Quantum Interference in Showering:

The LPM effect, what is is, and why its theoretical development is still interesting 60 years later

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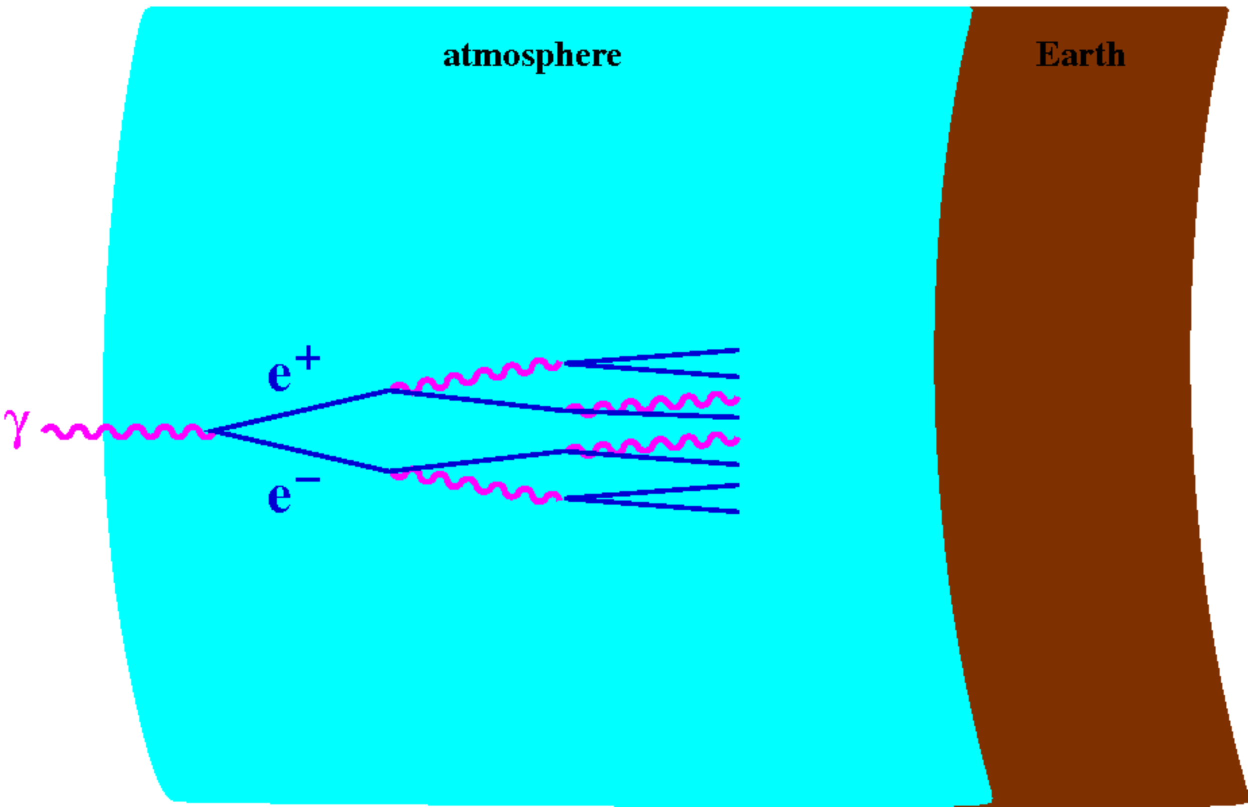
Reporting (eventually) on recent work with

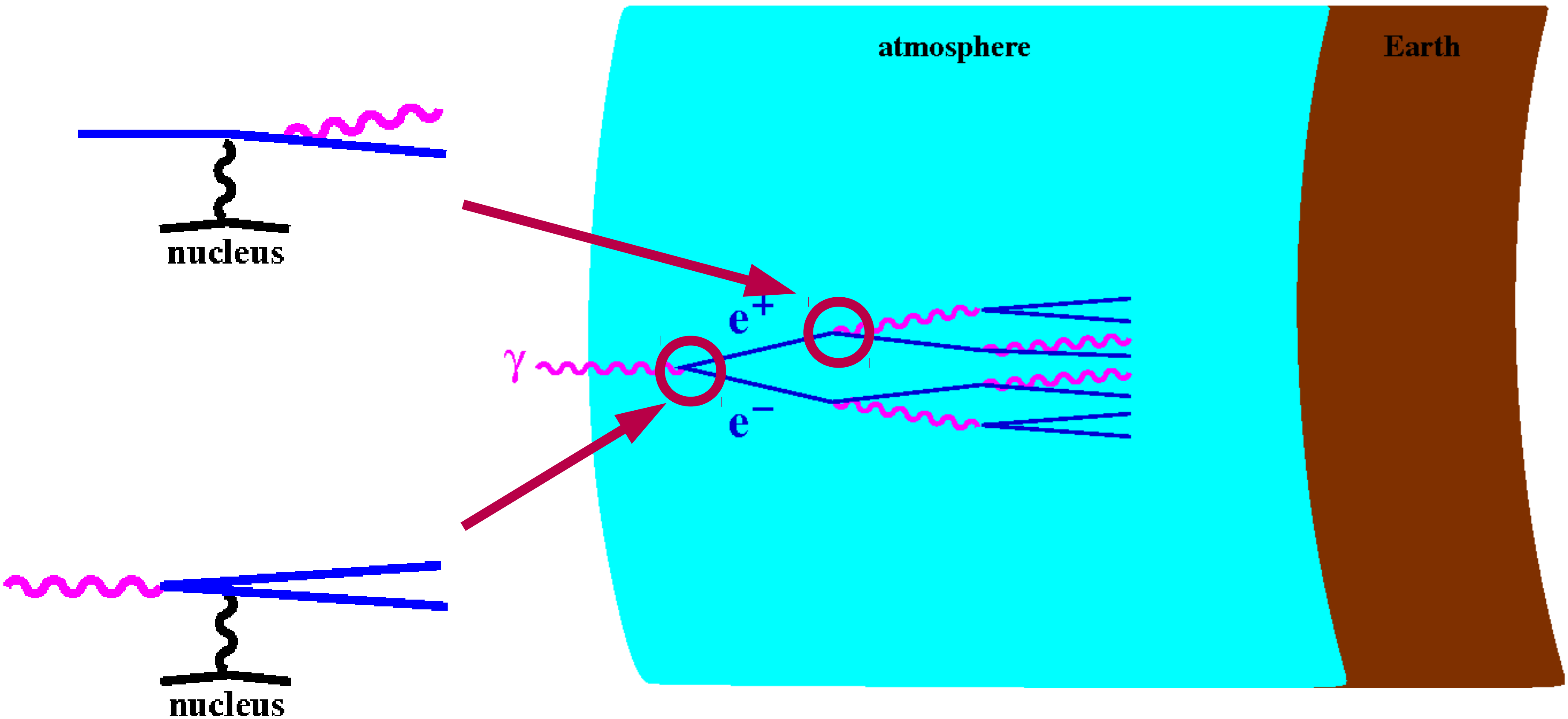
Shahin Iqbal

High energy particles traveling through matter lose energy via successive bremsstrahlung and pair production:



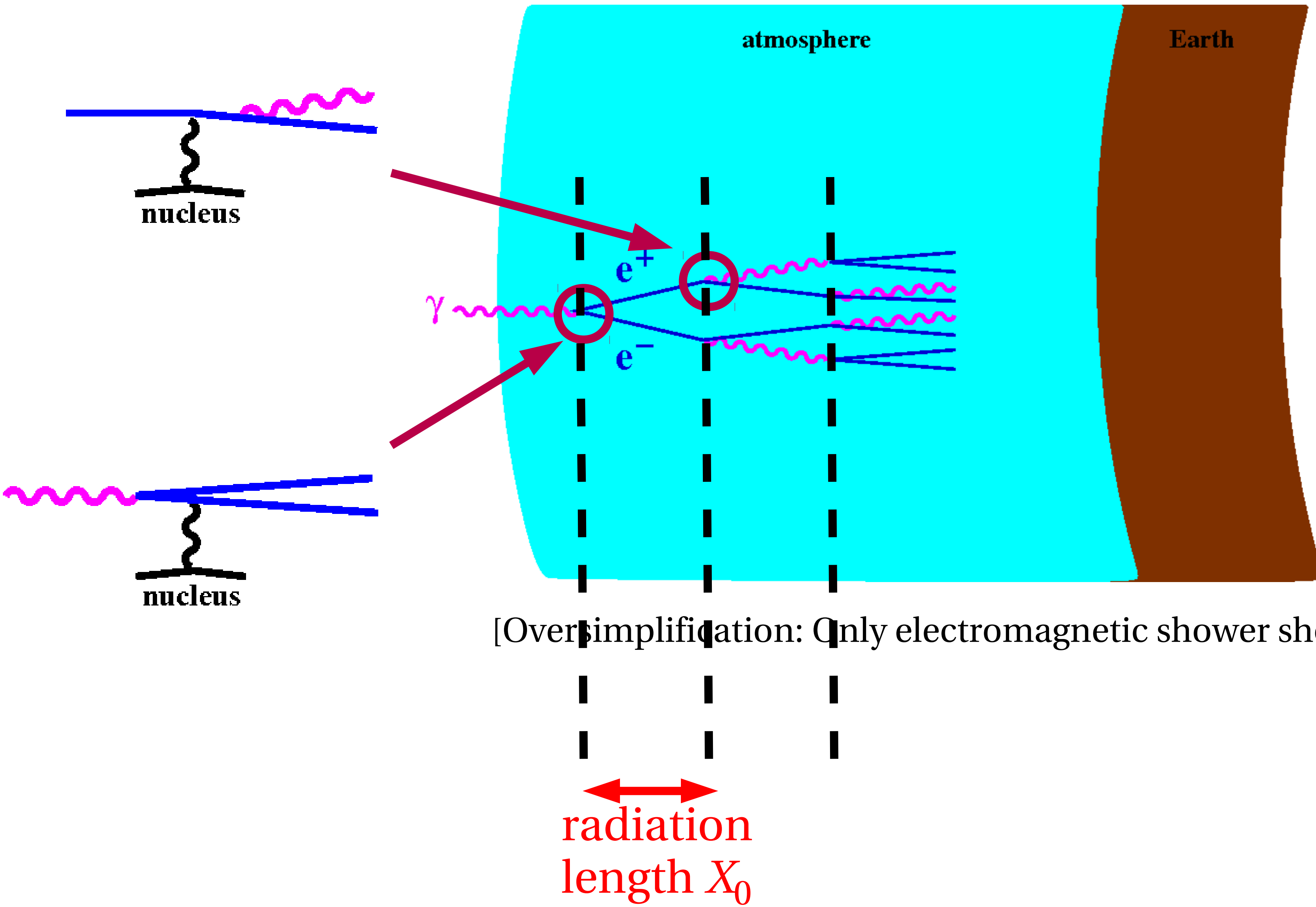
[Oversimplification: Only electromagnetic shower shown.]

High energy particles traveling through matter lose energy via successive bremsstrahlung and pair production:



[Oversimplification: Only electromagnetic shower shown.]

High energy particles traveling through matter lose energy via successive bremsstrahlung and pair production:



[Oversimplification: Only electromagnetic shower shown.]

Part 1

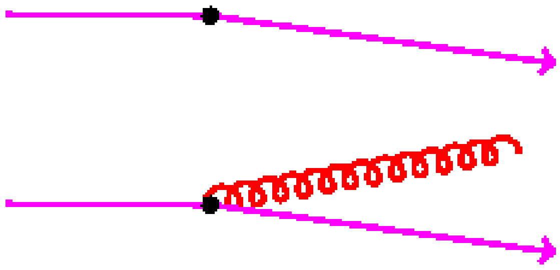
THE LPM EFFECT IN QED

[LPM = Landau, Pomeranchuk, Migdal]

Review of high-energy bremsstrahlung

Collisions with the medium

generate chances for bremsstrahlung



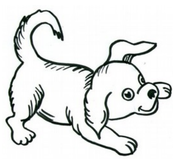
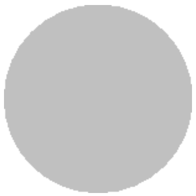
Naively,

prob of emission ~ α per collision

BUT

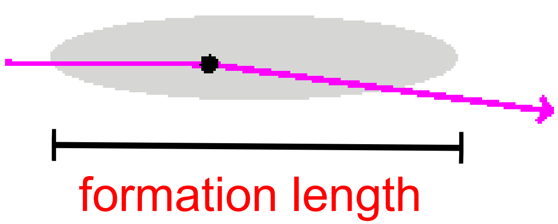
Light can't resolve features on small scales.

Non-relativistic:

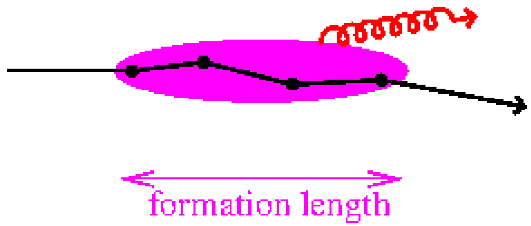
 and  both look like  if $\lambda >> d$.

Extremely relativistic, nearly-collinear motion:

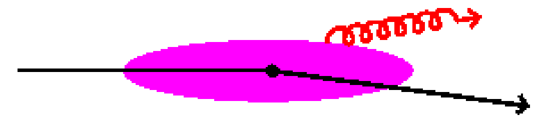
Similar effect, but size of fuzziness stretched out.



$$l_{\text{form}} \propto \sqrt{E}$$ (for fixed x)



indistinguishable from



So

prob of emission ~ α per formation length $l_{\rm form} \propto \sqrt{E}$

Calculated quantitatively by

LPM for QED (1950s)
BDMPS-Z for QCD (1990s)

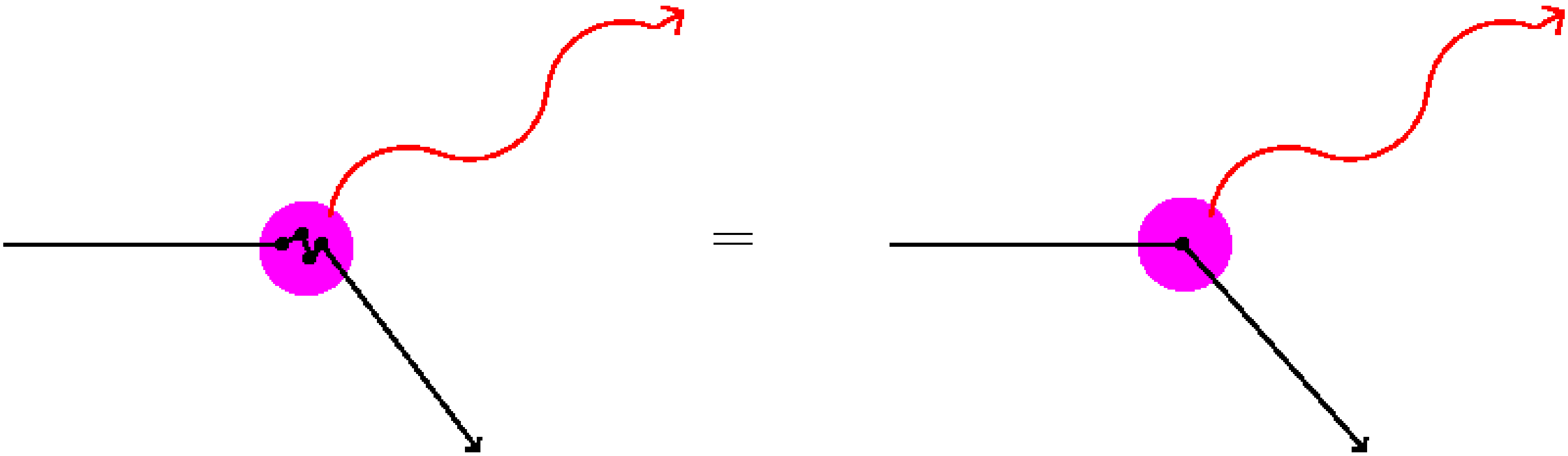
and investigated in many ways by many people since.

Consequence: At high enough energy, the effective bremsstrahlung rate in medium is reduced by factor $\propto \sqrt{E}$

[For QED, “high enough” energy means 200 PeV for air and 4 TeV for Lead for hard bremsstrahlung]

The LPM Effect (QED)

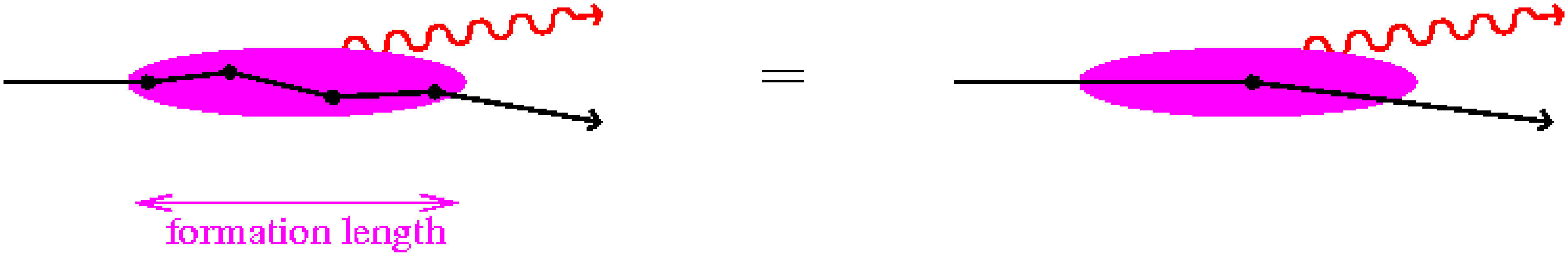
Warm-up: Recall that light cannot resolve details smaller than its wavelength.



[Photon emission from different scatterings have same phase → coherent.]

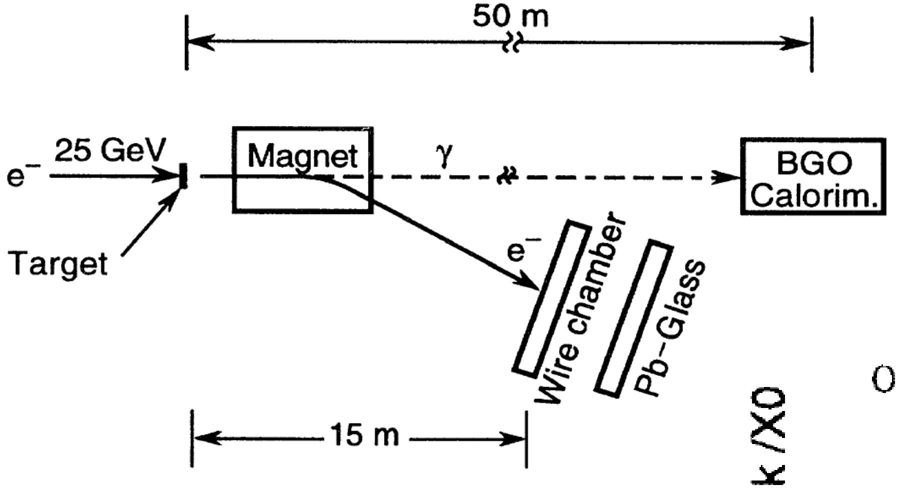
Now: Just Lorentz boost above picture by a lot!

The LPM Effect (QED)



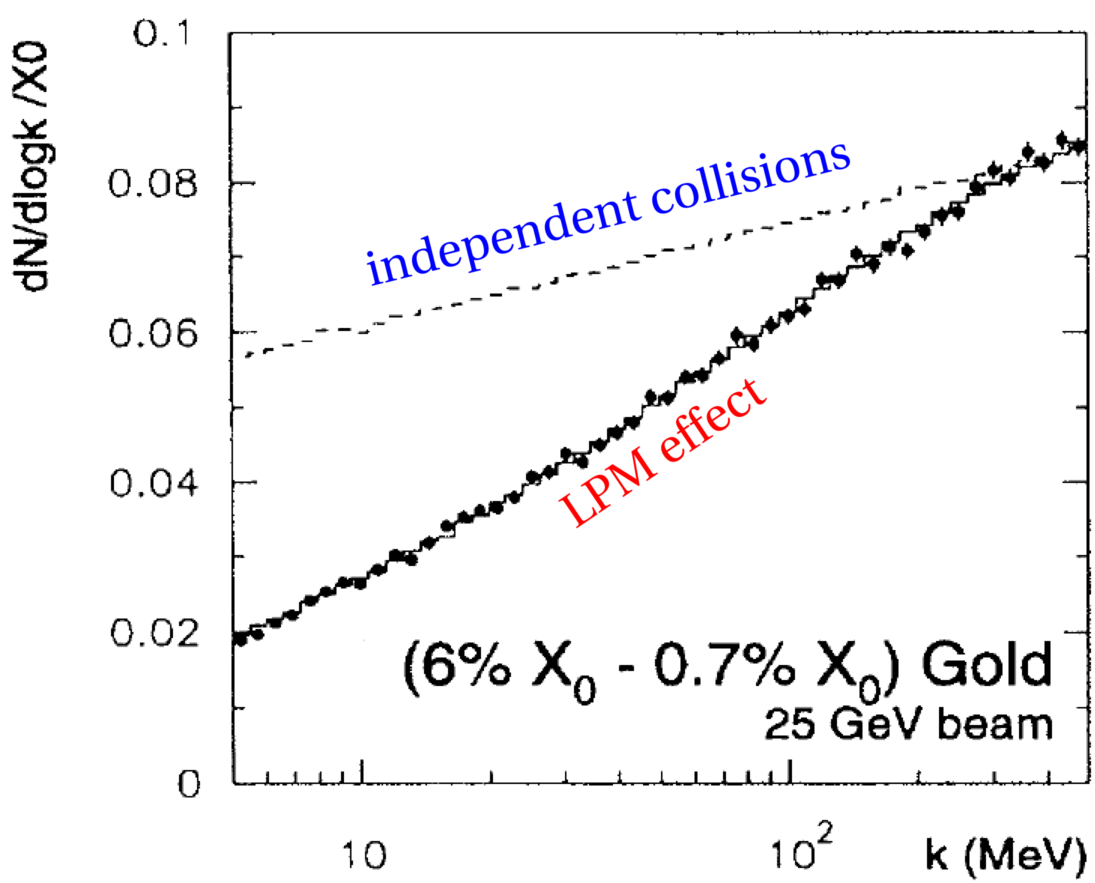
Note: (1) **bigger *E*** requires bigger boost → more time dilation → **longer formation length**
(2) big boost → this process is **very collinear.**

Experimental Measurement of LPM (QED)



SLAC E-146

Phys. Rev. Lett. **75** (1995) 2949.



Part 2

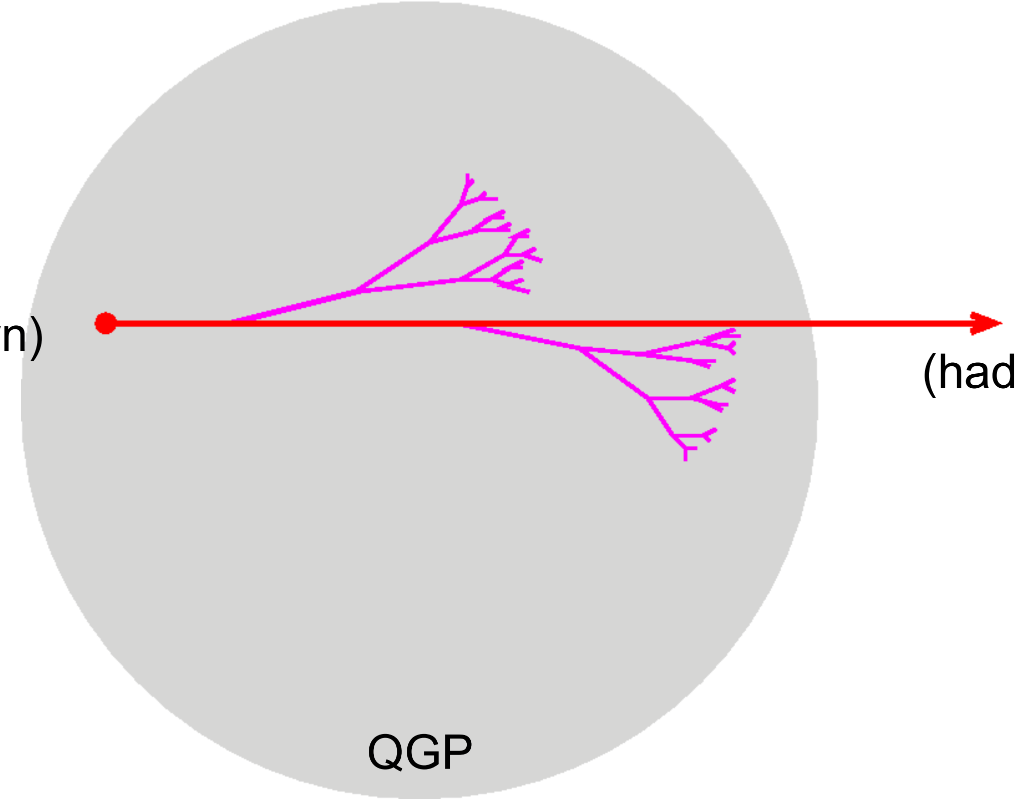
A new puzzle for LPM calculations in the 2010's

We could talk about this in QED, but we'll see that it's much more interesting to switch to a QCD application...

Consider cartoon of

In-medium evolution of a jet in a quark-gluon plasma

(initial vacuum-like radiation not shown)



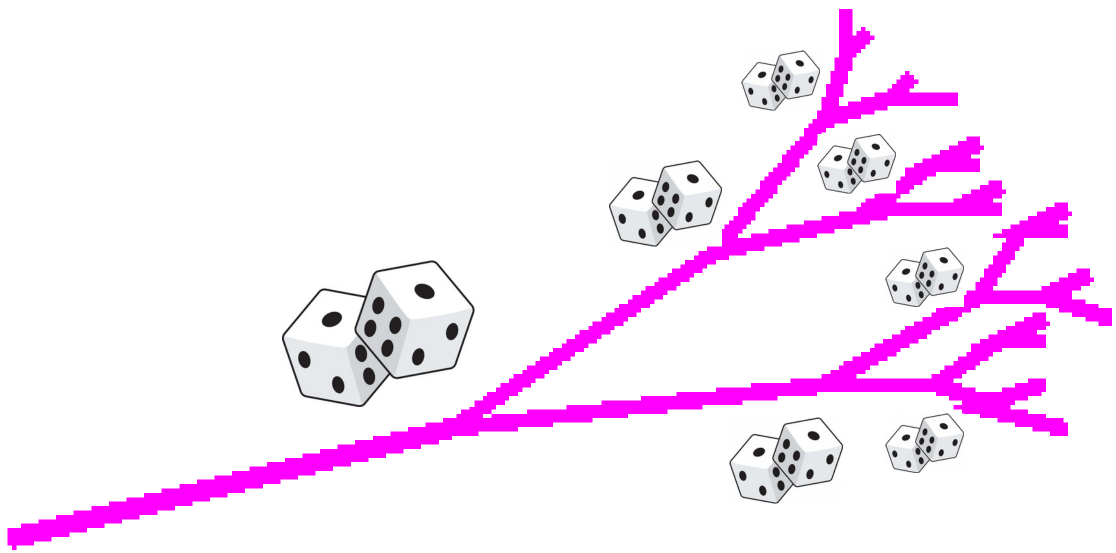
(hadronization not shown)

For this talk, simplify discussion by focusing on ...

Cascades that stop in-medium

- Qualitative points we'll discuss generalize.
- Formalism generalizable as well.

An idealized Monte Carlo picture of in-medium evolution



As time passes,

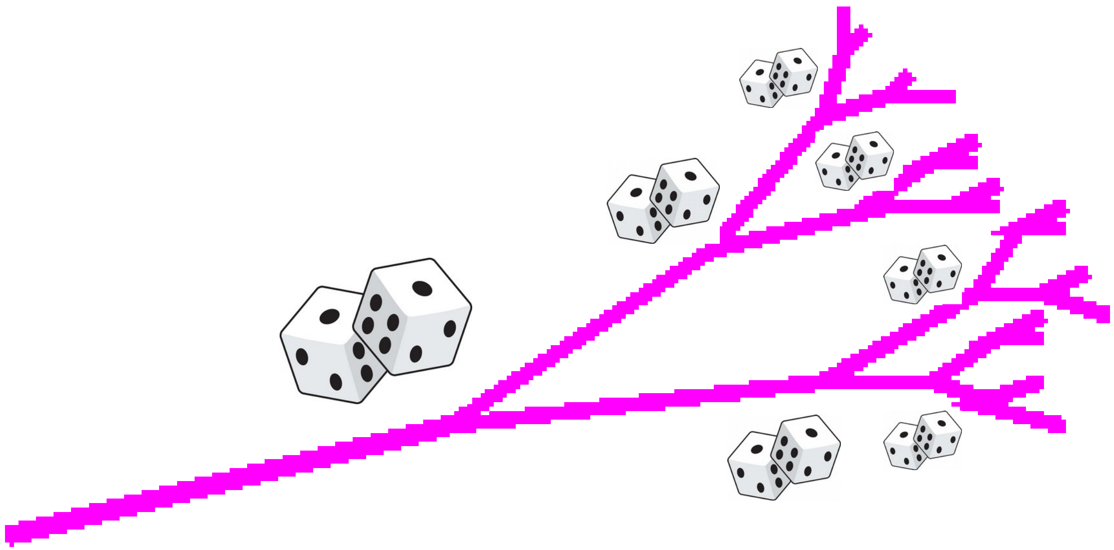
roll classical dice for probability of each splitting

weighted by the quantum calculation of the single splitting rate

$\frac{d\Gamma_{\rm brem}}{dx}$ for each vertex  shown above.

LPM effect included in this rate!

An idealized Monte Carlo picture of in-medium evolution



Built-in assumption:

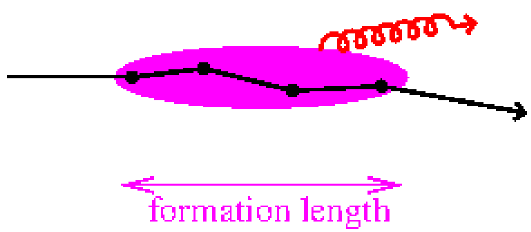
Consecutive splittings are quantum-mechanically independent.

(Are they ?)

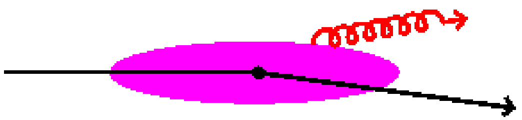
Remember from previous discussion:

Chance of brem ~ α per formation time

because



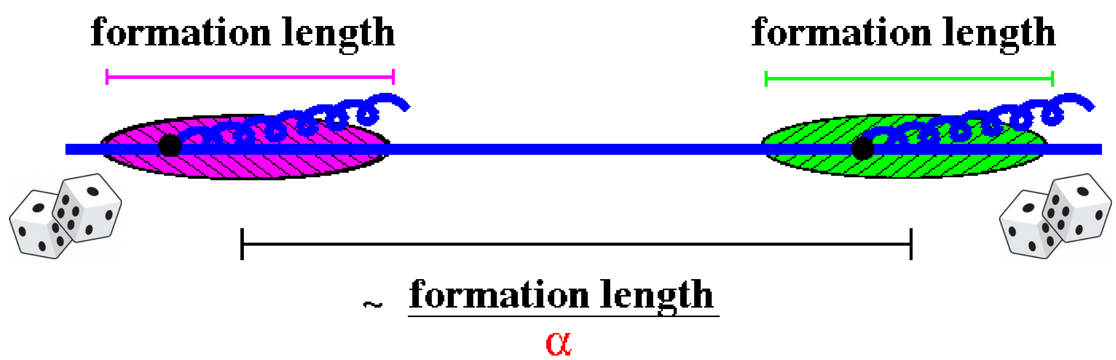
indistinguishable from



Consecutive emissions

Chance of brem ~ α per formation time

means that two consecutive splittings will typically look like



So chance of overlap (i.e. “rolling dice separately” breaking down) is

$\propto \alpha$

How big is “α” ??

How big is α_s?

Nothing to do with whether medium is

sQGP / perfect liquid

[$\alpha_s(T)$ big]

vs.

weakly-coupled QGP

[$\alpha_s(T)$ small]

α_s on previous slide associated with emission vertex:

costs roughly $\alpha_s(Q_\perp)$ with $Q_\perp \sim (\hat{q}E)^{1/4} \lesssim$ a few GeV

panic and/or fool around with AdS/CFT energy loss

[$\alpha_s(Q_\perp)$ big]

vs.

LPM-based analysis

[$\alpha_s(Q_\perp)$ small]

Does the wisdom of the ages tell us if α_s(few GeV) is small?

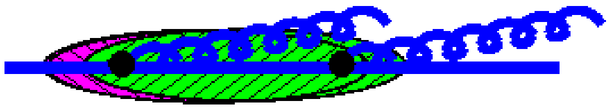
Particle physics in vacuum:

Small for some things, like matching lattice calculations to continuum MS-bar α_s

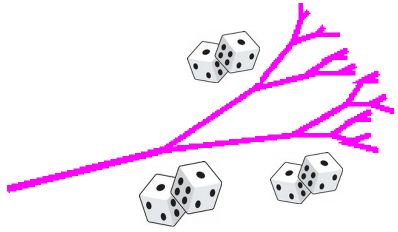
High-temperature physics:

Bad news (except possibly if one does sophisticated resummations of perturbation series)

Overlapping formation times effects on cascade:

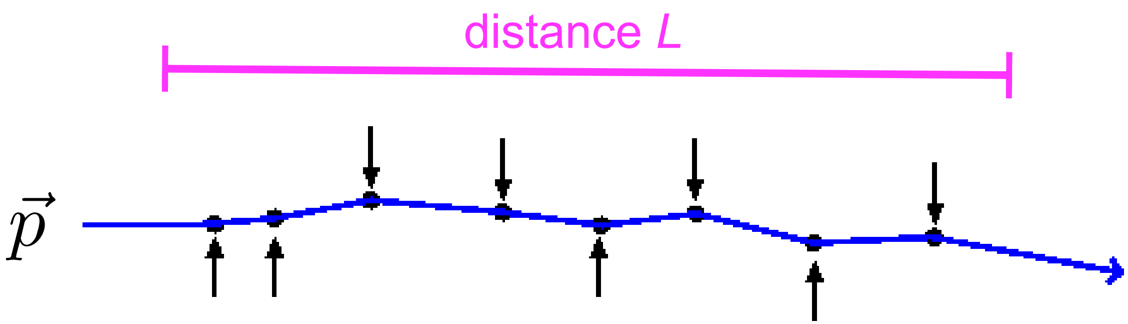


$\propto \alpha$ effect on

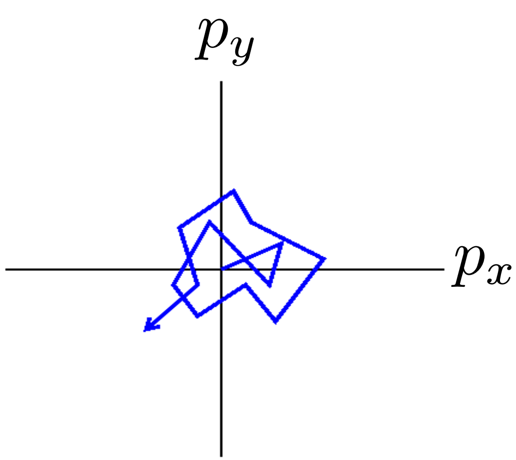


We should calculate it and see.

Characterizing the medium: $\hat{q}$



Random kicks from medium change p_T by tiny amounts << E



→ Random walk in transverse momentum plane:

$$(p_\perp)_{\text{rms}} \propto \sqrt{N_{\text{kicks}}} \propto \sqrt{L}$$

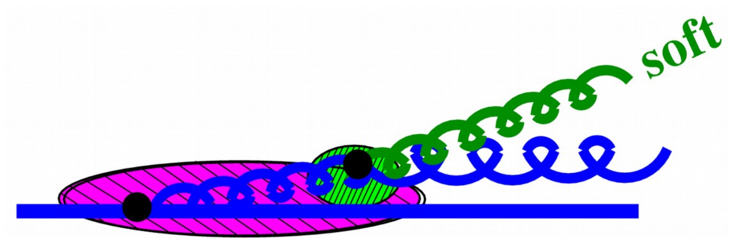
$$\langle p_\perp^2 \rangle = \hat{q} L$$

$\hat{q}$ defined as this proportionality constant

It's the only characteristic of the medium that matters for the problem under discussion.

Soft emission

Soft emissions are generally enhanced by logs.
Path-breaking authors found small-*x*-like double logs in this case,



$$\propto \alpha_s \ln^2 \left(\frac{E}{\hat{q}\tau_{\rm mfp}} \right)$$

Blaizot & Mehtar-Tani; Iancu; Wu (2014)

This is a BIG effect for large *E*.
But they found soft emission effects could be absorbed into the medium parameter

$$\hat{q} \to \hat{q}_{\rm eff}(E) \propto E^{\#\sqrt{\alpha_s}}$$

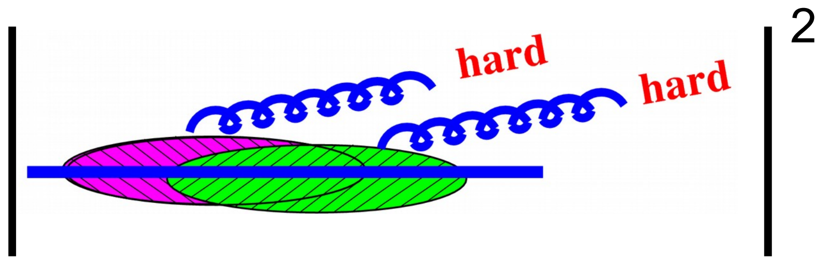
following Liou, Mueller, Wu (2013)

Refined question

What about overlap effects that *can't* be absorbed into $\hat{q}$?

Our program

Compute the effect of the overlap for hard emissions

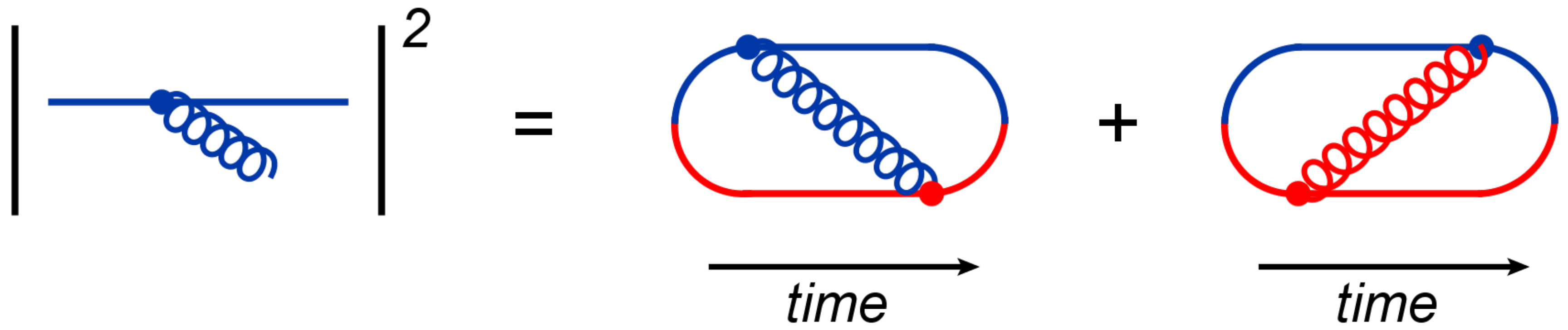


⇒ relative $O(\alpha_s)$ correction due to overlap effects

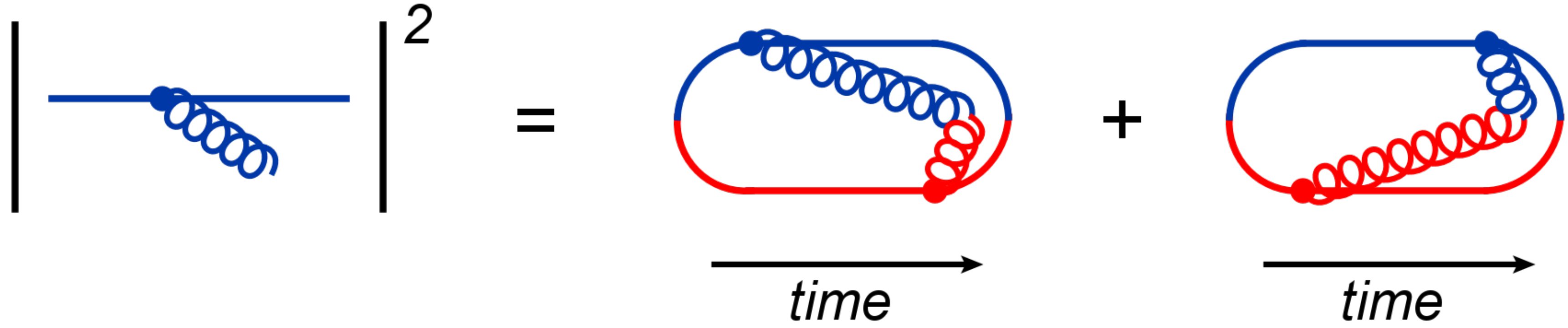
In broad brush: interesting and fun field theory problem.

In calculational detail: a pain in the ass.

First: How we draw diagrams



First: How we draw diagrams



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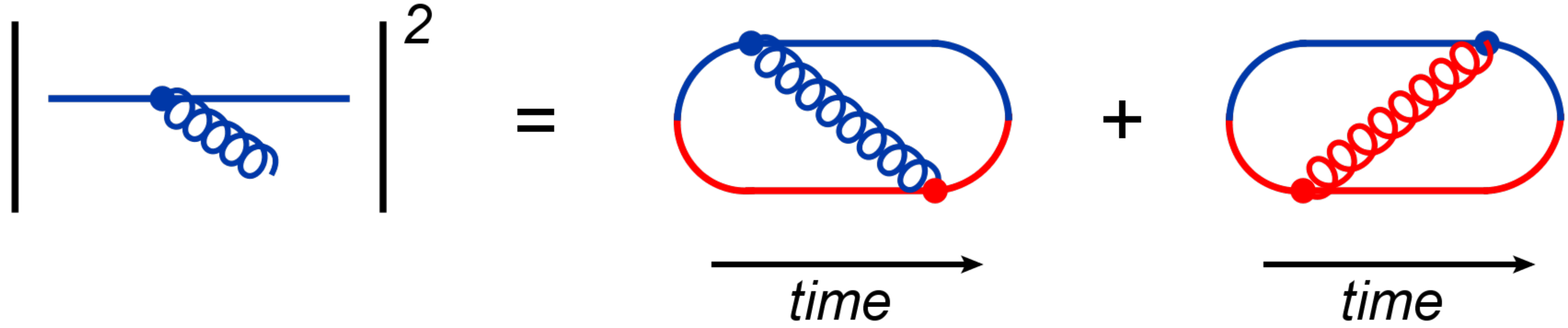

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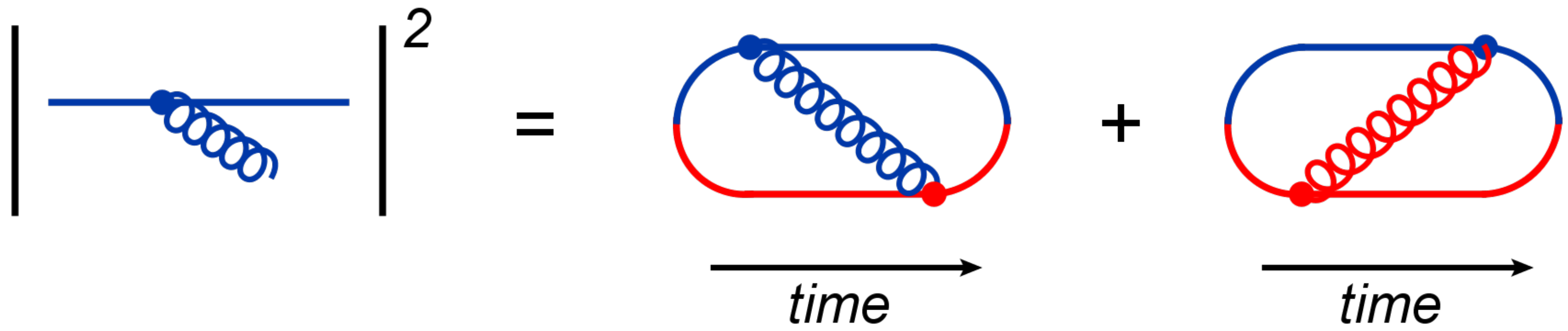
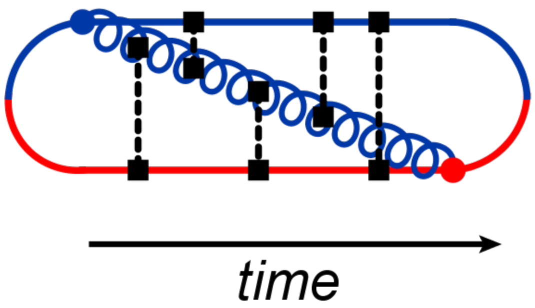


implicitly including interactions with the medium (in invisible ink above):



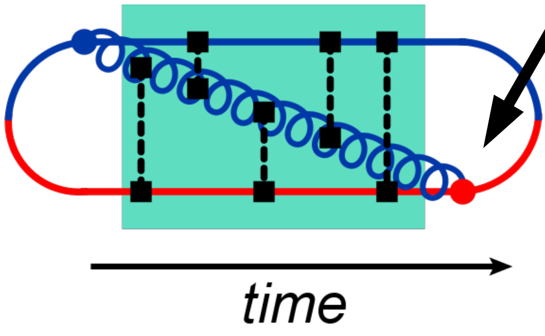
■ = interaction with medium

- - - - = correlations in medium (relatively localized in time) taken from

- perturbation theory
- AdS/CFT
- or phenom. fit to $\hat{q}$

Medium-averaged evolution can be treated (at high energy) as (non-Hermitian) 2-dim quantum mechanics problem in transverse plane.

High-energy splitting vertices can be taken from QFT (DGLAP splitting amplitudes).



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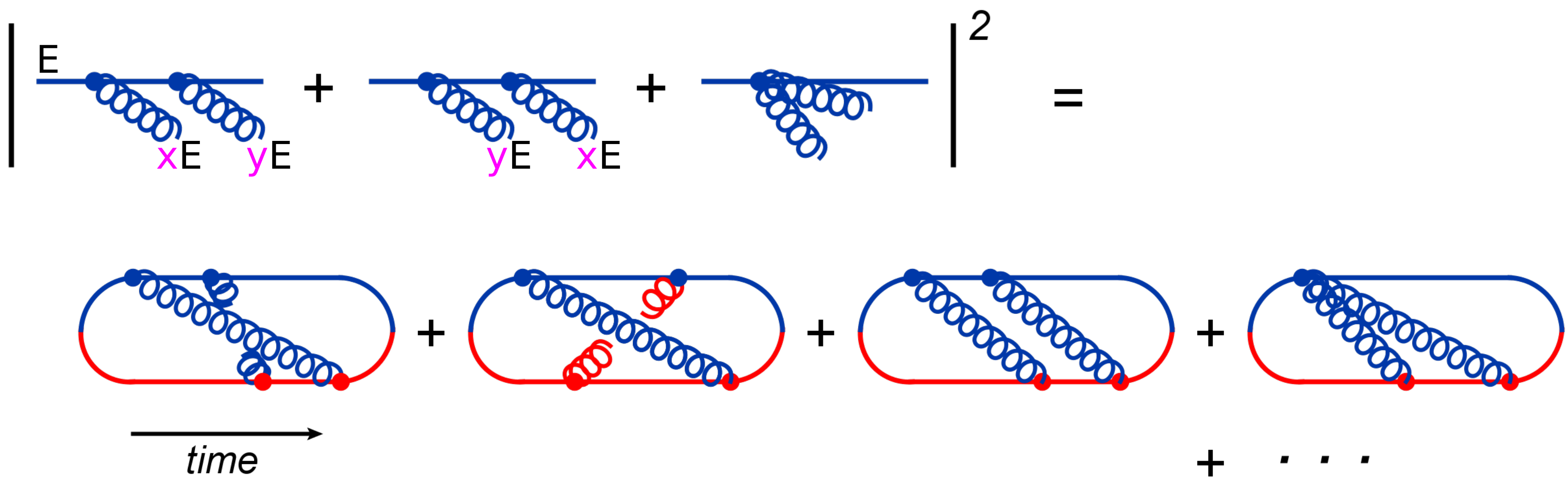
Double Splitting Diagrams



[calculated with Shahin Iqbal and Han-Chih Chang]

<u>Infrared Issue:</u>

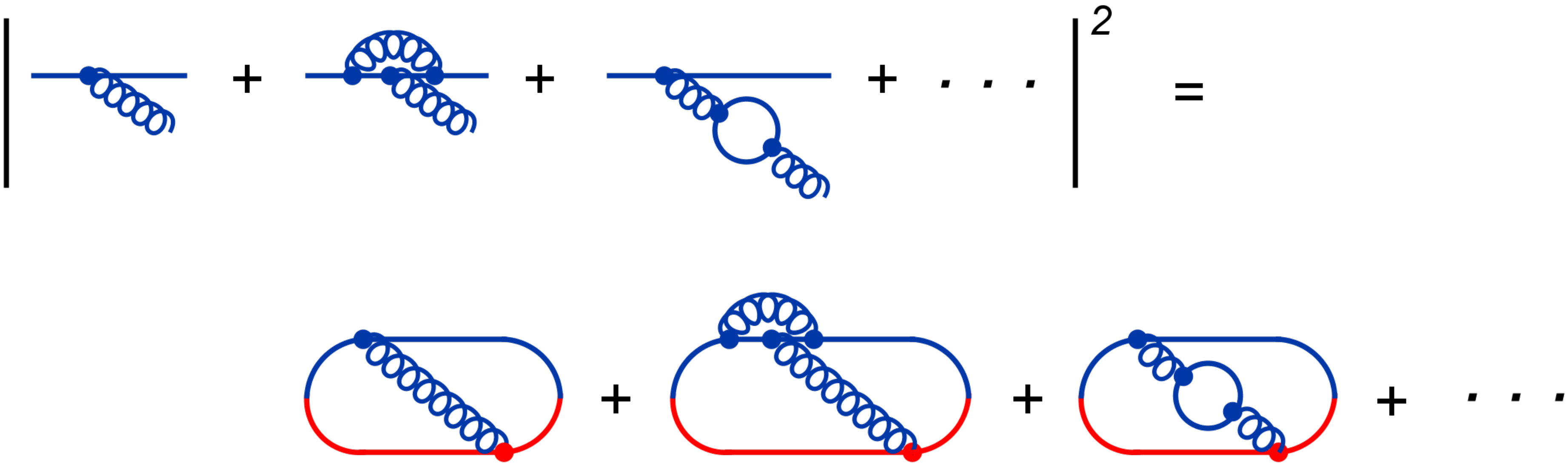
$$\frac{d\Gamma}{dx\,dy} \sim \frac{\alpha_{\rm s}^2}{xy^{3/2}}\sqrt{\frac{\hat q}{E}} \qquad (\text{for } y \lesssim x),$$

giving power-law IR-divergent contributions to energy loss, etc.

Part 2

VIRTUAL CORRECTIONS

Need virtual corrections to single splitting

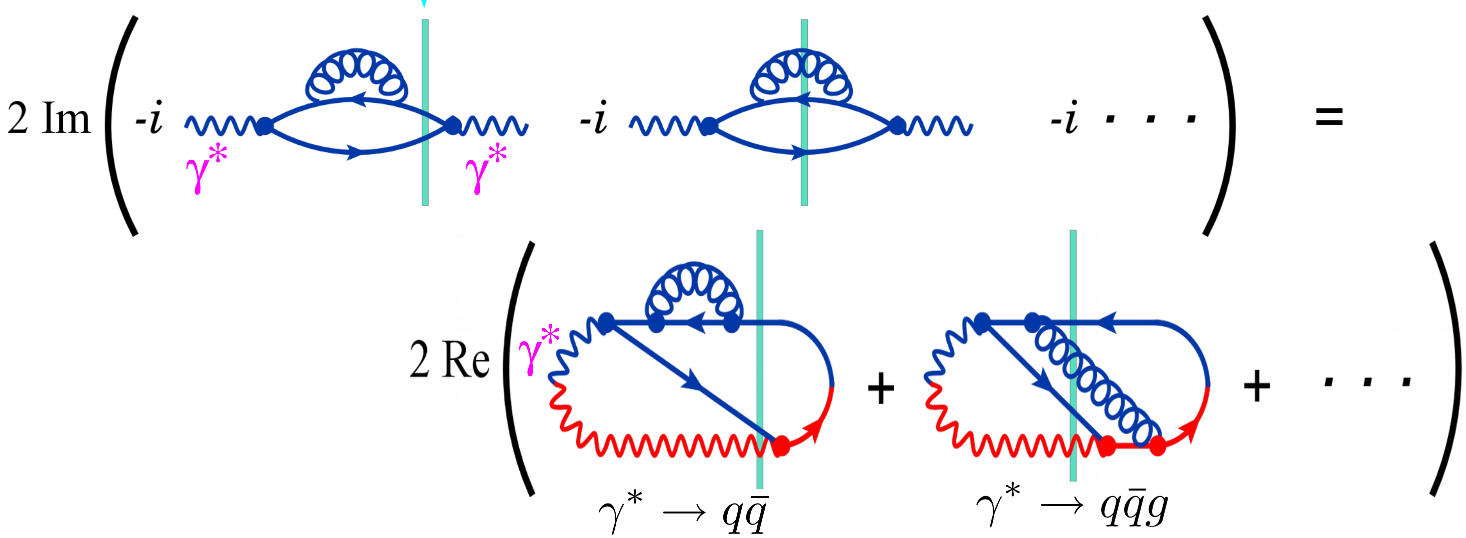


These have UV divergences that renormalize α in leading diagram.

Our calculations vs. small-x DIS

Small-x Deep Inelastic Scattering: Hänninen, Lappi, Paatelainen (2016,2017); Beuf (2016,2017)

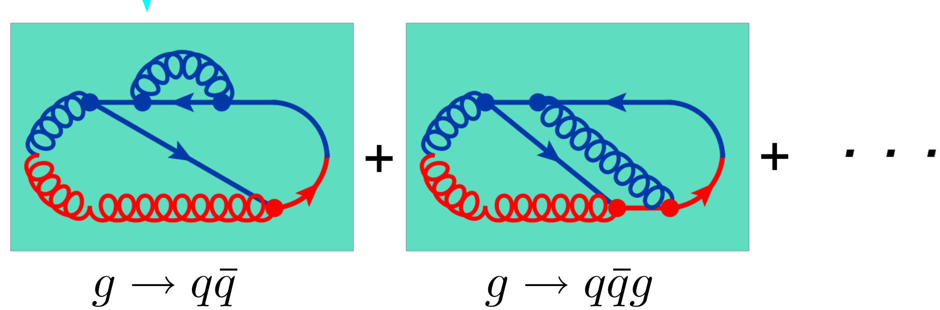
very Lorentz-contracted medium (medium width << formation length)



(in our own notation)

Our problem: (e.g.)

all propagators in medium! (medium width << formation length)

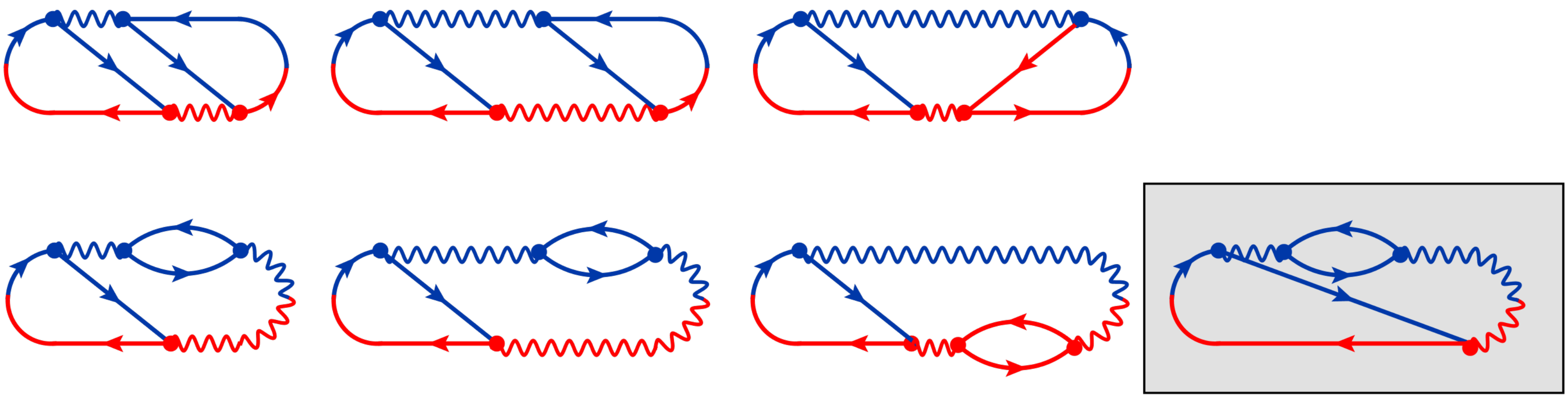


What we've actually done, as a warm-up [arXiv:1806.08796, two weeks ago]:

Large-N_f QED

$e \longrightarrow e\bar{e}e$

$e \longrightarrow \gamma e$

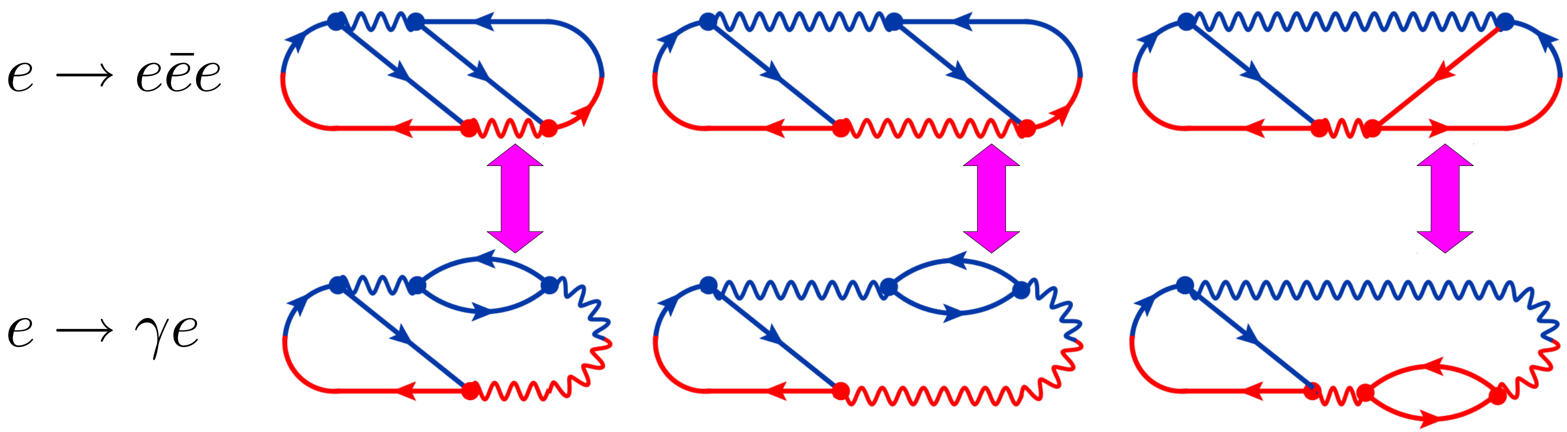


Calculate these diagrams using dimensional regularization.

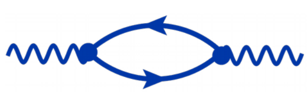
Remember: All time evolution is in medium background, statistically averaged over medium fluctuations.

What we've actually done, as a warm-up [arXiv:1806.yesterday]:

Large-N_f QED



Related by **conservation of probability** because for *these* diagrams the important piece of

 ~ probability photon does **not** split

What we've actually done, as a warm-up [arXiv:1806.yesterday]:

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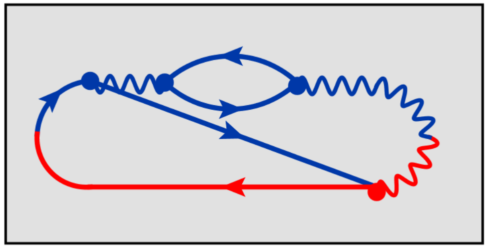
$e \longrightarrow e\bar{e}e$

$e \longrightarrow \gamma e$



But this (UV-divergent) diagram is **complicated** and *also* generates the **renormalization of α**:

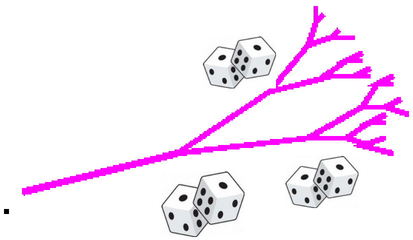
$$\alpha \longrightarrow \alpha(\mu)$$

with

$$\mu \sim (\text{hard particle transverse separation})^{-1}$$
$$\sim Q_\perp \sim (\hat{q} t_{\text{formation}})^{1/2} \sim (\hat{q} E)^{1/4}$$

Conclusion

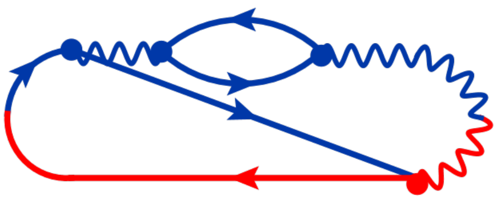
Reminder



Ultimate goal: figure out whether rolling independent dice for in-medium QCD shower is good, bad, or ugly for slightly-small α_s.

Our Recent Progress

Using large-N_f QED as an example, we've shown we can compute necessary virtual corrections to single emission.



+ . . .

Sanity check: The divergent part of these calculations correctly reproduces the known renormalization of α. ✔

What about QCD?

We *think* that we have now done everything difficult and that results for (large-Nc) QCD can now be obtained by a combination of

(I) simple and simple-ish transformations of other QCD diagrams previously calculated, the simplest transformation being the "conservation of probability" ⇕

(2) adaptation of large-Nf QED diagrams now calculated by just changing to QCD group factors, QCD DGLAP splitting functions, etc.

Stay tuned.

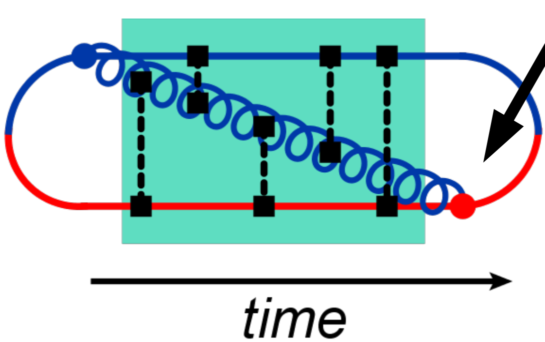
Super fun bonus material!

Packet A

Framework for LPM calculations

Medium-averaged evolution can be treated (at high energy) as (non-Hermitian) 2-dim quantum mechanics problem in transverse plane.

High-energy splitting vertices can be taken from QFT (DGLAP splitting amplitudes).



■ = interaction with medium

- - - - = correlations in medium (relatively localized in time) taken from

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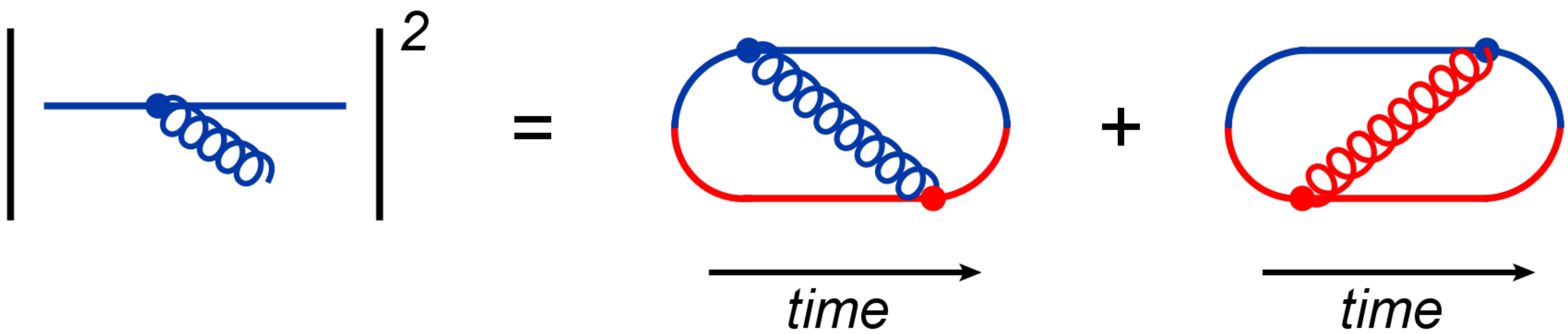
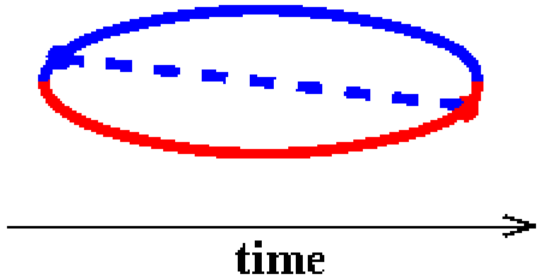
Formalism for LPM: single brem

Recall

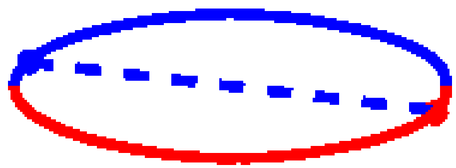


Focus on first diagram, and use simpler graphics:



Can (formally) interpret this as 3 particles moving forward in time [Zakharov 1990's]:

2 particles from the amplitude (evolving with e^{-iHt})
1 particle from the conjugate amplitude (evolving with e^{+iHt})

Will show that evolution in  can be described by

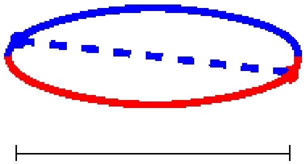
3-particle non-relativistic Quantum Mechanics in 2 dimensions

$$\mathcal{H}_{\mathrm{eff}} = \frac{p_{\perp 1}^2}{2m_1} + \frac{p_{\perp 2}^2}{2m_2} + \frac{p_{\perp 3}^2}{2m_3} + V(b_1, b_2, b_3)$$

with weird properties:

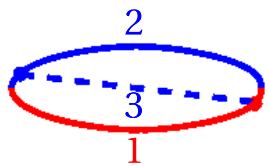
- $m_1 + m_2 + m_3 = 0$
- $V \propto -i$ (i.e. $\mathcal{H}$ is non-Hermitian)

⇨ interference vanishes as $\Delta t \to \infty$, as it must!



Kinetic terms:

Energy of a high-p_z particle: $$\epsilon_p = \sqrt{p_z^2 + p_\perp^2} \simeq p_z + \frac{p_\perp^2}{2p_z}$$

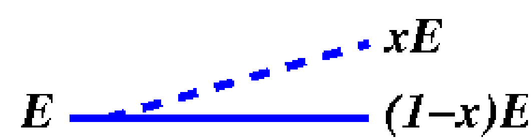
Evolution of 

is $e^{-i\mathcal{H}t}$ with

$$\mathcal{H}_{\rm kin} = -\epsilon_{p_1} + \epsilon_{p_2} + \epsilon_{p_3} \simeq -\frac{p_{\perp 1}^2}{2p_{z1}} + \frac{p_{\perp 2}^2}{2p_{z2}} + \frac{p_{\perp 3}^2}{2p_{z3}}$$

$$\simeq -\frac{p_{\perp 1}^2}{2E} + \frac{p_{\perp 2}^2}{2(1-x)E} + \frac{p_{\perp 3}^2}{2xE}$$

conjugate evolves with $e^{+i\mathcal{H}t}$



This is 2-dimensional non-relativistic QM with

$$(m_1, m_2, m_3) = (-E, (1-x)E, xE)$$

As promised,

$$m_1 + m_2 + m_3 = 0$$

Potential term:

$V(b_1, b_2, b_3)$ incorporates (statistically averaged) effect of collisions with the medium.

Potential terms:

To motivate form, think of something else...

A classical Boltzman analysis of scattering:

$$\frac{d}{dt} f(p_\perp) = \underbrace{\int_{q_\perp} f(p_\perp - q_\perp) \frac{d\Gamma_{\rm el}}{dq_\perp}}_{\text{gain term}} - \underbrace{f(p_\perp) \int_{q_\perp} \frac{d\Gamma_{\rm el}}{dq_\perp}}_{\text{loss term}}$$

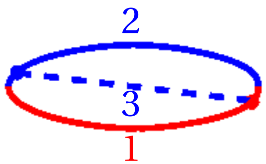
Fourier transform:

$$\frac{d}{dt} f(b) = f(b) \left[\Gamma_{\rm el}(b) - \Gamma_{\rm el}(0)\right] \quad \text{with} \quad \Gamma_{\rm el}(b) \equiv \int_{q_\perp} \frac{d\Gamma_{\rm el}}{dq_\perp} e^{-ib\cdot q_\perp}$$

This looks like a Schrodinger-ish equation:

$$i \frac{d}{dt} f = \mathcal{H}_{\rm boltz} f \qquad \text{with} \qquad \mathcal{H}_{\rm boltz} = -i\left[\Gamma_{\rm el}(0) - \Gamma_{\rm el}(b)\right]$$

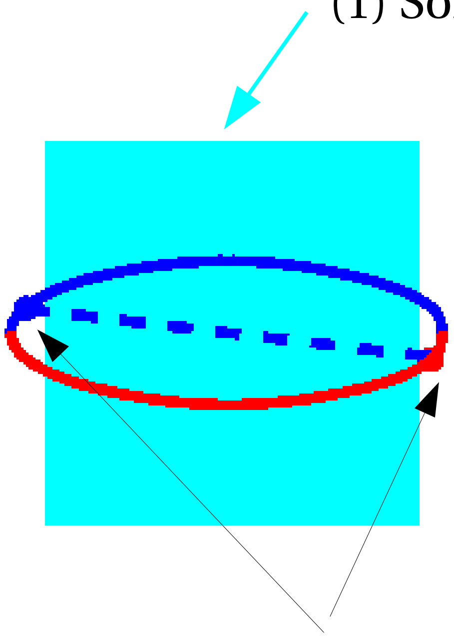
In our problem, this physics gives V:



$$V = -i\left[\Gamma_{\rm el}(0) - \Gamma_{\rm el}(b_2 - b_1)\right] \qquad \text{(QED)}$$

How to put the calculation together:

(1) Solve for propagation in 3-particle QM in shaded region.



(2) Tie together with QFT matrix elements for vertices

$$\propto \sqrt{\textbf{DGLAP splitting functions}}$$

$$\propto \sqrt{P_{i\to j}(x)}$$

Simplifcation: 3-particle QM → 1-particle QM

Can use various symmetries of problem to get rid of 2 d.o.f.

$$\mathcal{H} = \frac{P_B^2}{2M} + V(B)$$

[BDMPS-Z (1990's)]

Method 1. Can solve numerically. [Zakharov (2004+); Caron-Huot & Gale (2010)]

Simplifcation: Harmonic Oscillator

Method 2. High energies → very collinear → b's small.

So make small B approximation to $V(B)$ → a harmonic oscillator problem

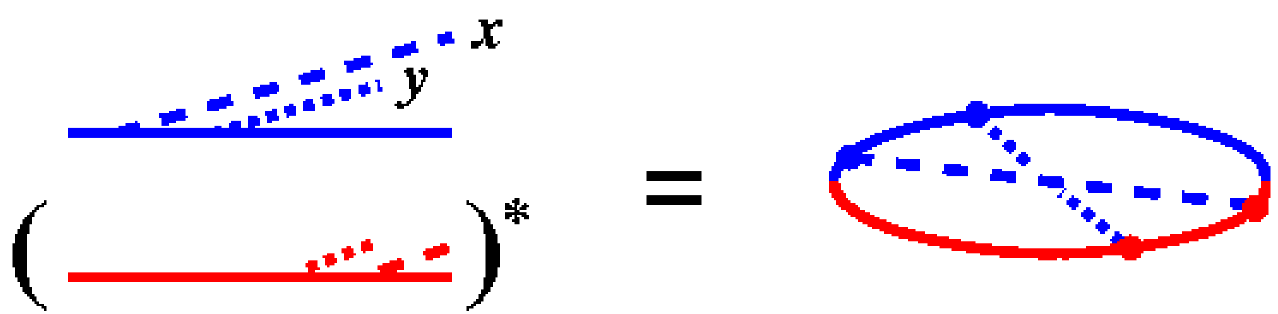
$$\mathcal{H} = \frac{P_B^2}{2M} + \frac{1}{2} M \Omega_0^2 B^2$$

[Baier *et al*. (1998)]

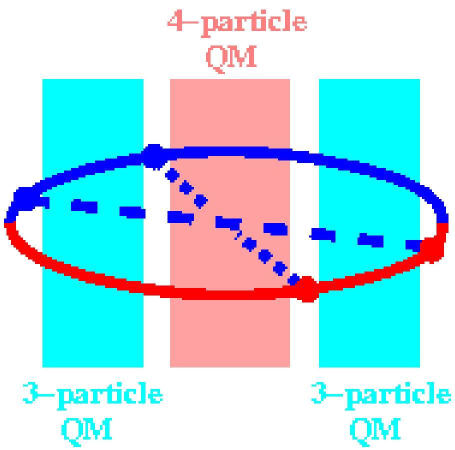
(a non-Hermitian one: $\Omega_0^2 \propto -i$)

Formalism for LPM: double brem

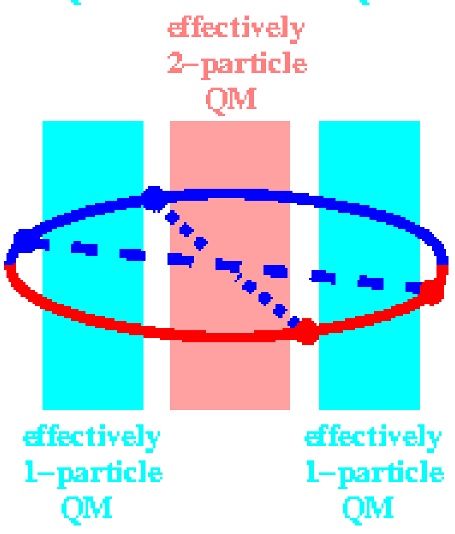
Example of an interference contribution:



To compute: Sew together QFT matrix element for vertices with QM evolution in between.



Simplify: Using symmetries, as before.

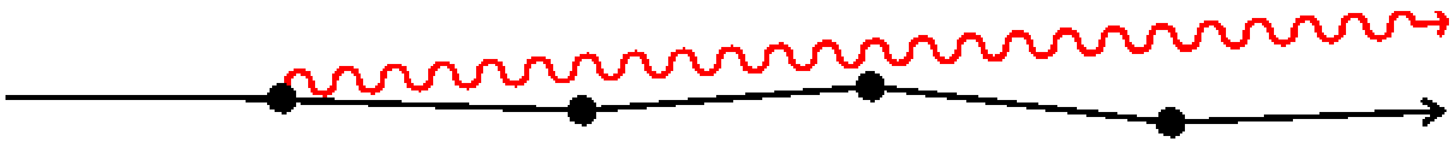


Super fun bonus material!

Packet B

QED vs. QCD (qualitative)

An alternative picture of LPM Effect (QED)



versus

Are these two possibilities in phase? Or does the interference average to zero?

IN PHASE if (i) everything is nearly collinear ✓
(ii) particle and photon have nearly same velocity ✓ (*speed of light*)

The LPM Effect (QCD)

There is a qualitative difference for ***soft*** bremsstrahlung.:

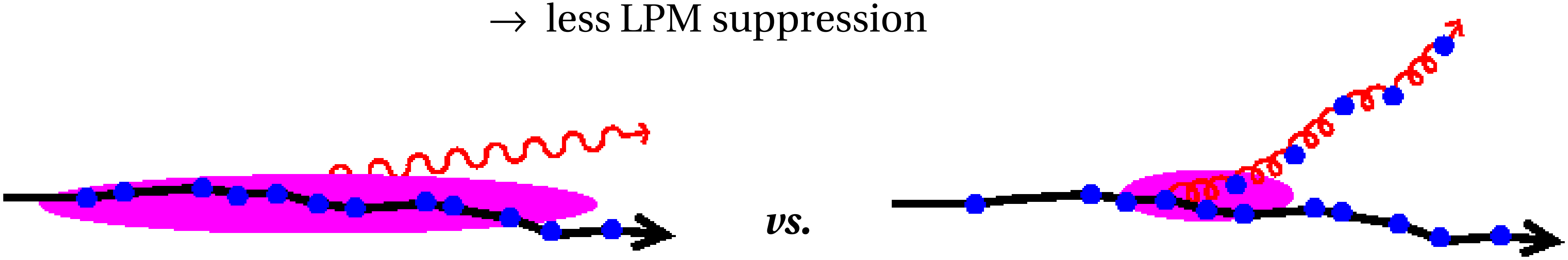
QED

Softer brem photon $\rightarrow$ longer wavelength
$\rightarrow$ less resolution
$\rightarrow$ more LPM suppression

QCD

Unlike a brem photon, a brem gluon can easily scatter from the medium.

Softer brem gluon $\rightarrow$ easier for brem gluon to scatter
$\rightarrow$ less collinearity
$\rightarrow$ less LPM suppression

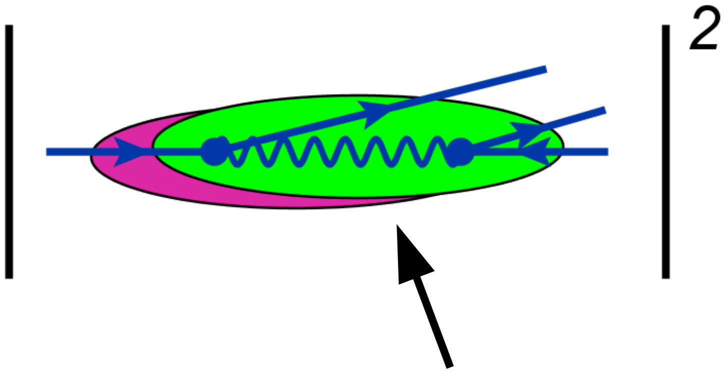


Upshot: Soft brem more important in QCD than in QED (for high-E particles in a medium)

Super fun bonus material!

Packet C

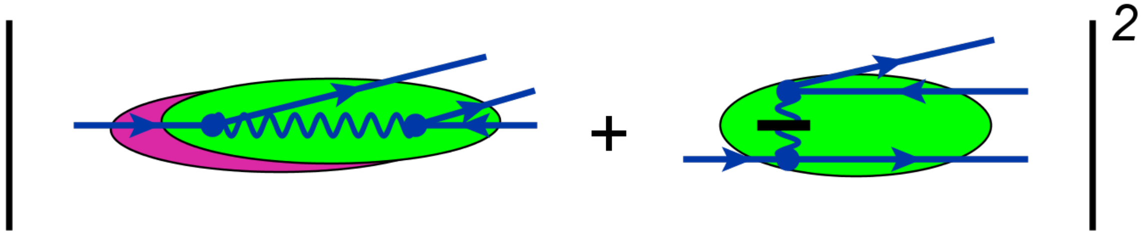
Light Cone Perturbation Theory



Transverse polarization... what about longitudinal?

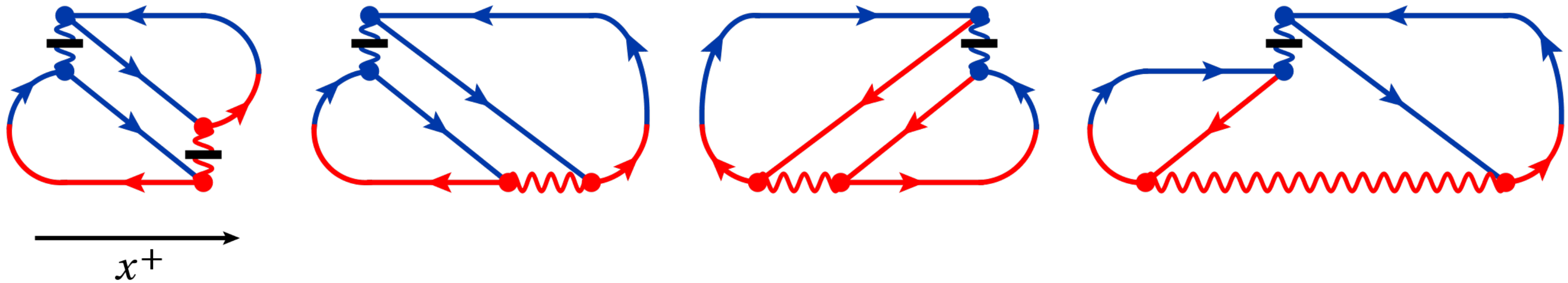
To work with only transverse photons, need to integrate out longitudinal ones.

Light-cone gauge → new interactions that are instantaneous in light-cone time x^+
→ need

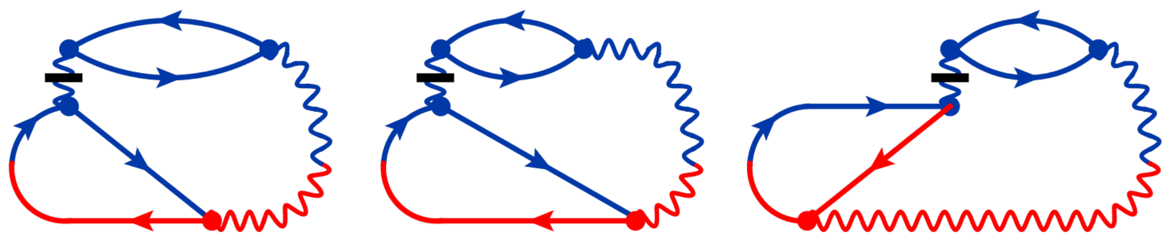


C2

$e \longrightarrow e\bar{e}e$

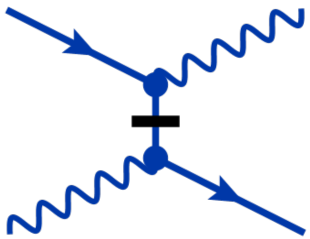


$e \longrightarrow \gamma e$

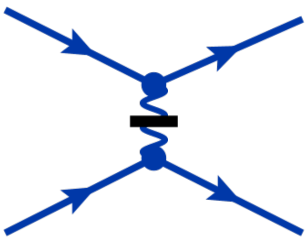


Yet more diagrams?

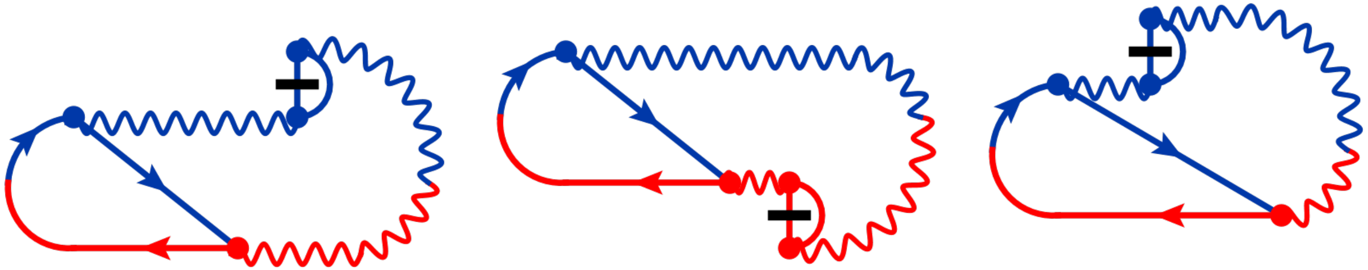
When you integrate out all the non-physical polarizations, Light-Cone Perturbation Theory also has x^+-instantaneous interactions



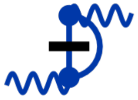
in addition to the previous



which generates the additional loop diagrams



Fortunately, Lappi and Paatelainen (2016) taught me that, when masses are ignorable,

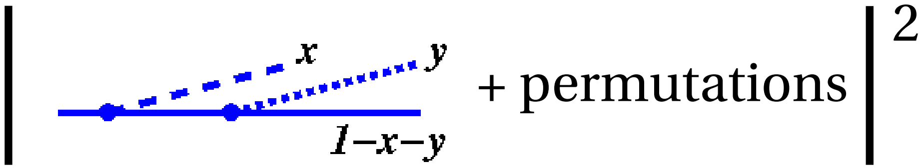
 = 0 in dimensional regularization in *vacuum*.

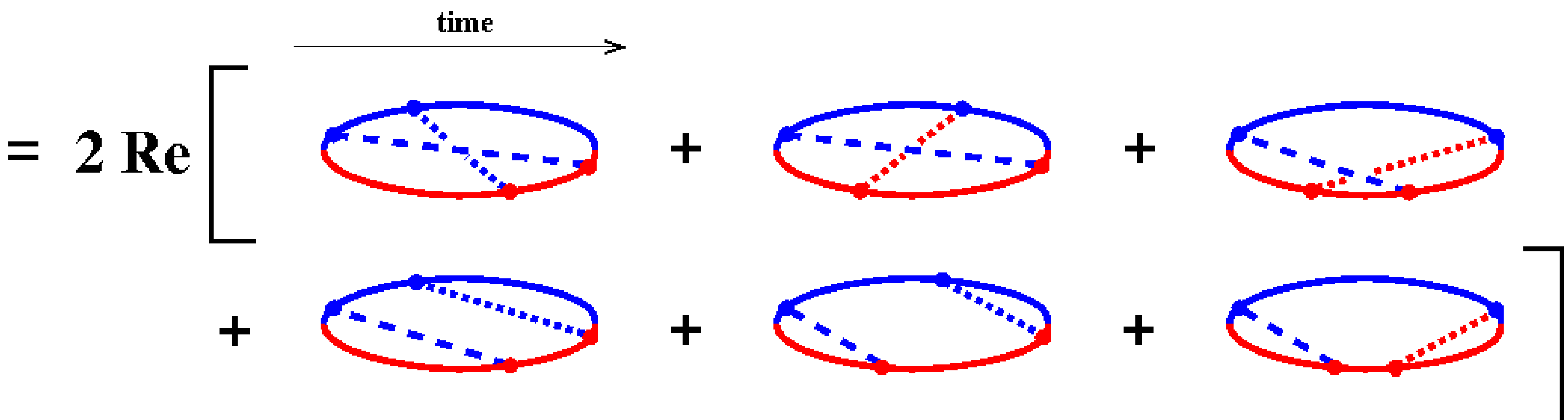
In *medium*, one can argue that such loops are suppressed by some power of $1/E$.

Other random back-up slides

Published Work

[all for $g \to gg \to ggg$]

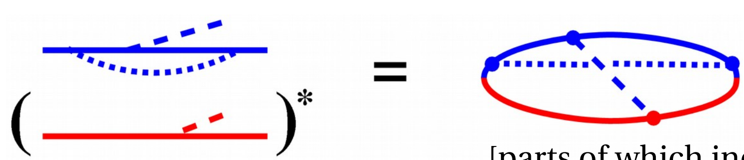




+ appropriate permutations of (x, y, 1-x-y)

- 4-gluon vertices, e.g. 

Still in progress

- virtual corrections, e.g.  correct single brem rate

[parts of which included in $y \ll x \ll 1$ work of earlier refs.]

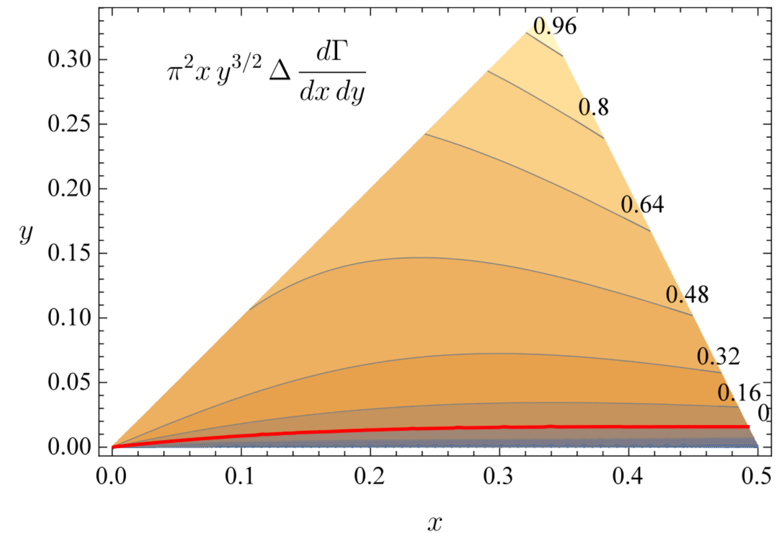
- Putting it all together to compute physical, infrared-safe characteristics of shower development (including earlier authors' resummation of soft bremsstrahlung).

Results

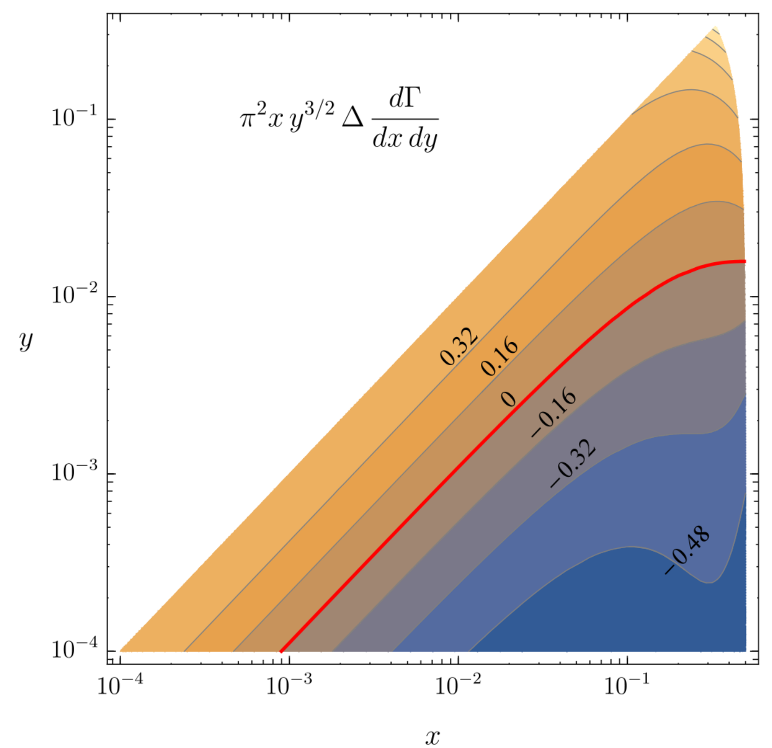
$$\color{red}{\Delta}\frac{d\Gamma}{dx\,dy} \equiv$$ correction to double brem due to overlapping formation times

$$= f(x,y)\,\frac{C_A^2\alpha_s^2}{\pi^2 x y^{3/2}}\sqrt{\frac{\hat{q}_A}{E}}$$

$$(y < x < 1-x-y)$$

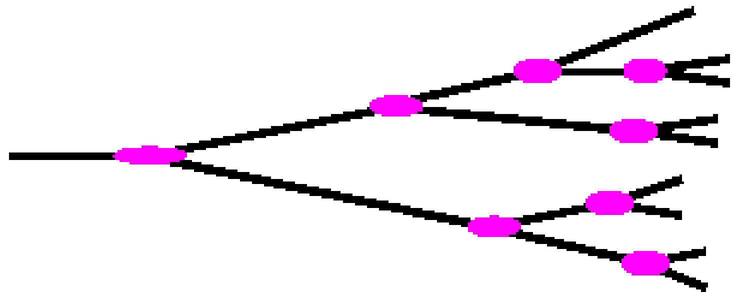


where *f*(*x*,*y*) varies from 1.05 to -0.90 and is shown on the right.

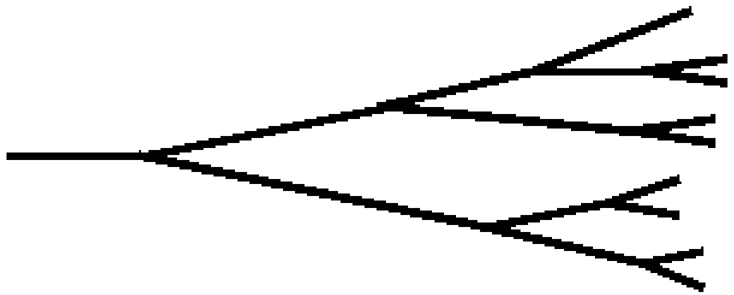


Qualitative Point

Effect of overlapping formation times enhances the rate except when one gluon is very soft.

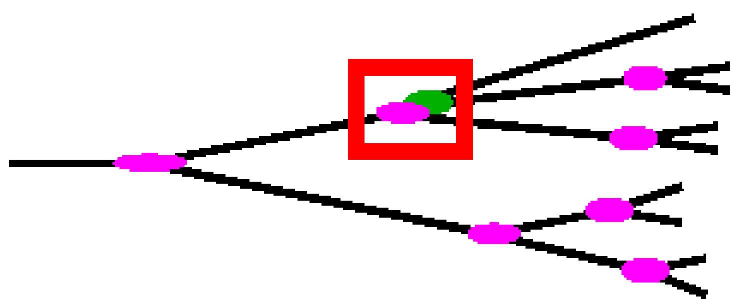


vs



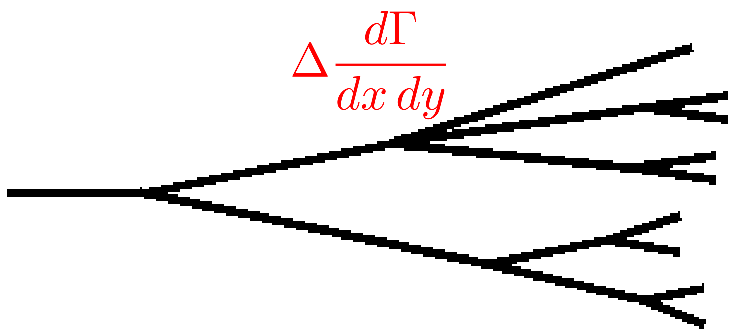
Monte Carlo (MC)

How to account for correction from

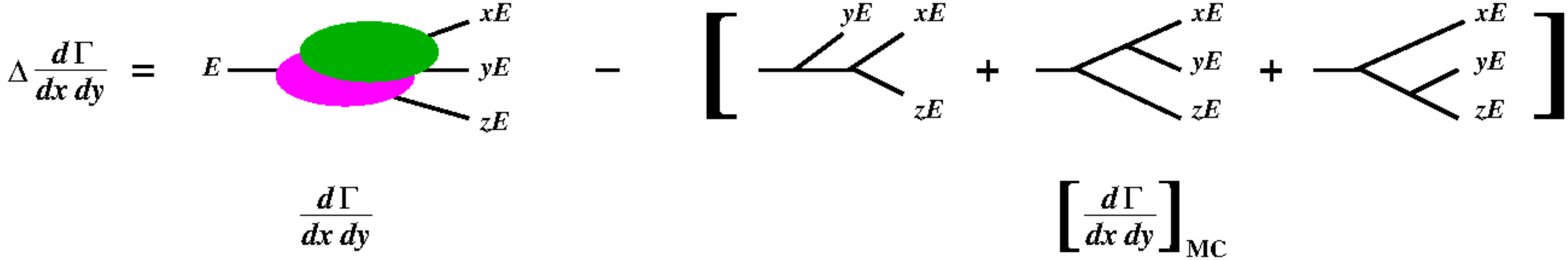


?

Add a $g \rightarrow ggg$ Monte Carlo possibility to account for correction:



where



$\Delta \frac{d\Gamma}{dx\,dy}$ =
E
xE
yE
zE
$\frac{d\Gamma}{dx\,dy}$
−
yE
xE
zE
+
xE
yE
zE
+
xE
yE
zE
$\left[\frac{d\Gamma}{dx\,dy}\right]_{\mathrm{MC}}$