Transverse spin structure of octet baryons using lattice QCD

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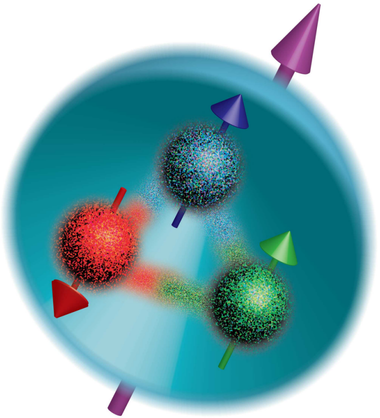
- One of the major goals of the nuclear physics community is to understand the structure and behaviour of strongly interacting matter
- We wish to understand this in terms of its most basic constituents - Quarks and Gluons
- Important to this goal is understanding the internal structure of the nucleon and how all these internal constituents interact.



(EIC white paper [1212.1701])

Hadron Structure

Why are we Interested

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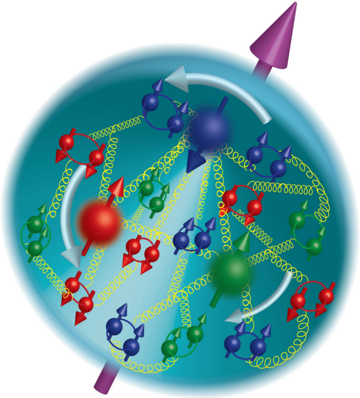

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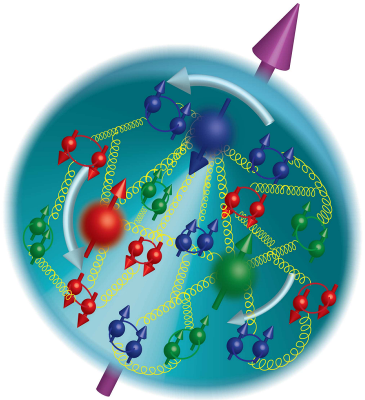

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Jefferson Lab

The Jefferson Lab's Continuous Electron Beam Accelerator Facility explores the internal structure of hadronic through

- Electric and magnetic Elastic form factors
- Deeply Virtual Compton scattering
- Parity Violation
- and many more experiments

Jefferson Lab

Experiments on Hadron Structure

COMPASS

The CERN COMPASS experiment at the Super Proton Synchrotron aims to

- Discover more about how the property of spin arises in protons and neutrons
- how much spin is contributed by the gluons
- Uses Muons fired at polarized targets

Experiments on Hadron Structure

Mainz

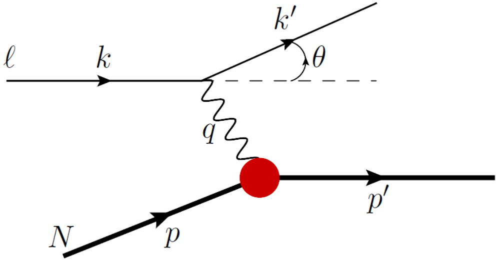
The A1 - Electron Scattering group at Mainz uses three high-resolution focussing magnetic spectrometers investigating

- Form Factors in electron-proton elastic scattering
- Radiative inelastic scattering, with response described in terms of polarizabilities and spatial distributions

Hadron Structure

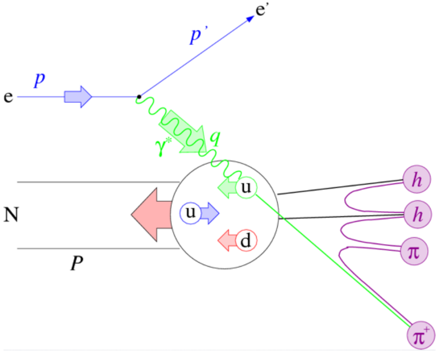
Two types of $e-p$ scattering are

Elastic Scattering



$\Rightarrow$ Form Factors

Deep Inelastic Scattering

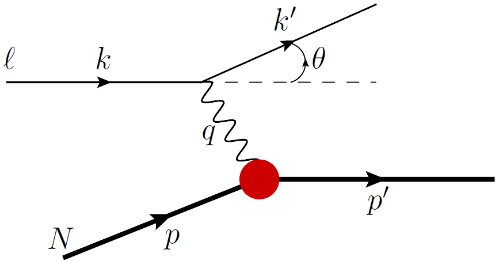


$\Rightarrow$ Structure Functions

Hadron Structure

Two types of $e-p$ scattering are

Elastic Scattering



Elastic Scattering

- Maps out the charge and density distributions inside the nucleon
- 4-momentum transfer $q = k - k' = P' - P$
- Final state of the nucleon remains intact with recoil
- We compare the cross section with that of a point particle

$$\frac{d\sigma}{d\Omega} = \left(\frac{d\sigma}{d\Omega}\right)_{point} |F(q^2)|^2$$

Elastic Scattering

- Elastic scattering cross-section from spin-1/2 target with extended structure

$$\frac{d\sigma}{d\Omega} = \left(\frac{d\sigma}{d\Omega}\right)_{Mott} \left[\frac{G_E^2(Q^2) + \tau G_M^2(Q^2)}{1+\tau} + 2\tau G_M^2(Q^2) tan^2 \frac{\theta}{2} \right]$$

where $\tau = \frac{Q^2}{4M^2}$

- Here

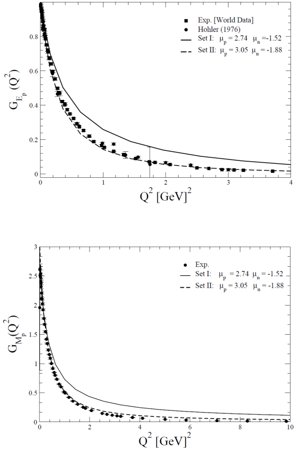
$$G_E(Q^2) = F_1(Q^2) - \tau F_2(Q^2)$$
$$G_M(Q^2) = F_1(Q^2) + F_2(Q^2)$$

are the Sachs electric and magnetic form factors

- By rewriting this in terms of the virtual photon's longitudinal polarisation and using Rosenbluth separation to vary scattering angles, we can extract G_E and G_M

Elastic Scattering

- $G_E \neq G_M \rightarrow$ different charge and magnetisation distributions
- Initial and final states have the same internal state $\rightarrow$ Fourier transforms of these form factors are the density distributions
- But M is finite so we need to consider nucleon recoil effects of elastic scattering $\rightarrow$ initial and final states now measured in different frames
- One of the ways we get around this is by considering the Infinite momentum frame



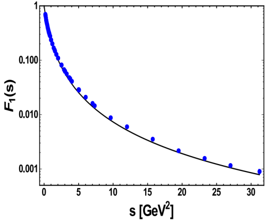
arXiv:1707.00168v2 [hep-ph]

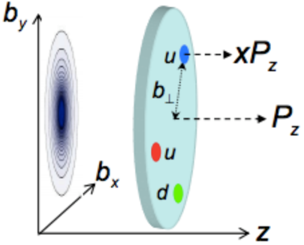
Density Distributions

- Considering the Infinite momentum frame where $|P| = |P'|$
- The initial and final states have momenta with equal magnitude,to a Lorentz contraction
- $F_1(Q^2)$ can be interpreted as the Fourier transform of the charge distribution

$$f(b_\perp^2) \equiv \int \frac{d^2\Delta_\perp}{(2\pi^2)} e^{-ib_\perp.\Delta_\perp} f(t = -\Delta_\perp^2)$$

where $\Delta_\perp$ is the transverse momentum transfer





Transverse Spin Density

Transverse Spin Density Equation

In order to determine the spin density, we require each of the following form factors in terms of $b_\perp$

M. Diehl and P. Hagler [hep-ph/0504175]

$$\rho^n(b_\perp, s_\perp, S_\perp) = \int_{-1}^{1} dx\, x^{n-1} \rho(x, b_\perp, s_\perp, S_\perp)$$

$$= \frac{1}{2}\{A_{n0}(b_\perp^2) + s_\perp^i S_\perp^i \left(A_{Tn0}(b_\perp^2) - \frac{1}{4m^2}\Delta_{b_\perp}\tilde{A}_{Tn0}(b_\perp^2)\right)$$

$$+ \frac{b_\perp^j \epsilon^{ji}}{m}\left(S_\perp^i B'_{n0}(b_\perp^2) + s_\perp^i \overline{B}'_{Tn0}(b_\perp^2)\right) + s_\perp^i \left(2 b_\perp^i b_\perp^j - b_\perp^2 \delta^{ij}\right) S_\perp^j \frac{1}{m^2}\tilde{A}''_{Tn0}(b_\perp^2)\}$$

- Where $b_\perp$ is the distance from the center of momentum
- $s_\perp$ is the transverse spin of the quark
- $S_\perp$ is the transverse spin of the nucleon

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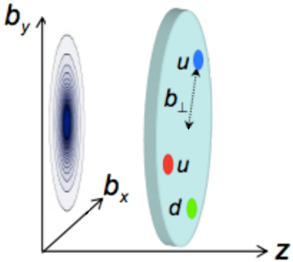


Unpolarised

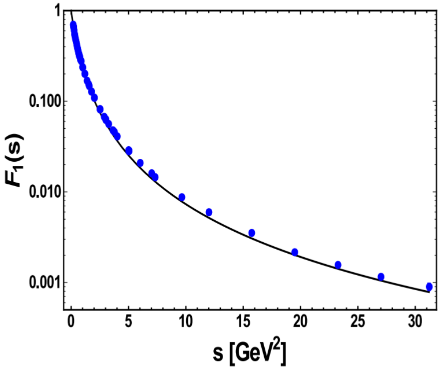
Transverse Spin Density Equation

In order to determine the spin density, we require each of the following form factors in terms of $b_\perp$

For $n = 1$, this generalised form factor $A_{10}(b_\perp^2)$ is the fourier transformed Dirac Electric form factor

$$A_{10}(Q^2) = F_1(Q^2)$$

$$\left\langle N(p', s') \middle| j_\mu(q) \middle| N(p, s) \right\rangle$$
$$= \overline{u}(p', s') \left[\gamma_\mu F_1(Q^2) + \frac{i\sigma_{\mu\nu} q^\nu}{2m_B} F_2(Q^2) \right] u(p, s)$$



arXiv:1611.07265v2 [hep-ph]

Transverse Spin Density Equation

In order to determine the spin density, we require each of the following form factors in terms of $b_\perp$

Flavour Form Factors

- Experimental results are for the proton
- We wish to separate form factors into individual quark contributions
- To obtain these we:
  - Assume Charge Symmetry

$$F^{p,u}_{1/2} = F^{n,d}_{1/2}$$

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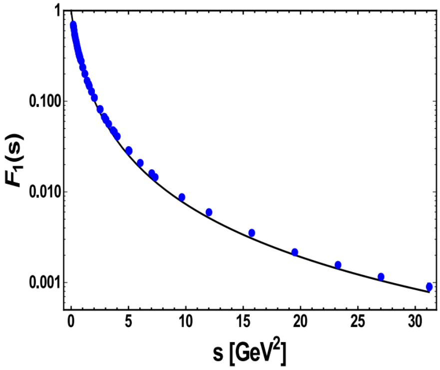

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Transverse Spin Density Equation

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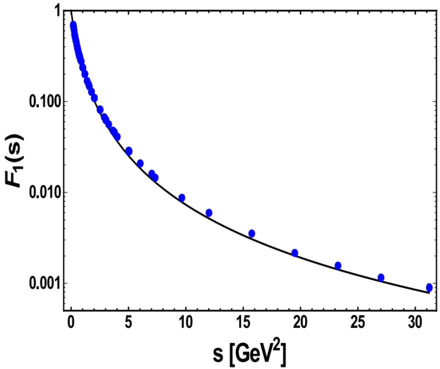
Flavour Form Factors

- Experimental results are for the proton
- We wish to separate form factors into individual quark contributions
- To obtain these we:
  - Assume Strange FF = 0
  - decompose p and n FFs

$$F^p_{1/2} = \frac{2}{3}F^u_{1/2} - \frac{1}{3}F^d_{1/2}$$

$$F^n_{1/2} = \frac{2}{3}F^d_{1/2} - \frac{1}{3}F^u_{1/2}$$

- Solve for u and d form factors



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&= \frac{1}{2}\Big[A_{n0}(b_\perp^2) + s_\perp^i S_\perp^i \Big(A_{Tn0}(b_\perp^2) - \frac{1}{4m^2}\Delta_{b_\perp} \tilde{A}_{Tn0}(b_\perp^2)\Big) \\
&+ \frac{b_\perp^j \epsilon^{ji}}{m}\Big(S_\perp^i B'_{n0}(b_\perp^2) + s_\perp^i \overline{B}'_{Tn0}(b_\perp^2)\Big) \\
&+ s_\perp^i \Big(2b_\perp^i b_\perp^j - b_\perp^2 \delta^{ij}\Big) S_\perp^j \frac{1}{m^2} \tilde{A}''_{Tn0}(b_\perp^2)\Big]
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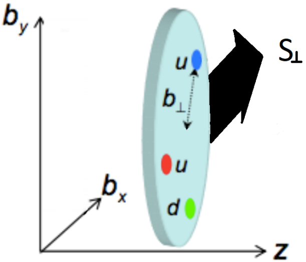


Nucleon Spin Polarisation

Transverse Spin Density Equation

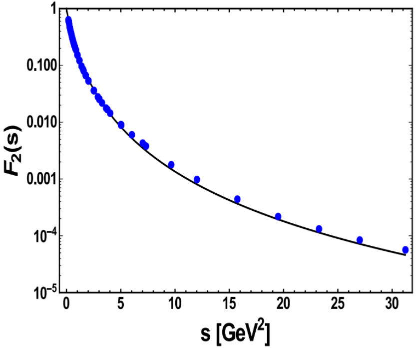
In order to determine the spin density, we require each of the following form factors in terms of $b_\perp$

For $n = 1$, this form factor $B'_{10}(b_\perp^2)$ is the first derivative of the Fourier transformed Pauli Magnetic form factor $B_{10}(Q^2) = F_2(Q^2)$

$$\left\langle N(p',s') \middle| j_\mu(q) \middle| N(p,s) \right\rangle$$

$$= \overline{u}(p',s')\left[\, \gamma_\mu F_1(Q^2) + \frac{i\sigma_{\mu\nu} q^\nu}{2m_B} F_2(Q^2) \,\right] u(p,s)$$

$$B'_{10}(b_\perp^2) = \frac{\partial}{\partial b^2} B_{10}(b_\perp^2)$$



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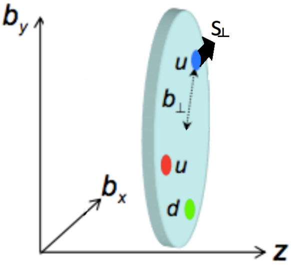
$$



Quark Spin Polarisation

Transverse Spin Density Equation

In order to determine the spin density, we require each of the following form factors in terms of $b_\perp$

For n=1, the form factor $\overline{B}'_{T10}(b_\perp^2)$ is the first derivative of a fourier transformed combination of the tensor form factors $\overline{B}_{T10} \approx 2\tilde{A}_{T10} + B_{T10}$ and acts like an anomalous tensor magnetic moment similar to $F_2(Q^2)$

$$\left\langle N(p', s') \middle| \overline{\psi}(0) i\sigma^{\mu\nu} \psi(0) \middle| N(p, s) \right\rangle =$$

$$\bar{u}(p', s') \left[i\sigma^{\mu\nu} A_{T10}(Q^2) + \frac{\bar{P}^{[\mu} \Delta^{\nu]}}{m^2} \tilde{A}_{T10}(Q^2) + \frac{\gamma^{[\mu} \bar{P}^{\nu]}}{2m} B_{T10}(Q^2) \right] u(p, s)$$

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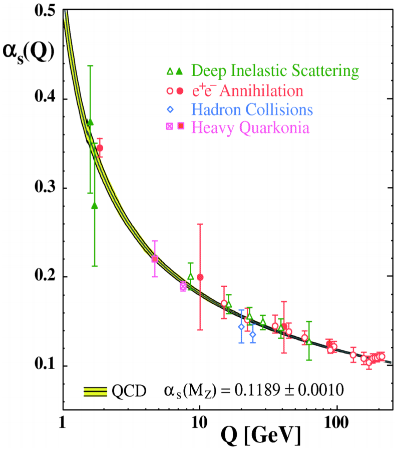
Not enough experimental data

Lattice QCD

QCD

- As a result of gluon self interactions, the QCD coupling $\alpha_s = g^2/4\pi$ becomes very small at high energies acting almost like a theory of free partons
- This asymptotic freedom of the QCD theory allows perturbative methods to be used at small distance scales for high energy reactions
- Low energy reactions however, employ strong QCD interactions and thus perturbation theory is no longer applicable and we must numerically discretise the QCD equation using Lattice



arXiv:hep-ph/0607209

Quantum Chromodynamics

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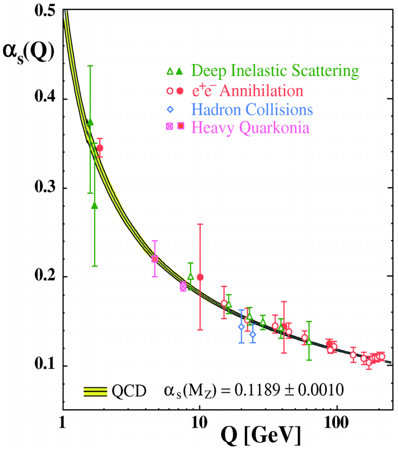

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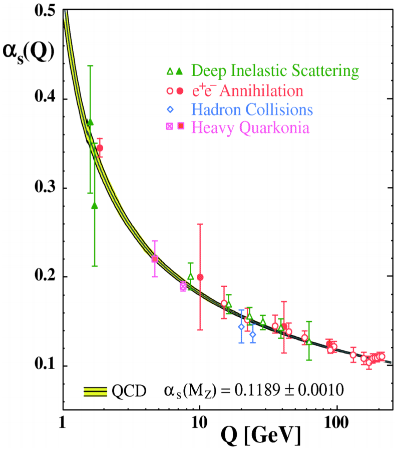
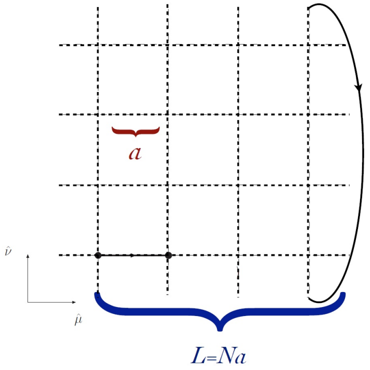


arXiv:hep-ph/0607209

The Lattice

- We work in Euclidean space where $t \to i\tau$
- Discretise space-time with a separation of the lattice spacing a
- We have a finite lattice, so we introduce periodic boundary conditions
- Discretise 3-momenta, $\vec{q}^2$ given by $(n_x^2 + n_y^2 + n_z^2) \times \left(\frac{2\pi}{La}\right)^2$
- The values of the 4-momentum transfer q^2 vary with the baryon mass M_B by the dispersion relation $q^2 = \left(\sqrt{M_B^2 + \vec{q}^2} - M_B\right)^2 - \vec{q}^2$
- We formulate the theory on the 4-torus



$$\mathbb{L} \subset a\mathbb{Z}^4 = \{x | x^\mu = an^\mu, n \in \mathbb{Z}^4\}$$

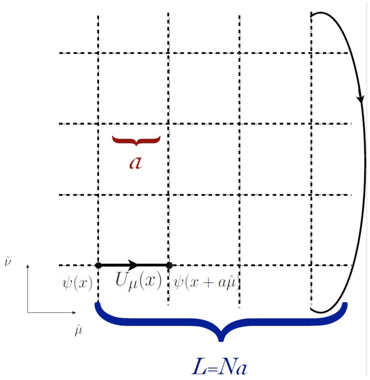
The Lattice

- Quark fields reside on the sites of the lattice $\psi(x)$
- Gauge fields on the links $U_\mu(x) = e^{-iagA_\mu(x)}$
- We discretise the QCD action and path integrals

$$\langle \mathcal{O} \rangle = \frac{1}{Z} \int \mathcal{D}A\mathcal{D}\overline{\psi}\mathcal{D}\psi\mathcal{O}[A,\overline{\psi},\psi]e^{-S[A,\overline{\psi},\psi]}$$

- We approximate using Monte Carlo methods weighted by the action

$$\langle \mathcal{O} \rangle \simeq \frac{1}{N_{conf}} \sum_{i}^{N_{conf}} \mathcal{O}([U^{[i]}])$$

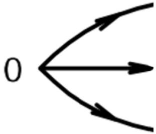


Three-Point Correlation Functions

Generating Three-point Functions

We create a state at time $t = 0$

$$\langle \Omega | T[\chi_\alpha(t,\vec{x}_2)\mathcal{O}(\tau,\vec{x}_1)\overline{\chi}_\beta(0)] | \Omega \rangle$$

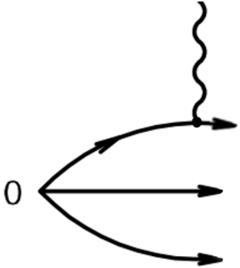


Three-Point Correlation Functions

Generating Three-point Functions

Insert an operator with momentum $\vec{q}$ at some later time τ

$$\langle\, \Omega \,|\, T\,[\, \chi_\alpha(t, \vec{x}_2) \mathcal{O}(\tau, \vec{x}_1) \overline{\chi}_\beta(0) \,]\, |\, \Omega \,\rangle$$

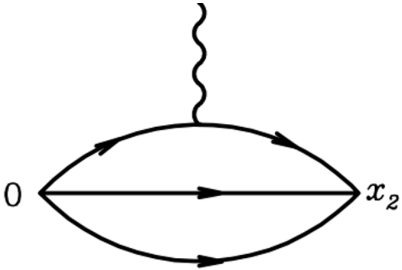


Three-Point Correlation Functions

Generating Three-point Functions

Annihilate the state at a final time t

$$\langle \Omega \,|\, T\left[\, \chi_\alpha(t,\vec{x}_2)\mathcal{O}(\tau,\vec{x}_1)\overline{\chi}_\beta(0)\,\right] |\Omega\,\rangle$$



Ratio of Correlation Functions

Thus the Three-point function

$$C_{3pt}(t,\tau;\vec{p},\vec{p}') = \sum_{s,s'} e^{-E_{p'}(t-\tau)} e^{-E_p \tau} \Gamma_{\beta\alpha}$$

$$\langle \Omega | \chi_\alpha(0) | N(\vec{p}', s') \rangle \langle N(\vec{p}', s') | \mathcal{O}(q) | N(\vec{p}, s) \rangle \langle N(\vec{p}, s) | \overline{\chi}_\beta(0) | \Omega \rangle$$

Using Two-point Functions in the form

$$C_{2pt}(t,\vec{p}) = \sum_{s} e^{-E_p t} \Gamma_{\beta\alpha} \langle \Omega | \chi_\alpha | N(\vec{p}, s) \rangle \left\langle N(\vec{p}, s) \middle| \overline{\chi}_\beta \middle| \Omega \right\rangle$$

We construct a ratio of Two-point and Three-point correlation functions

$$R(t,\tau;\vec{p},\vec{p}') \approxeq \frac{C_{3pt}(t,\tau;\vec{p}',\vec{p})}{C_{2pt}(t,\tau;\vec{p}',\vec{p})}$$

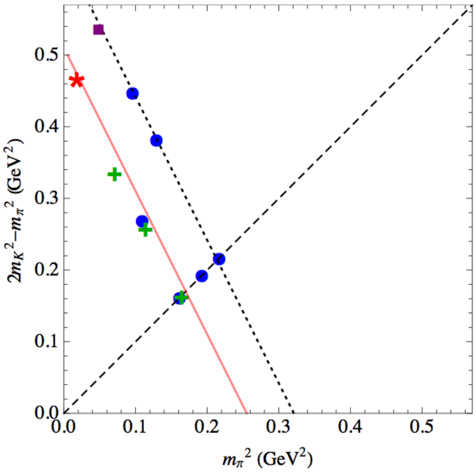
Which allows us to remove the time dependence and solve matrix elements.

Lattice Parameters

- $N_f = 2+1$ $O(a)$-improved Clover Fermions
- Lattice spacing $a = 0.074fm$
- Novel method for tuning the quark masses
  - Keep the singlet quark mass fixed

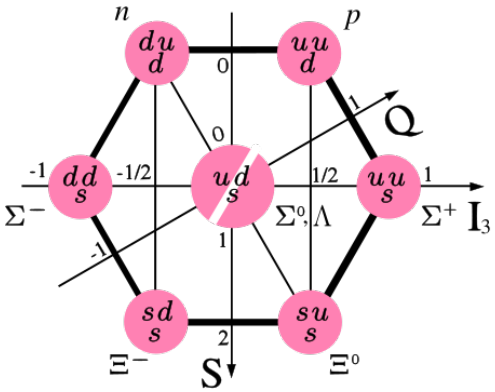
$$\overline{m}^R = \frac{1}{3}\left(2m_l^R + m_s^R\right)$$

  - At its physical value $\overline{m}^{R*}$
- Using multiple Lattice volume sizes including $32^3 \times 64$, $48^3 \times 96$



Lattice Parameters

- Using $N_f = 2 + 1$ flavour configurations allows us to simulate the octet baryons
- These are represented by doubly and singly light and heavy quarks since $m_l = m_u = m_d$
- An advantage of the lattice is that we can directly obtain the quark contributions to these quantities through the light and strange quarks



Form Factors

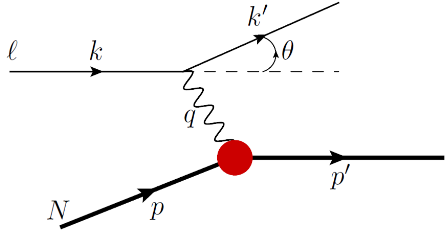
Electromagnetic Form Factors

The Dirac $F_1(Q^2)$ and Pauli $F_2(Q^2)$ form factors are obtained from the decomposition of matrix elements from the electromagnetic current j_μ where

$$\left\langle N(p',s')\middle| j_\mu(q) \middle| N(p,s)\right\rangle = \overline{u}(p',s')\left[\gamma_\mu F_1(Q^2) + \frac{i\sigma_{\mu\nu}q^\nu}{2m_B}F_2(Q^2)\right]u(p,s)$$

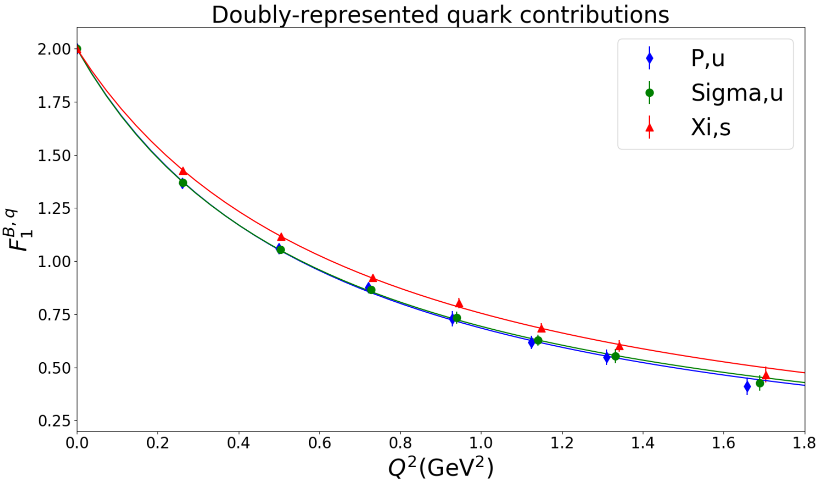
Here

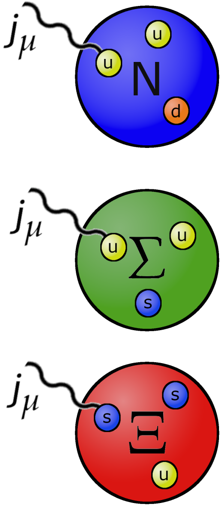
- $u(p,s)$ are Dirac spinors with momentum p and spin polarisation s
- the transfer momentum $q = p' - p$ and $Q^2 = -q^2$
- and the mass of the baryon is m_B.



Electromagnetic Form Factors

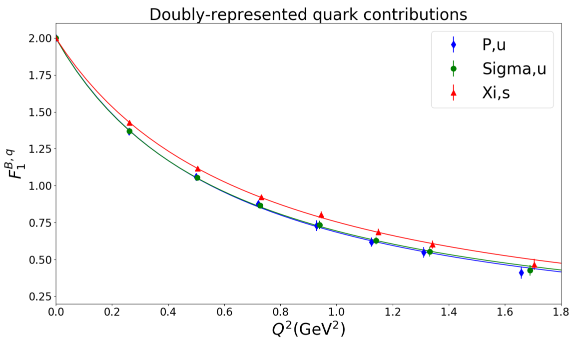
$F_1 = A_{10}$ Dirac Form Factor $\qquad (m_\pi, m_K) = (330, 435)MeV$

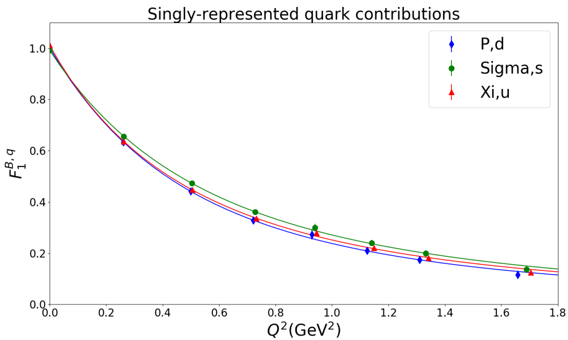




Electromagnetic Form Factors

$$F_1 = A_{10} \text{ Dirac Form Factor}$$





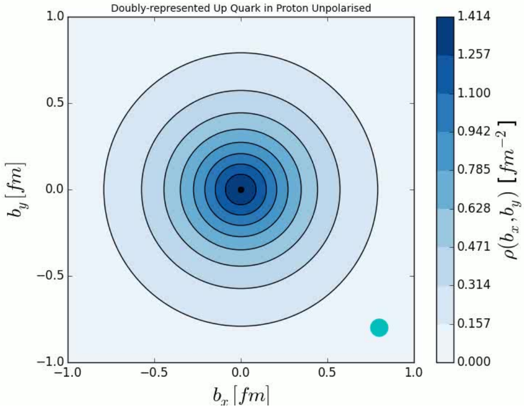
Transverse Spin Density Equation

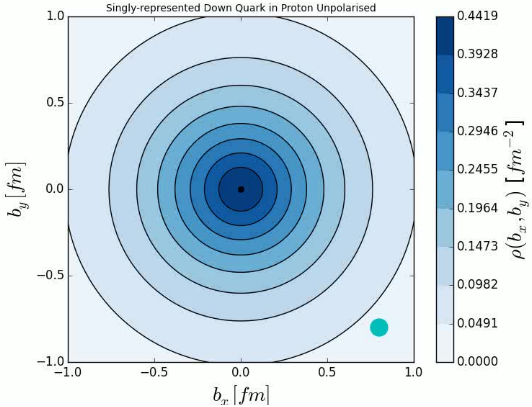
A reminder of the equation and required form factors

$$\rho(b_\perp, s_\perp, S_\perp) = \int_{-1}^{1} dx \;\; \rho(x, b_\perp, s_\perp, S_\perp)$$
$$= \frac{1}{2}\{A_{10}(b_\perp^2) + s_\perp^i S_\perp^i \left(A_{T10}(b_\perp^2) - \frac{1}{4m^2}\triangle_{b_\perp}\tilde{A}_{T10}(b_\perp^2)\right)$$
$$+ \frac{b_\perp^j \epsilon^{ji}}{m}\left(S_\perp^i B'_{10}(b_\perp^2) + s_\perp^i \overline{B}'_{T10}(b_\perp^2)\right) + s_\perp^i \left(2b_\perp^i b_\perp^j - b_\perp^2 \delta^{ij}\right) S_\perp^j \frac{1}{m^2}\tilde{A}''_{T10}(b_\perp^2)\}$$

Unpolarised

Unpolarised Quark Densities



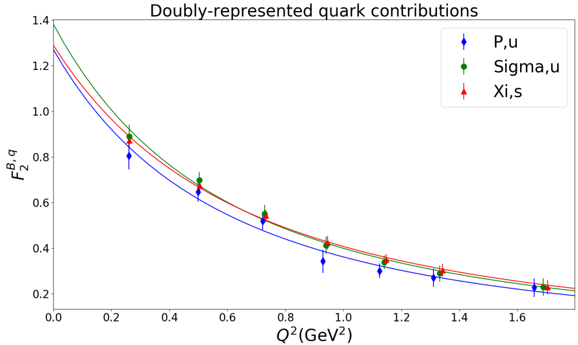


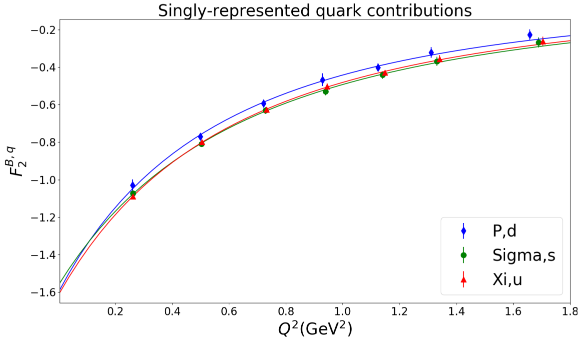
Doubly represented unpolarised up quark in the unpolarised proton.

Singly represented unpolarised down quark in the unpolarised proton.

Electromagnetic Form Factors

$$F_2 = B_{10} \text{ Pauli Magnetic Form Factor}$$





Transverse Spin Density Equation

A reminder of the equation and required form factors

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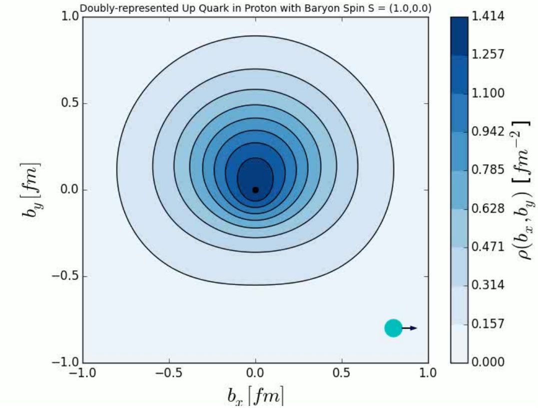
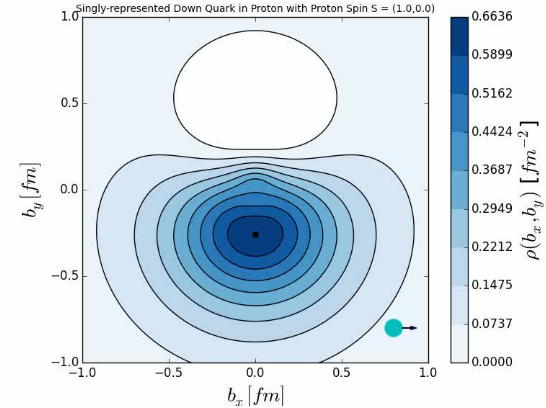
$$+ \frac{b_\perp^j \epsilon^{ji}}{m}\left(S_\perp^i B_{10}'(b_\perp^2) + s_\perp^i \overline{B}_{T10}'(b_\perp^2)\right) + s_\perp^i \left(2b_\perp^i b_\perp^j - b_\perp^2 \delta^{ij}\right) S_\perp^j \frac{1}{m^2}\tilde{A}_{T10}''(b_\perp^2)$$

Hadron Spin Polarisation

Proton Quark Densities with Nucleon spin polarisation



Doubly represented up quark in the proton with polarised Nucleon spin.



Singly represented down quark in the proton with polarised Nucleon Spin.

Tensor Form Factors

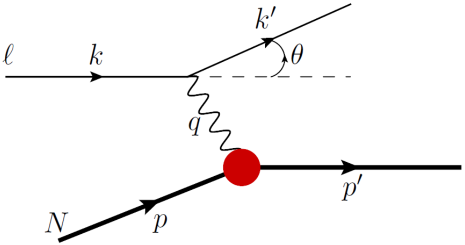
Similar to the electromagnetic form factor, we calculate the tensor form factors using a new insertion operator $i\sigma_{\mu\nu}$

$$\left\langle N(p',s') \left| \overline{\psi}(0) i\sigma^{\mu\nu}\psi(0) \right| N(p,s) \right\rangle =$$

$$\bar{u}(p',s')\left[i\sigma^{\mu\nu} A_{T10}(Q^2) + \frac{\bar{P}^{[\mu}\Delta^{\nu]}}{m^2}\tilde{A}_{T10}(Q^2) + \frac{\gamma^{[\mu}\bar{P}^{\nu]}}{2m} B_{T10}(Q^2) \right] u(p,s)$$

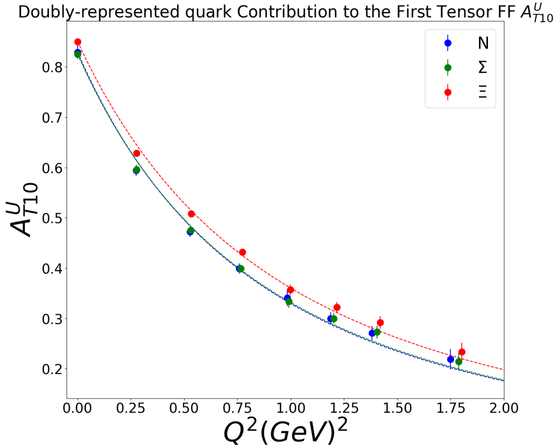
where here

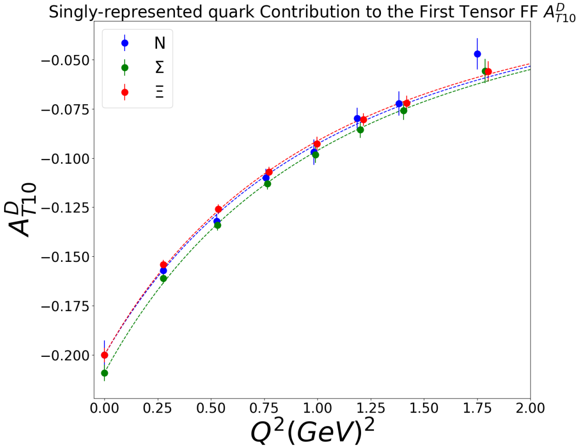
- $\gamma^{[\mu}\bar{P}^{\nu]} \equiv \gamma^{\mu}\bar{P}^{\nu} - \gamma^{\nu}\bar{P}^{\mu}$
- $\Delta = p' - p$, $\bar{P} = \frac{p'+p}{2}$
- $i\sigma^{\mu\nu} = i\gamma^{\mu}\gamma^{\nu}$



Tensor Form Factors

$$A_{T10}(Q^2) \qquad \text{where } A_{T10}(Q^2=0)=g_T$$





Transverse Spin Density Equation

A reminder of the equation and required form factors

$$\rho(b_\perp, s_\perp, S_\perp) = \int_{-1}^{1} dx \;\; \rho(x, b_\perp, s_\perp, S_\perp)$$

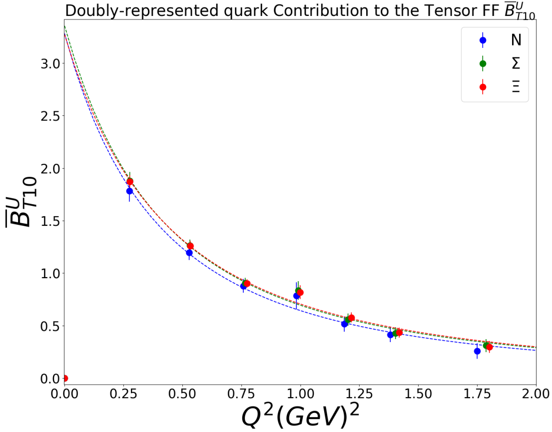
$$= \frac{1}{2}\{A_{10}(b_\perp^2) + s_\perp^i S_\perp^i \left(A_{T10}(b_\perp^2) - \frac{1}{4m^2} \triangle_{b_\perp} \tilde{A}_{Tn0}(b_\perp^2) \right)$$

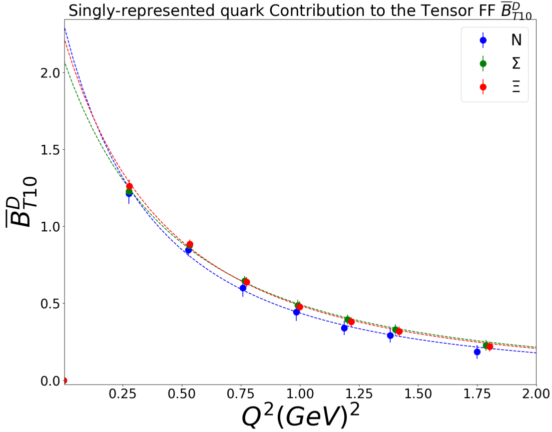
$$+ \frac{b_\perp^j \epsilon^{ji}}{m} \left(S_\perp^i B_{10}'(b_\perp^2) + s_\perp^i \overline{B}_{T10}'(b_\perp^2) \right) + s_\perp^i \left(2 b_\perp^i b_\perp^j - b_\perp^2 \delta^{ij} \right) S_\perp^j \frac{1}{m^2} \tilde{A}_{T10}''(b_\perp^2)$$

Quark Spin Polarisation

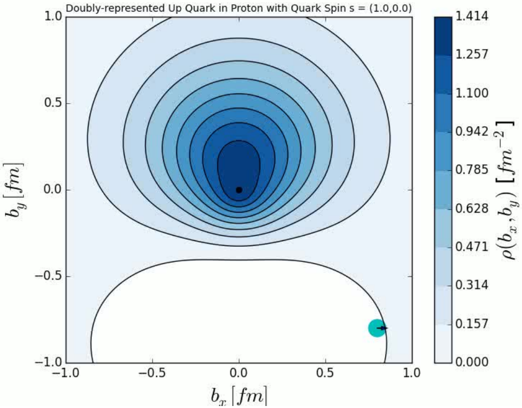
Tensor Form Factors

$$\overline{B}_{T10}(Q^2) \approx 2\tilde{A}_{T10} + B_{T10}$$

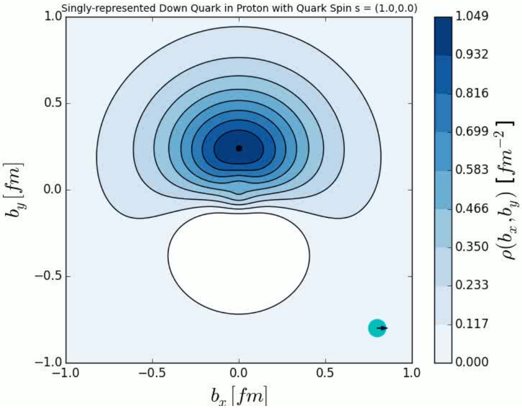




Proton Quark Densities with Quark spin polarisation



Doubly represented up quark in the proton with polarised Quark spin.



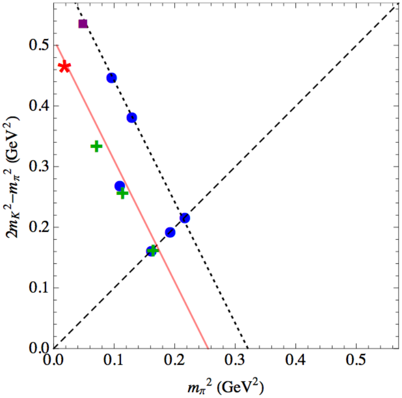
Singly represented down quark in the proton with polarised Quark Spin.

SU(3) Flavour Symmetry Breaking Expansion

SU(3)-Flavour Symmetry Breaking

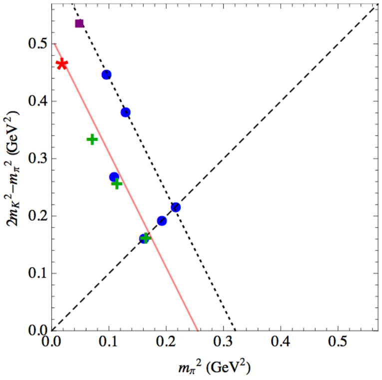
Mass Tuning

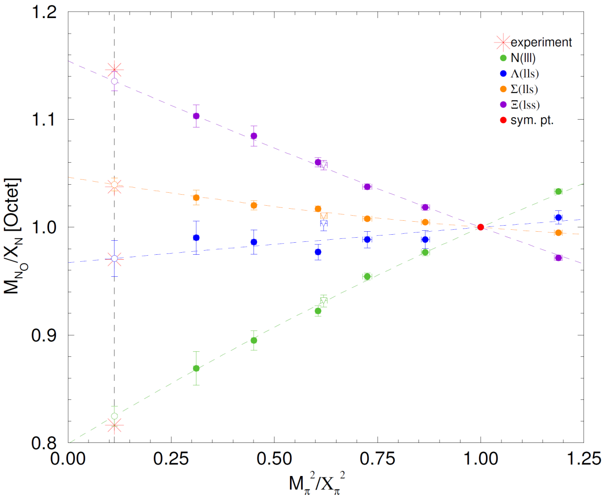
- A Feature of the gauge configurations used is that the simulation trajectory follows a line of constant singlet mass $m_q = (2m_l + m_s)/3$.
- This is achieved by first finding the SU(3)-flavour-symmetric point where flavour-singlet quantities take their physical values, then varying the individual quark masses about that point.
- By doing so we find a Flavour-symmetry breaking effect due to the differences between the strange and light quark masses



SU(3)-Flavour Symmetry Breaking

Fan plot of generalised tensor form factor A_{T10} at $Q^2 = 0$





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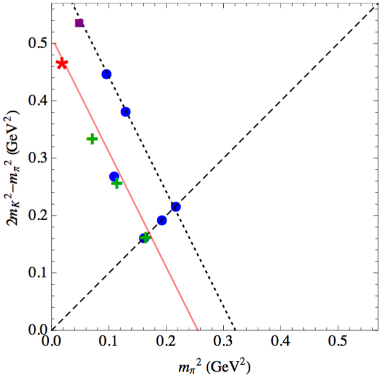
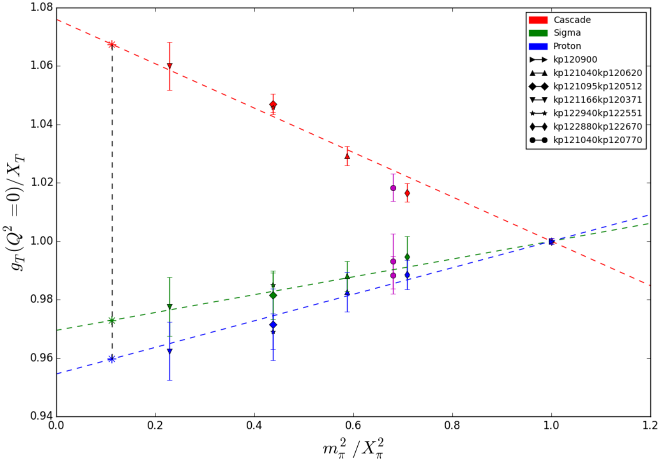


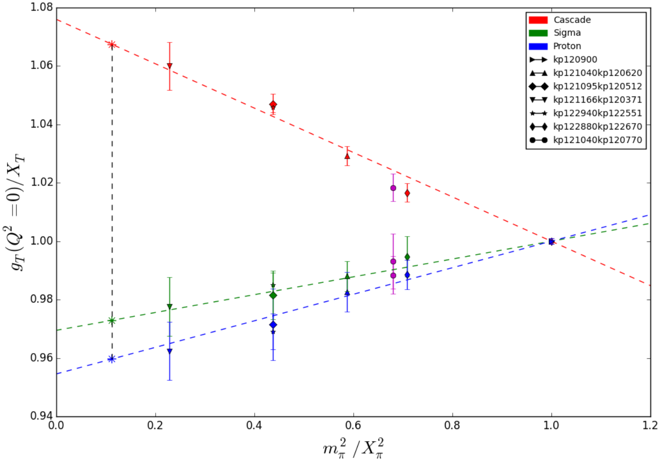


SU(3)-Flavour Symmetry Breaking

Fan plot of generalised tensor form factor A_{T10} at $Q^2 = 0$

Generating Fan plots

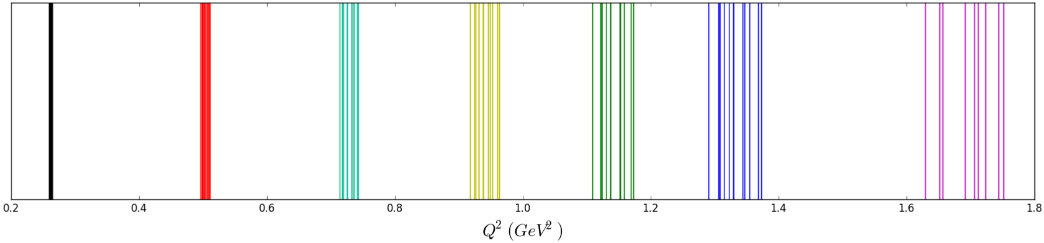
- Scale X-axis by $X_\pi^2 = (2M_K^2 + M_\pi^2)/3$
- Scale Y-axis by flavour singlet quantity X_T constructed from average diagonal amplitudes



Physical Expansion

Binning

- The 4-momentum $Q^2 = -q^2$ is dependent on M_B
- We bin the Q^2 values from each ensemble into separate bins.
- From each Q^2 bin we take an average value for the bin
- Using this average we then shift each ensemble to fit the average Q^2 value such that we can compare and create fan plots at each Q^2 bin.



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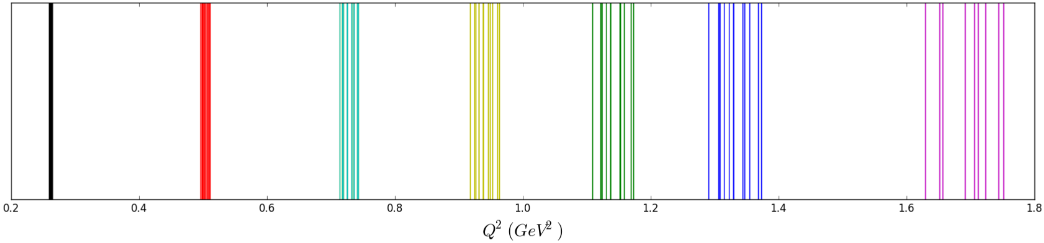

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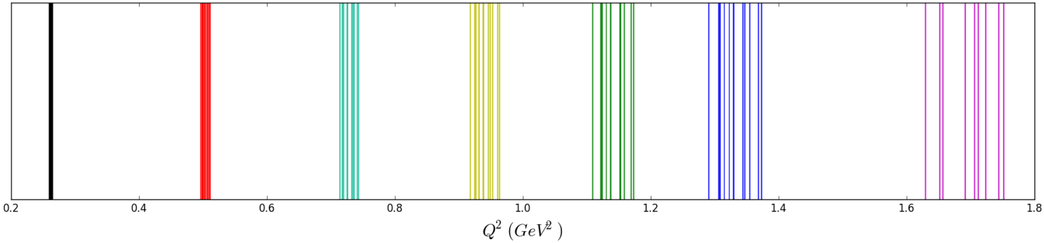

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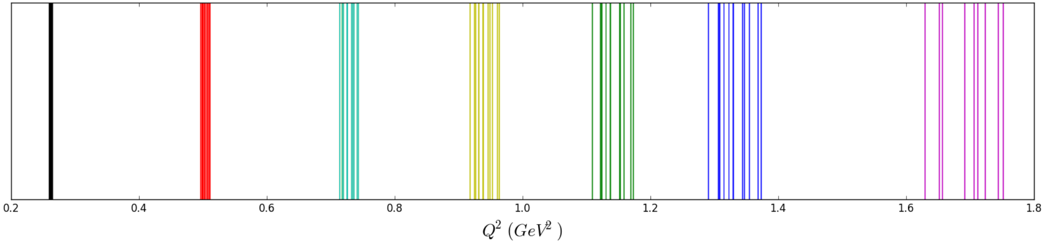
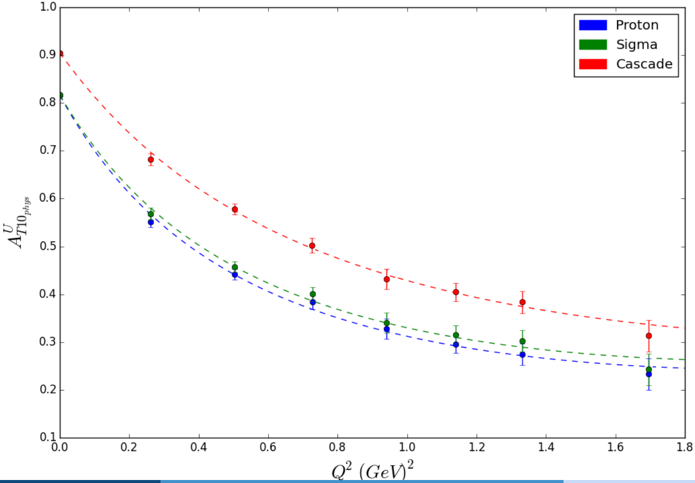


Physical Form Factors

A_{T10} First Tensor form factor

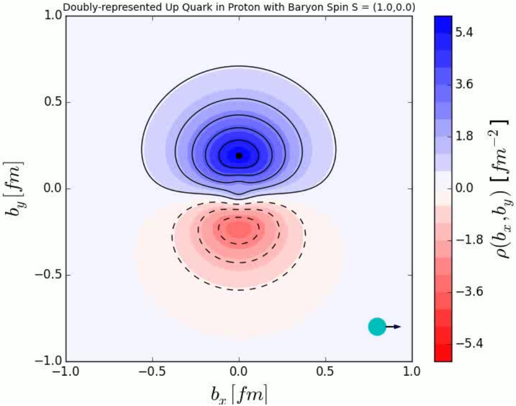


Physical Mass Results

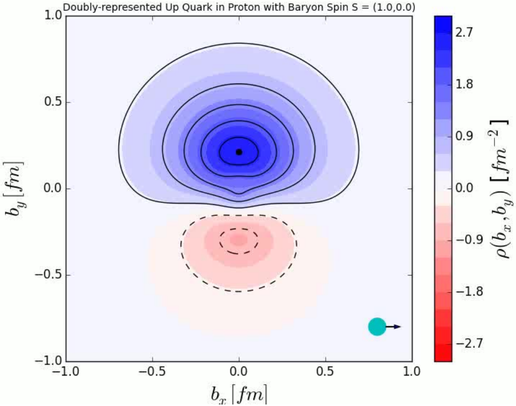
Comparing Baryon Spin Densities

Difference between the doubly represented up quarks in the Proton and Sigma

Up Quarks in Proton

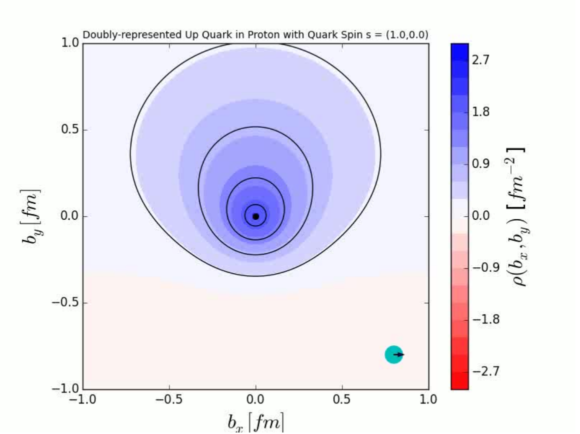


Up Quarks in Sigma

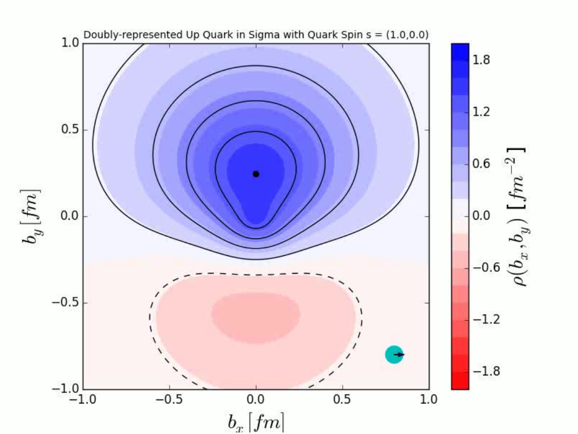


Varying Nucleon Spin Polarisation

Up Quarks in Proton



Up Quarks in Sigma



Varying Quark Spin Polarisation

Transverse Spin Density equation

$$\begin{aligned}\rho(b_\perp, s_\perp, S_\perp) &= \int_{-1}^{1} dx \;\; \rho(x, b_\perp, s_\perp, S_\perp) \\ &= \frac{1}{2}\{A_{10}(b_\perp^2) + s_\perp^i S_\perp^i \left(A_{T10}(b_\perp^2) - \frac{1}{4m^2}\triangle_{b_\perp}\tilde{A}_{T10}(b_\perp^2)\right) \\ &+ \frac{b_\perp^j \epsilon^{ji}}{m}\left(S_\perp^i B'_{10}(b_\perp^2) + s_\perp^i \overline{B}'_{T10}(b_\perp^2)\right) + s_\perp^i \left(2b_\perp^i b_\perp^j - b_\perp^2 \delta^{ij}\right) S_\perp^j \frac{1}{m^2}\tilde{A}''_{Tn0}(b_\perp^2)\}\end{aligned}$$

$$f' = \frac{\partial}{\partial b^2} f, \qquad f'' = \left(\frac{\partial}{\partial b^2}\right)^2 f, \qquad \Delta_b f = 4\frac{\partial}{\partial b^2}\left(b^2 \frac{\partial}{\partial b^2}\right) f$$

Transverse Spin Density equation

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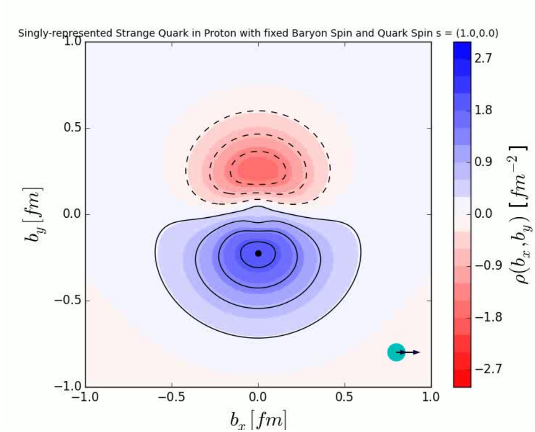
Comparing Baryon Spin Densities

Allowing both $S_\perp$ and $s_\perp$ to be non-zero

Fixed Nucleon Spin varying Quark Spin

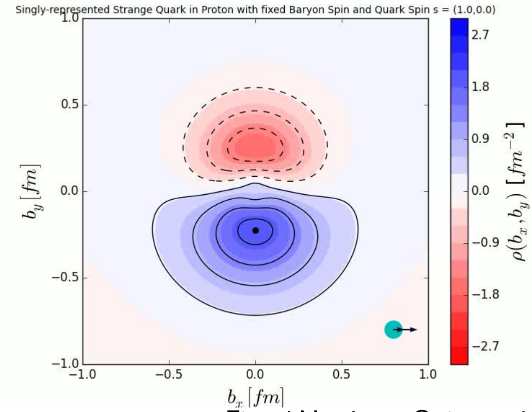


Down Quark in the Proton

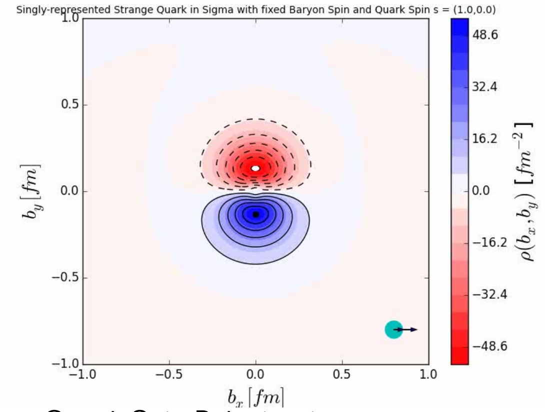
Comparing Baryon Spin Densities

Difference between the singly represented down quark and strange quark in the Proton and Sigma

Down Quark in the Proton



Strange Quark in the Sigma



Fixed Nucleon Spin and varying Quark Spin Polarisation

Ongoing Work

Ongoing Work

Investigating the Second Moment of the Nucleon's g_1 and g_2 Structure Functions

Structure Functions

- The Nucleon's second spin-dependent structure function g_2 at leading order in Q^2 recieves contribution from both twist-2 and twist-3 operators.
- Using equations of motion $g_2(x)$ can be expressed as a sum of a piece that is entirely determined in terms of $g_1(x)$ plus an interaction dependent twist-3 part that involes quark gluon correlations

$$g_2(x,Q^2) = g_2^{WW}(x,\mu^2) + \overline{g}_2^q(x,\mu^2)$$

$$g_2^{WW}(x,Q^2) = -g_1(x,\mu^2) + \int_x^1 \frac{dy}{y} g_1^q(y,\mu^2)$$

Where g_2^{WW} is the 'Wandzura-Wilczek' relation which depends only on the first spin-dependent structure function $g_1(x,\mu^2)$ with twist-2 contributions

Using neglecting quark mass for simplicity, the x^2 moment of $\overline{g}_2$ yields

$$\int dx x^2 \overline{g}_2^{(f)} = \frac{1}{3} d_2^{(f)}$$

Using leading order Operator Product Expansion (OPE) we find the operator

$$\langle \vec{p}, \vec{s} | \mathcal{O}^{5(f)}_{[\sigma\{\mu_1]\mu_2\}} | \vec{p}, \vec{s} \rangle = \frac{1}{3} d_2^{(f)}$$

where this operator is

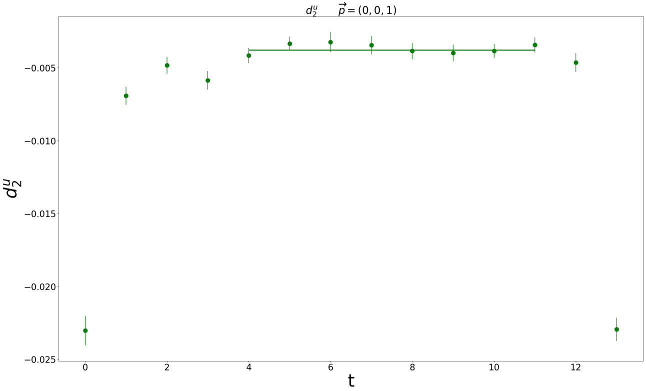
$$\mathcal{O}^{5(f)}_{[\sigma\{\mu_1]\mu_2\}} = \frac{-1}{4} \overline{\psi} \gamma_\sigma \gamma_5 \overleftrightarrow{D}_{\mu_1} \overleftrightarrow{D}_{\mu_2} \psi$$

where $\overleftrightarrow{D} = \overrightarrow{D} - \overleftarrow{D}$

Ongoing Work

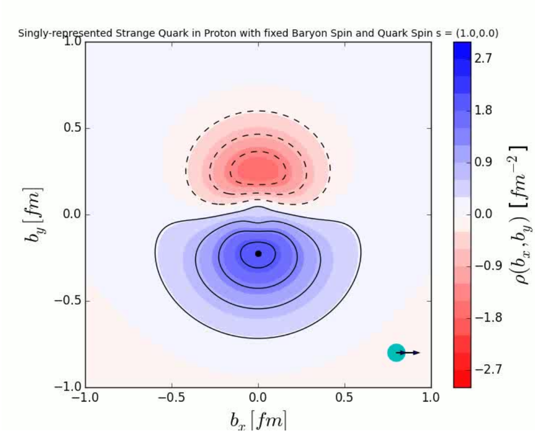
Investigating the Second Moment of the Nucleon's g_1 and g_2 Structure Functions

- Here we have the forward matrix element for the u quark contribution to the d_2
- Involves the same linear combination of lorentz components bewteen electro-magnetism and the gluon strength tensor
- A better interpretation of these matrix elements would be a 'color' lorentz force



Thank you for Listening

Fixed Nucleon Spin varying Quark Spin



Down Quark in the Proton