Using a Quadrupole Deformed Generalized Woods-Saxon plus Spin-Orbit Potential to Describe the Unpolarized and Polarized Interaction $^{7}Li + {}^{12}C$ at $34\ MeV$.

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Seminar Pesentation
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June 2018

Outline of the presentation

1. Motivation for this study.
2. Characteristics of the reactions studied.
3. Theoretical formalism of the analyzing powers.
4. Analysing powers in polarized cross sections.
5. Potentials employed.
6. Coupled channels Schrödinger equation.
7. Results.
8. Conclusions.
9. References.

Reactions with beams of ^{7}Li with energies of tens of MeVs on targets of ^{12}C have been thoroughly studied.

So...

Why did I do this?

- The elastic differential cross section is well explained, but all of **its observables of polarization have not been properly described**.

- There exist **many potentials**, and **combinations among them**, proposed for this interaction. Radial (spherical and deformed) and tensor forms.

- There are **several models** for the ^{7}Li internal structructure. Colective, cluster and single nucleon motion.

- Polarized reactions **offer a lot of information** about the nuclei of the reaction and its mechanisms of interaction.

Elastic unpolarized reaction

The differential cross section of the reaction ${}^{12}C\left({}^{7}Li, {}^{7}Li\right){}^{12}C$ at $E_{lab}\left({}^{7}Li\right) = 34\ MeV$ in the $0^{\circ} < \theta_{c.m.} < 180^{\circ}$ angular region.

Elastic/Inelastic polarized reaction

The analysing powers (of order 1, 2 and 3) of the polarized reactions

- ${}^{12}C\left({}^{7}\vec{Li}, {}^{7}Li\right){}^{12}C$
- ${}^{12}C\left({}^{7}\vec{Li}, {}^{7}Li^{*}_{(1/2^{-},\, 0.4776\ MeV)}\right){}^{12}C$

at $E_{lab}\left({}^{7}\vec{Li}\right) = 34\ MeV$ in the $\theta_{c.m.} < 90^{\circ}$ angular region.

The experiment

The spin of a polarized nucleus can be spatially manipulated in the laboratory by means of rotations using magnetic fields.

The theory

The density matrix formalism is used to describe the polarization in a projectile-target system.

$$\Downarrow$$

Every element of the density matrix will contain certain information about the polarization of the system.

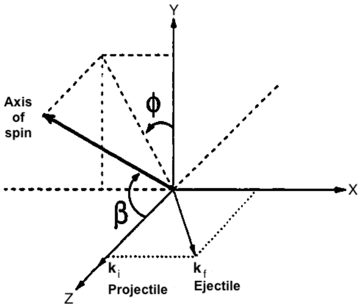
The numerical calculations

The computational code Fresco of coupled reaction channel calculations was employed to generate the adjustments to the data.

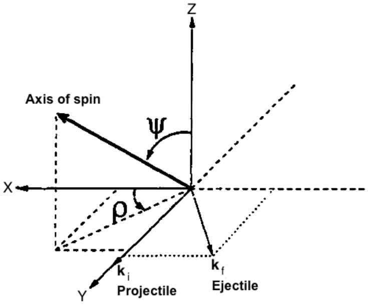
Madison and Transverse reference systems

Systems of reference used to orient the axis of spin (projectile's spin **"direction"** in the laboratory).

Madison system



Transverse system



Wave function of a completely polarized nucleus - Pure states

The wave function of a nucleus, or projectile, completely polarized is pure

$$|\psi_{i_p}\rangle = \sum_{m_{i_p}=-i_p}^{i_p} a^{i_p}_{m_{i_p}} |i_p, m_{i_p}\rangle$$

where the probability amplitudes are given by

$$a^{i_p}_{m_{i_p}} = \langle i_p, m_{i_p} | \psi_{i_p}\rangle$$

Density matrix

$$\rho = \sum_{m_{i_p}=-i_p}^{i_p} \sum_{m'_{i_p}=-i_p}^{i_p} \rho_{m_{i_p},m'_{i_p}} |i_p, m_{i_p}\rangle\langle i_p, m'_{i_p}|$$

whose elements are

$$\rho_{m_{i_p},m'_{i_p}} = a^{i_p}_{m_{i_p}} {a^{i_p}_{m'_{i_p}}}^*$$

The axis of spin direction diagonalizes the density matrix of the polarized nuclei in any reference system when it coincides with its z axis.

The axis of spin can be rotated according to a reference system (active rotation) or the system can be rotated (passive rotation).

Rotation operator - Active rotation

$$\mathcal{R}^a(\alpha,\beta,\gamma) = \exp\left(-\mathrm{i}\alpha\frac{I_z}{\hbar}\right)\exp\left(-\mathrm{i}\beta\frac{I_y}{\hbar}\right)\exp\left(-\mathrm{i}\gamma\frac{I_z}{\hbar}\right)$$

Elements of the Wigner D-matrix - Active rotation

$$D^{i_p}_{\mu_{i_p},m_{i_p}}(\alpha,\beta,\gamma) = \langle i_p,\mu_{i_p}|\mathcal{R}^a(\alpha,\beta,\gamma)|i_p,m_{i_p}\rangle$$

α, β and γ are the Euler angles.

Probability amplitudes of a rotated wave function - Active rotation

$$a'^{i_p}_{m_{i_p}} = \sum_{\mu_{i_p}=-i_p}^{i_p} D^{i_p}_{\mu_{i_p}, m_{i_p}}(\alpha, \beta, \gamma) a^{i_p}_{m_{i_p}}$$

Rotated elements of the density matrix - Active rotation

$$\rho'_{\mu_{i_p}, \mu'_{i_p}} = \sum_{m_{i_p}=-i_p}^{i_p} \sum_{m'_{i_p}=-i_p}^{i_p} D^{i}_{\mu_{i_p}, m_{i_p}}(\alpha, \beta, \gamma) D^{i_p}_{\mu'_{i_p}, m'_{i_p}}(\alpha, \beta, \gamma)^* \rho_{m_{i_p}, m'_{i_p}}$$

An inconvenient

Rotations of the density matrix are more **"complicated"** than rotations of the wave function.

How to deal with it

Irreducible tensor operators are used instead of the density matrix of the system because they are **"easier"** to rotate.

Irreducible tensor operator - Definition

An active rotation of one of the $2k+1$ independent element $O_{k,q}$ of an irreducible spherical tensor operator $\mathcal{O}^k$ of rank k is written as

$$\mathcal{R}^a(\alpha,\beta,\gamma)O_{k,q}\mathcal{R}^a(\alpha,\beta,\gamma)^{-1} = \sum_{q'=-k}^{k} D^k_{q,q'}(\alpha,\beta,\gamma)^* O_{k,q'}$$

Wigner-Eckart theorem

$$\langle l,m|O_{k,q}|l',m'\rangle \equiv \frac{1}{\sqrt{2l+1}}\langle l',m';k,q|l,m\rangle\langle l||\mathcal{O}^k||l'\rangle$$

where

$$|l-l'| \leq k \leq l+l' \quad \text{y} \quad m = m'+q$$

Therefore, the elements $O_{k,q}$ are decomposed in:

- Geometric part $\rightarrow$ Clebsch-Gordan coeficient.
- Part that depends on the internal dynamics of the system $\rightarrow$ Reduced matrix element.

The reduced matrix element is independent of the angular momentum orientation.

Elements of the tensor of polarization t^k

Using the Wigner-Eckart theorem and properties of the Clebsch-Gordan coefficients, the elements of the polarization spherical tensor t^k of the system are written as

$$t_{k,q} = \sqrt{2i_p + 1} \sum_{m_{i_p}=-i_p}^{i_p} \sum_{m'_{i_p}=-i_p}^{i_p} (-1)^{i_p - m_{i_p}} \langle i_p, m'_{i_p}; i_p, -m_{i_p} | k, q \rangle \rho_{m_i, m'_i}$$

where the elements of a geometric tensor $\tau_{k,q}$ can be defined as

$$(\tau_{k,q})_{m'_{i_p}, m_{i_p}} = \sqrt{2i_p + 1} (-1)^{i_p - m_{i_p}} \langle i_p, m'_{i_p}; i_p, -m_{i_p} | k, q \rangle$$

therefore

$$t_{k,q} = Tr[\rho\, \tau_{k,q}] = \langle \tau_{k,q} \rangle$$

Rotated element of the polarization tensor - Active rotation

$$t'_{k,q} = \sum_{q'=-k}^{k} D^k_{q,q'}(\alpha, \beta, \gamma)^* t_{k,q'}$$

Components of the polarization tensor ($i_p = 3/2$) - Axis of spin parallel to Z

$$t_{1,0} = \sqrt{\frac{9}{5}} \left(\rho_{\frac{3}{2},\frac{3}{2}} - \rho_{-\frac{3}{2},-\frac{3}{2}}\right) + \frac{1}{\sqrt{5}} \left(\rho_{\frac{1}{2},\frac{1}{2}} - \rho_{-\frac{1}{2},-\frac{1}{2}}\right)$$

$$t_{2,0} = \left(\rho_{\frac{3}{2},\frac{3}{2}} + \rho_{-\frac{3}{2},-\frac{3}{2}}\right) - \left(\rho_{\frac{1}{2},\frac{1}{2}} + \rho_{-\frac{1}{2},-\frac{1}{2}}\right)$$

$$t_{3,0} = \frac{1}{\sqrt{5}} \left(\rho_{\frac{3}{2},\frac{3}{2}} - \rho_{-\frac{3}{2},-\frac{3}{2}}\right) - \sqrt{\frac{9}{5}} \left(\rho_{\frac{1}{2},\frac{1}{2}} - \rho_{-\frac{1}{2},-\frac{1}{2}}\right)$$

Polarization in the source

The $\rho_{m_{i_p},m_{i_p}}$ is related to the probability of finding projectiles with spin projection m_{i_p}.

Population fractions - Experimental measurement

$$N_{m_{i_p}} = \rho_{m_{i_p},m_{i_p}}$$

What are the analysing powers?

The analysing powers ($T_{k,q}$) show how sensitivity are reaction channels with respect to polarization states.

$\Downarrow$

They quantify the effect of the $t_{k,q}$ on the scattering.

Probability of finding a state of polarization

$$\omega\left(\mathsf{t}^k\right) = C \sum_{k=0}^{2i_p} \sum_{q=-k}^{k} t_{k,q} {T_{k,q}}^*$$

where C is a constant of normalization.

Polarized differential cross section - Just projectile polarization

$$\frac{d\sigma_p}{d\Omega} = \frac{d\sigma_{np}}{d\Omega} \sum_{k=0}^{2i_p} \sum_{q=-k}^{k} t_{k,q} \, {T_{k,q}}^*$$

Where:

- $\frac{d\sigma_p}{d\Omega} \leftrightarrow$ Polarized differential cross section.
- $\frac{d\sigma_{np}}{d\Omega} \leftrightarrow$ Unpolarized differential cross section.
- $t_{k,q} \leftrightarrow$ Elements of the tensor of polarization.
- $T_{k,q} \leftrightarrow$ Analysing powers.

Polarized differential cross section - Madison system

Due to properties of the $t_{k,q}$ and $T_{k,q}$ we have that

$$\frac{d\sigma_p}{d\Omega} = \frac{d\sigma_{np}}{d\Omega} \sum_{k=0}^{3} \sum_{q=-k}^{k} \sqrt{\frac{4\pi}{2k+1}} \,\mathfrak{Re}\left(\varepsilon_k Y_{k,q}(\beta,\phi) t^Z_{k,0}\right) \varepsilon_k T_{k,q}$$

where

$$\varepsilon_k = \begin{cases} 1 & \text{if } k \text{ is even} \\ \mathrm{i} & \text{if } k \text{ is odd} \end{cases}$$

$$\begin{aligned}
\frac{\frac{d\sigma_p}{d\Omega}}{\frac{d\sigma_{np}}{d\Omega}} = & 1 + \sqrt{2}\sin(\beta)\cos(\phi) t^Z_{1,0}\mathrm{i}T_{1,1} + \frac{1}{2}\left(3\cos^3(\beta) - 1\right) t^Z_{2,0} T_{2,0} + \\
& \sqrt{\frac{3}{2}}\sin(2\beta)\sin(\phi) t^Z_{2,0} T_{2,1} - \sqrt{\frac{3}{2}}\sin^2(\beta)\cos(2\phi) t^Z_{2,0} T_{2,2} + \\
& \frac{\sqrt{3}}{2}\sin(\beta)\left(5\cos^2(\beta) - 1\right)\cos(\phi) t^Z_{3,0}\mathrm{i}T_{3,1} + \\
& \sqrt{\frac{15}{8}}\sin(\beta)\sin(2\beta)\sin(2\phi) t^Z_{3,0}\mathrm{i}T_{3,2} - \frac{\sqrt{5}}{2}\sin^3(\beta)\cos(3\phi) t^Z_{3,0}\mathrm{i}T_{3,3}
\end{aligned}$$

The $t^Z_{k,0}$ represent the polarization of the beam at the "source" and β and ϕ are measured at the target using the Madison system.

Polarized differential cross section - Transverse system

Due to properties of the ${}^{T}t_{k,q}$ and ${}^{T}T_{k,q}$ we have that

$$\frac{d\sigma_p}{d\Omega} = \frac{d\sigma_{np}}{d\Omega} \sum_{k=0}^{3} \sum_{q=-k}^{k} (-1)^q \sqrt{\frac{4\pi}{2k+1}} Y_{k,q}(\Psi,\rho)\, {}^{T}t^{Z}_{k,0}\, {}^{T}T_{k,-q}$$

$$\begin{aligned}
\frac{\frac{d\sigma_p}{d\Omega}}{\frac{d\sigma_{np}}{d\Omega}} =& 1 + \cos(\Psi)\, {}^{T}t^{Z}_{1,0}\, {}^{T}T_{1,0} + \frac{1}{2}\left(3\cos^2(\Psi) - 1\right)\, {}^{T}t^{Z}_{2,0}\, {}^{T}T_{2,0} + \\
& \frac{\sqrt{6}}{4} \sin^2(\Psi)\, {}^{T}t^{Z}_{2,0} \left(\exp(-2i\rho)^{T}T_{2,2} + \exp(2i\rho)^{T}T_{2,-2}\right) + \\
& \frac{1}{2}\left(5\cos^3(\Psi) - 3\cos(\Psi)\right)\, {}^{T}t^{Z}_{3,0}\, {}^{T}T_{3,0} + \\
& \frac{\sqrt{30}}{4} \sin^2(\Psi)\cos(\Psi)\, {}^{T}t^{Z}_{3,0} \left(\exp(-2i\rho)^{T}T_{3,2} + \exp(2i\rho)^{T}T_{3,-2}\right)
\end{aligned}$$

The ${}^{T}t^{Z}_{k,0}$ represent the polarization of the beam at the “source” and Ψ and ρ are measured at the target using the Transverse system.

Asymptotic form of the scattered wave function of a reaction

$$|\Psi_\beta(\boldsymbol{r}_\beta, \theta_{cm}, \phi_{cm})\rangle \xrightarrow{r_\beta \to \infty} A_0 \mathfrak{f}(\theta_{cm}, \phi_{cm}) \frac{\exp(ik_\beta r_\beta)}{r_\beta} \left(|\psi_E\rangle \otimes |\psi_R\rangle\right)$$

where $\mathfrak{f}(\theta_{cm}, \phi_{cm})$ is the scattering amplitude.

Theoretical differential cross section

$$\frac{d\sigma_\beta}{d\Omega}(\theta_{cm}, \phi_{cm}) \propto |\mathfrak{f}(\theta_{cm}, \phi_{cm})|^2$$

Theoretical analysing powers

$$T_{k,q}(\theta_{cm}) = \frac{Tr\left[\mathfrak{f}(\theta_{cm}, \phi_{cm})\, \tau_{k,q}\, \mathfrak{f}(\theta_{cm}, \phi_{cm})^\dagger\right]}{Tr\left[\mathfrak{f}(\theta_{cm}, \phi_{cm})\, \mathfrak{f}(\theta_{cm}, \phi_{cm})^\dagger\right]}$$

To deform a nuclear potential, its real and imaginary average radius of interaction are deformed by an expansion in spherical harmonics.

Deformed interaction radius

$$R_{i,j}^{def} = R_{i,j} + R_i^{Proj} \sum_{\lambda=0}^{\infty} \beta_\lambda Y_{\lambda,0}\left(\hat{\boldsymbol{r}}'_\alpha\right)$$

where

$$R_i^{Proj} = \frac{U_i\, r_{i,u} + W_i\, r_{i,w}}{U_i + W_i} A_P^{1/3}$$

and λ is related to the type of the deformation.

The deformation length

$$\delta_\lambda^i = \beta_\lambda^{Proj}\, R_i^{Proj}$$

The subscript i represents the type of potential (c, v, s and so) and j its real or imaginary part (v and w).

Deformed nuclear radius and potential

$$V_{def} = V_{sph} + \Delta V$$

Where

$$V_{sph} = \sum_{i=c,v,s,so} V_i(r)$$

and if only quadrupole deformation ($\lambda = 2$) are supposed

$$\Delta V = -\sum_{i=v,s} \frac{d\,V_i(r)}{dr} \sum_{\mu=-2}^{2} \delta^i_{\lambda=2}\, D^{\lambda=2}_{\mu,0}(\alpha,\beta,0)\, Y_{\lambda=2,\mu}\,(\theta,\varphi)$$

The subscripts c, v, s and so represent the Coulomb, volumetric Woods-Saxon, superficial Woods-Saxon and Spin-Orbit potential respectively.

Due to limitations in the computational code, just the volumetric and superficial Woods-Saxon potential were deformed.

Woods-Saxon distribution function

$$F(r, R_{i,j}, a_{i,j}) = \frac{1}{1 + \exp\left(\frac{r - R_{i,j}}{a_{i,j}}\right)}$$

Where:

- $r \leftrightarrow$ relative position of nuclei.
- $R_{i,j} = r_{i,j}(A_P^{1/3} + A_T^{1/3}) \leftrightarrow$ mean interaction radius.
- $a_{i,j} \leftrightarrow$ diffuseness of the potential.

Coulomb potential

$$V_c(r) = \begin{cases} \frac{1}{4\pi\varepsilon_0}\frac{Z_P Z_T e^2}{2R_c^T}\left(3 - \left(\frac{r}{R_c^T}\right)^2\right) & \text{if } r \leq R_c^T \\ \frac{1}{4\pi\varepsilon_0}\frac{Z_P Z_T e^2}{r} & \text{if } r > R_c^T \end{cases}$$

Volumetric Woods-Saxon potential

$$V_v(r) = -U_v F(r, R_{v,v}, a_{v,v}) - \mathrm{i} W_v F(r, R_{v,w}, a_{v,w})$$

Superficial Woods-Saxon potential

$$V_s(r) = -\mathrm{i}4 W_s a_{s,w} \frac{d}{dr} F(r, R_{s,w}, a_{s,w})$$

Spin-Orbit potential

$$V_{so}(r) = -\left(\frac{\hbar}{m_\pi c}\right)^2 \frac{1}{2r}\left(U_{so}\frac{d}{dr}F(r, R_{so,v}, a_{so,v}) + \mathrm{i} W_{so}\frac{d}{dr}F(r, R_{so,w}, a_{so,w})\right)\boldsymbol{L}_P \cdot \boldsymbol{I}_P$$

Coupling scheme of angular momenta

$$\boldsymbol{L}_P + \boldsymbol{I}_P = \boldsymbol{J}_P \text{ and } \boldsymbol{J}_P + \boldsymbol{I}_T = \boldsymbol{J}_T$$

Where other types of couplings are not recommendable because they do not diagonalize the spin-orbit potential.

Coupled channel Schrödinger equation

$$\left(E_{p,t} + T_{l_p} - V_{sph}\right)|R_{J_T,J_p,l_p}\rangle = \sum_{l_p{}',i_p{}'} \langle (l_p, i_p)J_p|\Delta V|(l_p{}', i_p{}')J_p\rangle \, |R_{J_T,J_p,l_p{}'}\rangle$$

Being

$$E_{p,t} = E_{cm} - \epsilon_p - \epsilon_t$$

and

$$T_{l_p} = \frac{\hbar^2}{2m}\left(\frac{d^2}{dr^2} - \frac{l_p(l_p+1)}{r^2}\right)$$

Coupling potential - Reorientations and transitions (quadrupole deformation)

$$\langle (l_p, i_p) J_p | \Delta V | (l_p{}', i_p{}') J_p \rangle = \mathfrak{c}^{J_p, \lambda=2}_{l_p, i_p; l_p{}', i_p{}'} \sum_{i=v,s} \frac{dV_i(r)}{dr} \langle i_p || \hat{\delta}^i_{\lambda=2} || i_p{}' \rangle$$

Where

$$\mathfrak{c}^{J_p, \lambda=2}_{l_p, i_p; l_p{}', i_p{}'} = \frac{1}{\sqrt{4\pi}} (-1)^{1+J_p - i_p{}' - l_p + l_p{}'} \sqrt{2l_p + 1} \sqrt{2l_p{}' + 1}$$
$$\langle l_p, 0; l_p{}', 0 | \lambda = 2, 0 \rangle \, W\left(l_p, l_p{}', i_p, i_p{}'; \lambda = 2, J_p\right)$$

If the ^7Li is consider to have a quadrupole deformation ($\lambda = 2$) with a deformation parameter of $\beta_2^{Proj} = -0.934$ (reported by Weller et al.) and its four lowest states belong to a rotational band with $K = 1/2$ (as sugested by El-Batanoni and Kresnin), then

$$\langle i_p || \hat{\delta}^i_{\lambda=2} || i_p{}' \rangle \equiv f_{i_p, i_p{}'} \sqrt{2i_p{}' + 1} \, \langle i_p{}', K = 1/2; \lambda = 2, 0 | i_p, K = 1/2 \rangle \, \delta^i_{\lambda=2}$$

and

$$f_{i_p, i_p{}'} = (-1)^{\left(i_p{}' - i_p + \left| i_p{}' - i_p \right|\right)/2}$$

Effect of the generalized Woods-Saxon potential and the coupled channel calculation employing the rotational model on the elastic and inelastic analysing powers.

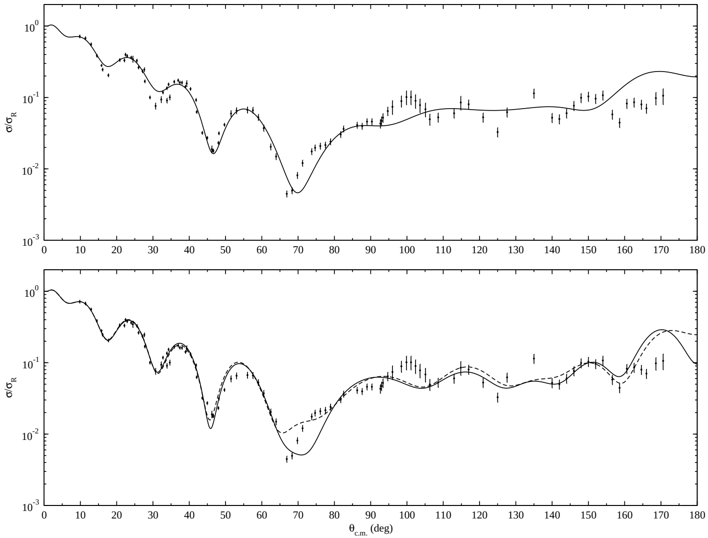


Figure: Experimental elastic differential cross section reported by Vineyard et al. Upper panel: Adjustment of the complex Woods-Saxon volumetric potential plus a real spin-orbit potential reported by Vineyard et al. and Momotyuk et al. respectively. Lower panel: Adjustment of the couple channel calculation hereby presented, the solid and dash line represent the adjustments with all the couplings to the 3 and 4 internal state of ^7Li respectively.

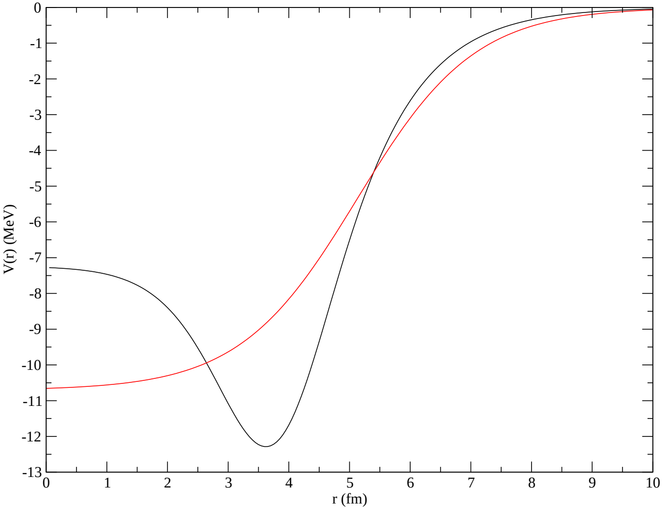


Figure: Imaginary potentials. The black line represents the addition of the Woods-Saxon volumetric and superficial imaginary spherical potentials hereby presented. The red line represents the imaginary potential reported by Vineyard et al.

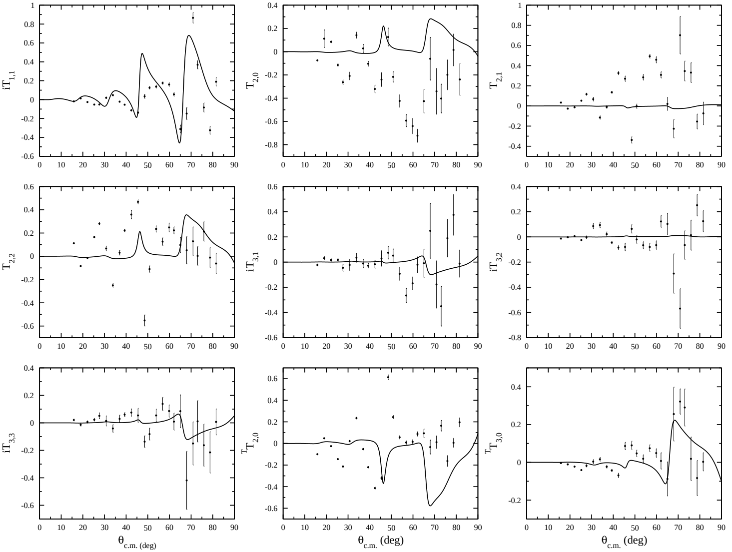


Figure: Experimental elastic analysing powers reported by Bartosz et al. The theoretical adjustments correspond to the complex Woods-Saxon volumetric potential plus a real spin-orbit potential reported by Vineyard et al. and Momotyuk et al. respectively.

Potential parameters - Other works

Potential	$U\ (MeV)$	$r_v\ (fm)$	$a_v\ (fm)$	$W\ (MeV)$	$r_w\ (fm)$	$a_w\ (fm)$
WS v. *Vineyard et al.*	290.0	0.64	0.64	10.71	1.22	0.97
SO *Bartosz et al.*	1.75	1.2	0.45	-	-	-

Potential parameters - This work

Potential	$U\ (MeV)$	$r_v\ (fm)$	$a_v\ (fm)$	$W\ (MeV)$	$r_w\ (fm)$	$a_w\ (fm)$	$\beta_{\lambda=2}^{Proj}$ of $^7Li_{g.s.}$
WS v.	290.0	0.7	0.69	7.25	1.16	1.0	-0.934
WS s.	-	-	-	6.75	0.9	0.65	-0.934
SO	2.75	0.925	0.525	0.1	1.075	0.4	-

The difference in the parameter W in the Woods-Saxon volumetric potentials led to the introduction of the superficial potential.

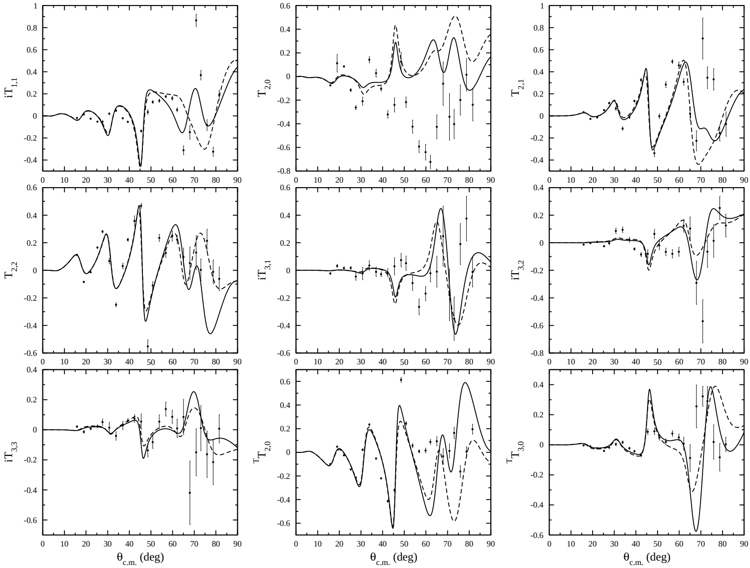


Figure: Experimental elastic analysing powers reported by Bartosz et al. The solid and dash line represent the adjustments with all the couplings to the 3 and 4 internal state of ^{7}Li respectively.

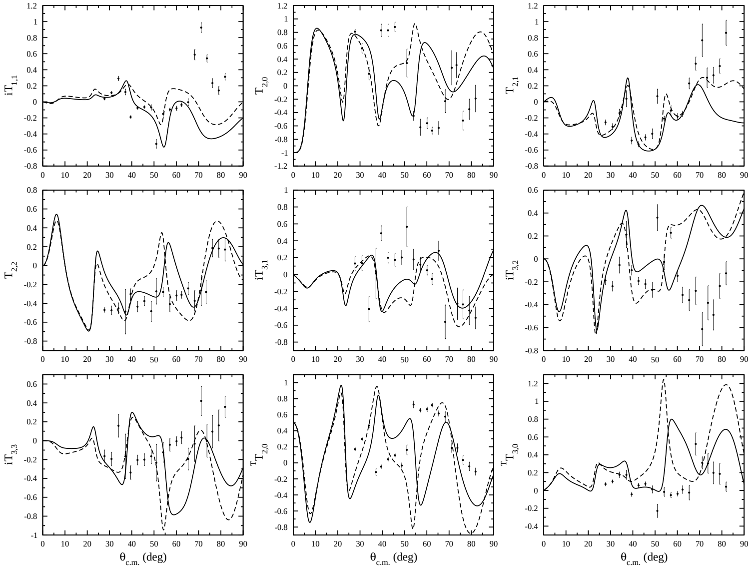


Figure: Experimental inelastic analysing powers reported by Bartosz et al. The solid and dash line represent the adjustments with all the couplings to the 3 and 4 internal state of ^{7}Li respectively.

Aye, there's the rub...

The theoretical calculations of the elastic analysing powers $T_{2,0}$, $T_{2,1}$, $T_{2,2}$, ${}^{T}T_{2,0}$, $iT_{3,1}$, $iT_{3,2}$, $iT_{3,3}$, ${}^{T}T_{3,0}$ had to be multiplied by -1 after the introduction of the reorientation.

So, is this the tie?...

Wave functions corresponding to odd A nuclei for the ^7Li must be used in the calculation of the reduced matrix element in the coupling potentials.

or

The ^7Li has a prolate form in its ground state.

- The scattering of ^{7}Li on ^{12}C at the energy studied requires a surface potential because its interaction is stronger at the surface of these nuclei.

- The ground state reorientation of the ^{7}Li plays an essential role in the description of the second rank analysing powers, **as it has been reported in other works using other interaction models**.

- Couplings to the lowest 3 and 4 internal states of ^{7}Li using the rotational model help to describe the third rank analysing powers.

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