Constraints from Analyticity in Hadron Spectroscopy and the JLab program

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Indiana University
-
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INDIANA UNIVERSITY

Jefferson Lab
Thomas Jefferson National Accelerator Facility

Outline

- **Resume**
- **Accomplishments**
- **Research plan @JLab**

Resume

- **Master in Engineering (UMons, 2004)**
- **PhD in Physics (UMons, 2008)**
- **Postdoc at Valencia U. (2009-11)**
- **Postdoc at ECT*-Trento (2011-2013)**
- **Postdoc at Indiana U. (2013-present)**

- **35 publications (~ 600 citations) with 31 collaborators**
- **1 invited review**
- **1 white paper editor (42 authors)**

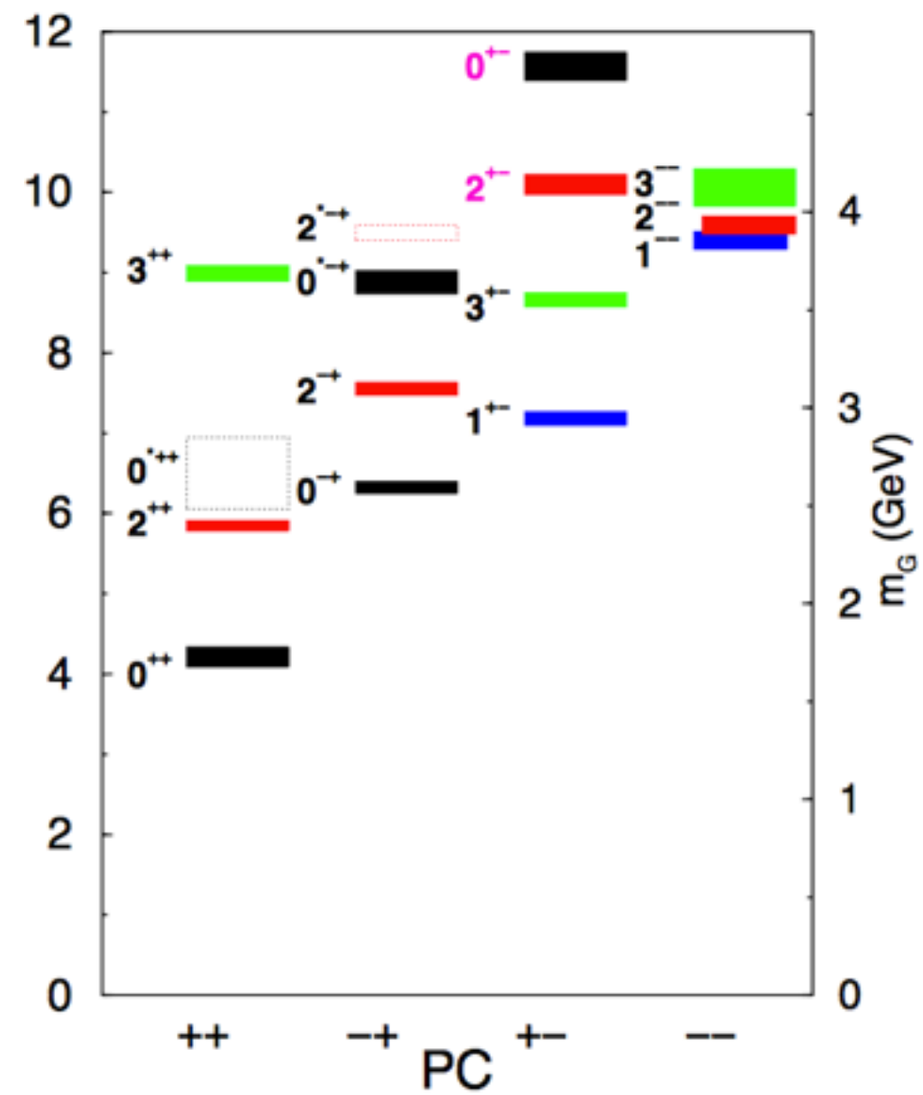
- **Referee for PRL, PRC&D, PLB, JHEP, …**
- **Organizers of 3 summer schools and 3 conferences**
- **~ 15 talks/year in meetings and physics departments**

- **Joint Physics Analysis Center website: http://www.indiana.edu/~jpac/**

Bound States of Gluons

- **Evidence from lattice calculations of "Glueballs" in pure gauge theory**

$$\mathcal{L}_{QCD} = -\frac{1}{4}\left(\partial\cdot\vec{A} + g\vec{A}\times\vec{A}\right)^2 \cancel{+\sum_f \bar{q}_f\left(\gamma\cdot\partial - ig\gamma\cdot\vec{A} - m_f\right)q_f}$$



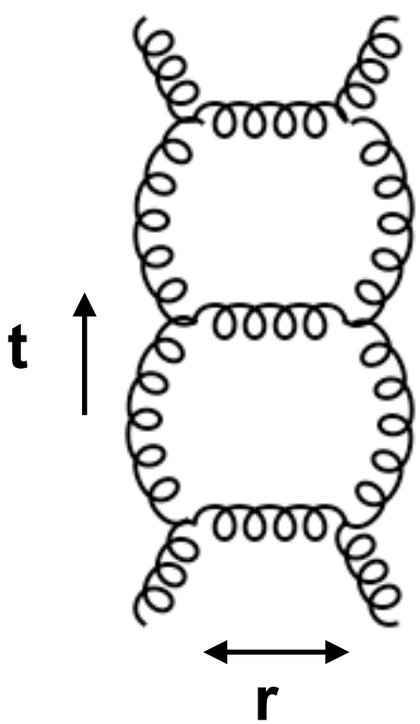
Morningstar and Peardon 1999

Bound States of Gluons

- **Evidence from lattice calculations of "Glueballs" in pure gauge theory**

$$\mathcal{L}_{QCD} = -\frac{1}{4}\left(\partial\cdot\vec{A} + g\vec{A}\times\vec{A}\right)^2 \cancel{+\sum_f \bar{q}_f\left(\gamma\cdot\partial - ig\gamma\cdot\vec{A} - m_f\right)q_f}$$

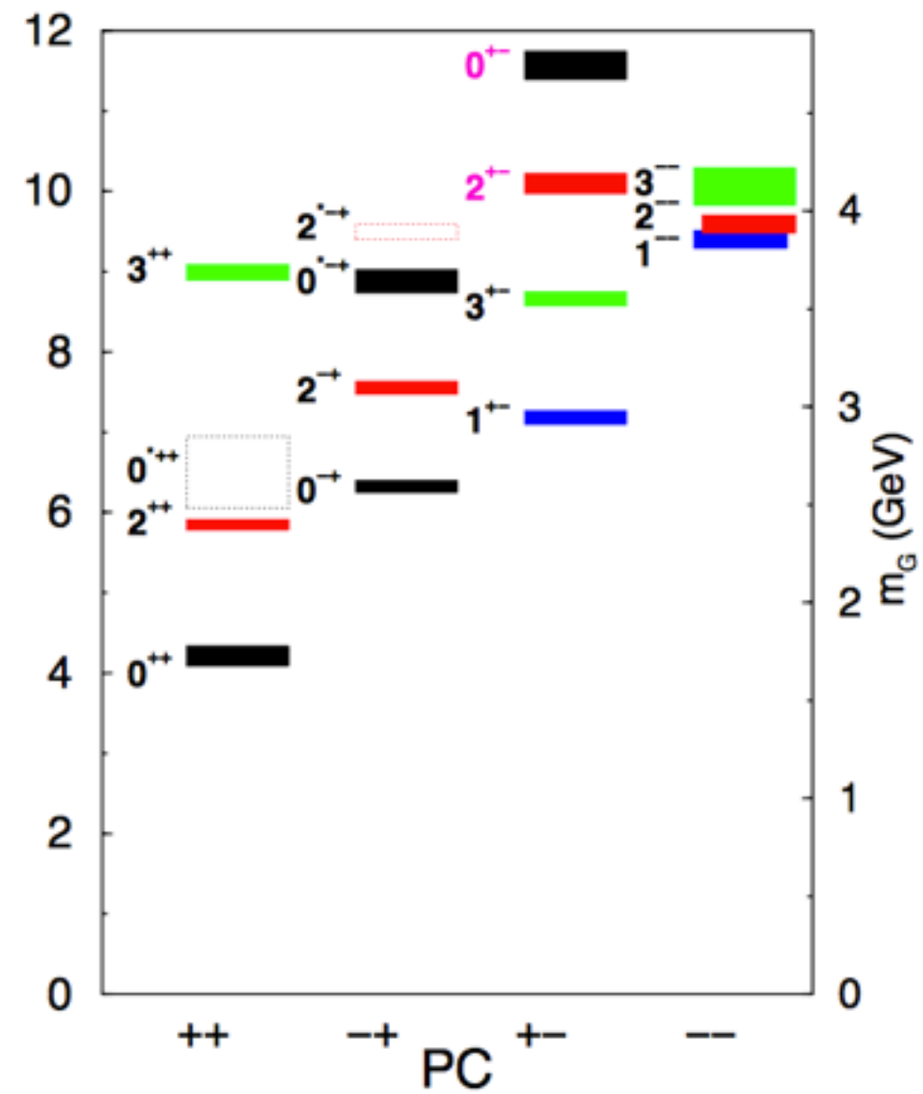
How to understand them with effective gluons ?



Schrödinger's equation

$$\left[\frac{\nabla^2}{2m} + V(r)\right]\Psi = E\Psi$$

$$V(r) = \sigma r + \cdots$$



Morningstar and Peardon 1999

Bound States of Gluons

Publication list on glueballs (during PhD in Mons):

- **On #gluon in glueballs**

  Boulanger, Buisseret, VM and Semay EPJA38 (2008)

- **On 2-gluon C = + glueballs**

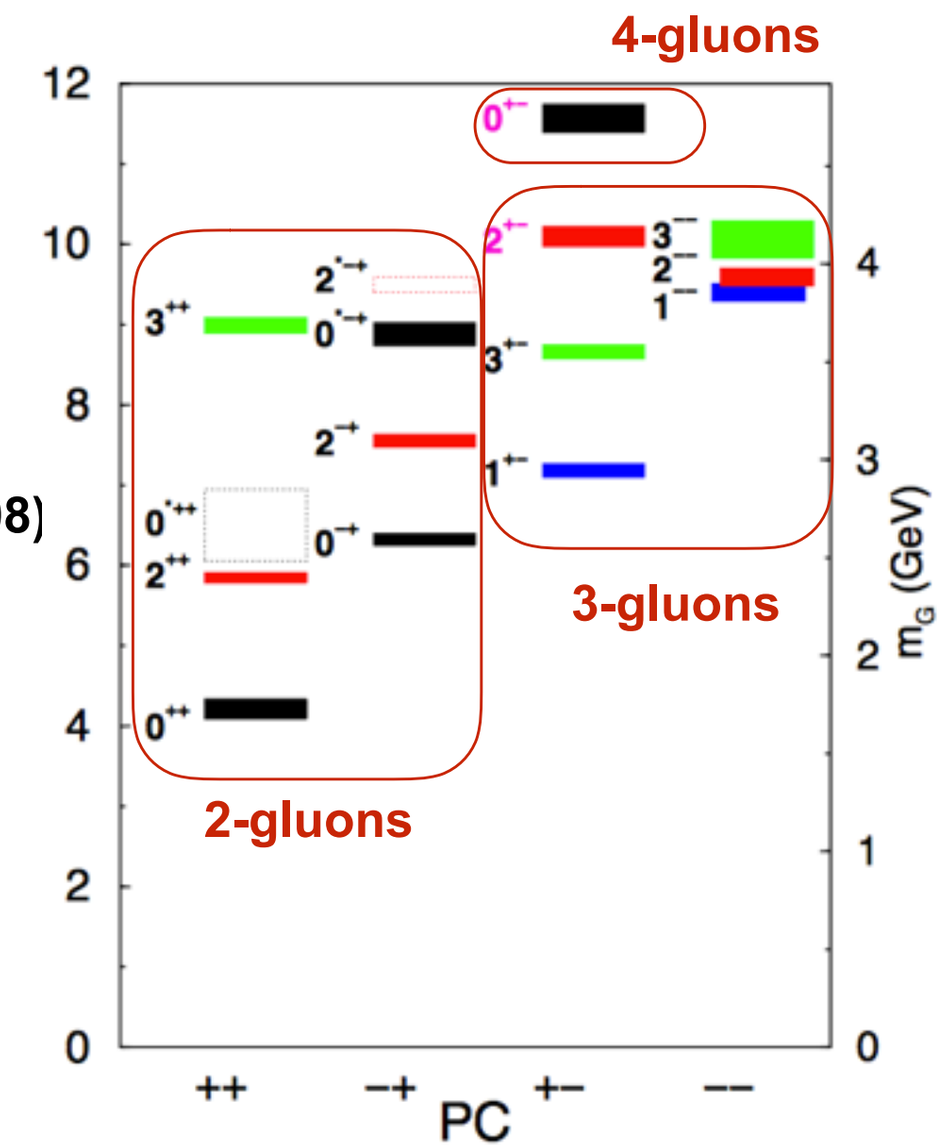
  VM, Buisseret and Semay PRD77 (2008), PRD80 (2009)

- **On 3-gluon C = - glueballs**

  VM, Semay and Silvestre-Brac PRD74 (2006), PRD77 (2008)

- **Invited review:**

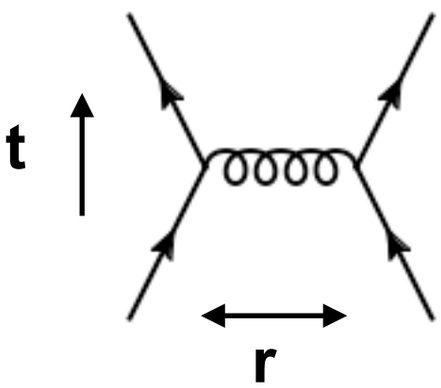
VM, Kochelev and Vento Int. J. Mod. Phys. E18 (2009) 1-49



Strong Interaction

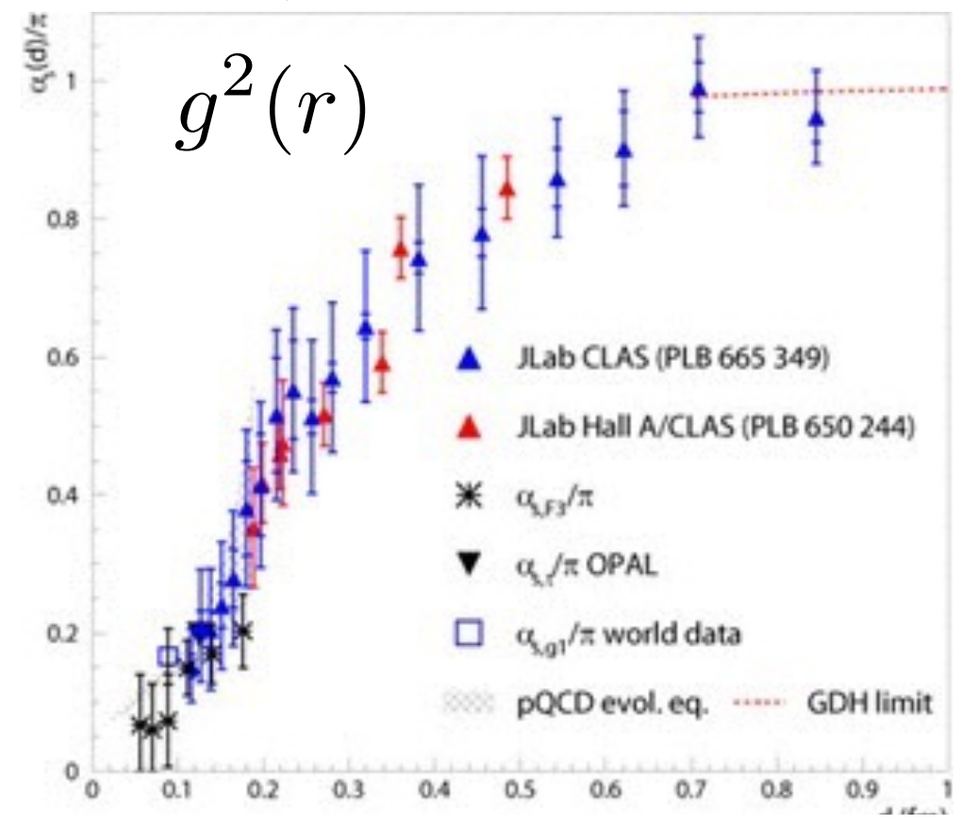
$$\mathcal{L}_{QCD} = -\frac{1}{4}\left(\partial\cdot\vec{A} + g\vec{A}\times\vec{A}\right)^2 + \sum_f \bar{q}_f\left(\gamma\cdot\partial - ig\gamma\cdot\vec{A} - m_f\right)q_f$$

quark interaction via gluon exchange



strength of interaction related to gluon propagator

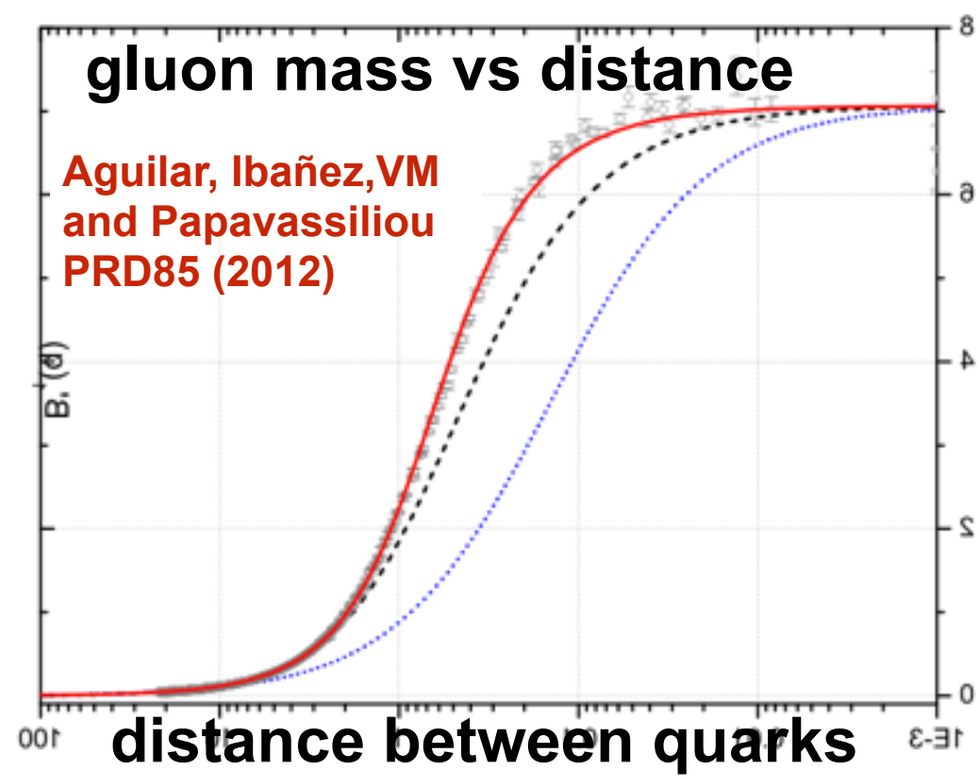
How can we understand that ?



Evidence from lattice calculation of IR saturation of gluon propagator

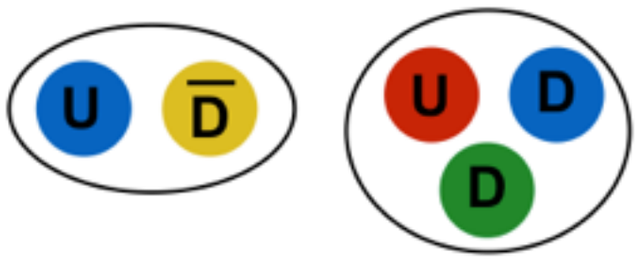
→ **dynamical gluon mass generation**

Gluon mass consistent with gauge invariance thanks to Schwinger mechanism

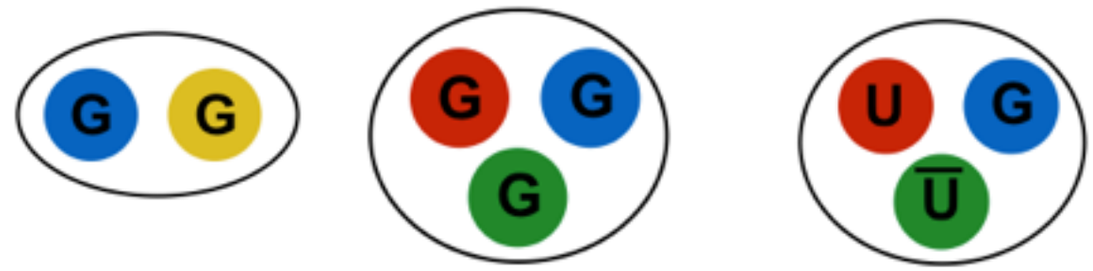


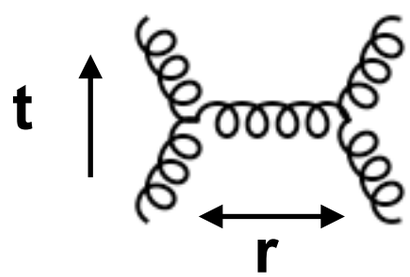
Matching Theory and Experiments

Ordinary matter



Exotic matter





Theory:
Quantum Chromodynamics

constituent models, lattice

Resonances properties

Matching Theory and Experiments

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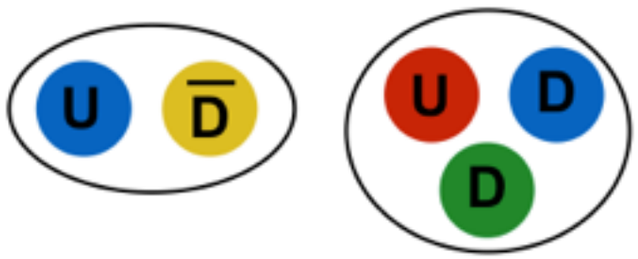

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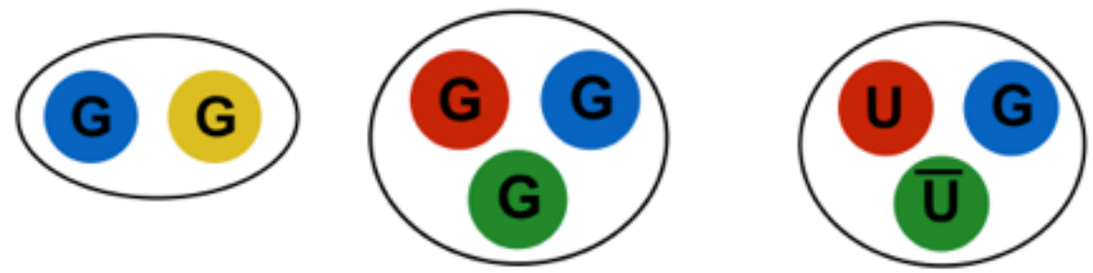

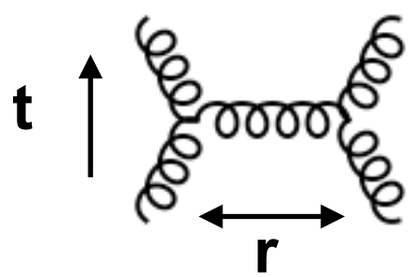

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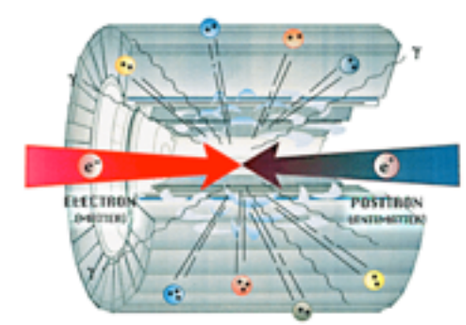
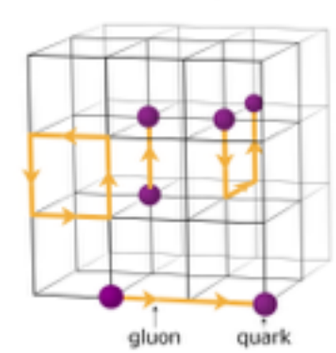
Experiments: JLab, CERN,…

amplitude analysis





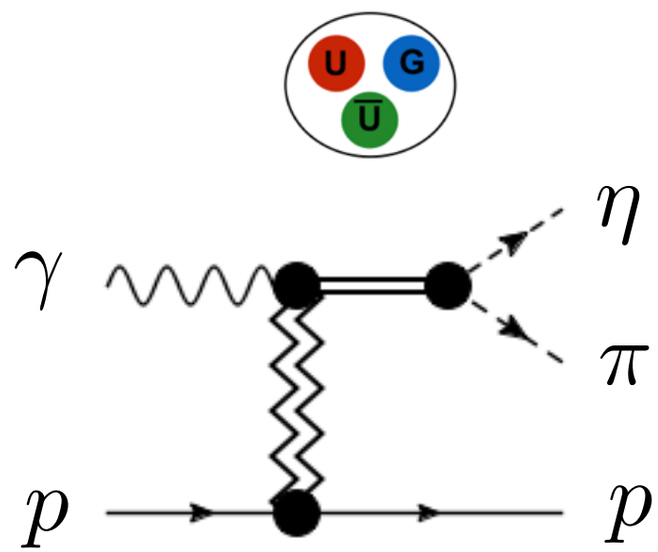
Experimental identification of exotic matter is difficult

→ **begin with simple case**

Resonances properties

Experiments: JLab, CERN,…

Exotic matter is produced diffractively



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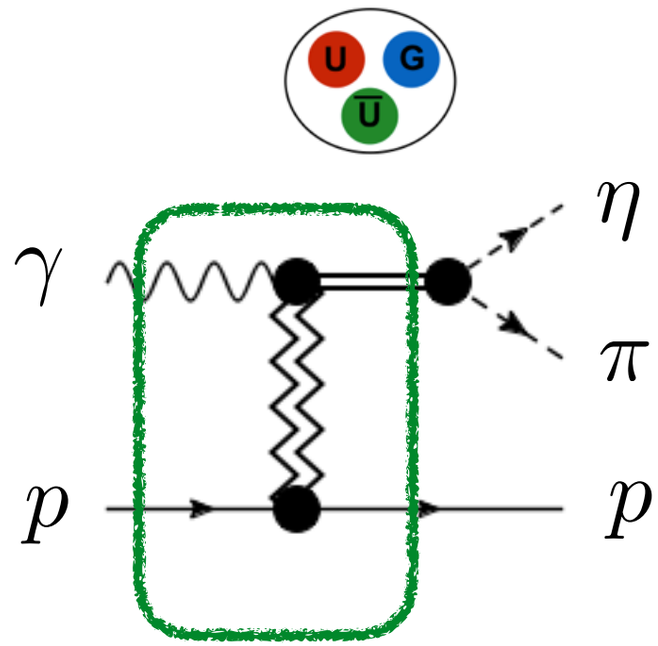

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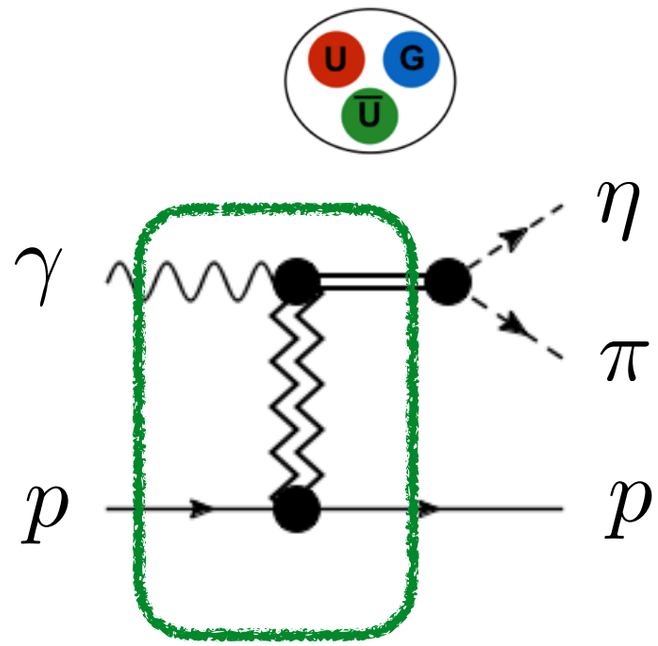
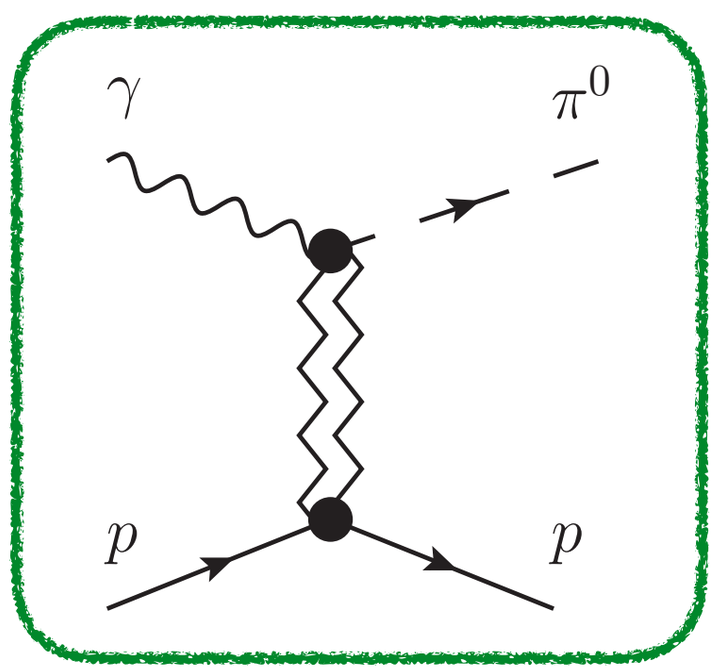


Simplest cases:



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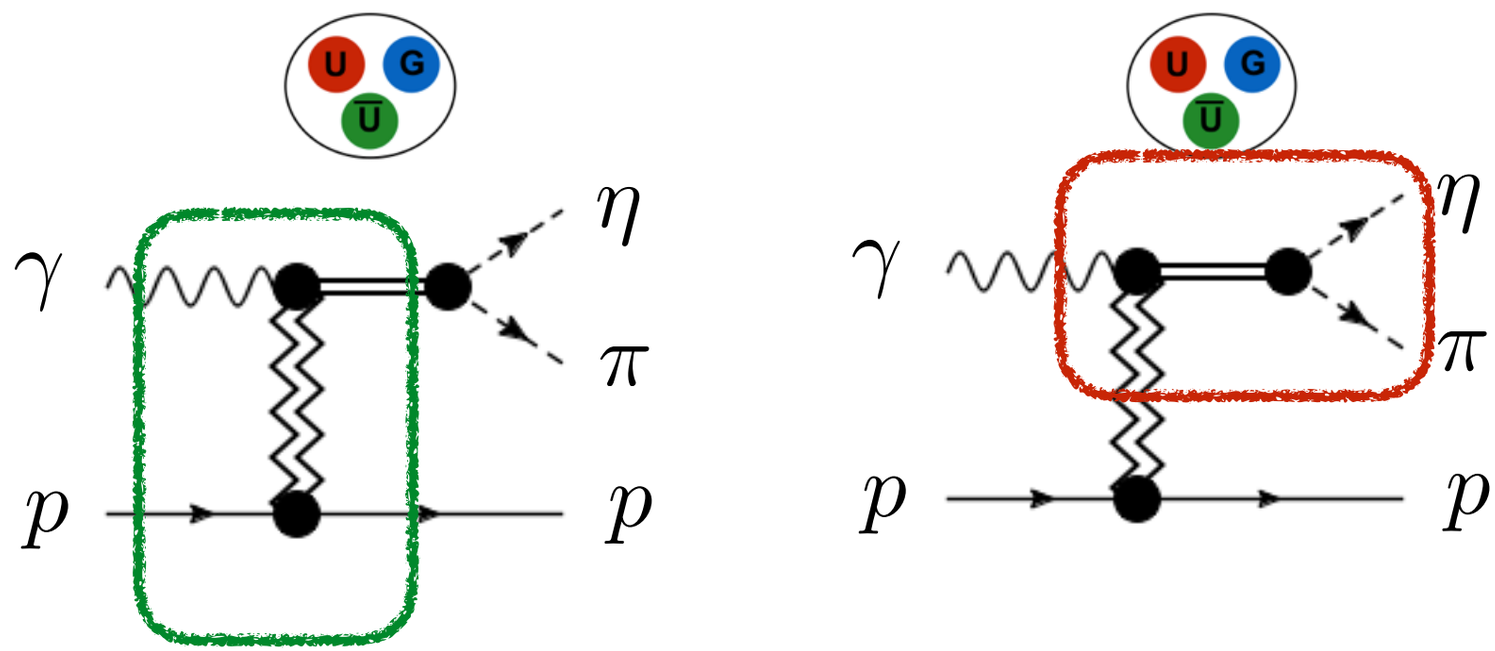

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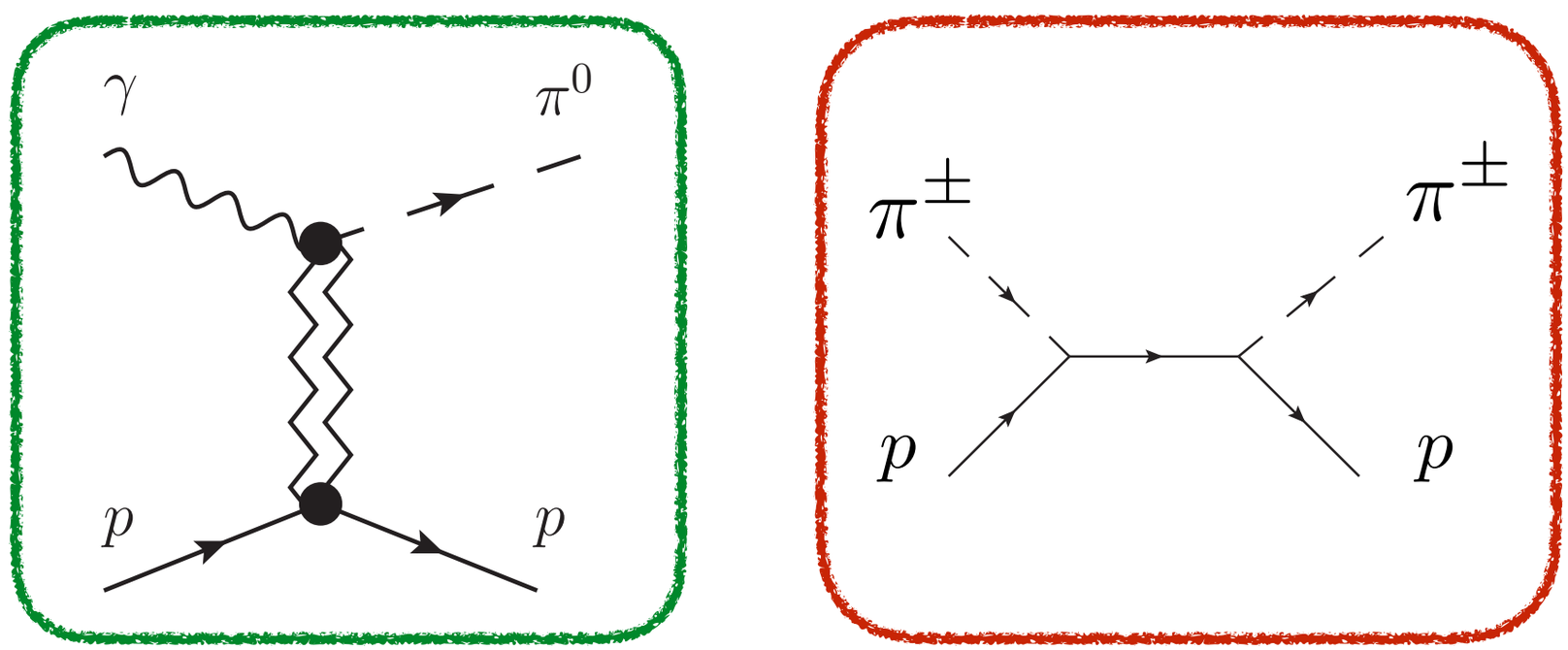
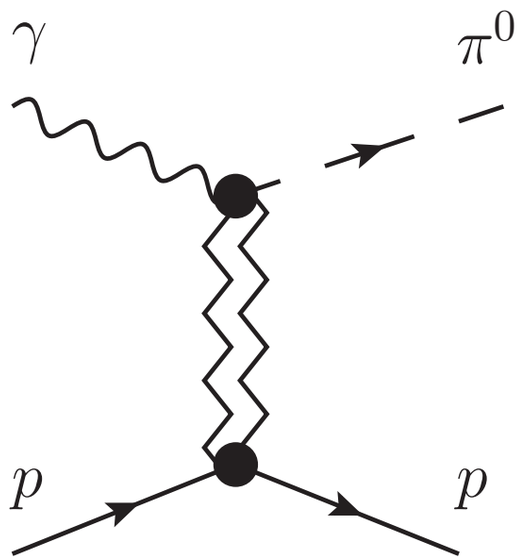
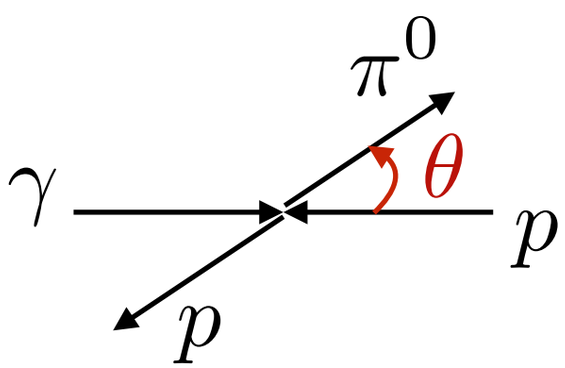
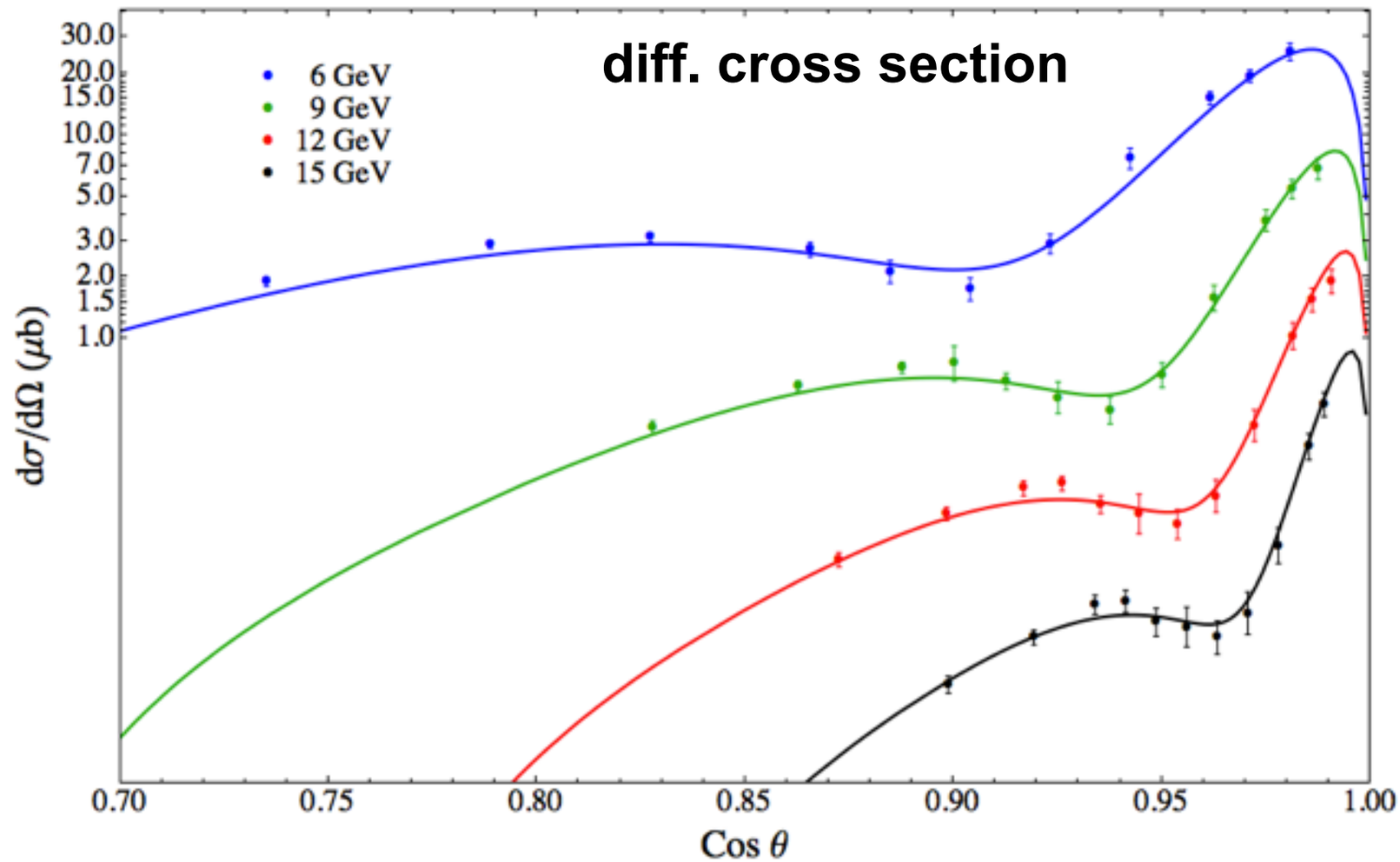


Photo-production of a Pion







VM, Szczepaniak and Fox PRD92 (2015)

Simple model reproduces data

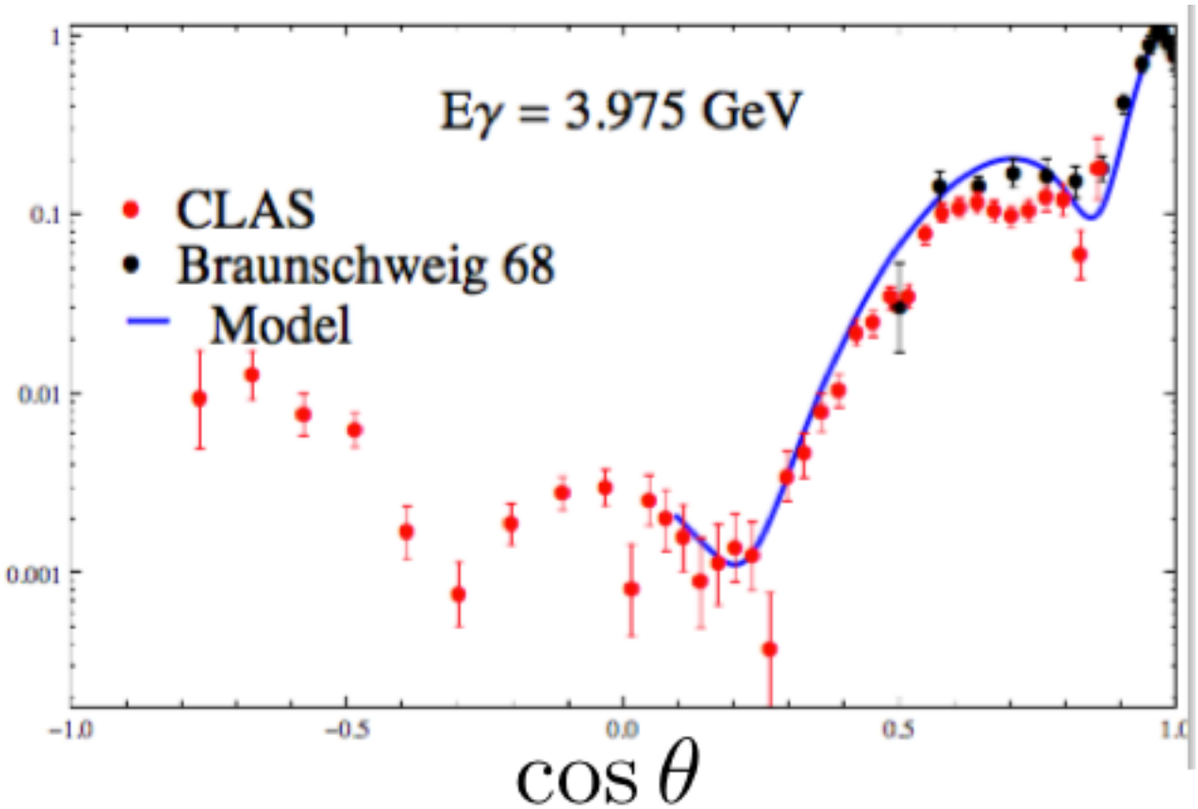
correct identification of relevant d.o.f.

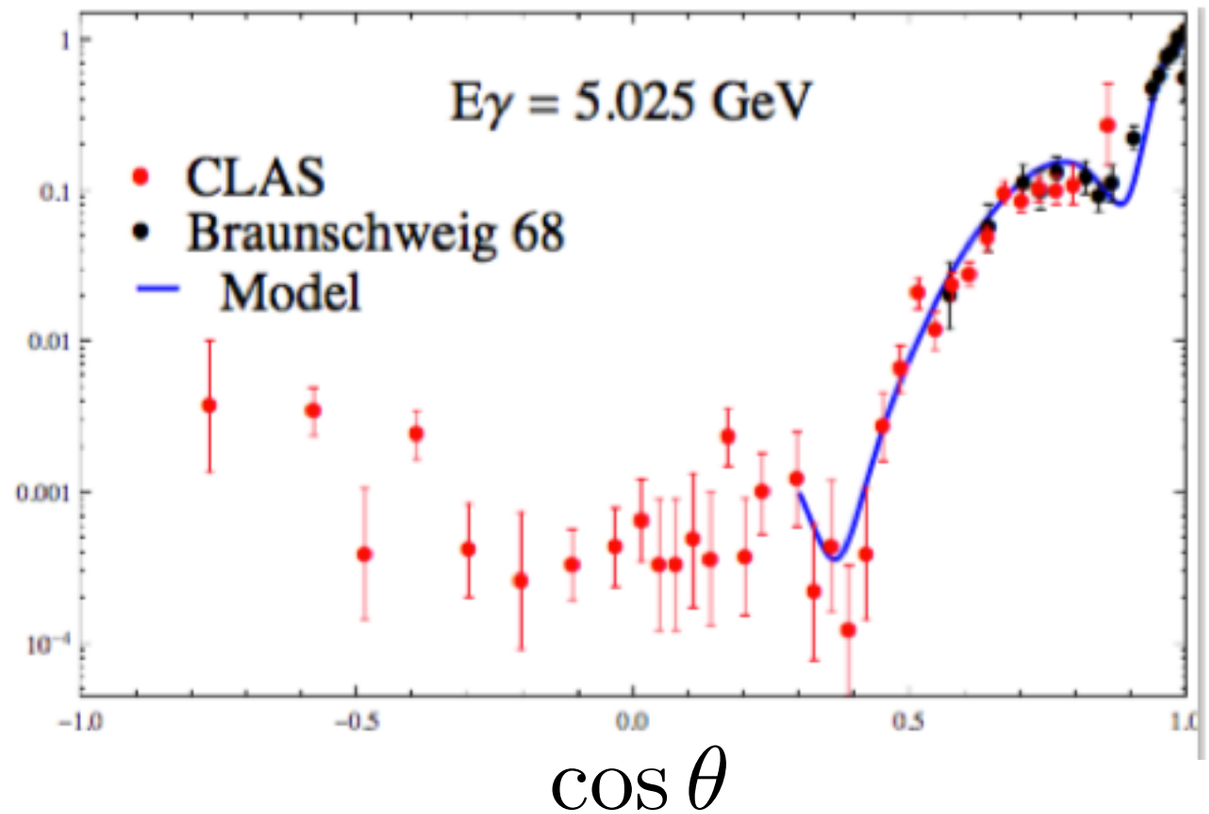
Can the model makes predictions ?

Photo-production of a Pion

Blue lines = model from VM, Szczepaniak and Fox PRD92 (2015)

Red points = new data from CLAS CLAS and VM, in preparation





How to share material with experimentalists:

Interactive webpage: http://www.indiana.edu/~jpac/index.html

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Jefferson Lab
Thomas Jefferson National Accelerator Facility

THE GEORGE WASHINGTON UNIVERSITY
WASHINGTON, DC

Joint Physics Analysis Center

HOME PROJECTS PUBLICATIONS LINKS

Resources

- **Publication:** [Mat15a]
- **Fortran:** Fortran file, Input file, Output file
- **C/C++:** AmpTools class, C/C++ file, AmpTools class header
- **Mathematica:** notebook , converted in text
- **Data:** Anderson, All data
- **Contact person:** Vincent Mathieu
- **Last update:** November 2015

Description of the Fortran code: [show/hide]
Description of the C/C++ code: [show/hide]

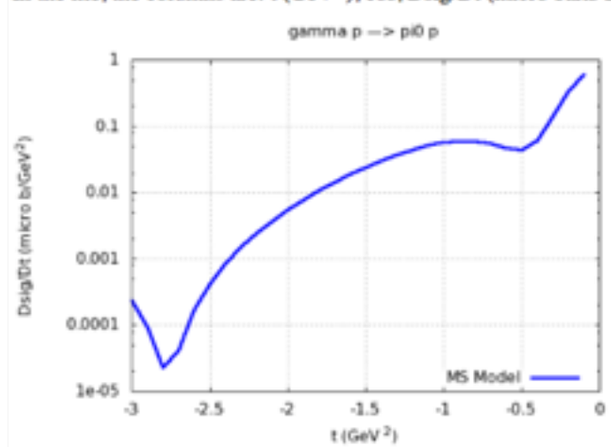
Run the code

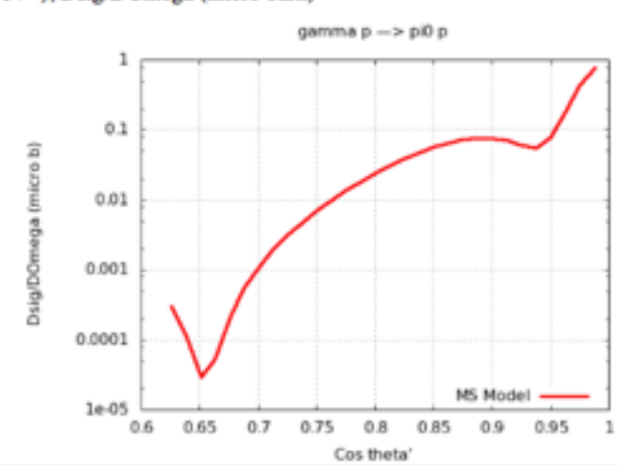
Choose the beam energy in the lab frame E_γ, the other variable (t or $\cos\theta$) and its minimal, maximal, and increment values.
If you choose t (cos) only the min, max and step values of t ($\cos\theta$) are read.

E_γ in GeV 9
t cos
t in GeV^2 (min max step) -3 -0.1 0.1
$\cos\theta$ (min max step) 0.5 0.95 0.01
Start reset

beam energy: 9 GeV
Observable: differential cross section
X variable: t with interval -3:0.1:-0.1

Download the output file, the plot with Ox=t , the plot with Ox=cos .
In the file, the columns are: t (GeV^2), cos, Dsig/Dt (micro barn/GeV^2), Dsig/DOmega (micro barn)





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→ **begin with simple case**

Resonances properties

Experiments: JLab, CERN,…

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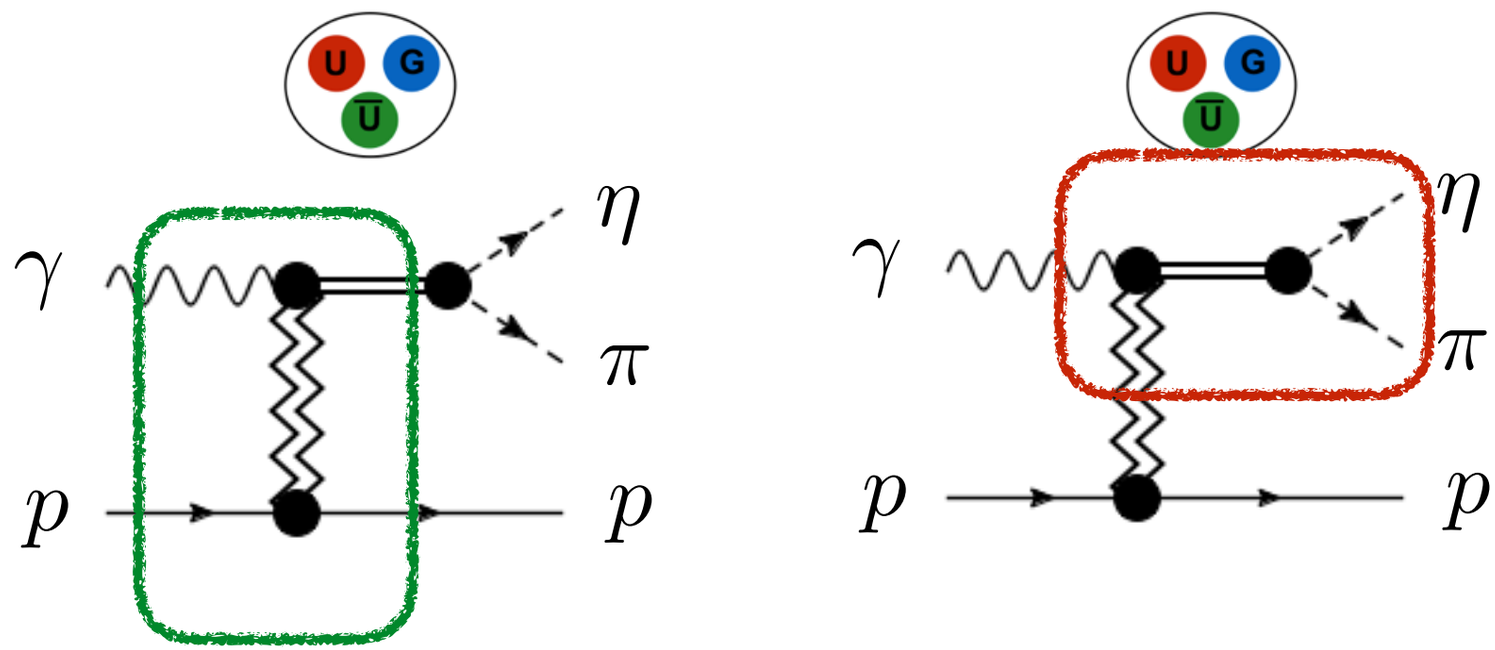

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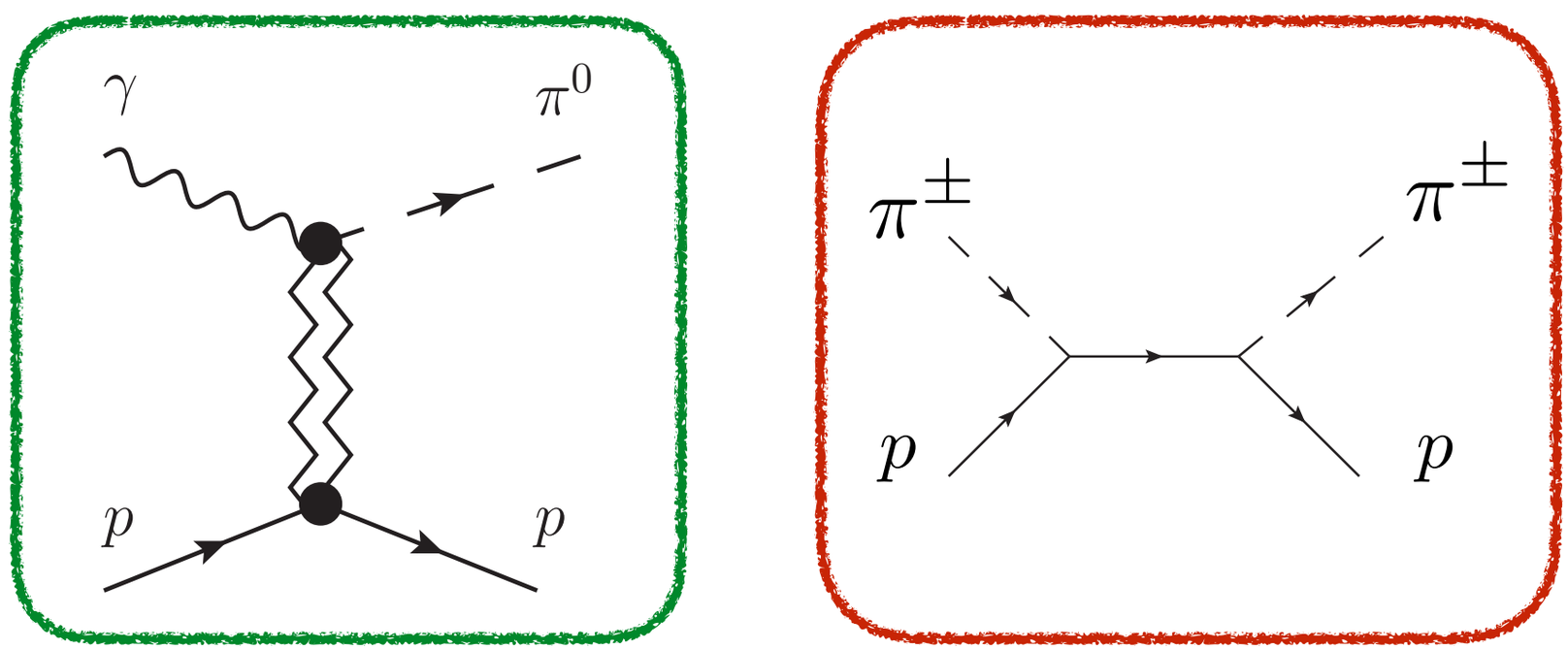
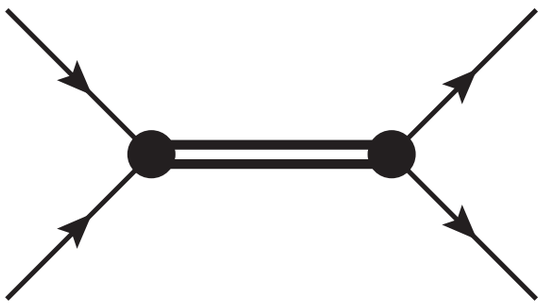
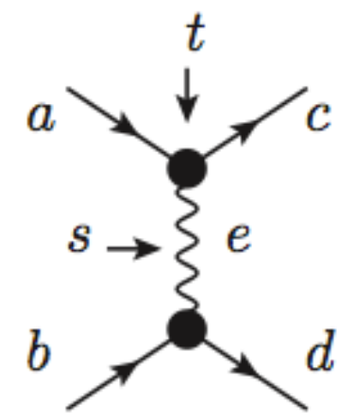


$\pi^- p \to \pi^0 n$

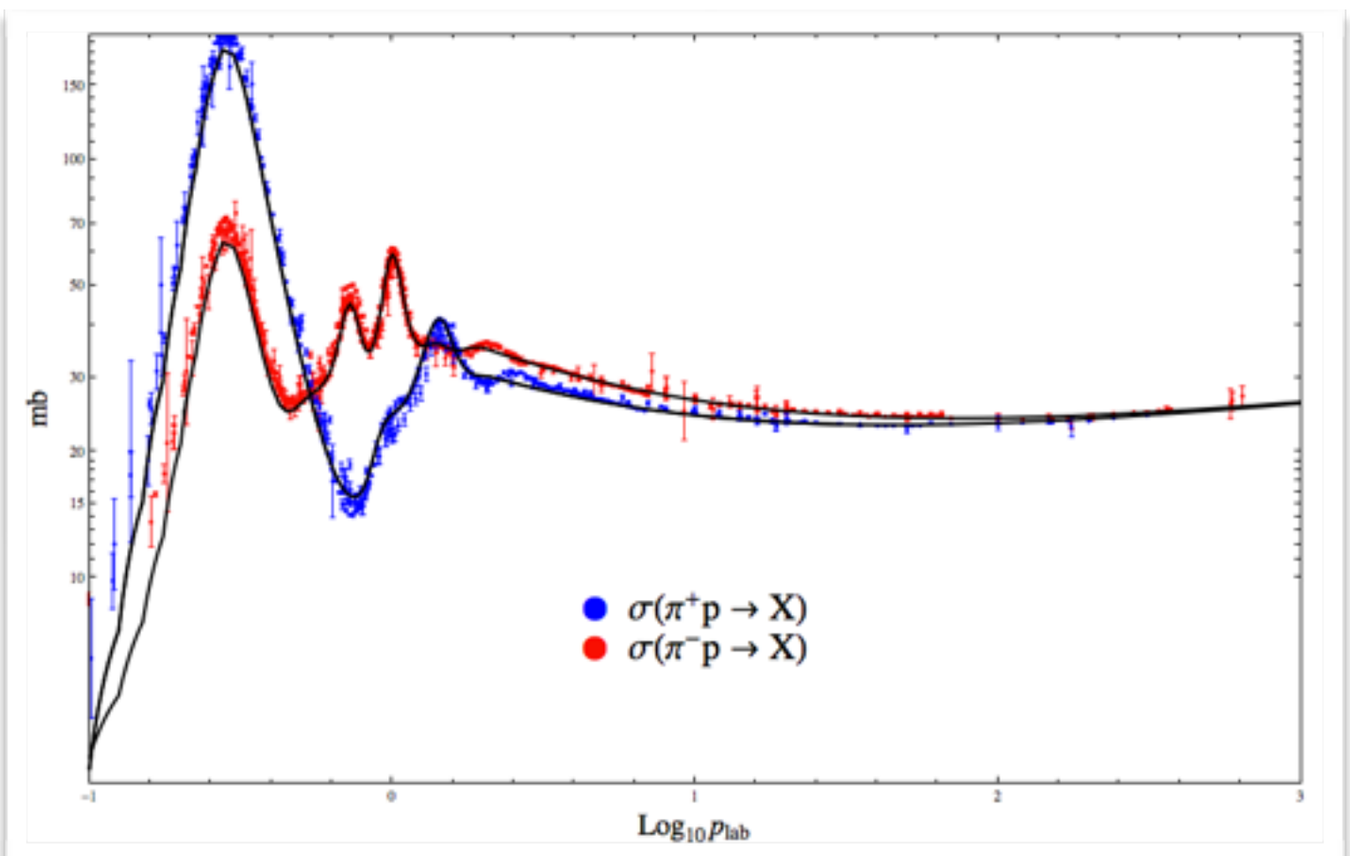
Low energy: **baryon resonances**



High energy: **Regge exchange**



Total cross section



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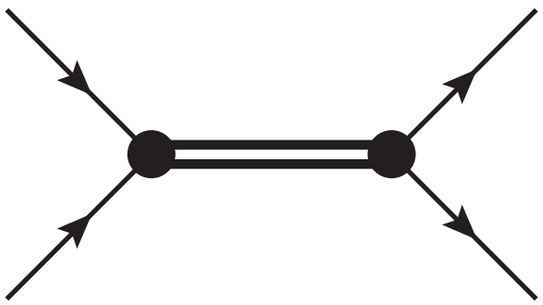

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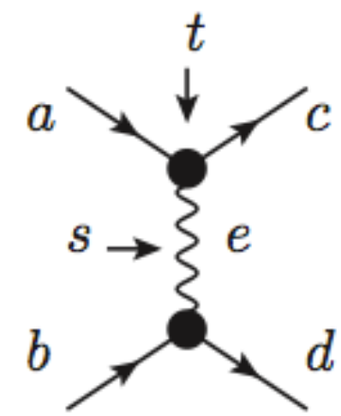

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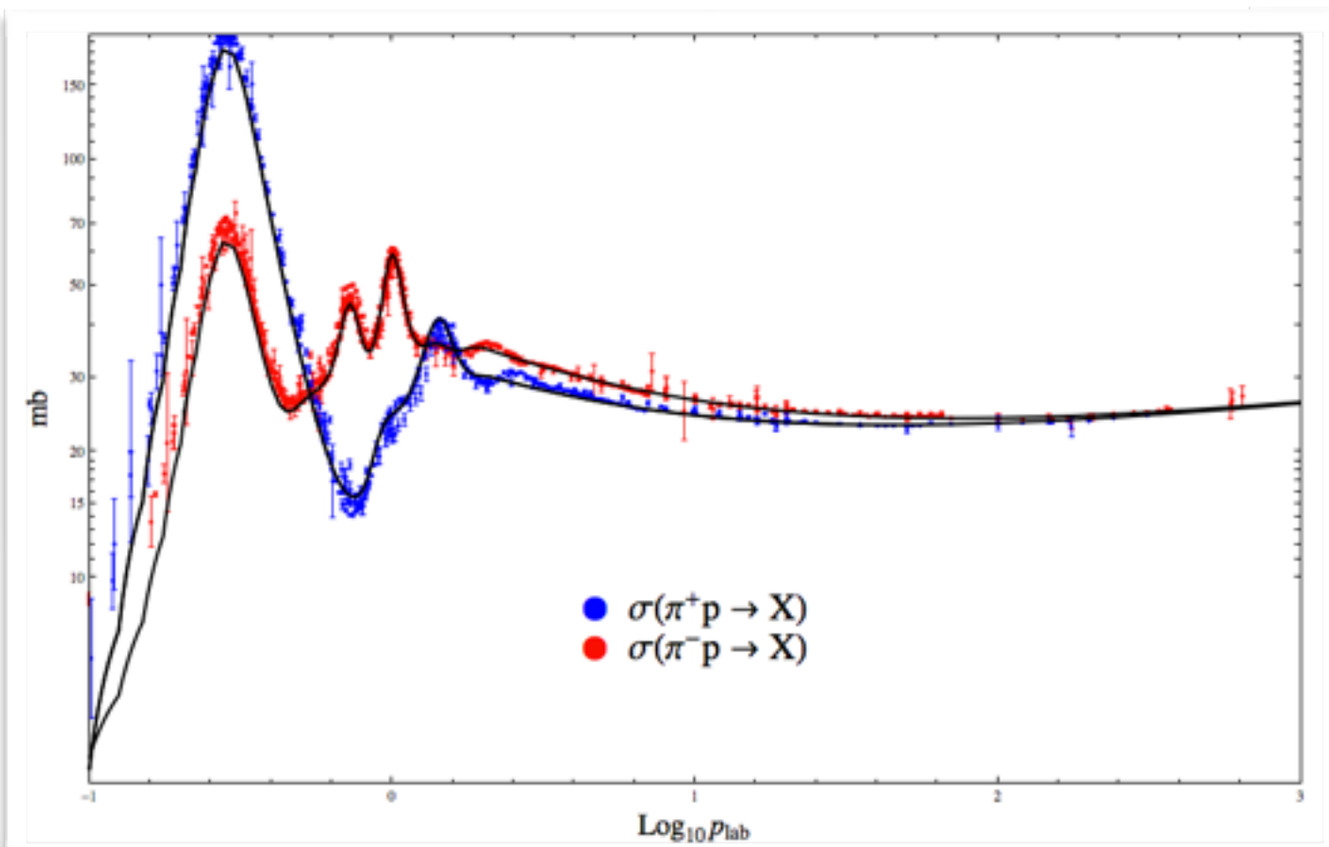
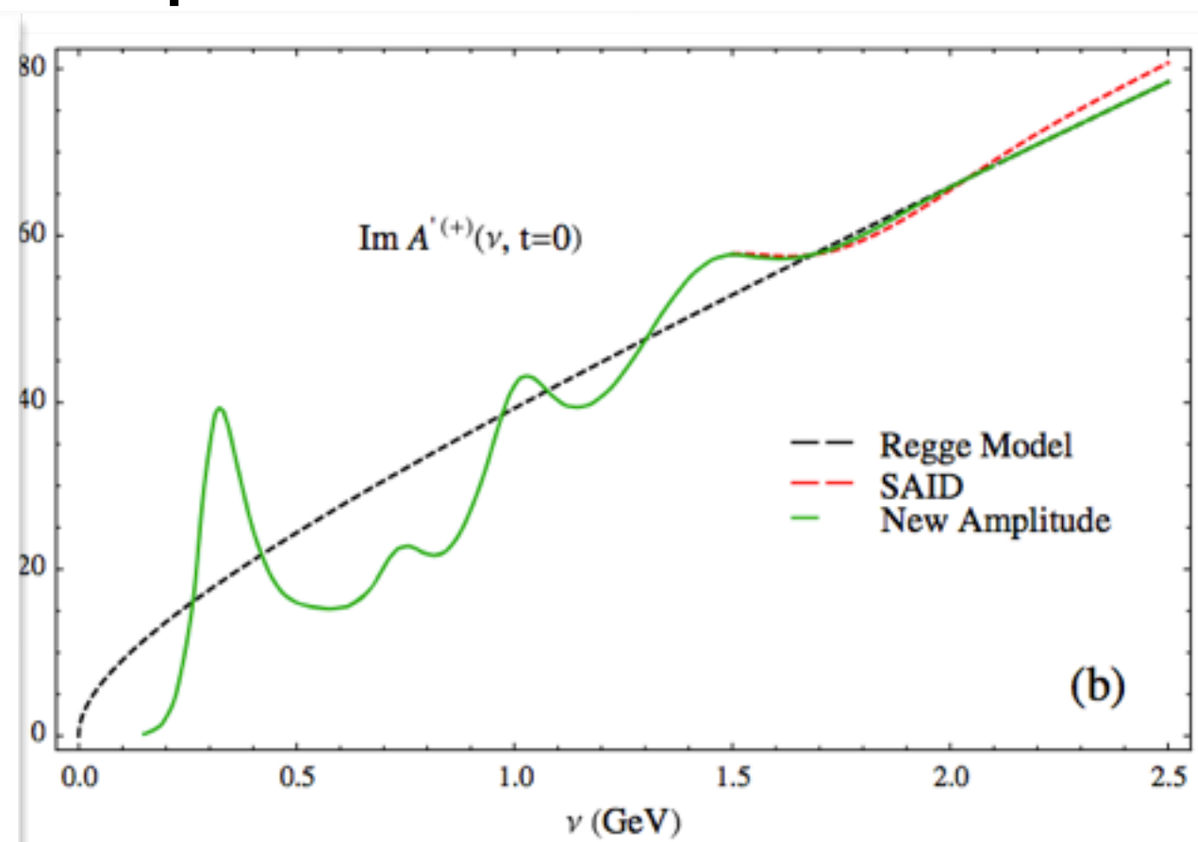
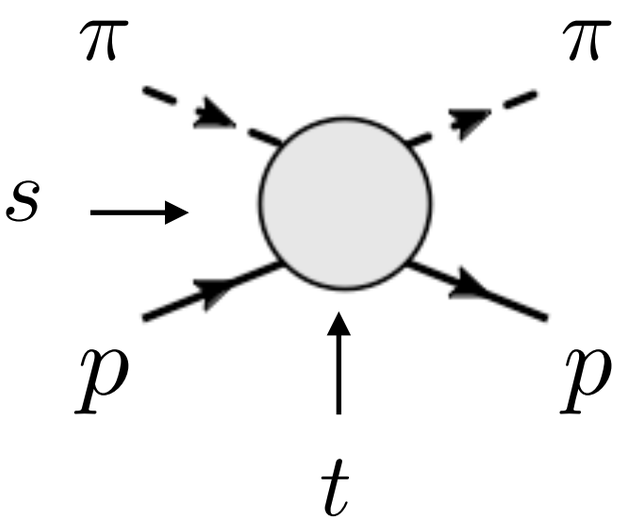


Amplitude



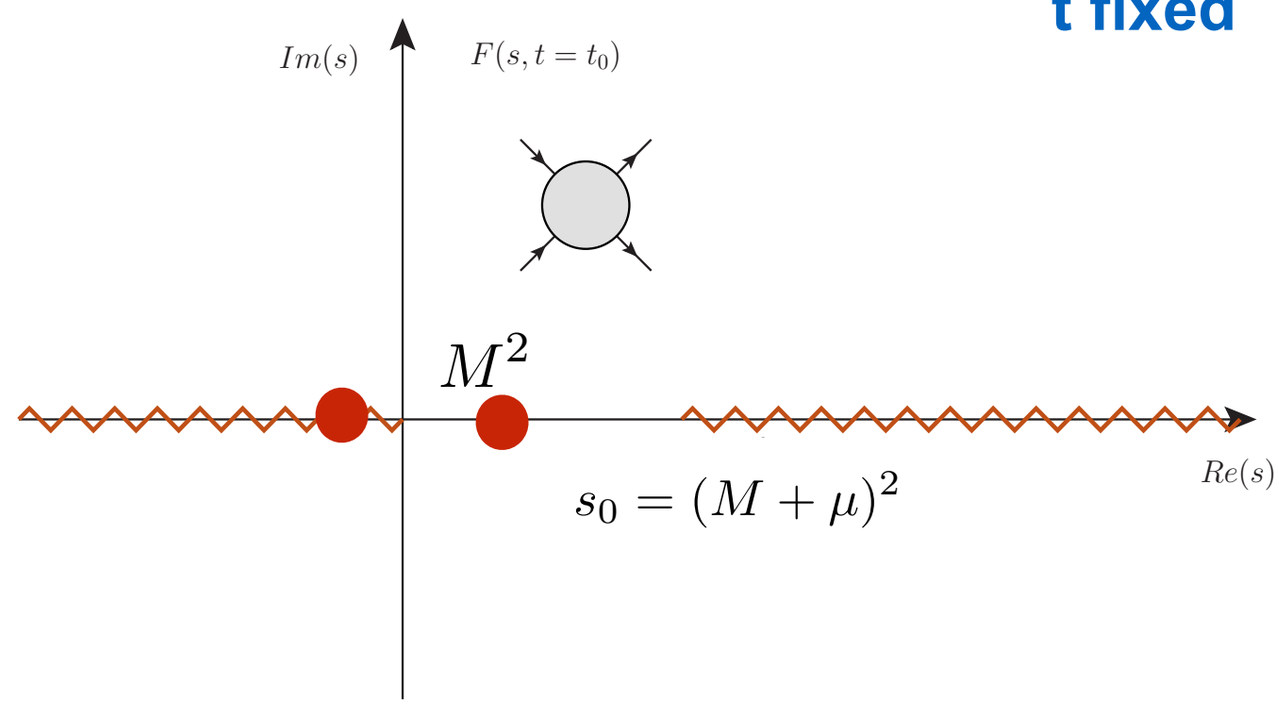
Dispersion Relation



Decompose into scalar function

$$T = \bar{u}(p_4, \lambda_4)\left(A + \frac{1}{2}(\not{p}_1 + \not{p}_3)\, B\right) u(p_2, \lambda_2)$$

t fixed



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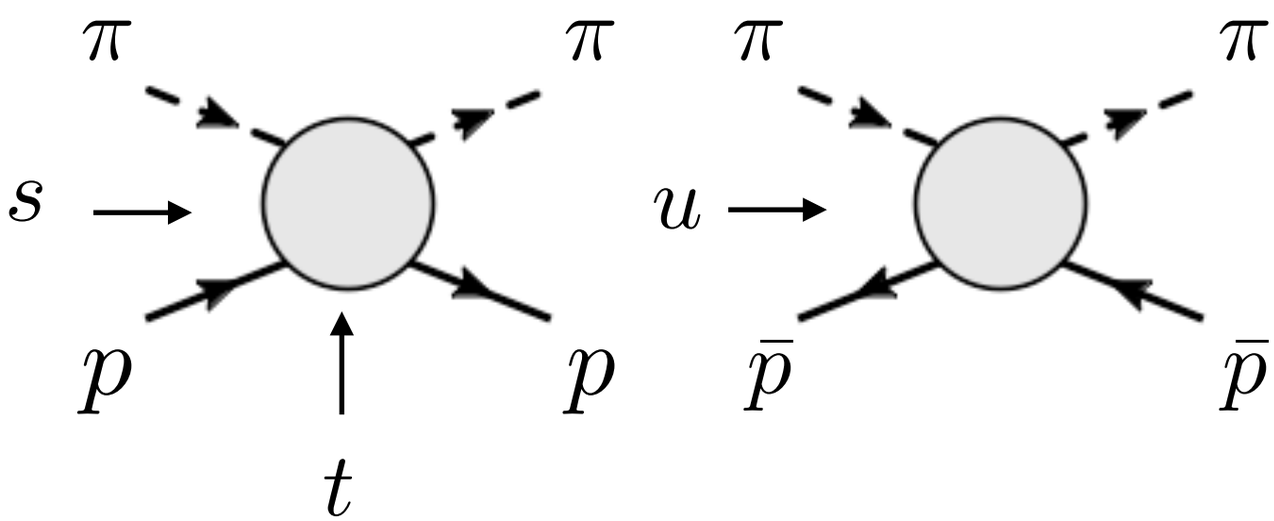


Conjugation charge relate

πp **and** $\pi \bar{p}$ **scatterings**

Decompose into scalar function

$$T = \bar{u}(p_4, \lambda_4) \left(A + \frac{1}{2} (\not{p}_1 + \not{p}_3) B \right) u(p_2, \lambda_2)$$

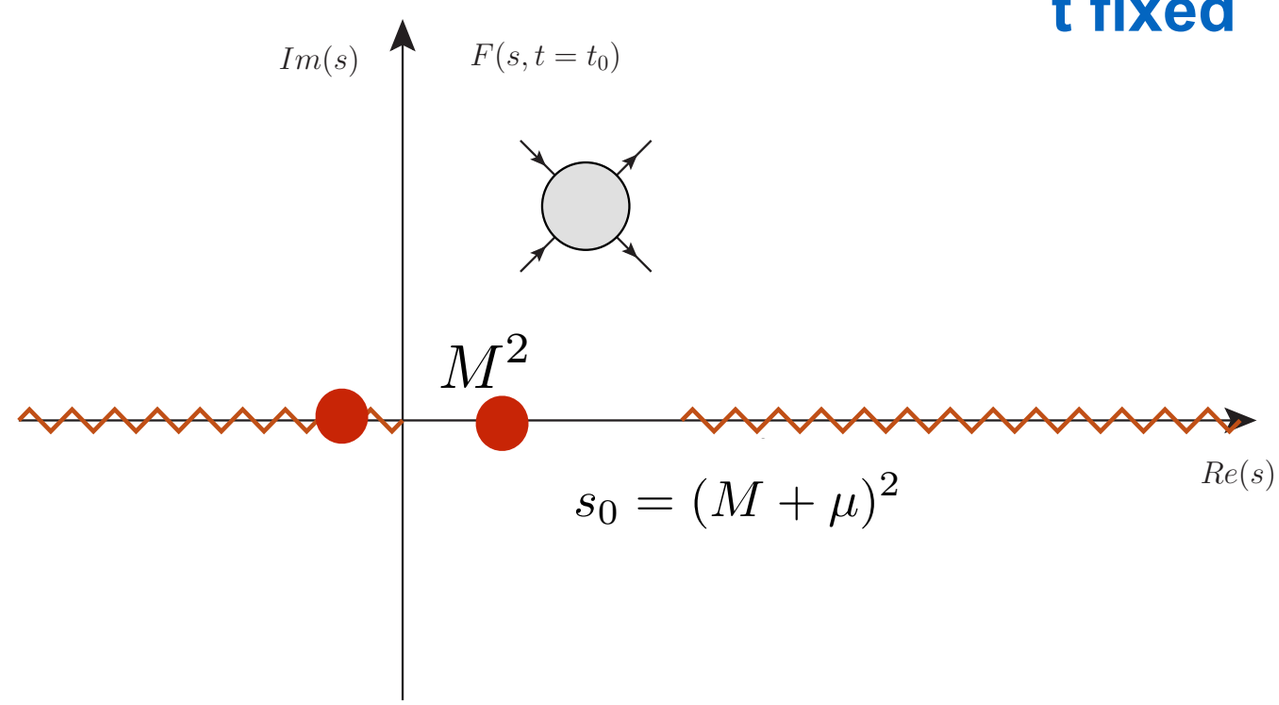
Introduce the crossing variable

$$\nu = \frac{s - u}{2}$$

$$u(s,t) = -s - t + 2M^2 + 2\mu^2$$

and symmetrize the two cuts

t fixed



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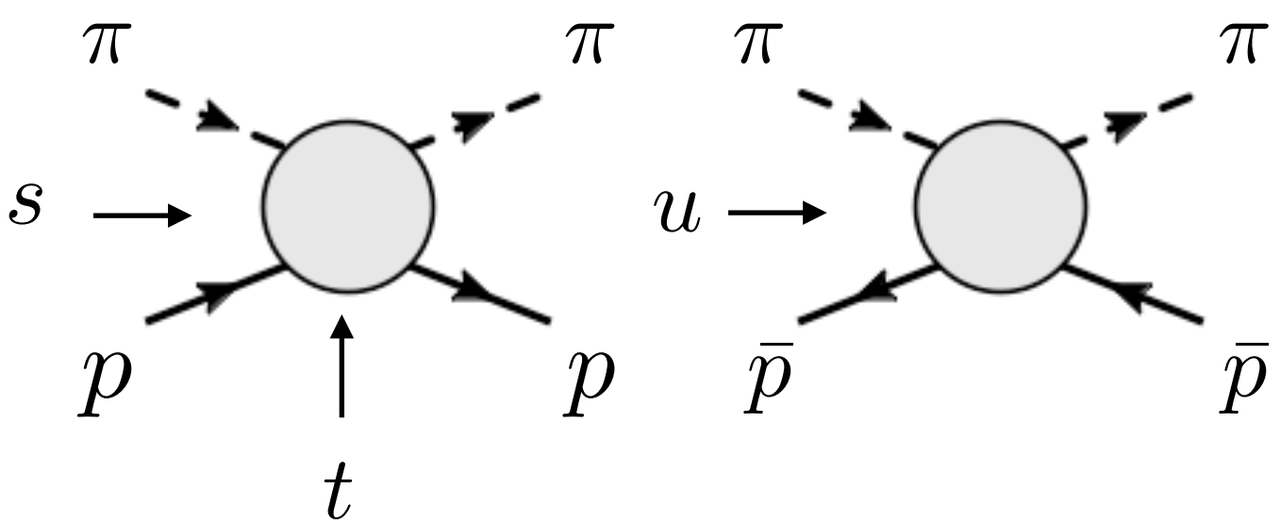

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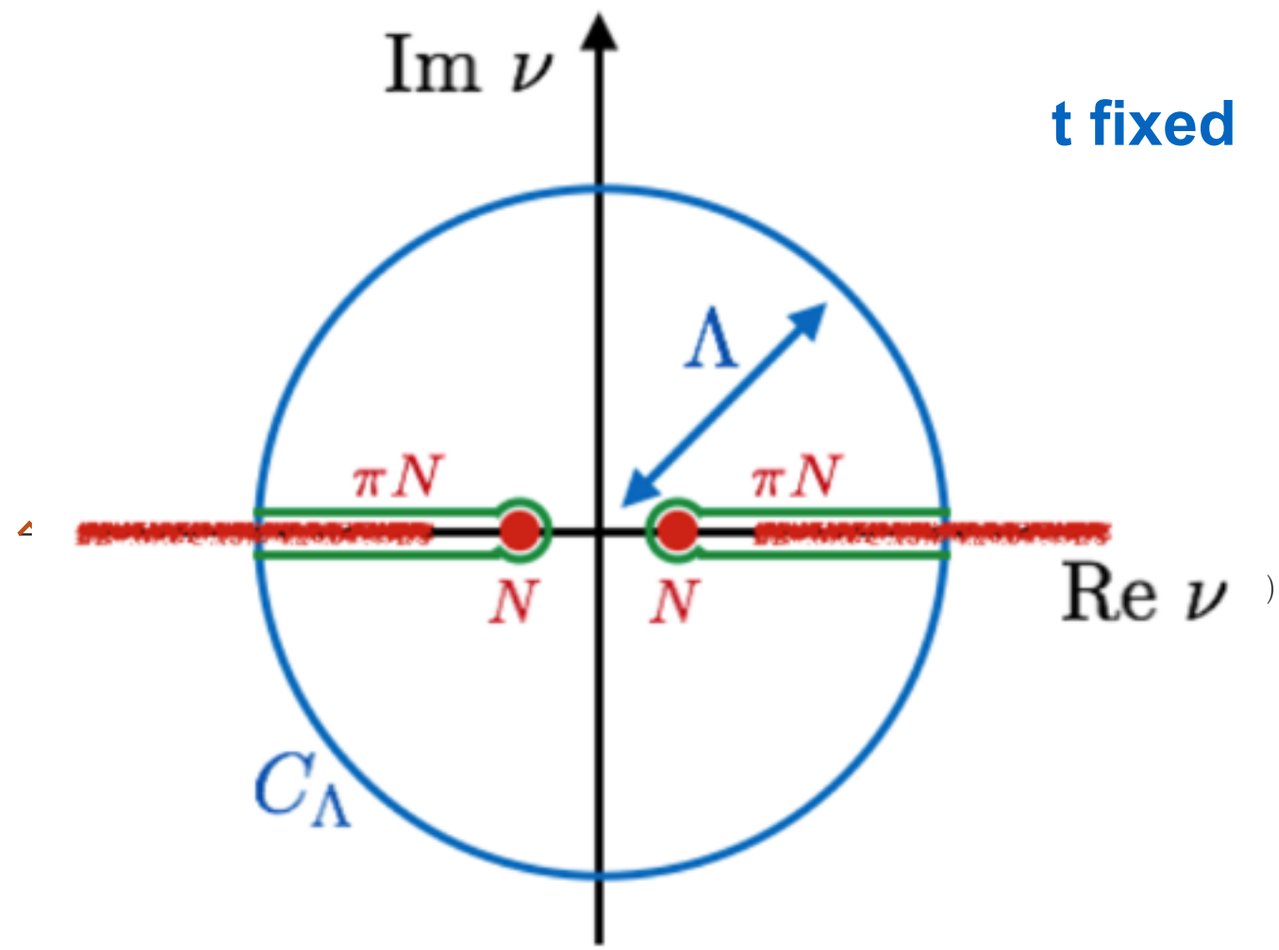
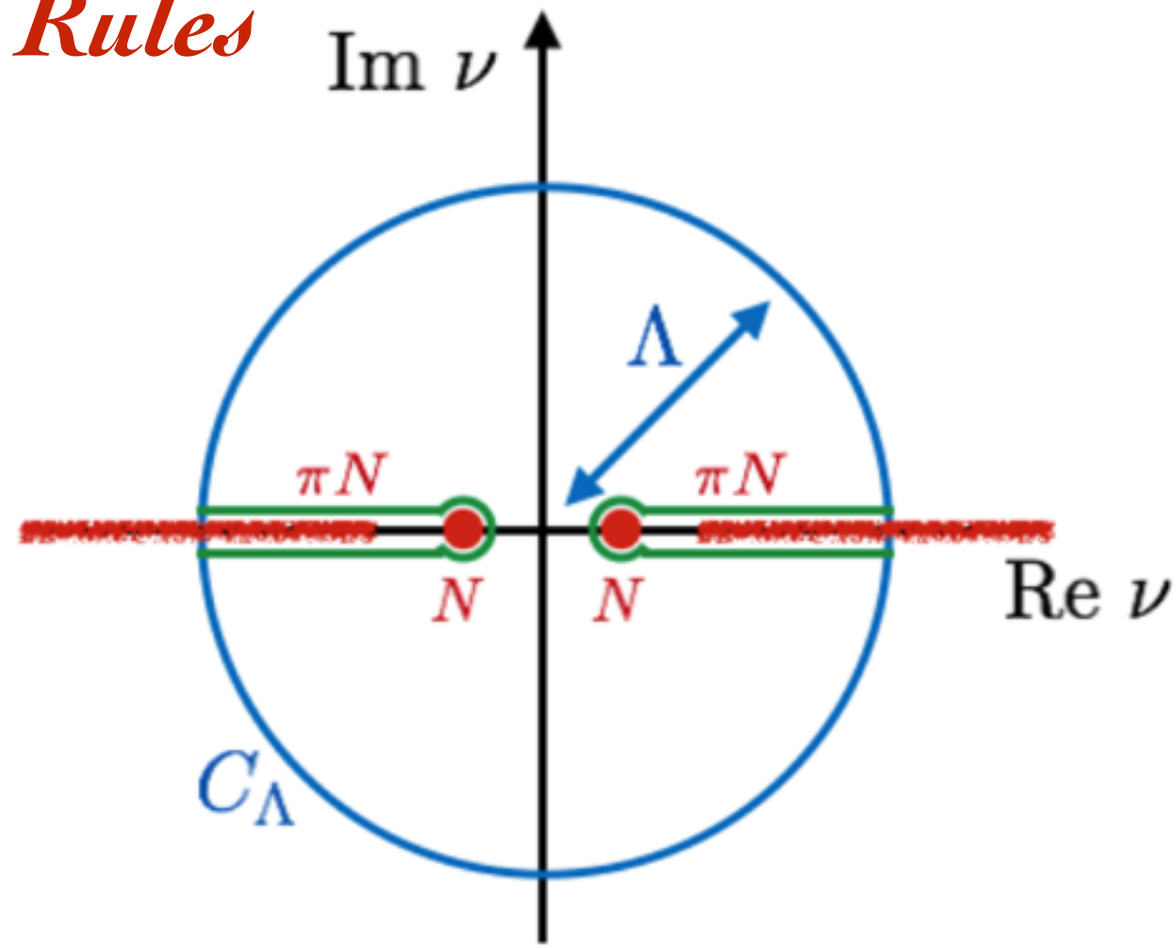


Finite Energy Sum Rules

Cauchy contour

$$\oint_C A(\nu, t) d\nu = 0$$



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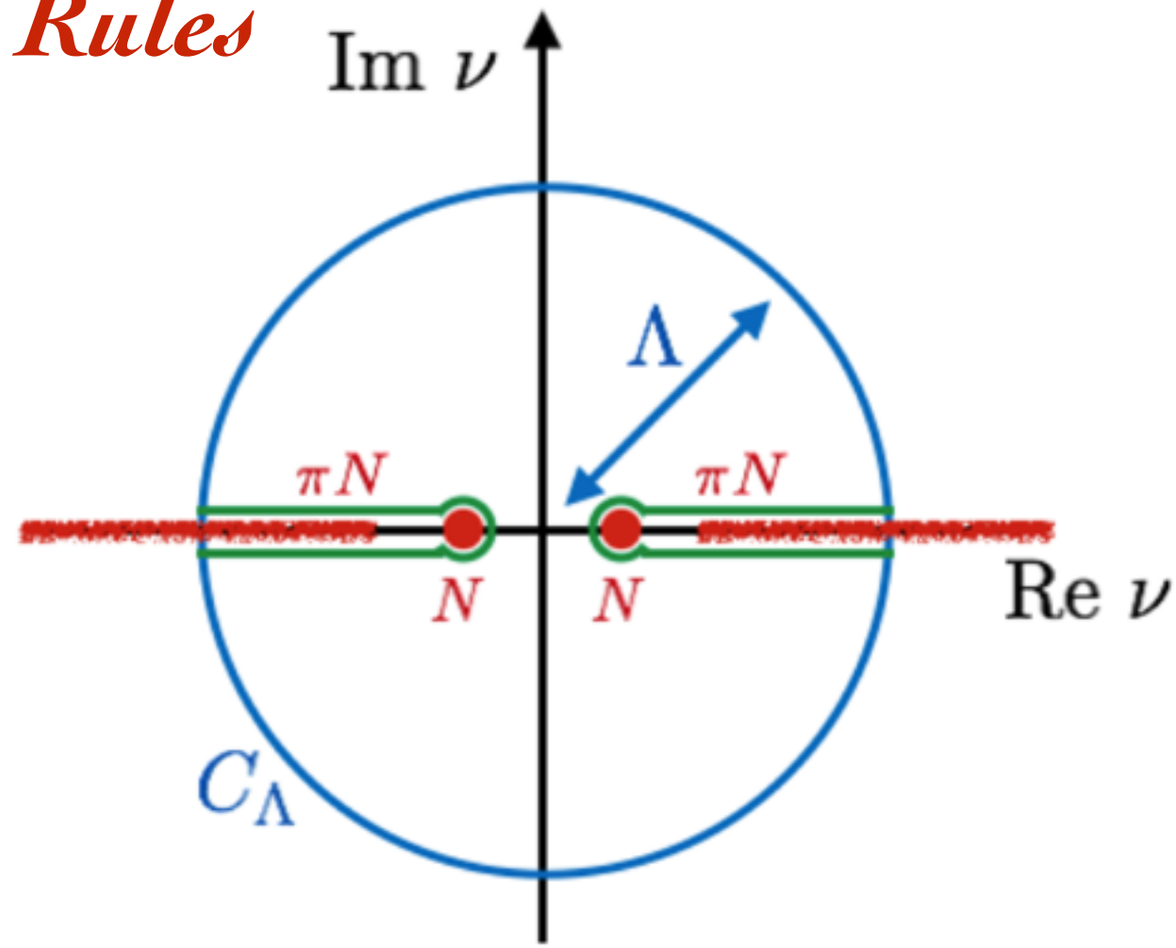
$$2i \int_{\nu_0}^{\Lambda} \mathrm{Im}\ A(\nu, t) d\nu = -\oint_{C_\Lambda} A(\nu, t) d\nu$$



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Cauchy contour

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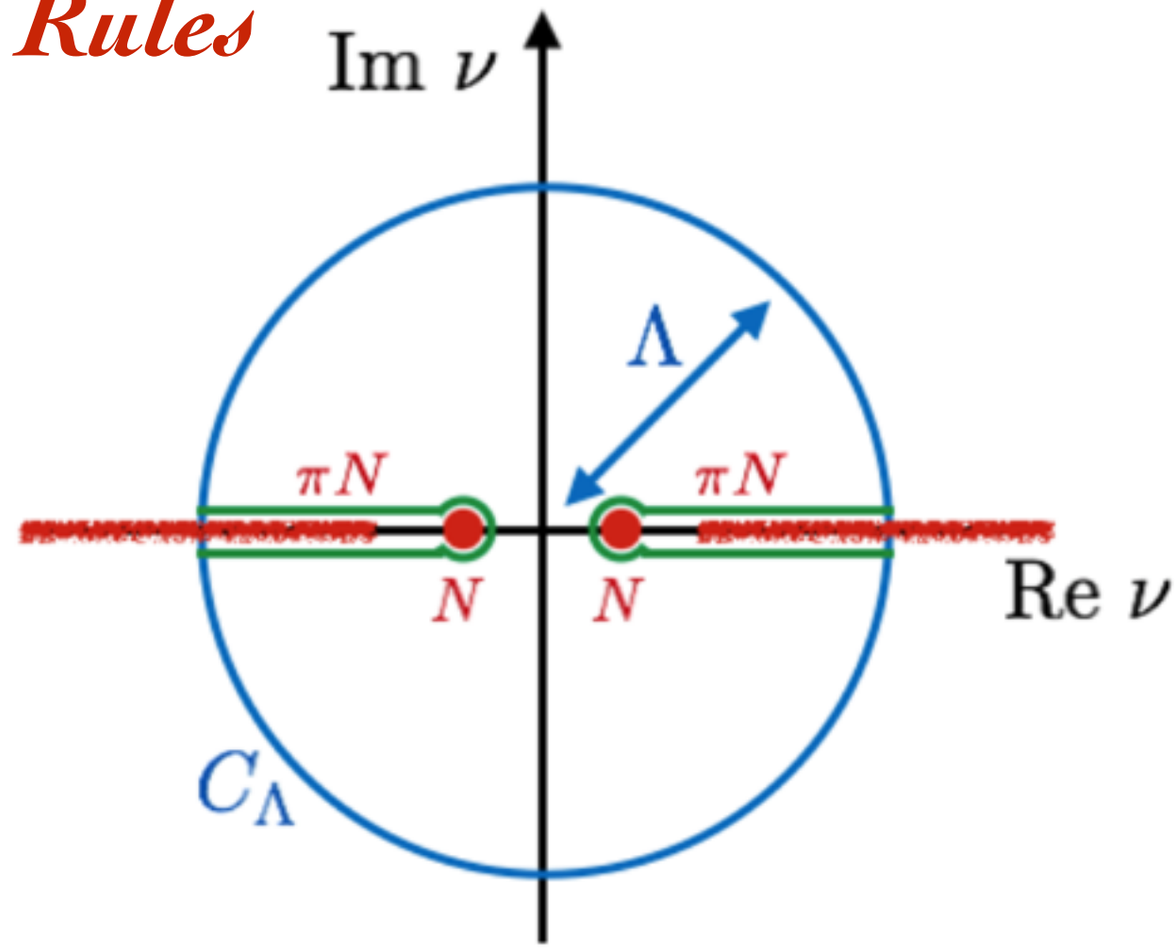
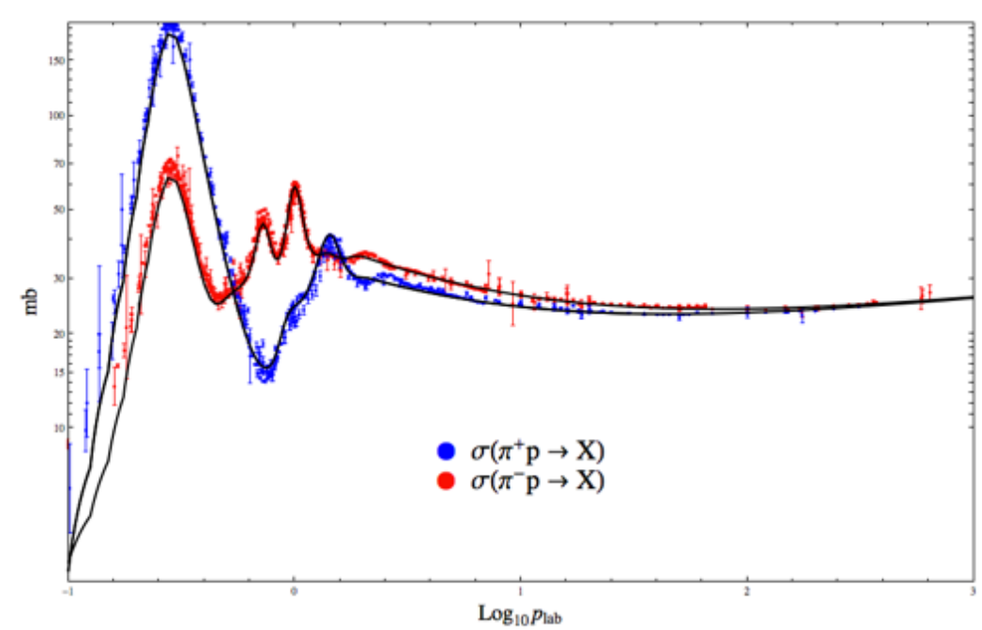


$$2i\int_{\nu_0}^{\Lambda} \mathrm{Im}\ A(\nu,t)d\nu = -\oint_{C_\Lambda} A(\nu,t)d\nu$$

Assume Regge form at $\nu = \Lambda$ **:**

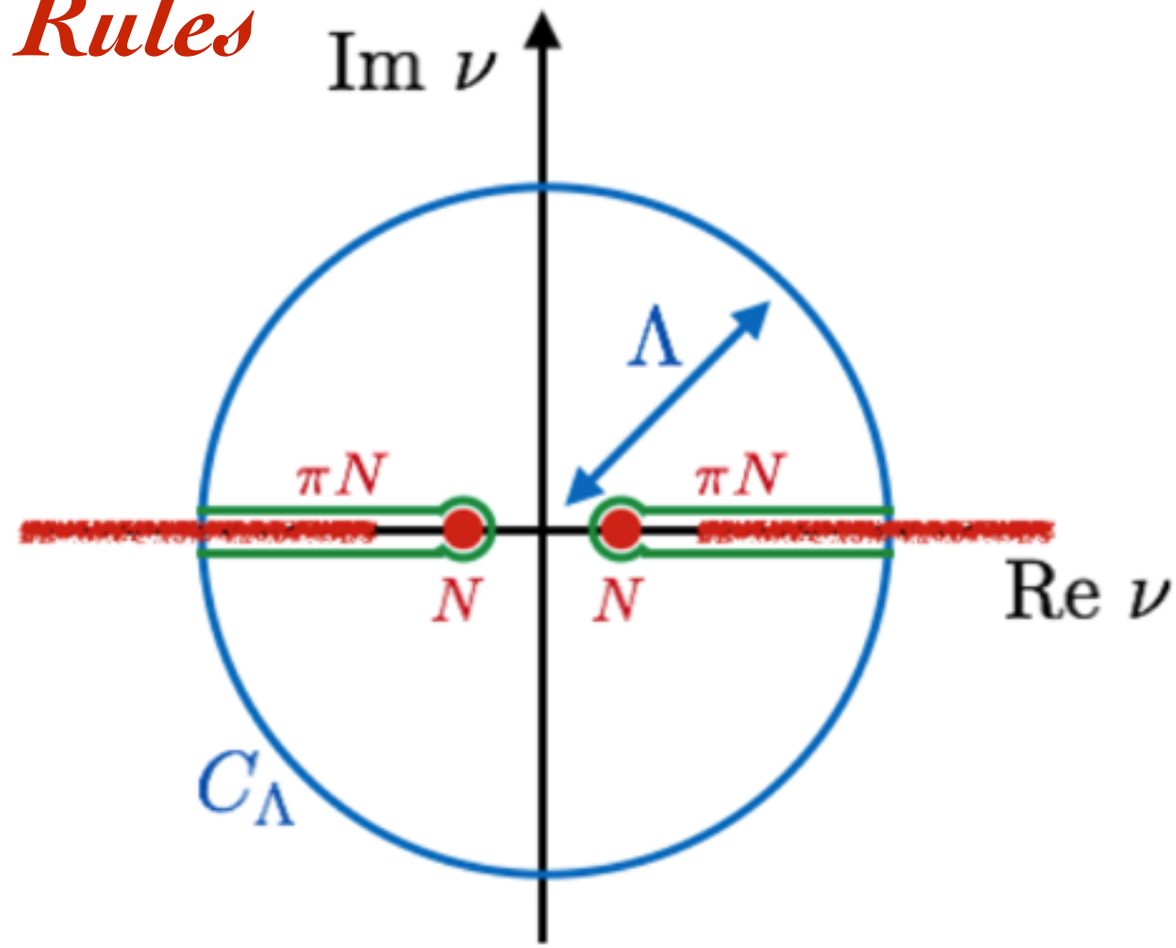
$$A(\nu,t) = \beta(t)\frac{\pm 1 - e^{-i\pi\alpha(t)}}{\sin\pi\alpha(t)}\nu^{\alpha(t)}$$



Finite Energy Sum Rules

Cauchy contour

$$\oint_C A(\nu,t)d\nu = 0$$



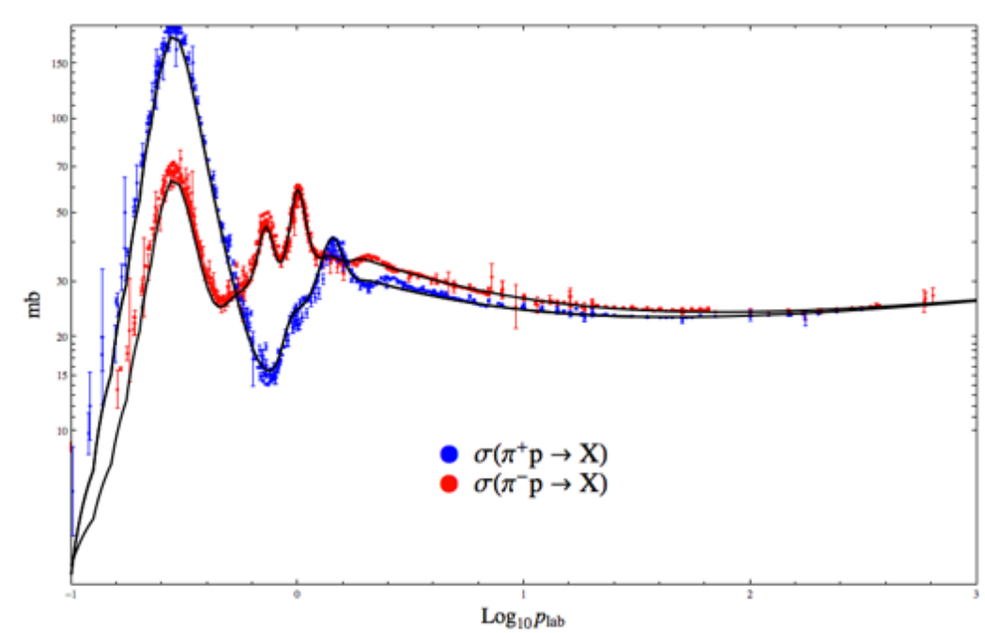
$$2i\int_{\nu_0}^{\Lambda} \mathrm{Im}\ A(\nu,t)d\nu = -\oint_{C_\Lambda} A(\nu,t)d\nu$$

Assume Regge form at $\nu = \Lambda$ **:**

$$A(\nu,t) = \beta(t)\frac{\pm 1 - e^{-i\pi\alpha(t)}}{\sin\pi\alpha(t)}\nu^{\alpha(t)}$$

$$\int_{\nu_0}^{\Lambda} \mathrm{Im}\ A(\nu,t)d\nu = \beta(t)\frac{\Lambda^{\alpha(t)+1}}{\alpha(t)+1}$$

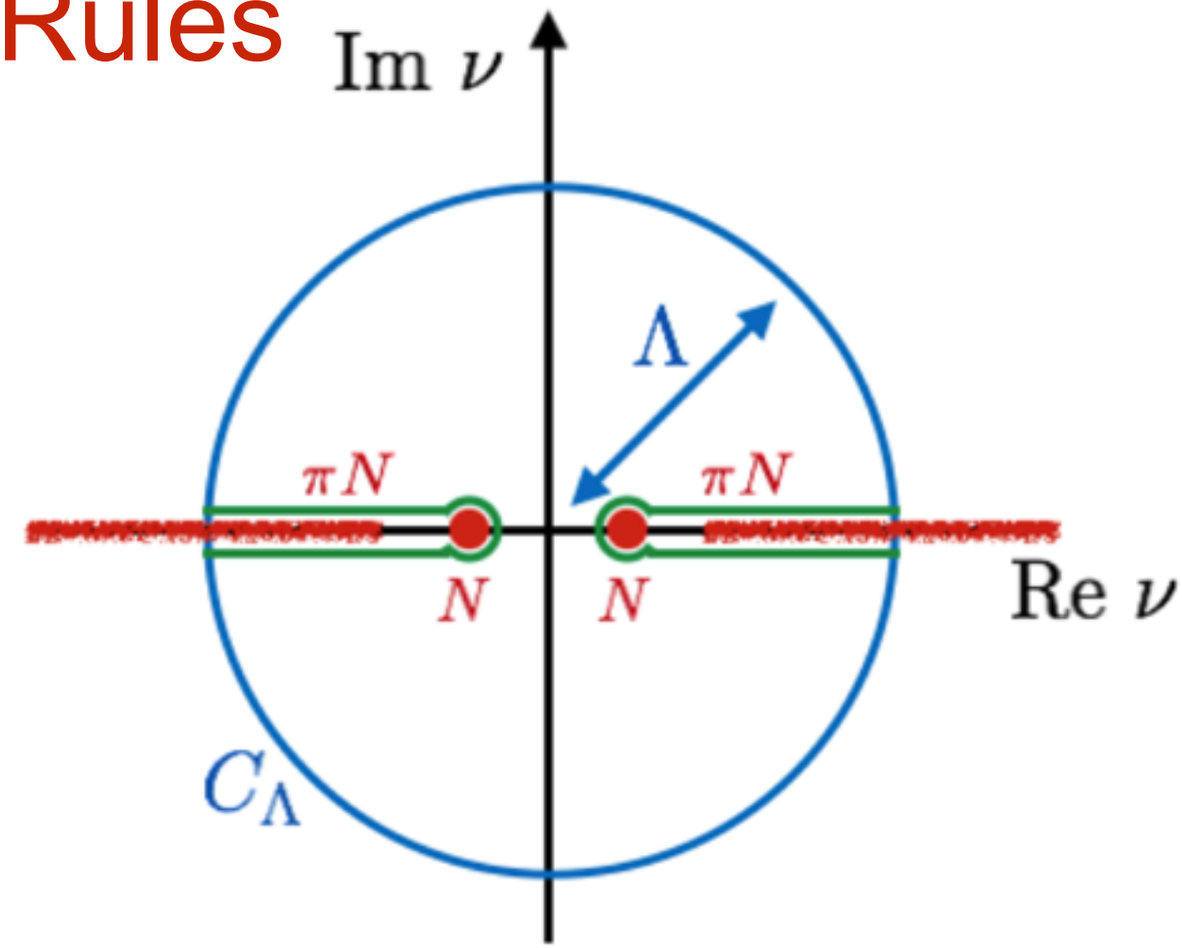
t fixed



Finite Energy Sum Rules

Cauchy contour

$$\oint_C A(\nu,t)\nu^k d\nu = 0$$



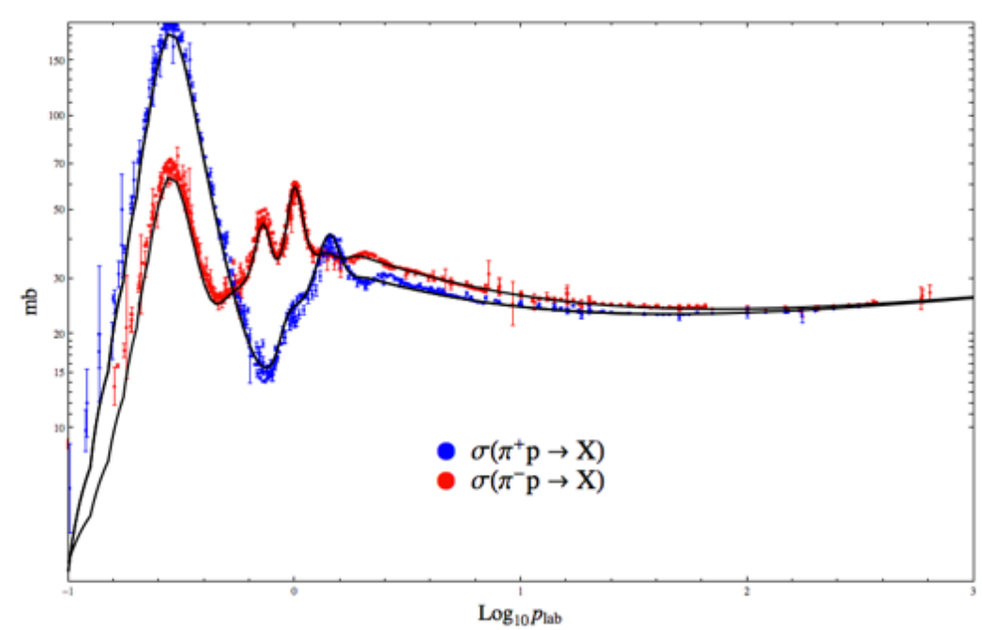
$$2i\int_{\nu_0}^{\Lambda} \mathrm{Im}\ A(\nu,t)d\nu = -\oint_{C_\Lambda} A(\nu,t)d\nu$$

Assume Regge form at $\nu = \Lambda$ **:**

$$A(\nu,t) = \beta(t)\frac{\pm 1 - e^{-i\pi\alpha(t)}}{\sin\pi\alpha(t)}\nu^{\alpha(t)}$$

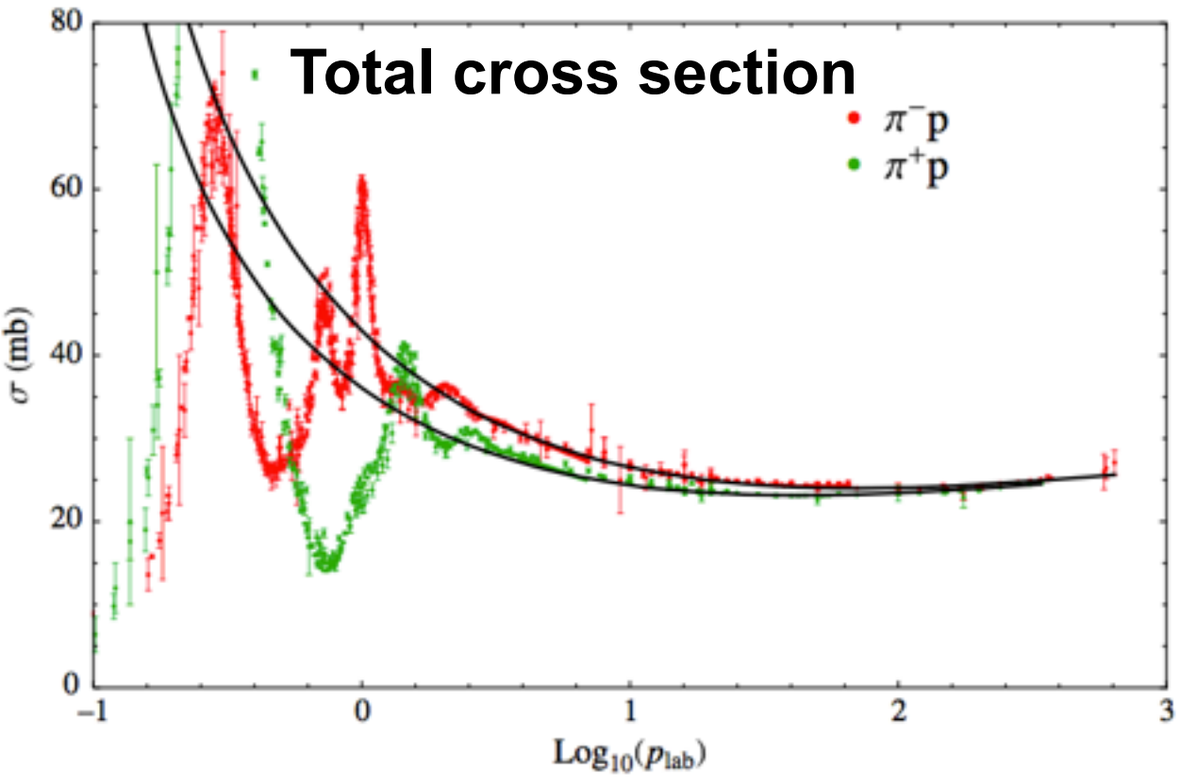
$$\frac{1}{\Lambda^k}\int_{\nu_0}^{\Lambda} \mathrm{Im}\ A(\nu,t)\nu^k d\nu = \frac{\beta(t)\Lambda^{\alpha(t)+1}}{\alpha(t)+k+1}$$

t fixed



Application to πN : High Energy Fit

VM et al (JPAC) PRD92
arXiv:1506.01764



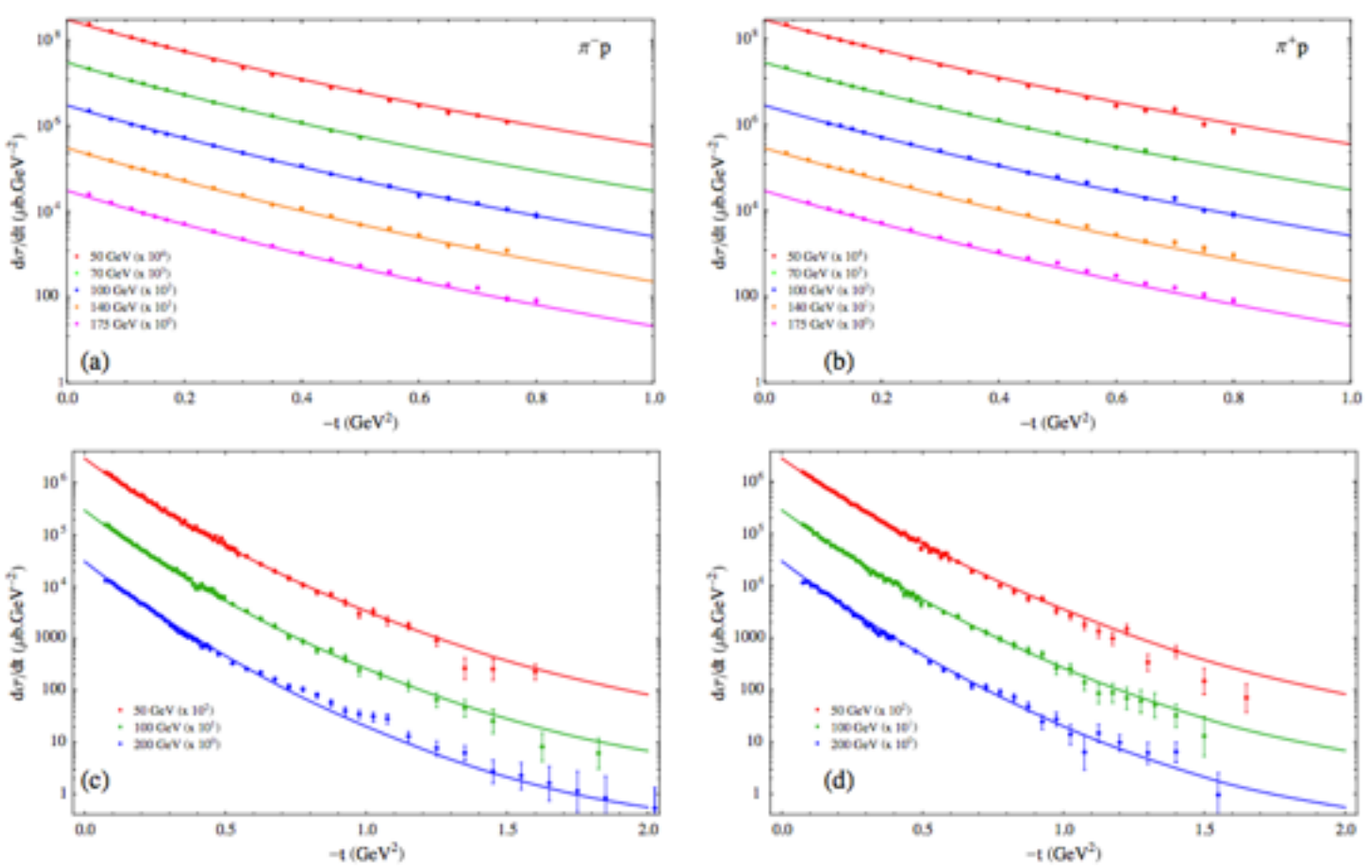
Fit to the world data on

$$\pi^{\pm} p \to \pi^{\pm} p$$

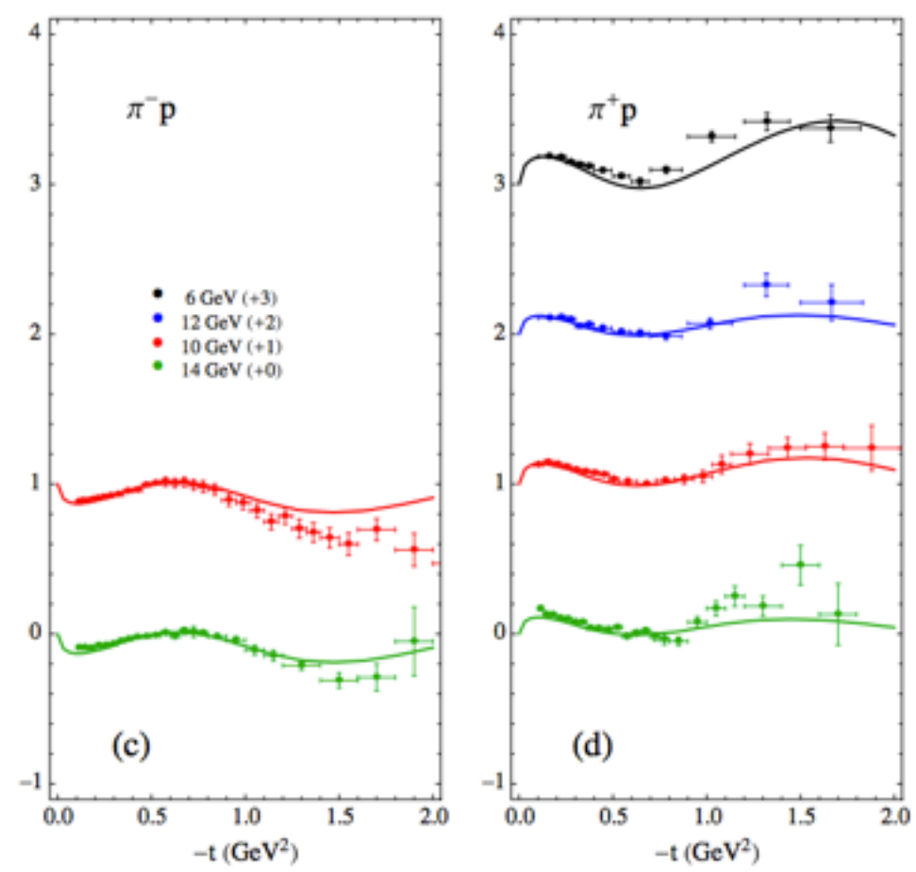
$$\pi^{-} p \to \pi^{0} n$$

for beam energy > 2 GeV

Differential cross section



Polarization observable

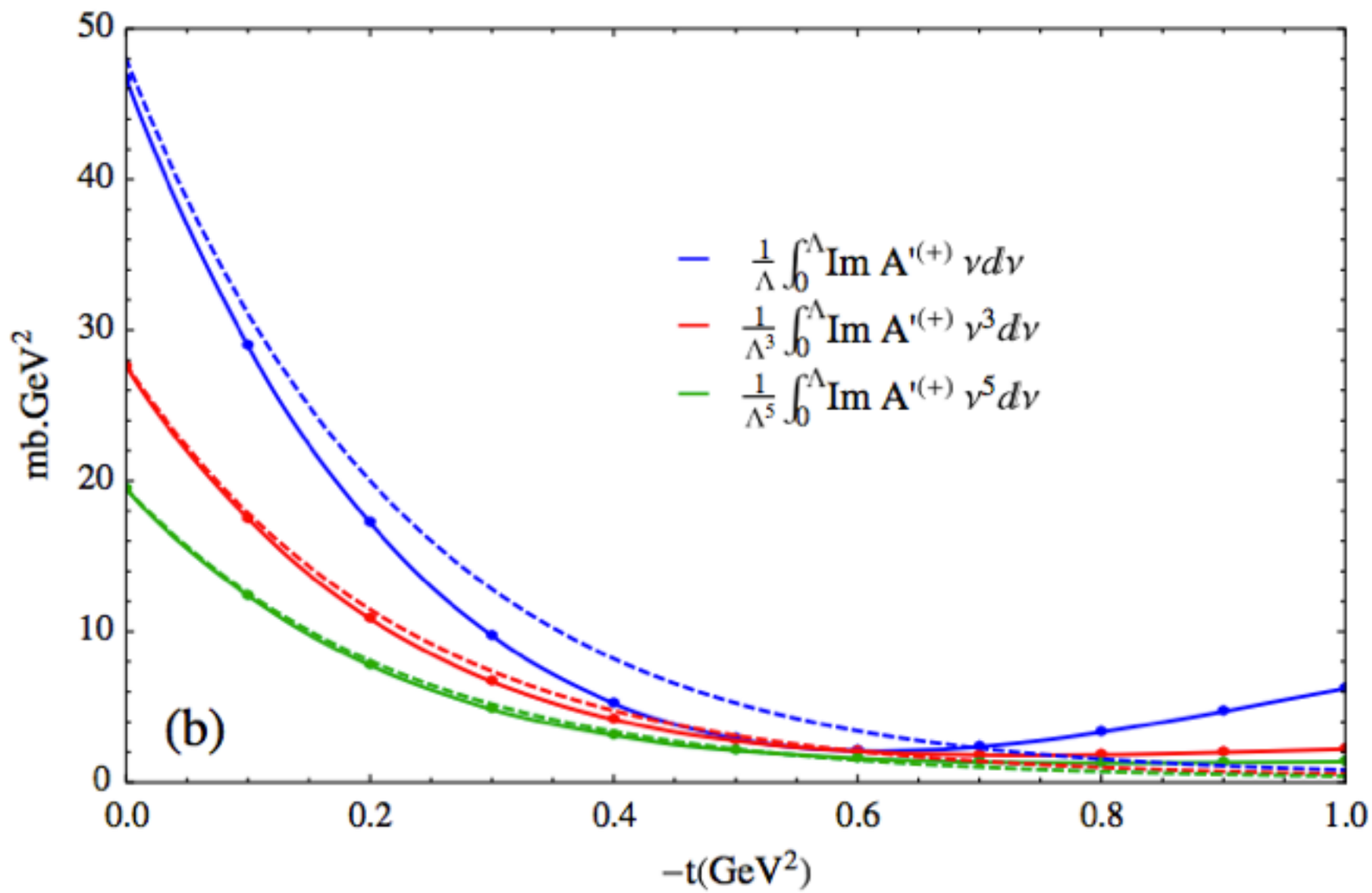


Let's compare both side of the sum rule

Solid line: SAID

Dashed line: Regge

excellent match !

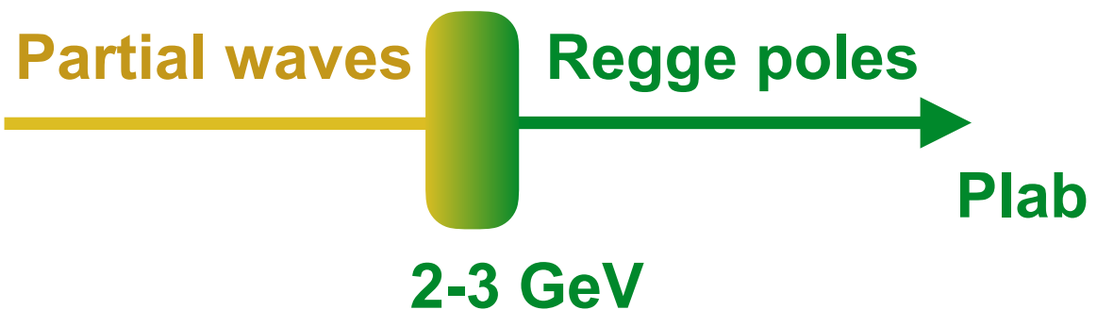


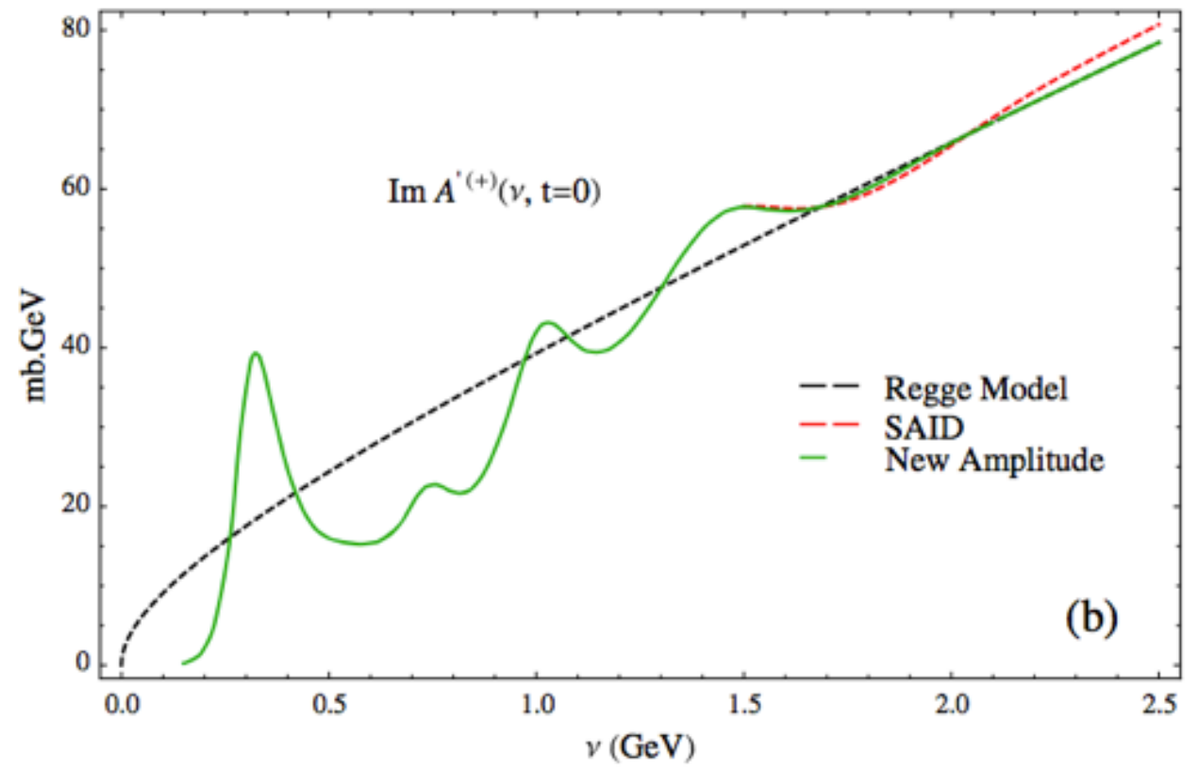
$$\frac{1}{\Lambda^k}\int_{\nu_0}^{\Lambda} \mathrm{Im}\ A(\nu,t)\nu^k d\nu = \frac{\beta(t)\Lambda^{\alpha(t)+1}}{\alpha(t)+k+1}$$

VM et al (JPAC) PRD92
arXiv:1506.01764

Checking Analyticity

Match low energy (PW) and high energy (Regge) imaginary parts





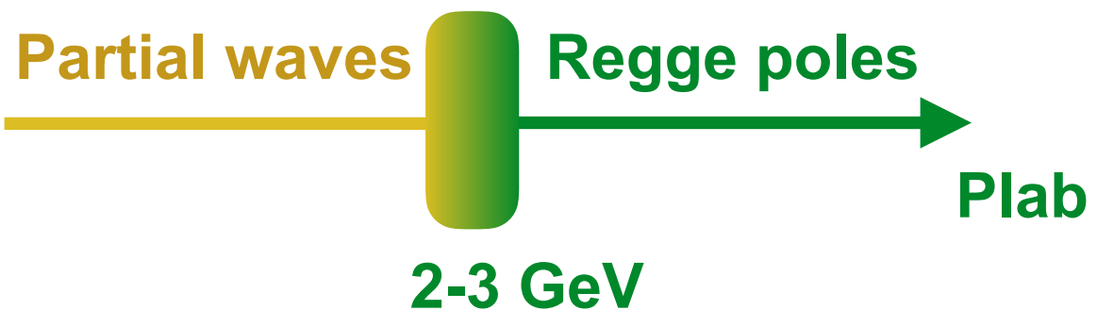
Reconstruct the real part from the dispersion relation

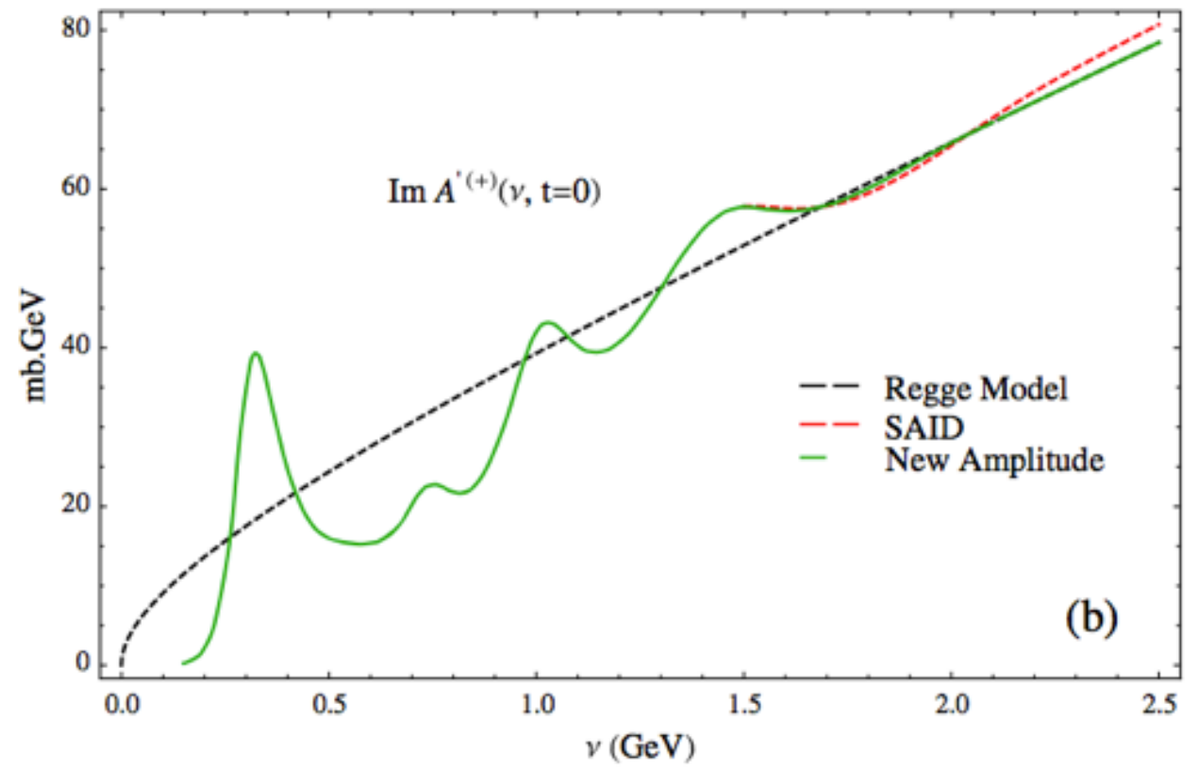
$$A(\nu,t) = \frac{2}{\pi}\int_{\nu_0}^{\infty} \frac{\mathrm{Im}\, A(\nu',t)}{\nu'^2-\nu^2}\nu' d\nu'$$

VM et al (JPAC) PRD92 arXiv:1506.01764

Checking Analyticity

Match low energy (PW) and high energy (Regge) imaginary parts





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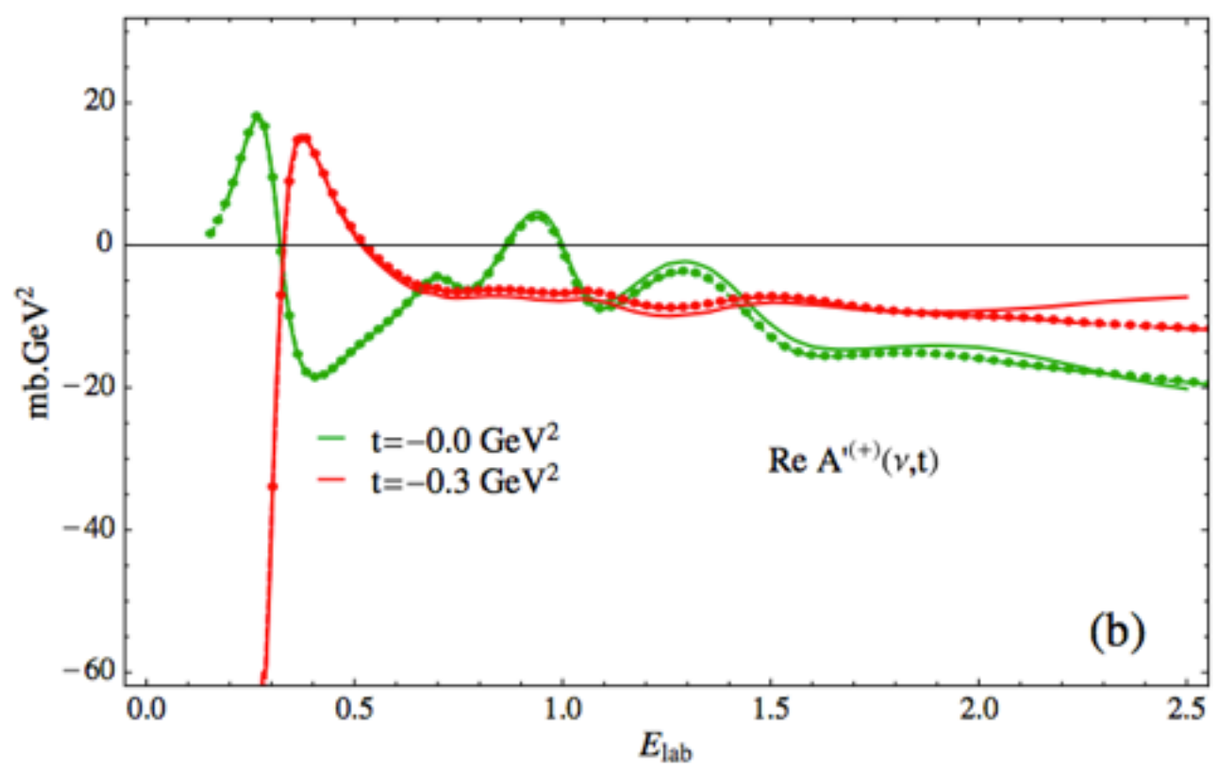
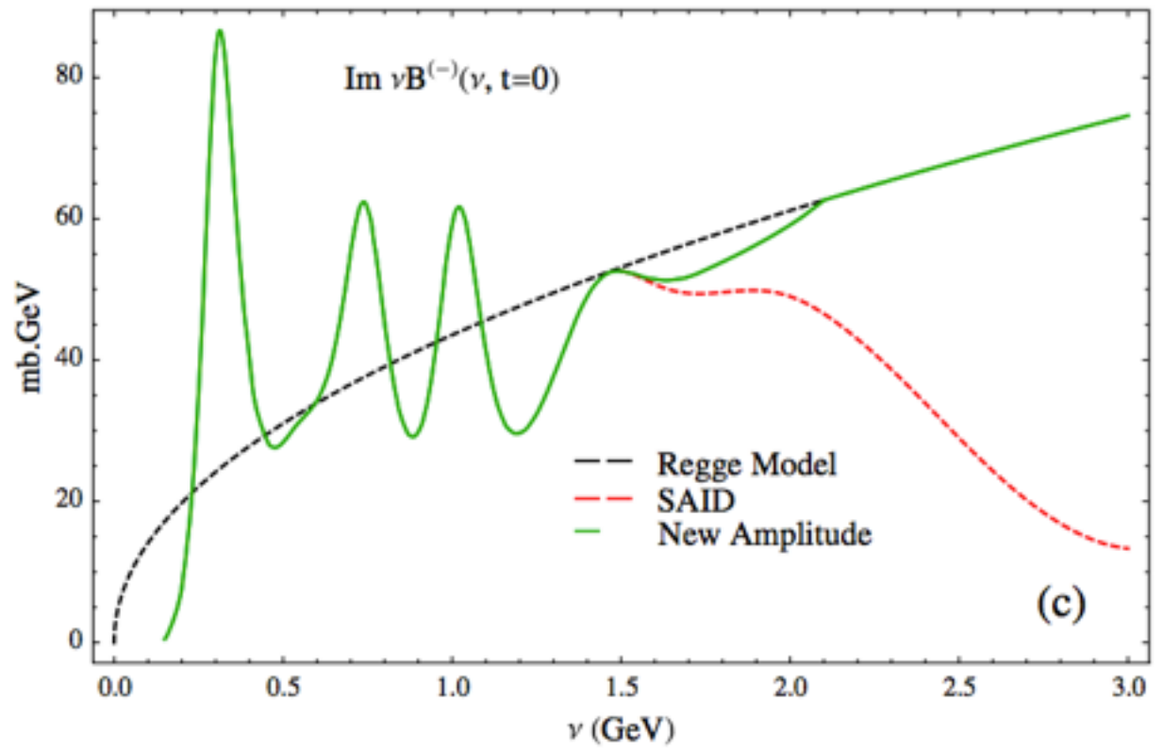


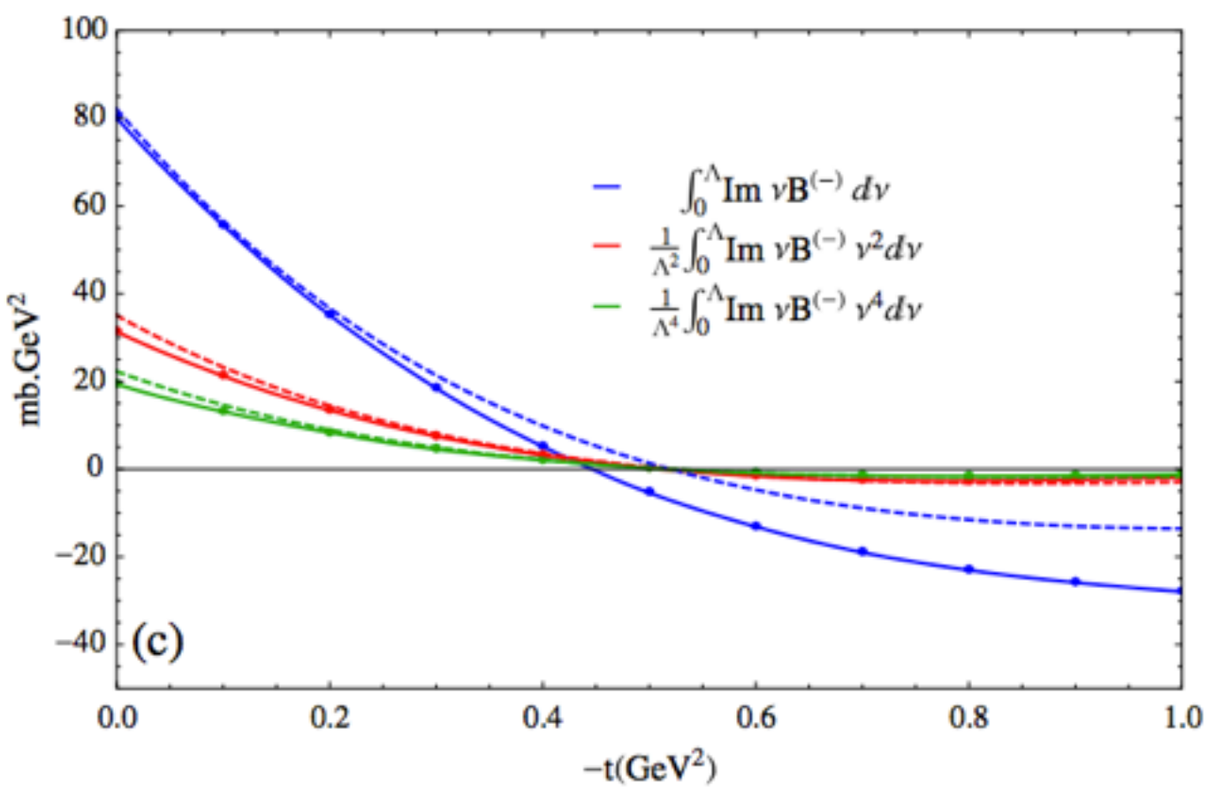
VM et al (JPAC) PRD92 arXiv:1506.01764

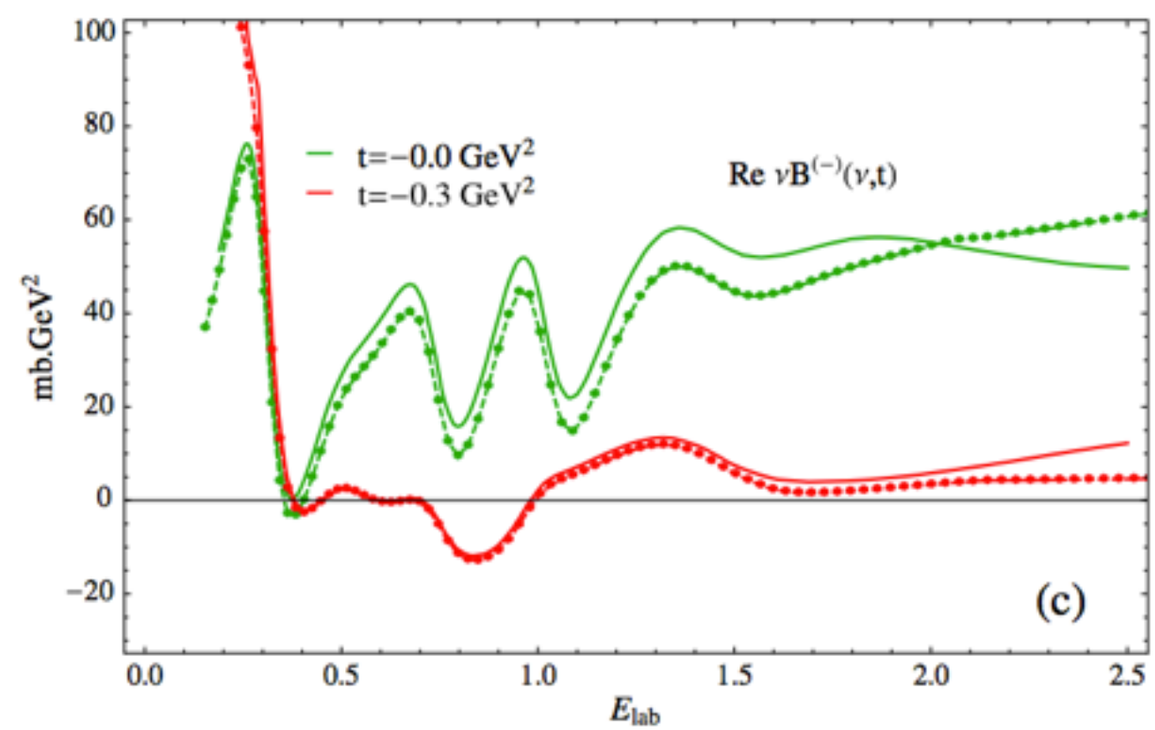
VM et al (JPAC) PRD92
arXiv:1506.01764

Similar results for the other amplitude

$$T = \bar{u}(p_4, \lambda_4)\left(A + \frac{1}{2}(\not{p}_1 + \not{p}_3)B\right)u(p_2, \lambda_2)$$







Application to Pseudoscalar Photoproduction

$$\gamma N \to \pi N$$

$$(\pm 1)\left(\pm\frac{1}{2}\right) \longrightarrow 0\left(\pm\frac{1}{2}\right)$$

8 helicity configurations related by pair via parity

4 indep. helicity configurations

use CGLN basis $A_1, \ldots, A_4$

Isospin symmetry:
every amplitude has an isospin index (+,-,0)

$$A^a_{ji} = A^{(+)}\delta^{a3}\delta_{ji} + A^{(-)}\frac{1}{2}[\tau^a, \tau^3]_{ji} + A^{(0)}\tau^a_{ji}$$

12 indep. helicity/isospin configurations

$$\gamma p \to \pi^+ n : \sqrt{2}\left(A^{(0)} + A^{(-)}\right)$$

$$\gamma n \to \pi^- p : \sqrt{2}\left(A^{(0)} - A^{(-)}\right)$$

$$\gamma p \to \pi^0 p : \qquad A^{(+)} + A^{(0)}$$

$$\gamma n \to \pi^0 n : \qquad A^{(+)} - A^{(0)}$$

Application to Pseudoscalar Photoproduction

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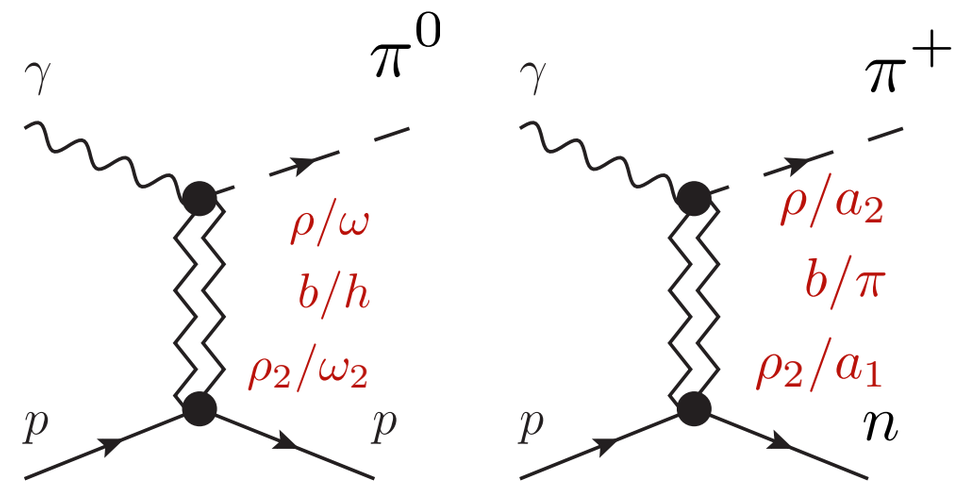

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$$\gamma n \to \pi^0 n : \qquad A^{(+)} - A^{(0)}$$

A_i	(0)	$(+)$	$(-)$
A_1	ρ	ω	a_2
A_2	b	h	π
A_3	ρ_2	ω_2	a_1
A_4	ρ	ω	a_2

VM et al (JPAC), in preparation

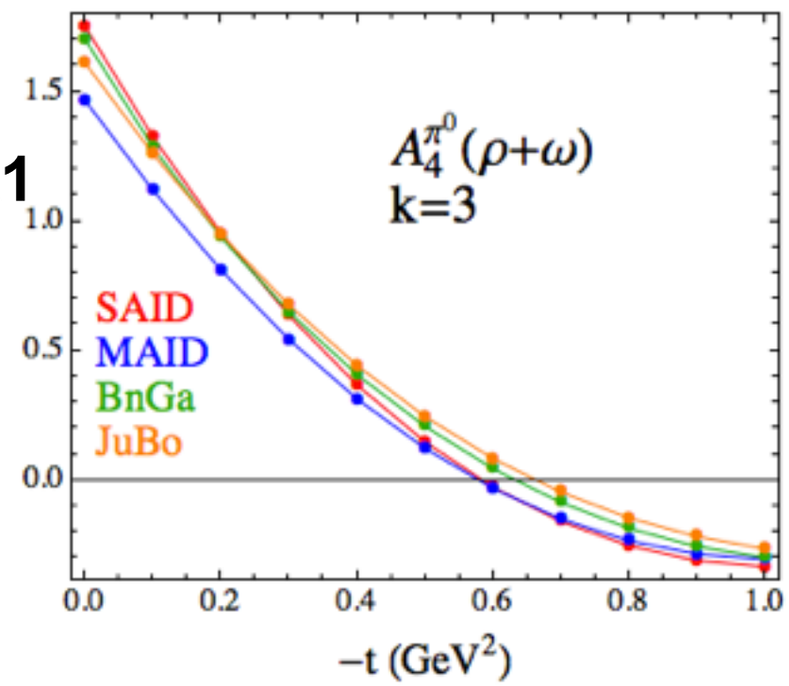
Use CGLN basis $A_{1,2,3,4}$

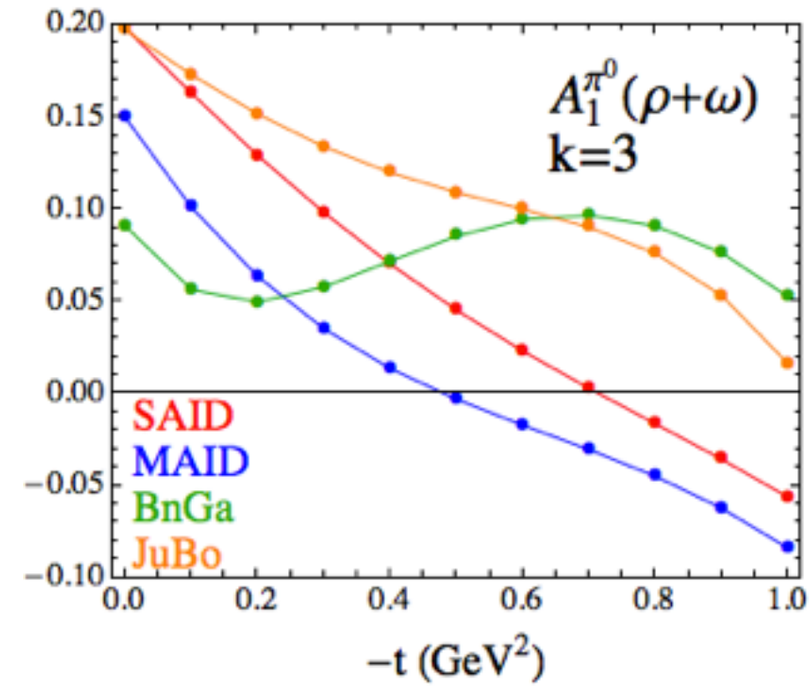
$$\gamma p \to \pi^0 p$$

Agreed on dominant A4

Variation on subdominant A1

$$\frac{1}{\Lambda^k}\int_{\nu_0}^{\Lambda} \mathrm{Im}\ A(\nu,t)\nu^k d\nu = \frac{\beta(t)\Lambda^{\alpha(t)+1}}{\alpha(t)+k+1}$$





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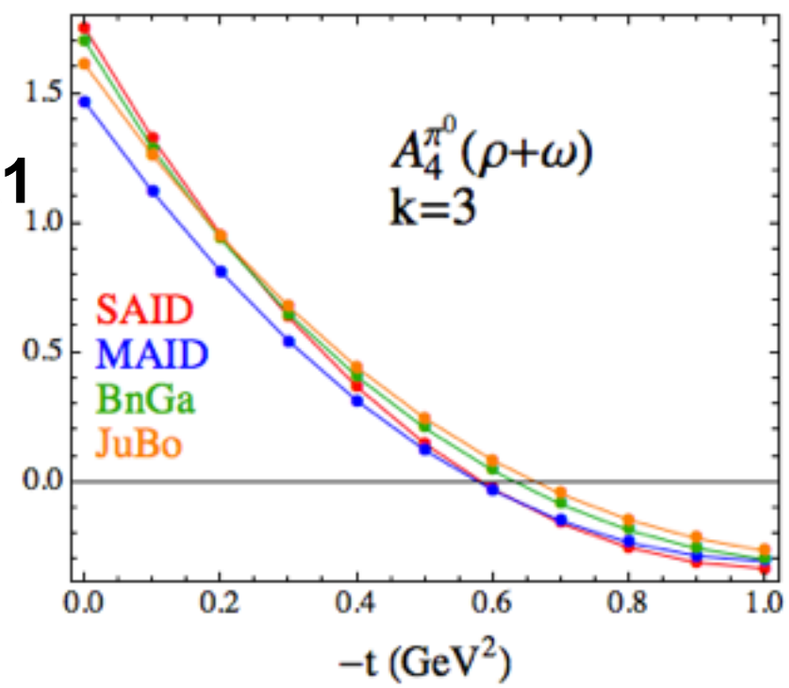
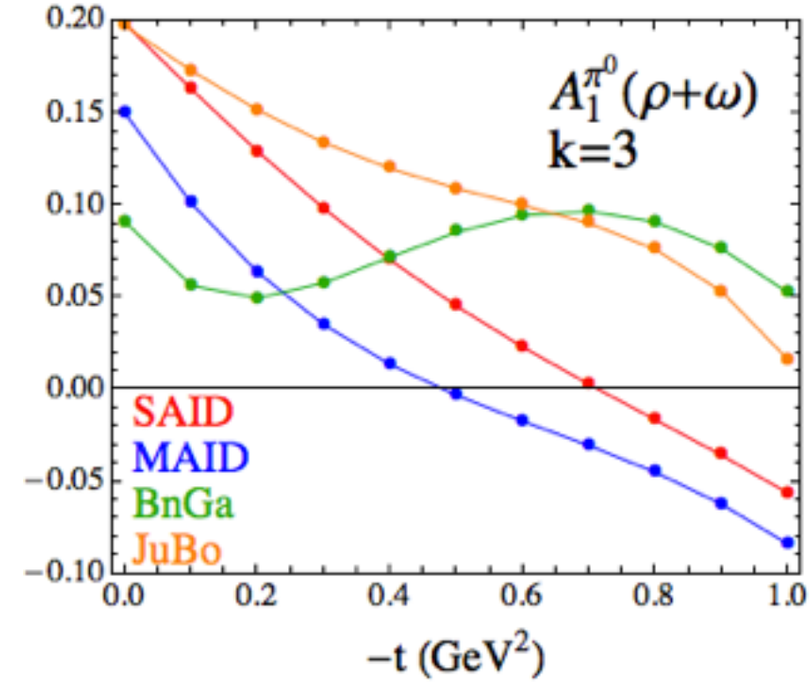
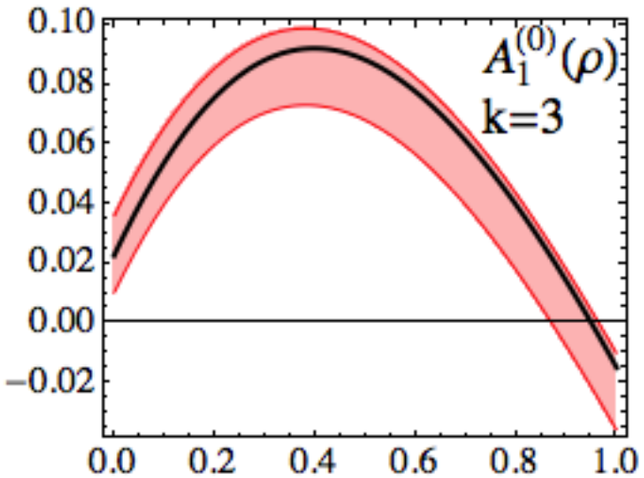
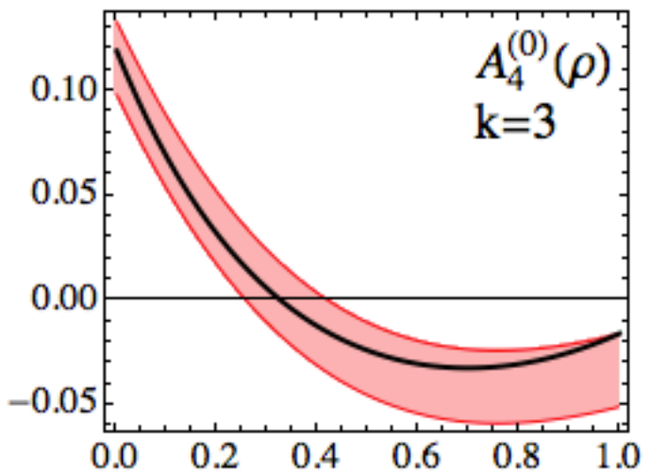
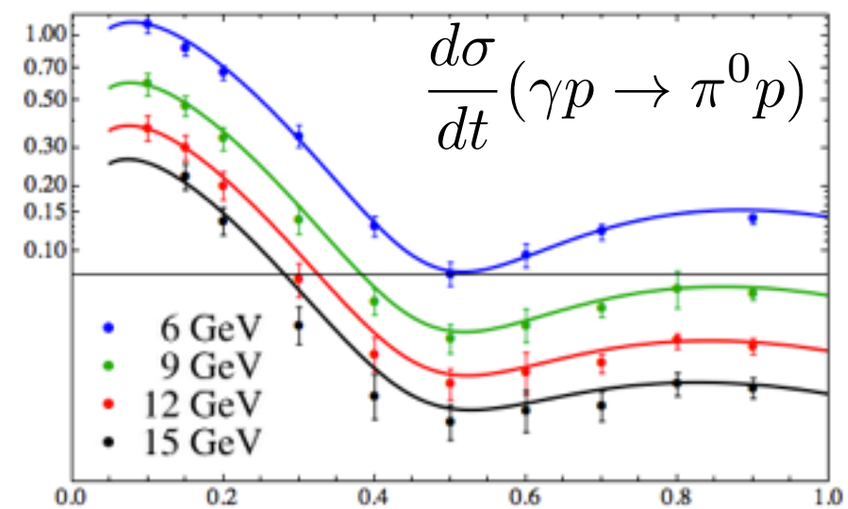
Use SAID to fit FESR and observable

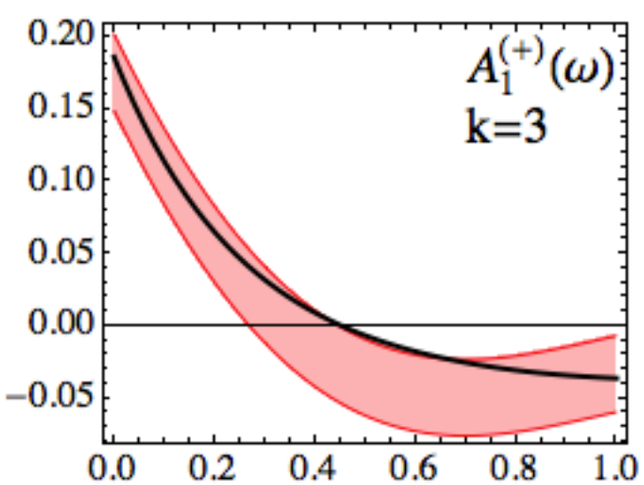


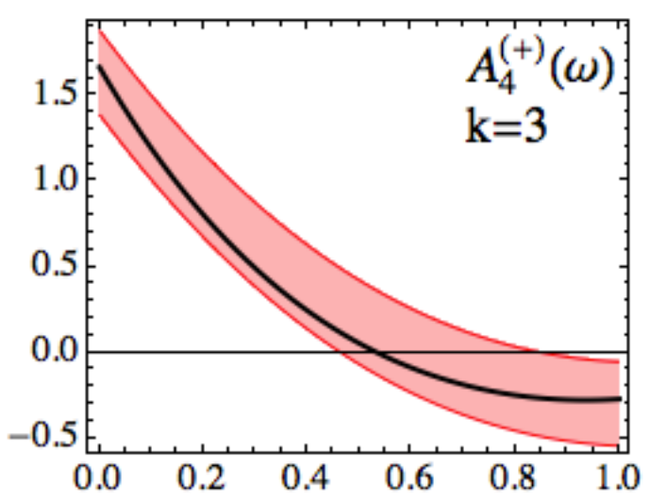


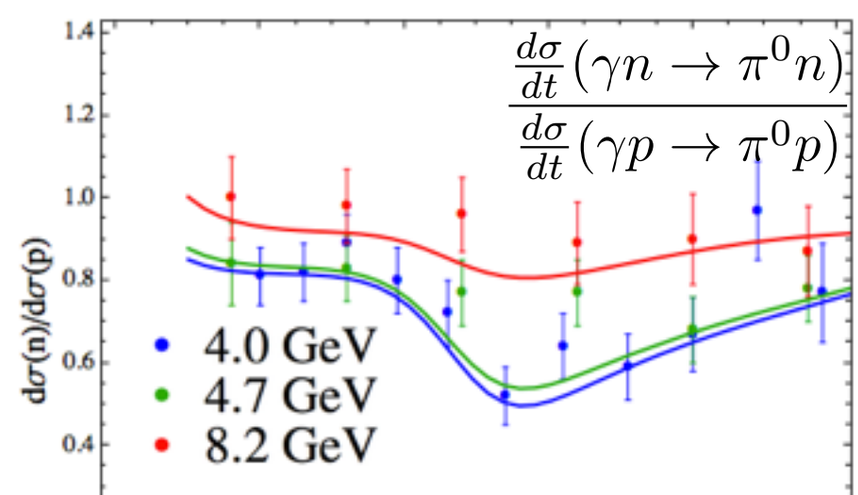




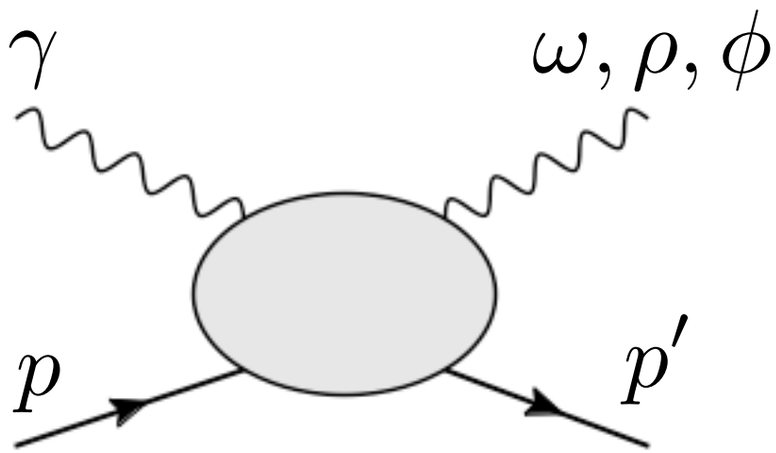




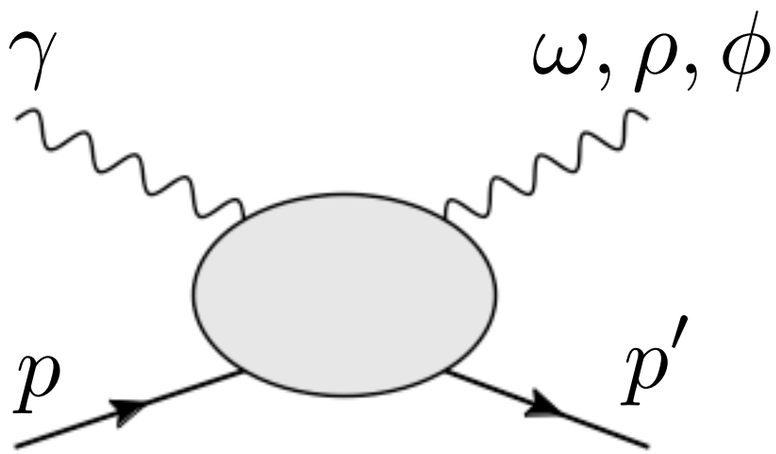




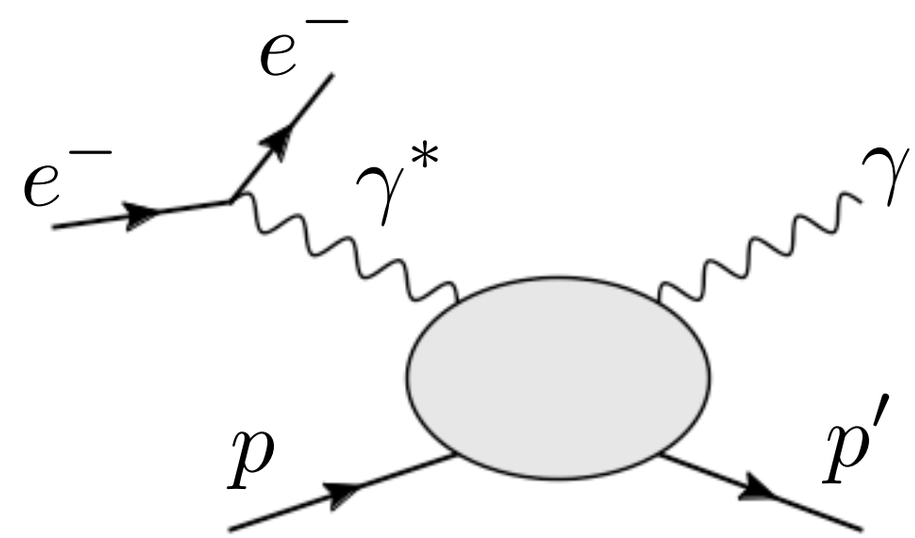
Vector Photoproduction



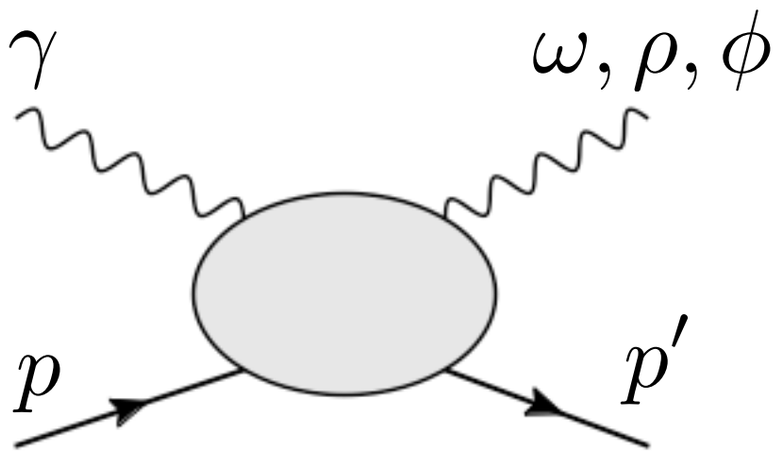
Vector Photoproduction



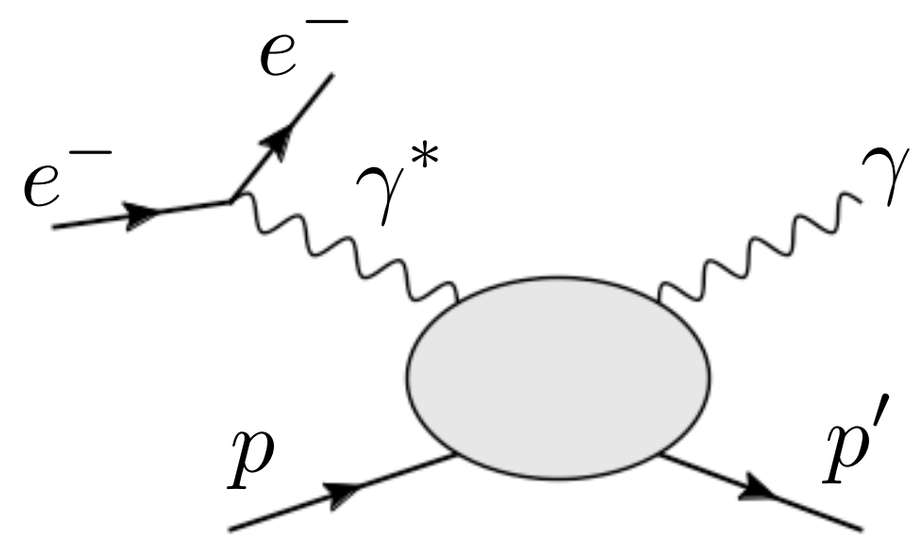
Virtual Compton Scattering



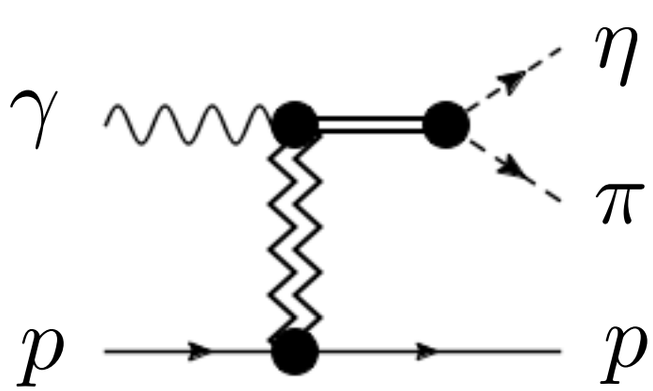
Vector Photoproduction



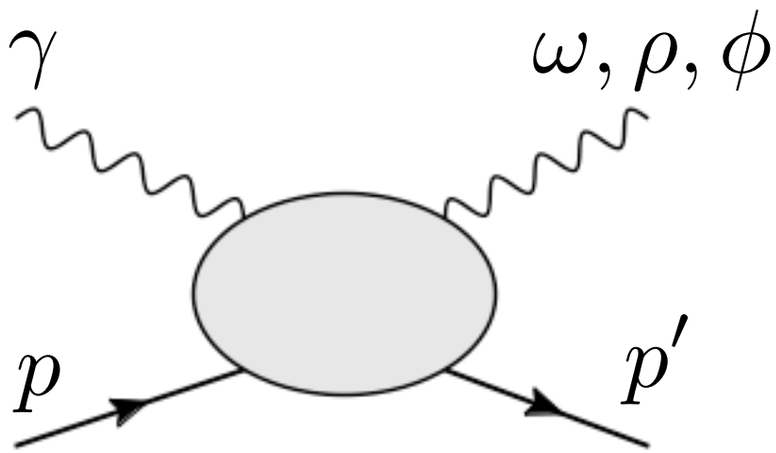
Virtual Compton Scattering



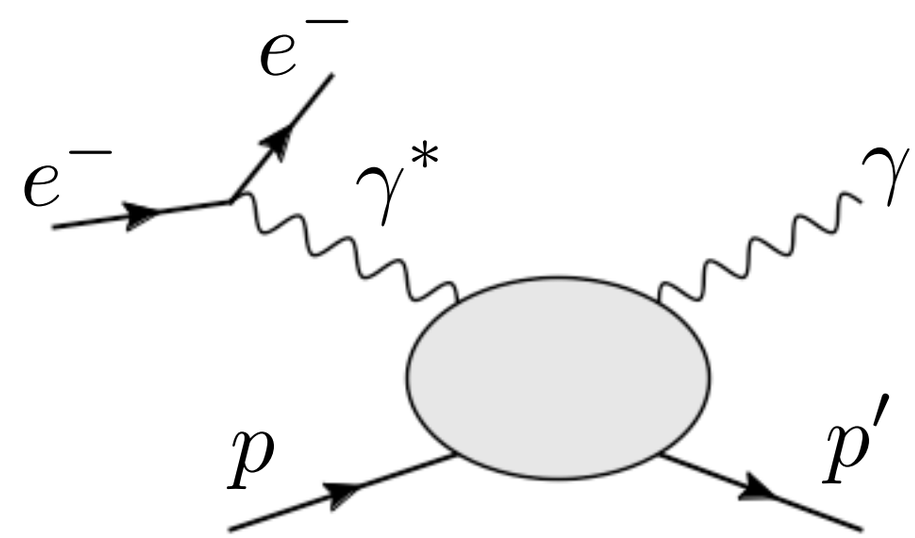
FESR for Diffractive Prod.



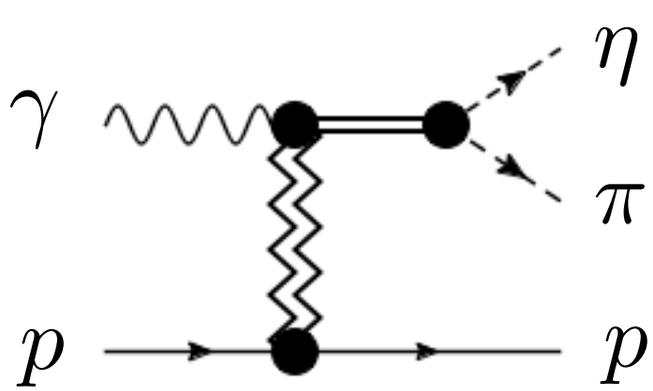
Vector Photoproduction



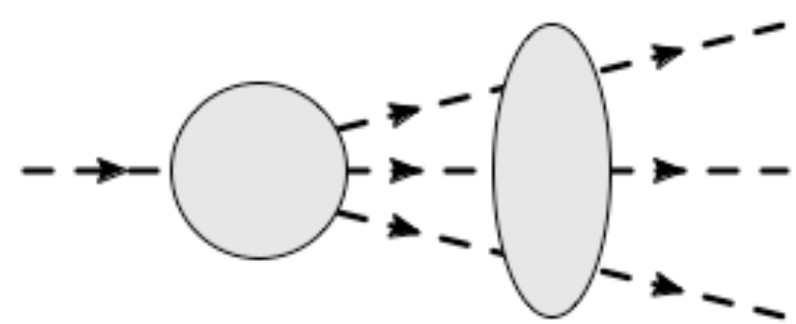
Virtual Compton Scattering



FESR for Diffractive Prod.



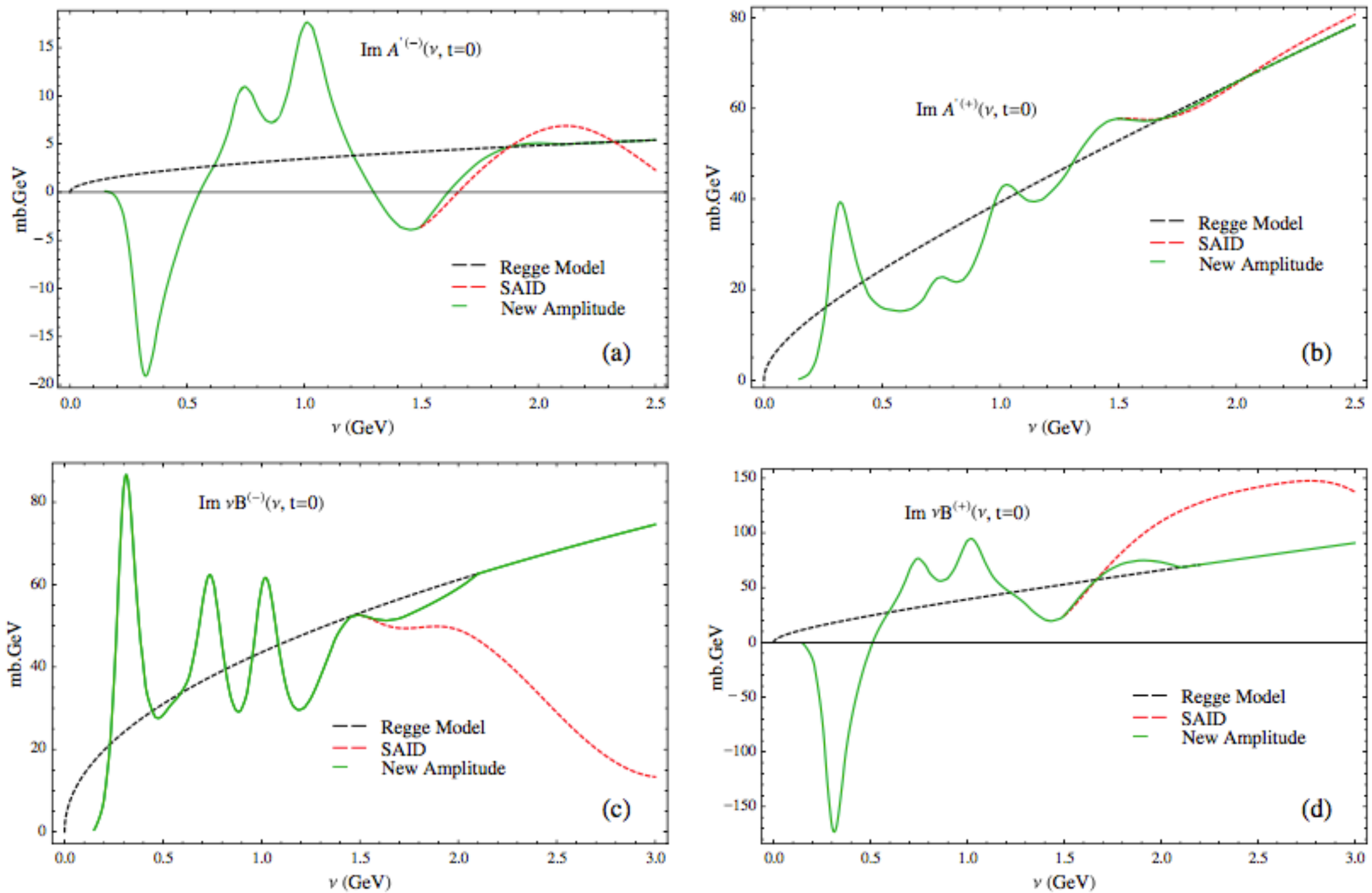
S-Matrix Constraints on the Lattice



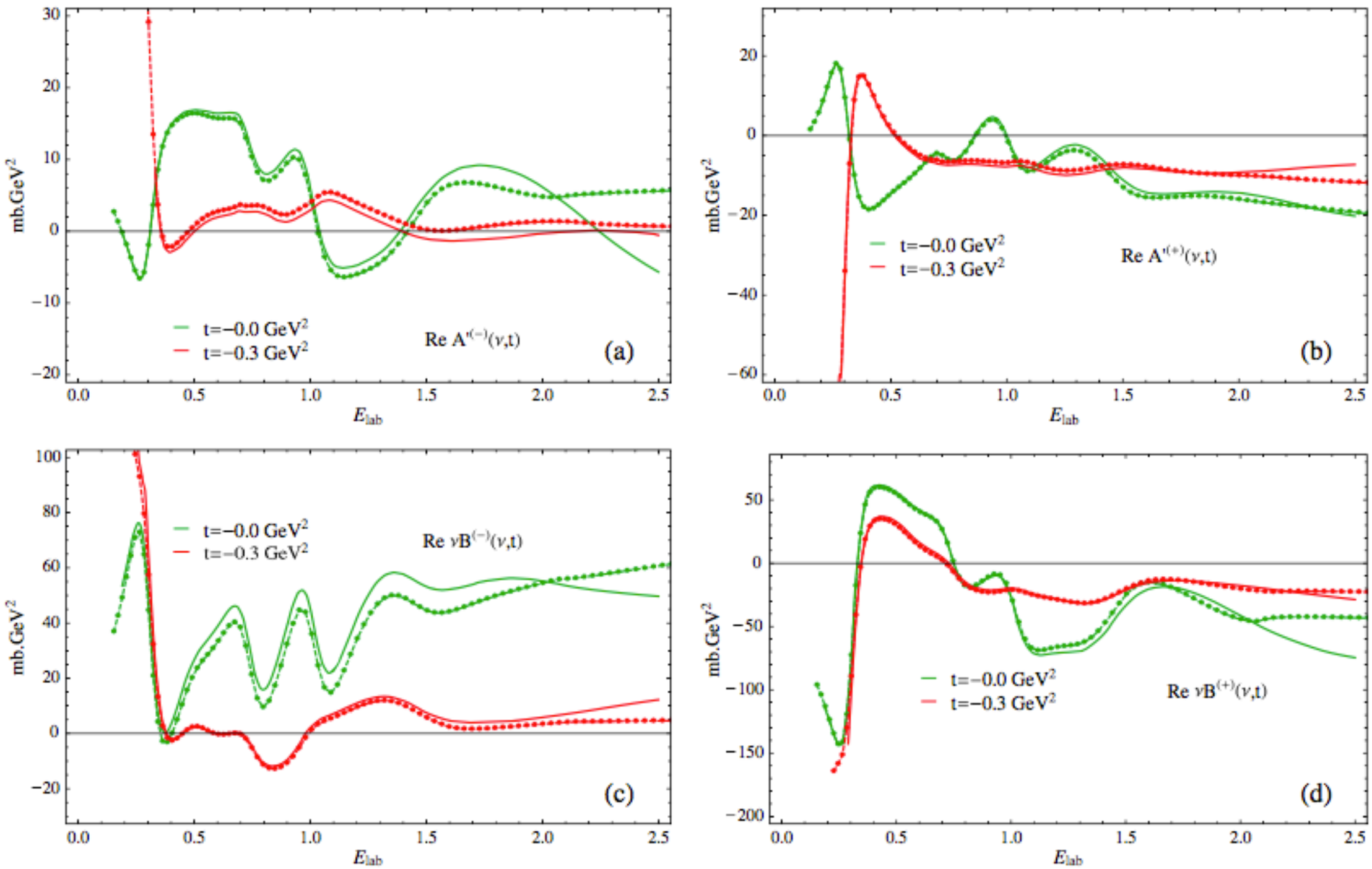
Thank You !

Backup Slides

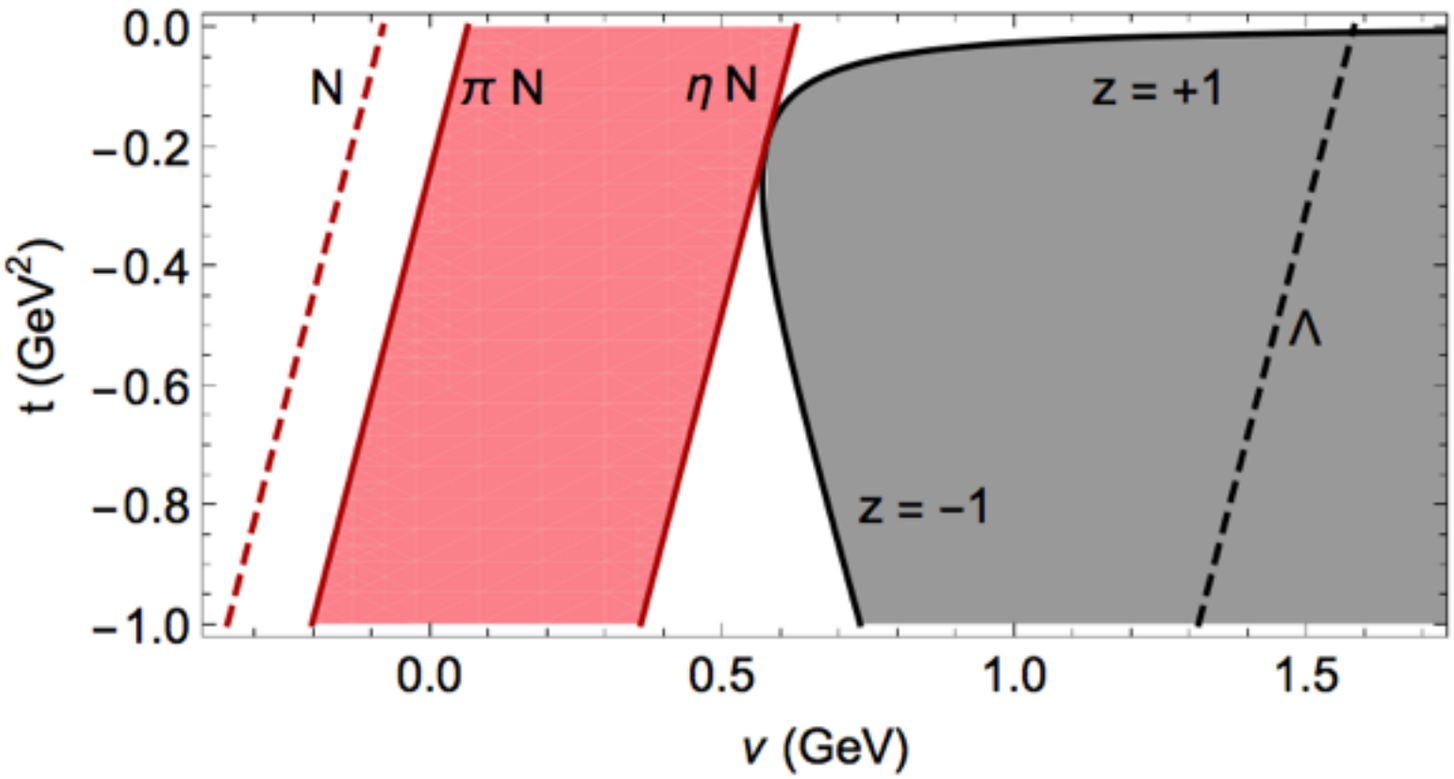
$\pi N \to \pi N$

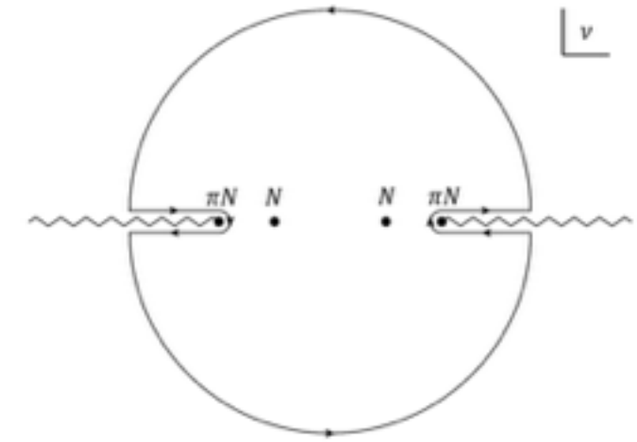

Im $A^{(-)}(\nu, t=0)$
Im $A^{(+)}(\nu, t=0)$
Im $\nu B^{(-)}(\nu, t=0)$
Im $\nu B^{(+)}(\nu, t=0)$
mb.GeV
ν (GeV)
Regge Model
SAID
New Amplitude
(a)
(b)
(c)
(d)

$\pi N \to \pi N$

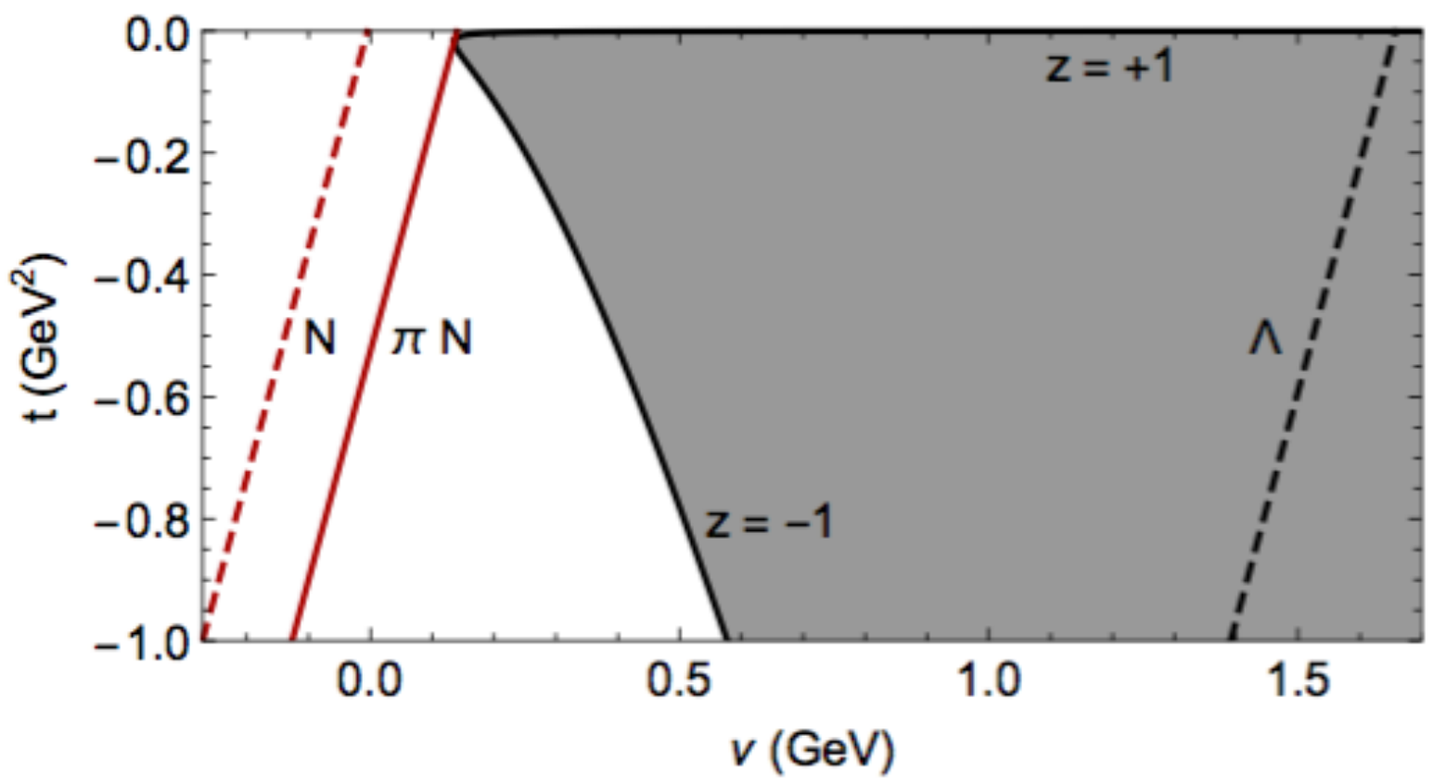

mb.GeV^2
E_{lab}
t=−0.0 GeV^2
t=−0.3 GeV^2
Re A$^{(-)}$(ν,t)
(a)
Re A$^{(+)}$(ν,t)
(b)
Re νB$^{(-)}$(ν,t)
(c)
Re νB$^{(+)}$(ν,t)
(d)

Integration Region

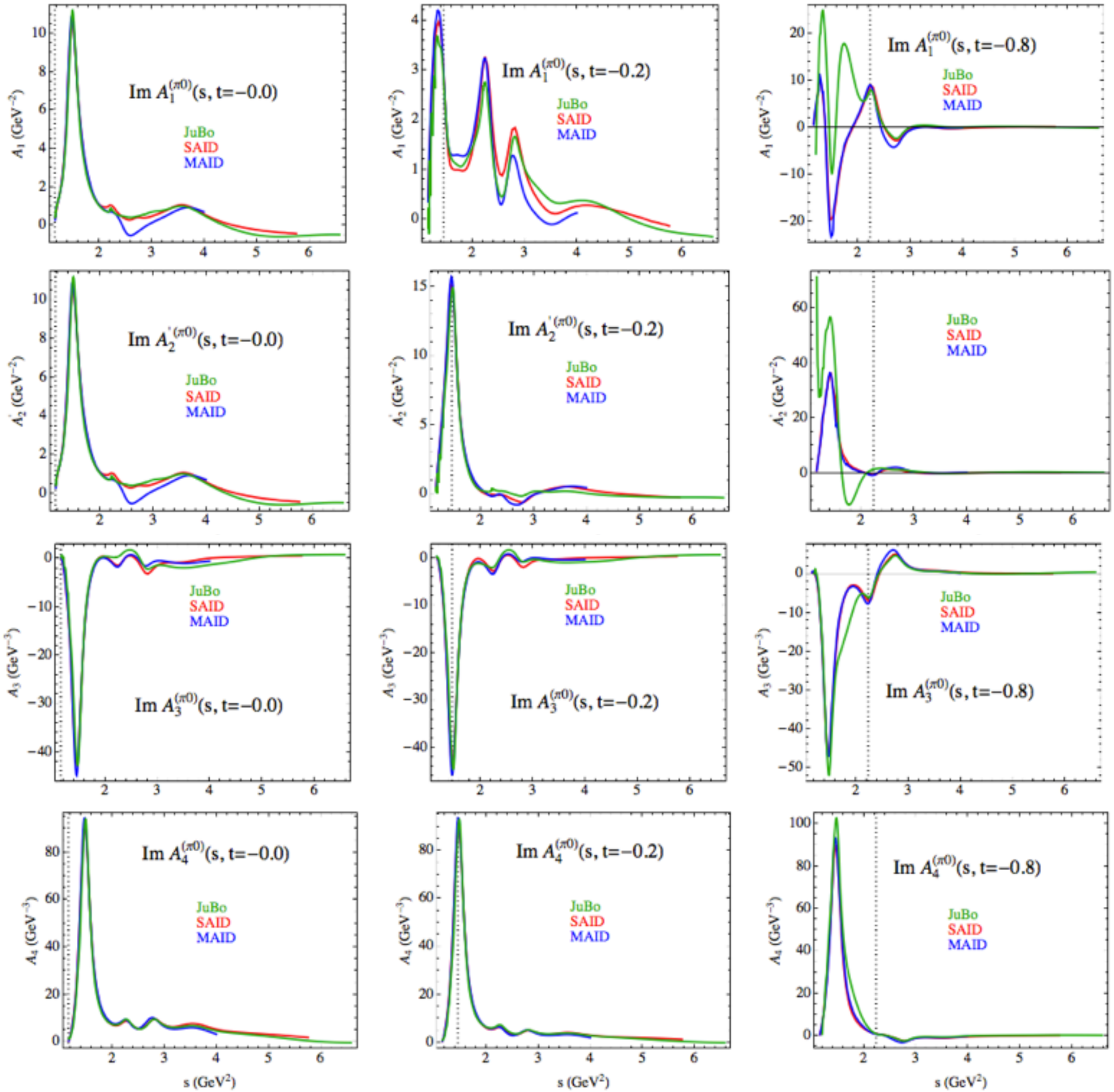




$$\gamma N \to \eta N$$

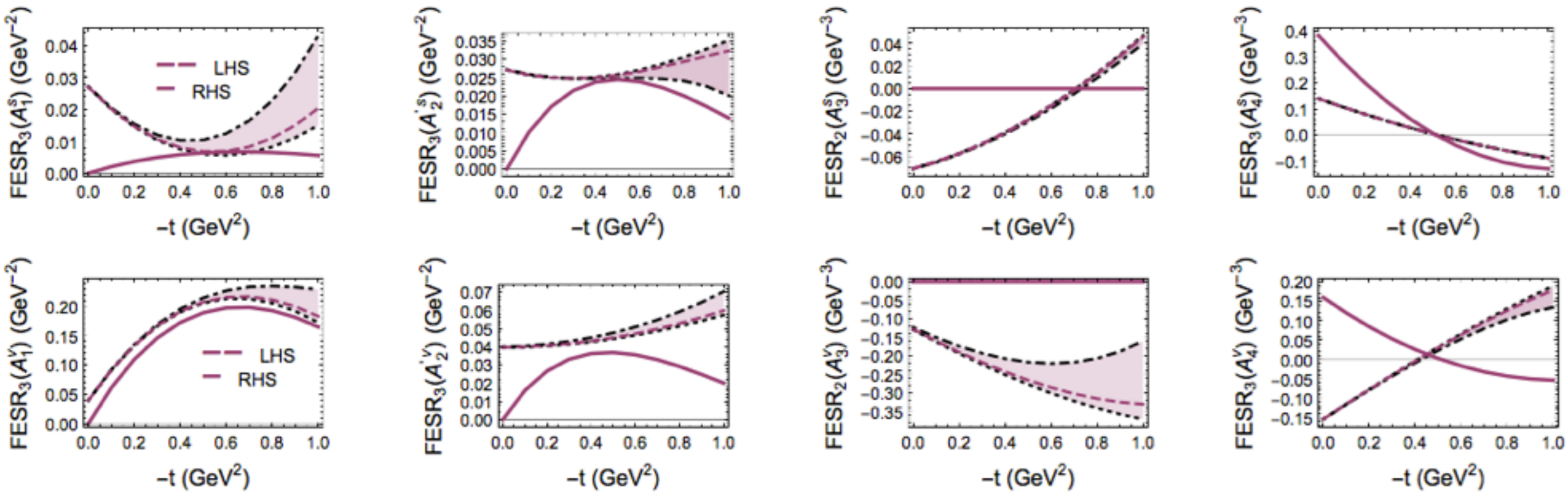


$$\gamma N \to \pi N$$


Im $A_1^{(\pi 0)}$(s, t=−0.0)
Im $A_1^{(\pi 0)}$(s, t=−0.2)
Im $A_1^{(\pi 0)}$(s, t=−0.8)
Im $A_2^{'(\pi 0)}$(s, t=−0.0)
Im $A_2^{'(\pi 0)}$(s, t=−0.2)
Im $A_3^{(\pi 0)}$(s, t=−0.0)
Im $A_3^{(\pi 0)}$(s, t=−0.2)
Im $A_3^{(\pi 0)}$(s, t=−0.8)
Im $A_4^{(\pi 0)}$(s, t=−0.0)
Im $A_4^{(\pi 0)}$(s, t=−0.2)
Im $A_4^{(\pi 0)}$(s, t=−0.8)
JuBo
SAID
MAID
A_1 (GeV^{-2})
A_2' (GeV^{-2})
A_3 (GeV^{-3})
A_4 (GeV^{-3})
s (GeV^2)

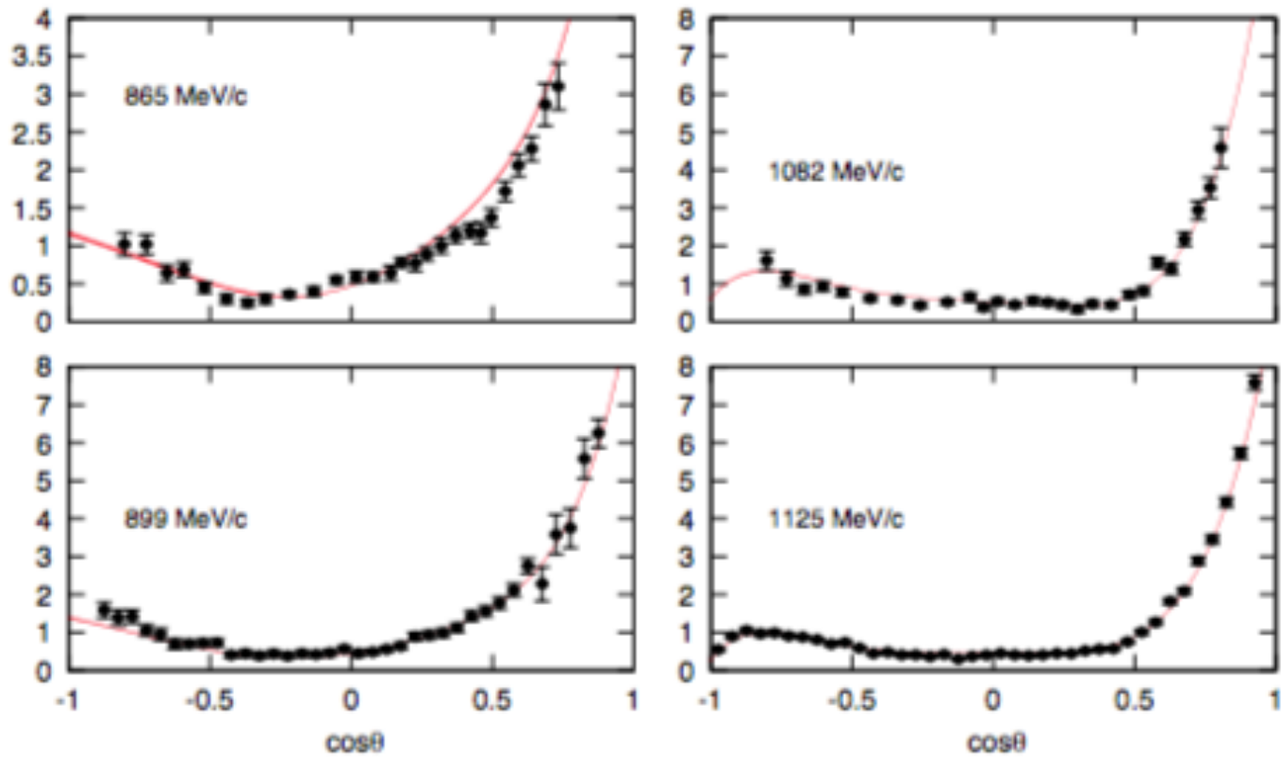
$\gamma N \to \eta N$

A_1 A_2 A_3 A_4

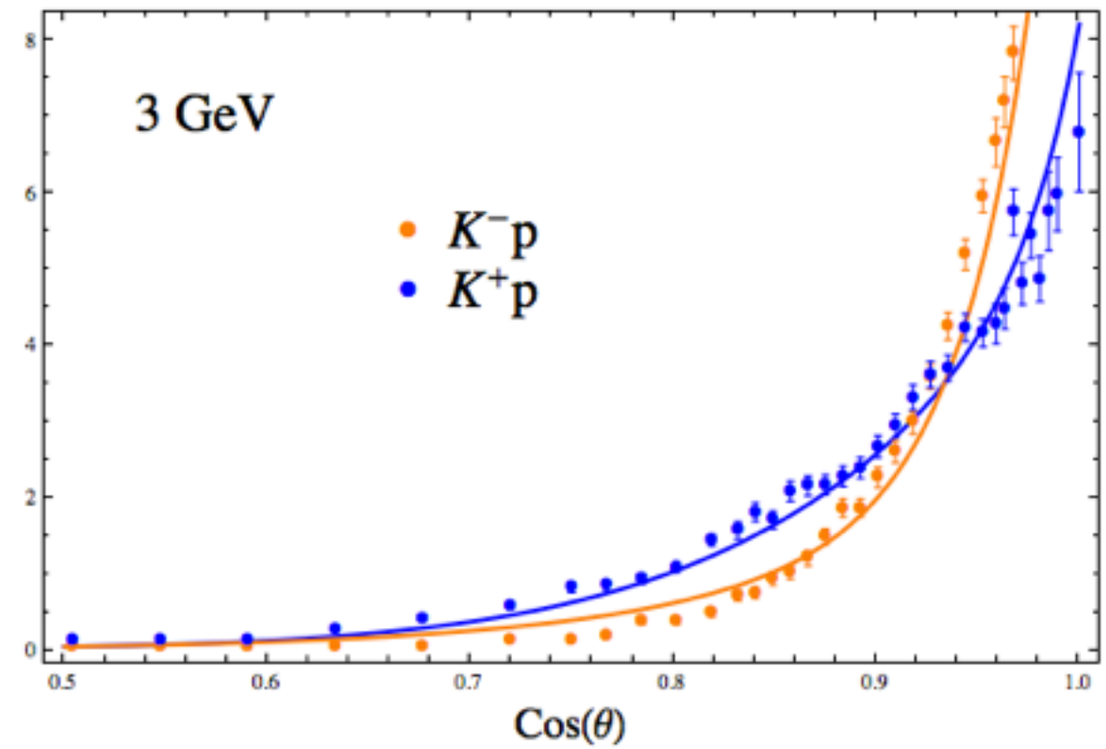


$K^- p \to K^- p$ *Energy Evolution*

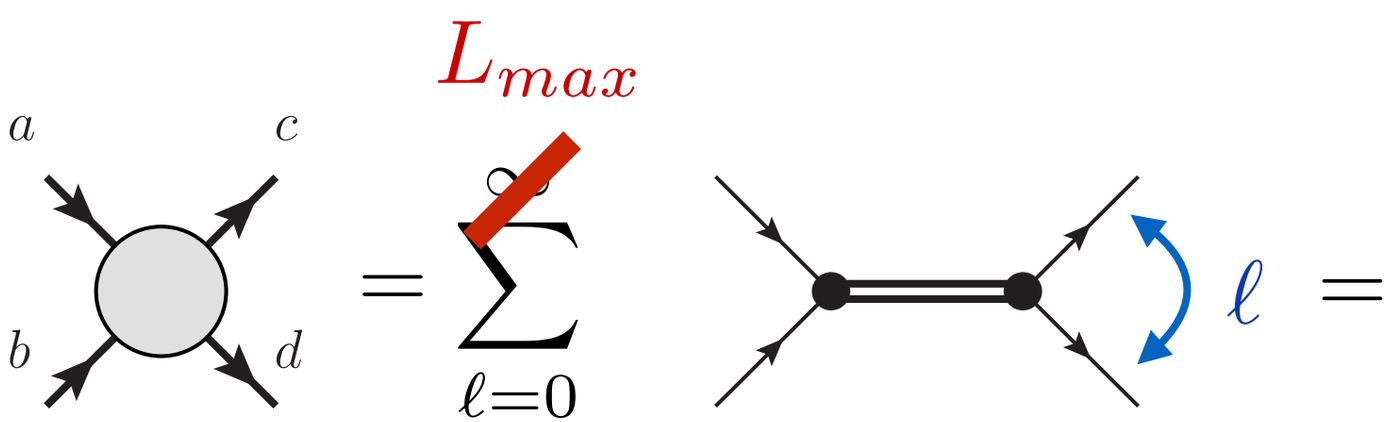
C. Fernandez-Ramirez et al. (JPAC) ArXiv:1510:07065



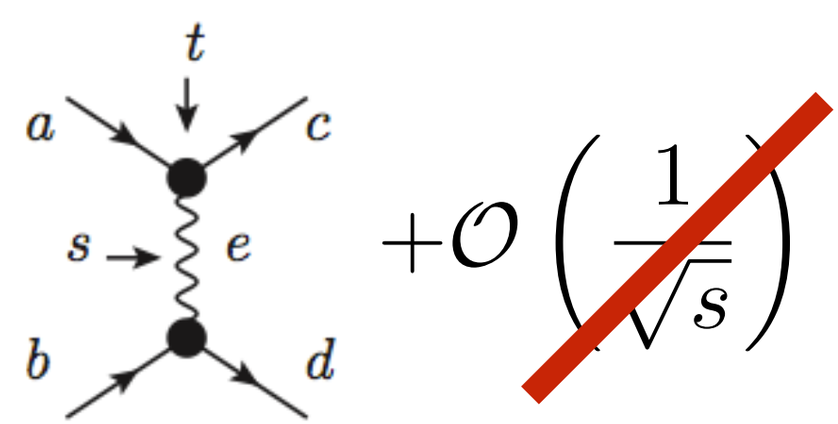
VM (unpublished)



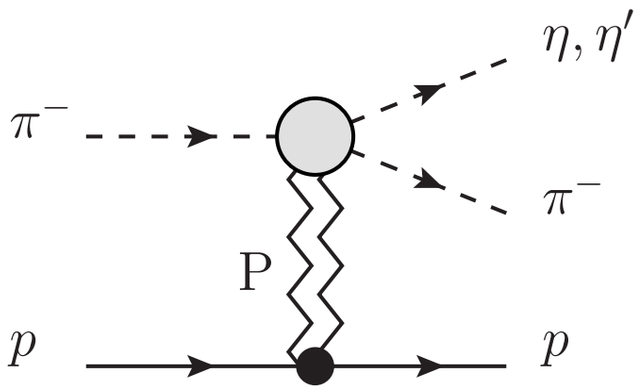
Partial wave expansion

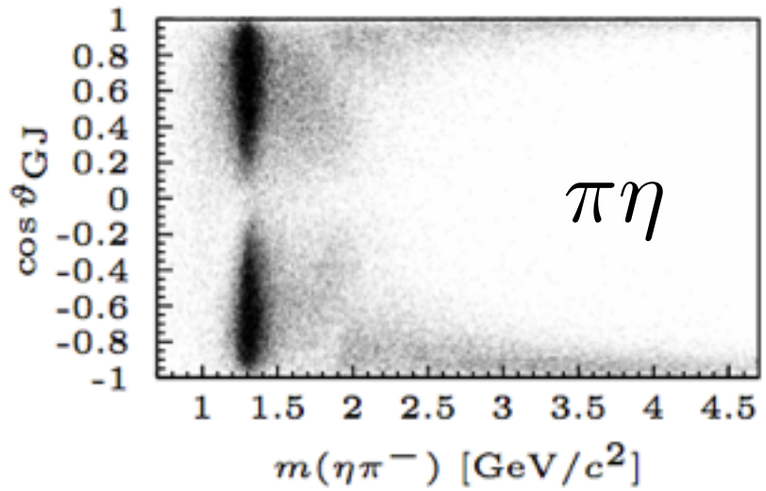


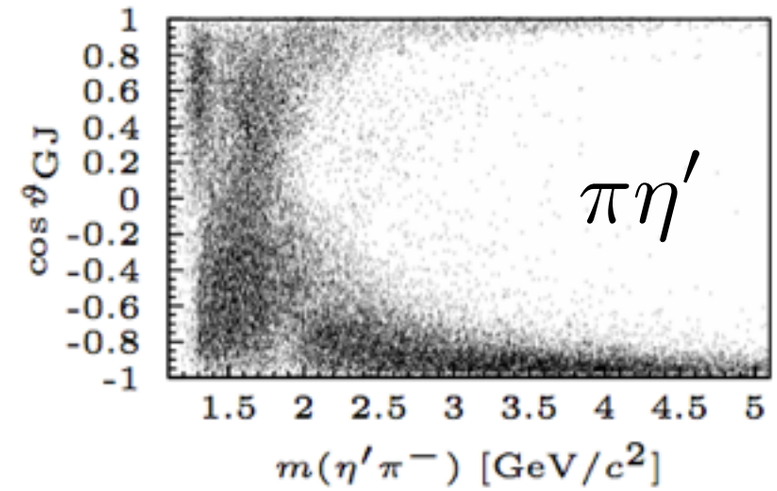
Regge pole expansion



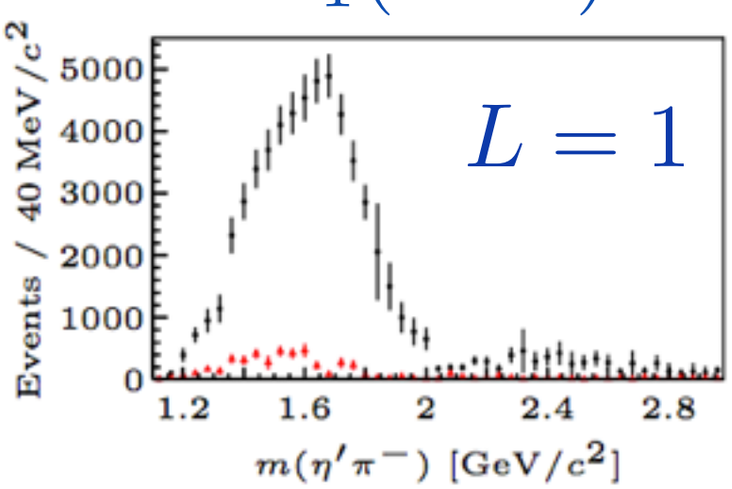
Discovering (?) New Resonances: Eta(')-Pi @COMPASS



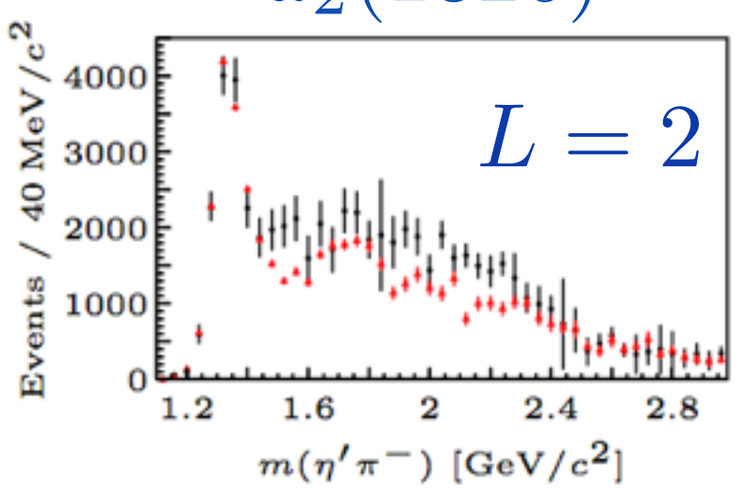




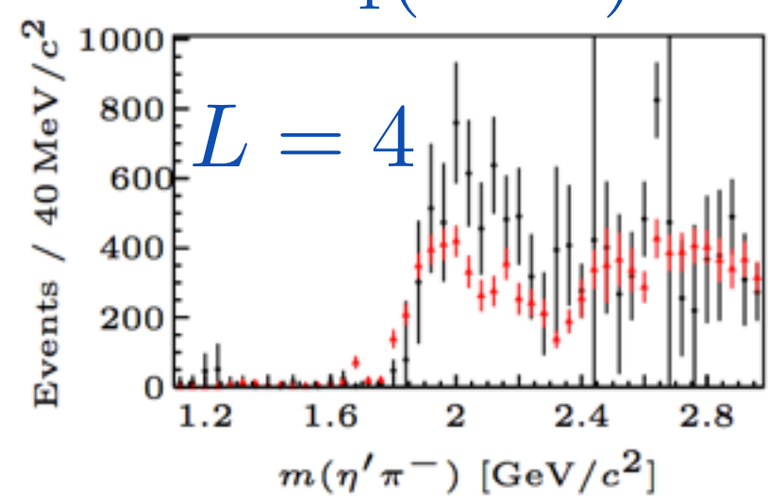
$\pi_1(1600)$?

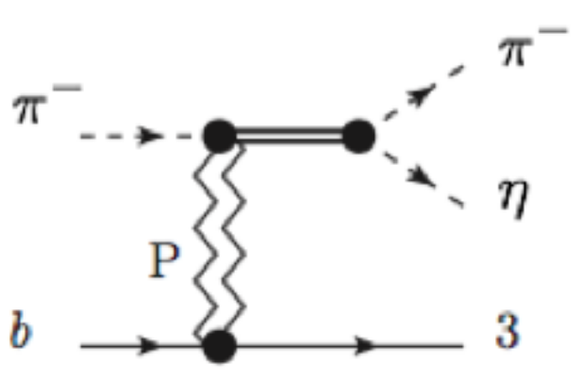


$a_2(1320)$



$a_4(2040)$





black: $\pi\eta'$

red: $\pi\eta$ **(scaled)**

Resonance in angular mom. L = 1 ?

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