Correlators of twist-2 light-ray operators in the BFKL limit

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Outline

1. Setting the stage: correlator of 3 light-ray operators
   - Light-ray operators as analytic continuation of local operators
   - Three-point correlator of local operators:
   - Three-point correlator of LR operators: what to expect
2. CF of two light-ray operators in the BFKL limit
   - Regge limit in a conformal theory.
   - An example of NLO calculation of 4-point CF
   - Correlation function of two “Wilson frames” in the BFKL approximation.
   - CF of two light-ray operators.
3. 3-point CF in the BFKL limit
   - 3-point CF of “Wilson frames” from the BK equation
   - Structure constants at the “BFKL point”
   - Conclusions
   - Work in progress: triple Regge limit

Gluon operators of leading twist

$$F^l \simeq F^a_{-i}\nabla_-^{l-2}F^{ai}_-(x)$$

Anomalous dimension (in gluodynamics)

$$\gamma_l = \frac{2}{\pi}\alpha_s N_c\Big[-\frac{1}{l(l-1)} - \frac{1}{(l+1)(l+2)} + \psi(l+1) + \gamma_E - \frac{11}{12}\Big] + O(\alpha_s^2)$$

BFKL and anomalous dimensions of twist-two operators

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BFKL gives $\gamma(n,\alpha_s)$ at the non-physical point $l \to 1$

$$\gamma_l = \Big[A_l^{\text{LO BFKL}} + \omega B_l^{\text{NLO BFKL}} + ...\Big]\Big(\frac{\alpha_s N_c}{\pi\omega}\Big)^l \qquad \omega \equiv l-1$$

LO: Jaroszewicz (1982), NLO: Lipatov, Fadin, Camici, Ciafaloni (1998)
NNLO (in $N=4$ SYM) ; Gromov, Caron-Hout (2017)

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Q: which one? A: gluon light-ray (LR) operator

Light-ray operators

Gluon light-ray (LR) operator of twist 2

$$F^a_{-i}(x'_+ + x_\perp)[x'_+, x_+]^{ab} F^{b\ i}_-(x_+ + x_\perp)$$

Forward matrix element - gluon parton density

$$z^\mu z^\nu \langle p|F^a_{\mu\xi}(z)[z,0]^{ab} F^{b\xi}_\nu(0)|p\rangle^\mu \stackrel{z^2=0}{=} 2(pz)^2 \int_0^1 dx_B\ x_B D_g(x_B,\mu)\cos(pz)x_B$$

Evolution equation (in gluodynamics)

$$\mu^2 \frac{d}{d\mu^2} F^a_{-i}(x'_+ + x_\perp)[x'_+, x_+]^{ab} F^{b\ i}_-(x_+ + x_\perp)$$
$$= \int_{x_+}^{x'_+} dz'_+ \int_{x_+}^{z'_+} dz_+\ K(x'_+, x_+; z'_+, z_+; \alpha_s) F^a_{-i}(z'_+ + x_\perp)[z'_+, z_+]^{ab} F^{b\ i}_-(z_+ + x_\perp)$$

“Forward” LR operator

$$F(L_+, x_\perp) = \int dx_+\ F^a_{-i}(L_+ + x_+ + x_\perp)[L_+ + x_+, x_+]^{ab} F^{bi}_-(x_+ + x_\perp)$$

Light-ray operators

Expansion in ("forward") local operators

$$F(L_+,x_\perp)\ =\ \sum_{n=2}^{\infty}\frac{L_+^{n-2}}{(n-2)!}\mathcal{O}^g_n(x_\perp),\qquad \mathcal{O}^g_n\ \equiv\ \int dx_+ F^a_{-i}\nabla_-^{n-2}F^{ai}_-(x_+,x_\perp)$$

Evolution equation for $F(L_+,x_\perp)$

$$\mu\frac{d}{d\mu}F(L_+,x_\perp)\ =\ \int_0^1 du\ K_{gg}(u,\alpha_s)F(uL_+,x_\perp)$$

$$\Rightarrow\ \ \gamma_n(\alpha_s)\ =\ -\int_0^1 du\ u^{n-2}K_{gg}(u,\alpha_s)\qquad \mu\frac{d}{d\mu}\mathcal{O}^g_n\ =\ -\gamma_n(\alpha_s)\mathcal{O}^g_n$$

$u^{-1}K_{gg}$ - DGLAP kernel

$$u^{-1}K_{gg}(u)=\frac{2\alpha_sN_c}{\pi}\Big(\bar{u}u+\Big[\frac{1}{\bar{u}u}\Big]_+-2+\frac{11}{12}\delta(\bar{u})\Big)+\text{higher orders in }\alpha_s$$

Light-ray operators

Conformal LR operator ($j=\frac{3}{2}+i\nu$)

$$F^\mu_j(x_\perp) = \int_0^\infty dL_+\, L_+^{1-j}F^\mu(L_+,x_\perp)$$

Evolution equation for "forward" conformal light-ray operators

$$\Rightarrow\ \mu^2\frac{d}{d\mu^2}F_j(z_\perp) = \int_0^1 du\, K_{gg}(u,\alpha_s)u^{j-2}F_j(z_\perp)$$

$\Rightarrow \gamma_j(\alpha_s)$ is an analytical continuation of $\gamma_n(\alpha_s)$

Correlation functions of (local) operators in a conformal theory

Conformal theory: theory invariant under Lorentz transformations, rescaling $x_\mu \to \lambda x_\mu$, and inversion $x_\mu \to \frac{x_\mu}{x^2}$.

Mathematically, combinations of $\hat{P}_\mu, \hat{M}_{\mu\nu}, \hat{D}$, and $\hat{K}_\mu$ form $SO(d+1,1)$ group. (or $SO(d,2)$ in Minkowski space).

In a conformal theory, two-point and three-point CFs of local operators are fixed.

2-point CF of scalar operators:

$$\langle \hat{\mathcal{O}}_1(x)\mathcal{O}_2(y)\rangle \;=\; \delta_{\Delta_1\Delta_2}\frac{\text{const}\times\mu^{-2\gamma_1}}{|x-y|^{2\Delta_1}}$$

$\Delta = d+\gamma$ is the (canonical + anomalous) dimension of the operator, μ - normalization point

$$[\hat{D},\hat{\mathcal{O}}(x)] \;=\; \big(x_\mu\partial^\mu + \Delta\big)\hat{\mathcal{O}}(x), \qquad \mu\frac{d}{d\mu}\hat{\mathcal{O}}(x) \;=\; -\gamma\hat{\mathcal{O}}(x)$$

Correlation functions of (local) operators in a conformal theory

3-point CF

$$\langle \hat{\mathcal{O}}_1(x)\hat{\mathcal{O}}_2(y)\hat{\mathcal{O}}_3(z)\rangle = \frac{C(\Delta_1,\Delta_2,\Delta_3)\mu^{-\gamma_1-\gamma_2-\gamma_3}}{|x-y|^{\Delta_1+\Delta_2-\Delta_3}|y-z|^{\Delta_2+\Delta_3-\Delta_1}|x-z|^{\Delta_1+\Delta_3-\Delta_2}}$$

$C(\Delta_1,\Delta_2,\Delta_3) \equiv$ structure constants.

Anomalous dimensions and structure constants define all of the dynamics of a CFT.

Four-point CFs (for simplicity, $\Delta_1 = \Delta_2 = \Delta_3 = \Delta_4 = \Delta$)

$$\langle \hat{\mathcal{O}}_1(x)\hat{\mathcal{O}}_2(y)\hat{\mathcal{O}}_3(z)\hat{\mathcal{O}}_4(w)\rangle = \frac{\mu^{-4\gamma}}{|x-y|^{-2\Delta}|y-w|^{-2\Delta}}F(R_1,R_2)$$

R_1, R_2 - conformal ratios

$$R_1 = \frac{x-z)^2(y-w)^2}{(x-y)^2(z-w)^2}, \qquad R_2 = \frac{x-w)^2(y-z)^2}{(x-y)^2(z-w)^2}$$

Supermultiplet of twist-2 operators in $\mathcal{N}=4$ SYM

SU_4 singlet operators. (Korchemsky et al)

$$\tilde{S}^l_{1n}(z) = \tilde{F}^l_n(z) + \frac{l-1}{24}\tilde{\Lambda}^l_n(z) + \frac{l(l-1)}{24}\tilde{\Phi}^l_n(z)$$
$$\tilde{S}^l_{2n}(z) = \tilde{F}^l_n(z) - \frac{1}{24}\tilde{\Lambda}^l_n(z) - \frac{l(l+1)}{72}\tilde{\Phi}^l_n(z)$$
$$\tilde{S}^l_{3n}(z) = \tilde{F}^l_n(z) - \frac{l+2}{12}\tilde{\Lambda}^l_n(z) + \frac{(l+1)(l+2)}{24}\tilde{\Phi}^l_n(z)$$

$$\tilde{F}^l_n(z) \equiv i^{l-2}\mathrm{tr}\, F_{\mu n}\partial_n^{l-2}C^{\frac{5}{2}}_{l-2}\Big(\frac{\overleftarrow{\nabla}_n + \overrightarrow{\nabla}_n}{\partial_n}\Big)F^\mu_{\ n} + O(g^2)$$
$$\tilde{\Lambda}^l_n(z) \equiv i^{l-1}\mathrm{tr}\, \bar{\lambda}\partial_n^{l-1}C^{\frac{3}{2}}_{l-1}\Big(\frac{\overleftarrow{\nabla}_n + \overrightarrow{\nabla}_n}{\partial_n}\Big)\lambda(z) + O(g^2)$$
$$\tilde{\Phi}^l_n(z) \equiv i^{l}\mathrm{tr}\, \bar{\phi}^I\partial_n^{l-1}C^{\frac{3}{2}}_{l}\Big(\frac{\overleftarrow{\nabla}_n + \overrightarrow{\nabla}_n}{\partial_n}\Big)\phi^I(z) + O(g^2)$$

$C^\lambda_l(x)$ - Gegenbauer polynomials, $n^2=0$, and $F^\mu_n \equiv F^{\mu\nu}n_\nu$ etc.

All operators have the same anomalous dimension

$$\gamma^{S_1}_l(\alpha_s) \equiv \gamma_l(\alpha_s) = \frac{2\alpha_s}{\pi}N_c[\psi(l-1)+C] + O(\alpha_s^2), \qquad \gamma^{S_2}_l = \gamma^{S_1}_{l+1}, \qquad \gamma^{S_3}_l = \gamma^{S_1}_{l+2}$$

3-point CF of local operators with spin

Rychkov et al: CF of 3 operators with spin ($n_1^2 = n_2^2 = n_3^2 = 0$)

$$< \mathcal{O}_{n_1}^{l_1}(x)\mathcal{O}_{n_2}^{l_2}(y)\mathcal{O}_{n_3}^{l_3}(z) >= \sum_{m_{12},m_{13},m_{23}\geq 0} \lambda_{m_{12},m_{23},m_{13}} \begin{bmatrix} \Delta_1 & \Delta_2 & \Delta_3 \\ l_1 & l_2 & l_3 \\ m_{23} & m_{13} & m_{12} \end{bmatrix}$$

The sum runs over

$$m_1 = l_1 - m_{12} - m_{13} \geq 0\,, \quad m_2 = l_2 - m_{12} - m_{23} \geq 0\,, \quad m_3 = l_3 - m_{13} - m_{23} \geq 0$$

where Δ_i is dimension and l_i is Lorentz spin.

$$\begin{bmatrix} \Delta_1 & \Delta_2 & \Delta_3 \\ l_1 & l_2 & l_3 \\ m_{23} & m_{13} & m_{12} \end{bmatrix} = \frac{(V_{1,23})^{l_1-m_{12}-m_{13}}(V_{2,31})^{l_2-m_{12}-m_{23}}(V_{3,12})^{l_3-m_{13}-m_{23}}(H_{12})^{m_{12}}(H_{13})^{m_{13}}(H_{23})^{m_{23}}}{|x-y|^{\Delta_1+\Delta_2-\Delta_3}|x-z|^{\Delta_1+\Delta_3-\Delta_2}|y-z|^{\Delta_2+\Delta_3-\Delta_1}}$$

V and H - some tensor structures

If we define "forward" operators

$$\Phi^l_n(x_\perp) = \int du\ \bar\phi^a_{AB}\nabla^l_n\phi^{ABa}(un+x_\perp),$$
$$\Lambda^l_n(x_\perp) = \int du\ i\bar\lambda^a_A\nabla^{l-1}_n\sigma_n\lambda^a_A(un+x_\perp)$$
$$F^l(x_\perp) = \int du\ F^a_{ni}\nabla^{l-2}_n F^{ai}_n(un+x_\perp),$$

the renorm-invariant operators are

$$S^l_{1n} = F^l_n + \frac{1}{4}\Lambda^l_n - \frac{1}{2}\Phi^l_n, \quad S^l_{2n} = F^l_n - \frac{1}{4(l-1)}\Lambda^l_n + \frac{(l+1)}{6(l-1)}\Phi^l_n$$
$$S^l_{3n} = F^l_n - \frac{l+2}{2(l-1)}\Lambda^l_n - \frac{(l+1)(l+2)}{2l(l-1)}\Phi^l_n$$

and Rychkov's structures reduce to one ($x_\perp\cdot n_1 = x_\perp\cdot n_2 = x_\perp\cdot n_3 = 0$)

$$\langle S^{l_1}_{n_1}(x_{1_\perp})S^{l_2}_{n_2}(x_{2_\perp})S^{k_3}_{n_3}(x_{3_\perp})\rangle =$$
$$= C(g^2,l_i)\frac{(2n_1\cdot n_2)^{\frac{l_1+l_2-l_3-1}{2}}(2n_1\cdot n_3)^{\frac{l_1+l_3-l_2-1}{2}}(2n_2\cdot n_3)^{\frac{l_2+l_3-l_1-1}{2}}}{|x_{12_\perp}|^{\Delta_1+\Delta_2-\Delta_3-1}|x_{13_\perp}|^{\Delta_1+\Delta_3-\Delta_2-1}|x_{23_\perp}|^{\Delta_2+\Delta_3-\Delta_1-1}}$$

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Our aim is to find the structure constants $C(g^2,l_i)$ in the “BFKL limit” $l_i\to 1$

Supermultiplet of LR operators

Gluino and scalar LR operators

$$\Lambda(L_+,x_\perp) = \frac{i}{2}\int dx'_+ \, [\bar\lambda^a(L_+ + x_+ + x_\perp)[x'_+ + x_+, x_+]^{ab}\sigma_-\nabla_-\lambda^b(x_+ + x_\perp) + \mathrm{c.c}]$$

$$\Phi(L_+,x_\perp) = \int dx'_+ \, \phi^{a,I}(L_+ + x_+ + x_\perp)[x'_+ + x_+, x_+]^{ab}\nabla_-^2\phi^{b,I}(x_+ + x_\perp)$$

$$\Lambda_j(x_\perp) = \int_0^\infty dL_+ \, L_+^{-j+1}\Lambda(L_+,x_\perp), \qquad \Phi_j(x_\perp) = \int_0^\infty dL_+ \, L_+^{-j+1}\Phi(L_+,x_\perp)$$

SU_4 singlet LR operators.

$$S_{1j}(x_\perp) = F_j(x_\perp) + \frac{j-1}{8}\Lambda_j(x_\perp) - \frac{j(j-1)}{8}\Phi_j(x_\perp)$$

$$S_{2j}(x_\perp) = F_j(x_\perp) - \frac{1}{8}\Lambda_j(x_\perp) + \frac{j(j+1)}{24}\Phi_j(x_\perp)$$

$$S_{3j}(x_\perp) = F_j(x_\perp) - \frac{j+2}{4}\Lambda_j(x_\perp) - \frac{(j+1)(j+2)}{8}\Phi_j(x_\perp)$$

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$$\gamma_j^{S_1}(\alpha_s) \equiv \gamma_j(\alpha_s) = \frac{2\alpha_s}{\pi}N_c[\psi(j-1) + C] + O(\alpha_s^2), \qquad \gamma_j^{S_2} = \gamma_{j+1}^{S_1}, \qquad \gamma_j^{S_3} = \gamma_{j+2}^{S_1}$$

Correlators of LR operators

Since LR operators are “analytic continuation” of local operators, we expect ($j_1 = \frac{3}{2} + i\nu_1, j_2 = \frac{3}{2} + i\nu_2$)

$$\langle S^{j_1}_{n_1}(x_{1_\perp}) S^{j_2}_{n_2}(x_{2_\perp})\rangle \; = \; \delta(\nu_1 - \nu_2) f(\alpha_s, j) \frac{(2n_1 \cdot n_2)^{\omega_1} (\mu^2)^{-\gamma(j_1,\alpha_s)}}{|x_{12_\perp}|^{\Delta(\alpha_s, j_1)}}$$

for 2-point CF and similarly ($j_i \equiv 1 + \omega_i$)

$$\langle S^{j_1}_{n_1}(x_{1_\perp})\; S^{j_2}_{n_2}(x_{2_\perp})\; S^{j_2}_{n_3}(x_{3_\perp})\rangle \; = \; \frac{F(\alpha_s, \omega_1, \omega_2, \omega_3)}{(\omega_1 + \omega_2 - \omega_3)(\omega_1 + \omega_3 - \omega_2)(\omega_2 + \omega_3 - \omega_1)}$$
$$\times \; \frac{(2n_1 \cdot n_2)^{\frac{\omega_1 + \omega_2 - \omega_3}{2}}}{|x_{12_\perp}|^{\Delta_1 + \Delta_2 - \Delta_3}} \frac{(2n_1 \cdot n_3)^{\frac{\omega_1 + \omega_3 - \omega_2}{2}}}{|x_{13_\perp}|^{\Delta_1 + \Delta_3 - \Delta_2}} \frac{(2n_2 \cdot n_3)^{\frac{\omega_2 + \omega_3 - \omega_1}{2}}}{|x_{23_\perp}|^{\Delta_2 + \Delta_3 - \Delta_1}} \mu^{-\gamma(j_1) - \gamma(j_2) - \gamma(j_3)}$$

for the 3-point CF ($\Delta \; = \; j + \gamma(j) = 1 + \omega + \gamma_\omega$ - dimension).

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$$\times \frac{(2n_1 \cdot n_2)^{\frac{\omega_1 + \omega_2 - \omega_3}{2}}}{|x_{12_\perp}|^{\Delta_1 + \Delta_2 - \Delta_3}} \frac{(2n_1 \cdot n_3)^{\frac{\omega_1 + \omega_3 - \omega_2}{2}}}{|x_{13_\perp}|^{\Delta_1 + \Delta_3 - \Delta_2}} \frac{(2n_2 \cdot n_3)^{\frac{\omega_2 + \omega_3 - \omega_1}{2}}}{|x_{23_\perp}|^{\Delta_2 + \Delta_3 - \Delta_1}} \mu^{-\gamma(j_1) - \gamma(j_2) - \gamma(j_3)}$$

for the 3-point CF ($\Delta \ = \ j + \gamma(j) = 1 + \omega + \gamma_\omega$ - dimension).

Our aim is to calculate $f(\alpha_s, j)$ and $F(\alpha_s, j_1, j_2, j_3)$ at $j_i = 1 + \omega_i$ in the “BFKL limit” $g^2 \to 0$, $\omega \to 0$, and $\frac{g^2}{\omega}$ = fixed

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for 2-point CF and similarly $(j_i \equiv 1 + \omega_i)$

$$\langle S^{j_1}_{n_1}(x_{1_\perp})\, S^{j_2}_{n_2}(x_{2_\perp})\, S^{j_2}_{n_3}(x_{3_\perp})\rangle \;=\; \frac{F(\alpha_s, \omega_1, \omega_2, \omega_3)}{(\omega_1 + \omega_2 - \omega_3)(\omega_1 + \omega_3 - \omega_2)(\omega_2 + \omega_3 - \omega_1)}$$
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for the 3-point CF ($\Delta \;=\; j + \gamma(j) = 1 + \omega + \gamma_\omega$ - dimension).

Our aim is to calculate $f(\alpha_s, j)$ and $F(\alpha_s, j_1, j_2, j_3)$ at $j_i = 1 + \omega_i$ in the “BFKL limit” $g^2 \to 0$, $\omega \to 0$, and $\frac{g^2}{\omega}$ = fixed

BK equation for evolution of color dipoles $\Rightarrow$
$F(\alpha_s, 1 + \omega_1, 1 + \omega_2, 1 + \omega_3)$ at $\omega_i \to 0$ and $\omega_1 = \omega_2 + \omega_3$

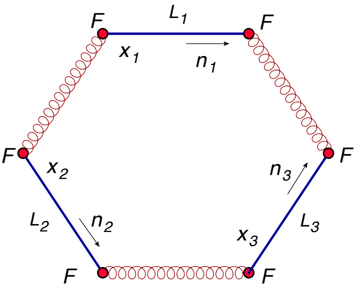
Warm-up exercise: LO

Since LR operators are "analytic continuation" of local operators, we expect

$$\langle S^{j_1}_{n_1}(x_{1_\perp})\, S^{j_2}_{n_2}(x_{2_\perp})\, S^{j_2}_{n_3}(x_{3_\perp})\rangle \;=\; \frac{F(\alpha_s,\omega_1,\omega_2,\omega_3)}{(\omega_1+\omega_2-\omega_3)(\omega_1+\omega_3-\omega_2)(\omega_2+\omega_3-\omega_1)}$$
$$\times\;\frac{(2n_1\cdot n_2)^{\frac{\omega_1+\omega_2-\omega_3}{2}}}{|x_{12_\perp}|^{\Delta_1+\Delta_2-\Delta_3}}\,\frac{(2n_1\cdot n_3)^{\frac{\omega_1+\omega_3-\omega_2}{2}}}{|x_{13_\perp}|^{\Delta_1+\Delta_3-\Delta_2}}\,\frac{(2n_2\cdot n_3)^{\frac{\omega_2+\omega_3-\omega_1}{2}}}{|x_{23_\perp}|^{\Delta_2+\Delta_3-\Delta_1}}$$

$\Delta\;=\;j+\gamma(j)$ - dimension

Warm-up exercise: LO



+ *perm.*

Warm-up exercise: LO

$$\langle \mathcal{S}_{n_1}^{1+\omega_1}(x_{1_\perp})\mathcal{S}_{n_2}^{1+\omega_2}(x_{2_\perp})\mathcal{S}_{n_3}^{1+\omega_3}(z_{3_\perp})\rangle = -\frac{(N_c^2-1)F(\alpha_s,\omega_1,\omega_2,\omega_3)}{4\pi^6(\omega_1+\omega_2-\omega_3)(\omega_1+\omega_3-\omega_2)(\omega_2+\omega_3-\omega_1)}$$
$$\times \frac{1}{x_{12}^2x_{13}^2x_{23}^2}\Big(\frac{2n_1\cdot n_2}{x_{12}^2}\Big)^{\frac{\omega_1+\omega_2-\omega_3}{2}}\Big(\frac{2n_1\cdot n_3}{x_{13}^2}\Big)^{\frac{\omega_1+\omega_3-\omega_2}{2}}\Big(\frac{2n_2\cdot n_3}{x_{23}^2}\Big)^{\frac{\omega_2+\omega_3-\omega_1}{2}}$$

$$F(\alpha_s,\omega_1,\omega_2,\omega_3) = \Gamma\Big(1+\frac{\omega_1+\omega_2-\omega_3}{2}\Big)\Gamma\Big(1+\frac{\omega_2+\omega_3-\omega_1}{2}\Big)\Gamma\Big(1+\frac{\omega_1+\omega_3-\omega_2}{2}\Big)$$
$$\times \prod_i \Gamma(1-\omega_i)\Gamma\Big(\frac{\omega_1+\omega_2-\omega_3}{2}+2\Big)\Gamma\Big(\frac{\omega_2+\omega_3-\omega_1}{2}+2\Big)\Gamma\Big(\frac{\omega_1+\omega_3-\omega_2}{2}+2\Big)$$
$$\times \Big\{\Big(e^{i\pi\omega_3}-1\Big)\Big[e^{i\pi(\omega_1-\omega_2)}+e^{i\pi(\omega_2-\omega_1)}-2e^{-i\pi\omega_3}\Big]+\Big(e^{i\pi\omega_1}-1\Big)\Big[e^{i\pi(\omega_2-\omega_3)}+e^{i\pi(\omega_3-\omega_2)}$$
$$-2e^{-i\pi\omega_1}\Big]+\Big(e^{i\pi\omega_2}-1\Big)\Big[e^{i\pi(\omega_3-\omega_1)}+e^{i\pi(\omega_1-\omega_3)}-2e^{-i\pi\omega_2}\Big]$$
$$+e^{i\pi(\omega_1+\omega_2-\omega_3)}+e^{i\pi(\omega_2+\omega_3-\omega_1)}+e^{i\pi(\omega_1+\omega_3-\omega_2)}-e^{i\pi(\omega_1+\omega_2+\omega_3)}-2\Big\}$$

At small ω's

$$F(\omega_1,\omega_2,\omega_3) \simeq -2\pi^2(\omega_1^2+\omega_2^2+\omega_3^2)-\pi^2(\omega_1^2+\omega_2^2+\omega_3^2-2\omega_1\omega_2-2\omega_1\omega_3-2\omega_2\omega_3)$$

In higher orders one should expect

$$\langle \mathcal{S}^{1+\omega_1}_{n_1}(x_{1_\perp})\mathcal{S}^{1+\omega_2}_{n_2}(x_{2_\perp})\mathcal{S}^{1+\omega_3}_{n_3}(z_{3_\perp})\rangle =$$

$$= -\frac{N_c^2-1}{4\pi^6 x_{12}^2 x_{13}^2 x_{23}^2}\frac{1}{(\omega_1+\omega_2-\omega_3)(\omega_1+\omega_3-\omega_2)(\omega_2+\omega_3-\omega_1)}$$

$$\times \frac{(2n_1\cdot n_2)^{\frac{\omega_1+\omega_2-\omega_3}{2}}}{(x_{12}^2)^{\frac{\Delta_1+\Delta_2-\Delta_3}{2}}}\frac{(2n_1\cdot n_3)^{\frac{\omega_1+\omega_3-\omega_2}{2}}}{(x_{13}^2)^{\frac{\Delta_1+\Delta_3-\Delta_2}{2}}}\frac{(2n_2\cdot n_3)^{\frac{\omega_2+\omega_3-\omega_1}{2}}}{(x_{23}^2)^{\frac{\Delta_2+\Delta_3-\Delta_1}{2}}}F(\omega_1,\omega_2,\omega_3;g^2),$$

$$F(\omega_1,\omega_2,\omega_3;g^2) \simeq F(\omega_1,\omega_2,\omega_3)\Big[1+\sum c_n\big(\frac{g^2}{\omega_i}\big)^n\Big]$$

It could be obtained from the CF of three color dipoles with long sides collinear to n_1, n_2, and n_3 and transverse short sides.

This means analyzing QCD (or N=4 SYM) in the triple Regge limit which is work in progress.
Triple Regge limit: scattering of 3 particles moving with speed $\sim c$ in x, y, and z directions.

In higher orders one should expect

$$\langle \mathcal{S}^{1+\omega_1}_{n_1}(x_{1_\perp})\mathcal{S}^{1+\omega_2}_{n_2}(x_{2_\perp})\mathcal{S}^{1+\omega_3}_{n_3}(z_{3_\perp})\rangle =$$

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This means analyzing QCD (or N=4 SYM) in the triple Regge limit which is work in progress.
Triple Regge limit: scattering of 3 particles moving with speed $\sim c$ in x, y, and z directions.

What we can do in a meantime is to take $n_3 \to n_2$ and consider the CF of a dipole in $n_1 = n_+$ direction and two dipoes in $n_2 = n_3 = n_-$ directions which can be obtained using the BK evolution.

High-energy amplitudes in a conformal theory

Conformal four-point amplitude

$$A(x,y,x',y') = (x-y)^4(x'-y')^4 N_c^2 \langle \mathcal{O}(x)\mathcal{O}^\dagger(y)\mathcal{O}(x')\mathcal{O}^\dagger(y')\rangle$$

$\mathcal{O} = \mathrm{Tr}\{Z^2\}$ ($Z = \frac{1}{\sqrt{2}}(\phi_1 + i\phi_2)$) - chiral primary operator
In a conformal theory the amplitude is a function of two conformal ratios

$$A = F(R,R')$$
$$R = \frac{(x-y)^2(x'-y')^2}{(x-x')^2(y-y')^2}, \qquad R' = \frac{(x-y)^2(x'-y')^2}{(x-y')^2(x'-y)^2}$$

At large N_c

$$A(x,y,x',y') = A(\lambda; x,y,x',y') \qquad \lambda = g^2 N_c = 4\pi\alpha_s N_c \ - \text{ ‘t Hooft coupling}$$

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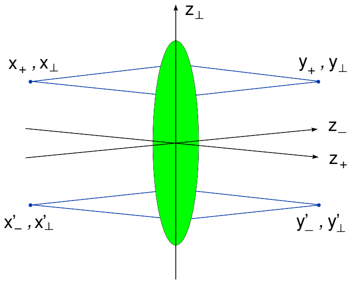
Our goal is perturbative expansion and resummation of $(\lambda \ln s)^n$ at large energies

$$(\lambda \ln s)^n (c_n^{\mathrm{LO}} + \lambda c_n^{\mathrm{NLO}} + \ldots)$$

Regge limit in the coordinate space

Regge limit: $x_+ \to \rho x_+,\ x'_+ \to \rho x'_+, y_- \to \rho' y_-,\ y'_- \to \rho' y-_-$ $\quad \rho, \rho' \to \infty$



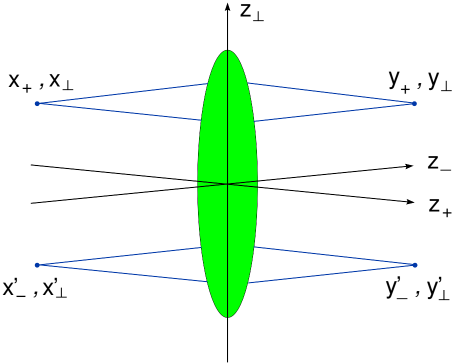
Full 4-dim conformal group: $A = F(R, r)$

$$R = \frac{(x-y)^2(x'-y')^2}{(x-x')^2(y-y')^2} \to \frac{\rho^2\rho'^2 x_+x'_+y_-y'_-}{(x-x')^2_\perp(y-y')^2_\perp} \to \infty$$

$$r = \frac{[(x-y)^2(x'-y')^2 - (x'-y)^2(x-y')^2]^2}{(x-x')^2(y-y')^2(x-y)^2(x'-y')^2}$$

$$\to \frac{[(x'-y')^2_\perp x_+y_- + x'_+y'_-(x-y)^2_\perp + x_+y'_-(x'-y)^2_\perp + x'_+y_-(x-y')^2_\perp]^2}{(x-x')^2_\perp(y-y')^2_\perp x_+x'_+y_-y'_-}$$

Regge limit: $x_+ \to \rho x_+,\ x'_+ \to \rho x'_+, y_- \to \rho' y_-,\ y'_- \to \rho' y-_-$
$\rho, \rho' \to \infty$



Regge limit symmetry: 2-dim conformal group $SL(2,C)$ formed from P_1, P_2, M^{12}, D, K_1 and K_2 which leave the plane $(0,0,z_\perp)$ invariant.

Pomeron in a conformal theory

$$A(x,y;x',y') \overset{s\to\infty}{=} \frac{i}{2}\int d\nu\, f_+(\aleph(\lambda,\nu))F(\lambda,\nu)\Omega(r,\nu)R^{\aleph(\lambda,\nu)/2}$$

L. Cornalba (2007)

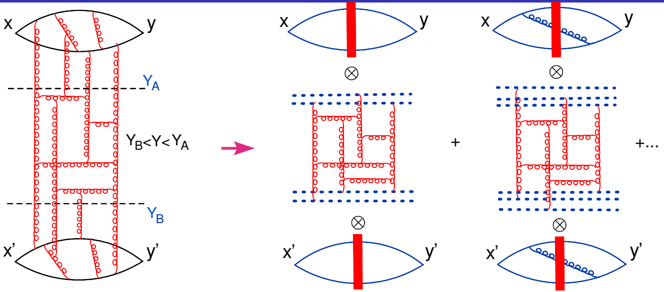
$f_+(\omega) = \frac{e^{i\pi\omega}-1}{\sin\pi\omega}$ - signature factor

$$\Omega(r,\nu) = \frac{\sin\nu\rho}{\sinh\rho}, \qquad \cosh\rho = \frac{\sqrt{r}}{2}$$

- solution of the eqn $(\Box_{H_3} + \nu^2 + 1)\Omega(r,\nu) = 0$

The dynamics is described by:
$\aleph(\lambda,\nu)$ - pomeron intercept,
and
$F(\lambda,\nu)$ - “pomeron residue”.

NLO Amplitude in $\mathcal{N}$=4 SYM theory: factorization in rapidity



$$(x-y)^4(x'-y')^4\langle T\{\hat{\mathcal{O}}(x)\hat{\mathcal{O}}^\dagger(y)\hat{\mathcal{O}}(x')\hat{\mathcal{O}}^\dagger(y')\}\rangle$$
$$= \int d^2z_{1\perp}d^2z_{2\perp}d^2z'_{1\perp}d^2z'_{2\perp}\,\mathrm{IF}^{a_0}(x,y;z_1,z_2)[\mathrm{DD}]^{a_0,b_0}(z_1,z_2;z'_1,z'_2)\mathrm{IF}^{b_0}(x',y';z'_1,z'_2)$$

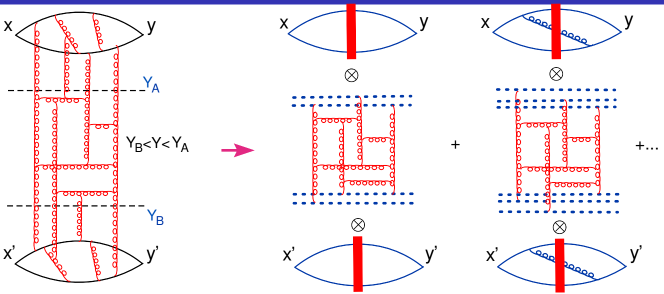
$y_A = \frac{1}{2}\ln a_0,\ y_B = \frac{1}{2}\ln b_0$ - “rapidity dividers”

From conformal invariance we choose

$a_0 = \frac{x_+y_+}{(x-y)^2},\ b_0 = \frac{x'_-y'_-}{(x'-y')^2}$ $\Leftrightarrow$ impact factors do not scale with energy

$\Rightarrow$ all energy dependence is contained in $[\mathrm{DD}]^{a_0,b_0}$ $(a_0b_0 = R)$

NLO Amplitude in $\mathcal{N}$=4 SYM theory: factorization in rapidity



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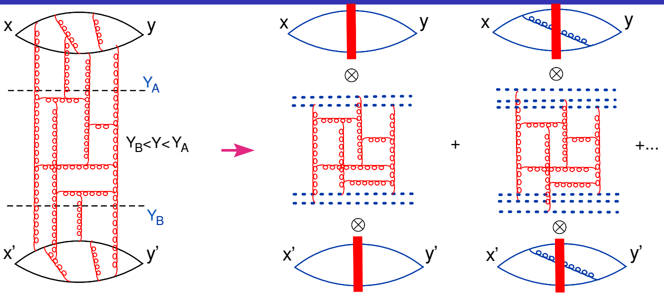
Impact factor ($\mathcal{R}=\frac{(x-y)^2z_{12}^2}{x_+y_+\mathcal{Z}_1\mathcal{Z}_2}$ - conf. ratio)

$$\mathrm{IF}^{a_0}=\int d\nu\int dz_0\,\mathcal{R}^{\frac{1}{2}+i\nu}\Big(1+\frac{\alpha_sN_c}{\pi}\Big[\frac{2\pi^2}{3}+\frac{4\chi(\nu)-8}{1+4\nu^2}-\frac{2\pi^2}{\cosh^2\pi\nu}\Big]\Big)\mathcal{U}^{a_0}(z_0,\nu)$$

$$\mathcal{U}^a(z_0,\nu)=\frac{1}{\pi^2}\int\frac{d^2z_1d^2z_2}{z_{12}^4}\Big(\frac{z_{12}^2}{z_{10}^2z_{20}^2}\Big)^{\frac{1}{2}-i\nu}\mathcal{U}(z_1,z_2),\qquad \mathcal{U}\equiv 1-\frac{1}{N_c}\mathrm{Tr}\{U_{z_1}U^\dagger_{z_2}\}$$



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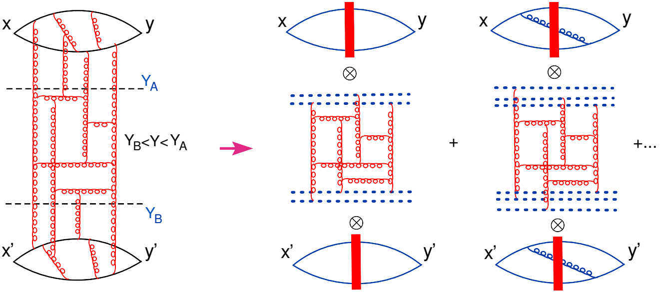
Dipole-dipole scattering $\quad \chi(\gamma)\equiv 2C-\psi(\gamma)-\psi(1-\gamma),\ \gamma\equiv\frac{1}{2}+i\nu$

$$[DD]=\int d\nu\int dz_0\ \Big(\frac{z_{12}^2}{z_{10}^2z_{20}^2}\Big)^{\frac{1}{2}+i\nu}\Big(\frac{z_{12}^2}{z_{10}^2z_{20}^2}\Big)^{\frac{1}{2}-i\nu}D\Big(\frac{1}{2}+i\nu;\lambda\Big)R^{\omega(\nu)/2}$$

$$D(\gamma;\lambda)\ =\ \frac{\Gamma(-\gamma)\Gamma(\gamma-1)}{\Gamma(1+\gamma)\Gamma(2-\gamma)}\Big\{1+\frac{\alpha_sN_c}{2\pi}\Big[\frac{4\chi(\nu)}{1+4\nu^2}-\frac{\pi^2}{3}+i\pi\frac{N_c^2-4}{2N_c^2}\Big]+O(g^4)\Big\}$$

NLO Amplitude in $\mathcal{N}$=4 SYM theory: factorization in rapidity



$$(x-y)^4(x'-y')^4\langle T\{\hat{\mathcal{O}}(x)\hat{\mathcal{O}}^\dagger(y)\hat{\mathcal{O}}(x')\hat{\mathcal{O}}^\dagger(y')\}\rangle$$
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Result : (G.A. Chirilli and I.B.)

$$F(\nu) = \frac{N_c^2}{N_c^2-1}\frac{4\pi^4\alpha_s^2}{\cosh^2\pi\nu}\left\{1+\frac{\alpha_s N_c}{\pi}\left[\frac{\pi^2}{2}-\frac{2\pi^2}{\cosh^2\pi\nu}-\frac{8}{1+4\nu^2}+i\pi\frac{N_c^2-4}{N_c^2}\right]\right\}$$

CF of two light-ray operators

$$S_j^+(x_\perp) \;=\; \int dx_+ \int_0^\infty dL_+ \; L_+^{1-j} F^a_{-i}(L_+ + x_+ + x_\perp)[L_+ + x_+, x_+]^{ab} F^{bi}_-(x_+ + x_\perp)$$
$$+ \text{ gluinos } + \text{ scalars}$$
$$S_{j'}^-(y_\perp) \;=\; \int dy_- \int_0^\infty dL_- \; L_-^{1-j'} F^a_{-i}(L_- + y_- + y_\perp)[L_- + y_-, y_-]^{ab} F^{bi}_-(y_- + y_\perp)$$
$$+ \text{ gluinos } + \text{ scalars}$$

A general formula for the correlation function of two LR operators reads

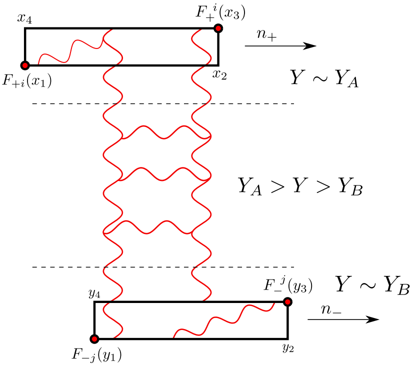
$$\langle S^+_{j=\frac{3}{2}+i\nu}(x_\perp) S^-_{j'=\frac{3}{2}+i\nu'}(y_\perp)\rangle = \frac{\delta(\nu-\nu')a(j,\alpha_s)(\mu^2)^{-\gamma(j,\alpha_s)}}{((x-y)^2_\perp)^{j+1+\gamma(j,\alpha_s)}}$$

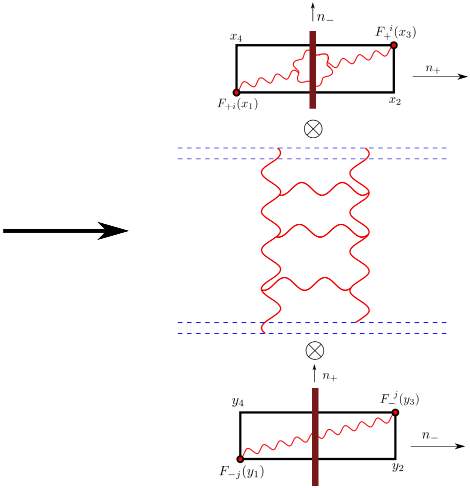
$\delta(\nu-\nu')$ reflects boost invariance: $x_+ \to \lambda x_+$, $x_- \to \frac{1}{\lambda}x_-$ does not change the correlation functions which depend on x_+y_-.

We need to reproduce it and find $\gamma(j,\alpha_s)$ and $a(j,\alpha_s)$ as $j \to 1$.

Correlator of two "Wilson frames"

"Wilson frame": light-ray operator with point-splitting in the transverse direction





Correlator of two Wilson frames

$$\langle \mathbf{U}^{\sigma_-}(\mathbf{x_{1\perp}}, \mathbf{z}_\perp)\mathbf{V}^{\sigma_+}(\mathbf{y_{1\perp}}, \mathbf{w}_\perp)\rangle =$$

$$= -\frac{8g^4}{N^2}\int\int \frac{d\nu\,\nu^2\ d^2z_0}{(\frac{1}{4}+\nu^2)^2}\left(\frac{(x_1-z)_\perp^2}{(x_1-z_0)_\perp^2(z-z_0)_\perp^2}\right)^{\frac{1}{2}+i\nu}$$

$$\left(\frac{(y_1-w)_\perp^2}{(y_1-z_0)_\perp^2(w-z_0)_\perp^2}\right)^{\frac{1}{2}-i\nu}\left(\frac{\sigma_+\sigma_-}{\sigma_{+0}\sigma_{-0}}\right)^{\aleph(\nu)}.$$

From experience with 4-point CFs in the BFKL limit

$$\left(\frac{\sigma_+\sigma_-}{\sigma_{+0}\sigma_{-0}}\right)^{\aleph(\nu)} \to$$

$$\to \frac{i}{\sin\pi\aleph(\nu)}\left(\frac{((x_1-y_3)^2)^{\frac{\aleph(\nu)}{2}}((x_3-y_1)^2)^{\frac{\aleph(\nu)}{2}}}{(x_{13}^2)^{\frac{\aleph(\nu)}{2}}(y_{13}^2)^{\frac{\aleph(\nu)}{2}}} - \frac{((x_1-y_1)^2)^{\frac{\aleph(\nu)}{2}}((x_3-y_3)^2)^{\frac{\aleph(\nu)}{2}}}{(x_{13}^2)^{\frac{\aleph(\nu)}{2}}(y_{13}^2)^{\frac{\aleph(\nu)}{2}}}\right).$$

(For small $x_{13}^\perp$ and $y_{13}^\perp$ Wilson frames are approximately conformally invariant)

$$\langle \breve{S}^{2+\omega_1}_{gl+} \breve{S}^{2+\omega_2}_{gl-} \rangle = -i\frac{N^2 g^4}{4\pi^3} \int d\nu (\Delta_\perp^2)^{\aleph(\nu)-\omega} B(-\omega, \omega - \aleph(\nu)) \frac{1 - e^{i\pi(2\aleph(\nu)-\omega)}}{\sin \pi\aleph(\nu)} \cdot$$

$$\times \frac{\nu^2}{(\frac{1}{4}+\nu^2)^2} \frac{\delta(\omega_1 - \omega_2)}{(|x_{13}|_\perp^2 |y_{13}|_\perp^2)^{1+\frac{\aleph(\nu)}{2}}} \left(\frac{(|x_{13}|_\perp^2)^{\frac{1}{2}+i\nu} (|y_{13}|_\perp^2)^{\frac{1}{2}+i\nu}}{(|x-y|_\perp^2)^{1+2i\nu}} G(\nu) + (\nu \to -\nu) \right)$$

where

$$G(\nu) = -i\frac{4^{-1-2i\nu}\pi^3 (i-2\nu)^2}{\Gamma^2(\frac{3}{2}-i\nu)\Gamma^2(1+i\nu)\sinh(2\pi\nu)} \, .$$

Now we can carry out the last integration over ν as the pole contribution at $\omega = \aleph(\nu)$.

We pick here the first pole Ψ-functions in pomeron intercept which corresponds to the operator with the lowest possible twist=2.

$$\langle S_+^{1+\omega_1}(x_{1\perp},x_{3\perp})S_-^{1+\omega_2}(y_{1\perp},y_{3\perp})\rangle \xrightarrow[x_{13\perp},\, y_{13\perp}\to 0]{} \delta(\omega_1-\omega_2)\Upsilon(\tilde\gamma)\frac{(x_{13\perp}^2)^{\frac{\tilde\gamma}{2}-\frac{\omega}{2}}(y_{13\perp}^2)^{\frac{\tilde\gamma}{2}-\frac{\omega}{2}}}{((x-y)_\perp^2)^{2+\tilde\gamma}},$$

$$\Upsilon(\tilde\gamma) = -N^2g^4\frac{2^{-1-2\tilde\gamma}\pi}{\tilde\gamma^2\Gamma^2(1-\frac{\tilde\gamma}{2})\Gamma^2(\frac12+\frac{\tilde\gamma}{2})\sin(\pi\tilde\gamma)\hat\aleph'(\tilde\gamma)}$$

$\tilde\gamma = -1+2i\nu$ is the solution of $\omega = \hat\aleph(\tilde\gamma)$
We use the point-splitting regularization in the orthogonal direction for our light-ray operators $\Rightarrow$ cutoffs are defined as $\Lambda_x = \frac{1}{|x_{13\perp}|}$ and $\Lambda_y = \frac{1}{|y_{13\perp}|}$
Rewrite

$$\langle S_+^{1+\omega_1}(x_{1\perp},x_{3\perp})S_-^{1+\omega_2}(y_{1\perp},y_{3\perp})\rangle \xrightarrow[x_{13\perp},\, y_{13\perp}\to 0]{} \delta(\omega_1-\omega_2)\Upsilon(\gamma+\omega)\frac{(x_{13\perp}^2)^{\frac{\gamma}{2}}(y_{13\perp}^2)^{\frac{\gamma}{2}}}{((x-y)_\perp^2)^{2+\omega+\gamma}}.$$

The anomalous dimension $\gamma = \tilde\gamma-\omega$ satisfies

$$\omega = \hat\aleph(\gamma+\omega) = \hat\aleph(\gamma)+\hat\aleph'(\gamma)\hat\aleph(\gamma)+o(g^4)\,.$$

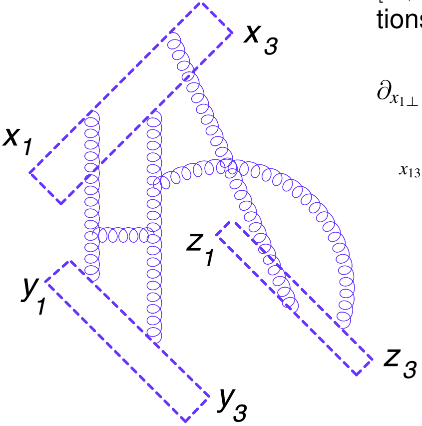
- Lipatov-Fadin formula

Result for anomalous dimension

$$\gamma_j = -2\frac{\alpha_s N_c}{\pi(j-1)} \; + \; [0 + \zeta(3)(j-1)]\Big(\frac{\alpha_s N_c}{\pi(j-1)})\Big)^3 + ...$$

is an anomalous dimension of light-ray operator $F\nabla^{j-2}F(x_\perp)$

CF of three Wilson frames: one in "+" direction and two in "-"



$[x_1, x_3]_\Box \equiv$ Wilson frame (without F insertions)

$$\partial_{x_{1\perp}} \cdot \partial_{x_{3\perp}} \int dx_{1-} dx_{3-} (x_{1-} - x_{3-})^{-2-\omega} [x_1, x_3]_\Box \to$$

$$\underset{x_{13\perp} \to 0,\, \omega \to 0}{\to} |x_{13\perp}|^{\gamma_J} c(g_{YM}^2, N_c, \omega) \breve{\mathcal{S}}^{2+\omega}(x_{1\perp}).$$

$$[x_1, x_3]_\Box \equiv \mathbf{U}^{\sigma_{1-}}(x_{1\perp}, x_{3\perp})$$

with some cutoff σ_{1-}

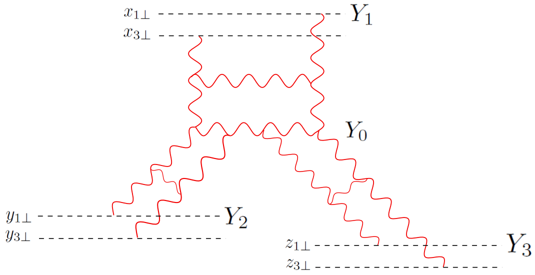
Using decomposition over Wilson lines we get:

$$\langle S_+^{2+\omega_1}(x_{1\perp},x_{3\perp})S_-^{2+\omega_2}(y_{1\perp},y_{3\perp})S_-^{2+\omega_3}(z_{1\perp},z_{3\perp})\rangle =$$

$$= \mathcal{D}_\perp \int_{-\infty}^{\infty} dx_{1-} \int_{x_{1-}}^{\infty} dx_{3-}\, x_{31-}^{-2-\omega_1} \int_{-\infty}^{\infty} dy_{1+} \int_{y_{1+}}^{\infty} dy_{3+}\, y_{31+}^{-2-\omega_2} \int_{-\infty}^{\infty} dz_{1+} \int_{z_{1+}}^{\infty} dz_{3+}\, z_{31+}^{-2-\omega_3} \times$$

$$\times \langle \mathbf{U}^{\sigma_{1-}}(x_{1\perp},x_{3\perp})\mathbf{V}^{\sigma_{2+}}(y_{1\perp},y_{3\perp})\mathbf{W}^{\sigma_{3+}}(z_{1\perp},z_{3\perp})\rangle,$$

where $\mathcal{D}_\perp = -\frac{N^3}{c(\omega_1)c(\omega_2)c(\omega_3)}(\partial_{x_{1\perp}}\cdot\partial_{x_{3\perp}})(\partial_{y_{1\perp}}\cdot\partial_{y_{3\perp}})(\partial_{z_{1\perp}}\cdot\partial_{z_{3\perp}})$.



- BK equation:

$$\sigma \frac{d}{d\sigma} \mathbf{U}^{\sigma}(z_1, z_2) = \mathcal{K}_{\mathrm{BK}} * \mathbf{U}^{\sigma}(z_1, z_2),$$

where $\mathcal{K}_{\mathrm{BK}}$ in LO approximation:

$$\mathcal{K}_{\mathrm{LO\,BK}} * \mathbf{U}(z_1, z_2) =$$
$$= \frac{2g^2}{\pi} \int d^2 z_3 \frac{z_{12}^2}{z_{13}^2 z_{23}^2} \left[\mathbf{U}(z_1, z_3) + \mathbf{U}(z_3, z_2) - \mathbf{U}(z_1, z_2) - \mathbf{U}(z_1, z_3)\mathbf{U}(z_3, z_2)\right].$$

- Schematically calculation of correlation function of 3 dipoles can be wrote as:

$$\int dY_0 (\mathbf{U}^{Y_1} \to \mathbf{U}^{Y_0}) \otimes (\text{BK vertex at } Y_0) \otimes \begin{pmatrix} \langle \mathbf{U}^{Y_0} \mathbf{V}^{Y_2} \rangle \\ \langle \mathbf{U}^{Y_0} \mathbf{W}^{Y_3} \rangle \end{pmatrix}$$

where we introduced rapidity $Y_i = \ln \sigma_i$

Planar contribution

$$
\begin{aligned}
&\langle \mathbf{U}^{Y_1}(x_{1\perp},x_{3\perp})\mathbf{V}^{Y_2}(y_{1\perp},y_{3\perp})\mathbf{W}^{Y_3}(z_{1\perp},z_{3\perp})\rangle_{pl}\\
&= -\frac{2g^2}{\pi}\int dY_0\int d\nu_1\int d^2x_0\frac{\nu_1^2}{\pi^2}E_{\nu_1}(x_{10},x_{30})e^{\aleph(\nu_1)Y_{10}}\times\\
&\times\frac{1}{\pi^2}\int\frac{d^2\alpha d^2\beta d^2\gamma}{|\gamma-\beta|^2|\gamma-\alpha|^2|\beta-\alpha|^2}E^*_{\nu_1}(\gamma-x_0,\beta-x_0)\\
&\times(\langle \mathbf{U}^{Y_0}(\gamma,\alpha)\mathbf{V}^{Y_2}(y_{1\perp},y_{3\perp})\rangle\langle \mathbf{U}^{Y_0}(\alpha,\beta)\mathbf{W}^{Y_3}(z_{1\perp},z_{3\perp})\rangle\\
&+\langle \mathbf{U}^{Y_0}(\gamma,\alpha)\mathbf{W}^{Y_3}(z_{1\perp},z_{3\perp})\rangle\langle \mathbf{U}^{Y_0}(\alpha,\beta)\mathbf{V}^{Y_2}(y_{1\perp},y_{3\perp})\rangle)
\end{aligned}
$$

where

$$
\begin{aligned}
&\langle \mathbf{U}^{Y_0}(\gamma,\alpha)\mathbf{V}^{Y_2}(y_{1\perp},y_{3\perp})\rangle =\\
&=\frac{8g^4(1-N_c^2)}{N_c^4}\int d^2y_0\int\frac{d\nu_2\nu_2^2e^{Y_{02}\aleph(\nu_2)}}{(\frac14+\nu_2^2)^2}E_{\nu_2}(\gamma-y_0,\alpha-y_0)E^*_{\nu_2}(y_{10},y_{30}) \qquad (1)\\
&\langle \mathbf{U}^{Y_0}(\alpha,\beta)\mathbf{W}^{Y_3}(z_{1\perp},z_{3\perp}) =\\
&=\frac{8g^4(1-N_c^2)}{N_c^4}\int d^2z_0\int\frac{d\nu_3\nu_3^2e^{Y_{03}\aleph(\nu_3)}}{(\frac14+\nu_3^2)^2}E_{\nu_3}(\gamma-z_0,\alpha-z_0)E^*_{\nu_3}(z_{10},z_{30})
\end{aligned}
$$

Planar contribution

As we learned in case of two-point correlator we can choose the rapidity cutoff using anharmonic ratios:

$$e^{Y_{12}\aleph(\nu)} \rightarrow$$

$$\rightarrow \frac{-i}{\sin\pi\aleph(\nu)}\left(\frac{((x_1-y_3)^2)^{\frac{\aleph(\nu)}{2}}((x_3-y_1)^2)^{\frac{\aleph(\nu)}{2}}}{(x_{13}^2)^{\frac{\aleph(\nu)}{2}}(y_{13}^2)^{\frac{\aleph(\nu)}{2}}} - \frac{((x_1-y_1)^2)^{\frac{\aleph(\nu)}{2}}((x_3-y_3)^2)^{\frac{\aleph(\nu)}{2}}}{(x_{13}^2)^{\frac{\aleph(\nu)}{2}}(y_{13}^2)^{\frac{\aleph(\nu)}{2}}}\right)$$

In the LO approximation we can take just an asymptotic:

$$e^{Y_{12}\aleph(\nu)} \rightarrow \left(\frac{x_{31-}y_{31+}}{\Lambda^2}\right)^{\aleph(\nu)},$$

where Λ is a cutoff whose precise value is irrelevant for us. Using this identification and introducing L_0 for the intermediate rapidity $Y_0 = \log\frac{L_0}{\Lambda}$ we can identify all rapidities in the following way:

$$Y_{10} = \log\frac{x_{31-}}{L_0}, \quad Y_{02} = \log\frac{L_0 y_{31+}}{\Lambda^2}, \quad Y_{03} = \log\frac{L_0 z_{31+}}{\Lambda^2}$$

Integral over rapidities reads as:

$$\int L_1^{-1-\omega_1}\int L_2^{-1-\omega_2}\int L_3^{-1-\omega_3}\int dY_0\, e^{Y_{10}\aleph_1+Y_{02}\aleph_2+Y_{03}\aleph_3}\theta(Y_{10})\theta(Y_0-max(Y_2,Y_3))$$

The structure of 3-point correlator in 2d - $\perp$ space

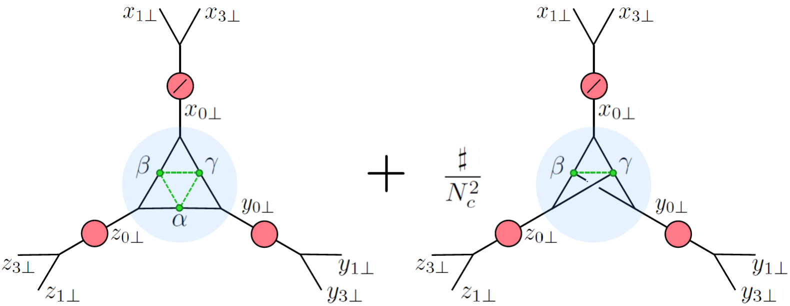


Figure: The structure of 3-point correlator. Red circles correspond to BFKL propagators (the crossed one has extra multiplier $(\frac{1}{4} + \nu_1^2)^2$). The blue blob corresponds to the 3-point functions of 2-dimensional BFKL CFT. The triple veritces correspond to E-functions. The $\alpha\beta\gamma$-triangle in the first, planar, term and $\beta\gamma$-link in the second, nonplanar, term correspond to triple pomeron vertex.

Result:

$$\langle \mathcal{S}_{n_1}^{1+\omega_1}(x_{1\perp}, x_{3\perp}) \mathcal{S}_{n_2}^{1+\omega_2}(y_{1\perp}, y_{3\perp}) \mathcal{S}_{n_2}^{1+\omega_3}(z_{1\perp}, z_{3\perp}) \rangle =$$

$$= -ig^{10} \frac{\delta(\omega_1 - \omega_2 - \omega_3)}{c(\omega_1)c(\omega_2)c(\omega_3)} H \frac{\Psi(\nu_1^*, \nu_2^*, \nu_3^*) |x_{13}|^{\gamma_1} |y_{13}|^{\gamma_2} |z_{13}|^{\gamma_3}}{|x-y|^{2+\gamma_1+\gamma_2-\gamma_3} |x-z|^{2+\gamma_1+\gamma_3-\gamma_2} |y-z|^{2+\gamma_2+\gamma_3-\gamma_1}}$$

where ν_i^* is a solution of BFKL equation for anomalous dimensions $\omega_i = \aleph(\nu_i^*)$

$$H = \frac{2^{10}(N_c^2-1)^2}{\pi^2 N_c^5} \gamma_1^2 (2+\gamma_1)^4 (2+\gamma_2)^2 (2+\gamma_3)^2 \frac{G(\nu_1^*)}{\aleph'(\nu_1^*)} \frac{G(\nu_2^*)}{\aleph'(\nu_2^*)} \frac{G(\nu_3^*)}{\aleph'(\nu_3^*)},$$

$\gamma_i = \gamma(j_i)$ - anomalous dimension ($j_i = 1 + \omega_i$) and

$$G(\nu) = \frac{\nu^2}{(\frac{1}{4}+\nu^2)^2} \frac{\pi \Gamma^2(\frac{1}{2}+i\nu)\Gamma(-2i\nu)}{\Gamma^2(\frac{1}{2}-i\nu)\Gamma(1+2i\nu)},$$

$$\Psi(\nu_1, \nu_2, \nu_3) = \Omega(h_1, h_2, h_3) - \frac{2\pi}{N_c^2} \Lambda(h_1, h_2, h_3) \text{Re}(\psi(1) - \psi(h_1) - \psi(h_2) - \psi(h_3)),$$

where $h_i = \frac{1}{2} + i\nu_i = 1 + \frac{\gamma_i}{2}$.

Expression for Ω and Λ was obtained by G.Korchemsky in terms of higher hypergeometric and Meijer G-functions.

$n_2 \to n_3$ limit

To identify the function $\Psi(\nu_1^*, \nu_2^*, \nu_3^*)$ with structure constants of CF of three LR operators we need to consider limit $n_2 \to n_3$ in the formula

$$\langle S_{n_1}^{j_1}(x_{1_\perp})\, S_{n_2}^{j_2}(x_{2_\perp})\, S_{n_3}^{j_2}(x_{3_\perp})\rangle \;=\; \frac{C(\alpha_s, \omega_1, \omega_2, \omega_3)}{(\omega_1+\omega_2-\omega_3)(\omega_1+\omega_3-\omega_2)(\omega_2+\omega_3-\omega_1)}$$
$$\times\; \frac{(n_1\cdot n_2)^{\frac{\omega_1+\omega_2-\omega_3}{2}}}{|x_{12_\perp}|^{\Delta_1+\Delta_2-\Delta_3-1}} \frac{(n_1\cdot n_3)^{\frac{\omega_1+\omega_3-\omega_2}{2}}}{|x_{13_\perp}|^{\Delta_1+\Delta_3-\Delta_2-1}} \frac{(n_2\cdot n_3)^{\frac{\omega_2+\omega_3-\omega_1}{2}}}{|x_{23_\perp}|^{\Delta_2+\Delta_3-\Delta_1-1}}$$

The limit $n_2 \to n_3$ is tricky:
in the limit $n_2 \to n_3$ we get a “zero mode” coming from boost invariance at $n_2 = n_3$

$$\frac{1}{\omega_1-\omega_2-\omega_3}\Big(\frac{(n_2,n_3)}{s}\Big)^{\frac{\omega_2+\omega_3-\omega_1}{2}} \;\overset{n_2\to n_3}{\to}\; \int d\xi e^{-\xi(\omega_1-\omega_2-\omega_3)} \;=\; \delta(\omega_1-\omega_2-\omega_3)$$

Limit $n_2 \to n_3$

Rapidity integral at $n_2 = n_3$

$$\int dY_1 dY_2 dY_3 \int dY_0\, \theta(Y_1 - Y_0)\theta(Y_0 + Y_2)\theta(Y_0 + Y_3)e^{-\omega_1 Y_1 - \omega_2 Y_2 - \omega_3 Y_3}$$

$$e^{\aleph_1(Y_1 - Y_0) + \aleph_2(Y_0 + Y_2) + \aleph_3(Y_0 + Y_3)} \;=\; \frac{\delta(\omega_1 - \omega_2 - \omega_3)}{(\omega_1 - \aleph_1)(\omega_2 - \aleph_2)(\omega_3 - \aleph_3)}$$

Let us take $n_2 \neq n_3$ but $n_1 \cdot n_2 \simeq n_1 \cdot n_3$. We can use our formulas for $n_2 = n_3$ case until longitudinal distances between frames "2" and "3" are smaller than typical transverse separation $\Delta_\perp^2$, i.e. when $(y_1 - z_1)^2 \leq \Delta_\perp^2 \;\Leftrightarrow\; l_2 l_3 n^{23} \leq \Delta_\perp^2$.
In terms of rapidities $Y_2 = \ln l_2 \frac{\sqrt{n^{12}}}{\Delta_\perp},\; Y_3 = \ln l_3 \frac{\sqrt{n^{12}}}{\Delta_\perp}$ this restriction means $Y_2 + Y_3 \;\leq\; r,\; r \equiv\; \ln \frac{n_1 \cdot n_2}{n_2 \cdot n_3}$.

Limit $n_2 \to n_3$

Rapidity integral with restriction $Y_2 + Y_3 \ \leq \ r, \ r \equiv \ \ln \frac{n_1 \cdot n_2}{n_2 \cdot n_3}$.

$$\int dY_1 dY_2 dY_3 \int dY_0 \ \theta(Y_1 - Y_0)\theta(Y_0 + Y_2)\theta(Y_0 + Y_3)\theta\big(Y_2 + Y_3 < r\big)$$

$$e^{-\omega_1 Y_1 - \omega_2 Y_2 - \omega_3 Y_3 + \aleph_1(Y_1 - Y_0) + \aleph_2(Y_0 + Y_2) + \aleph_3(Y_0 + Y_3)}$$

$$= \frac{e^{-\frac{r}{2}(\omega_2 + \omega_3 - \omega_1)}}{(\omega_1 - \omega_2 - \omega_3)(\omega_1 - \aleph_1)\big(\omega_2 - \aleph_2 + \frac{\omega_1 - \omega_2 - \omega_3}{2}\big)\big(\omega_3 - \aleph_3 + \frac{\omega_1 - \omega_2 - \omega_3}{2}\big)}$$

$$\stackrel{\omega_2 + \omega_3 \to \omega_1}{\to} \quad \frac{\left(\frac{n_2 \cdot n_3}{n_1 \cdot n_2}\right)^{\omega_2 + \omega_3 - \omega_1}}{(\omega_1 - \omega_2 - \omega_3)(\omega_1 - \aleph_1)(\omega_2 - \aleph_2)(\omega_3 - \aleph_3)}$$

$$\Rightarrow \quad \frac{1}{\omega_1 - \omega_2 - \omega_3}\left(\frac{(n_2, n_3)}{s}\right)^{\frac{\omega_2 + \omega_3 - \omega_1}{2}} \quad \stackrel{n_2 \to n_3}{\to} \quad \delta(\omega_1 - \omega_2 - \omega_3)$$

Finally for normalized structure constant $c_{\omega_1,\omega_2,\omega_3} = \frac{C_{+--}(\{\Delta_i\},\{1+\omega_i\})}{\sqrt{b_{1+\omega_1} b_{1+\omega_2} b_{1+\omega_3}}}$ we get:

$$C_{\omega_1,\omega_2,\omega_3} = i^{3/2} g^4 \frac{2}{\pi^5} \frac{\sqrt{N_c^2-1}}{N_c^2} \gamma_1^2 (2+\gamma_1)^2 \sqrt{\frac{G(\nu_1^*)}{\aleph'(\nu_1^*)} \frac{G(\nu_2^*)}{\aleph'(\nu_2^*)} \frac{G(\nu_3^*)}{\aleph'(\nu_3^*)}} \Psi(\nu_1^*, \nu_2^*, \nu_3^*),$$

where $\omega_i = \aleph(\nu_i^*)$ and

$$G(\nu) = \frac{\nu^2}{(\frac{1}{4}+\nu^2)^2} \frac{\pi \Gamma^2(\frac{1}{2}+i\nu)\Gamma(-2i\nu)}{\Gamma^2(\frac{1}{2}-i\nu)\Gamma(1+2i\nu)},$$

$$\Psi(\nu_1,\nu_2,\nu_3) = \Omega(h_1,h_2,h_3) - \frac{2\pi}{N_c^2} \Lambda(h_1,h_2,h_3) \mathrm{Re}(\psi(1)-\psi(h_1)-\psi(h_2)-\psi(h_3))$$

with notation $h_i = \frac{1}{2} + i\nu_i = 1 + \frac{\gamma_i}{2}$, $\omega_i = \aleph(\nu_i)$

The structure of the formula is $C_{\omega_1,\omega_2,\omega_3} = g \frac{\sqrt{N_c^2-1}}{N_c^2} f(\frac{g^2}{\omega_1}, \frac{g^2}{\omega_2}, \frac{g^2}{\omega_3})$

In the limit $\frac{g^2}{\omega_i} \to 0$ we get the asymptotics:

$$\Omega(h_1^*, h_2^*, h_3^*) \to -\frac{16\pi^3}{\gamma_1^2\gamma_2^2\gamma_3^2} \cdot [\gamma_1^2(\gamma_2+\gamma_3)+\gamma_2^2(\gamma_1+\gamma_3)+$$

$$+\gamma_3^2(\gamma_1+\gamma_2)+\gamma_1\gamma_2\gamma_3)(1+O(g^2/\omega_i))$$

$$\Lambda(h_1^*, h_2^*, h_3^*) \to \frac{8\pi^2(\gamma_1+\gamma_2+\gamma_3)}{\gamma_1\gamma_2\gamma_3}(1+O(g^2/\omega_i))$$

$\gamma_i = -\frac{8g^2}{\omega_i} + o(\frac{g^2}{\omega_i})$

$$C_{\omega_1,\omega_2,\omega_3} = -ig^2\frac{\sqrt{N_c^2-1}}{\sqrt{2\pi}N_c^2}\frac{1}{\omega_1^{\frac{5}{2}}\omega_2^{\frac{1}{2}}\omega_3^{\frac{1}{2}}}(\omega_1^2(\omega_2+\omega_3)+$$

$$\omega_2^2(\omega_1+\omega_3)+\omega_3^2(\omega_1+\omega_2)+\omega_1\omega_2\omega_3)(1+O(g^2))$$

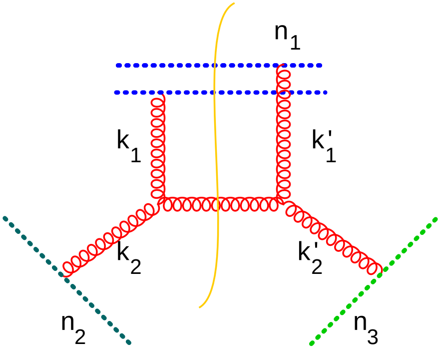
Conclusions and Outlook

Conclusions

- We calculated QCD structure constants in the "BFKL limit" $\omega_i \to 0$ at $\omega_1 = \omega_2 + \omega_3$

Outlook

- Structure constants in the triple Regge limit ($\omega_i \neq \omega_j + \omega_k$)



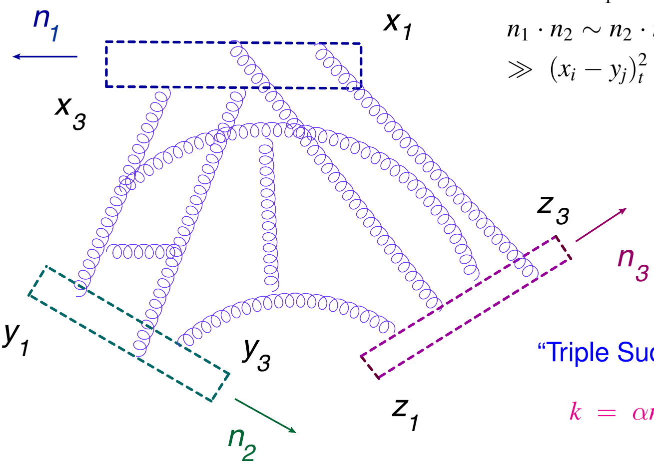
$$k = \alpha n_1 + \beta n_2 + \gamma n_3 + k_t$$

At $\alpha_1 \gg \alpha_2, \alpha_3$
- BFKL logarithms $g^2 \ln \frac{\alpha_{\max}}{\alpha_{\min}}$

$$\frac{1}{n_{12}^2} L(k_1,k_2)L(k'_1,k'_2) = (k_1 - k'_1)_t^2 + \frac{s_{12}s_{23}}{s_{13}}(\beta_1 - \beta'_1)^2$$

$$+ \frac{\left(k_{1t}^2 + \frac{s_{12}s_{23}}{s_{13}}\beta_1^2\right)\left({k'_{2t}}^2 + \frac{s_{12}s_{23}}{s_{13}}{\beta'}_2^2\right)}{(k_1+k_2)_t^2 + \frac{s_{12}s_{23}}{s_{13}}(\beta_1+\beta_2)^2} + \frac{\left(k_{2t}^2 + \frac{s_{12}s_{23}}{s_{13}}\beta_2^2\right)\left({k'_{1t}}^2 + \frac{s_{12}s_{23}}{s_{13}}{\beta'}_1^2\right)}{(k_1+k_2)_t^2 + \frac{s_{12}s_{23}}{s_{13}}(\beta_1+\beta_2)^2} + O\left(\frac{k_t^2}{\alpha_1 s}\right)$$

Wilson frames in triple Regge limit



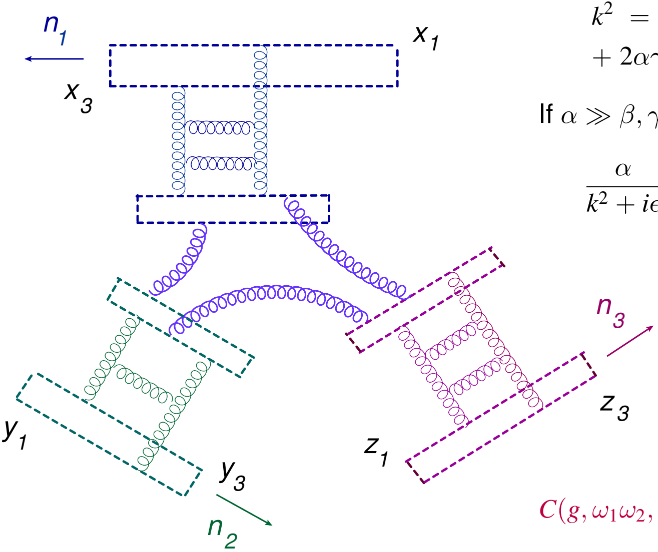
$$n_1^2 = n_2^2 = n_3^2 = 0,$$

$$n_1 \cdot n_2 \sim n_2 \cdot n_3 \sim n_1 \cdot n_3 \gg$$

$$\gg (x_i - y_j)_t^2 \sim (x_i - z_j)_t^2 \sim (y_i - z_j)_t^2$$

"Triple Sudakov variables":

$$k = \alpha n_1 + \beta n_2 + \gamma n_3 + k_t$$



$$k^2 = -k_t^2 + 2\alpha\beta(n_1 \cdot n_2) + 2\alpha\gamma(n_1 \cdot n_3) + 2\beta\gamma(n_2 \cdot n_3)$$

If $\alpha \gg \beta, \gamma$ - eikonal

$$\frac{\alpha}{k^2 + i\epsilon} \to \frac{1}{\beta + \gamma + i\epsilon\alpha}$$

$$C(g, \omega_1\omega_2, \omega_3) = g^2 \times (?)$$