Constraints from Analyticity in Hadron Spectroscopy and the JLab program

Vincent MATHIEU

Indiana University

Joint Physics Analysis Center

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Outline

• Resume
• Accomplishments
• Research plan @JLab
Resume

- Master in Engineering (UMons, 2004)
- PhD in Physics (UMons, 2008)
- Postdoc at Valencia U. (2009-11)
- Postdoc at ECT*-Trento (2011-2013)
- Postdoc at Indiana U. (2013-present)

- 35 publications (~ 600 citations) with 31 collaborators
- 1 invited review
- 1 white paper editor (42 authors)

- Referee for PRL, PRC&D, PLB, JHEP, ...
- Organizers of 3 summer schools and 3 conferences
- ~ 15 talks/year in meetings and physics departments

- Joint Physics Analysis Center website:
  http://www.indiana.edu/~jpac/
Bound States of Gluons

- Evidence from lattice calculations of “Glueballs” in pure gauge theory

\[ \mathcal{L}_{QCD} = -\frac{1}{4} \left( \nabla \cdot \vec{A} + g \vec{A} \times \vec{A} \right)^2 + \sum_f \bar{q}_f \left( \gamma \cdot \nabla - i g \gamma \cdot \vec{A} - m_f \right) q_f \]
Bound States of Gluons

- Evidence from lattice calculations of “Glueballs” in pure gauge theory

\[ \mathcal{L}_{QCD} = -\frac{1}{4} \left( \partial \cdot \vec{A} + g \vec{A} \times \vec{A} \right)^2 + \sum_f \bar{q}_f \left( \gamma \cdot \partial - ig \gamma \cdot \vec{A} - m_f \right) q_f \]

How to understand them with effective gluons?

Schrödinger’s equation

\[
\left[ \frac{\nabla^2}{2m} + V(r) \right] \Psi = E \Psi
\]

\[ V(r) = \sigma r + \cdots \]

Morningstar and Peardon 1999
Bound States of Gluons

Publication list on glueballs (during PhD in Mons):

- **On #gluon in glueballs**
  Boulanger, Buisseret, VM and Semay EPJA38 (2008)

- **On 2-gluon C = + glueballs**
  VM, Buisseret and Semay PRD77 (2008), PRD80 (2009)

- **On 3-gluon C = - glueballs**

- **Invited review:**
Strong Interaction

\[ \mathcal{L}_{QCD} = -\frac{1}{4} \left( \partial \cdot \vec{A} + g \vec{A} \times \vec{A} \right)^2 + \sum_f \bar{q}_f \left( \gamma \cdot \partial - ig \gamma \cdot \vec{A} - m_f \right) q_f \]

quark interaction via gluon exchange

strength of interaction related to gluon propagator

How can we understand that?

Evidence from lattice calculation of IR saturation of gluon propagator

dynamical gluon mass generation

Gluon mass consistent with gauge invariance thanks to Schwinger mechanism
Matching Theory and Experiments

Ordinary matter

Exotic matter

Theory: Quantum Chromodynamics

Resonances properties

constituent models, lattice
Matching Theory and Experiments

Ordinary matter

```
U  D
```

Exotic matter

```
G  G
G  G
U  U
```

Theory: Quantum Chromodynamics

constituent models, lattice

Resonances properties

amplitude analysis

Experiments: JLab, CERN,…
Experimental identification of exotic matter is difficult. Begin with a simple case.

Exotic matter is produced diffractively.
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Simplest cases:
Experimental identification of exotic matter is difficult. Begin with simple case.

Exotic matter is produced diffractively.

Experiments: JLab, CERN, ...

Simplest cases:
Photo-production of a Pion

Simple model reproduces data

correct identification of relevant d.o.f.

Can the model makes predictions?

VM, Szczepaniak and Fox PRD92 (2015)
Photo-production of a Pion

Blue lines = model from VM, Szczepaniak and Fox PRD92 (2015)

Red points = new data from CLAS CLAS and VM, in preparation

How to share material with experimentalists:

Interactive webpage:  http://www.indiana.edu/~jpac/index.html
Resources

- **Publication:** [Mat15a]
- **Fortran:** Fortran file, Input file, Output file
- **C/C++:** AmpTools class, C/C++ file, AmpTools class header
- **Mathematica:** notebook, converted in text
- **Data:** Anderson, All data
- **Contact person:** Vincent Mathieu
- **Last update:** November 2015

Description of the Fortran code: [show/hide]
Description of the C/C++ code: [show/hide]

Interactive webpage:  http://www.indiana.edu/~jpac/index.html
Experimental identification of exotic matter is difficult begin with simple case

Exotic matter is produced diffractively

Simplest cases:

Experiments: JLab, CERN,…

Resonances properties
\[ \pi^- p \rightarrow \pi^0 n \]

**Low energy:** baryon resonances

**High energy:** Regge exchange

**Total cross section**
\[ \pi^- p \rightarrow \pi^0 n \]

**Low energy:** baryon resonances

**High energy:** Regge exchange

**Total cross section**

**Amplitude**
Dispersion Relation

Decompose into scalar function

\[ T = \bar{u}(p_4, \lambda_4) \left( A + \frac{1}{2} (p_1 + p_3) B \right) u(p_2, \lambda_2) \]

\[ s_0 = (M + \mu)^2 \]

\[ t \text{ fixed} \]
**Dispersion Relation**

Decompose into scalar function

\[
T = \bar{u}(p_4, \lambda_4) \left( A + \frac{1}{2} (p_1 + p_3) B \right) u(p_2, \lambda_2)
\]

Introduce the crossing variable

\[
\nu = \frac{s - u}{2}
\]

\[
u(s, t) = -s - t + 2M^2 + 2\mu^2
\]

and symmetrize the two cuts

**Conjugation charge relate**

\[
\pi p \quad \text{and} \quad \pi \bar{p} \quad \text{scatterings}
\]

\[
M^2
\]

\[
s_0 = (M + \mu)^2
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Dispersion Relation

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Conjugation charge relate \( \pi \rho \) and \( \pi \bar{\rho} \) scatterings
Finite Energy Sum Rules

Cauchy contour

\[
\oint_{C} A(\nu, t) d\nu = 0
\]
Finite Energy Sum Rules

Cauchy contour

\[ \oint_C A(\nu, t) d\nu = 0 \]

\[ 2i \int_{\nu_0}^{\Lambda} \text{Im} \ A(\nu, t) d\nu = -\oint_{C_\Lambda} A(\nu, t) d\nu \]
Finite Energy Sum Rules

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\[ 2i \int_{\nu_0}^{\Lambda} \text{Im} \ A(\nu, t) \, d\nu = - \oint_{C_{\Lambda}} A(\nu, t) \, d\nu \]

Assume Regge form at \( \nu = \Lambda \):

\[ A(\nu, t) = \beta(t) \frac{\pm 1 - e^{-i\pi \alpha(t)}}{\sin \pi \alpha(t)} \nu^{\alpha(t)} \]
Finite Energy Sum Rules

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\[ \int_{\nu_0}^{\Lambda} \text{Im} \ A(\nu, t) d\nu = \beta(t) \frac{\Lambda^{\alpha(t)+1}}{\alpha(t) + 1} \]

\( t \) fixed
Finite Energy Sum Rules

Cauchy contour

\[
\oint_C A(\nu, t) \nu^k d\nu = 0
\]

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2i \int_{\nu_0}^{\Lambda} \text{Im } A(\nu, t) d\nu = -\oint_{C_\Lambda} A(\nu, t) d\nu
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\]

\[
\frac{1}{\Lambda^k} \int_{\nu_0}^{\Lambda} \text{Im } A(\nu, t) \nu^k d\nu = \frac{\beta(t) \Lambda^{\alpha(t)+1}}{\alpha(t) + k + 1}
\]

t fixed
Application to $\pi N$ : High Energy Fit

Fit to the world data on

$$\pi^\pm p \rightarrow \pi^\pm p$$
$$\pi^- p \rightarrow \pi^0 n$$

for beam energy $> 2$ GeV

Total cross section

Differential cross section

Polarization observable
Let’s compare both side of the sum rule

Solid line: SAID

Dashed line: Regge

excellent match !

\[
\frac{1}{\Lambda^k} \int_{\nu_0}^{\Lambda} \text{Im } A(\nu, t) \nu^k d\nu = \frac{\beta(t)\Lambda^{\alpha(t)+1}}{\alpha(t) + k + 1}
\]
Checking Analyticity

Match low energy (PW) and high energy (Regge) imaginary parts

Partial waves → Regge poles → Plab

2-3 GeV

Reconstruct the real part from the dispersion relation

\[ A(\nu, t) = \frac{2}{\pi} \int_{\nu_0}^{\infty} \frac{\text{Im} \, A(\nu', t)}{\nu'^2 - \nu^2} \nu' \, d\nu' \]

VM et al (JPAC) PRD92 arXiv:1506.01764
Checking Analyticity

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VM et al (JPAC) PRD92 arXiv:1506.01764
Similar results for the other amplitude

\[ T = \bar{u}(p_4, \lambda_4) \left( A + \frac{1}{2} (p_1 + p_3) B \right) u(p_2, \lambda_2) \]
Application to Pseudoscalar Photoproduction

\[ \gamma N \rightarrow \pi N \]

\[ (\pm 1) \left( \pm \frac{1}{2} \right) \rightarrow 0 \left( \pm \frac{1}{2} \right) \]

8 helicity configurations related by pair via parity

4 indep. helicity configurations

use CGLN basis \( A_1, \ldots, A_4 \)

Isospin symmetry:

every amplitude has an isospin index (+, -, 0)

\[ A^a_{ji} = A^{(+)} \delta^a \delta_{ji} + A^{(-)} \frac{1}{2} [\tau^a, \tau^3]_{ji} + A^{(0)} \tau^a_{ji} \]

12 indep. helicity/isospin configurations

\[ \gamma p \rightarrow \pi^+ n : \sqrt{2} \left( A^{(0)} + A^{(-)} \right) \]

\[ \gamma n \rightarrow \pi^- p : \sqrt{2} \left( A^{(0)} - A^{(-)} \right) \]

\[ \gamma p \rightarrow \pi^0 p : A^{(+)} + A^{(0)} \]

\[ \gamma n \rightarrow \pi^0 n : A^{(+)} - A^{(0)} \]
Application to Pseudoscalar Photoproduction

\[ \gamma N \rightarrow \pi N \]
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12 indep. helicity/isospin configurations

<table>
<thead>
<tr>
<th>(A_i)</th>
<th>(0)</th>
<th>(+)</th>
<th>(-)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(A_1)</td>
<td>(\rho)</td>
<td>(\omega)</td>
<td>(a_2)</td>
</tr>
<tr>
<td>(A_2)</td>
<td>(b)</td>
<td>(h)</td>
<td>(\pi)</td>
</tr>
<tr>
<td>(A_3)</td>
<td>(\rho_2)</td>
<td>(\omega_2)</td>
<td>(a_1)</td>
</tr>
<tr>
<td>(A_4)</td>
<td>(\rho)</td>
<td>(\omega)</td>
<td>(a_2)</td>
</tr>
</tbody>
</table>

8 helicity configurations related by pair via parity

4 indep. helicity configurations

Use CGLN basis \(A_1, \ldots, A_4\)
Use CGLN basis $A_{1,2,3,4}$

Agreed on dominant A4

Variation on subdominant A1

$\frac{1}{A^k} \int_{\nu_0}^{A} \text{Im} \ A(\nu, t) \nu^k d\nu = \frac{\beta(t) A^{\alpha(t)+1}}{\alpha(t) + k + 1}$

$\gamma p \rightarrow \pi^0 p$

VM et al (JPAC), in preparation
Use CGLN basis $A_{1,2,3,4}$

Agreed on dominant A4

Variation on subdominant A1

\[
\frac{1}{k!} \int_{\nu_0}^{\Lambda} \text{Im} \ A(\nu, t) \nu^k d\nu = \frac{\beta(t) \Lambda^{\alpha(t)+1}}{\alpha(t) + k + 1}
\]

Use SAID to fit FESR and observable

VM et al (JPAC), in preparation
Vector Photoproduction

\[ \gamma \rightarrow \omega, \rho, \phi \]

\[ p \rightarrow p' \]
Vector Photoproduction

Virtual Compton Scattering
Vector Photoproduction

Virtual Compton Scattering

FESR for Diffractive Prod.
Vector Photoproduction

Virtual Compton Scattering

FESR for Diffractive Prod.

S-Matrix Constraints on the Lattice
Thank You !
Backup Slides
\[ \pi N \rightarrow \pi N \]
\[ \pi N \rightarrow \pi N \]
\[ \gamma N \rightarrow \eta N \]
\[ \gamma N \rightarrow \pi N \]
\( \gamma N \rightarrow \eta N \)
$K^- p \rightarrow K^- p$  \textbf{Energy Evolution}

C. Fernandez-Ramirez et al. (JPAC) ArXiv:1510:07065

VM (unpublished)

\begin{equation}
\begin{aligned}
L_{\text{max}} &= \sum_{\ell=0}^{a} \lambda \ell \Rightarrow \\
\end{aligned}
\end{equation}

\begin{equation}
\begin{aligned}
+ O \left( \frac{1}{\sqrt{s}} \right)
\end{aligned}
\end{equation}

\textbf{Partial wave expansion}

\textbf{Regge pole expansion}
Discovering (?) New Resonances: Eta(‘)-Pi @COMPASS

\[ \pi^{-} \rightarrow \eta, \eta' \]

\[ \eta' \rightarrow \pi^{-} \]

\[ p \rightarrow \pi^{-}, \pi^{-} \]

\[ m(\eta' \pi^{-}) \text{ [GeV/c}^2\text{]} \]

\[ m(\pi \eta') \text{ [GeV/c}^2\text{]} \]

\[ m(\pi \eta) \text{ [GeV/c}^2\text{]} \]

\[ \pi_1(1600)? \]

\[ L = 1 \]

\[ \alpha_2(1320) \]

\[ L = 2 \]

\[ \alpha_4(2040) \]

\[ L = 4 \]

black: \( \pi \eta' \)

red: \( \pi \eta \) (scaled)

Resonance in angular mom. \( L = 1 \) ?