int_{k_{\perp,c}^2} d\sigma_H \) will exceed the total $pp$ cross section at the LHC for \( k_{\perp,c} \lesssim 10 \text{ GeV} \).

There are more than one partonic interaction per $pp$-collision

\[ \langle n \rangle(k_{\perp,c}) = \frac{\int_{k_{\perp,c}^2} d\sigma_H}{\sigma_{tot}} \]
The trick in PYTHIA is to treat everything as if it is perturbative.

\[
\frac{d\hat{\sigma}_{Hij}}{dk^2_{\perp}} \rightarrow \frac{d\hat{\sigma}_{Hij}}{dk^2_{\perp}} \times \left( \frac{\alpha_s(k^2_{\perp} + k^2_{\perp0})}{\alpha_s(k^2_{\perp})} \cdot \frac{k^2_{\perp}}{k^2_{\perp} + k^2_{\perp0}} \right)^2
\]

Where \( k^2_{\perp0} \) is motivated by colour screening and is dependent on collision energy.

\[
k_{\perp0}(E_{CM}) = k_{\perp0}(E_{CM}^{ref}) \times \left( \frac{E_{CM}}{E_{CM}^{ref}} \right)^\epsilon
\]

with \( \epsilon \sim 0.16 \) with some handwaving about the the rise of the total cross section.
The total and non-diffractive cross section is put in by hand (or with a Donnachie–Landshoff parameterization).

- Pick a hardest scattering according to \( d\sigma_H / \sigma_{ND} \)
  (for small \( k_\perp \), add a Sudakov-like form factor).
- Pick an impact parameter, \( b \), from the overlap function
  (high \( k_\perp \) gives bias for small \( b \)).
- Generate additional scatterings with decreasing \( k_\perp \)
  according to \( d\sigma_H(b) / \sigma_{ND} \)
**Hadronic matter distributions**

We assume that we have factorization

\[ \mathcal{L}_{ij}(x_1, x_2, b, \mu_F^2) = \mathcal{O}(b)f_i(x_1, \mu_F^2)f_j(x_2, \mu_F^2) \]

\[ \mathcal{O}(b) = \int dt \int dxdydz \rho(x, y, z)\rho(x + b, y, z + t) \]

Where \( \rho \) is the matter distribution in the proton (note: general width determined by \( \sigma_{ND} \))

- A simple Gaussian
- Double Gaussian
- \( x \)-dependent Gaussian
Each scattering consumes momentum from the proton, and eventually we will run out of energy.

- Continue generating MI’s with decreasing $k_\perp$, until we run out of energy.
- Or rescale the PDF’s after each additional MI. (Taking into account flavour conservation).

Note that also initial-state showers take away momentum from the proton.
Interleaved showers

When do we shower?

- First generate all MI’s, then shower each?
- Generate shower after each MI?

Is it reasonable that a low-$k_\perp$ MI prevents a high-$k_\perp$ shower emission? Or vice versa?

- Include MI’s in the shower evolution
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After the primary scattering we can have

- Initial-state shower splitting, $P_{\text{ISR}}$
- Final-state shower splitting, $P_{\text{ISR}}$
- Additional scattering, $P_{\text{MI}}$
- Rescattering of final-state partons, $P_{\text{RS}}$

Let them compete

$$\frac{dP_a}{dk_{\perp}^2} = \frac{dP_a}{dk_{\perp}^2} \times \exp - \left( \int k_{\perp}^2 \left( dP_{\text{ISR}} + dP_{\text{FSR}} + dP_{\text{MI}} + dP_{\text{RS}} \right) \right)$$
At high energies we can have many additional scatterings per collision.

- Many strings
- Overlapping in space
- Break-down of Jet Universality
Strings $\rightarrow$ Ropes

Take the simplest case of two simple, un-correlated, completely overlapping strings, with opposite colour flow.

\[ q \quad \leftarrow \quad \bar{q} \]
\[ \bar{q}' \quad \rightarrow \quad q' \]

- 1/9: A colour-singlet
- 8/9: A colour-octet

The string tension is proportional to the Casimir operator:

\[ C_2^{(8)} = \frac{9}{4} C_2^{(3)}. \]
Get into the swing

The singlet case is dealt with by introducing a final-state swing.

\[ 1(q) \rightarrow 2(\bar{q}) \]

Swing probability \( \propto \frac{m_{12}^2 m_{34}^2}{m_{14}^2 m_{32}^2} \)
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\[
\begin{array}{c}
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\end{align*}
\]

Swing probability \( \propto \frac{m_{12}^2 m_{34}^2}{m_{14}^2 m_{32}^2} \)
But we may also have overlapping strings with parallel colour flows

- 1/3: An anti-triplet
- 2/3: A colour-sextet

And Junctions!
A model for rope-fragmentation

Strings are still broken up by tunneling out one $q\bar{q}$-pair at the time. But the string tension depends on the overlap with other strings in the vicinity of the overlap.

The total string tension before the break-up in the case of two anti-parallel strings in a colour-octet rope is given by

$$\kappa_{\text{sum}} = \frac{C_2^{(8)}}{C_2^{(3)}} \kappa_0.$$ 

What influences the physics in the string model is the change in the tension from the breaking — i.e. the energy released giving the mass and transverse momenta to the $q\bar{q}$.

Hence the first breakup has an effective tension

\[ \kappa_{\text{eff}} = \left( C_2^{(8)} - C_2^{(3)} \right) \frac{C_2^{(3)}}{\kappa_0} = 1.25 \kappa_0 \]

while the break-up in the string that is left is given by the standard \( \kappa_0 \).

The higher string tension affects several fragmentation parameters in a non-trivial way, basically increasing the probability to create heavy quarks or di-quarks in string break-ups.

Generalize to several strings giving a random walk in colour-space, and partly overlapping strings using the impact-parameter picture from DIPSY.
Perturbative Precision
Non-perturbative models
Heavy Ions

Swing
Ropes
Shoving

Event Generators
Leif Lönnblad
Lund University
String shoving giving flow effects

- Ridges linked to flow seen in AA, pA and pp.
- Very well described by hydrodynamics.
A microscopic model for expansion

- Idea: The overlapping regions will generate a transverse pressure.
- This will "shove" the strings apart.

arXiv:1612.05132
Two–particle correlations

- Shoving produces a "ridge".
- Currently for events consisting of long, soft strings only.
- Working towards a complete description.
Generating Heavy Ion Collisions — Reviving Fritiof

Simple picture of pp collisions

- Flat rapidity plateau.
- High mass diffractions $d\sigma/dM_X^2 \propto M_X^{-2(1+\epsilon)}$ were $\epsilon$ is small.
- Works surprisingly well for $\sqrt{s} \leq$ ISR.
- Fritiof + Glauber gives heavy ion collisions.
- Works very well at low energies.
- At higher energies we have multiple semi-hard parton–parton scatterings.
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Generating Heavy Ion Collisions — Reviving Fritiof

multiple soft gluon exchanges

- Flat rapidity plateau.
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Generating Heavy Ion Collisions — Reviving Fritiof

Longitudinal excitation of both protons

- Flat rapidity plateau.
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Generating Heavy Ion Collisions — Reviving Fritiof

String ends evenly distributed in rapidity

- Flat rapidity plateau.
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Generating Heavy Ion Collisions — Reviving Fritiof

Hadronises as if doubly diffractive excitation

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Generating Heavy Ion Collisions — Reviving Fritiof

The picture in pA

- Flat rapidity plateau.
- High mass diffractions $d\sigma/dM_X^2 \propto M_X^{-2(1+\epsilon)}$ were $\epsilon$ is small.
- Works surprisingly well for $\sqrt{s} \leq \text{ISR}$.
- Fritiof + Glauber gives heavy ion collisions.
- Works very well at low energies.
- At higher energies we have multiple semi-hard parton–parton scatterings.
- We need hard parton scatterings
- We need MPI
- Which nucleons are actually diffractively excited?
- How are the particles from the non-diffractively wounded nucleons distributed?
- We need hard parton scatterings – \texttt{PYTHIA8}
- We need MPI – \texttt{PYTHIA8}
- Which nucleons are actually diffractively excited?
- How are the particles from the non-diffractively wounded nucleons distributed?
How do we estimate nuclear effects?

- Estimate number distribution of \textit{wounded/participating} nucleons \textit{using Glauber}.
- Find a \textit{centrality} observable that should be sensitive to the number of hit nucleons.
- Build up a reference sample by \textit{stacking pp-events}, fudging them a bit to fit the centrality distribution.

The centrality measure is typically defined in terms of some multiplicity or energy in the nucleus direction.
The black disk approach, clearly cannot properly take diffractive excitation into account

It can be fudged, by setting e.g. $\sigma = \sigma_{\text{in}}$ or $\sigma = \sigma_{\text{abs}}$, but we want to do better.

We want to be able to see which nucleons that contributes to the centrality observable.

- absorptively wounded nucleons.
- diffractively wounded nucleons.
- …Not elastically scattered nucleons.

$$\sigma_w \equiv \sigma_{\text{abs}} + \sigma_{\text{Dt}} + \sigma_{\text{DD}} = \sigma_{\text{tot}} - \sigma_{\text{el}} - \sigma_{\text{Dp}}$$
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Note that this definition of a wounded cross section does not depend on the fluctuation in the target nucleon.

But setting \( \sigma = \sigma_w \) in a fixed black-disk is not enough since we want to model also the fluctuations in the projectile.
\[ \sigma_w \equiv \sigma_{\text{abs}} + \sigma_{\text{Dt}} + \sigma_{\text{DD}} = \sigma_{\text{tot}} - \sigma_{\text{el}} - \sigma_{\text{Dp}} \]

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But setting \( \sigma = \sigma_w \) in a fixed black-disk is not enough since we want to model also the fluctuations in the projectile.
Glauber–Gribov and Colour Fluctuations

Arguable the most advanced model is by Strickman et al. (GG)

Assume a fluctuating cross section:

$$
\sigma_{\text{tot}} = \int d\sigma P_{\text{tot}}(\sigma) = \int d\sigma \rho \frac{\sigma}{\sigma + \sigma_0} \exp \left\{ -\frac{(\sigma/\sigma_0 - 1)^2}{\Omega^2} \right\}
$$

Experiments typically use this together with a black disk, 
$$
T(b) = \Theta(\sqrt{\sigma/\pi} - b), \text{ with } \Omega = 1.01 \text{ or } 0.55, \text{ with the } P_{\text{tot}} \text{ scaled to get the total inelastic cross section}
$$

$$
\sigma_{\text{in}} = \int d\sigma P_{\text{in}}(\sigma) \equiv \int d\sigma P_{\text{tot}}(\sigma/\lambda_{\text{in}}), \quad \lambda_{\text{in}} = \sigma_{\text{in}}/\sigma_{\text{tot}}
$$
GG vs. DIPSY

Let’s analyse what GG does by comparing with DIPSY.

Assuming the GG distribution corresponds to fluctuations in the projectile only.
DIPSY

Heavy Ions
Summary/Outlook

Distribution of wounded nucleons

Leif Lönnblad
Lund University
Distribution of wounded nucleons

We can now use our modified GG tuned to experimental data and obtain a distribution of wounded nucleons.
Generating final states in pA

How much does a wounded nucleon contribute to particle production?

An absorptive pN scattering would distribute particles evenly in $\eta$. But what about projectile wounding two nucleons absorptively?

For a diffractively wounded nucleon, we expect a high-mass tail:

$$\frac{d\sigma}{dM_X^2} \propto \frac{1}{M_X^{2(1+\epsilon)}}$$

with $0 < \epsilon < 0.2$. 
The old Fritiof model assumed that each wounded nucleon contributed with a string with a mass distribution corresponding to $\epsilon = 0$.

This worked very well for low energies, where perturbative effects were smallish. It does not work for LHC.
Non-perturbative models

Heavy Ions

Summary/Outlook

Distribution of wounded nucleons

Generating HI final states

Results

Event Generators 59

Leif Lönnblad

Lund University
The New Model

- Generate $N_{abs}$ and $N_w$.
- Let PYTHIA8 generate one absorptive pp event, but bias the hard ME with a factor $N_{abs}$.
- Stack another $N_w - 1$ diffractive events using $\epsilon = 0$ (the default in PYTHIA8).
- Make sure energy and momentum is conserved.
Centrality

![Graph showing centrality distribution](image)

Sum $E_\perp$, $|\eta|<3.2$, $p_\perp>0.1$ GeV

- FritiofP8
- Absorptive
- DIPSY
Non-perturbative models

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Event Generators

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Results

\[ \frac{1}{(2\pi p_\perp)} \frac{d^2 N}{dp_\perp/d\eta} \text{ (GeV/c)^2} \]

\[ \frac{1}{(2\pi p_\perp)} \frac{d^2 N}{dp_\perp/d\eta} \text{ (GeV/c)^2} \]

\[ \text{pPb @ 5.02 TeV, Inclusive charged } 1.3 < \eta < 1.8 \]

\[ \text{pPb @ 5.02 TeV, Inclusive charged } -1.3 < \eta < -0.8 \]

Data

FritiofP8

Absorptive
Summary

- Event Generators are not about simple tuning – they are all about physics
- State of the art is multi-leg NLO matching with Parton Showers
  NNLO with NLL showers is coming.
- Non-perturbative modelling is getting better
- High multiplicity pp is a challenge
- On our way towards a full microscopical model for heavy ion collisions
RIVET

A program for publishing measurements in a way usable for everyone.

A set of tools that can be used to build analysis routines for comparison of a given measurement with event generators.

Philosophy:

- The actual analysis (with all kinematical cuts)
- Detector-independent (unfolding)
- **Model-independent**

It is ok to publish data as a function of $N_{\text{part}}$ or $N_{\text{coll}}$ — this is an interpretation of the data.

But it is not independent of theoretical models.
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But it is not independent of theoretical models.
If you want to make your measurements useful for others — publish the *actual* measurement in RIVET!
The MCnet Collaboration

EU-funded collaboration between all general purpose event generators and more

- Herwig7
- Pythia8
- Sherpa
- Ariadne/DIPSY, HEJ
- Rivet, Professor, MCplots

Manchester (CERN), Durham (SLAC), Glasgow, Göttingen (Heidelberg), Karlsruhe (industry), Louvain, Lund (Monash, FNAL), UCL (experiments)
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Monte Carlo training scholarships

3-6 month fully funded scholarships for current PhD students at one of the MCnet nodes. An excellent opportunity to really understand and improve the Monte Carlos you use!

Application rounds every 3 months.

for details go to: www.montecarlonet.org
Backup slides?
Hadron scattering cross sections

Working with the optical theorem in impact parameter space

\[ T = -iA_{el} \]

averaging over initial states of the target and projectile \( T_{pt}(b) \):

\[
\frac{d\sigma_{\text{tot}}}{d^2 b} = 2 \langle T(b) \rangle \\
\frac{d\sigma_{\text{el}}}{d^2 b} = \langle T(b) \rangle^2 \\
\frac{d\sigma_{\text{in}}}{d^2 b} = 2 \langle T(b) \rangle - \langle T(b) \rangle^2
\]
Diffractive excitation

Following Good–Walker

\[
\frac{d\sigma_{\text{DP}}}{d^2b} = \langle T^{2}_{pt}(b) \rangle_{t}^{2} - \langle T_{pt}(b) \rangle_{pt}^{2}
\]

\[
\frac{d\sigma_{\text{DT}}}{d^2b} = \langle T^{2}_{pt}(b) \rangle_{p}^{2} - \langle T_{pt}(b) \rangle_{pt}^{2}
\]

\[
\frac{d\sigma_{\text{DD}}}{d^2b} = \langle T^{2}_{pt}(b) \rangle_{pt}^{2} - \langle T^{2}_{pt}(b) \rangle_{t}^{2} - \langle T_{pt}(b) \rangle_{pt}^{2} + \langle T_{pt}(b) \rangle_{pt}^{2}
\]

Diffractive excitation is related to fluctuations.

\[
\frac{d\sigma_{\text{in,ND}}}{d^2b} \equiv \frac{d\sigma_{\text{abs}}}{d^2b} = 2 \langle T_{pt}(b) \rangle_{pt} - \langle T^{2}_{pt}(b) \rangle_{pt}
\]

Also the non-diffractive cross section depends on fluctuations.
Which nucleons are participating/wounded

The simplest Glauber model uses nucleons distributed with a Wood–Saxon and treating them as solid, fixed-size black disks.

Looking at the cross sections for a projectile on a single nucleon, we have \( T(b) = \Theta(\sqrt{\sigma/\pi} - b) \), and for \( \sigma = \sigma_{\text{tot}} \) we get

\[
\sigma_{\text{tot}} = \int d^2 b \ 2 \langle T(b) \rangle
\]
\[
\sigma_{\text{el}} = \int d^2 b \ \langle T(b) \rangle^2 = \sigma_{\text{tot}}/2
\]
\[
\sigma_{\text{abs}} = \int d^2 b \ (2 \langle T(b) \rangle - \langle T(b) \rangle^2) = \sigma_{\text{tot}}/2
\]
\[
\sigma_{\text{diff}} = \int d^2 b \ (\langle T^2(b) \rangle - \langle T(b) \rangle^2) = 0
\]
The two-radii model

A simple extension of the black-disk Glauber model is to assume that the target and nucleus fluctuates between two states with different radii, $R$ and $r$, with the probability $c$ and $1 - c$ respectively.

Since we have four independent cross sections $\sigma_{\text{abs}}$, $\sigma_{\text{el}}$, $\sigma_{\text{Dp}} = \sigma_{\text{Dt}}$, and $\sigma_{\text{DD}}$, we introduce a fourth transparency parameter, $\alpha$ with

$$T_{\text{pt}}(b) = \alpha \Theta(r_p + r_t - b).$$

$\alpha$, $c$, $r$ and $R$ can now be fit to reproduce all relevant cross sections.

But the fluctuations are very crude.
Assuming that GG describes the fluctuations in the projectile only, we can fit the parameters to three cross section:

\[
\begin{align*}
\sigma_{\text{tot}} &= \int d^2b \int d\sigma P_{\text{tot}}(\sigma) 2T(\sigma, b) \\
\sigma_{\text{el}} &= \left| \int d^2b \int d\sigma P_{\text{tot}}(\sigma) T(\sigma, b) \right|^2 \\
\sigma_{\text{w}} &= \int d^2b \int d\sigma P_{\text{tot}}(\sigma) \left( 2T(\sigma, b) - T^2(\sigma, b) \right)
\end{align*}
\]

We have tried

\[
T(\sigma, b) \propto \exp(-b^2/2B), \quad (B \propto 1/\sigma) \quad \text{and} \quad T(\sigma, b) = \alpha \Theta(\sqrt{\sigma/2\pi\alpha} - b).
\]

But only with the latter was it possible to fit experimental cross sections.
But we also need the absorptively wounded nucleons

Using the simple 2-radii model to estimate target fluctuations
The popcorn model is implemented as diquark break-ups, with a probability (parameter) to have a meson produced in-between the baryon and anti-baryon.

Other baryon parameters:

- probability of having diquark in breakup
- probability of having spin-one vs. spin-zero diquarks
- strange diquarks vs. light
- ... 

Should $a$ be the same for mesons and baryons? (parameter!)

How about strange and light mesons? (parameter!!)
Also $p_{\perp Z}$ is LO. To get NLO we need to start with $Z+\text{jet}$ at Born-level.

But for small $p_{\perp Z}$ the NLO cross section diverges due to $L^{2n}\alpha_s^n$, $L = \log(p_{\perp Z}/\mu_R)$.

If $L^2\alpha_s \sim 1$, the $\alpha_s^2$ corrections are parametrically as large as the NLO corrections.

Can be alleviated by clever choices for $\mu_R$, but in general you need to resum.
Assume we have a generator capable of doing three jets to NLO \((B_3 + V_3 + B_4)\)

Azimuth angle between the two *hardest* jets

Not always obvious when NLO stops being NLO
Unlike the cluster model, the string model does not have a simple way of selecting which meson to produce. Again we use parameters.

E.g. the probability to form a $\rho$ rather than a $\pi$ should be large due to the number of spin states, but it is suppressed since the $\rho$ is a much broader resonance. We introduce a parameter (vector-to-pseudoscalar), which may differ for light and strange mesons.