Lorentz Violation in Deep Inelastic Scattering

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April 10, 2017
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E.L., A. Kostelecky and A. Vieira, PLB 769 (272) 2017 (1610.08755)

Outline

- Introductory remarks
- Symmetries: observer vs particle transformations
- The minimal Standard Model Extension (SME)
- Constraints
- Focus on Lorentz Violation in the quark sector
- LV effects are clouded by confinement and non-perturbative effects:
  - low energy effects
  - high-energy effects
- Basics on Deep Inelastic Scattering and Parton Distribution Functions
- Focus on LV effects in $ep \to eX$ (last seen at HERA) but easy to generalize to pp collisions
- Sensitivity of sidereal time studies of HERA data

Introductory Remarks

- The Standard Model is the most general Lorentz invariant and renormalizable theory given its matter content and its symmetry group, $SU(3)\times SU(2)\times U(1)$:

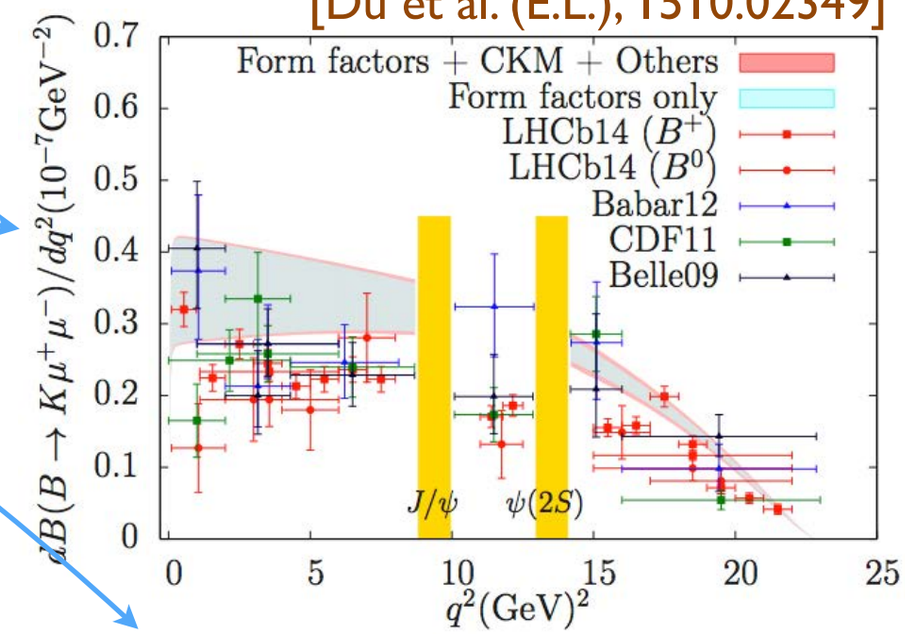
$$\begin{aligned}\mathcal{L}_{SM} \;=\; & -\frac{1}{4}B_{\mu\nu}B^{\mu\nu} - \frac{1}{4}F^{i}_{\mu\nu}F^{i,\mu\nu} - \frac{1}{4}G^{a}_{\mu\nu}G^{a,\mu\nu} + \theta_{\mathrm{QCD}}\frac{g_s^2}{64\pi^2}\epsilon^{\mu\nu\rho\sigma}G^{a}_{\mu\nu}G^{a}_{\rho\sigma} \\ & + \bar{L}_L\, i\not{D}\, L_L + \bar{Q}_L\, i\not{D}\, Q_L + \bar{E}_R\, i\not{D}\, E_R + \bar{D}_R\, i\not{D}\, D_R + \bar{U}_R\, i\not{D}\, U_R \\ & + |D_\mu\phi|^2 - V(\phi) \\ & - \left[\bar{L}_L\,\phi\,\hat{y}^e\, E_R + \bar{Q}_L\,\phi\,\hat{y}^d\, D_R + \bar{Q}_L\,\tilde{\phi}\,\hat{y}^u\, U_R + \text{h.c.}\right]\end{aligned}$$

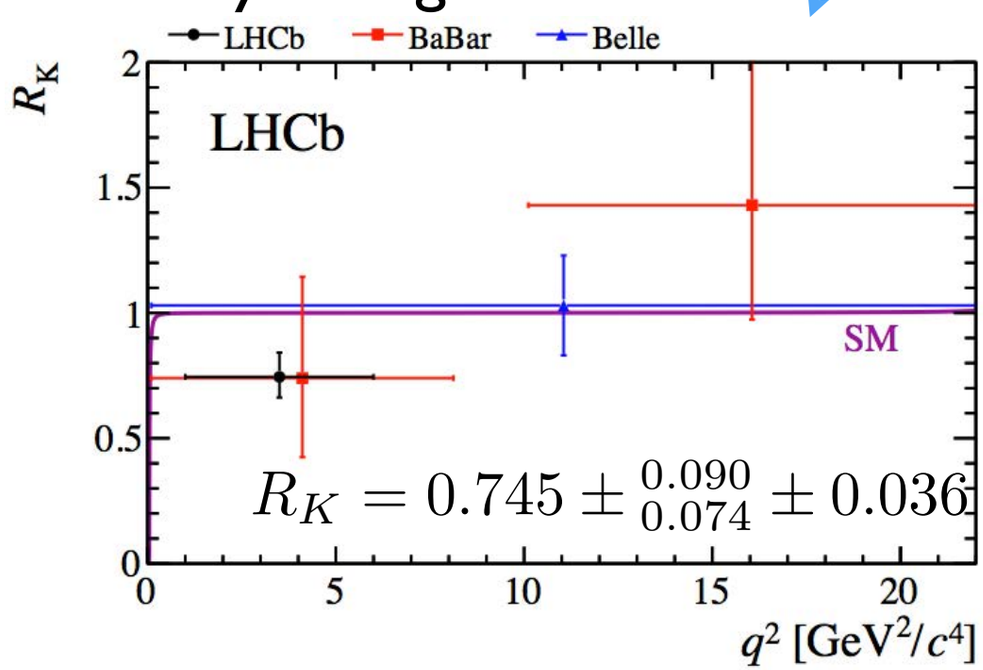
- We know that the SM has to be replaced at least at scales close to M_{Planck} in order to include gravity
- Unfortunately the Wilson analysis of Renormalization Group effects shows that at energies (p_{ext}) much smaller than the cut-off of this theory ($\Lambda \approx M_{\mathrm{Planck}}$) only renormalizable operators in the high energy theory matter
- The converse is also true: deviations from the SM predictions at energies currently tested(between $100\ GeV$ and $100\ TeV$ for flavor/collider experiments) point to new physics at nearby scales

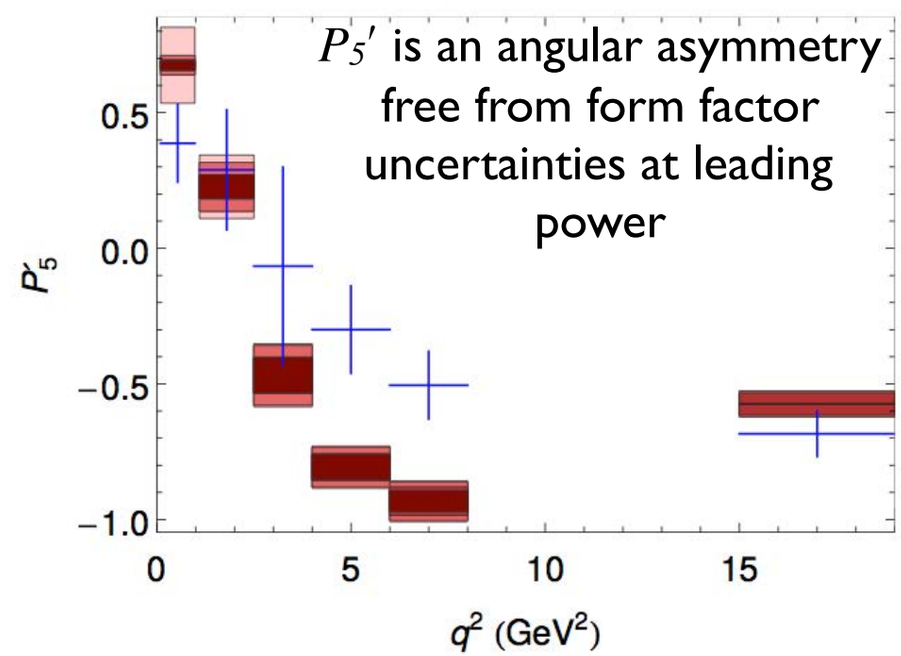
Introductory Remarks

- Apart from big-item issues like dark matter and dark energy (that do not point conclusively to a particular NP scale), there are a number of "tensions" in low energy experiments:
  - $\delta(g_\mu - 2)/2 = (27.4 \pm 7.6) \times 10^{-10}$
  - $b \to s\ell\ell$ anomalies
  - $R_K = \dfrac{\mathcal{B}(B^+ \to K^+\mu\mu)}{\mathcal{B}(B^+ \to K^+ee)} = 1 + \mathcal{O}(10^{-4})$
  - $R_{D^{(*)}} = \dfrac{\Gamma(B \to D^{(*)}\tau\nu)}{\Gamma(B \to D^{(*)}\ell\nu)}$
  - Unitarity triangle fits

[Du et al. (E.L.), 1510.02349]







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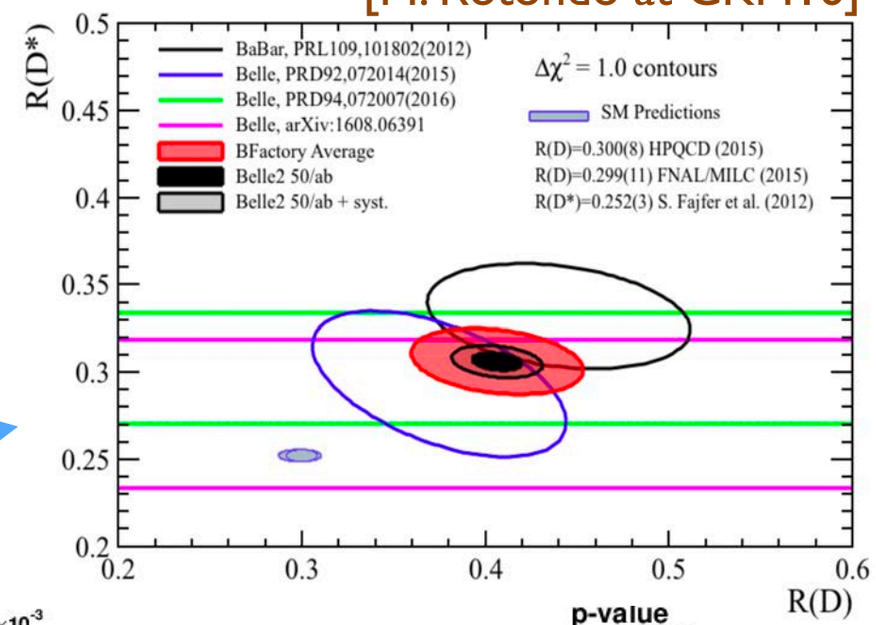
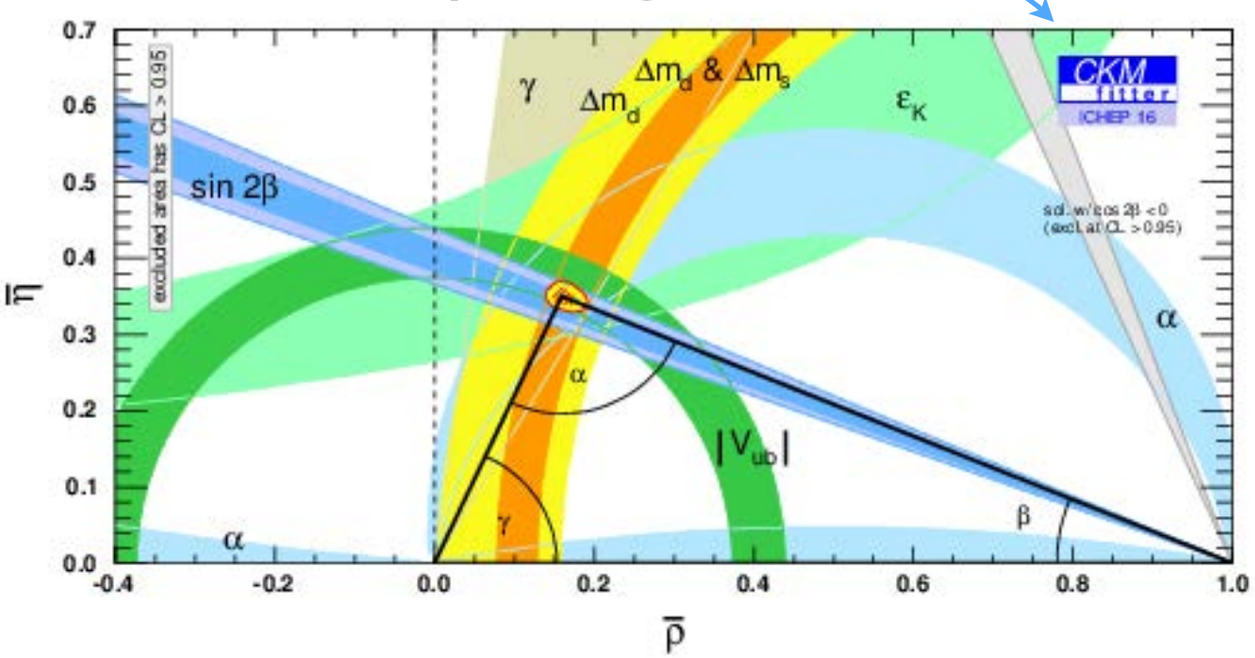
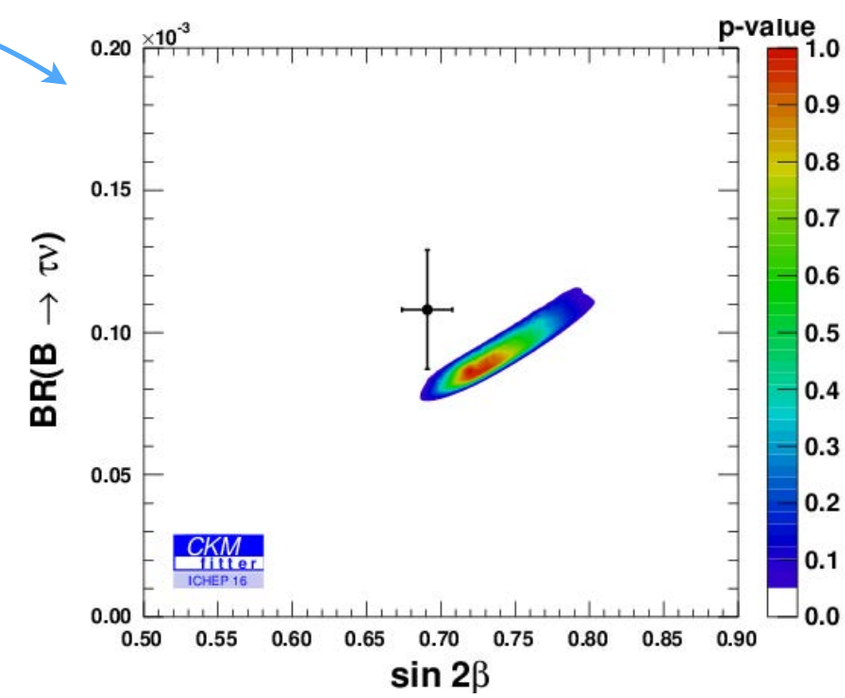
[M. Rotondo at CKM16]



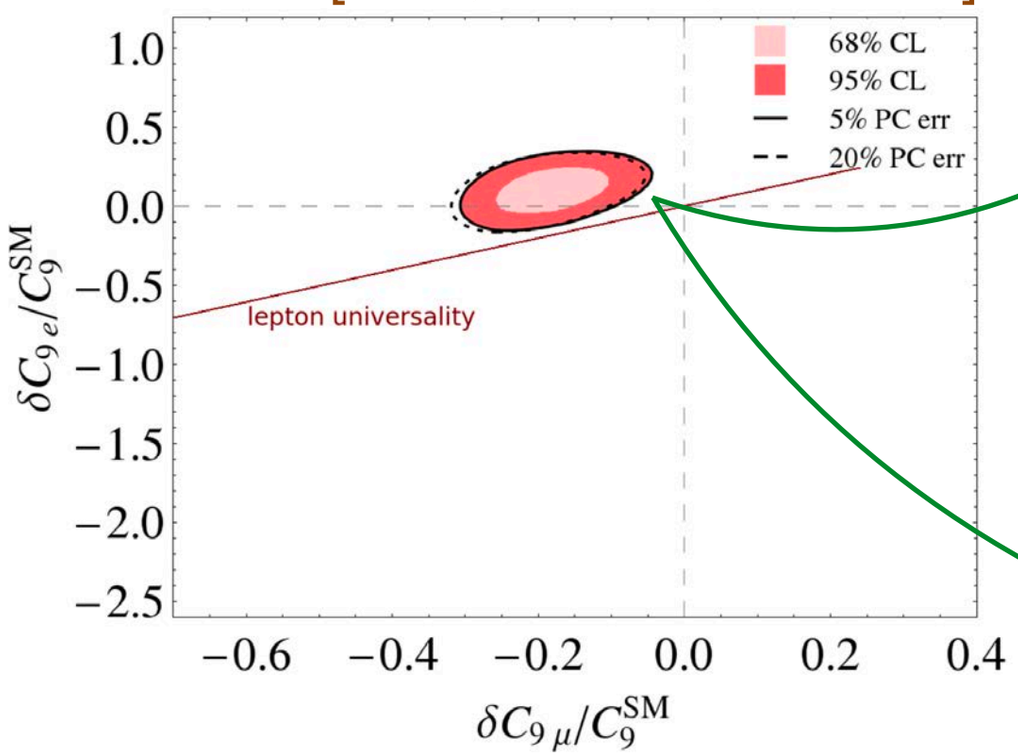




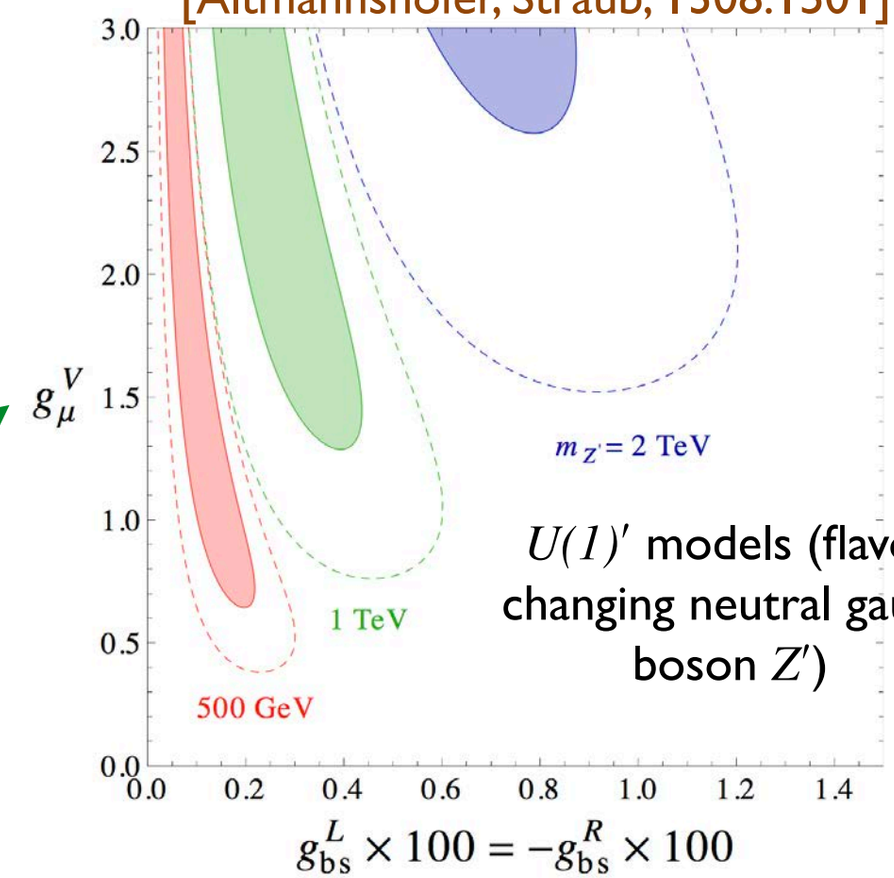
Introductory Remarks

- Interpretation of these anomalies requires new physics at relatively low scales
  - $[\delta(g_\mu - 2)/2]_{\rm SUSY} \sim 24 \times 10^{-10} \frac{\tan\beta}{10} \frac{(250\ {\rm GeV})^2}{M_2\ \mu}$
  - $b \to s\ell\ell$ anomalies

[Hurth, Mahmoudi, 1611.05060]



[Altmannshofer, Straub, 1308.1501]



U(1)′ models (flavor changing neutral gauge boson *Z′*)

$$\delta C_9^{\rm MSSM} \sim 0.1 \frac{m_t^2}{m_{\tilde t_R}^2} - 0.04 \frac{1}{\tan^2\beta} \frac{m_t^2}{m_{H^\pm}^2} - 0.12 \frac{m_t^2}{m_{\tilde W}^2} \delta_{ct}^L - 1.2 \frac{m_t^2}{m_{\tilde g}^2} \delta_{bs}^L$$

- $\mathcal{R}(D) \Longrightarrow \tan\beta/m_H = (0.44 \pm 0.02)\ {\rm GeV}^-1$

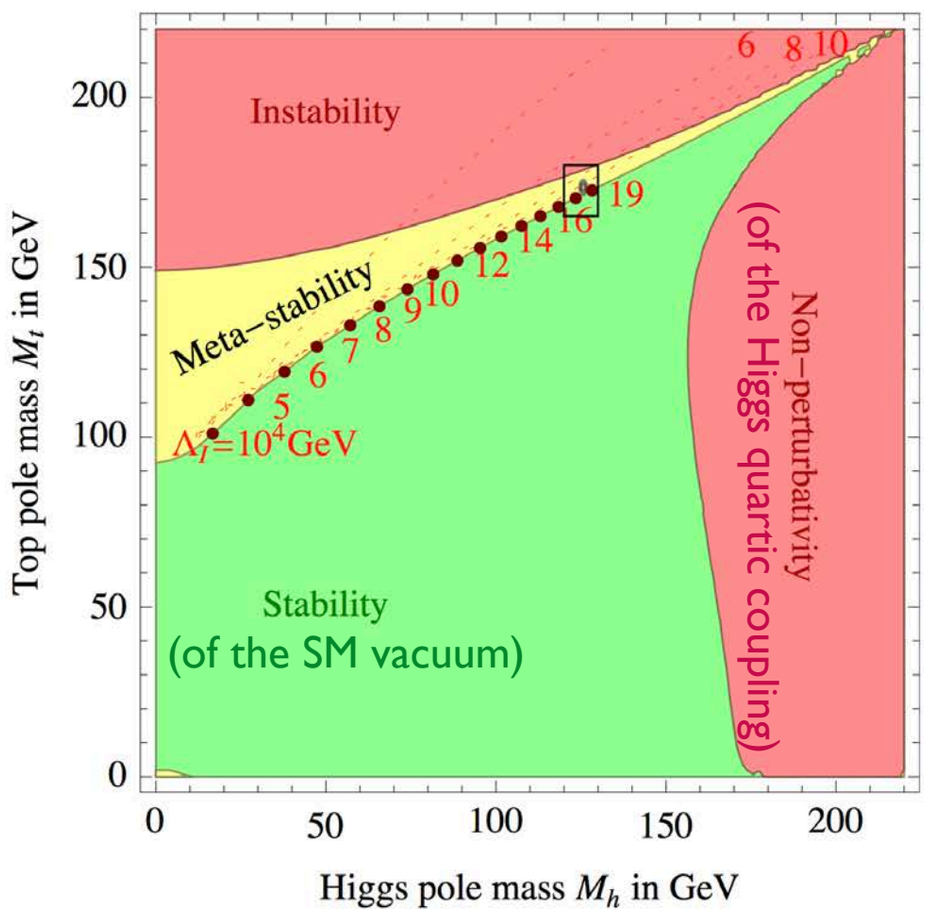
  $\mathcal{R}(D^*) \Longrightarrow \tan\beta/m_H = (0.75 \pm 0.04)\ {\rm GeV}^-1$

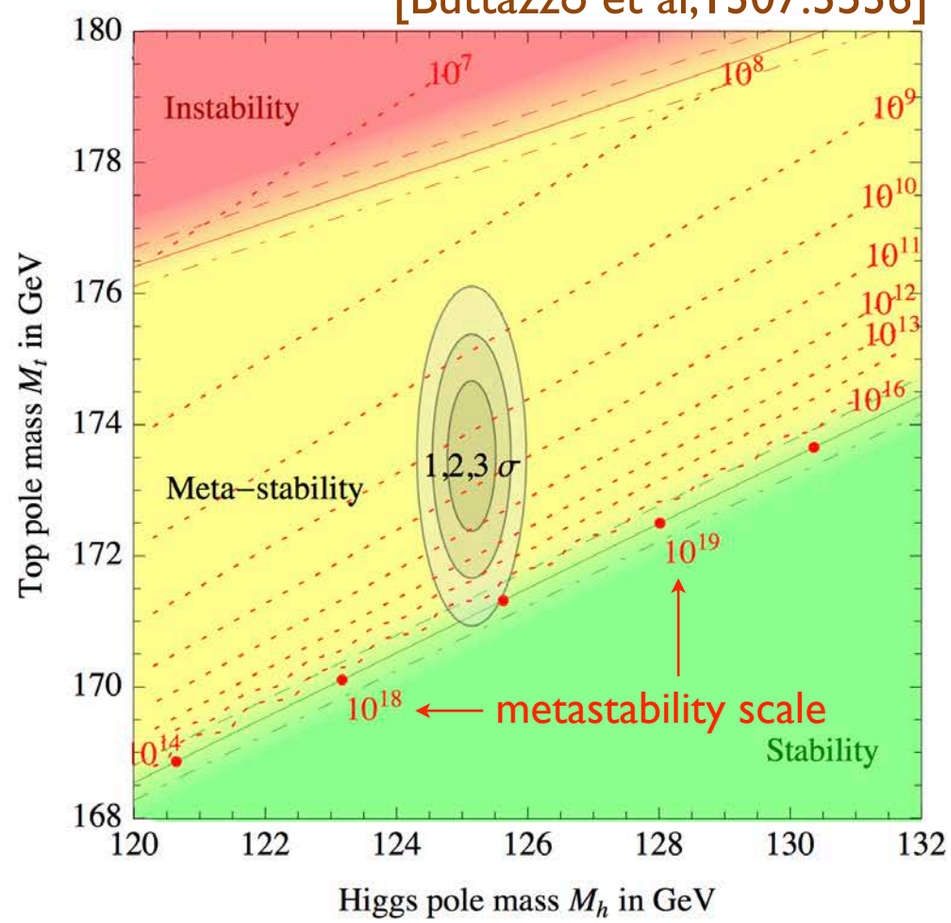
  [M. Rotondo at CKM16]

Introductory Remarks

- The discovery of the Higgs boson was a great success but was also a warning sign!

[Buttazzo et al,1307.3536]



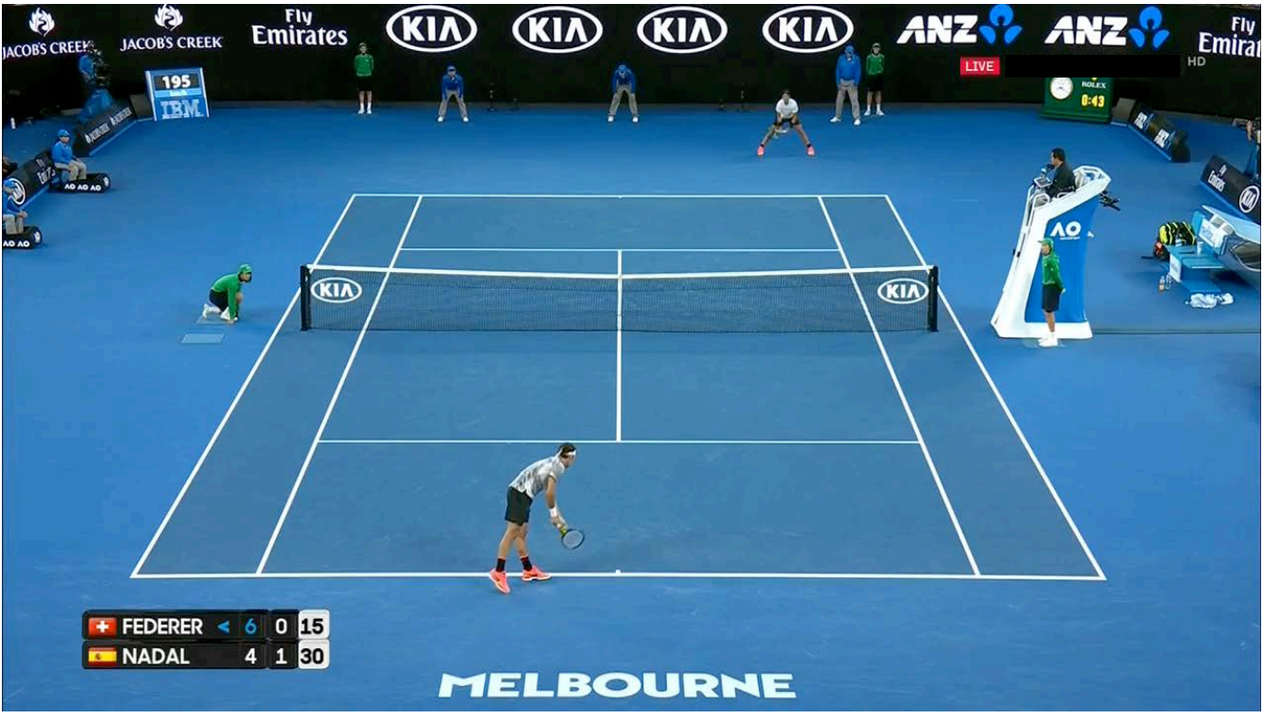


- The Higgs mass is compatible with extending the SM without modifications through scales of order M_{Planck}!
- If this is the case, all above mentioned tensions are destined to disappear
- One possibility is to find new renormalizable interactions with "unusual" signatures: this is possible if we are willing to give up Lorentz invariance

Symmetry Transformations: Observer vs Particle

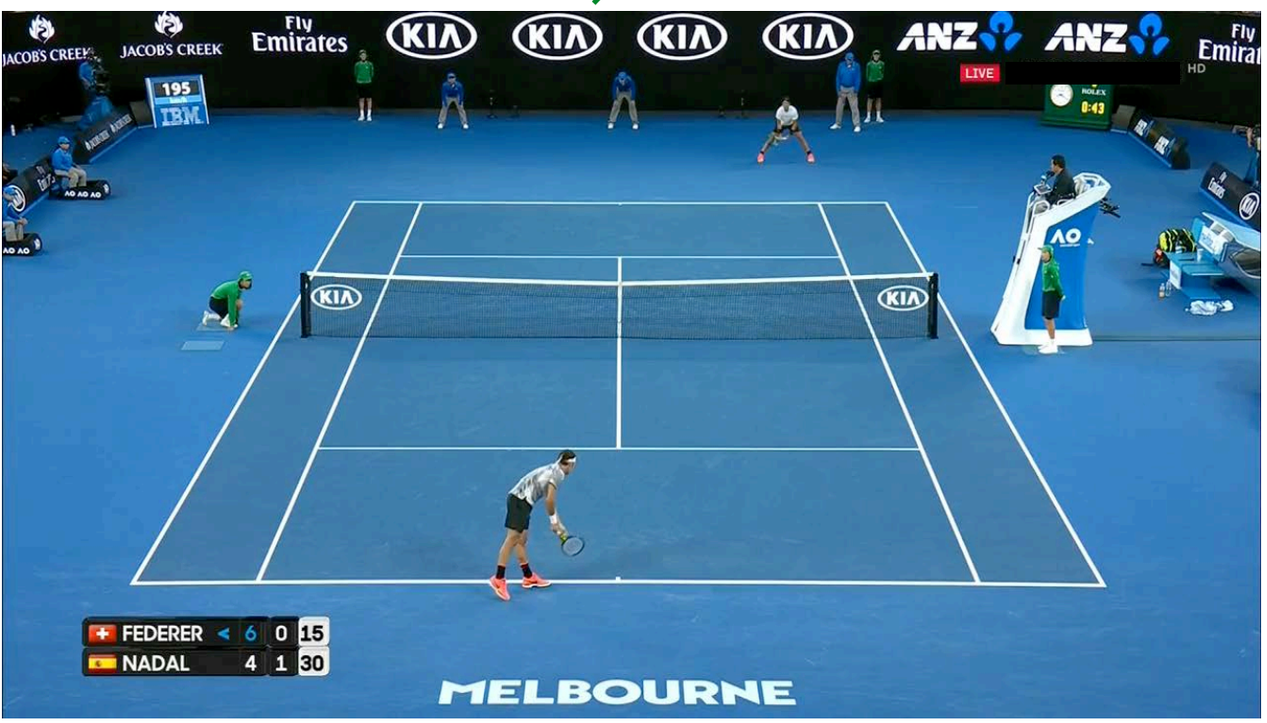
- In order to discuss Lorentz Violation it is important to separate ***observer*** and ***particle*** transformations

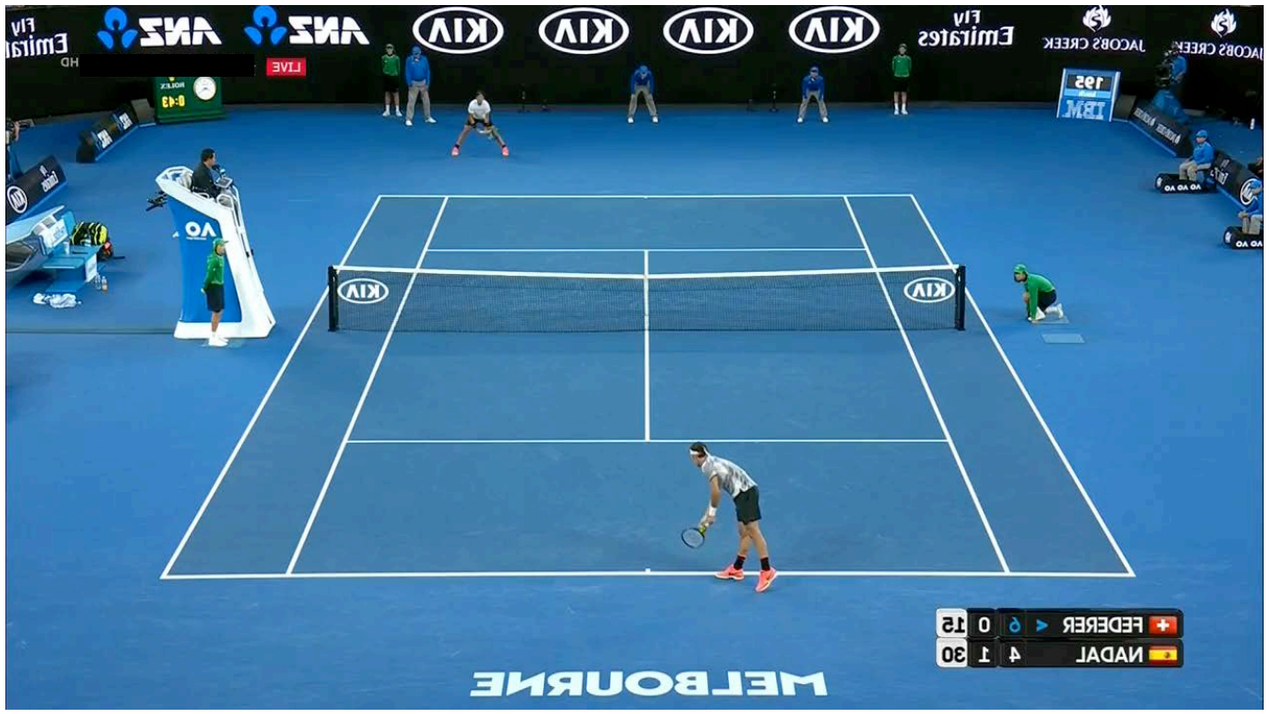
Symmetry Transformations: Parity



Symmetry Transformations: Parity

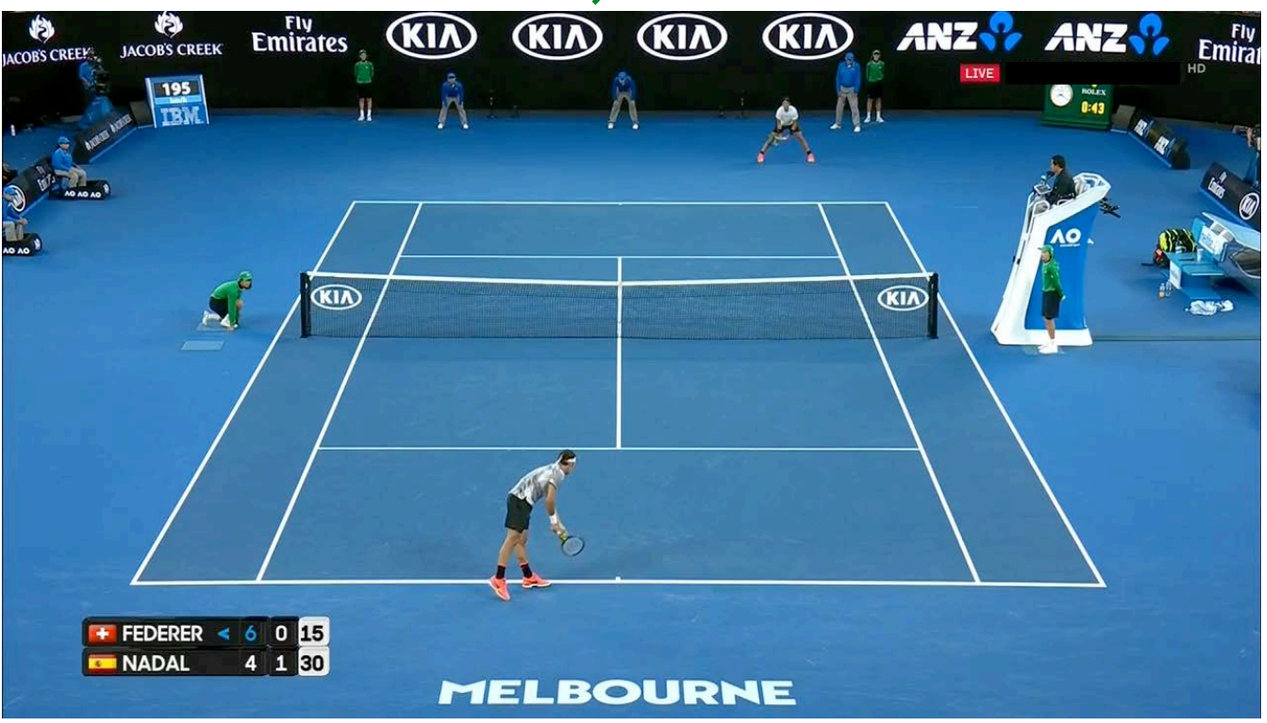
Observer transformation

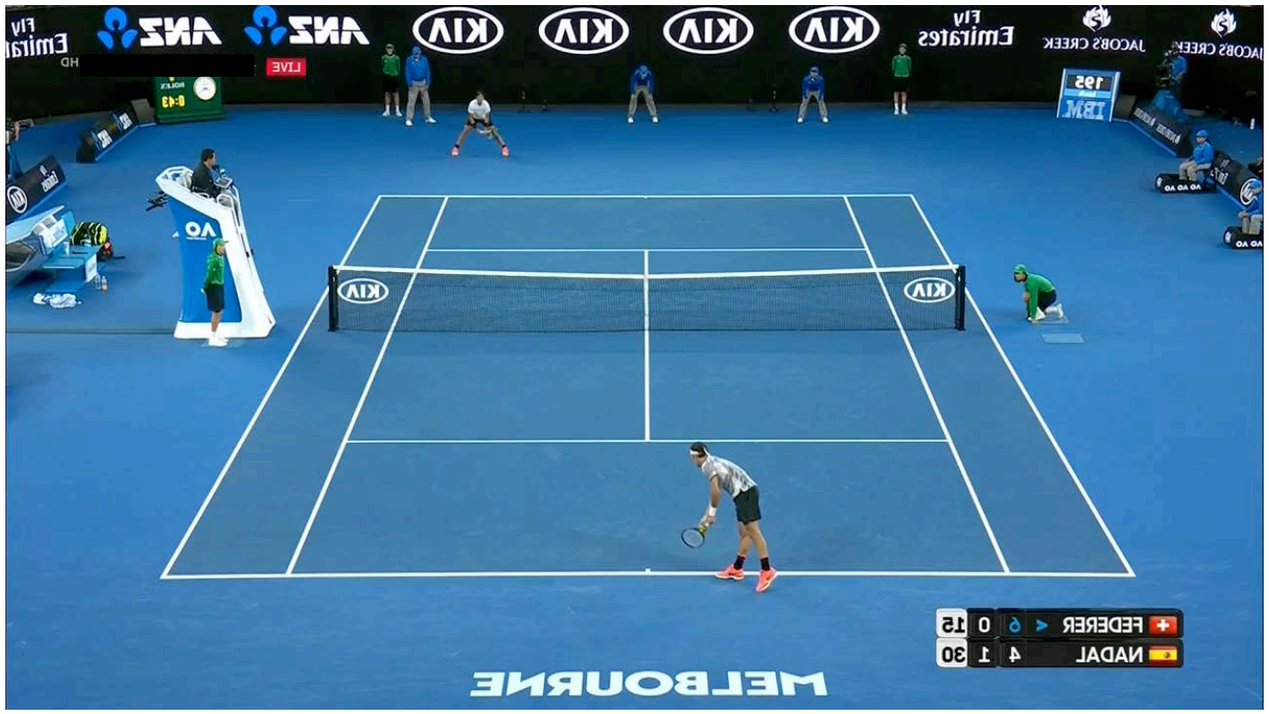




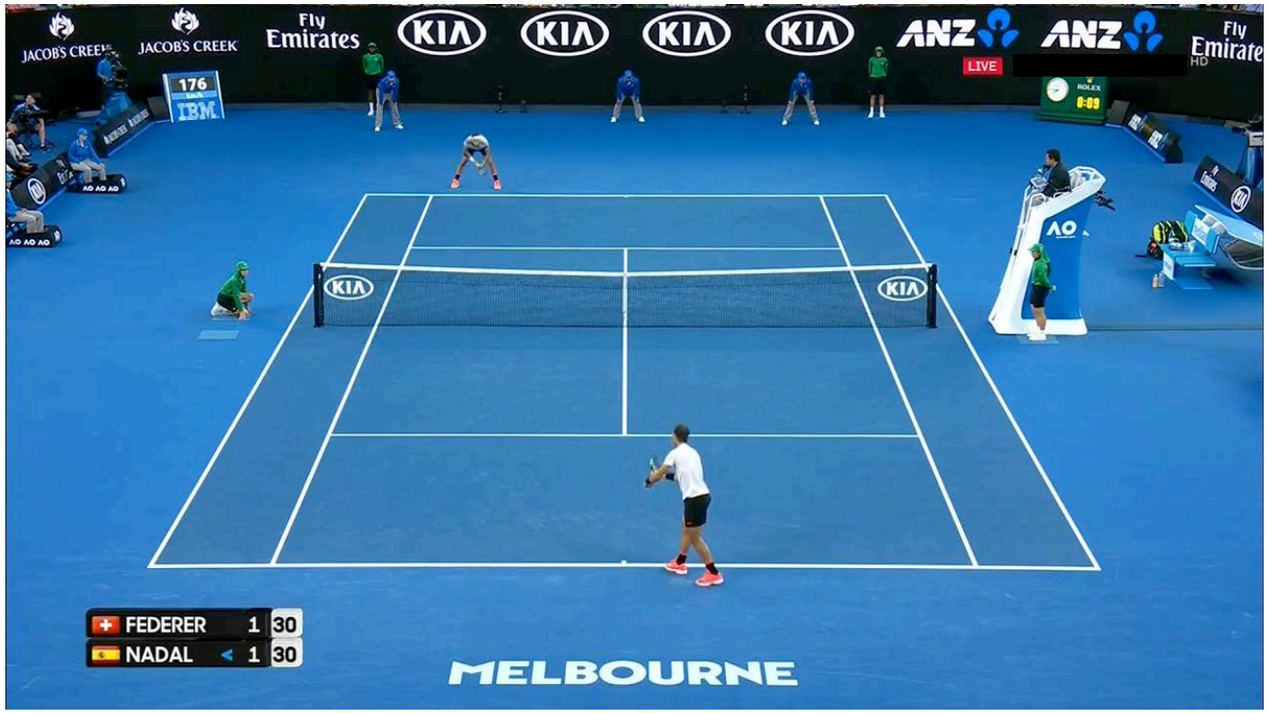
Symmetry Transformations: Parity

Observer transformation





Particle transformation



Symmetry Transformations: Observer vs Particle

- In order to discuss Lorentz Violation it is important to separate ***observer*** and ***particle*** transformations
- ***Observer transformations***
  - Act on the observer reference frame while leaving the system unchanged
  - Only observable quantities transform
  - Might or might not have "physical meaning"
    - Parity: look at the system in a mirror
    - Lorentz: look at the system from a rotated/boosted reference frame
    - Cartesian to polar change of coordinates: no real life equivalent
- ***Particle transformations***
  - Act on the system while leaving the observer frame unchanged
  - All quantities the appear in a theory (fields, couplings, …) and have to be assigned transformation properties in order to achieve invariance
    - e.g. you could define a parity, which is a not a symmetry of the strong interactions, in which the pion has positive intrinsic parity

Lorentz Transformations: Observer vs Particle

- A muon decaying at rest has a certain lifetime that changes if the muon is observed in a frame boosted in some direction or if the muon itself is boosted in the opposite direction:
  if observer Lorentz transformations describe correctly the effect of boosting one reference frame and particle transformations are a symmetry of nature, these two lifetimes are identical (relativity principle)
- One could <u>modify observer Lorentz transformations</u> and construct a <u>modified particle transformation that leaves the physics invariant</u>.
  - This is what happened in the transition from Galilean to Lorentz invariance. Note that even though kinematical effects vanish for small velocities, the change in metric is dramatic (also, what about magnetic effects?)
  - We will not pursue this route
- ***We will preserve observer Lorentz transformations but spontaneously break particle Lorentz invariance***
  - The muon lifetime *(1)* of a boosted muon and *(2)* of a muon at rest measured in a boosted frame will differ (breakdown of the relativity principle)
  - Care must be paid in case Lorentz violation affects the measurement
  - Spontaneous breaking is preferred because explicit breaking is inconsistent, upon inclusion of gravity, with Riemann geometry (Bianchi identities issues)

The Standard Model Extension (SME)

- We adopt the general parametrization offered by the ***Standard Model Extension***: *an effective field theory which has exact observer Lorentz covariance and that contains explicit preferred directions*

  [hep-ph/9703464; Colladay, Kostelecky]
  [hep-ph/9809521; Colladay, Kostelecky]
  [hep-th/0312310; Kostelecky]
- As discussed in the introduction, we will focus on renormalizable interactions: the resulting theory is known as ***minimal SME***
- The advantages of this approach include preservation of:
  - Standard Quantization
  - Microcausality
  - Spin-Statistic Theorem
  - Observer Lorentz covariance
  - Hermiticity
  - Positivity of the Energy
  - Power counting renormalizability
  - Conservation of Energy-Momentum for constant Lorentz Violating vacuum expectations values
- The coefficients for LV are dimensionless, need renormalization and are arbitrary. If a SSB mechanism is specified, say at the Planck scale, their natural size is going to be suppressed by powers of M_{Planck}.
- Note that some non-renormalizable LV terms yield signatures that are not achievable within the minimal SME

The Standard Model Extension (SME)

- Our phenomenological investigation focuses on electron-proton Deep Inelastic Scattering: only the *SU(3)×U(1)* sector of the theory is relevant
- The *SU(3)×U(1)* gauge, lepton and quark sectors are (ψ=*u*,*d*,*e*):

$$\mathcal{L}_{\rm SM} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \frac{1}{4}G^a_{\mu\nu}G^{a,\mu\nu} + \bar{\psi}(\gamma^\mu iD_\mu - m_\psi)\psi$$

$$\delta\mathcal{L}_{\rm SME} = -\frac{1}{4}\kappa_F^{\kappa\lambda\mu\nu}F_{\kappa\lambda}F_{\mu\nu} - \frac{1}{4}\kappa_G^{\kappa\lambda\mu\nu}G^a_{\kappa\lambda}G^a_{\mu\nu} + \bar{\psi}(\Gamma^\mu iD_\mu - M)\psi$$

where $\Gamma^\mu = c^{\mu\nu}\gamma_\nu + d^{\mu\nu}\gamma_\nu + e^\mu + if^\mu\gamma^5 + \frac{1}{2}g^{\alpha\beta\mu}\sigma_{\alpha\beta}$

$$M = a_\mu\gamma^\mu + b_\mu\gamma^5\gamma^\mu + \frac{1}{2}H^{\mu\nu}\sigma_{\mu\nu}$$

D_μ is the standard QCD & QED covariant derivative

- Under observer transformations the SME Lagrangian is a scalar: for example a_μ and $\bar{\psi}\gamma^\mu\psi$ are both 4-vectors
- Under particle transformations $\bar{\psi}\gamma^\mu\psi$ is a 4-vector and a_μ are 4 scalars
- There are additional gauge sector terms which we neglect because they generate instabilities associated with negative contributions to the energy

Physical coefficients

- Not all coefficients introduced above are physical
- Some coefficients can be eliminated via a field redefinitions like:

  $\psi(x) \to e^{if(x)}\psi(x)$

  $\psi(x) \to [1 + v(x)\cdot\Gamma]\psi(x) \quad \text{with} \quad \Gamma = \gamma^\alpha,\ \gamma_5\gamma^\alpha,\ \sigma^{\alpha\beta}$

  $\Rightarrow$ in this way a_μ and the antisymmetric part of $c_{\mu\nu}$ can be eliminated

- Some parts of the coefficients are not LV. For instance, even after removing its antisymmetric part we have: $c_{\mu\nu} = \underbrace{[c_{\mu\nu}]_{traceless\ \&\ symmetric}}_{\text{L.V.}} + \underbrace{\alpha\ \eta_{\mu\nu}}_{\text{L.I.}}$

- Some coefficients can be eliminated via a choice of coordinates:

$$\mathcal{L} = -\frac{1}{4}\left(\kappa^{\kappa\lambda\mu\nu} + \eta^{\kappa\mu}\eta^{\lambda\nu}\right)F_{\kappa\lambda}F_{\mu\nu} + \left(\eta^{\mu\nu} + c^{\mu\nu}\right)\bar{\psi}\gamma_\mu iD_\nu\psi$$

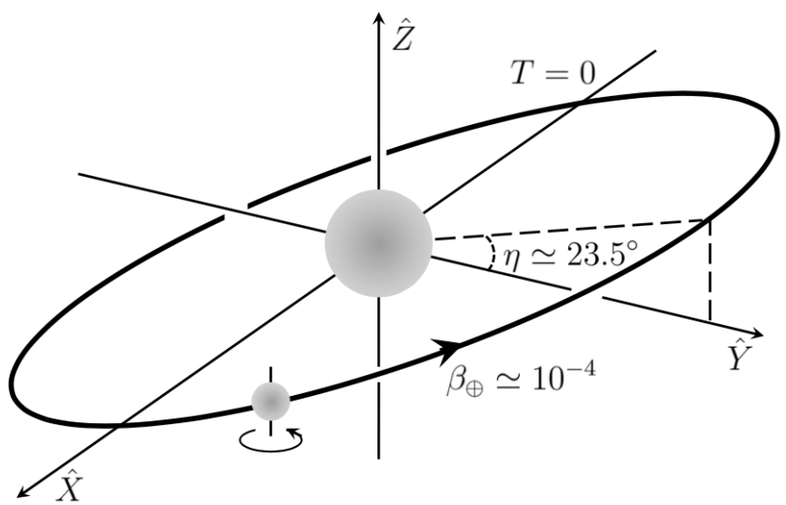
$$\Big\downarrow\ x^\mu \to x^\mu - \frac{1}{2}\kappa^{\alpha\mu}{}_{\alpha\nu}x^\nu$$

$$\mathcal{L} = -\frac{1}{4}F^{\mu\nu}F_{\mu\nu} + \left(\eta^{\mu\nu} + c^{\mu\nu} + \frac{1}{2}\kappa^{\alpha}{}_{\mu\alpha\nu}\right)\bar{\psi}\gamma_\mu iD_\nu\psi$$

We can choose one sector of the SME to define the scales of the four coordinates

Existing Constraints

- All coefficients for Lorentz-Violation are defined with respect to a Sun-centered celestial-equatorial frame (which is the "most" inertial frame that is accessible [0801.0287; Kostelecky, Russell]



- Experiments that focus on the properties of stable particles (electrons, muons, protons, neutrons, photons) yield very strong constraints:

$$\kappa_F^{\alpha\beta\mu\nu} < \left[10^{-14} - 10^{-32}\right]$$
$$c_{\text{electron}}^{\mu\nu} < \left[10^{-17} - 10^{-21}\right]$$
$$c_{\text{proton}}^{\mu\nu} < \left[10^{-20} - 10^{-28}\right]$$
$$c_{\text{neutron}}^{\mu\nu} < \left[10^{-13} - 10^{-29}\right]$$
$$c_{\text{muon}}^{\mu\nu} < 10^{-11}$$

- Note that extreme care has to be used in interpreting these bounds. Experiments can only place limits on "physical" combinations of coefficients.
- Coefficients in the quark sectors are almost completely unconstrained due to the difficulty of accessing quark level transitions directly. The strongest constraint comes from absence of LV Cherenkov radiation in high energy gamma rays: $c_u^{TT}, c_d^{TT} < 1.8 \times 10^{-21}$ and $(\kappa_G)_{\text{tr}} < 2 \times 10^{-13}$

LV in the quark sector: observables

- The main problem is that the Hilbert space of QCD does not contain quarks and gluons but mesons and baryons. The latter are bound states controlled by non-perturbative effects that are (almost) only accessible in lattice-QCD simulations.
- There are several avenues that one can pursue:
  - LV impact on hadron properties
    Impact of the $c_{\mu\nu}$ coefficients on proton, neutron and pion properties (dispersion relation, mass, spin, interactions). E.g. A spurion analysis of how the $c_{\mu\nu}$ coefficients can enter into the chiral Lagrangian suggests that existing constraint on various hadrons could be used (up to non perturbative QCD effects) to place strong bounds on some combinations of $c_{\mu\nu}$ coefficients [Kamand, Altschul, Schindler; 1608.06503]
  - Hadronic properties sensitive to short distance physics
    e.g.: meson-antimeson mixing [hep-ph/9809572; Kostelecky. Berger, Di Domenico, Van Kooten, van Tilburg in proceedings of the 7th meeting on CPT and Lorentz Symmetry]
  - Focus on high-energy hadronic interactions where, using factorization, it is possible to (partially) bypass non-perturbative problems and directly relate observables to the underlying quark dynamics: Deep Inelastic Scattering

LV in the quark sector: our setup

- We will focus on Deep Inelastic electron-proton Scattering: we are affected by coefficients that appear in the electron ($c_e^{\mu\nu}$), photon ($\kappa_F^{\alpha\beta\mu\nu}$), quark ($c_{u,d}^{\mu\nu}$) and gluon ($\kappa_G^{\alpha\beta\mu\nu}$) sectors.
  - We adopt coordinates in which the photon does not show any spin-independent Lorentz violation ($\kappa_F{}^{\alpha}{}_{\mu\alpha\nu} = 0$)
  - All coefficients in the electron sector are strongly constrained and do not contribute appreciably to electron-proton DIS
  - We set the gluon coefficients to zero and focus on the quarks
- Assuming spontaneous Lorentz violation at scales of order M_{Planck}, we expect "natural" size for most coefficients to be given by the ratio of some low energy mass to M_{Planck}. Nevertheless the need to perform direct experimental searches should not be understated.
- Note that the quark coefficients contribute via divergent loops to LV in the photon and electron sector. This does not change our fundamental set up but might rise issues of fine tuning/naturalness in the electron sector. We leave a detailed investigation of this point to a forthcoming analysis.

LV in the quark sector: general considerations

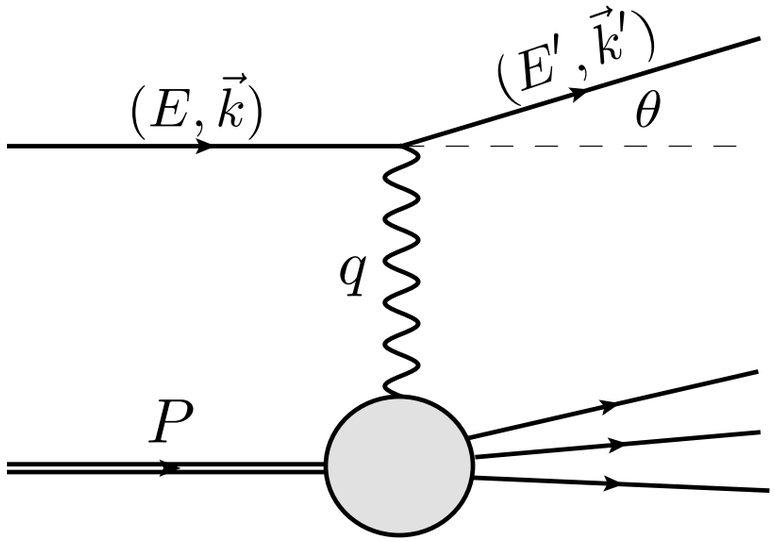
- In our case: $\mathcal{L} = \bar{q}\left[\frac{1}{2}\gamma_\mu(g^{\mu\nu} + c^{\mu\nu})i\overleftrightarrow{D}_\nu - m\right]q$
- The quark dispersion relation is modified:

$$0 = \tilde{p}^\mu \tilde{p}_\mu - m^2 = p_\mu(\eta^{\mu\nu} + c^{\mu\nu})(\eta_{\nu\lambda} + c_{\nu\lambda})p^\lambda - m^2$$

  - Velocity ($\vec{v} = \vec{\nabla}_p E(\vec{p})$) and momentum are not parallel anymore
  - Sums and averages over quark spins are affected
  - The cross section flux factor is also affected (must use velocities of colliding particles!)
  - In our case the relevant flux factor involves the electron and proton, both of which receive negligible Lorentz violating effects (given the kind of constraints that we will be able to achieve)
- We treat the traceless tensor $c_{\mu\nu}$ as a small perturbation resulting in a standard Feynman diagram expansion

Deep Inelastic electron-proton Scattering

- A general parametrization of ep scattering is given in terms of two form factors:

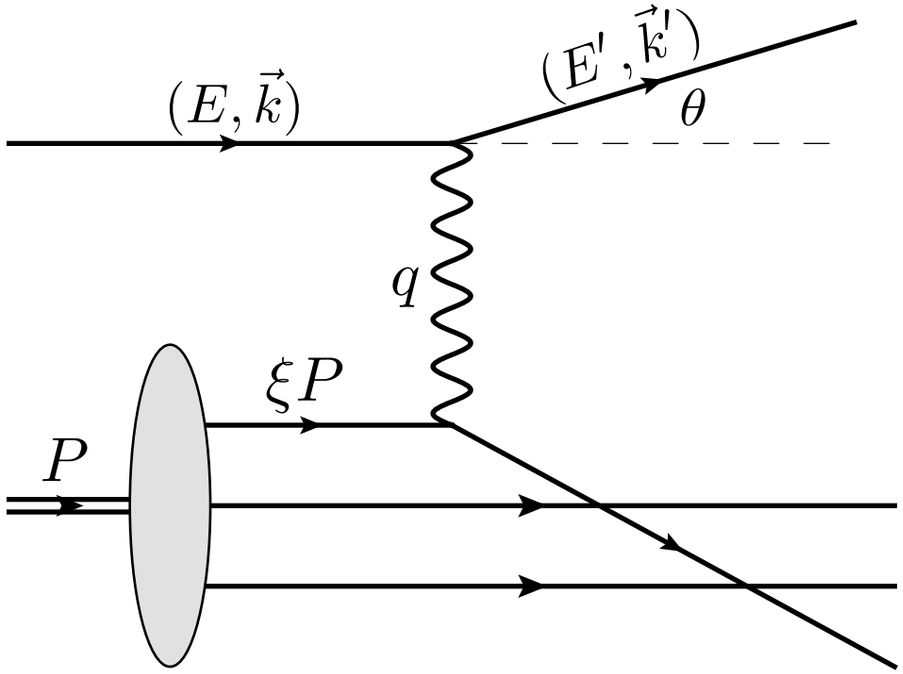


$$\left(\frac{d\sigma}{d\Omega\, dE'}\right)_{\text{lab}} = \frac{\alpha_{\text{em}}^2}{8\pi E^2 \sin^4\frac{\theta}{2}}\left[\frac{m_p}{2}W_2(x,Q)\cos^2\frac{\theta}{2} + \frac{1}{m_p}W_1(x,Q)\sin^2\frac{\theta}{2}\right]$$

$$Q^2 = -q^2 > 0$$

$$x = \frac{Q^2}{2P\cdot q} \in [0,1]$$

- At large Q^2 the parton model picture emerges:



partonic cross section
(parton momentum is $p_i^\mu = \xi p^\mu$)

$$\sigma(ep \to eX) = \sum_i \int_0^1 d\xi\, \boxed{f_i(\xi)}\, \boxed{\hat{\sigma}(ep_i \to eX)}$$

parton distribution function (PDF):
"probability" of finding parton with
momentum ξp^μ inside the proton)

- The PDF's are independent of Q
- Perturbative and non-perturbative physics factorize
- The latter is universal

Deep Inelastic electron-proton Scattering

- At tree-level one finds $\hat{\sigma}(ep_i \to eX) \propto \delta(\xi - x)$:

$$W_1(x,Q) = 2\pi \sum_i Q_i^2 f_i(x)$$

[Independent of Q: Bjorken scaling]

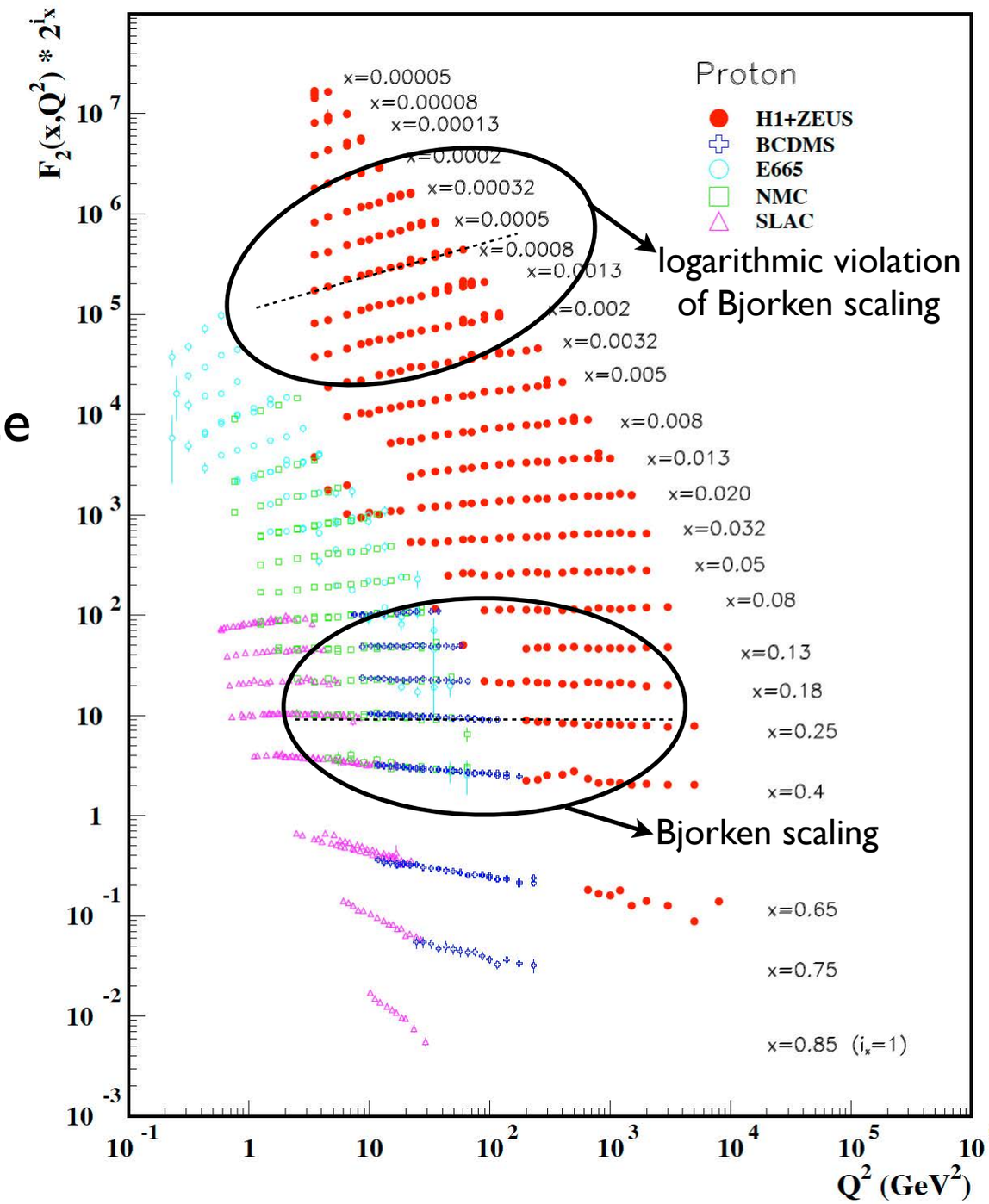
$$W_2(x,Q) = 2\pi \frac{x^2}{Q^2} \sum_i Q_i^2 f_i(x) = \frac{4x^2}{Q^2} W_1(x,Q)$$

[Callan-Gross relation]

- QCD corrections (known at NNLO) induce logarithmic corrections to Bjorken scaling
- **These experimental results are used to measure the PDFs**

- After time averaging, LV effects modify the Q^2 dependence of the cross section and of the DGLAP equations.
  Some effects are absorbed in the PDFs!

- Sidereal time analyses are more promising



Deep Inelastic electron-proton Scattering

- In the case of e-p DIS this (somewhat heuristic) discussion can be formalized in the language of an Operator Product Expansion:

$$d\sigma \sim |M(ep \to eX)|^2 \sim \mathrm{Im}[M(ep \to ep)] \sim \mathrm{Im}\langle p|TJ_\mu(x)J_\nu(0)|p\rangle \sim C^{\mu\nu\mu_1\cdots\mu_n}\,\langle p|O_{\mu_1\cdots\mu_n}|p\rangle$$

optical theorem — em current — OPE — Wilson Coefficients — local operators

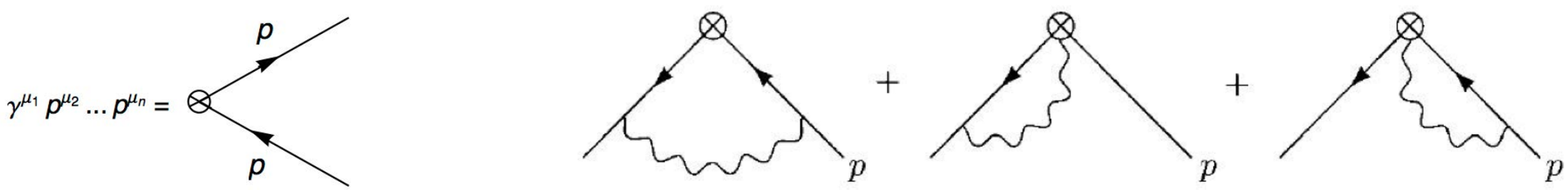
- Schematically: $C^{\mu\nu\mu_1\cdots\mu_n} \sim \dfrac{q^{\mu_1}\cdots q^{\mu_n}}{Q^{2n}}\left(\dfrac{q^\mu q^\nu}{Q^2} - g^{\mu\nu}\right) + \cdots$

$$O_{\mu_1\cdots\mu_n} = \bar{q}\gamma_{\mu_1} iD_{\mu_2}\cdots iD_{\mu_n} q + \text{symmetrizations} - \text{traces}$$

$$\langle p|O_{\mu_1\cdots\mu_n}|p\rangle = \mathcal{A}^n P_{\mu_1}\cdots P_{\mu_n} - \text{traces}$$

- Connection with PDFs: $f(x) = \dfrac{1}{\pi}\sum \dfrac{\mathcal{A}^n}{x^n}$

- The DGLAP evolution is reproduced by the standard RGE's for the the operators:



Lorentz Violating corrections to e-p scattering

- The DIS cross section is $d\sigma \sim L^{\mu\nu} W_{\mu\nu}$, where $L^{\mu\nu}$ and $W^{\mu\nu}$ are the leptonic and hadronic tensors (the latter is expressed in terms of W_1 and W_2)
- In the SM: $$W^{\mu\nu} \sim \int_0^1 \frac{f_i(\xi)}{\xi} Q_i^2 \, \xi P_\alpha (\xi P_\beta + q_\beta) \frac{\mathrm{Tr}[\gamma^\alpha \gamma^\mu \gamma^\beta \gamma^\nu]}{(\xi P + q)^2 + i\epsilon} + (\mu \leftrightarrow \nu, q \leftrightarrow -q)$$
- In the SME: $$W^{\mu\nu} \sim \int_0^1 \frac{f_i(\xi)}{\xi} Q_i^2 \, \xi P_\alpha (\xi P_\beta + q_\beta) \frac{\mathrm{Tr}[\Gamma^\alpha \Gamma^\mu \Gamma^\beta \Gamma^\nu]}{[(\xi P_\beta + q_\beta)\Gamma^\beta]^2 + i\epsilon} + (\mu \leftrightarrow \nu, q \leftrightarrow -q)$$

  where $\Gamma^\mu = \gamma^\mu + c^{\mu\nu}\gamma_\nu$

  - The trace in the numerator is simply expanded keeping only linear terms in $c^{\mu\nu}$
  - The denominator is slightly trickier because we need its imaginary part. Introducing $\tilde{p}^\mu \equiv (g^{\mu\nu} + c^{\mu\nu}) p_\nu$ one gets:

$$\frac{1}{\pi} \mathrm{Im} \frac{1}{[(\xi P_\beta + q_\beta)\Gamma^\beta]^2 + i\epsilon} = \delta\left(\tilde{q}^2 + 2\xi \tilde{P} \cdot \tilde{q} + \xi^2 \tilde{q}^2\right)$$

$$\sim \frac{1}{2P \cdot q} \left[\delta(\xi - x) + \delta'(\xi - x) c^{\mu\nu} H_{\mu\nu}\right]$$

Yields terms proportional to the derivative of the PDFs

Lorentz Violating corrections to e-p scattering

- The final expression for the double differential decay rate (γ exchange only) is:

$$\frac{d\sigma}{dx dy d\phi} = \frac{\alpha^2}{q^4} \sum_f Q_f^2 \, x'_f \, f_f(x'_f) \left[\frac{ys^2}{\pi}(1+(1-y)^2)\delta_f + \frac{y^2 s}{x} x_f \right.$$
$$- \frac{4M^2}{s}(c_f^{kk'} + c_f^{k'k}) + 4(c_f^{k'P} + c_f^{Pk'}) + \frac{4}{x}(1-y)c_f^{kk}$$
$$\left. - 4xyc_f^{PP} - \frac{4}{x}c_f^{k'k'} + 4(1-y)(c_f^{kP} + c_f^{Pk}) \right]$$

large effects at low x

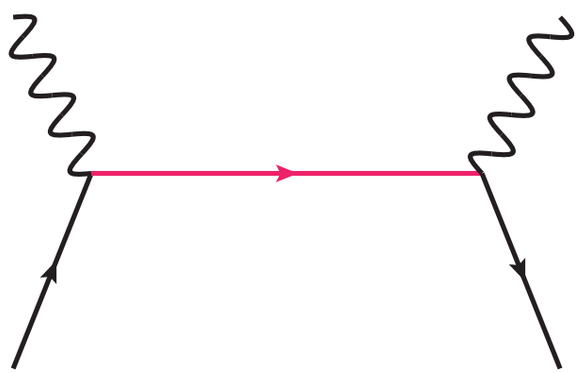
where

- $P^\mu = E_p(1, -\hat{k})$, $k^\mu = E(1, \hat{k})$ and $k'^\mu = E'(1, \hat{k}')$ are the proton, incoming and outgoing electron momenta in the **lab frame** (e.g. for HERA $E = 27.5\ GeV$ and $E_p = 920\ GeV$). s is the center-of-mass energy of the collision.
- $y = \frac{P \cdot q}{P \cdot k} = \frac{Q^2}{4E_p E\, x}$
- $\delta_f = \frac{\pi}{ys}\left(1 - \frac{2}{ys}(c_f^{Pq} + c_f^{qP} + 2xc_f^{PP})\right)$

  $x'_f = x - x_f = x - \frac{2}{ys}(c_f^{qq} + xc_f^{qP} + xc_f^{Pq} + x^2 c_f^{PP})$
- $c_f^{pq} \equiv p_\mu c_f^{\mu\nu} q_\nu$
- In our numerical results we include also Z boson exchange diagrams

Lorentz Violating corrections to e-p scattering: OPE

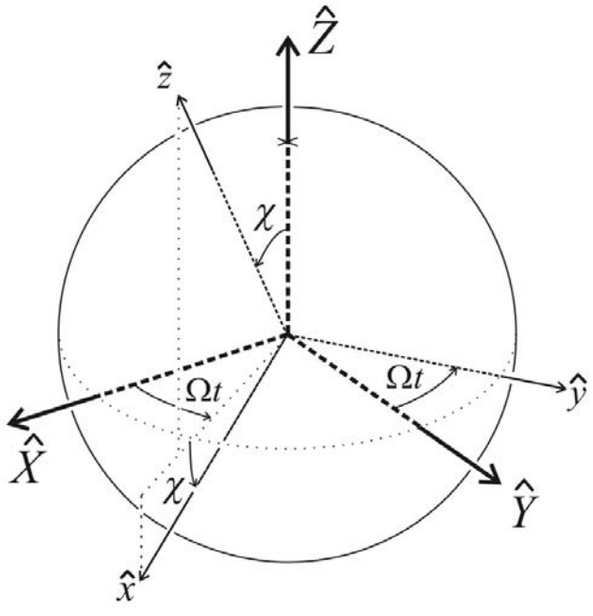
- The preceding discussion extended the intuitive parton model picture to a LV framework. It is important to verify whether this is reproduced by a more rigorous OPE calculation.
- We seek the OPE for the product of two electromagnetic currents:



$$\bar{\psi}_f(x)\Gamma_f^\mu \frac{i(i\tilde{\not\partial} + \tilde{\not q})}{(i\tilde{\partial} + \tilde{q})^2}\Gamma_f^\nu \psi_f(0)$$

- Expand: $\frac{1}{(i\tilde{\partial} + \tilde{q})^2} = \frac{1}{\tilde{q}^2}\sum_{n=0}^{\infty}\left(-\frac{2i\tilde{q}\cdot\tilde{\partial}}{\tilde{q}^2}\right)^n + O(\tilde{\partial}^2/\tilde{q}^2)$
- Operator basis: $\langle p|\bar{\psi}_f \gamma^{(\mu_1} i\tilde{\partial}^{\mu_2}\cdots i\tilde{\partial}^{\mu_n)}\psi_f|p\rangle = 2A_f^n\, \tilde{p}^{\mu_1}\cdots p^{\mu_n}$
- The matrix elements A_f^n are the moments of the quarks PDFs.
- The resulting DIS cross section is identical to our parton model calculation

Lorentz Violating corrections to e-p scattering



- The tensor $c_{\mu\nu}$ as it appears in our equations is related to the corresponding tensor in the non-rotating inertial frame by a spatial rotation:

$$\mathcal{R} = \begin{pmatrix} \pm 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & \mp 1 & 0 \end{pmatrix} \begin{pmatrix} \cos\varphi & \sin\varphi & 0 \\ -\sin\varphi & \cos\varphi & 0 \\ 0 & 0 & 1 \end{pmatrix}$$
$$\times \begin{pmatrix} \cos\chi\cos\omega_\oplus T_\oplus & \cos\chi\sin\omega_\oplus T_\oplus & -\sin\chi \\ -\sin\omega_\oplus T_\oplus & \cos\omega_\oplus T_\oplus & 0 \\ \sin\chi\cos\omega_\oplus T_\oplus & \sin\chi\sin\omega_\oplus T_\oplus & \cos\chi \end{pmatrix}$$

where χ=*36.4°* is the colatitude of the HERA, $\omega_\oplus$=$2\pi/(23h56m)$ is the sidereal frequency, $T_\oplus$ is the local sidereal time, φ is the orientation of the experiments (φ_{ZEUS}=*20° NoE*, φ_{H1}=*20° SoW*).

- The c_f^{ij} and c_f^{0i} components of the $c_f^{\mu\nu}$ tensor are given by $c_f^{KL} R_{Ki} R_{Lj}$ and $c_f^{TK} R_{iK}$, where c_f^{AB} $(A, B = T, X, Y, Z)$ is the tensor in the Sun-centered frame.
- The structure of the time dependent DIS cross section is:

$$\sigma(T_\oplus) = \sigma_{\rm SM}\Big[1 + (c_f^{TT},\ c_f^{TZ},\ c_f^{ZZ}) + (c_f^{TX},\ c_f^{TY},\ c_f^{YZ},\ c_f^{XZ})(\cos\omega_\oplus T_\oplus,\ \sin\omega_\oplus T_\oplus)$$
$$+ (c_f^{XY},\ c_f^{XX} - c_f^{YY})(\cos 2\omega_\oplus T_\oplus,\ \sin 2\omega_\oplus T_\oplus)\Big]$$

Constraints on LV couplings

- Terms that contribute to the time averaged cross section ($c_f^{TT}, c_f^{TZ}, c_f^{ZZ}$) are hard to constrain because DIS measurements are used to define the PDFs
  - Nevertheless LV corrections introduce a novel Q^2 dependence into the cross section that leads to a tree-level violation of Bjorken scaling
  - It might be possible to constrain these coefficients by disentangling the weak logarithmic Q^2 dependence introduced by the DGLAP equations and the strong power Q^2 dependence of the LV terms.
- On the other hand, terms that do not contribute to the time averaged cross section ($c_f^{TX}, c_f^{TY}, c_f^{YZ}, c_f^{XZ}, c_f^{XY}, c_f^{XX} - c_f^{YY}$) can be constrained in a straightforward way by employing a sidereal time analysis of the cross section.
- We calculated the expected constraints that can be obtained from a 4-bin sidereal time analysis of the whole *ZEUS+H1* DIS combined results [arXiv:1506.06042].

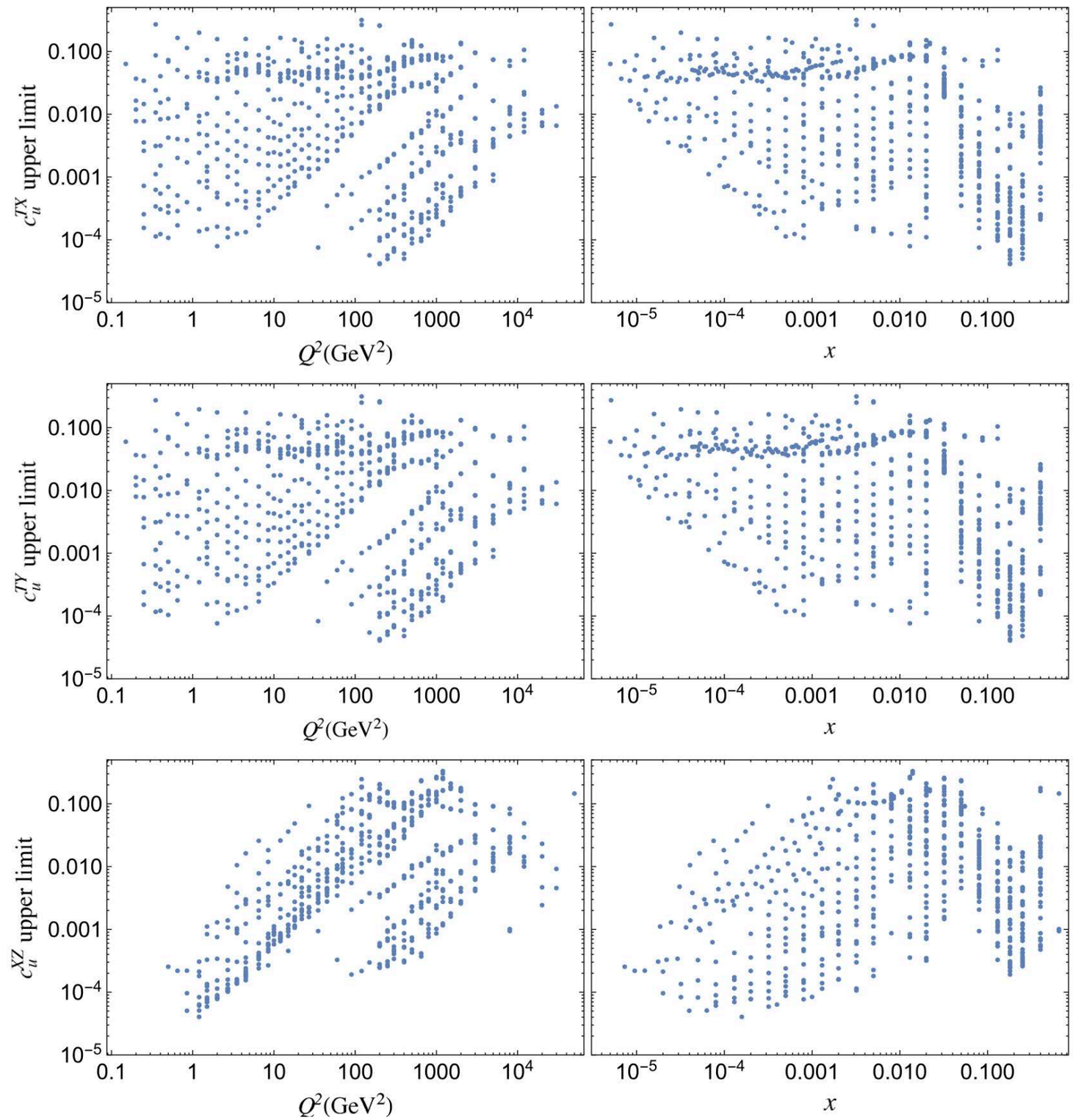
Expected constraints on LV couplings: HERA data

- We consider 644 neutral current measurements performed by *ZEUS* and *H1* [arXiv:1506.06042]
- ***For each measurement (i.e. each value/bin of x and Q^2):***
  - we estimate how the uncertainty increases due to a sidereal binning (4 bins)
  - the functional form that we assume for the binned theoretical cross section is $\sigma_i^{\rm th}(x,Q) = \alpha(x,Q)\,(1 + c_{AA}\,\beta_i(x,Q))$ where i indexes the sidereal bin and $\alpha(x,Q)$ and c_{AA} are fit parameters.
    The tree-level functions $\beta_i(x,Q)$ are our central result.
  - we generate a set of 10^3 possible experimental results assuming a normal distribution and the absence of LV effects
  - for each set we extract the frequentist 95% C.L. upper limit using a standard chi-squared (4 measurements and 2 fit variables)
  - the expected upper limit is the median of the upper limits over the set
- ***We also extract an expected upper limit by considering simultaneously all 644 measurements. In this case the chi-squared has 4×644 measurements and 644+1 fit variables***

Expected constraints on LV couplings: HERA data

- Expected constraints as a function of Q^2 and x
- Best expected limits:

$$|c^u_{TX}| \lesssim 4 \times 10^{-5}$$
$$|c^u_{TY}| \lesssim 4 \times 10^{-5}$$
$$|c^u_{XZ}| \lesssim 4 \times 10^{-5}$$
$$|c^u_{YZ}| \lesssim 4 \times 10^{-5}$$
$$|c^u_{XY}| \lesssim 4 \times 10^{-5}$$
$$|c^u_{XX} - c^u_{YY}| \lesssim 1 \times 10^{-5}$$



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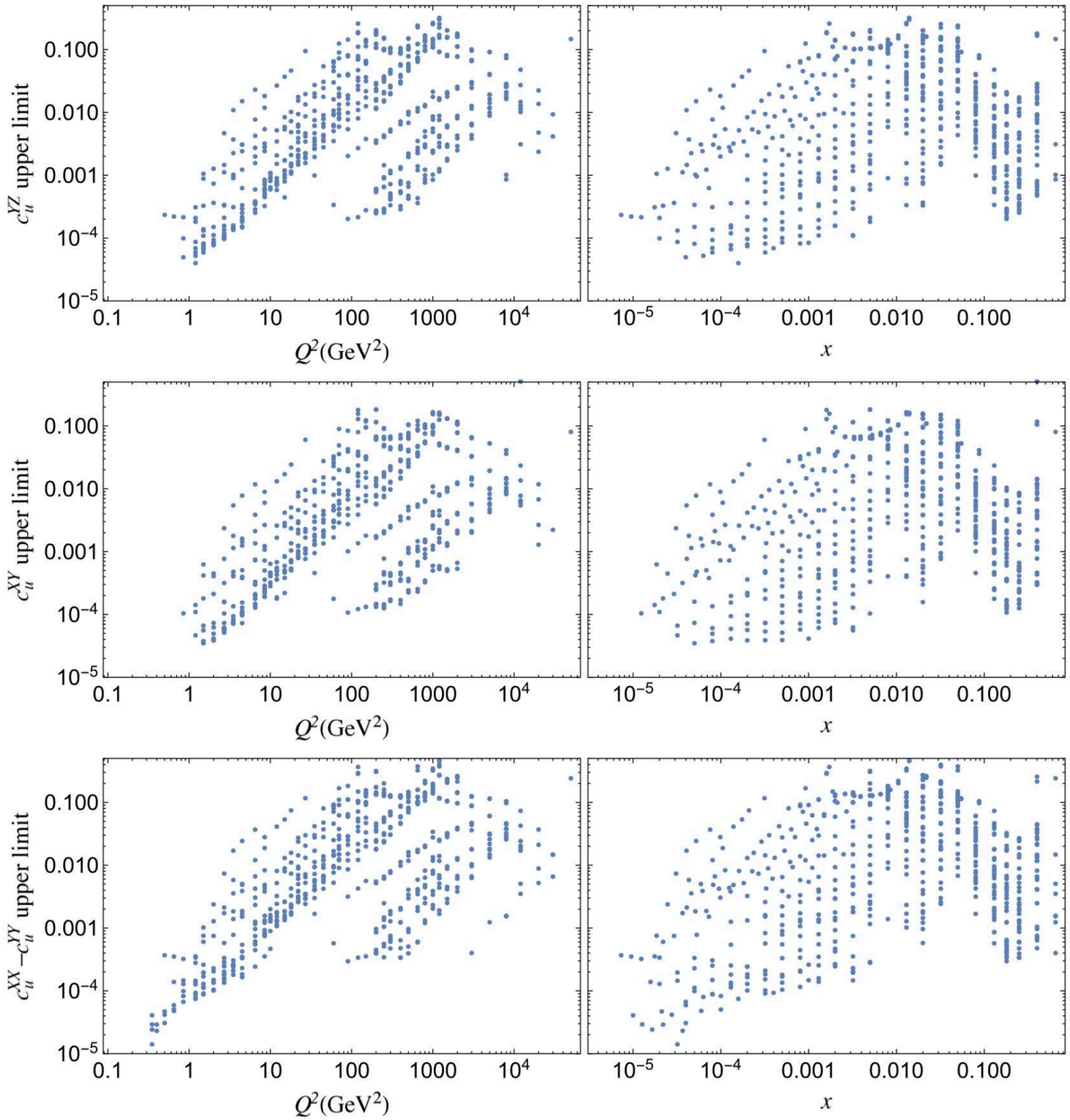

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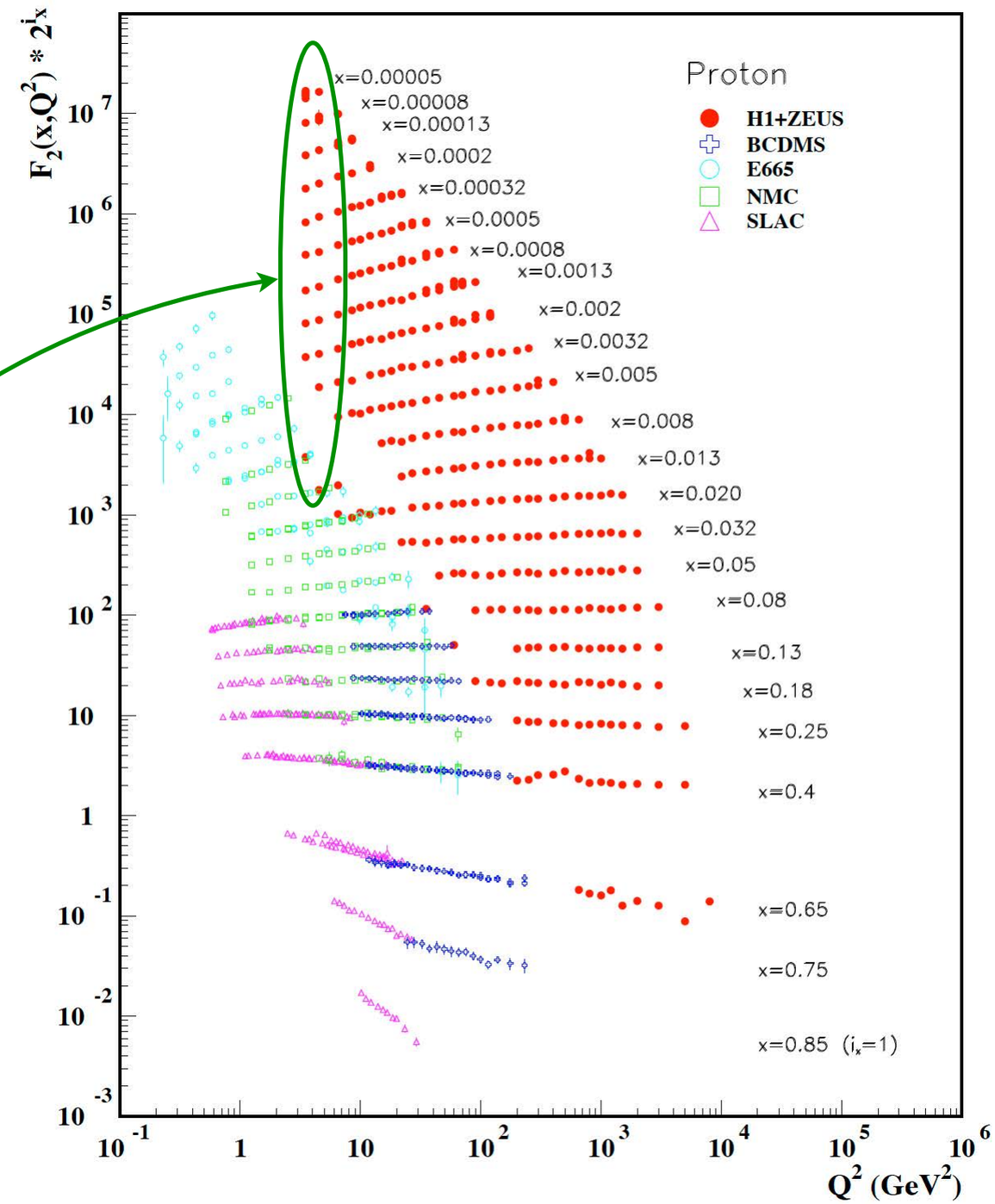
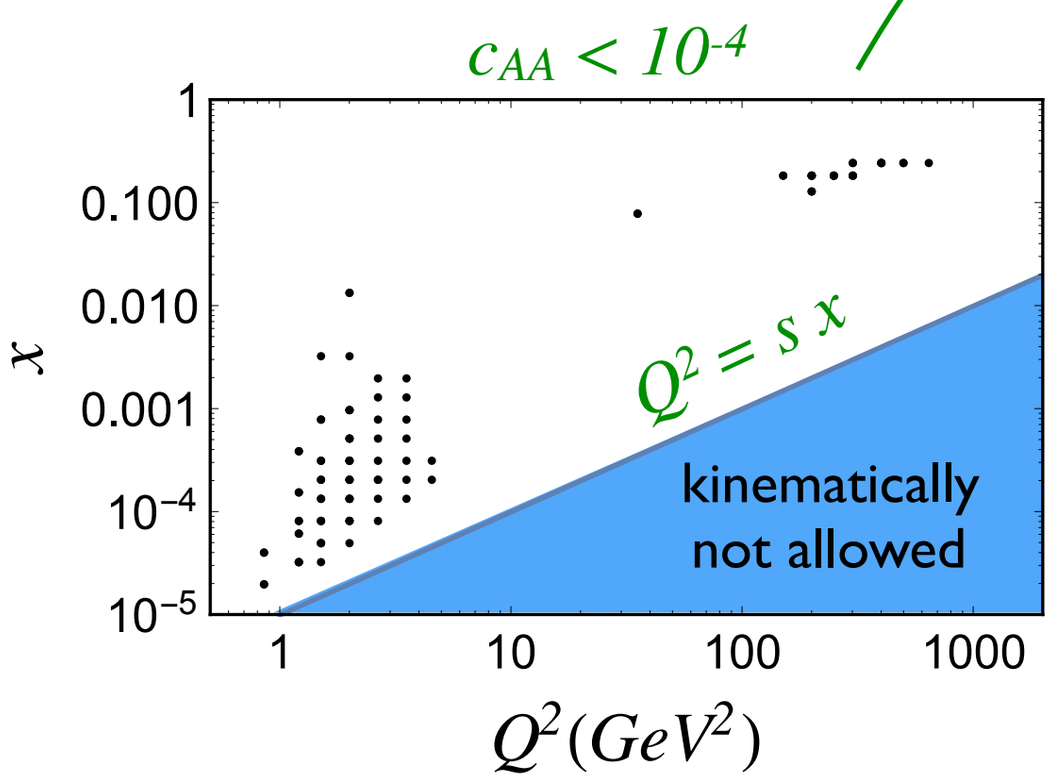


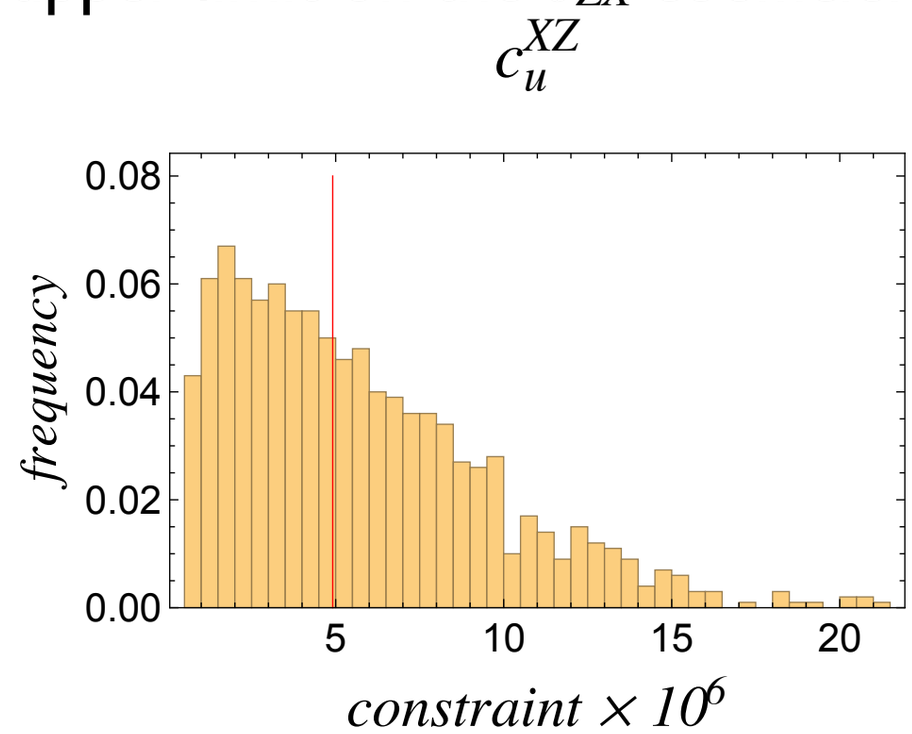


Expected constraints on LV couplings: HERA data

- Expected constraints extracted from the global analysis:

$$|c^u_{TX}| \lesssim 1 \times 10^{-5}$$
$$|c^u_{TY}| \lesssim 1 \times 10^{-5}$$
$$|c^u_{XZ}| \lesssim 5 \times 10^{-6}$$
$$|c^u_{YZ}| \lesssim 5 \times 10^{-6}$$
$$|c^u_{XY}| \lesssim 3 \times 10^{-6}$$
$$|c^u_{XX} - c^u_{YY}| \lesssim 8 \times 10^{-6}$$

- Distribution of expected 95% C.L. upper limit on the c_{ZX} coefficient:



$$|c^u_{XZ}| \lesssim (5 \pm 5) \times 10^{-6}$$

- As expected the simultaneous fit of the whole HERA dataset yields much tighter expected constraints.

Conclusions

- The Standard Model Extension is a generic extension of the SM that incorporates particle Lorentz Violation while preserving Lorentz covariance
- Coefficients in the photon, electron, muon, proton and neutron sectors are strongly constrained.
- The quark sector is much harder to constraint because of the nature of QCD
- We focused on electron-proton Deep Inelastic Scattering
- Coefficients that induce a time dependence on the cross section can be constrained by a straightforward sidereal binning analysis
- We calculated the expected sensitivity of such a study and found potential LV coefficients bounds at the 10^{-5} level
- Similar studies can be performed on Tevatron ($p\bar{p}$) and LHC (pp) observables