Sensitive tests of the standard model from *K* mesons and lattice QCD

Jefferson Laboratory

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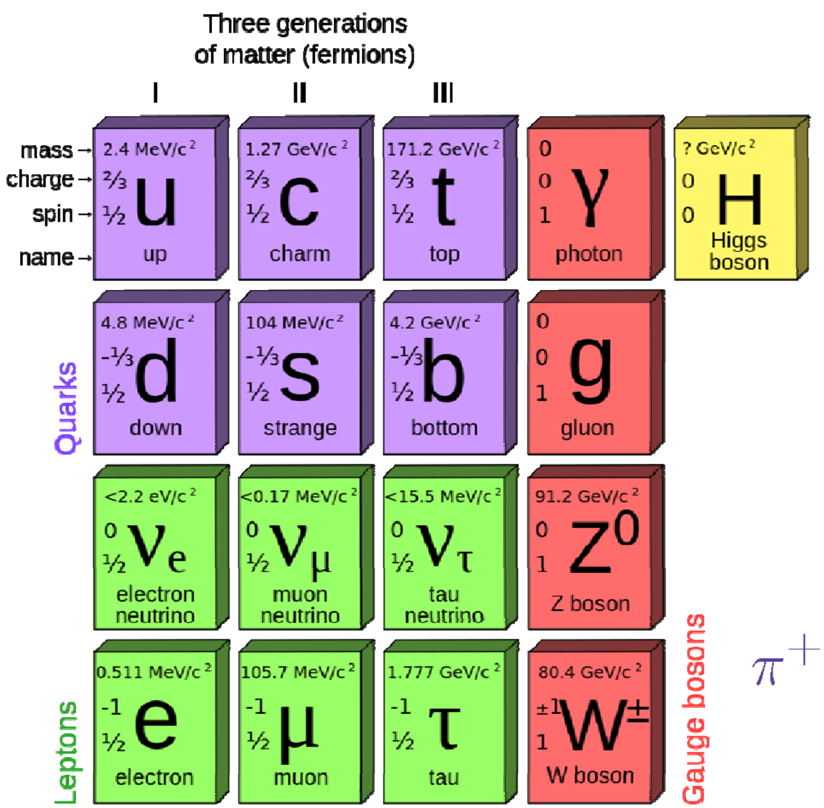
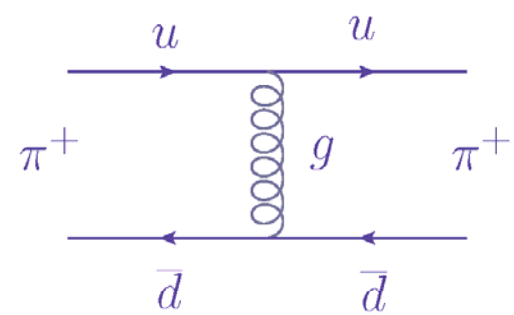
RBC and UKQCD Collaborations

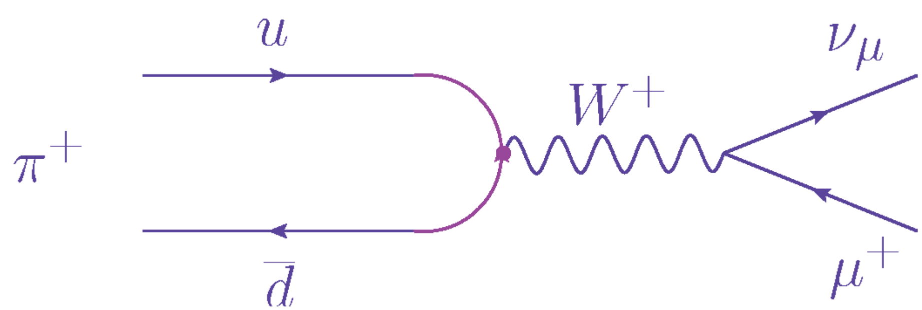
Outline

- Quick review of the standard model
- Lattice QCD: methods and status
- Four precision tests of the standard model:
  1) $K \to \pi\pi$ decay and direct ~~CP~~: ε'
  2) $K_L - K_S$ mass difference
  3) Long distance contribution to ε_K
  4) Long distance contribution to rare kaon decay: $K^+ \to \pi^+ \nu \bar{\nu}$

Standard Model





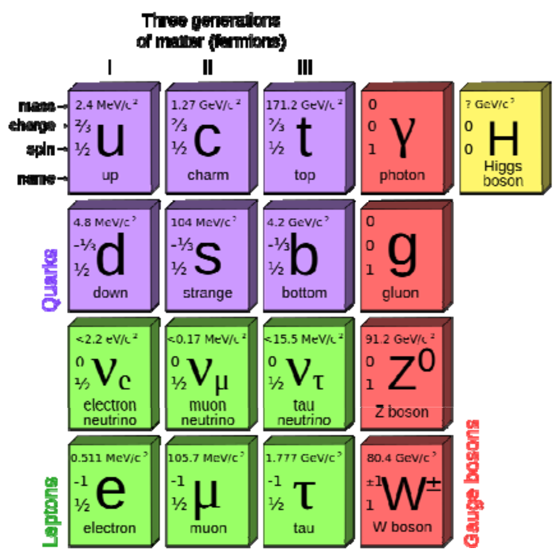


Examples of *g* (gluon) and *W*+ (weak) exchange

Cabibbo-Kobayashi-Maskawa mixing

- $W^{\pm}$ emission scrambles the quark flavors

$$\begin{pmatrix} u \\ c \\ t \end{pmatrix} \overset{W^{\pm}}{\longleftrightarrow} \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \begin{pmatrix} d \\ s \\ b \end{pmatrix}$$



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$$\begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} = \begin{pmatrix} 1-\frac{\lambda^2}{2} & \lambda & A\lambda^3(\overline{\rho}-i\overline{\eta}) \\ -\lambda & 1-\frac{\lambda^2}{2} & A\lambda^2 \\ A\lambda^3(1-\overline{\rho}-i\overline{\eta}) & -A\lambda^2 & 1 \end{pmatrix} + \mathcal{O}(\lambda^4)$$

$$\lambda = 0.22535 \pm 0.00065\,, \qquad A = 0.811^{+0.022}_{-0.012}\,,$$

$$\bar{\rho} = 0.131^{+0.026}_{-0.013}\,, \qquad \bar{\eta} = 0.345^{+0.013}_{-0.014}\,.$$

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CP violation!

$$\begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} = \begin{pmatrix} 1-\frac{\lambda^2}{2} & \lambda & A\lambda^3(\overline{\rho}-i\overline{\eta}) \\ -\lambda & 1-\frac{\lambda^2}{2} & A\lambda^2 \\ A\lambda^3(1-\overline{\rho}-i\overline{\eta}) & -A\lambda^2 & 1 \end{pmatrix} + \mathcal{O}(\lambda^4)$$

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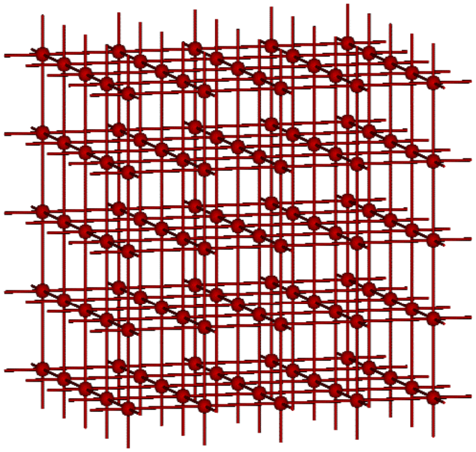
State-of-the-art

Lattice QCD

Lattice QCD



- Introduce a space-time lattice.
- Evaluate the Euclidean Feynman path integral
  - Study $e^{-H_{QCD}\,t}$
  - Precise non-perturbative formulation
  - Capable of numerical evaluation

$$\sum_n \langle n|e^{-H(T-t)}\mathcal{O}e^{-Ht}|n\rangle = \int d[U_\mu(n)]e^{-\mathcal{A}[U]}\det(D+m)\mathcal{O}[U](t)$$

- Evaluate using Monte Carlo importance sampling with hybrid, molecular dynamics/Langevin evolution.

Lattice QCD – 2018

- Physical quark masses (ChPT not needed)
- Chiral quarks (doubling problem solved)
- Large physical volumes: $(6 - 10 \text{ fm})^3$
- Small lattice spacing: $1/a = 2.4$ GeV
  - $(\Lambda_{QCD}\, a)^2$ effects < 1% ☺
  - $(m_{charm}\, a)^2$ effects ~ 15% ☹

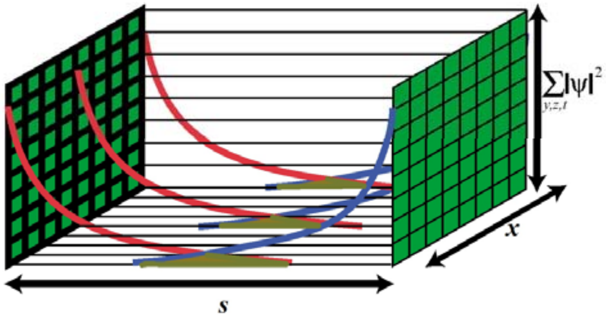
QCD in Euclidean space

- Euclidean $e^{-H_{QCD}\,t}$ projects onto the ground state.

- Treat two-particle states using Luscher's finite-volume analysis
  - Finite-volume energy shifts determine scattering phase shifts.
  - Must work below multi-particle thresholds
  - Two-particle state of interest may not be the lowest energy state
  - Hansen and Sharpe working on 3-particle states – making progress but difficult.
- Extra problems for second-order weak calculations

Elaborate methods required

- Use 5-D, domain wall lattice fermions – physical quarks bound to 4D boundaries



- Measurements on 64^3 x 128 lattice
- Compute 2000 lowest Dirac eigenvectors to speed up Dirac operator inversion.
- KNL chip has 68 cores, each with 4 threads and two 512-bit wide, pipelined FPUs.
- Broad collaboration and substantial funding needed.

Lattice QCD

$$\sum_n \langle n | e^{-H(T-t)} \mathcal{O} e^{-Ht} | n \rangle = \int d[U_\mu(n)] e^{-\mathcal{A}[U]} \det(D+m) \mathcal{O}[U](t)$$

- Very large computational challenge:
  - For a 64^3 x128 lattice: Integrate over one billion variables
  - Spin-1/2 quarks are represented as 4-D states on the boundary of a 5-D volume.
  - Integrand contains the determinant of a (10 Billion) x (10 Billion) matrix

- Fast code running on 32K nodes of Mira sustains one Petaflops [10^{15} (adds + mults)/sec]

The RBC & UKQCD collaborations

BNL and RBRC

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Hiroshi Ohki
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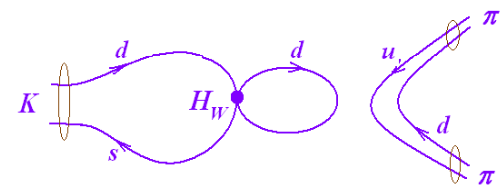
Precision tests of the Standard Model

Direct CP violation $K \rightarrow \pi\,\pi$

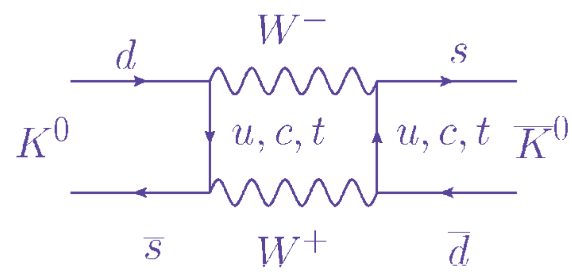
$|\varepsilon'| = 3.70(53) \times 10^{-6}$



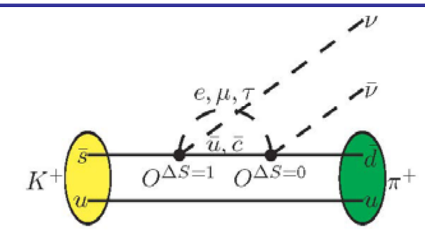
Indirect CP violation $K \rightarrow \pi\,\pi$

$|\varepsilon| = 0.002228\ (11)$

$m_{K_L} - m_{K_S} = 3.19(41)(96) \times 10^{-12}$ MeV



$K^+ \rightarrow \pi^+ \nu\, \bar{\nu}$, BR=$1.73\ ^{+1.15}_{-1.05} \times 10^{-10}$



$K \rightarrow \pi\pi$ Decay

$K^0 - \overline{K^0}$ mixing

- ΔS=1 weak decays allow K^0 and $\overline{K^0}$ to decay to the same $\pi\pi$ state.
- Resulting mixing described by Wigner-Weisskopf

$$i\frac{d}{dt}\begin{pmatrix} K^0 \\ \overline{K}^0 \end{pmatrix} = \left\{ \begin{pmatrix} M_{00} & M_{0\bar{0}} \\ M_{\bar{0}0} & M_{\bar{0}\bar{0}} \end{pmatrix} - \frac{i}{2}\begin{pmatrix} \Gamma_{00} & \Gamma_{0\bar{0}} \\ \Gamma_{\bar{0}0} & \Gamma_{\bar{0}\bar{0}} \end{pmatrix} \right\} \begin{pmatrix} K^0 \\ \overline{K}^0 \end{pmatrix}$$

- Decaying states are mixtures of K^0 and $\overline{K^0}$

$$|K_S\rangle = \frac{K_+ + \overline{\epsilon} K_-}{\sqrt{1+|\overline{\epsilon}|^2}}$$

$$|K_L\rangle = \frac{K_- + \overline{\epsilon} K_+}{\sqrt{1+|\overline{\epsilon}|^2}}$$

$$\overline{\epsilon} = \frac{i}{2}\left\{ \frac{\mathrm{Im} M_{0\bar{0}} - \frac{i}{2}\mathrm{Im}\Gamma_{0\bar{0}}}{\mathrm{Re} M_{0\bar{0}} - \frac{i}{2}\mathrm{Re}\Gamma_{0\bar{0}}} \right\}$$

Indirect CP violation

CP violation

- CP violating, experimental amplitudes:

$$\eta_{+-} \equiv \frac{\langle \pi^+\pi^-|H_w|K_L\rangle}{\langle \pi^+\pi^-|H_w|K_S\rangle} = \epsilon + \epsilon'$$

$$\eta_{00} \equiv \frac{\langle \pi^0\pi^0|H_w|K_L\rangle}{\langle \pi^0\pi^0|H_w|K_S\rangle} = \epsilon - 2\epsilon'$$

- Where: $\epsilon = \overline{\epsilon} + i\frac{\mathrm{Im}\boldsymbol{A}_0}{\mathrm{Re}\boldsymbol{A}_0}$

Indirect: $|\varepsilon| = (2.228 \pm 0.011) \times 10^{-3}$

Direct: $\mathrm{Re}(\varepsilon'/\varepsilon) = (1.66 \pm 0.23) \times 10^{-3}$

$K \to \pi\pi$ and CP violation

- Final $\pi\pi$ states can have I = 0 or 2.

$$\langle \pi\pi(I=2)|H_w|K^0\rangle = A_2 e^{i\delta_2} \qquad \Delta I = 3/2$$

$$\langle \pi\pi(I=0)|H_w|K^0\rangle = A_0 e^{i\delta_0} \qquad \Delta I = 1/2$$

- CP symmetry requires A_0 and A_2 be real.
- Direct CP violation in this decay is characterized by:

$$\epsilon' = \frac{i\,e^{\delta_2-\delta_0}}{\sqrt{2}} \left|\frac{A_2}{A_0}\right| \left(\frac{\mathrm{Im}A_2}{\mathrm{Re}A_2} - \frac{\mathrm{Im}A_0}{\mathrm{Re}A_0}\right)$$

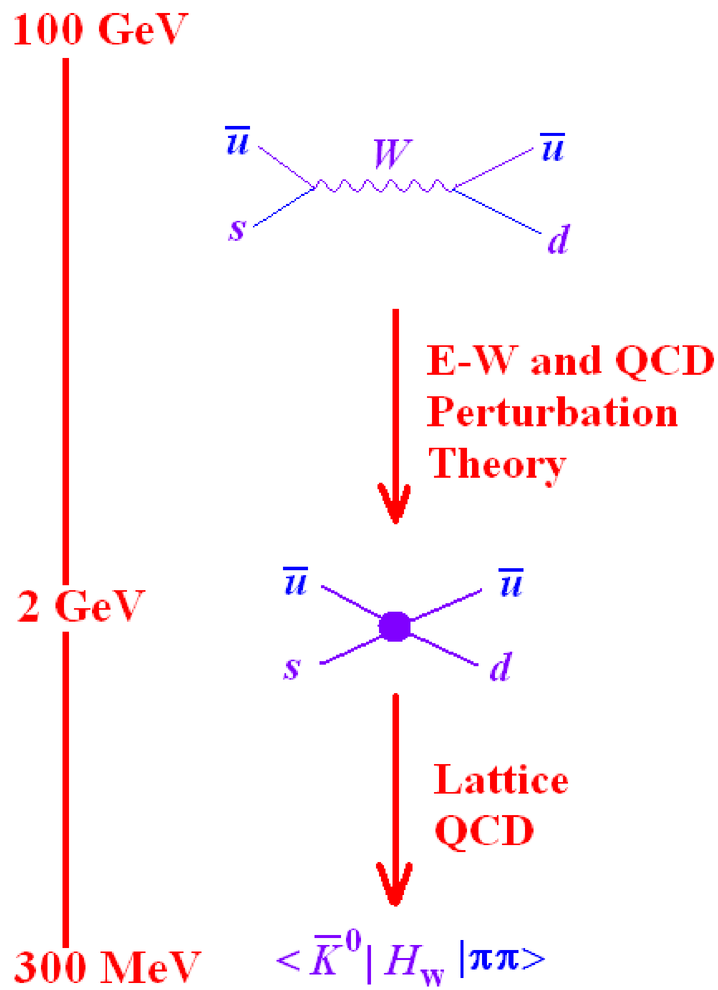
Direct CP violation

Low Energy Effective Theory

- Represent weak interactions by local four-quark Lagrangian

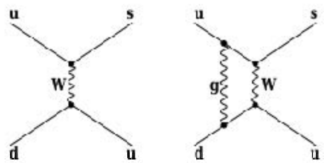
$$\mathcal{H}^{\Delta S=1} = \frac{G_F}{\sqrt{2}} V_{ud} V_{us}^* \left\{ \sum_{i=1}^{10} \left[z_i(\mu) + \tau y_i(\mu) \right] Q_i \right\}$$

- $\tau = -\frac{V_{td} V_{ts}^*}{V_{ud} V_{us}^*} = (1.543 + 0.635i) \times 10^{-3}$
- $V_{qq'}$ – CKM matrix elements
- z_i and y_i – Wilson Coefficients
- Q_i – four-quark operators



Local four quark operators

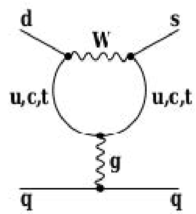
- **Current-current operators**



$$Q_1 \equiv (\bar{s}_\alpha d_\alpha)_{V-A} (\bar{u}_\beta u_\beta)_{V-A}$$

$$Q_2 \equiv (\bar{s}_\alpha d_\beta)_{V-A} (\bar{u}_\beta u_\alpha)_{V-A}$$

- **QCD Penguins**



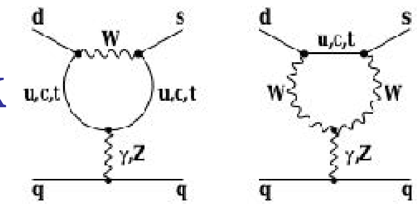
$$Q_3 \equiv (\bar{s}_\alpha d_\alpha)_{V-A} \sum_{q=u,d,s} (\bar{q}_\beta q_\beta)_{V-A}$$

$$Q_4 \equiv (\bar{s}_\alpha d_\beta)_{V-A} \sum_{q=u,d,s} (\bar{q}_\beta q_\alpha)_{V-A}$$

$$Q_5 \equiv (\bar{s}_\alpha d_\alpha)_{V-A} \sum_{q=u,d,s} (\bar{q}_\beta q_\beta)_{V+A}$$

$$Q_6 \equiv (\bar{s}_\alpha d_\beta)_{V-A} \sum_{q=u,d,s} (\bar{q}_\beta q_\alpha)_{V+A}$$

- **Electro-Weak Penguins**



$$Q_7 \equiv \frac{3}{2} (\bar{s}_\alpha d_\alpha)_{V-A} \sum_{q=u,d,s} e_q (\bar{q}_\beta q_\beta)_{V+A}$$

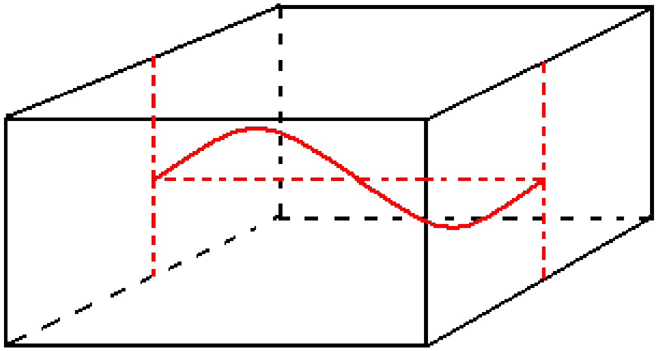
$$Q_8 \equiv \frac{3}{2} (\bar{s}_\alpha d_\beta)_{V-A} \sum_{q=u,d,s} e_q (\bar{q}_\beta q_\alpha)_{V+A}$$

$$Q_9 \equiv \frac{3}{2} (\bar{s}_\alpha d_\alpha)_{V-A} \sum_{q=u,d,s} e_q (\bar{q}_\beta q_\beta)_{V-A}$$

$$Q_{10} \equiv \frac{3}{2} (\bar{s}_\alpha d_\beta)_{V-A} \sum_{q=u,d,s} e_q (\bar{q}_\beta q_\alpha)_{V-A}$$

Physical $\pi\pi$ states – Lellouch-Luscher

- Euclidean e^{-Ht} projects onto $|\pi\pi(\vec{p}=0)>$
- Use finite-volume quantization.
- Adjust volume so 1st or 2nd excited state has correct *p.*

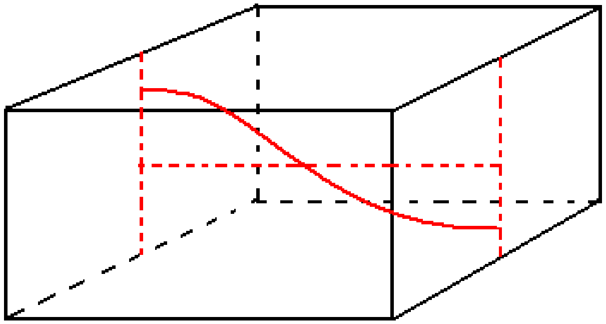


- Include $\pi - \pi$ interactions with leading $1/L^3$ finite-volume correction.
- Requires extracting signal from non-leading large-***t*** behavior:

$$G(t) \sim c_0 e^{-E_0 t} + c_1 e^{-E_1 t}$$

Exploit boundary conditions

- Remove $\pi\pi$ states with $E_{\pi\pi} < M_K$ by imposing anti-periodic boundary conditions:



$$2\sqrt{3\left(\frac{\pi}{L}\right)^2 + M_\pi^2} = M_K$$ → L = 5.2 fm

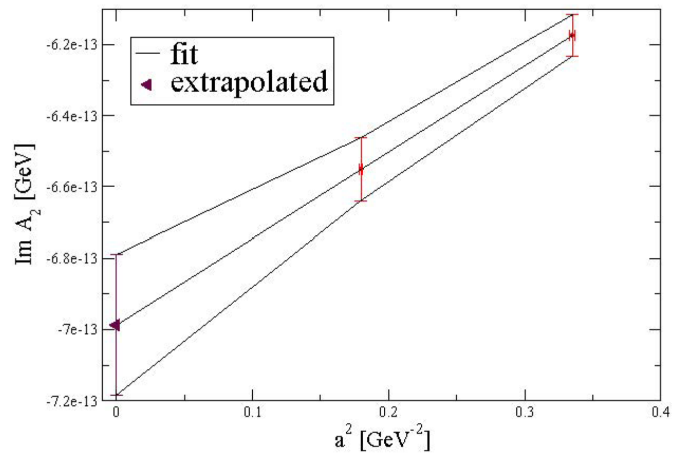
- $I = 2$, Repulsive, L → 5.7 fm
  – Work with $\pi^+\pi^+$ state, impose anti-periodic BC on *d* quark
  – $|\pi^+\pi^+>$ unique, charge-two state, does not mix
- $I = 0$, Attractive, L → 4.5 fm
  – Must distinguish $I = 0$ state: $|\pi^+\pi^-> -2\,|\pi^0\pi^0> + |\pi^-\pi^+>$
  – Impose *G*-parity BC, $G = C\, e^{i\pi I_y}$; $[G, \vec{I}\,] = 0$

Calculation of A_2

ΔI = 3/2 – Continuum Results

(M. Lightman, E. Goode T. Janowski)

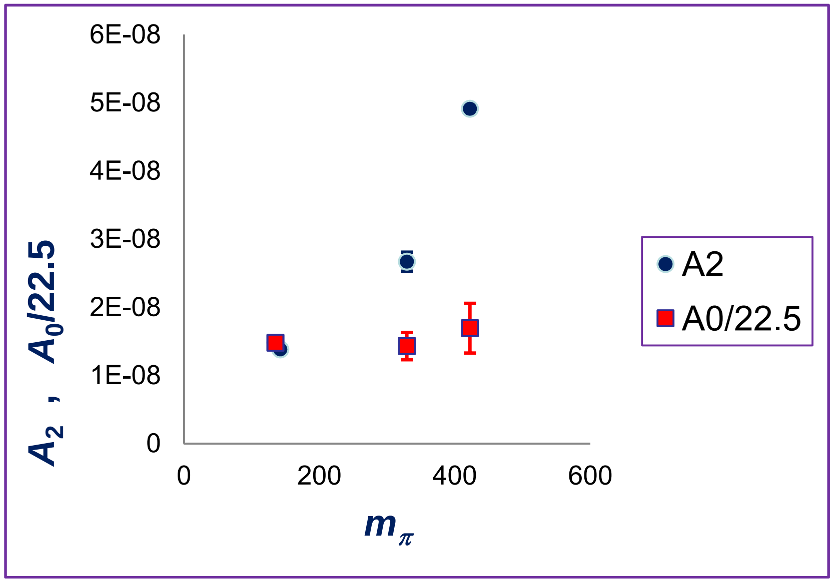
- Use two large ensembles to remove a^2 error (m_π=135 MeV, L=5.4 fm)
  - 48^3 x 96, 1/a=1.73 GeV
  - 64^3 x 128, 1/a=2.28 GeV



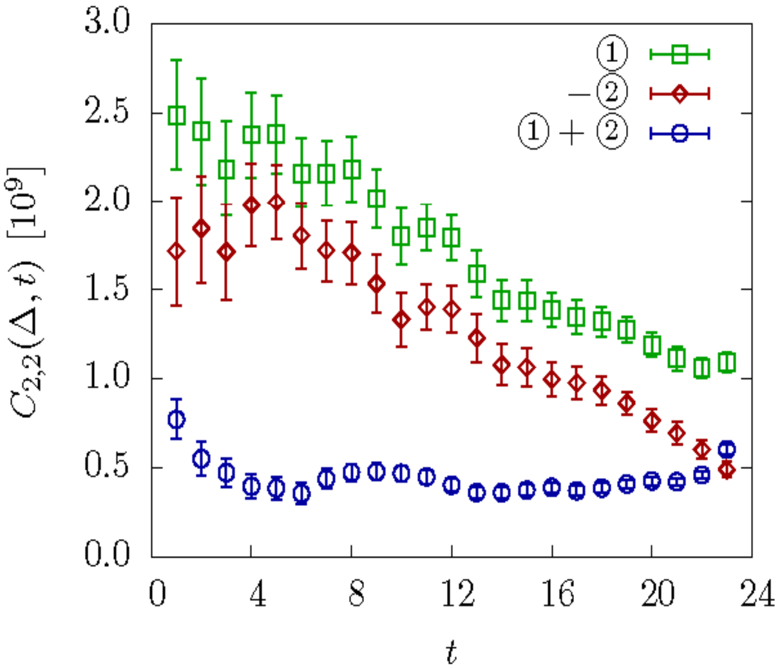
- Continuum results:
  - $\mathrm{Re}(A_2)$ = 1.50(0.04$_{\mathrm{stat}}$) (0.14)$_{\mathrm{syst}}$$\times 10^{-8}$ GeV
  - $\mathrm{Im}(A_2)$ = - 6.99(0.20)$_{\mathrm{stat}}$ (0.84)$_{\mathrm{syst}}$$\times 10^{-13}$ GeV
- Experiment: $\mathrm{Re}(A_2)$ = 1.479(4) 10^{-8} GeV
- $E_{\pi\pi}$ → $\delta_2 = -11.6(2.5)(1.2)^o$
- [Phys.Rev. **D91**, 074502 (2015)]

$\Delta I = 1/2$ Rule
(Qiu Liu)

Compare A_2 and $A_0/22.5$



Cancellation in A_2



- 50 year puzzle resolved!
- A dynamical QCD effect – no more explanation needed?

  [Phys. Rev. Lett. 108 (2012) 141601]

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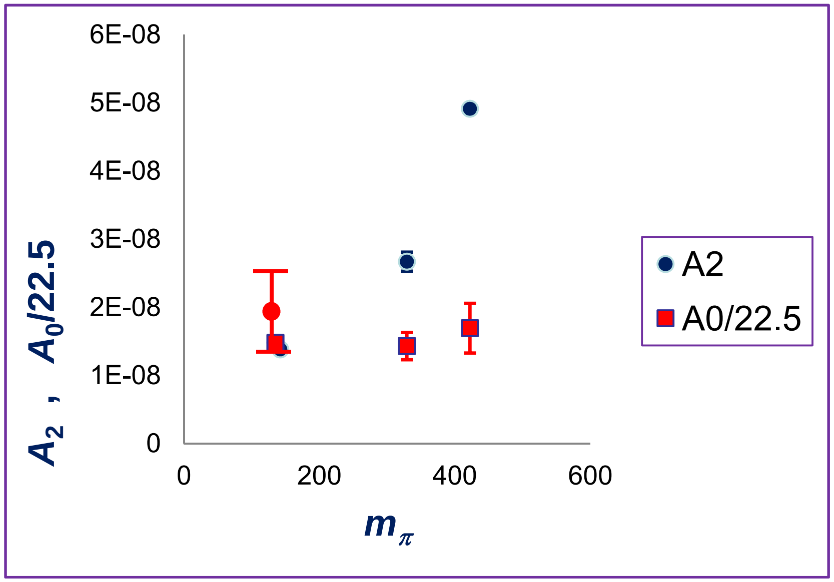

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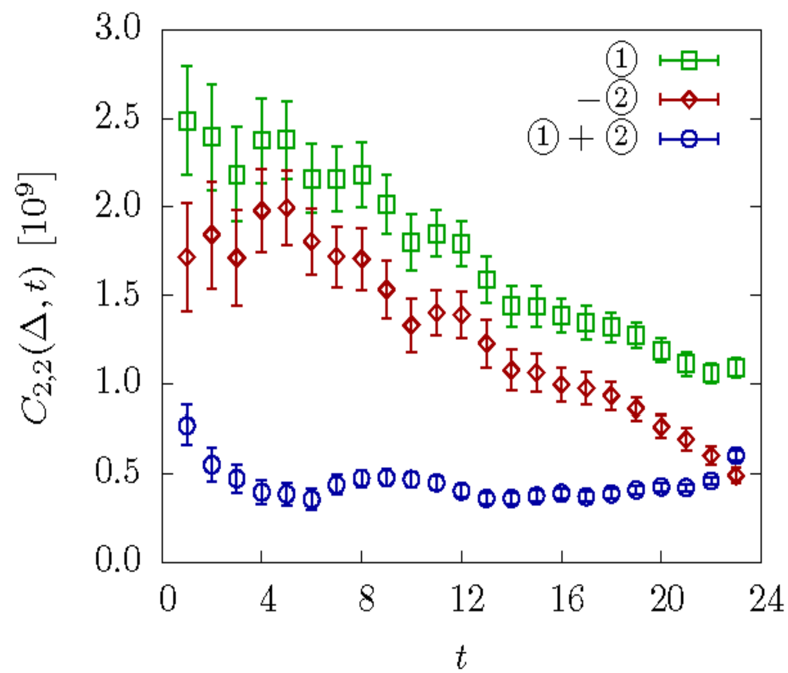

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[Phys. Rev. Lett. 108 (2012) 141601]

Calculation of A_0 and ε'

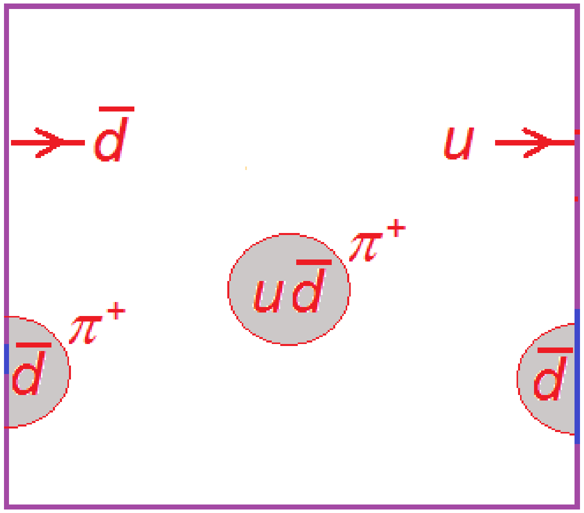
Overview of calculation

(Chris Kelly and Daiqian Zhang)

- Use 32^3 x 64 ensemble
  - *1/a* = 1.3784(68) GeV, *L* = 4.53 fm.
  - G-parity boundary condition in 3 directions
  - 216 configurations separated by 4 time units
  - 900 low modes for all-to-all propagators
  - Solve for $\pi\pi$ and kaon sources on each of 64 time slices
- Achieve essentially physical kinematics:
  - M_π = 143.1(2.0)
  - M_K = 490.6(2.2) MeV
  - $E_{\pi\pi}$ = 498(11) MeV

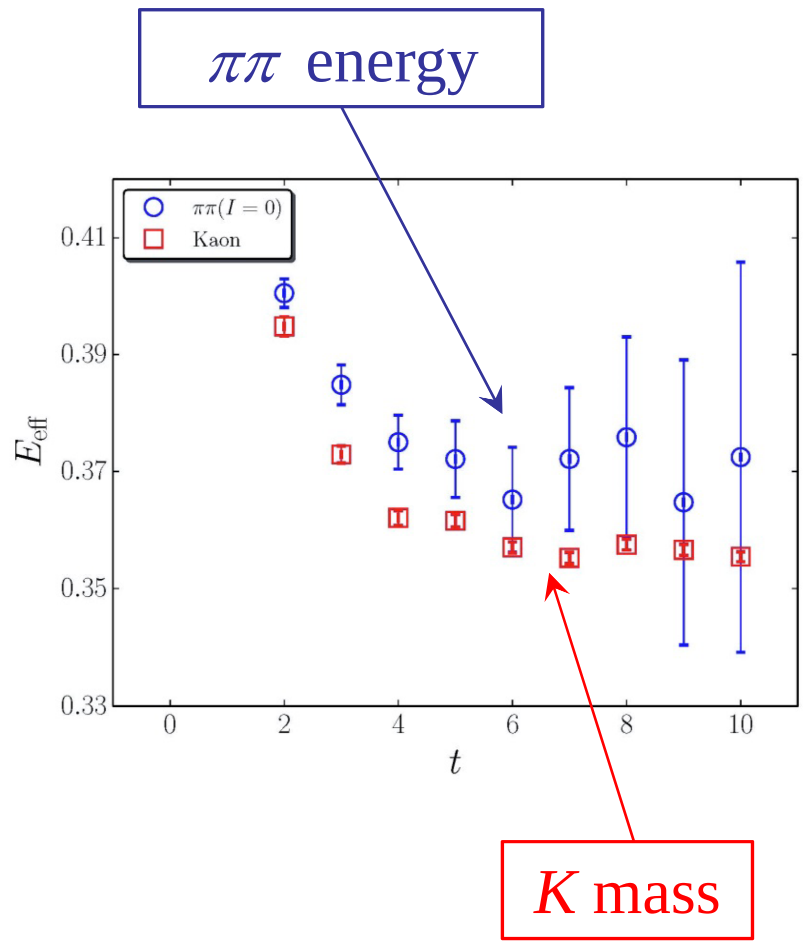
$\Delta I = ½\ K \to \pi\,\pi$ – above threshold
(Chris Kelly & Daiqian Zhang)

- Use **G-parity** BC to obtain p_π = 205 MeV (Changhoan Kim, hep-lat/0210003)
  - $G = C\, e^{i\pi I_y}$
  - Non-trivial: $\begin{pmatrix} u \\ d \end{pmatrix} \to \begin{pmatrix} \bar{d} \\ -\bar{u} \end{pmatrix}$
  - Gauge fields obey *C* BC
  - Extra *I* = 1/2, *s*′ quark adds $e^{-m_K L}$ error
  - Must take non-local square root of *s*-*s*′ determinant.
  - Tests: f_K and B_K correct within errors.



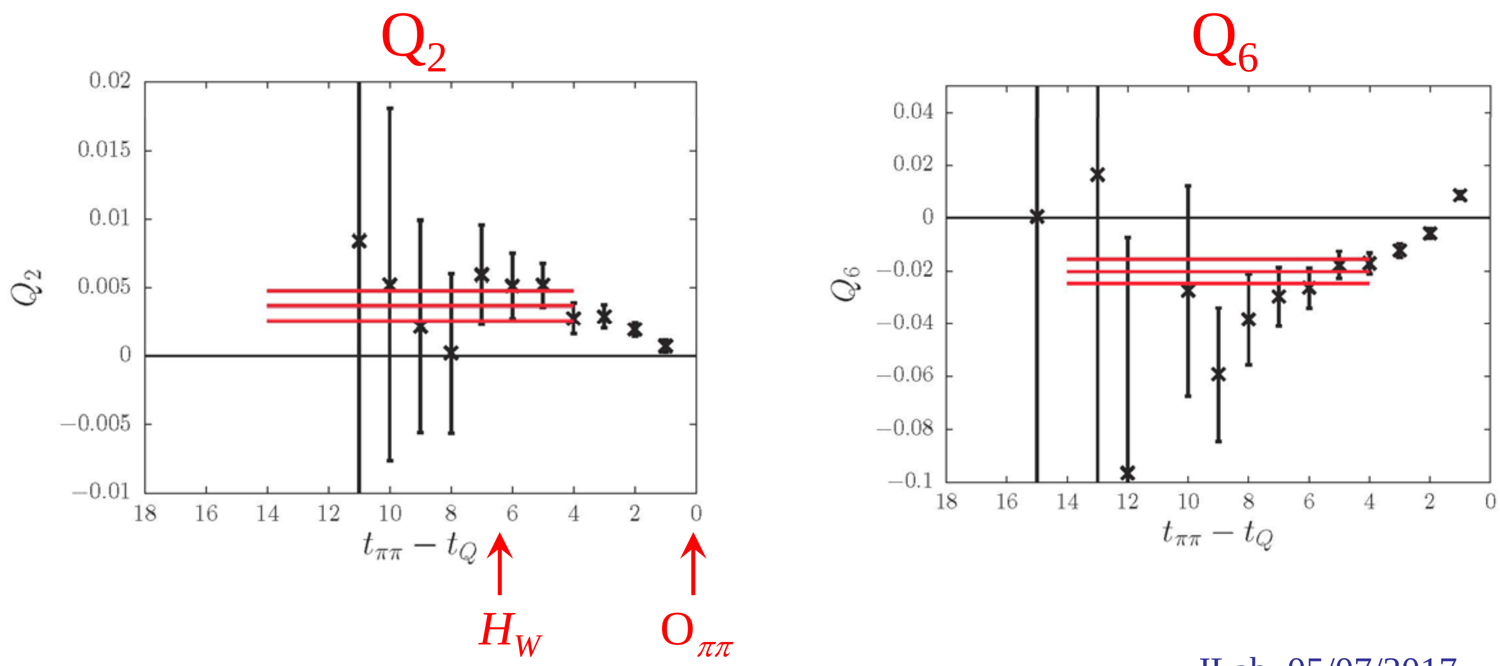
I = 0, $\pi\pi - \pi\pi$ correlator

- Determine normalization of $\pi\pi$ interpolating operator
- Determine energy of finite volume, I = 0, $\pi\pi$ state:
  $E_{\pi\pi}$ = 498(11) MeV
- Determine I = 0 $\pi\pi$ phase shift: δ_0 = 23.8(4.9)(2.2)^o
- Dispersion theory result:
  δ_0 = 38.0(1.3)^o [G. Colangelo]



$\Delta I = \frac{1}{2}$ $K \to \pi\pi$ matrix elements

- Vary time separation between H_W and $\pi\pi$ operator.
- Show data for all $K - H_W$ separations $t_Q - t_K \geq 6$ and $t_{\pi\pi} - t_K = 10, 12, 14, 16$ and 18.
- Fit correlators with $t_{\pi\pi} - t_Q \geq 4$
- Obtain consistent results for $t_{\pi\pi} - t_Q \geq 3$ or 5



Systematic errors

Description	Error
Operator renormalization	15%
Wilson coefficients	12%
Finite lattice spacing	12%
Lellouch-Luscher factor	11%
Finite volume	7%
Parametric errors	5%
Excited states	5%
Unphysical kinematics	3%
Total	27%

Results

Determine the complex ΔI=1/2 amplitude A_0

Re(A_0) = (4.66 ± 1.00_{stat} ± 1.26 $_{sys}$) x 10^{-7} GeV

Expt: (3.3201 ± 0.0018) x 10^{-7} GeV

Im(A_0) = (-1.90 ± 1.23_{stat} ± 1.08_{sys}) x 10^{-11} GeV

Calculate Re(ε'/ε):

Re(ε'/ε) = (1.38 ± 5.15_{stat} ± 4.59_{sys}) x 10^{-4}

Expt.: (16.6 ± 2.3) x 10^{-4}

2.1 σ difference

[Phys. Rev. Lett. 115 (2015) 212001]

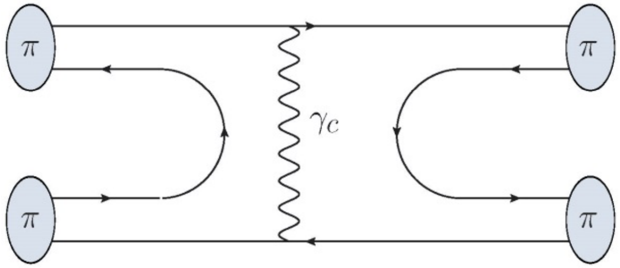
Extend and improve calculation
(Chris Kelly and Tianle Wang)

- ✓ Increase statistics: 216 → 1400 configs.
  - Reduce statistical errors
  - Allow in depth study of systematic errors
- ✓ Study operators neglected in our NPR implementation
- ✓ Use step-scaling to allow perturbative matching at a higher energy
- Use an expanded set of $\pi\pi$ operators
- Use X-space NPR to cross charm threshold (Masaaki Tomii).

Add E&M corrections

(Xu Feng)

- Avoid QED_L, instead use:
  - Use $$V_T(r) = \begin{cases} \frac{e^2}{r} & r \le R_T \\ 0 & r > R_T \end{cases}$$
  - Choose $R_{strong} < R_T < L/2$



- Hasen-Sharpe two-channel, finite-volume quantization/amplitude correction can be employed.
- Missing long-distance effects, including $\eta \ln(2kr)$ term cancel in the ratios η_{+-} , η_{00} or ε'

$$\eta_{+-} \equiv \frac{{}^{\text{out}}\langle(\pi\pi)^{\gamma}_{+-}|H_W|K_L\rangle}{{}^{\text{out}}\langle(\pi\pi)^{\gamma}_{+-}|H_W|K_S\rangle} \qquad \eta_{00} \equiv \frac{{}^{\text{out}}\langle(\pi\pi)^{\gamma}_{00}|H_W|K_L\rangle}{{}^{\text{out}}\langle(\pi\pi)^{\gamma}_{00}|H_W|K_S\rangle}$$

$$\epsilon' = \frac{1}{3}(\eta_{+-} - \eta_{00}) = \frac{\sin 2\theta}{\sin 2\theta^{\gamma}} \frac{i e^{i(\delta_2^{\gamma} - \delta_0^{\gamma})}}{\sqrt{2}} \frac{\text{Re}A_2^{\gamma}}{\text{Re}A_0^{\gamma}} \left(\frac{\text{Im}A_2^{\gamma}}{\text{Re}A_2^{\gamma}} - \frac{\text{Im}A_0^{\gamma}}{\text{Re}A_0^{\gamma}} \right)$$

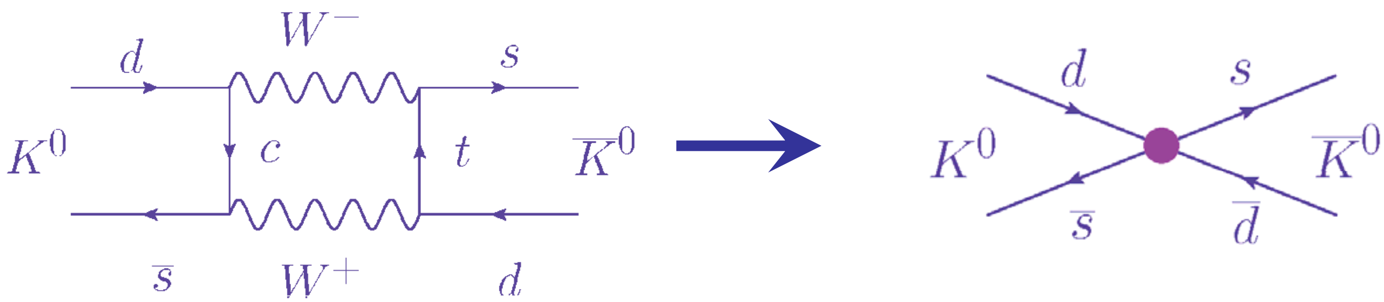
$K^0 - \overline{K^0}$ mixing

ΔM_K & ε_K

$K^0 - \overline{K^0}$ Mixing

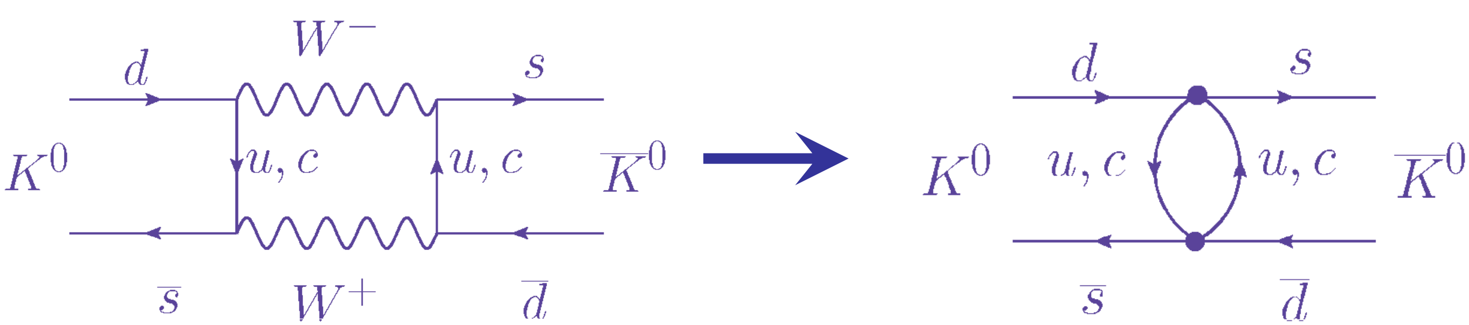
- CP violating: $p \sim m_t$

$$\epsilon_K = \frac{i}{2}\left\{\frac{\mathrm{Im}\,M_{0\bar{0}} - \frac{i}{2}\mathrm{Im}\,\Gamma_{0\bar{0}}}{\mathrm{Re}\,M_{0\bar{0}} - \frac{i}{2}\mathrm{Re}\,\Gamma_{0\bar{0}}}\right\} + i\,\frac{\mathrm{Im}\,A_0}{\mathrm{Re}\,A_0}$$



- CP conserving: $p \leq m_c$

$$m_{K_S} - m_{K_L} = 2\mathrm{Re}\{M_{0\bar{0}}\}$$



$K^0 - \overline{K^0}$ Mixing

- $\Delta S=1$ weak decay allows K^0 and $\overline{K^0}$ to decay to the same $\pi-\pi$ state
- Resulting mixing described by Wigner-Weisskopf:

$$i\frac{d}{dt}\begin{pmatrix} K^0 \\ \overline{K}^0 \end{pmatrix} = \left\{ \begin{pmatrix} M_{00} & M_{0\bar{0}} \\ M_{\bar{0}0} & M_{\bar{0}\bar{0}} \end{pmatrix} - \frac{i}{2}\begin{pmatrix} \Gamma_{00} & \Gamma_{0\bar{0}} \\ \Gamma_{\bar{0}0} & \Gamma_{\bar{0}\bar{0}} \end{pmatrix} \right\} \begin{pmatrix} K^0 \\ \overline{K}^0 \end{pmatrix}$$

where

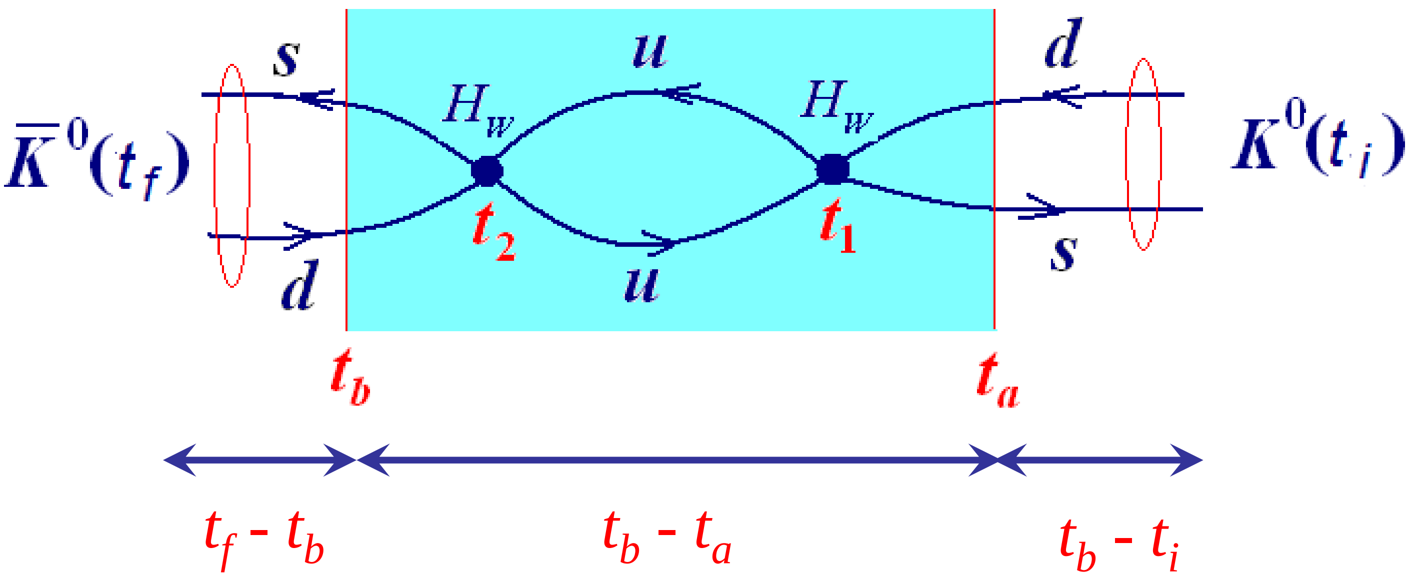
$$\Gamma_{ij} = 2\pi \sum_\alpha \int_{2m_\pi}^\infty dE \langle i|H_W|\alpha(E)\rangle\langle\alpha(E)|H_W|j\rangle\delta(E-m_K)$$

$$M_{ij} = \sum_\alpha \mathcal{P} \int_{2m_\pi}^\infty dE \frac{\langle i|H_W|\alpha(E)\rangle\langle\alpha(E)|H_W|j\rangle}{m_K - E}$$

Lattice Version

- Evaluate standard, Euclidean, 2nd order $\overline{K}^0 - K^0$ amplitude:

$$\mathcal{A} = \langle 0|T\left(K^0(t_f)\frac{1}{2}\int_{t_a}^{t_b} dt_2 \int_{t_a}^{t_b} dt_1 H_W(t_2) H_W(t_1) K^{0\dagger}(t_i)\right)|0\rangle$$



Interpret Lattice Result

$$\mathcal{A} = N_K^2 e^{-M_K(t_f-t_i)} \sum_n \frac{\langle \overline{K}^0 | H_W | n \rangle \langle n | H_W | K^0 \rangle}{M_K - E_n} \left(\underbrace{-(t_b - t_a)}_{1.} \underbrace{- \frac{1}{M_K - E_n}}_{2.} \underbrace{+ \frac{e^{(M_K-E_n)(t_b-t_a)}}{M_K - E_n}}_{3.} \right)$$

1. Δm_K^{FV}
2. Uninteresting constant
3. Growing or decreasing exponential: states with $E_n < m_K$ must be removed!

- Finite volume correction:

$$M_{K_L} - M_{K_S} = 2 \sum_n \frac{\langle \overline{K}^0 | H_W | n \rangle \langle n | H_W | K^0 \rangle}{M_K - E_n} - 2 \frac{d(\phi + \delta_0)}{dk} \Big|_{m_K} |\langle n_0 | H_W | K^0 \rangle|^2 \cot(\phi + \delta_0) \Big|_{M_K}$$

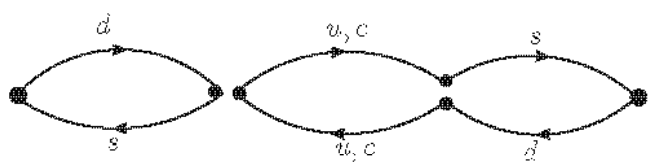
(N.H. Christ, X. Feng, G. Martinelli, C.T. Sachrajda, arXiv:1504.01170)

$K_L - K_S$
mass difference

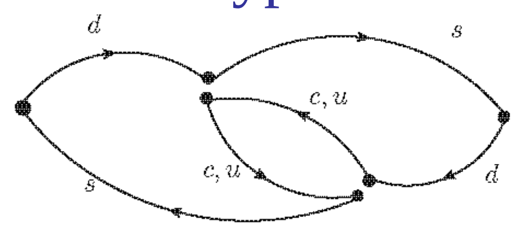
$K_L - K_S$ mass difference

- $M_{K_L} - M_{K_S}$ = 3.483(6) x 10^{-12} MeV: sensitive to 1000 TeV scale physics.
- Perturbative result integrates out charm and shows poor convergence (Brod and Gorbahn).
- Finite when charm quark is included (GIM).

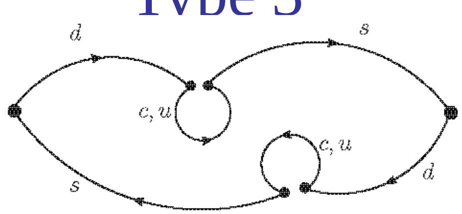
Type 1



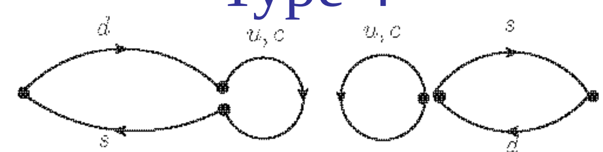
Type 2



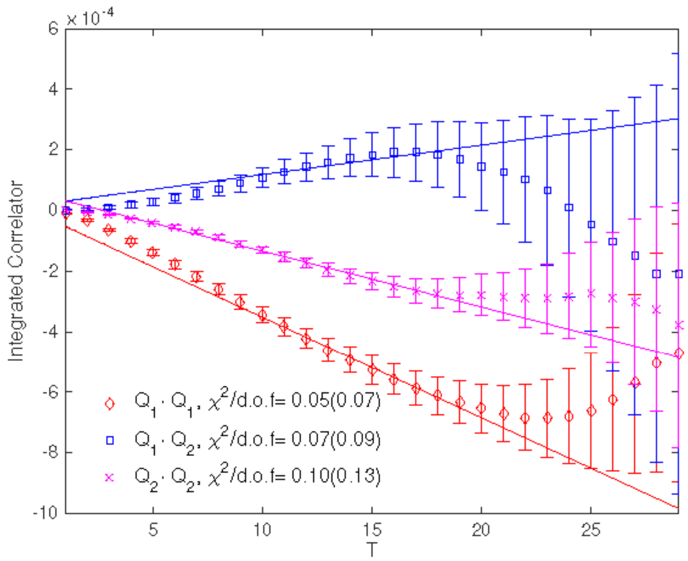
Type 3



Type 4



ΔM_K Preliminary Results
(Ziyuan Bai)



	ΔM_K x 10^{+12} MeV
Types 1-4	5.8(1.7)
Types 1-2	-1.1(1.2)
Δ_{FV}	0.27(18)
Expt.	3.483(6)

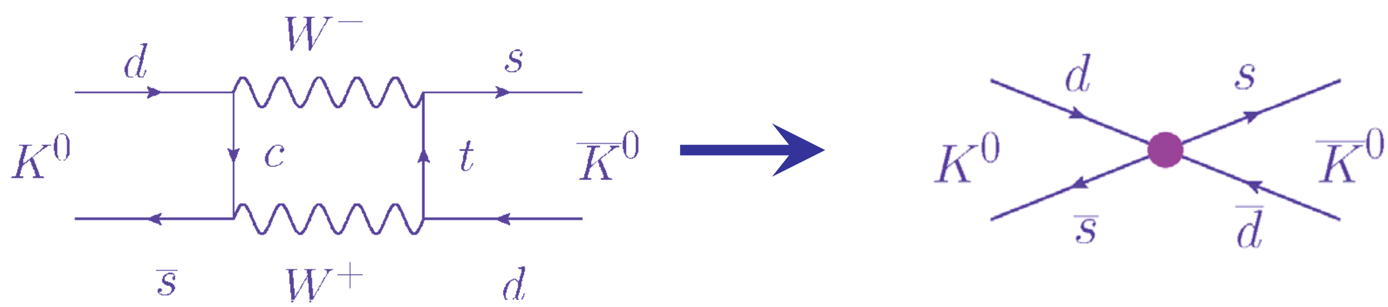
- $m_c^{\overline{MS}}$(2 GeV) ~ 1.2 MeV, M_π = 138 MeV
- 64^3 x 128, 1/a = 2.36 GeV
- Uncorrelated fit: $10 \leq T \leq 20$
- FV correction ~5%
- a^2 errors 5-10%

Long distance part of ε_K

$K^0 - \overline{K}^0$ mixing: Indirect CP Violation

- CP violating: $p \sim m_t$

$$\epsilon_K = \frac{i}{2}\left\{\frac{\mathrm{Im}\,M_{0\bar{0}} - \frac{i}{2}\mathrm{Im}\,\Gamma_{0\bar{0}}}{\mathrm{Re}\,M_{0\bar{0}} - \frac{i}{2}\mathrm{Re}\,\Gamma_{0\bar{0}}}\right\} + i\,\frac{\mathrm{Im}\,A_0}{\mathrm{Re}\,A_0}$$



- Where $|\varepsilon_K|$ = (2.228 ±0.011) x 10^{-3}
  - Short distance prediction [W.Lee, *et al*. 1710.06614]:
    $|\varepsilon_K|$ = 1.58 ± 0.16 (V_{cb} dominant error)
  - Long distance estimate [Buras, *et al*. 1002.3612] :
    results in 6% reduction

$\Delta S = 1$, four-flavor operators
(Ziyuan Bai)

- Choose appropriate $N_f = 4$ effective Hamiltonian:

$$H_W^{\Delta S=1;\Delta C=\pm 1,0} = \frac{G_F}{\sqrt{2}} \left\{ \sum_{q,q'=u,c} V_{q's}^* V_{qd} \sum_{i=1}^{2} C_i Q_i^{q'q} + V_{ts}^* V_{td} \sum_{i=3}^{6} C_i Q_i \right\}$$

$$\begin{aligned}
Q_1^{q'q} &= (\bar{s}_i q'_j)_{V-A} (\bar{q}_j d_i)_{V-A} \\
Q_2^{q'q} &= (\bar{s}_i q'_i)_{V-A} (\bar{q}_j d_j)_{V-A}
\end{aligned}$$

current x current

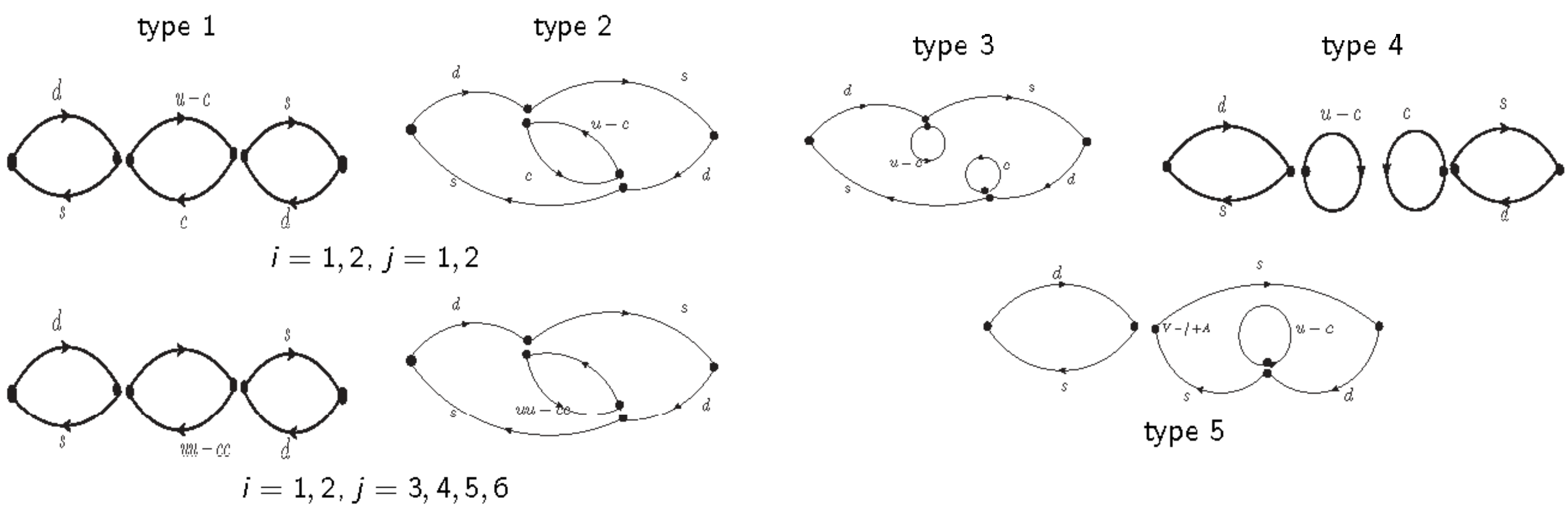
$$\begin{aligned}
Q_3 &= (\bar{s}_i d_i)_{V-A} \sum_{q=u,d,s,c} (\bar{q}_j q_j)_{V-A} \\
Q_4 &= (\bar{s}_i d_j)_{V-A} \sum_{q=u,d,s,c} (\bar{q}_j q_i)_{V-A} \\
Q_5 &= (\bar{s}_i d_i)_{V-A} \sum_{q=u,d,s,c} (\bar{q}_j q_j)_{V+A} \\
Q_6 &= (\bar{s}_i d_j)_{V-A} \sum_{q=u,d,s,c} (\bar{q}_j q_i)_{V+A}
\end{aligned}$$

QCD penguin

Diagrams for $\lambda_t \lambda_u$ contribution to ε_K

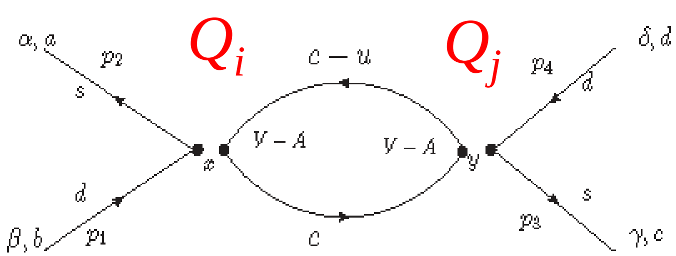
(Ziyuan Bai)

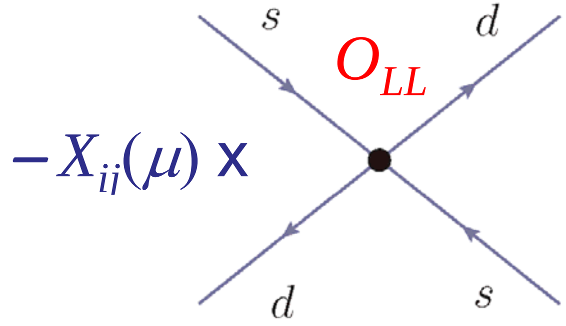
- Identify five types of diagrams



New ΔS = 2 counter term

(Ziyuan Bai)





- Subtract $X_{ij}(\mu)\ (\bar{s}\gamma^{\nu}(1\text{-}\gamma^5)d)\ (\bar{s}\gamma^{\nu}(1\text{-}\gamma^5)d)$ to make off-shell Greens function vanish at $p_i^2 = \mu_{RI}{}^2$
- Define infrared-safe Rome-Southampton normalization for bi-local operator.

Progress toward long-distance part of ε_K

(Ziyuan Bai)

- Compute NLO (one-loop) conversion from bilocal RI to MS
- Preliminary

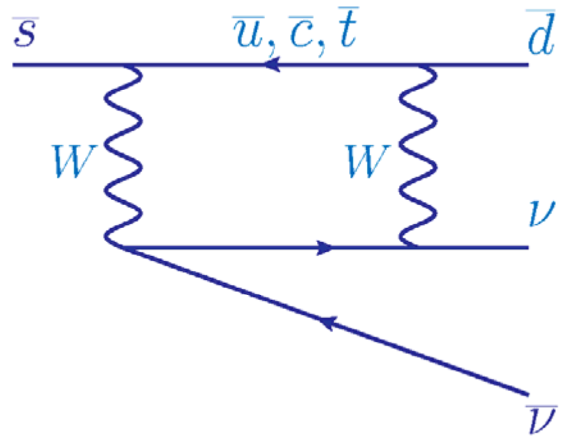
μ_{RI}	Im $M_{\bar{0}0}^{ut,RI}$	Im $M_{\bar{0}0}^{ut,RI\to\overline{MS}}$	Im $M_{\bar{0}0}^{ut,ld\,corr}$	$\epsilon_K^{ut,ld\,corr}$
1.54 GeV	-0.746(0.389)	0.282	-0.464 (0.389)	0.0911(0.076)
1.92 GeV	-0.912(0.389)	0.384	-0.527 (0.389)	0.104(0.076)
2.11 GeV	-0.986(0.389)	0.434	-0.551 (0.389)	0.108(0.076)
2.31 GeV	-1.050(0.390)	0.486	-0.565 (0.390)	0.111(0.077)
2.56 GeV	-1.115(0.390)	0.548	-0.568 (0.390)	0.111(0.077)

- $|\varepsilon_K| =$ 2.228(11) x 10^{-3} expt.

Rare Kaon Decays

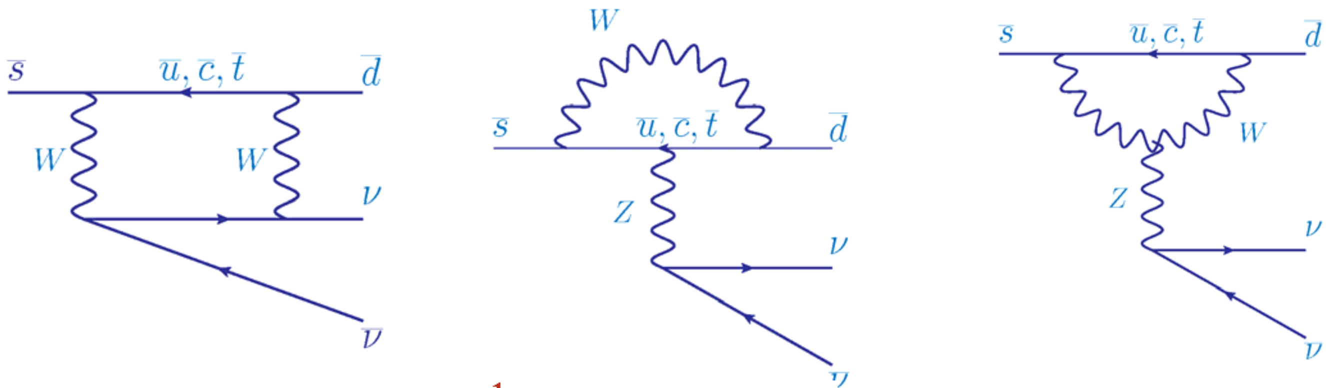
$$K^+ \rightarrow \pi^+ \nu \bar{\nu}$$

$K^+ \rightarrow \pi^+ \nu \bar{\nu}$ (Xu Feng)



- Flavor changing neutral current
  - Allowed in the Standard Model only in second order
  - Short distance dominated
- Target of NA62 at CERN
  - 100 events in 2-3 years
  - Test Standard Model prediction at 10% level
  - Use lattice for long distance part: 5% effect ?

$K^+ \to \pi^+ \nu \bar{\nu}$ in the Standard Model

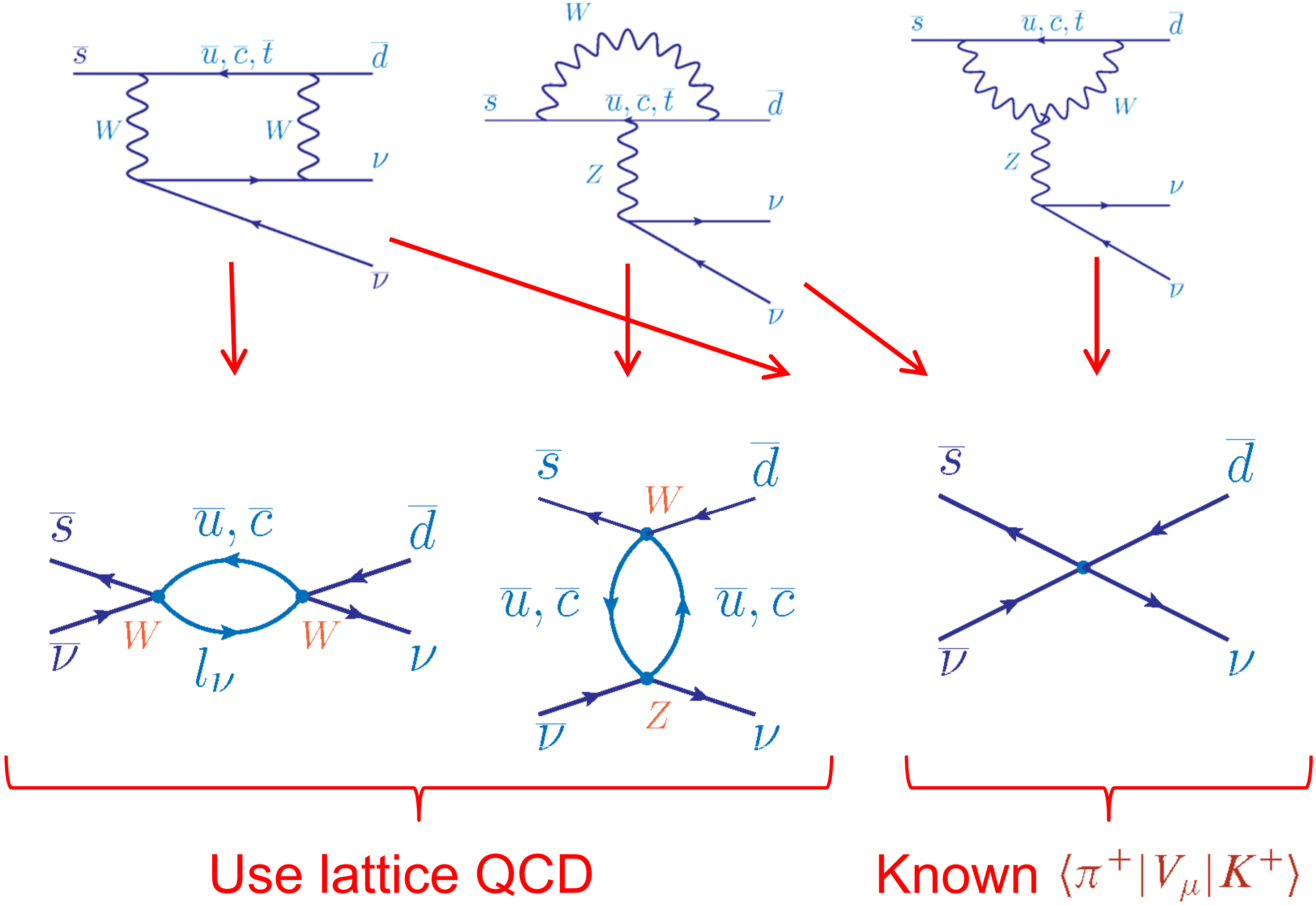


- Factors of $\frac{1}{M_W^4}$ or $\frac{1}{M_W^2 M_Z^2}$ force the largest contribution to come from short distance

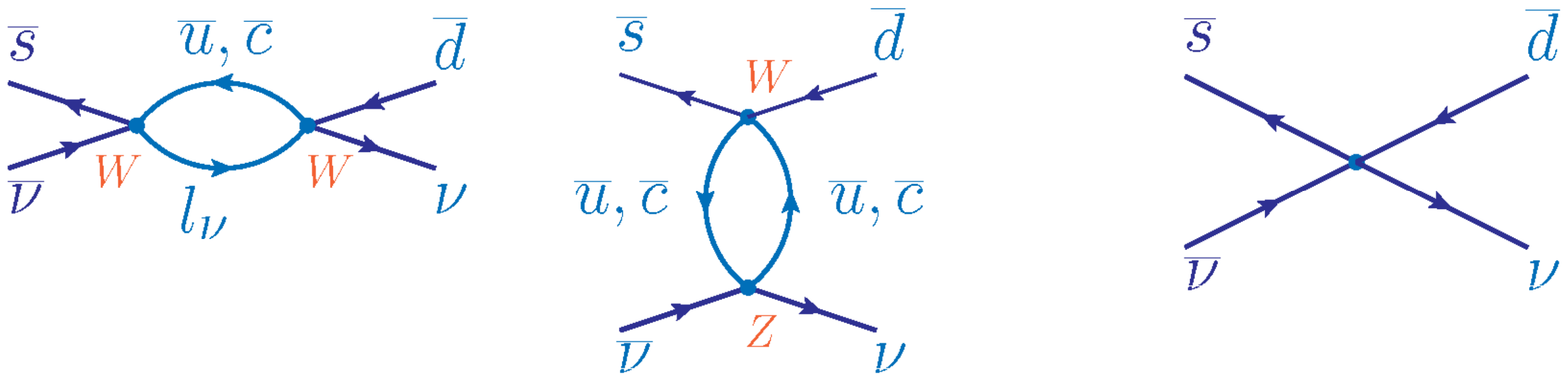
Pert. Th.
- Top quark contribution largest.
- GIM implies charm-up $\sim \frac{m_c^2 - m_u^2}{M_W^4} \ln(M_W^2 / m_c^2)$

Lattice
- Long distance part $\sim \frac{m_c^2}{M_W^4}$

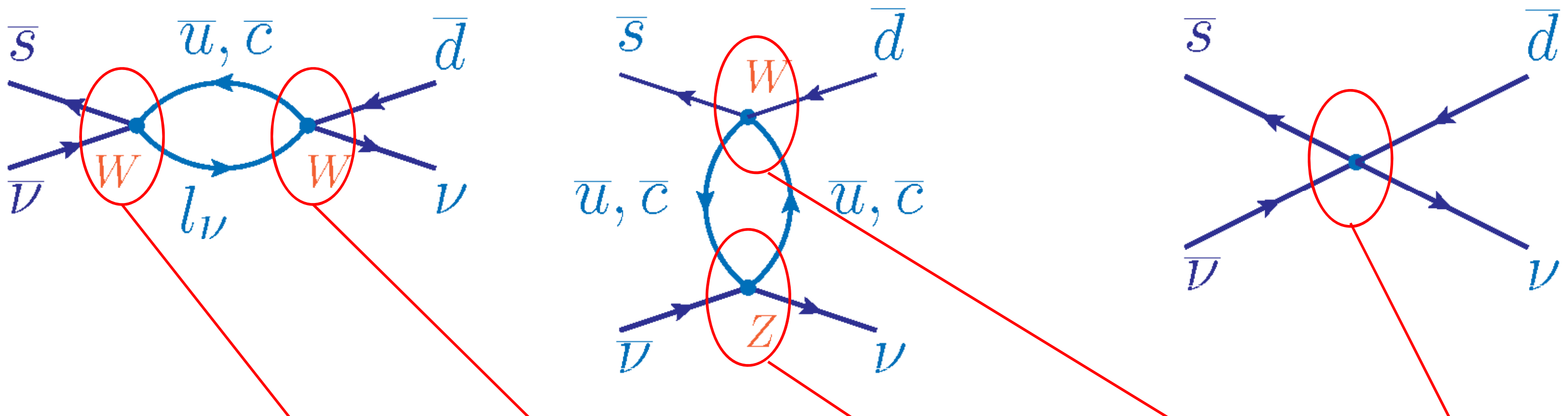
$K^+ \rightarrow \pi^+ \nu \bar{\nu}$ at long distance



H_{eff} for $K^+ \to \pi^+ \nu \bar{\nu}$

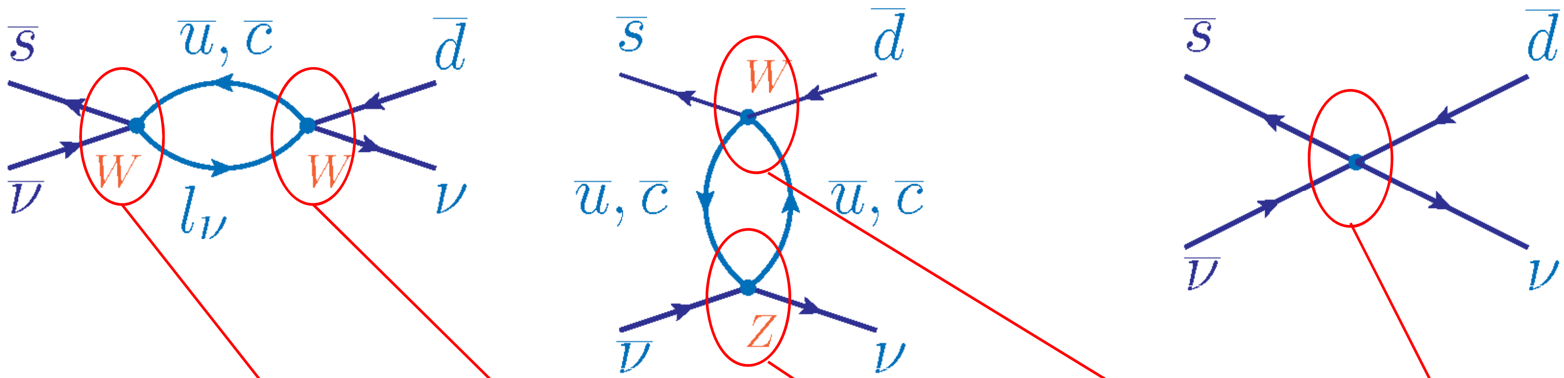


$H_{\rm eff}$ for $K^+ \to \pi^+ \nu \bar{\nu}$



$$\mathcal{H}_{\rm eff} = +\frac{G_F}{\sqrt{2}}\left\{ \sum_{\substack{q=u,c \\ \ell=e,\mu,\tau}} \left(V^*_{qs} O^{\Delta S=1}_{q\ell} + V_{qd} O^{\Delta S=0}_{q\ell} \right) + \sum_{\ell=e,\mu,\tau} O^Z_\ell + \sum_{q=u,c} \lambda_q O^W_q \right\} + O_0$$

$H_{\rm eff}$ for $K^+ \to \pi^+ \nu \bar\nu$



$$\mathcal{H}_{\rm eff} = +\frac{G_F}{\sqrt{2}}\left\{ \sum_{\substack{q=u,c\\ \ell=e,\mu,\tau}} \left(V^*_{qs} O^{\Delta S=1}_{q\ell} + V_{qd}\, O^{\Delta S=0}_{q\ell} \right) + \sum_{\ell=e,\mu,\tau} O^Z_\ell + \sum_{q=u,c} \lambda_q O^W_q \right\} + O_0$$

$$O^{\Delta S=1} = C_{\Delta S=1}(\overline{s}q)_{V-A}(\overline{\ell}\nu_\ell)_{V-A}$$

$$O^W_q = C_1(\overline{s}_a q_b)_{V-A}(\overline{q}_b d_a)_{V-A} + C_2(\overline{s}_a q_a)_{V-A}(\overline{q}_b d_b)_{V-A}$$

$$O^{\Delta S=0} = C_{\Delta S=0}(\overline{q}d)_{V-A}(\overline{\ell}\nu_\ell)_{V-A}$$

$$O_0 = C_0 \sum_{\ell=e,\mu,\tau} (\overline{s}d)_{V-A}(\overline{\nu}_\ell \nu_\ell)_{V-A}$$

$$O^Z_\ell = C_Z \sum_{q=u,c,d,s} \left(T^q_3 \overline{q}\gamma_\mu(1-\gamma_5)q - Q_{{\rm em},q}\sin^2\theta_W \overline{q}\gamma_\mu q \right) \overline{\nu_\ell}\gamma_\mu(1-\gamma_5)\ell$$

$K^+ \to \pi^+ \nu \bar{\nu}$: 2nd order effective theory

Bilocal

Local

$$\mathcal{A}(K^+ \to \pi^+ \nu\bar{\nu}) = \langle \pi^+ \nu\bar{\nu}| \, T\left\{\int d^4x \mathcal{H}'_{\rm eff}(x)\mathcal{H}'_{\rm eff}(0)\right\} + O_0(0) \, |K^+\rangle$$

$$\mathcal{H}_{\rm eff} = +\frac{G_F}{\sqrt{2}}\left\{\sum_{\substack{q=u,c\\ \ell=e,\mu,\tau}}\left(V^*_{qs}O^{\Delta S=1}_{q\ell} + V_{qd}O^{\Delta S=0}_{q\ell}\right) + \sum_{\ell=e,\mu,\tau} O^Z_\ell + \sum_{q=u,c}\lambda_q O^W_q\right\} + O_0$$

$$O^{\Delta S=1} = C_{\Delta S=1}(\bar{s}q)_{V-A}(\bar{\ell}\nu_\ell)_{V-A}$$

$$O^W_q = C_1(\bar{s}_a q_b)_{V-A}(\bar{q}_b d_a)_{V-A} + C_2(\bar{s}_a q_a)_{V-A}(\bar{q}_b d_b)_{V-A}$$

$$O^{\Delta S=0} = C_{\Delta S=0}(\bar{q}d)_{V-A}(\bar{\ell}\nu_\ell)_{V-A}$$

$$O_0 = C_0 \sum_{\ell=e,\mu,\tau}(\bar{s}d)_{V-A}(\bar{\nu}_\ell\nu_\ell)_{V-A}$$

$$O^Z_\ell = C_Z \sum_{q=u,c,d,s}\left(T^q_3\bar{q}\gamma_\mu(1-\gamma_5)q - Q_{{\rm em},q}\sin^2\theta_W\bar{q}\gamma_\mu q\right)\overline{\nu_\ell}\gamma_\mu(1-\gamma_5)\ell$$

$K^+ \to \pi^+ \nu \bar{\nu}$: Effect of bilocal operator

Bilocal — Local

$$\mathcal{A}(K^+ \to \pi^+ \nu\bar{\nu}) = \langle \pi^+ \nu\bar{\nu}| \; T\left\{\int d^4x \mathcal{H}'_{\text{eff}}(x)\mathcal{H}'_{\text{eff}}(0)\right\} + O_0(0) \; |K^+\rangle$$

- Standard continuum treatment
  - Replace bilocal term with (perturbative coefficient) x (local operator)
- Lattice treatment: Evaluate $H_{eff}(x)\ H_{eff}(0)$ product
  - Revolve logarithmic divergence as $x \to 0$
  - Deal with intermediate states with $E \le M_K$
    - Exponential Euclidean time dependence
    - Power-law finite volume corrections
    - Exploit methods from M_{K_L}-M_{K_S} calculation

Exploratory Lattice Calculation

- 16^3 x 32, RBC-UKQCD ensemble
  - 2+1 flavor DWF, $1/a$ = 1.73 GeV
  - M_π = 420 MeV, M_K = 540 MeV,
  - $m_c(2\text{ GeV})^{\overline{\text{MS}}}$ = 863 GeV
- Calculate all diagrams
- 800 configurations
- Low-mode deflation with 100 modes
- Place sources on 32 time slices
- Treat internal lepton as an overlap fermion moving in an ∞ time extent.

Compare lattice and pertubative:

- Decay rate is short distance dominated:

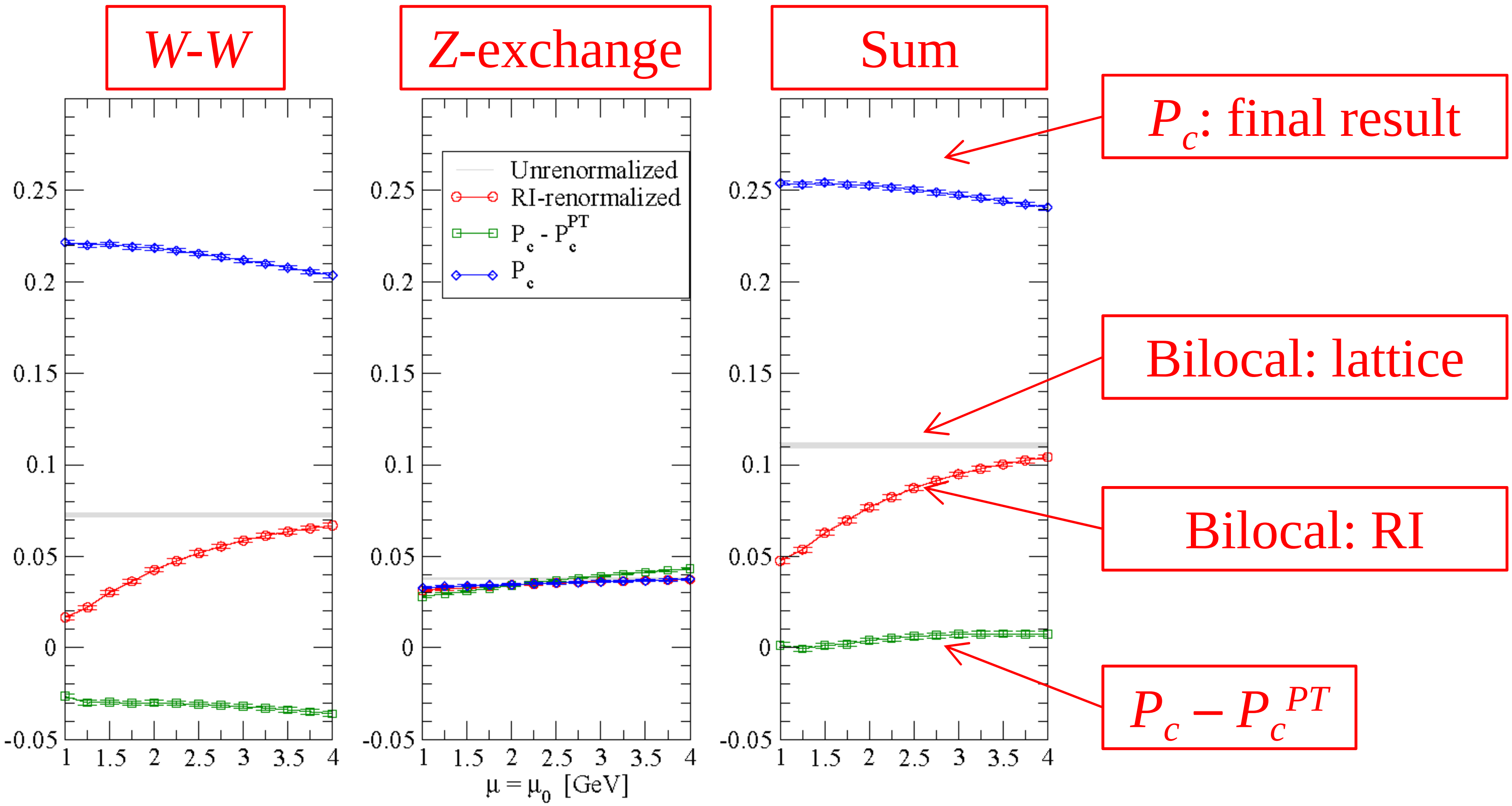
$$\mathrm{Br} = \kappa_+(1+\Delta_{\mathrm{EM}})\left[\left(\underbrace{\frac{\mathrm{Im}\lambda_t}{\lambda^4}X(x_t)}_{0.270\times 1.481}\right)^2 + \left(\underbrace{\frac{\mathrm{Re}\lambda_c}{\lambda}P_c}_{-0.974\times 0.365} + \underbrace{\frac{\mathrm{Re}\lambda_t}{\lambda^5}X(x_t)}_{-0.533\times 1.481}\right)^2\right]$$

- Charm contribution is less than top but is significant (removing charm lowers BR by 50%).
- Result for P_c :
  - Perturbation theory [Buras, et al.,1503.02693]: $P_c = 0.365(12)$
  - LD correction [Isidori, et al., hep-ph/0503107]: $\delta P_{cu} = 0.04(2)$
    (estimate of non-perturbative and $(\Lambda_{\mathrm{QCD}}/m_c)^2$ effects)
  - Exploratory lattice result:

$$P_c(\mu_{\overline{\mathrm{MS}}}) - P^{\mathrm{PT}}(\mu_{\overline{\mathrm{MS}}}) = 0.0040\,(\pm 13)_{\mathrm{stat}}\,(\pm 32)_{\mathrm{scale}}\,(-45)_{\mathrm{FV}}$$

(lattice evaluation of bilocal matrix element minus PT estimate)

Details of W-W – Z-exchange cancellation



Outlook

- Lattice QCD is now capable of 1st-principles calculation of:
  - $K \rightarrow \pi\pi$, $\Delta I = 3/2$ and $1/2$, ε'/ε.
  - $M_{K_L} - M_{K_S}$ and long distance contribution to ε.
  - Long distance parts of $K \rightarrow \pi \bar{l} l$, $K \rightarrow \pi \bar{\nu} \nu$.
- Physical quark mass calculations underway:
  - $M_{K_L} - M_{K_S}$
  - $K^+ \rightarrow \pi^+ \bar{\nu} \nu$
- With the new CORAL computers (Summit at ORNL) can perform $a^2 \rightarrow 0$ limit with charm.