Glue Spin and Helicity of Proton From Lattice QCD

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The decomposition of the proton spin

Quark spin/helicity: the integration of the quark helicity distribution

\[
\Delta q = \int_0^1 dx \Delta q(x) = \int_0^1 dx \int \frac{d\xi^-}{2\pi} e^{-ixP^+\xi^-} \langle PS|\psi_q(\xi^-)\gamma_5\gamma^+ L(\xi^-, 0)\psi_q(0)|PS\rangle
\]

Glue helicity: that of the quark helicity distribution

\[
\Delta G = \int_0^1 dx \Delta g(x) = \int_0^1 dx \frac{i}{2xP^+} \int \frac{d\xi^-}{2\pi} e^{-ixP^+\xi^-} \langle PS|F_{a\alpha}^+(\xi^-)\mathcal{L}^{ab}(\xi^-, 0)\tilde{F}_{a\alpha,b}^+(0)|PS\rangle
\]

The rest parts should be the orbital angular momentums,

\[
L_q + L_g = \frac{1}{2} - \frac{1}{2} \sum_{q=u,\bar{u},d,\bar{d},s,..} \Delta q - \Delta G
\]
Proton Spin decomposition

From the experiments

Longitudinal proton spin structure

$\Delta \Sigma (Q^2=10 \text{ GeV}^2) = 0.242$, de Florian et al., 2009

$\Delta G (Q^2=10 \text{ GeV}^2) \sim 0.2$, de Florian et al., 2014

Naïve spin sum rule:

$\frac{1}{2} = \frac{1}{2} \Delta \Sigma + \Delta G + l^z_q + l^z_q$
The global fit based on recent experimental data (2009 RHIC) shows evidence of nonzero polarization of gluon in the proton.

non-zero for x>0.05, but strongly scale dependent

Florian et. al, Phys.Rev.Lett. 113 (2014) no.1, 012001
Glue helicity

$\Delta g$ in the scaling violation of $\Delta q$


\[
\Delta f_k(x) = \eta_k \frac{\int_0^1 x^{\alpha_k} (1-x)^{\beta_k} (1+\gamma_k x) \, dx}{\int_0^1 x^{\alpha_k} (1-x)^{\beta_k} (1+\gamma_k x) \, dx}
\]

$\Delta f_k(x)$ ($k = 1\ldots4$) represents $\Delta q^S(x)$, $\Delta q_3(x)$, $\Delta q_8(x)$ and $\Delta g(x)$ and $\eta_k$ is the first moment of $\Delta f_k(x)$ at the reference scale $Q^2$.

For $\gamma_g = \gamma_S = 0$ ($\gamma_g = 0$ and $\gamma_S \neq 0$), a negative (positive) solution for $\Delta g(x)$ is obtained.
Glue helicity

The glue helicity is defined as,

$$\Delta G = \int_0^1 dx \Delta g(x) = \int_0^1 dx \frac{i}{2xP^+} \int \frac{d\xi^-}{2\pi} e^{-ixP^+\xi^-} \langle PS|F_{a+}^+(\xi^-)\mathcal{L}^{ab}(\xi^-, 0)\tilde{F}_{a,b}^+(0)|PS\rangle$$

and the glue helicity operator itself can be rewritten into,

$$\tilde{S}_g = \int dx \frac{i}{2xP^+} \int \frac{d\xi^-}{2\pi} e^{-ixP^+\xi^-} F_{a+}^+(\xi^-)\mathcal{L}^{ab}(\xi^-, 0)\tilde{F}_{a,b}^+(0)$$

$$= \int dx \frac{i}{2xP^+} \int \frac{d\xi^-}{2\pi} e^{-ixP^+\xi^-} 2\text{Tr}[F_{a+}^+(\xi^-)\mathcal{L}(\xi^-, 0)\tilde{F}_{a,b}^+(0)\mathcal{L}(0, \xi^-)]$$

$$= 2\text{Tr}[\tilde{E}(0) \times (\tilde{A}(0) - \int \frac{d\xi^-}{2\pi} \int dx \frac{e^{-ixP^+\xi^-}}{2xP^+} \mathcal{L}(0, \xi^-)\nabla A^+(\xi^-)\mathcal{L}(\xi^-, 0))]$$

$$\xrightarrow{\mathcal{A}^+ = 0} 2\text{Tr}[\tilde{E}(0) \times \tilde{A}(0)] = \tilde{E}^a(0) \times \tilde{A}^a(0)$$


Y. Hatta, Phys. Rev. D84, 041701 (2011),

But it can not be calculated on the lattice directly.
Outline

- The **large momentum effective theory (LaMET)** of the glue spin/helicity.

- The lattice calculation of the glue spin under **the coulomb gauge**, and **the glue helicity prediction** based on that with the **two-step matchings** from lattice to light-cone.

- **Discussions**
Find a finite frame operator $O_f$ which becomes the light cone operator $O$ in the large momentum limit.

In the large momentum limit, the IR behaviors of $O_f$ and $O$ should be the same while the UV one can be different.

$O = O^{\text{tree}} \left\{ 1 + \frac{\alpha_s}{4\pi} \left[ A \frac{1}{\epsilon'} + A\log \frac{\mu^2}{p^2} + B \right] + O(\alpha_s^2) \right\}$

$O_f = O_f^{\text{tree}} \left\{ 1 + \frac{\alpha_s}{4\pi} \left[ A_f \frac{1}{\epsilon'} + A_f \log \frac{\mu^2}{p^2} + B_f + (A - A_f) \log \frac{\vec{p}^2}{p^2} \right] + O(\alpha_s^2) \right\}$
The classical level

Find a finite frame operator $O_f$ which becomes the light cone operator $O$ in the large momentum limit.

In the large momentum limit, the **IR behaviors** of $O_f$ and $O$ should be the same while the **UV one** can be different.

\[
O = O_{\text{tree}} \{ 1 + \frac{\alpha_s}{4\pi} [A \frac{1}{\epsilon} + A \log \frac{\mu^2}{p^2} + B] + O(\alpha_s^2) \}
\]

\[
O_f = O_{f\text{tree}} \{ 1 + \frac{\alpha_s}{4\pi} [A_f \frac{1}{\epsilon} + A_f \log \frac{\mu^2}{p^2} + B_f + (A - A_f) \log \frac{p^2}{p^2}] + O(\alpha_s^2) \}
\]

Do the matching to fix the difference:

\[
O_f = O \{ 1 - \frac{\alpha_s}{4\pi} [(A - A_f)(\frac{1}{\epsilon} + A_f \log \frac{\mu^2}{p^2} + B - B_f)] + O(\alpha_s^2) \}
\]
LaMET

The available gauge conditions

- The **glue helicity** operator equivalent to $\text{ExA}$ under the light cone gauge.

$$O_{\Delta_G} = \left[ \vec{E}^a(0) \times (\vec{A}^a(0) - \frac{1}{\nabla^+} (\nabla A^{+,b}) L^{b,a}(\xi^-,0)) \right]^z = \vec{E}_{LC} \times \vec{A}_{LC}, \ A_{LC}^+ = 0$$

- We have to use the Lorentz in-covariant **glue spin (quasi-glue helicity)** operators to reach it in the large momentum limit:

$$O_{S_G^c} = \vec{E}^c \times \vec{A}^c, \ \partial_i A^c_i = 0 \quad \text{or} \quad O_{S_G^a} = \vec{E}^a \times \vec{A}^a, \ A^a_z = 0 \quad \text{or} \quad O_{S_G^t} = \vec{E}^t \times \vec{A}^t, \ A^t_0 = 0$$

**Coulomb gauge** \quad **Axial gauge** \quad **Temporal gauge**

or something else…

The glue spin

targets to the Light-cone glue helicity

- Glue spin under **Coulomb gauge** $S_G^c(|\vec{p}|, \mu)$
  - Lattice simulation is *straight forward*;

- Glue spin under **Temporal gauge** $S_G^t(|\vec{p}|, \mu)$
  - Lattice simulation is tricky (will be addressed latter),

+ proper matching to cancel the intrinsic frame dependence
to reach glue helicity $\Delta G(\mu)$.

\[
S_G^c(|\vec{p}|, \mu) = \left[ 1 + \frac{g^2 C_A}{16\pi^2} \left( \frac{7}{3} \log \left( \frac{\vec{p}^2}{\mu^2} \right) - 10.2098 \right) \right] \Delta G(\mu) + \frac{g^2 C_F}{16\pi^2} \left( \frac{4}{3} \log \left( \frac{\vec{p}^2}{\mu^2} \right) - 5.2627 \right) \Delta \Sigma(\mu) + O(g^4) + O\left( \frac{1}{(\vec{p})^2} \right)
\]

\[
S_G^t(|\vec{p}|, \mu) = \Delta G(\mu) + O(g^4) + O\left( \frac{1}{(\vec{p})^2} \right)
\]

**References**

From the **glue spin** on the lattice to the **glue helicity** in LC

1. Calculate the longitudinal glue spin matrix element in the moving proton on the lattice, $S_G(|\vec{p}|^2, 1/a^2)$.

2. Match the above matrix element to that under $\overline{\text{MS}}$ at $\mu^2=10$ GeV$^2$, $S_G^{\overline{\text{MS}}}(|\vec{p}|^2, \mu^2) = Z_{GG}(\mu^2 a^2) S_G(|\vec{p}|^2, 1/a^2) + Z_{QG}(\mu^2 a^2) \Delta \Sigma$.

3. Match again from the finite frame to IMF with LaMET, $\Delta G^{\overline{\text{MS}}}(\mu^2) = \tilde{Z}_{GG}(|\vec{p}|^2/\mu^2) S_G^{\overline{\text{MS}}}(|\vec{p}|^2, \mu^2) + \tilde{Z}_{QG}(|\vec{p}|^2/\mu^2) \Delta \Sigma$. 

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Converting the UV scale from 1/a to $\mu$

Converting the frame dependence to the scale $\mu$ dependence
The larger volume also allow us to simulate the quark corresponding to lighter pion mass correctly.

\begin{equation}
C(m_\pi, m_{\pi,\text{sea}}, a, L) = \\
C_{\text{phys}} + C_1(m_\pi^2 - m_{\pi,\text{phys}}^2) + C_2(m_{\pi,\text{sea}}^2 - m_{\pi,\text{phys}}^2) + C_3a^2 + C_4e^{-m_\pi L} + O(m_\pi^3, m_{\pi,\text{sea}}^3, a^4)...
\end{equation}
Glue Spin operator on the lattice

\[ \vec{S}_g = 2 \int d^3 x \ Tr(\vec{E}_c \times \vec{A}_c) \]

The gauge potential \( A_c \) defined by

\[ A_{c,\mu} = \left[ \frac{U_{\mu}^c(x) - U_{\mu}^c(x) + U_{\mu}^c(x - a\hat{\mu}) - U_{\mu}^c(x - a\hat{\mu})}{4i\text{diag}} \right]_{\text{traceless}} \]

with the discrete coulomb gauge fixing condition

\[ \sum_{\mu = x, y, z} \left[ U_{\mu}^c(x) - U_{\mu}^c(x - a\hat{\mu}) \right] = 0 \]

And the electric field \( E_i = F_{0i} \) is defined by the averaged plaquette,

\[ F_{\mu\nu}^c = \frac{i}{8a^2 g} \left( \mathcal{P}_{\mu,\nu} - \mathcal{P}_{\nu,\mu} + \mathcal{P}_{\nu,-\mu} - \mathcal{P}_{-\mu,\nu} ight) \]

\[ + \mathcal{P}_{-\mu,-\nu} - \mathcal{P}_{-\nu,-\mu} + \mathcal{P}_{-\nu,\mu} - \mathcal{P}_{\mu,-\nu} \]

\[ \mathcal{P}_{\mu,\nu} = U_{\mu}^c(x) U_{\nu}^c(x + a\hat{\mu}) U_{\mu}^{c\dagger}(x + a\hat{\nu}) U_{\nu}^{c\dagger}(x). \]
Glue Spin

example of the bare result

\[ R(t_f, t) = \frac{\langle 0 | \Gamma_m^m \int d^3y \, e^{-ip_3 y_3} \chi(\vec{y}, t_f) S_g^3(t) \bar{\chi}(\vec{0}, 0) | 0 \rangle}{\langle 0 | \Gamma_e \int d^3y \, e^{-ip_3 y_3} \chi(\vec{y}, t_f) \bar{\chi}(\vec{0}, 0) | 0 \rangle} \]

\[ = S_G + C_1 e^{-\Delta E(t_f-t)} + C_2 e^{-\Delta E \tau} + |C_3 e^{-\Delta E \tau f}|, \]

24I, unitary point.

From the **glue spin** on the lattice to the **glue helicity** in LC

1. Calculate the longitudinal glue spin matrix element in the moving proton on the lattice, $S_G(|\vec{p}|^2, 1/a^2)$.

2. Match the above matrix element to that under $\overline{\text{MS}}$ at $\mu^2=10$ GeV$^2$, $S_G^{\overline{\text{MS}}}(|\vec{p}|^2, \mu^2) = Z_{GG}(\mu^2 a^2) S_G(|\vec{p}|^2, 1/a^2) + Z_{QG}(\mu^2 a^2) \Delta \Sigma$,.

3. Match again from the finite frame to IMF with LaMET, $\Delta G^{\overline{\text{MS}}} (\mu^2) = \tilde{Z}_{GG}(|\vec{p}|^2/\mu^2) S_G^{\overline{\text{MS}}}(|\vec{p}|^2, \mu^2) + \tilde{Z}_{QG}(|\vec{p}|^2/\mu^2) \Delta \Sigma$. 

Converting the UV scale from 1/a to $\mu$

Converting the frame dependence to the scale $\mu$ dependence
The renormalization of a lattice bare quantity

The renormalization of the coulomb gluon self energy with the lattice regularization and under RI-MOM scheme is,

\[ Z_L^{\text{MOM}} = 1 + \frac{g^2}{16\pi^2} \left[ \left( \frac{2N_f}{3} - N_c \right) \log(a^2p^2) + N_f B_G^f - N_c B_G^L(z) \right] + O(g^4), \]

where \( B_Q^f \) and \( B_Q^L(z \equiv \frac{p^2}{p^2}) \) are the finite pieces which are sensitive to the lattice quark and gluon actions.

The continuum field renormalization with the dimensional regularization and under RI-MOM and \( \overline{MS} \) scheme is,

\[ Z_D^{\overline{MS}} = 1 - \frac{g^2}{16\pi^2} \left[ \left( \frac{2N_f}{3} - N_c \right) \frac{1}{\varepsilon} \right] + O(g^4), \]
\[ Z_D^{\text{MOM}} = 1 - \frac{g^2 N_f}{16\pi^2} \left[ \left( \frac{2N_f}{3} - N_c \right) \left( \frac{1}{\varepsilon} + \log(\mu^2/p^2) \right) + \frac{10N_f}{9} - N_c B_G^D(z) \right] + O(g^4). \]

So the final renormalization under \( \overline{MS} \) scheme for the lattice quantity is,

\[ Z_L^{\overline{MS}}(a,\mu) = \frac{Z_D^{\overline{MS}}}{Z_D^{\text{MOM}}} Z_L^{\text{MOM}}(a,\mu) = 1 + \frac{g^2 N_f}{16\pi^2} \left[ \left( \frac{2N_f}{3} - N_c \right) \log(\mu^2a^2) + N_f \left( \frac{10}{9} + B_G^f \right) - N_c (B_G^D(z) + B_G^L(z)) \right] + O(g^4), \]

where \( B_G^D(z) + B_G^L(z) \) should be independent to \( z \), and then the renormalization is independent to the momentum on the external legs.
The frame dependence of the finite pieces

\[ S_{G,(1)}^{MS} = \left\{ 1 - \frac{g^2}{16\pi^2} \left\{ N_f \left[ \frac{2}{3} \log(\mu^2 a^2) + B_{GG,f}^{DR} - B_{GG,f}^{LR} \right] \right. \right. \\
\left. \left. - C_A \left[ \frac{4}{3} \log(\mu^2 a^2) + B_{GG}^{DR} - B_{GG}^{LR} \right. \right. \right. \\
\left. \left. + B_{g}^{DR} - B_{g}^{LR} \right] \right\} S_{G,(1)}^{LR} \right. \\
+ \frac{g^2 C_F}{16\pi^2} \left\{ \frac{5}{3} \log(\mu^2 a^2) + B_{QG}^{DR} - B_{QG}^{LR} \right\} \sum_{q=u,d,s...} \Delta_q^{L,(1)} \\
+ O(g^4), \]

The IR dependence in the renormalization are fully canceled!

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<td>-16.919</td>
<td>-16.919</td>
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</table>
Glue spin

The renormalization and mixing

\[ S_{G,1}^{MS} = \left(1 - \frac{g^2 N_f}{16\pi^2} \left[ \frac{2}{3} \log(\mu^2 a^2) + 2.41 \right] + \frac{g^2 C_A}{16\pi^2} \left[ -\frac{4}{3} \log(\mu^2 a^2) + \sim 2 \right] \right) S_{G,1}^{Lat} \]

\[ + \frac{g^2 C_F}{16\pi^2} \left[ \frac{5}{3} \log(\mu^2 a^2) + 6.99 \right] \sum_{q=u,d,s,...} \Delta_q^{L,(1)} + O(a^2 p^2) + O(g^4) \]

The scale used by the experiment for the glue helicity is \( \mu^2 = 10 \text{ GeV}^2 \)

The overlap fermion and Iwasaki gluon with HYP smearing

\[ a^{-1} (\text{GeV}), \ \alpha_{s}^{\text{Lat}}(a^{-1}) \sim \alpha_{s}^{MS}(2a^{-1}) \]
The dependence of $m_{\pi}$, $a$, and $V$.

In the rest frame, the pion mass (both valence and sea), lattice spacing and volume dependences are mild.

$\mu^2 = 10$ GeV$^2$
From the **glue spin** on the lattice to the **glue helicity** in LC

1. Calculate the longitudinal glue spin matrix element in the moving proton on the lattice, $S_G(|\vec{p}|^2, 1/a^2)$.

   Converting the UV scale from $1/a$ to $\mu$

2. Match the above matrix element to that under $\overline{\text{MS}}$ at $\mu^2 = 10 \text{ GeV}^2$, $S_G^{\overline{\text{MS}}}(|\vec{p}|^2, \mu^2) = Z_{GG}(\mu^2a^2)S_G(|\vec{p}|^2, 1/a^2) + Z_{QG}(\mu^2a^2)\Delta\Sigma$.

   Converting the frame dependence to the scale $\mu$ dependence

3. Match again from the finite frame to IMF with LaMET, $\Delta G^{\overline{\text{MS}}}(\mu^2) = \tilde{Z}_{GG}(|\vec{p}|^2/\mu^2)S_G^{\overline{\text{MS}}}(|\vec{p}|^2, \mu^2) + \tilde{Z}_{QG}(|\vec{p}|^2/\mu^2)\Delta\Sigma$. 
From glue spin to helicity

with Large-momentum effective field theory

- The large finite pieces indicates a convergence problem
- Large frame dependence need re-summation.


\[
S_G(|\vec{p}|, \mu) = \left[ 1 + \frac{g^2 C_A}{16\pi^2} \left( \frac{7}{3} \log \frac{(\vec{p})^2}{\mu^2} - 10.2098 \right) \right] \Delta G(\mu) + \frac{g^2 C_F}{16\pi^2} \left( \frac{4}{3} \log \frac{(\vec{p})^2}{\mu^2} - 5.2627 \right) \Delta \Sigma(\mu) + O(g^4) + O\left(\frac{1}{(\vec{p})^2}\right).
\]

At \( \mu^2 = 10 \text{ GeV}^2 \) and \( |\vec{p}| = 1.5 \text{ GeV} \),

the factor before \( \Delta G \) is 0.22,

It means that \( \Delta G \) will be \( \sim 3 \)

if we apply this matching.
Glue spin

The glue spin at the large momentum limit for the renormalized value at $\mu^2=10$GeV$^2$:

$$S_G = 0.251(47)(16)$$

We neglect the matching and use the following empirical form to fit our data,

$$S_G(|\vec{p}|) = S_G(\infty) + \frac{C_1}{M^2 + |\vec{p}|^2} + C_2(m_{\pi,vv}^2 - m_{\pi,phys}^2)$$
$$+ C_3(m_{\pi,ss}^2 - m_{\pi,phys}^2) + C_4a^2$$

$$m_{\pi,phys} = 0.139 \text{ GeV} \quad M = 0.939 \text{ GeV}$$

Glue spin

The final result

Comparable with the present global analysis while have much better signal

The glue spin at the large momentum limit for the renormalized value at $\mu^2=10\text{GeV}^2$:

$$S_G=0.251(47)(16)$$


Florian et. al, Phys.Rev.Lett. 113 (2014) no.1, 012001
Glue spin

HIGHLIGHTED ARTICLES

Glue Spin and Helicity in the Proton from Lattice QCD
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\(\chi\)QCD Collaboration

Physics Viewpoint: Spinning Gluons in the Proton

Computer simulations indicate that about 50% of the proton’s spin comes from the spin of the gluons that bind its quark constituents.

“…sure to attract plenty of attention from the wider physics community.”

“…shed light on the composition of the nucleon.”

—from referees

“looks forward to having full calculations of the complete quark and gluon spin and orbital contributions to the proton’s spin”

—from Physics Viewpoint
The loopholes in the two-step matching

1. Calculate the longitudinal glue spin matrix element in the moving proton on the lattice, \( S_G(|\vec{p}|^2, 1/a^2) \).

Q1: Can we do the **non-perturbative renormalization**?

2. Match the above matrix element to that under \( \overline{\text{MS}} \) at \( \mu^2 = 10 \text{ GeV}^2 \),
\[
S_G^{\overline{\text{MS}}}(|\vec{p}|^2, \mu^2) = Z_{GG}(\mu^2 a^2) S_G(|\vec{p}|^2, 1/a^2) + Z_{QG}(\mu^2 a^2) \Delta \Sigma,
\]

Q2. Can we **avoid this matching** by using other quasi-glue helicity?

3. Match again from the finite frame to IMF with LaMET,
\[
\Delta G^{\overline{\text{MS}}}(|\vec{p}|^2) = \tilde{Z}_{GG}(|\vec{p}|^2 / \mu^2) S_G^{\overline{\text{MS}}}(|\vec{p}|^2, \mu^2) + \tilde{Z}_{QG}(|\vec{p}|^2 / \mu^2) \Delta \Sigma.
\]

Q3. Are the final results independent to the gauge we used?
Q1: Can we do the non-perturbative renormalization?

- Only the non-perturbative renormalization of the 2-quark/4-quark operators are developed so far.

- The equal-time propagator is investigated for the coulomb gauge on the lattice, but not the standard one.

- The quasi-temporal gauge propagator can be obtained on the lattice by applying the coulomb gauge fixing on the reference time slice:

\[
\vec{\partial} \cdot \vec{A}(\vec{x}, t_0) = 0 \quad \forall (\vec{x}, t_0).
\]

\[
A_0(x) = 0 \quad \forall x
\]

\[
\Omega(x) = \mathcal{P}e^{ig \int_{t_0}^{t} d\tau A_0(\vec{x}, \tau)} = \Omega(\vec{x}, t_0)P(\vec{x}; t_0, t)
\]


Q2: Can we avoid this matching by using other quasi-glue helicity?

Yes! The decomposition extended from the temporal gauge $A_0 = 0$ doesn’t require any large momentum matching at the 1-loop level.


\[
\begin{align*}
\langle P_h | \epsilon^{ij} F^{i+} A^j | P_h \rangle_{g} \bigg|_{A^+ = 0} &= \left[ 1 + \frac{\alpha_s}{4\pi} \left( \frac{\beta_0}{\varepsilon_v} + \frac{103N_c - 10N_f}{9} \right) \right] \langle P_h | \epsilon^{ij} F^{i+} A^j | P_h \rangle_{g}^{\text{tree}} \\
\beta_0 &= \frac{11N_c}{3} - \frac{2N_f}{3}
\end{align*}
\]

\[
\begin{align*}
\langle P_h | \epsilon^{ij} F^{i0} A^j | P_h \rangle_{g} \bigg|_{A^0 = 0} &= \left[ 1 + \frac{\alpha_s}{4\pi} \left( \frac{\beta_0}{\varepsilon_v} + \frac{103N_c - 10N_f}{9} \right) \right] \langle P_h | \epsilon^{ij} F^{i0} A^j | P_h \rangle_{g}^{\text{tree}}
\end{align*}
\]

- No frame dependence in the finite pieces
- Need to be checked whether it is still hold in the off shell scheme and the finite momentum frame
Q3: Are the final results independent to the gauge we used?

It should be independent to whether we use the glue spin under the Coulomb gauge or temporal gauge, up to the higher order corrections. But,

• the convergence of the perturbative series for ExA under the Coulomb gauge can be bad;

• the quasi-temporal gauge is not exactly the same as the temporal gauge, and the matrix element of ExA in proton under the temporal gauge is <0.01 and consistent with zero (based on the simulation).

We may need to go back to the original definition and consider the simulation of the polarized PDF?

\[
\Delta G = \int_0^1 \Delta g(x) dx = \int dx \frac{i}{2xP^+} \int \frac{d\xi^-}{2\pi} e^{-ixP^+} \langle PS| F_a^{+\alpha}(\xi^-) \mathcal{L}^{ab}(\xi^-, 0) \tilde{F}^{+\alpha}_a(0) |PS \rangle
\]
The further challenges:

The orbital angular momentums of the quark and gluon

\[ \vec{J} = \int d^3x \frac{1}{2} \bar{\psi} \vec{\gamma} \gamma^5 \psi + \int d^3x \psi^\dagger \{ \vec{x} \times (i\vec{\nabla}) \} \psi \]

\[ + \int d^3x 2\text{Tr}[\vec{E} \times \vec{A}] + \int d^3x 2\text{Tr}[E^i \vec{x} \times \vec{\nabla} A^i] \]

- The quasi-OAM under Coulomb condition can be calculated on the lattice but the LaEMT matching has the convergence problem as the glue spin case.

- That under temporal/quasi-temporal condition needs further investigation, but the LaEMT matching would be trivial.
Summary

The glue spin under the Coulomb gauge in the MS-bar scheme is obtained with the 1-loop perturbative matching.

• We find the results to be fairly insensitive to the lattice spacing, quark masses and also momentum frame up to 1.5 GeV.

• The glue spin under Coulomb gauge at the large momentum limit is $S_G=0.251(47)(16)$.

• The glue spin and the other AM components under the temporal gauge condition can also be good quasi-quantity on the lattice to reach the light-cone components.