Glue Spin and Helicity of Proton From Lattice QCD

Yi-Bo Yang
Michigan state university

With Raza Sabbir Sufian, Andrei Alexandru, Terrence Draper, Michael J. Glatzmaier, Keh-Fei Liu, and Yong Zhao

Based on Phys. Rev. Lett. 118 (2017) 102001

Apr. 2017

The decomposition

of the proton spin

Quark spin/helicity: the integration of the quark helicity distribution

$$\Delta q = \int_0^1 dx \Delta q(x) = \int_0^1 dx \int \frac{d\xi^-}{2\pi} e^{-ixP^+\xi^-} \langle PS|\psi_q(\xi^-)\gamma_5\gamma^+\mathcal{L}(\xi^-,0)\psi_q(0)|PS\rangle$$

Glue helicity: that of the quark helicity distribution

$$\Delta G = \int_0^1 dx \Delta g(x) = \int_0^1 dx \frac{i}{2xP^+} \int \frac{d\xi^-}{2\pi} e^{-ixP^+\xi^-} \langle PS|F_a^{+\alpha}(\xi^-)\mathcal{L}^{ab}(\xi^-,0)\tilde{F}^+_{\alpha,b}(0)|PS\rangle$$

The rest parts should be the orbital angular momentums,

$$L_q + L_g = \frac{1}{2} - \frac{1}{2}\sum_{q=u,\bar{u},d,\bar{d},s..} \Delta q - \Delta G$$

Proton Spin decomposition

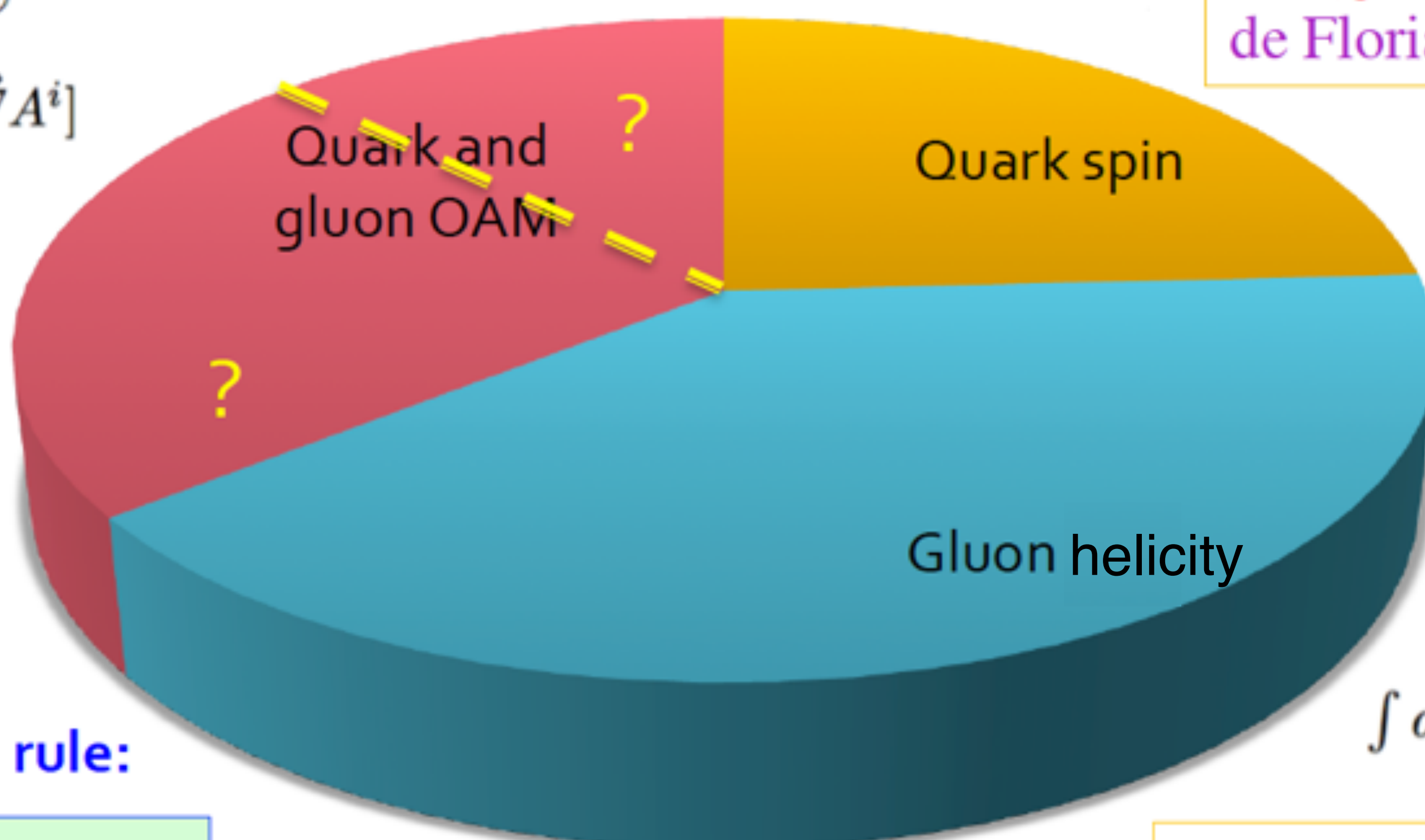
From the experiments

Longitudinal proton spin structure

$$\int d^3x \psi^\dagger \{\vec{x} \times (i\vec{\nabla})\} \psi$$
$$+ \quad \int d^3x 2\mathrm{Tr}[E^i \vec{x} \times \vec{\nabla} A^i]$$

$$\int d^3x \, \tfrac{1}{2} \overline{\psi} \vec{\gamma} \gamma^5 \psi$$

$\Delta\Sigma(Q^2=10$ GeV$^2) = 0.242$,
de Florian et al., 2009



SLAC
HERMES (DESY)
COMPASS (CERN)
JLab
RHIC

$$\int d^3x 2\mathrm{Tr}[\vec{E} \times \vec{A}]$$

Naïve spin sum rule:

$$\boxed{\frac{1}{2} = \frac{1}{2}\Delta\Sigma + \Delta G + l_q^z + l_q^z}$$

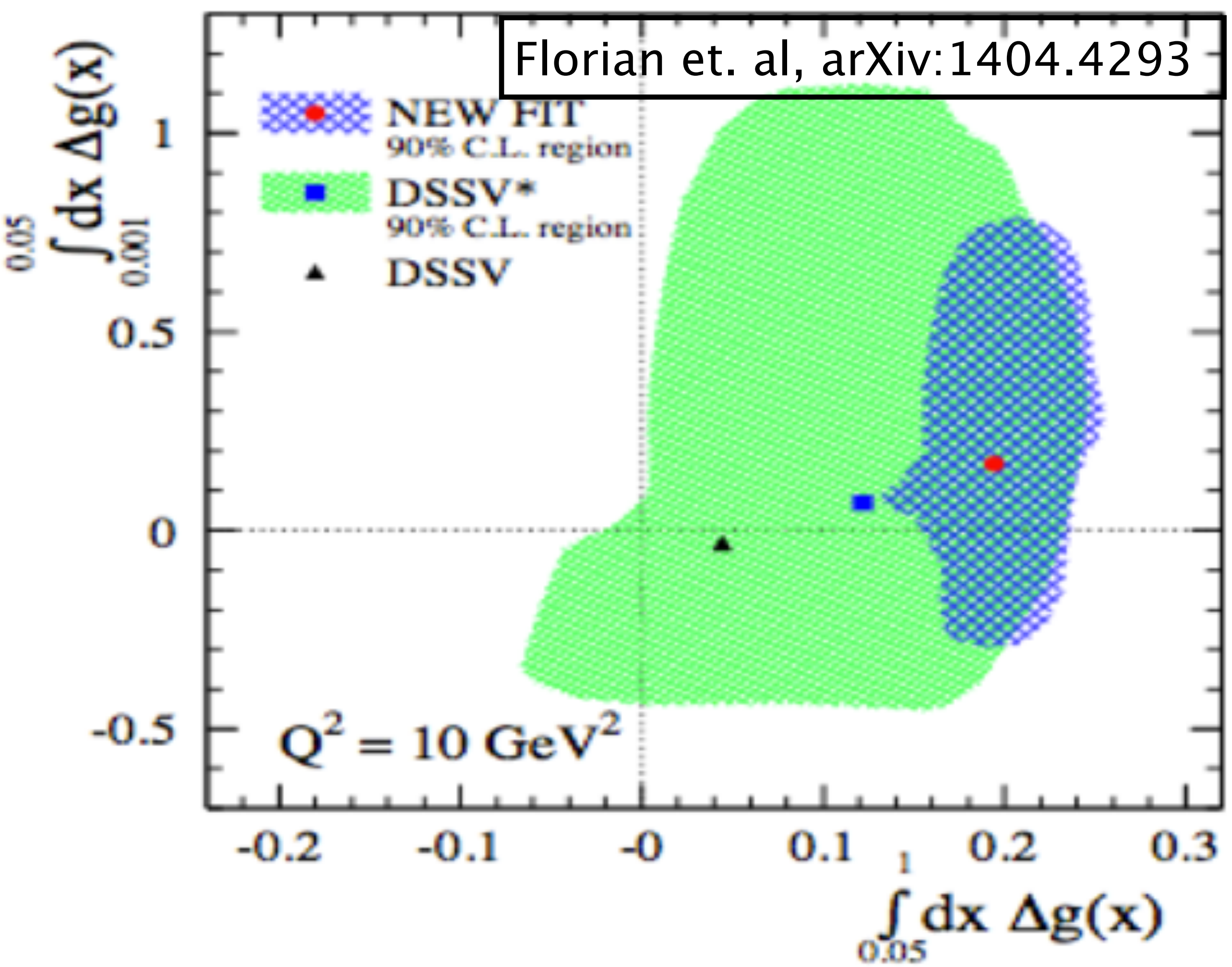
$\Delta G(Q^2=10$ GeV$^2) \sim 0.2$,
de Florian et al., 2014

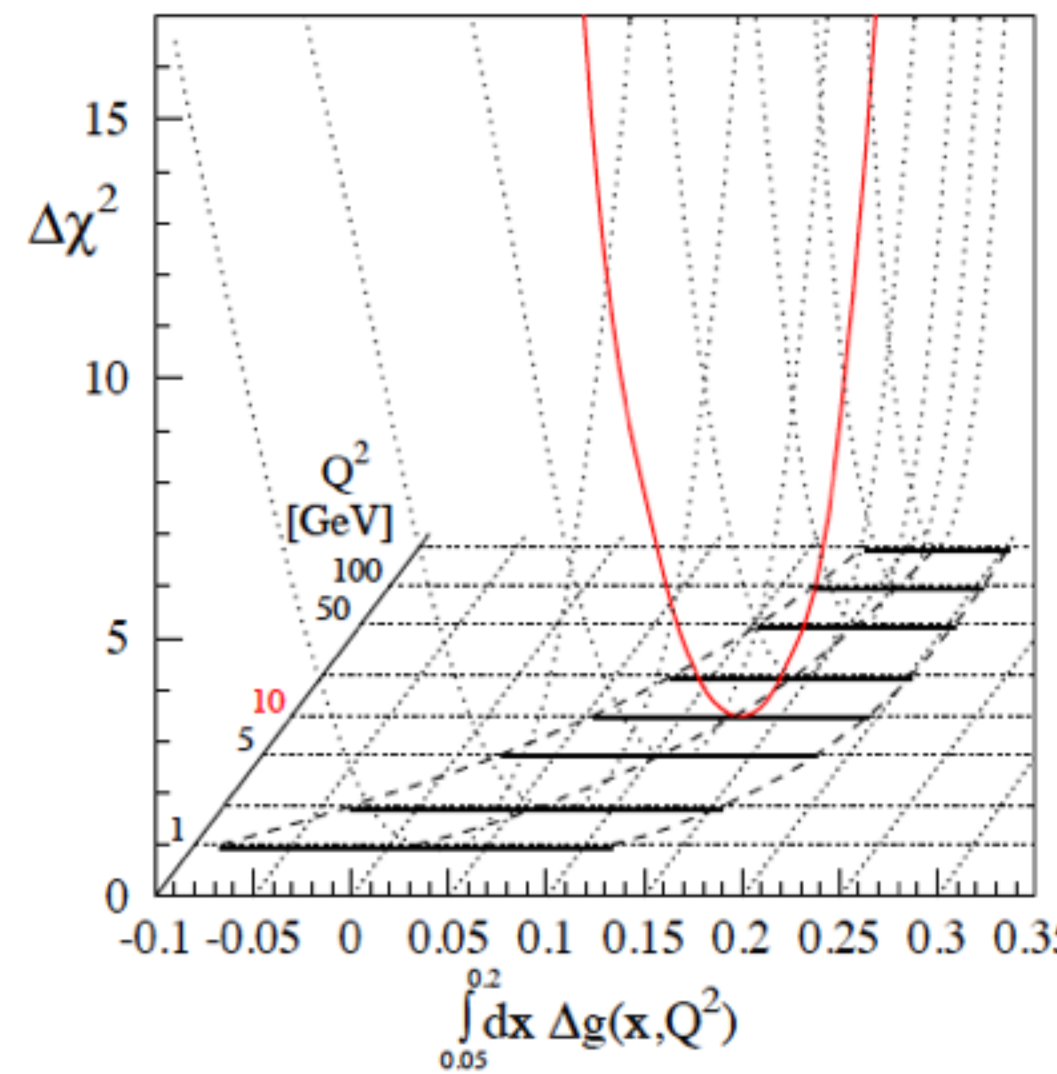
Glue helicity

Florian et. al, Phys.Rev.Lett. 113 (2014) no.1, 012001

First non-zero evidence

The global fit based on recent experimental data (2009 RHIC) shows evidence of nonzero polarization of gluon in the proton.





non-zero for x>0.05, but strongly scale dependent

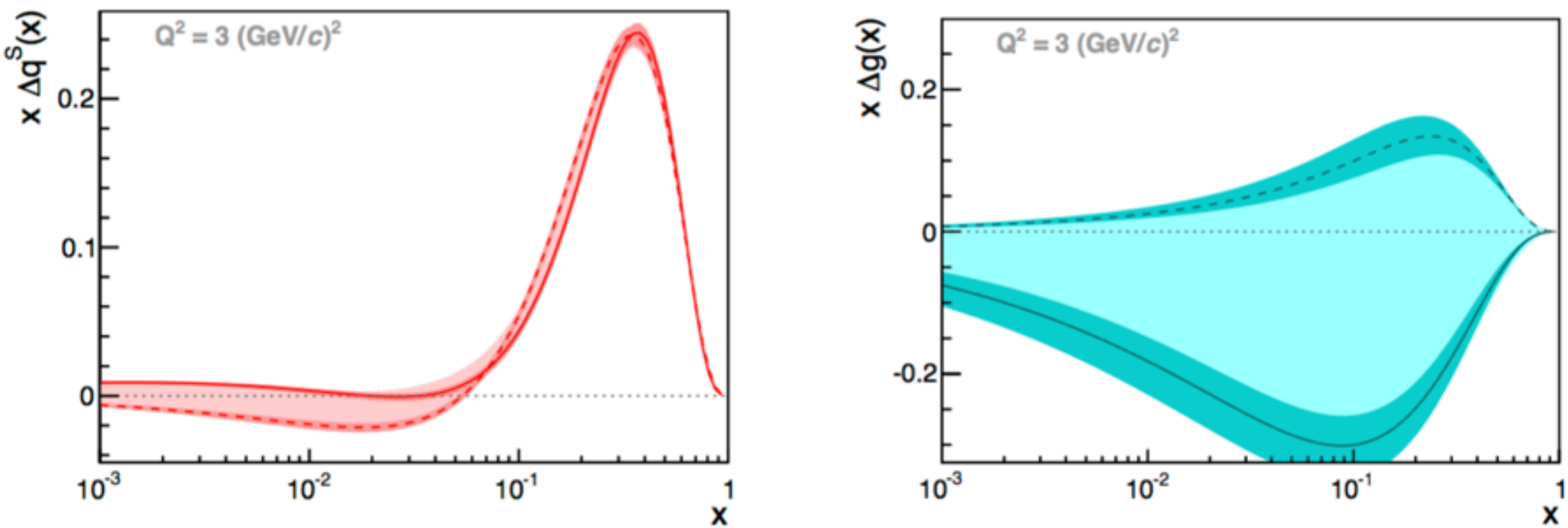
Glue helicity

COMPASS, Phys.Lett. B753 (2016) 18–28

Δg in the scaling violation of Δq

$$\Delta f_k(x) = \eta_k \, \frac{x^{\alpha_k}(1-x)^{\beta_k}(1+\gamma_k x)}{\int_0^1 x^{\alpha_k}(1-x)^{\beta_k}(1+\gamma_k x)\mathrm{d}x}$$

$\Delta f_k(x)$ $(k = 1\ldots4)$ represents $\Delta q^{\mathrm{S}}(x)$, $\Delta q_3(x)$, $\Delta q_8(x)$ and $\Delta g(x)$ and η_k is the first moment of $\Delta f_k(x)$ at the reference scale



For $\gamma_{\mathrm{g}} = \gamma_{\mathrm{S}} = 0$ ($\gamma_{\mathrm{g}} = 0$ and $\gamma_{\mathrm{S}} \neq 0$)

a negative (positive) solution for $\Delta g(x)$ is obtained.

Glue helicity

A possible simplification

The glue helicity is defined as,

A. V. Manohar, Phys. Lett. B255, 579 (1991)

$$\Delta G = \int_0^1 dx \Delta g(x) = \int_0^1 dx \frac{i}{2xP^+} \int \frac{d\xi^-}{2\pi} e^{-ixP^+\xi^-} \langle PS| F_a^{+\alpha}(\xi^-) \mathcal{L}^{ab}(\xi^-, 0) \tilde{F}^+_{\alpha,b}(0) |PS\rangle$$

and the glue helicity operator itself can be rewritten into,

Y. Hatta, Phys. Rev. D84, 041701 (2011),
X. Ji, J.H. Zhang, and Y. Zhao, Phys. Rev. Lett. 111 112002 (2013)

$$\begin{aligned}
\tilde{S}_g &= \int dx \frac{i}{2xP^+} \int \frac{d\xi^-}{2\pi} e^{-ixP^+\xi^-} F_a^{+\alpha}(\xi^-) \mathcal{L}^{ab}(\xi^-, 0) \tilde{F}^+_{\alpha,b}(0) \\
&= \int dx \frac{i}{2xP^+} \int \frac{d\xi^-}{2\pi} e^{-ixP^+\xi^-} 2\mathrm{Tr}[F^{+\alpha}(\xi^-) \mathcal{L}(\xi^-, 0) \tilde{F}^+_{\alpha,b}(0) \mathcal{L}(0, \xi^-)] \\
&= 2\mathrm{Tr}\left[\vec{E}(0) \times (\vec{A}(0) - \int \frac{d\xi^-}{2\pi} \int dx\, i \frac{e^{-ixP^+\xi^-}}{2xP^+} \mathcal{L}(0, \xi^-) \vec{\nabla} A^+(\xi^-) \mathcal{L}(\xi^-, 0)) \right] \\
&\xrightarrow[A^+=0]{} 2\mathrm{Tr}[\vec{E}(0) \times \vec{A}(0)] = \vec{E}^a(0) \times \vec{A}^a(0)
\end{aligned}$$

But it can not be calculated on the lattice directly.

Outline

- The **large momentum effective theory (LaMET)** of the glue spin/helicity.
- The lattice calculation of the glue spin under **the coulomb gauge**, and **the glue helicity prediction** based on that with the **two-step matchings** from lattice to light-cone.
- Discussions

LaMET

X. Ji, J.H. Zhang, and Y. Zhao, Phys. Rev. Lett. 111 112002 (2013)

of the glue spin/helicity

$\vec{E}^c \times \vec{A}^c$

The classical level

- Find a finite frame operator ***Of*** which becomes the light cone operator ***O*** in the large momentum limit.

$\vec{E}_{LC} \times \vec{A}_{LC}$

$$O_f^{tree} \xrightarrow[p_z \to \infty]{} O^{tree}$$

The quantum level

- In the large momentum limit, the **IR behaviors** of ***Of*** and ***O*** should be the same while the **UV one** can be different.

$$O = O^{tree}\{1 + \frac{\alpha_s}{4\pi}[A\frac{1}{\epsilon'} + A\log\frac{\mu^2}{p^2} + B] + O(\alpha_s^2)\}$$

$$O_f = O_f^{tree}\{1 + \frac{\alpha_s}{4\pi}[A_f\frac{1}{\epsilon'} + A_f\log\frac{\mu^2}{p^2} + B_f + (A - A_f)\log\frac{\vec{p}^2}{p^2}] + O(\alpha_s^2)\}$$

LaMET

X. Ji, J.H. Zhang, and Y. Zhao, Phys. Rev. Lett. 111 112002 (2013)

of the glue spin/helicity

$\vec{E}^c \times \vec{A}^c$

The classical level

- Find a finite frame operator ***O*_f** which becomes the light cone operator ***O*** in the large momentum limit.

$\vec{E}_{LC} \times \vec{A}_{LC}$

$$O_f^{tree} \xrightarrow[p_z \to \infty]{} O^{tree}$$

The quantum level

- In the large momentum limit, the **IR behaviors** of ***O*_f** and ***O*** should be the same while the **UV one** can be different.

$$O = O^{tree}\{1 + \frac{\alpha_s}{4\pi}[A\frac{1}{\epsilon'} + A\log\frac{\mu^2}{p^2} + B] + O(\alpha_s^2)\}$$

$$O_f = O_f^{tree}\{1 + \frac{\alpha_s}{4\pi}[A_f\frac{1}{\epsilon'} + A_f\log\frac{\mu^2}{p^2} + B_f + (A - A_f)\log\frac{\vec{p}^2}{p^2}] + O(\alpha_s^2)\}$$

- *Do the matching to fix the difference:*

$$O_f = O\{1 - \frac{\alpha_s}{4\pi}[(A - A_f)(\frac{1}{\epsilon'} + A_f\log\frac{\mu^2}{\vec{p}^2} + B - B_f] + O(\alpha_s^2)\}$$

LaMET

The available gauge conditions

- The **glue helicity** operator equivalent to ExA under the light cone gauge.

$$O_{\Delta_G} = \left[\vec{E}^a(0) \times (\vec{A}^a(0) - \frac{1}{\nabla^+}(\vec{\nabla} A^{+,b})\mathcal{L}^{ba}(\xi^-, 0))\right]^z = \vec{E}_{LC} \times \vec{A}_{LC},\ A^+_{LC} = 0$$

- We have to use the Lorentz in-covariant **glue spin (quasi-glue helicity)** operators to reach it in the large momentum limit:

Hatta, Ji and Zhao, Phys. Lett. B743, 180 (2015)

$\boxed{O_{S^c_G} = \vec{E}^c \times \vec{A}^c,\ \partial_i A^c_i = 0}$ or $\boxed{O_{S^a_G} = \vec{E}^a \times \vec{A}^a,\ A^a_z = 0}$ or $\boxed{O_{S^t_G} = \vec{E}^t \times \vec{A}^t,\ A^t_0 = 0}$

Coulomb gauge ***Axial gauge*** ***Temporal gauge***

or something else…

The glue spin

targets to the Light-cone glue helicity

- Glue spin under **Coulomb gauge** $S^c_G(|\vec{p}|,\mu)$
- Lattice simulation is *straight forward;*

Hatta, Ji and Zhao, Phys. Lett. B743, 180 (2015)

- Glue spin under **Temporal gauge** $S^t_G(|\vec{p}|,\mu)$
- Lattice simulation is tricky (will be addressed latter),

- **+** proper matching to cancel the intrinsic frame dependence to reach glue helicity $\Delta G(\mu)$.

X. Ji, J.H. Zhang, and Y. Zhao, Phys. Rev. Lett. 111 112002 (2013)

$$S^c_G(|\vec{p}|,\mu) = \left[1 + \frac{g^2 C_A}{16\pi^2}\left(\frac{7}{3}\log\frac{(\vec{p})^2}{\mu^2} - 10.2098\right)\right]\Delta G(\mu) + \frac{g^2 C_F}{16\pi^2}\left(\frac{4}{3}\log\frac{(\vec{p})^2}{\mu^2} - 5.2627\right)\Delta\Sigma(\mu) + O(g^4) + O\left(\frac{1}{(\vec{p})^2}\right)$$

$$S^t_G(|\vec{p}|,\mu) = \Delta G(\mu) + O(g^4) + O\left(\frac{1}{(\vec{p})^2}\right)$$

From the *glue spin* on the lattice to the *glue helicity* in LC

1. Calculate the longitudinal glue spin matrix element in the moving proton on the lattice, $S_G(|\vec{p}|^2, 1/a^2)$.

Converting the UV scale from 1/a to μ

2. Match the above matrix element to that under $\overline{\mathrm{MS}}$ at $\mu^2{=}10\ \mathrm{GeV}^2$,
$S_G^{\overline{\mathrm{MS}}}(|\vec{p}|^2, \mu^2) = Z_{GG}(\mu^2 a^2) S_G(|\vec{p}|^2, 1/a^2) + Z_{QG}(\mu^2 a^2)\Delta\Sigma$,

Converting the frame dependence to the scale μ dependence

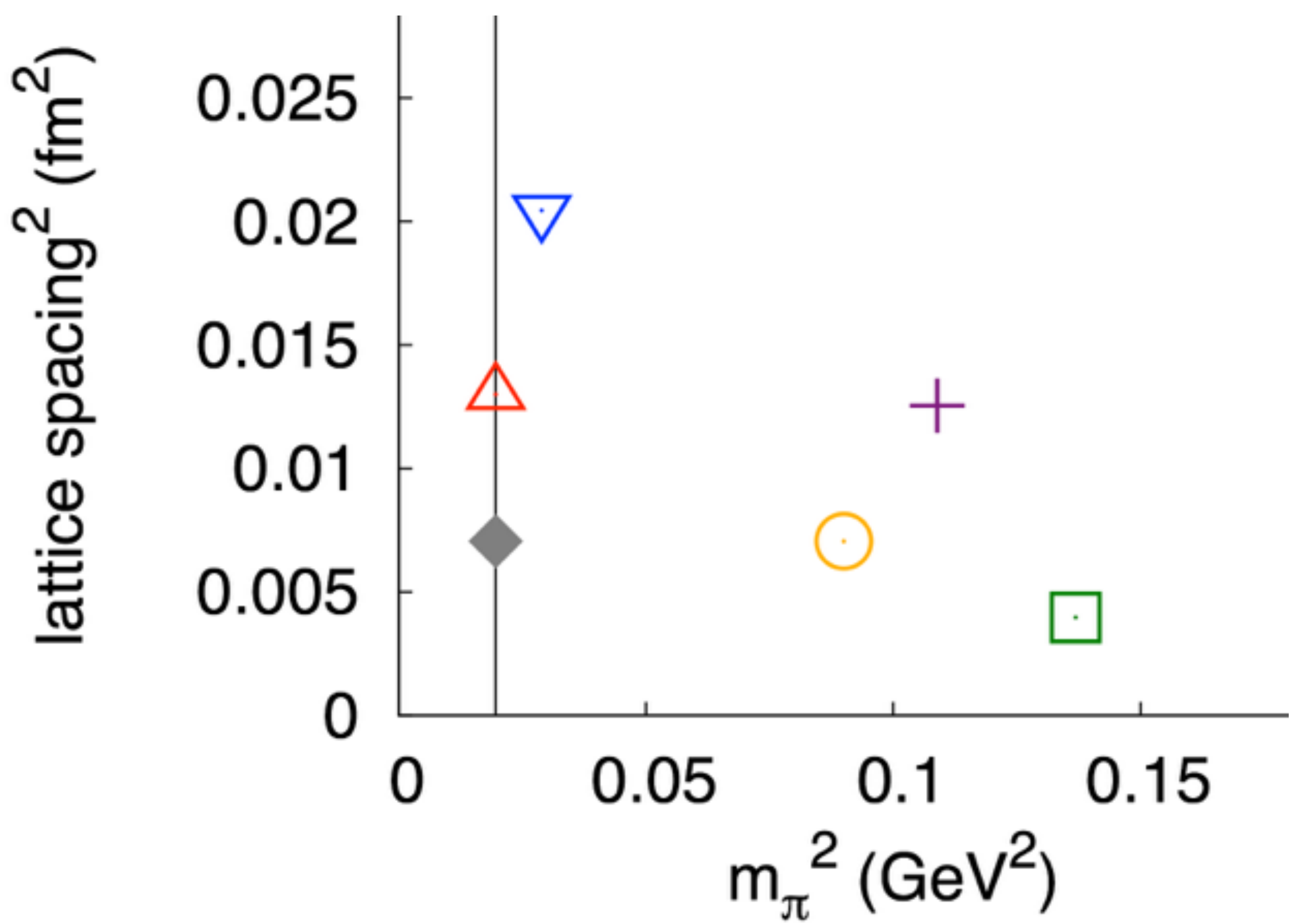
3. Match again from the finite frame to IMF with LaMET,
$\Delta G^{\overline{\mathrm{MS}}}(\mu^2) = \tilde{Z}_{GG}(|\vec{p}|^2/\mu^2) S_G^{\overline{\mathrm{MS}}}(|\vec{p}|^2, \mu^2) + \tilde{Z}_{QG}(|\vec{p}|^2/\mu^2)\Delta\Sigma$.

Glue Spin

Lattice setup

T. Blum et al. (RBC, UKQCD), Phys. Rev. D93, 074505 (2016)

2+1 flavor DWF configurations (RBC-UKQCD)



$$
\begin{aligned}
C(m_\pi, m_{\pi,sea}, a, L) = \\
& C_{\rm phys} + C_1(m_\pi^2 - m_{\pi,{\rm phys}}^2) \\
& + C_2(m_{\pi,sea}^2 - m_{\pi,{\rm phys}}^2) \\
& + C_3 a^2 + C_4 e^{-m_\pi L} \\
& + O(m_\pi^3, m_{\pi,sea}^3, a^4)...
\end{aligned}
$$

The **larger** volume also allow us to simulate the quark corresponding to **lighter** pion mass correctly.

Glue Spin

operator on the lattice

$$\vec{S}_g = 2\int d^3x\ \mathrm{Tr}(\vec{E}_c \times \vec{A}_c)$$

The gauge potential A_c defined by

$$A_{c,\mu} = \left[\frac{U^c_\mu(x) - U^{c\dagger}_\mu(x) + U^c_\mu(x-a\hat{\mu}) - U^{c\dagger}_\mu(x-a\hat{\mu})}{4iag}\right]_{traceless}$$

with the discrete coulomb gauge fixing condition $$\sum_{\mu=x,y,z}\left[U^c_\mu(x) - U^c_\mu(x-a\hat{\mu})\right] = 0$$

And the electric field $E_i=F_{0i}$ is defined by the averaged plaquette,

$$F^c_{\mu\nu} = \frac{i}{8a^2g}(\mathcal{P}_{\mu,\nu} - \mathcal{P}_{\nu,\mu} + \mathcal{P}_{\nu,-\mu} - \mathcal{P}_{-\mu,\nu} + \mathcal{P}_{-\mu,-\nu} - \mathcal{P}_{-\nu,-\mu} + \mathcal{P}_{-\nu,\mu} - \mathcal{P}_{\mu,-\nu})$$

$$\mathcal{P}_{\mu,\nu} = U^c_\mu(x)U^c_\nu(x+a\hat{\mu})U^{c\dagger}_\mu(x+a\hat{\nu})U^{c\dagger}_\nu(x).$$

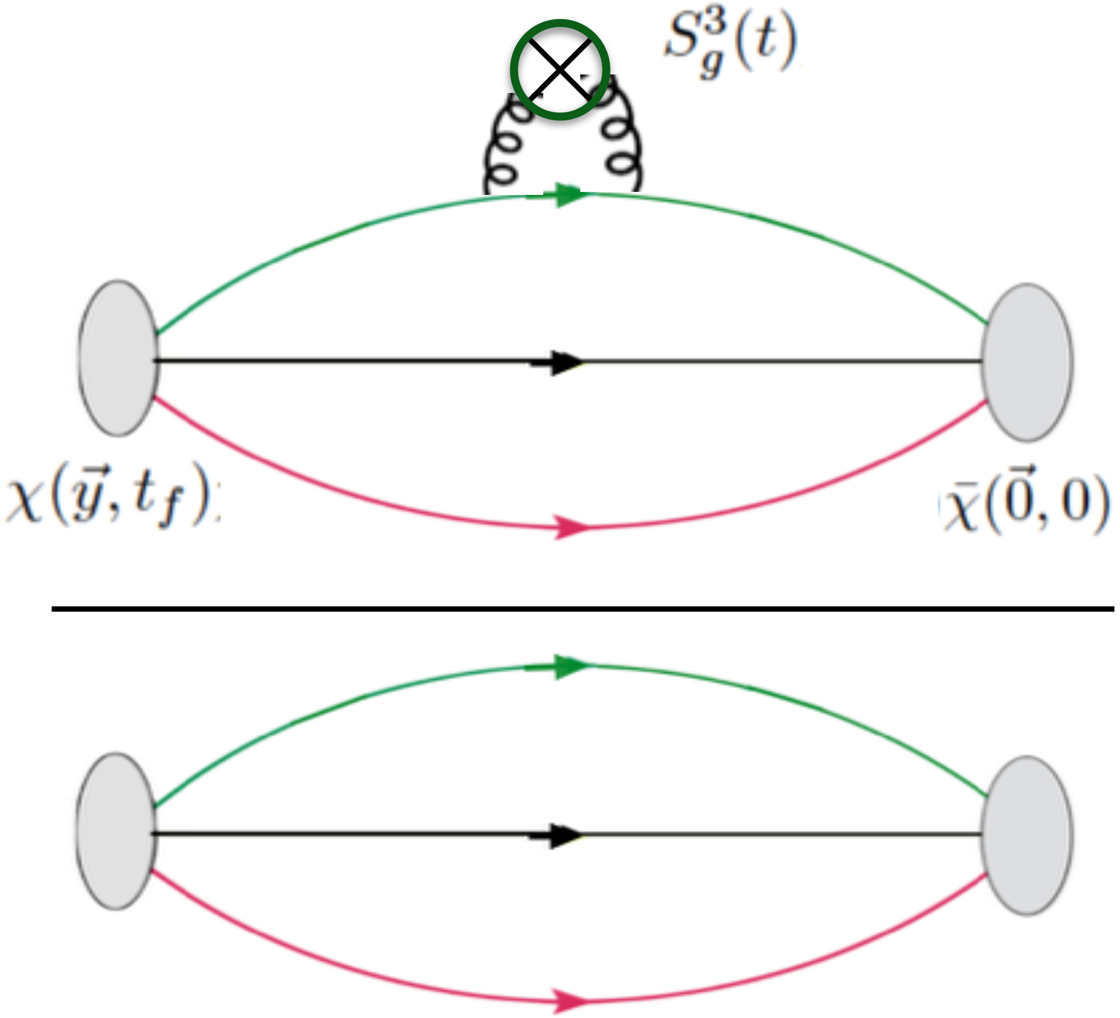
Glue Spin

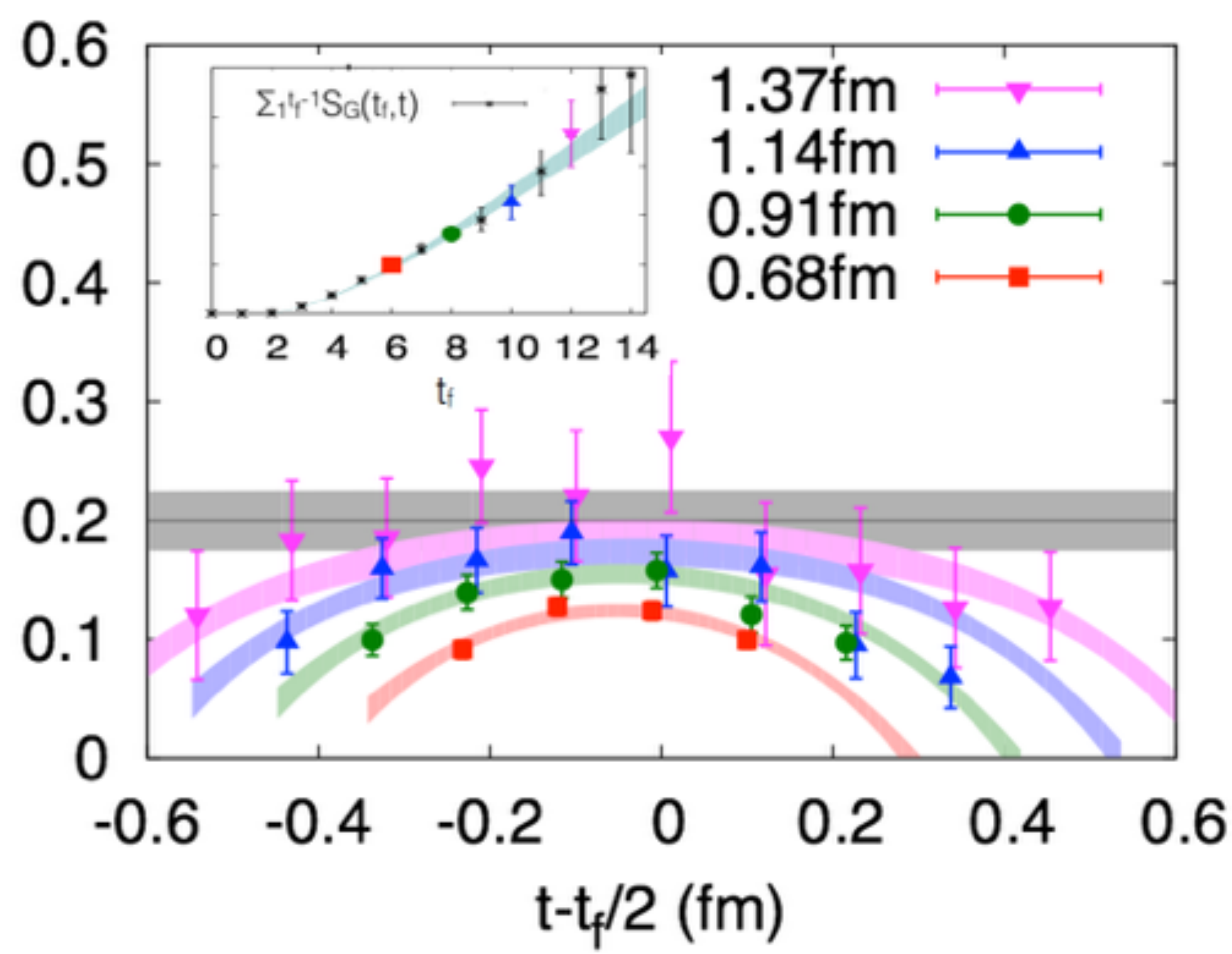
Y. Yang, R. S. Sufian, et al,
χQCD Collaboration,
Phys. Rev. Lett. 118 (2017) 102001

example of the bare result

$$R(t_f,t) = \frac{\langle 0|\Gamma_3^m \int d^3y\ e^{-ip_3y_3}\chi(\vec{y},t_f)S_g^3(t)\bar{\chi}(\vec{0},0)|0\rangle}{\langle 0|\Gamma^e \int d^3y\ e^{-ip_3y_3}\chi(\vec{y},t_f)\bar{\chi}(\vec{0},0)|0\rangle} = S_G + C_1 e^{-\Delta E(t_f-t)} + C_2 e^{-\Delta E t} + C_3 e^{-\Delta E t_f},$$

24I, unitary point.





From the *glue spin* on the lattice to the *glue helicity* in LC

1. Calculate the longitudinal glue spin matrix element in the moving proton on the lattice, $S_G(|\vec{p}|^2, 1/a^2)$.

Converting the UV scale from 1/a to μ

2. Match the above matrix element to that under $\overline{\mathrm{MS}}$ at $\mu^2{=}10$ GeV^2,
$S_G^{\overline{\mathrm{MS}}}(|\vec{p}|^2, \mu^2) = Z_{GG}(\mu^2 a^2) S_G(|\vec{p}|^2, 1/a^2) + Z_{QG}(\mu^2 a^2)\Delta\Sigma,$

Converting the frame dependence to the scale μ dependence

3. Match again from the finite frame to IMF with LaMET,
$\Delta G^{\overline{\mathrm{MS}}}(\mu^2) = \tilde{Z}_{GG}(|\vec{p}|^2/\mu^2) S_G^{\overline{\mathrm{MS}}}(|\vec{p}|^2, \mu^2) + \tilde{Z}_{QG}(|\vec{p}|^2/\mu^2)\Delta\Sigma.$

The renormalization of a lattice bare quantity

The renormalization of the coulomb gluon self energy with the lattice regularization and under RI-MOM scheme is,

$$Z_L^{MOM} = 1 + \frac{g^2}{16\pi^2}[(\frac{2N_f}{3} - N_c)\log(a^2p^2) + N_f B_G^f - N_c B_G^L(z)] + O(g^4),$$

where B_Q^f and $B_Q^L(z \equiv \frac{p_4^2}{p^2})$ are the finite pieces which are sensitive to the lattice quark and gluon actions.

The continuum field renormalization with the dimensional regularization and under RI-MOM and $\overline{MS}$ scheme is,

$$\begin{aligned} Z_D^{\overline{MS}} &= 1 - \frac{g^2}{16\pi^2}[(\frac{2N_f}{3} - N_c)\frac{1}{\tilde{\epsilon}}] + O(g^4), \\ Z_D^{MOM} &= 1 - \frac{g^2 N_f}{16\pi^2}[(\frac{2N_f}{3} - N_c)(\frac{1}{\tilde{\epsilon}} + \log(\mu^2/p^2)) + \frac{10N_f}{9} - N_c B_G^D(z)] + O(g^4). \end{aligned}$$

So the final renormalization under $\overline{MS}$ scheme for the lattice quantity is,

$$\begin{aligned} Z_L^{\overline{MS}}(a,\mu) &= \frac{Z_D^{\overline{MS}}}{Z_D^{MOM}} Z_L^{MOM}(a,\mu) \\ &= 1 + \frac{g^2 N_f}{16\pi^2}[(\frac{2N_f}{3} - N_c)\log(\mu^2 a^2) + N_f(\frac{10}{9} + B_G^f) - N_c(B_G^D(z) + B_G^L(z))] + O(g^4), \end{aligned}$$

where $B_G^D(z) + B_G^L(z)$ should be independent to z, and then the renormalization is independent to the momentum on the external legs.

The frame dependence of the finite pieces

Y. Yang, Y. Zhao, in preparation.

$$S^{\overline{MS}}_{G,(1)} = \{1 - \frac{g^2}{16\pi^2}\{N_f[\frac{2}{3}\log(\mu^2a^2) + B^{DR}_{GG,f} - B^{LR}_{GG,f}]$$
$$-C_A[\frac{4}{3}\log(\mu^2a^2) + B^{DR}_{GG} - B^{LR}_{GG}$$
$$+B^{DR}_{g} - B^{LR}_{g}]\}S^{LR}_{G,(1)}$$
$$+\frac{g^2C_F}{16\pi^2}[\frac{5}{3}\log(\mu^2a^2) + B^{DR}_{QG} - B^{LR}_{QG}] \sum_{q=u,d,s...} \Delta^{L,(1)}_q$$
$$+O(g^4),$$

The IR dependence in the renormalization are fully canceled !

$z = \frac{p_0^2}{p^2}$	0	$\frac{1}{4}$	$\frac{1}{2}$	$\frac{3}{4}$	1	$\frac{5}{4}$	$\frac{3}{2}$	$\frac{7}{4}$	2
B^{DR}_{QG}	5.191	5.080	5.021	4.986	4.966	4.953	4.946	4.942	4.940
$B^{LR,W}_{QG}$	2.913	2.803	2.743	2.708	2.688	2.675	2.668	2.664	2.662
$B^{DR}_{QG} - B^{LR,W}_{QG}$	2.278	2.278	2.278	2.278	2.278	2.278	2.278	2.278	2.278
B^{DR}_{GG}		2.413	1.995	1.667	1.402	1.180	0.992	0.828	0.684
$B^{LR,W}_{GG}$		6.192	5.774	5.446	5.181	4.959	4.771	4.608	4.463
$B^{DR}_{GG} - B^{LR,W}_{GG}$		-3.779	-3.779	-3.779	-3.779	-3.779	-3.779	-3.779	-3.779
B^{DR}_{g}	-1.233	-0.309	0.390	0.945	1.401	1.786	2.116	2.404	2.660
$B^{LR,W}_{g}$	15.686	16.609	17.308	17.864	18.320	18.704	19.035	19.323	19.578
$B^{DR}_{g} - B^{LR,W}_{g}$	-16.919	-16.919	-16.919	-16.919	-16.919	-16.919	-16.919	-16.919	-16.919

Wilson fermion and Wilson gluon

Glue spin

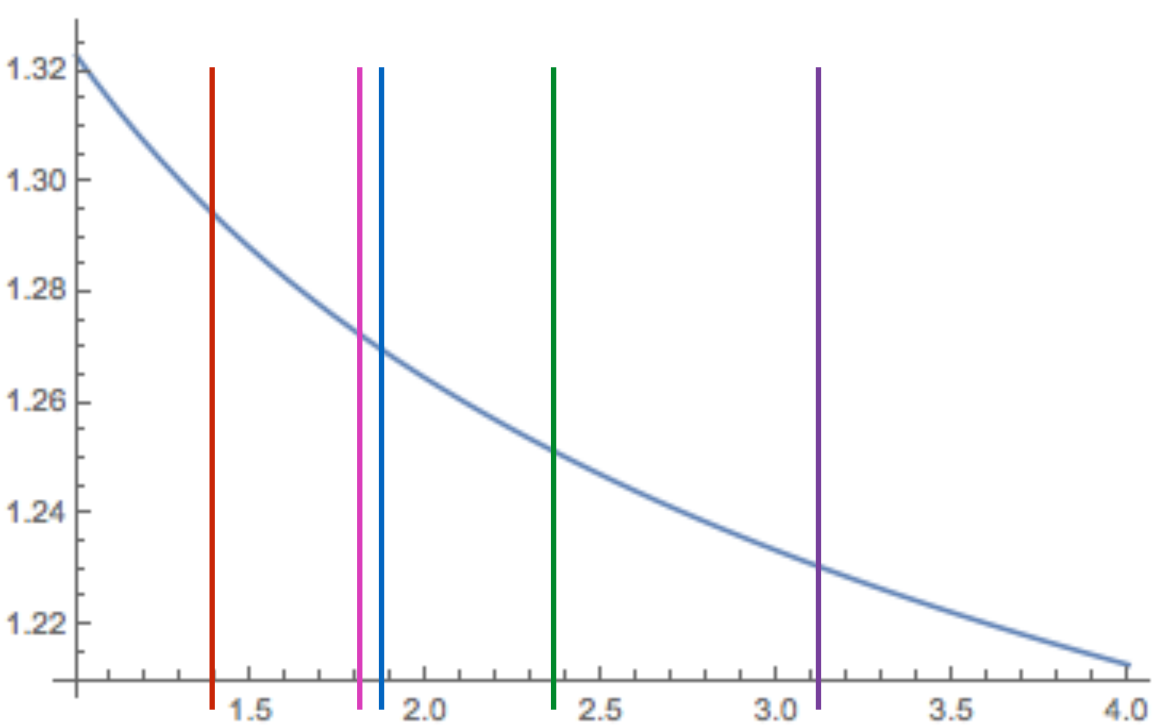
Y. Yang, Y. Zhao,
in preparation.

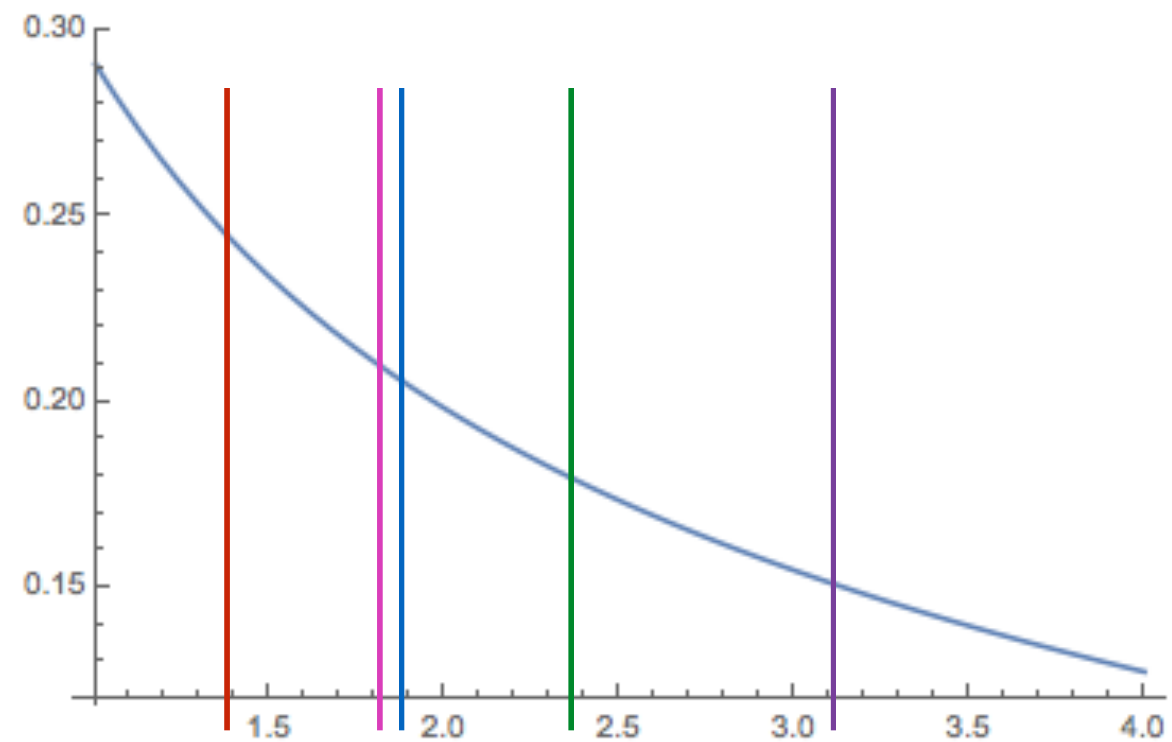
The renormalization and mixing

$$S_{G,(1)}^{\overline{MS}} = \left(1 - \frac{g^2 N_f}{16\pi^2}\left[\frac{2}{3}\log(\mu^2 a^2) + 2.41\right] + \frac{g^2 C_A}{16\pi^2}\left[-\frac{4}{3}\log(\mu^2 a^2) + \sim 2\right]\right) S_{G,(1)}^{L} + \frac{g^2 C_F}{16\pi^2}\left[\frac{5}{3}\log(\mu^2 a^2) + 6.99\right] \sum_{q=u,d,s\ldots} \Delta_q^{L,(1)} + O(a^2 p^2) + O(g^4)$$

The overlap fermion and Iwasaki gluon with HYP smearing

The scale used by the experiment for the glue helicity is ***μ^2=10 GeV^2***



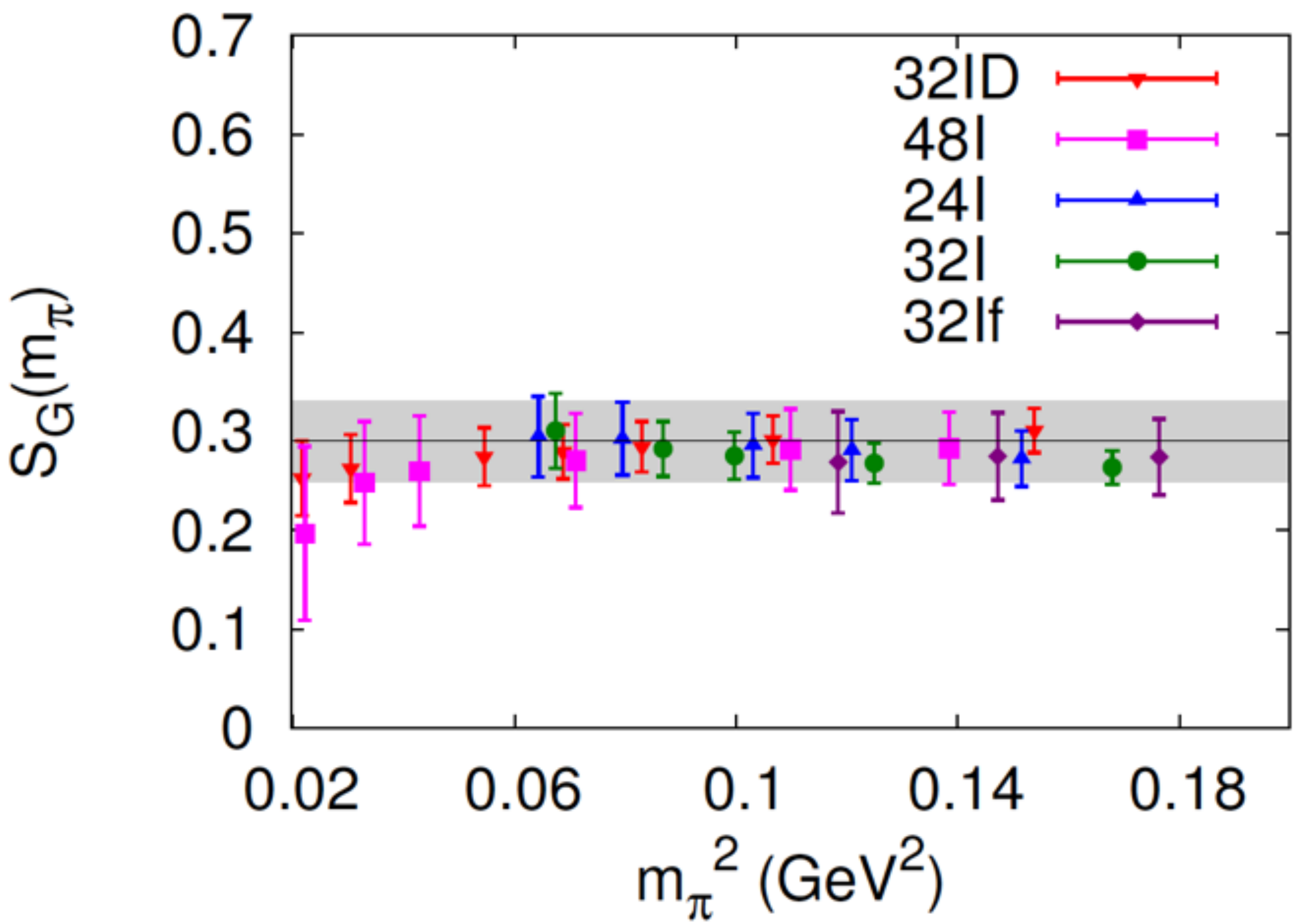


renormalization a^{-1} (GeV), $\alpha_s^{Lat}(a^{-1}) \sim \alpha_s^{MS}(2a^{-1})$ *mixing*

The dependence

Y. Yang, R. S. Sufian, et al,
χQCD Collaboration,
Phys. Rev. Lett. 118 (2017) 102001

of m_π, a, and V



μ^2=10 GeV^2

In the rest frame, *the pion mass (both valence and sea), lattice spacing and volume dependences are mild.*

From the *glue spin* on the lattice to the *glue helicity* in LC

1. Calculate the longitudinal glue spin matrix element in the moving proton on the lattice, $S_G(|\vec{p}|^2, 1/a^2)$.

$$\Big\downarrow \text{ Converting the UV scale from 1/a to } \mu$$

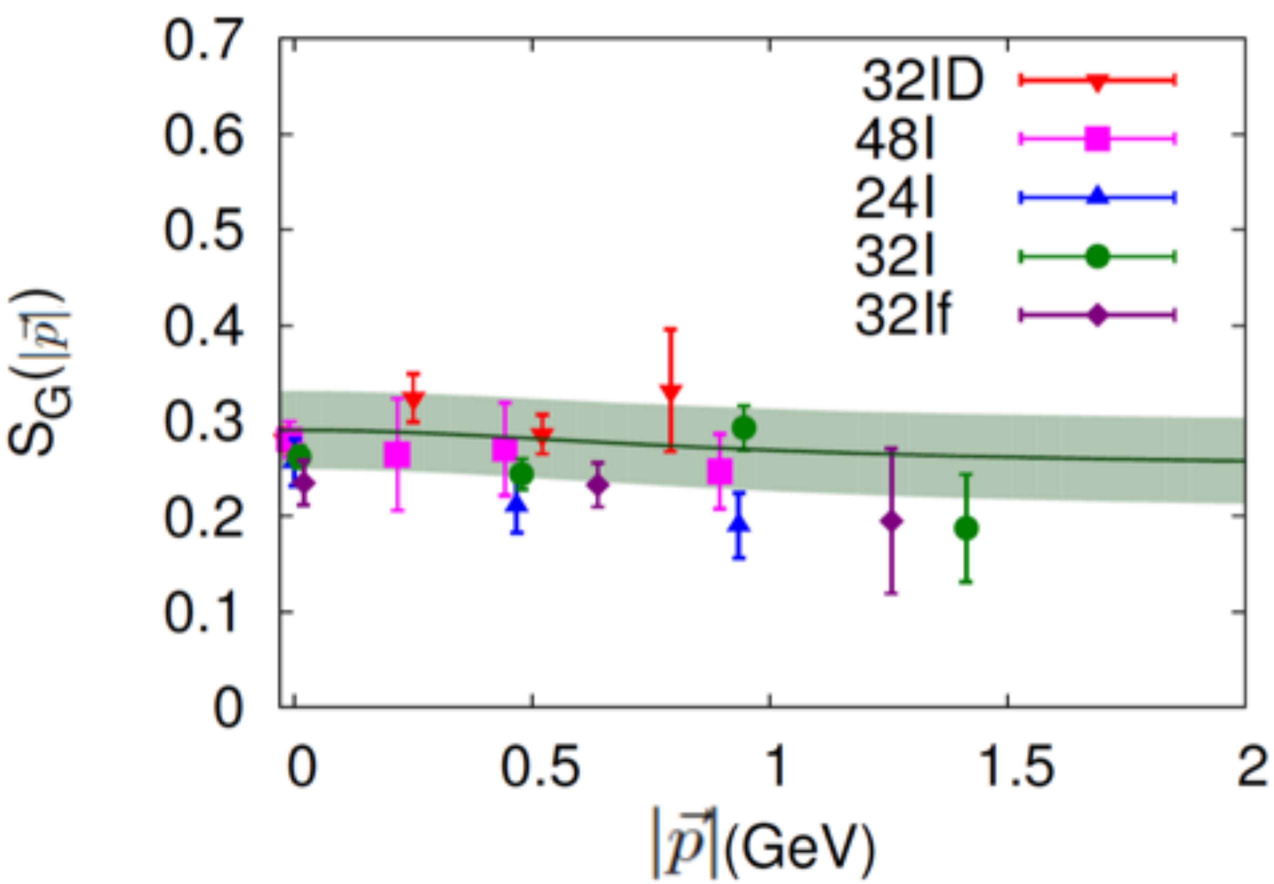
2. Match the above matrix element to that under $\overline{\mathrm{MS}}$ at μ^2=10 GeV^2,
$S_G^{\overline{\mathrm{MS}}}(|\vec{p}|^2, \mu^2) = Z_{GG}(\mu^2 a^2) S_G(|\vec{p}|^2, 1/a^2) + Z_{QG}(\mu^2 a^2)\Delta\Sigma$,

$$\Big\downarrow \text{ Converting the frame dependence to the scale } \mu \text{ dependence}$$

3. Match again from the finite frame to IMF with LaMET,
$\Delta G^{\overline{\mathrm{MS}}}(\mu^2) = \tilde{Z}_{GG}(|\vec{p}|^2/\mu^2) S_G^{\overline{\mathrm{MS}}}(|\vec{p}|^2, \mu^2) + \tilde{Z}_{QG}(|\vec{p}|^2/\mu^2)\Delta\Sigma$.

From glue spin to helicity

with *Large-momentum effective field theory*



X. Ji, J.-H. Zhang, and Y. Zhao, Phys. Lett. B743, 180 (2015)

$$S_G(|\vec{p}|,\mu) = \left[1 + \frac{g^2 C_A}{16\pi^2}\left(\frac{7}{3}\log\frac{(\vec{p})^2}{\mu^2} - 10.2098\right)\right]\Delta G(\mu)$$
$$+\frac{g^2 C_F}{16\pi^2}\left(\frac{4}{3}\log\frac{(\vec{p})^2}{\mu^2} - 5.2627\right)\Delta\Sigma(\mu)$$
$$+O(g^4) + O(\frac{1}{(\vec{p})^2}) \, .$$

At $\mu^2 = 10\ \text{GeV}^2$ and $|\vec{p}| = 1.5$ GeV,

the factor before ΔG is 0.22,

It means that ΔG will be ~ 3

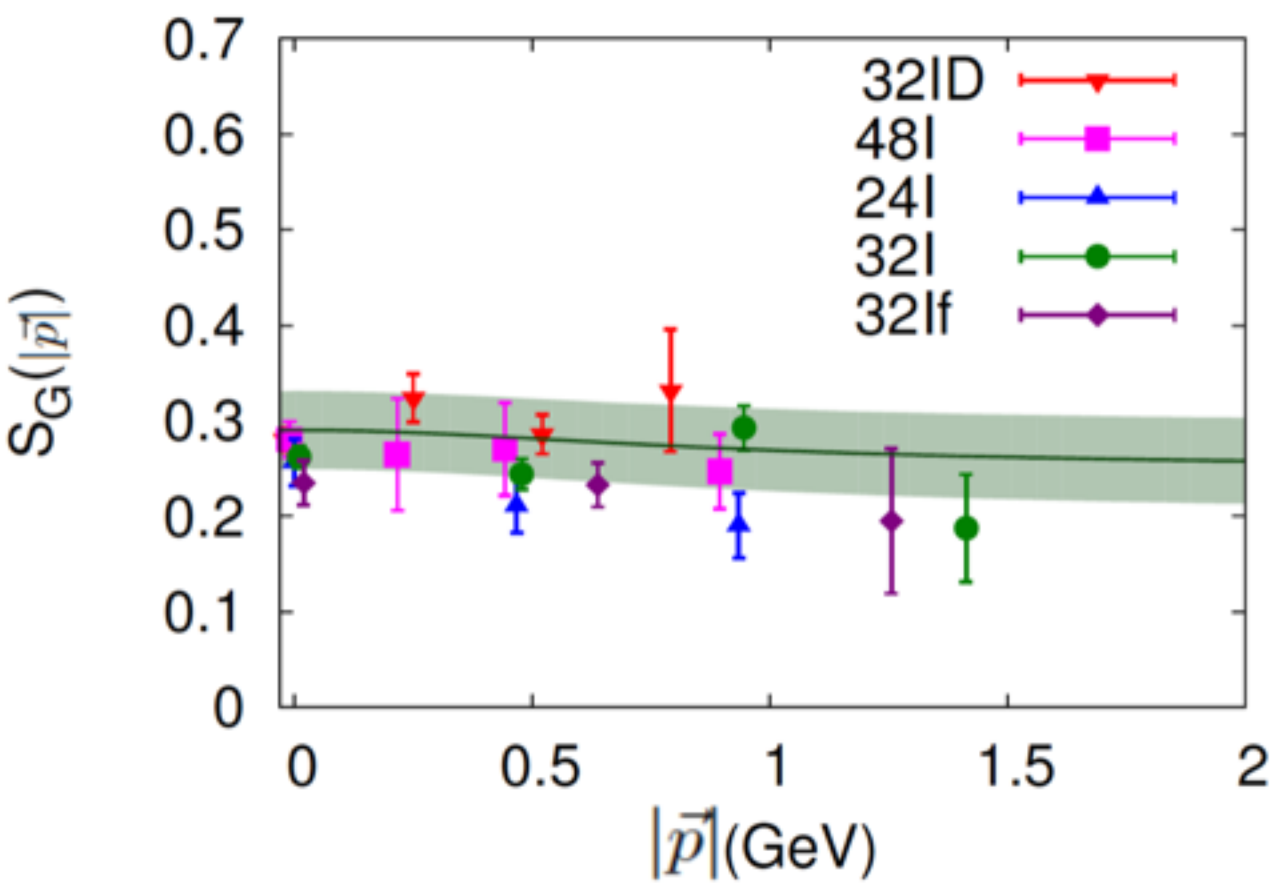
if we apply this matching.

- **The large finite pieces indicates a convergence problem**
- **Large frame dependence need re-summation.**

Glue spin

Y. Yang, R. S. Sufian, et al,
χQCD Collaboration,
Phys. Rev. Lett. 118 (2017) 102001

The final result



We neglect the matching and use the following empirical form to fit our data,

$$S_G(|\vec{p}|) = S_G(\infty) + \frac{C_1}{M^2 + |\vec{p}|^2} + C_2(m^2_{\pi,vv} - m^2_{\pi,phys})$$
$$+C_3(m^2_{\pi,ss} - m^2_{\pi,phys}) + C_4 a^2$$

$m_{\pi,phys}$=0.139 GeV $\quad M = 0.939$ GeV

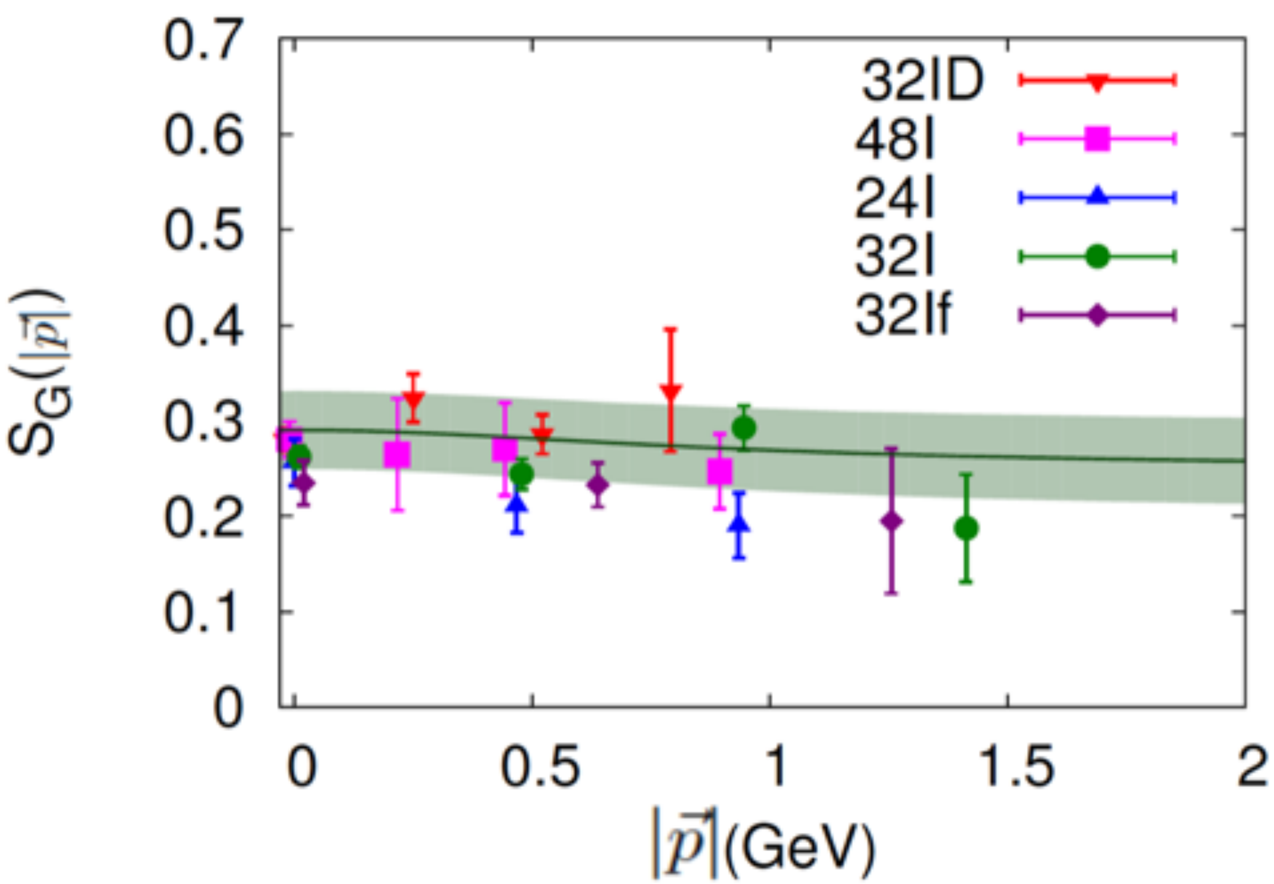
The glue spin at the large momentum limit for the renormalized value at μ^2=10GeV^2:

S_G=0.251(47)(16)

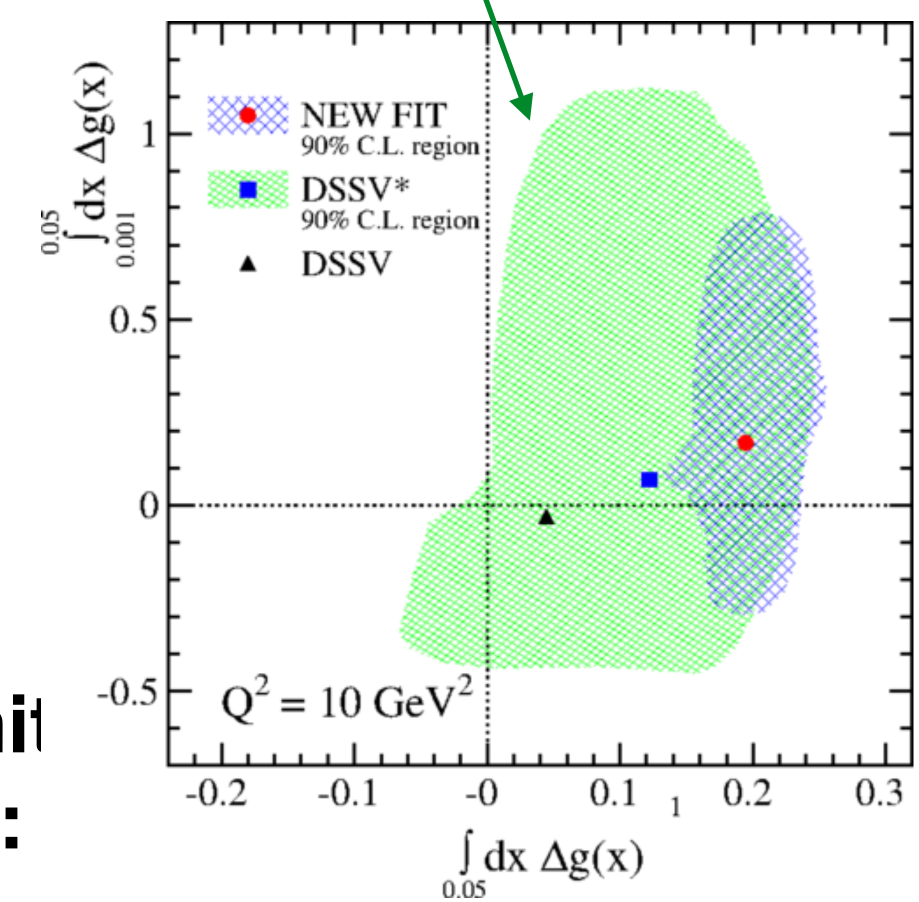
Glue spin

Y. Yang, R. S. Sufian, et al,
χQCD Collaboration,
Phys. Rev. Lett. 118 (2017) 102001

The final result



Comparable with the present global analysis while have much better signal



Florian et. al, Phys.Rev.Lett. 113 (2014) no.1, 012001

The glue spin at the large momentum limit for the renormalized value at μ^2=10GeV^2:

S_G=0.251(47)(16)

Glue spin

Y. Yang, R. S. Sufian, et al,
χQCD Collaboration,
Phys. Rev. Lett. 118 (2017) 102001

HIGHLIGHTED ARTICLES

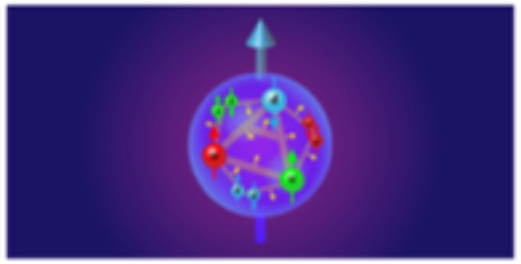
Featured in Physics | Editors' Suggestion

Glue Spin and Helicity in the Proton from Lattice QCD

Yi-Bo Yang, Raza Sabbir Sufian, Andrei Alexandru, Terrence Draper, Michael J. Glatzmaier, Keh-Fei Liu, and Yong Zhao (χQCD Collaboration)

Phys. Rev. Lett. **118**, 102001 (2017) – Published 6 March 2017

Physics Viewpoint: Spinning Gluons in the Proton



Computer simulations indicate that about 50% of the proton's spin comes from the spin of the gluons that bind its quark constituents.

Show Abstract +

PDF HTML

"…sure to attract plenty of attention from the wider physics community."

"…shed light on the composition of the nucleon."

—from referees

"looks forward to having full calculations of the complete quark and gluon spin and orbital contributions to the proton's spin"

—from Physics Viewpoint

The loopholes

in the two-step matching

1. Calculate the longitudinal glue spin matrix element in the moving proton on the lattice, $S_G(|\vec{p}|^2, 1/a^2)$.

Q1: Can we do the ***non-perturbative renormalization****?*

2. Match the above matrix element to that under $\overline{\mathrm{MS}}$ at $\mu^2{=}10$ GeV^2,
$S_G^{\overline{\mathrm{MS}}}(|\vec{p}|^2, \mu^2) = Z_{GG}(\mu^2 a^2) S_G(|\vec{p}|^2, 1/a^2) + Z_{QG}(\mu^2 a^2)\Delta\Sigma$,

Q2. Can we ***avoid this matching*** *by using other quasi-glue helicity?*

3. Match again from the finite frame to IMF with LaMET,
$\Delta G^{\overline{\mathrm{MS}}}(\mu^2) = \tilde{Z}_{GG}(|\vec{p}|^2/\mu^2) S_G^{\overline{\mathrm{MS}}}(|\vec{p}|^2, \mu^2) + \tilde{Z}_{QG}(|\vec{p}|^2/\mu^2)\Delta\Sigma$.

Q3. Are the final results independent to the gauge we used?

Q1: Can we do the **non-perturbative renormalization?**

- Only the non-perturbative renormalization of the 2-quark/4-quark operators are developed so far.

- The equal-time propagator is investigated for the coulomb gauge on the lattice, but not the standard one.

Y. Nakagawa, et al, Phys.Rev.D83:114503,2011

- The quasi-temporal gauge propagator can be obtained on the lattice by applying the coulomb gauge fixing on the reference time slice:

L. Conti, et al, Phys.Lett. B373 (1996) 164–170

$$\vec{\partial} \cdot \vec{A}(\vec{x}, t_0) = 0 \qquad \forall(\vec{x}, t_0).$$

+

$$A_0(x) = 0 \qquad \forall x \longleftarrow \Omega(x) = \mathcal{P}e^{ig\int_{t_0}^{t} d\tau A_0(\vec{x},\tau)} = \Omega(\vec{x}, t_0)P(\vec{x}; t_0, t)$$

Q2: Can we avoid this matching by using other quasi-glue helicity?

Yes! The decomposition extended from **the temporal gauge $A_0 = 0$** doesn't require any large momentum matching at the 1-loop level.

Hatta, Ji and Zhao, Phys. Lett. B743, 180 (2015)

$$\langle Ph|\epsilon^{ij}F^{i+}A^j|Ph\rangle_g\Big|_{\boxed{A^+=0}} = \left[1+\frac{\alpha_s}{4\pi}\left(\frac{\beta_0}{\varepsilon_v}+\frac{103N_c-10N_f}{9}\right)\right]\langle Ph|\epsilon^{ij}F^{i+}A^j|Ph\rangle_g^{tree}$$

$$\beta_0 = \frac{11N_c}{3}-\frac{2N_f}{3}$$

$$\langle Ph|\epsilon^{ij}F^{i0}A^j|Ph\rangle_g\Big|_{\boxed{A^0=0}} = \left[1+\frac{\alpha_s}{4\pi}\left(\frac{\beta_0}{\varepsilon_v}+\frac{103N_c-10N_f}{9}\right)\right]\langle Ph|\epsilon^{ij}F^{i0}A^j|Ph\rangle_g^{tree}$$

- *No frame dependence in the finite pieces*
- *Need to be checked whether it is still hold in the off shell scheme and the finite momentum frame*

Q3: Are the final results independent to the gauge we used?

It should be **independent** to whether we use the glue spin under the Coulomb gauge or temporal gauge, up to the higher order corrections. But,

- *the convergence of the perturbative series for ExA under the Coulomb gauge can be bad;*
- *the quasi-temporal gauge is not exactly the same as the temporal gauge, and the matrix element of ExA in proton under the temporal gauge is **<0.01 and consistent with zero** (based on the simulation).*

We may need to go back to the original definition and consider the simulation of the polarized PDF?

$$\Delta G = \int_0^1 \Delta g(x) dx = \int dx \frac{i}{2xP^+} \int \frac{d\xi^-}{2\pi} e^{-ixP^+\xi^-} \langle PS|F_a^{+\alpha}(\xi^-)\mathcal{L}^{ab}(\xi^-,0)\tilde{F}^{+}_{\alpha,b}(0)|PS\rangle$$

The further challenges:

the orbital angular momentums of the quark and gluon

$$\vec{J} = \underbrace{\boxed{\int d^3x\, \frac{1}{2}\, \overline{\psi}\, \vec{\gamma}\, \gamma^5\, \psi}}^{\text{quark spin}} + \overbrace{\boxed{\int d^3x \psi^\dagger \left\{ \vec{x} \times (i\vec{\nabla}) \right\} \psi}}^{\text{quark OAM}}$$

$$+ \underbrace{\boxed{\int d^3x 2\mathrm{Tr}[\vec{E} \times \vec{A}]}}_{\text{glue spin}} + \underbrace{\boxed{\int d^3x 2\mathrm{Tr}[E^i \vec{x} \times \vec{\nabla} A^i]}}_{\text{glue OAM}}$$

- The quasi-OAM under Coulomb condition can be calculated on the lattice but the LaEMT matching has the convergence problem as the glue spin case.
- That under temporal/quasi-temporal condition needs further investigation, but the LaEMT matching would be trivial.

Summary

- The glue spin under the Coulomb gauge in the MS-bar scheme is obtained with the 1-loop perturbative matching.

- We find the results to be fairly insensitive to the lattice spacing, quark masses and also momentum frame up to 1.5 GeV.

- **The glue spin under Coulomb gauge at the large momentum limit is S_G=0.251(47)(16).**

- **The glue spin and the other AM components under the temporal gauge condition can also be good quasi-quantity on the lattice to reach the light-cone components.**