Time to retire a myth in pQCD

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Summary

- Summarize standard presentation/argument for factorization
- What's wrong conceptually and physically?
- What should have been done instead?
- How do the conceptual errors not mess up standard hard-scattering phenomenology?
- Why should we care?

(See JCC, "Foundations of Perturbative QCD", Sec. 9.11.)

Review of factorization and its predictive power

Basis:

- Factorization for DIS at large Q:

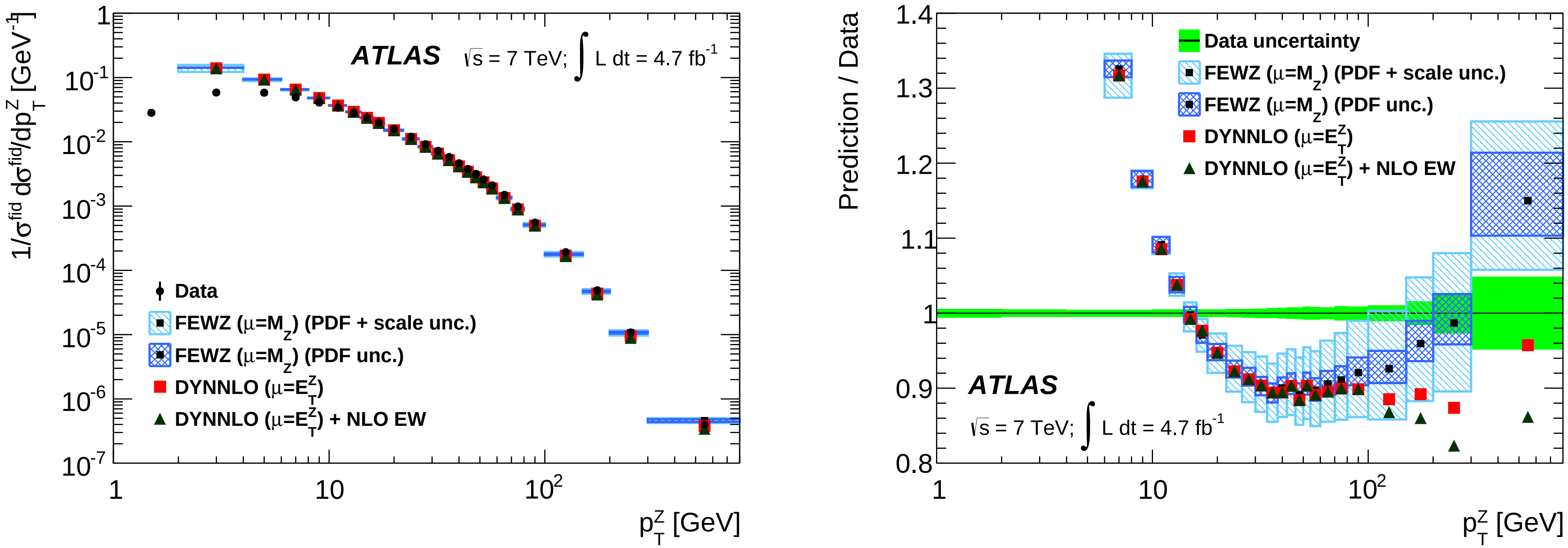
$$d\sigma_{\text{had.}}(x, Q) = f_{j/h} \otimes d\hat{\sigma}_{j,\text{hard}} + \text{power suppressed.}$$

- Etc for other processes, with parton densities, fragmentation functions
- Evolution equations for pdfs, ffs and α_s

Predictive power (evading difficulty of calculations in low-scale regions):

- pQCD calculation of hard scattering, DGLAP kernels, etc
- Measurement of pdfs, ffs, Λ_{QCD} (etc) from a limited set of data.
- Universality of pdfs, ffs, etc gives predictions for many other processes at all (high enough) Q.

Successful predictions, e.g., of Z production at LHC



From arXiv:1510.05427

Standard presentations of factorization have the following form:

1. *Assert* factorization in terms of "bare" pdfs and *unsubtracted* on-shell massless partonic cross section

$$\mathrm{d}\sigma_{\text{had}}(x,Q) \;=\; f_{j/h,\,\text{“bare”}} \otimes \mathrm{d}\hat{\sigma}_{j,\text{partonic}}$$

2. $\mathrm{d}\hat{\sigma}_{j,\,\mathrm{partonic}}$ has initial-state collinear divergences, which have been shown to factor

$$\mathrm{d}\hat{\sigma}_{j,\text{partonic}} \;=\; C \otimes \mathrm{d}\hat{\sigma}_{j,\text{finite}},$$

   so that

$$\mathrm{d}\sigma_{\text{had}}(x,Q) \;=\; f_{j/h,\text{“bare”}} \otimes C \otimes \mathrm{d}\hat{\sigma}_{j,\text{finite}}$$

3. Absorb collinear divergences into redefined pdfs,

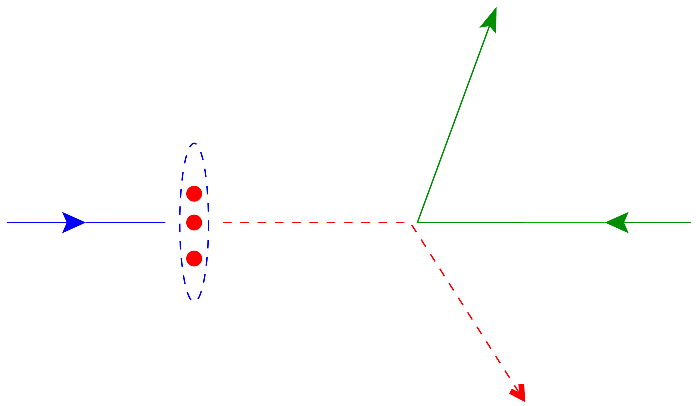
$$f_{j/h,\text{“ren”}} \;=\; f_{j/h,\text{“bare”}} \otimes C,$$

   to give final factorization formula

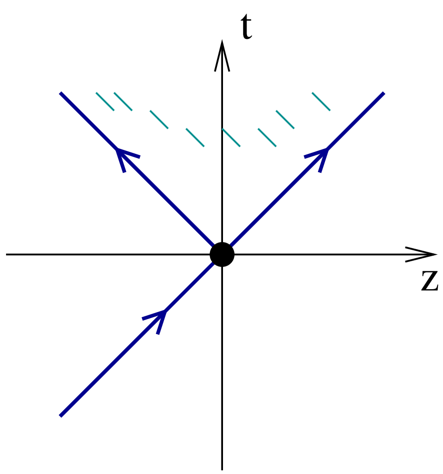
$$\mathrm{d}\sigma_{\text{had}}(x,Q) \;=\; f_{j/h,\text{“ren”}} \otimes \mathrm{d}\hat{\sigma}_{j,\text{finite}}\,.$$

[Observe: Final factorization formula here has same *structure* as factorization formula given earlier, but not necessarily with same definitions.]

Backtrack to parton model to *motivate* factorization/partons



DIS at large Q: Propose short distance collision of electron and point-like constituent of fast moving hadron, Lorentz contracted & time dilated.



Pdf probes correlation in target along $x^- \propto (t - z)$.
Long-time final-state interactions irrelevant for inclusive cross section.

- Gives $\mathrm{d}\sigma = \text{pdf} \otimes$ *Lowest order* hard sc.
- Coordinate space reasoning critical here. (N.B. Mismatch with mom. space work.)
- Implicit conjecture (pre-QCD!): separation of scales, etc.
- Implication: pdf is target expectation value of light-front number operator.

Standard presentation of factorization again:

1. *Assert* factorization in terms of "bare" pdfs and *unsubtracted* on-shell massless partonic cross section

$$\mathrm{d}\sigma_{\text{had}}(x, Q) \;=\; f_{j/h,\,\text{“bare”}} \otimes \mathrm{d}\hat{\sigma}_{j,\text{partonic}}$$

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$$\mathrm{d}\sigma_{\text{had}}(x, Q) \;=\; f_{j/h,\text{“ren”}} \otimes \mathrm{d}\hat{\sigma}_{j,\text{finite}}\,.$$

Problems

- Initial statement is just asserted, without motivation or justification:

$$\mathrm{d}\sigma_{\mathsf{had}}(x,Q) \;=\; f_{j/h,\,\text{“bare”}} \otimes \mathrm{d}\hat{\sigma}_{j,\mathsf{partonic}}$$

  with $\mathrm{d}\hat{\sigma}_{j,\mathsf{partonic}}$ including higher order correct. (E.g., $\Rightarrow$)
  (Possibly there's a reference to Feynman and parton model.)

- Bare pdfs not defined explicitly.

- Violent disagreement with parton model: Infinitely long distance processes in partonic cross section instead of pure short distance phenomena.

- The collinear divergences are treated as actually physically existing in QCD.

- Same treatment would apply in model QFT when all particles are massive and hence there are no true collinear divergences.

- By construction, pdfs with standard operator definition include all collinear physics, correctly.

- Hence formula is *wrong* and *unphysical* in any reasonable sense.

- For lattice gauge theory, need to know definition of pdf as operator matrix element (so same definition used in different subfields of QCD). How is absorbing collinear divergences to be implemented?

E.g., Dissertori, Knowles & Schmelling, "Quantum Chromodynamics" (OUP, 2003)

This parton cross section is related to the hadron cross section by weighting it by the hadron's p.d.f.s,

$$\mathrm{d}\sigma^{(\ell\mathrm{h})} = \sum_{f=q,\bar{q},g} \int_0^1 \mathrm{d}y\, f_{\mathrm{h}}(y)\, \mathrm{d}\hat{\sigma}^{(\ell f)}\left(\frac{x}{y}\right) . \tag{3.227}$$

Notes:

- No reason given.

A correct argument: Successive approximation

Strategy:

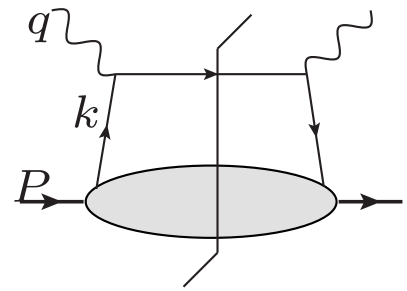
- Identify regions giving leading power (Libby-Sterman).
- Parton model approximation as start.
- Examine configurations in graphs where it fails.
- Set up these as contribution to factorization with NLO $\mathrm{d}\hat{\sigma}$, NNLO, etc
- But with subtractions for contributions already handled.

N.B. Shift of meaning of $\mathrm{d}\hat{\sigma}$ between equations, which I've indicated with appropriate labels ("hard", "partonic", "finite").

Successive approximation 1

Parton model for DIS converted to field theory starts with

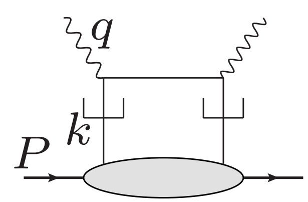
$$W^{\mu\nu} \simeq$$



with k of low virtuality and low k_T.

Parton model approximation neglects k^2, k_T and m in hard scattering (i.e., upper rung), to give on-shell massless quark scattering at LO:

$$W^{\mu\nu} \simeq W^{\mu\nu}_{\text{(LO)}} \stackrel{\text{def}}{=}$$

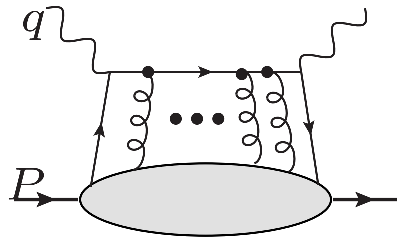


Gives operator definition of pdf with integration over *all* k^2 and k_T. Define resulting UV divergences to be renormalized. (First QCD complication.)

(One extension: next slide.)

Successive approximation 2

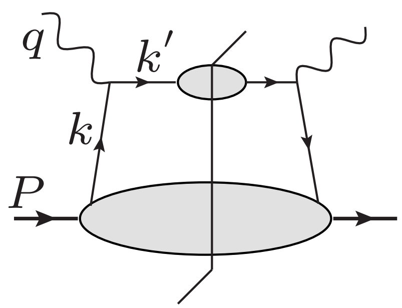
a. Extra collinear gluon exchanges (color field in target):



These reproduce Wilson line in gauge-invariant operator definition of pdf.

Wilson line gives effect (at leading power) of gluon field in target on outgoing struck quark. *No further changes in definition of pdf.*

b. There are long-distance final-state interactions, as in

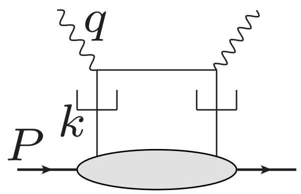


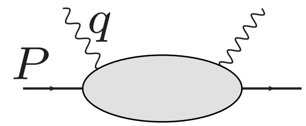
They cancel after sum over cuts, leaving only short distance remnant as part of NLO, NNLO etc corrections.

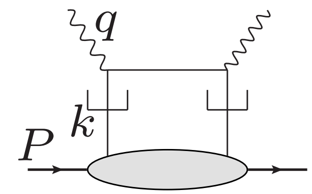
Successive approximation 3: Its principles

Write exact $W^{\mu\nu}$ as

$$W^{\mu\nu} = W^{\mu\nu}_{\text{(LO)}} + \left(W^{\mu\nu} - W^{\mu\nu}_{\text{(LO)}}\right)$$

$=$ 

$+ \Big($ 

$-$ 

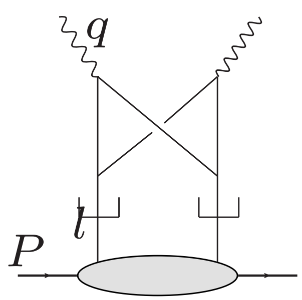
$\Big).$

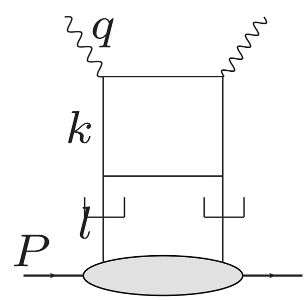
(N.B. Gluon exchanges and Wilson lines not shown.)

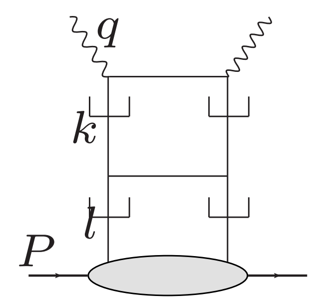
Hence write

$$W^{\mu\nu} = W^{\mu\nu}_{\text{(LO)}} + W^{\mu\nu}_{\text{(NLO)}} + \left(W^{\mu\nu} - W^{\mu\nu}_{\text{(LO)}} - W^{\mu\nu}_{\text{(NLO)}}\right),$$

etc, where

$W^{\mu\nu}_{\text{(NLO)}} =$ 

$+$ etc. $+$ 

$-$ 

Etc, for NNLO, NNLO, . . .

Conventional viewpoint does get correct $\overline{\mathrm{MS}}$ hard scattering, allowing correct phenomenology

Standard view

$$
\begin{aligned}
\mathrm{d}\sigma_{\text{had}}(x,Q) &= f_{j/h,\,\text{“bare”}} \otimes \mathrm{d}\hat{\sigma}_{j,\text{partonic}} \\
&= f_{j/h,\text{“bare”}} \otimes C \otimes \mathrm{d}\hat{\sigma}_{j,\text{finite}} \\
&= f_{j/h,\text{“ren”}} \otimes \mathrm{d}\hat{\sigma}_{j,\text{finite}}\,.
\end{aligned}
$$

Correctly derived factorization gives

$$
\mathrm{d}\sigma_{\text{had}}(x,Q) = f_{j/h} \otimes \mathrm{d}\hat{\sigma}_{j,\text{hard}}\,.
$$

Calculate $\mathrm{d}\hat{\sigma}_{j,\text{hard}}$ from perturbation theory and factorization for massless partonic cross section:

$$
\begin{aligned}
\mathrm{d}\sigma_{\text{parton }i}(x,Q) &= f_{j/i} \otimes \mathrm{d}\hat{\sigma}_{j,\text{hard}} \\
&= f_{j'/i,\text{bare}} \otimes Z_{j'j,\text{UV}\,\overline{\mathrm{MS}}} \otimes \mathrm{d}\hat{\sigma}_{j,\text{hard}} \\
&= Z_{ij,\text{UV}\,\overline{\mathrm{MS}}} \otimes \mathrm{d}\hat{\sigma}_{j,\text{hard}}
\end{aligned}
$$

(All perturbative integrals in f_{bare} are scale free, and therefore vanish.)

Hard scattering from partonic cross section

From

$$\begin{aligned} \mathrm{d}\sigma_{\text{parton } i} &= f_{j/i} \otimes \mathrm{d}\hat{\sigma}_{j,\text{hard}} \\ &= f_{j'/i,\text{bare}} \otimes Z_{j'j,\text{UV } \overline{\text{MS}}} \otimes \mathrm{d}\hat{\sigma}_{j,\text{hard}} \\ &= Z_{ij,\text{UV } \overline{\text{MS}}} \otimes \mathrm{d}\hat{\sigma}_{j,\text{hard}} \end{aligned}$$

we get

$$\mathrm{d}\hat{\sigma}_{j,\text{hard}} = Z^{-1}_{\text{UV } \overline{\text{MS}}} \otimes \mathrm{d}\sigma_{\text{parton } i}(x, Q)$$

Because of vanishing of scale-free integrals, the UV renormalization factor equals the collinear divergence factor, order by order in massless perturbation theory.

This generalizes to Drell-Yan, etc, all with standard collinear factorization in absence of heavy quarks.

Hence, with $\overline{\text{MS}}$ correctly defined hard scattering factors equal those from the conventional approach. These (and the DGLAP kernels) are the only predicted quantities that are used in standard phenomenology.

Why does the incorrectness of the standard presentation matter?

- Comparison with relevant lattice gauge theory calculations starts with the operator definition of pdfs. If the definition is messed up, everything goes wrong.

- Further developments need correct principles. E.g.,
  - Corrections to MC event generators.
  - Comparison of MC event generators and factorization.
  - Proper treatment of heavy quarks for all values of masses relative to Q.
  - Correct understanding of TMD factorization.
  - Analysis at moderately low Q, especially on nuclear targets.
  - Nuclear effects generally. Cf. Brodsky's work.
  - Target mass effects.
  - Other developments, e.g., jet structure, . . .

- Interfaces to nuclear physics, many body theory, and non-perturbative methods.

- View of our field by outsiders, e.g., condensed matter physicists.

  This and later items apply not just to the specific myth I analyzed in QCD, but quite broadly to the way derivations are made and presented in physics, including in textbooks.

. . .

- Discouragement of students.
- Miseducation of students in multiple directions. Impact of weak reasoning is widespread, beyond my chosen topic, I have found.
- Locating conceptual errors in derivations pinpoints areas of deficient understanding. Some of these can indicated important areas for research.

N.B. Subject is hard!

Exploration of new topic often requires fuzzy thinking, etc. But codification needs to be done much better, especially in mature areas.

APPENDIX

E.g., Ellis et al. NP B152, 285 (1979)

for any inclusive cross section which involves observed individual hadrons, the parton-model description requires a parton cross section for an "observed" individual parton. Schematically, the connection between the parton cross section, $d\sigma^{\text{parton}}$, and the corresponding hadronic cross section $d\sigma$ is as follows [3]:

$$d\sigma(P_i) \approx \int \prod_i f_i(\xi_i)\, d\xi_i d\sigma^{\text{parton}}(p_i)\,, \tag{1.1}$$

[3] R.P. Feynman, Photon-hadron interactions (Benjamin, Reading, MA, 1972).

Notes:

- No justification, other than reference to Feynman, is given.
- In parton model à la Feynman (pre-QCD), partonic $\mathrm{d}\sigma^{\text{parton}}$ is *lowest order* only.
- Feynman did not say anything about a real QFT and certainly not for QCD.
- But Ellis et al. use formula with all orders $\mathrm{d}\sigma^{\text{parton}}$.

E.g., Ellis, Stirling & Webber, "QCD and Collider Physics" (CUP, 1996)

In order to obtain a proton structure function we must convolute the quark structure function $\hat{F}_2$ of Eq. (4.68) with a 'bare' distribution q_0 of a quark in a proton and sum over quark flavours, as we did in Section 4.1 for the naive parton model. This gives

$$F_2(x, Q^2) = x \sum_{q,\bar{q}} e_q^2 \left[q_0(x) + \frac{\alpha_S}{2\pi} \int_x^1 \frac{d\xi}{\xi} q_0(\xi) \left\{ P\left(\frac{x}{\xi}\right) \ln \frac{Q^2}{\kappa^2} + C\left(\frac{x}{\xi}\right) \right\} + \ldots \right], \tag{4.77}$$

Notes:

- They say "we must convolute . . . ", but no reason given.
- κ is IR cut off.

E.g., Dissertori, Knowles & Schmelling, "Quantum Chromodynamics" (OUP, 2003)

This parton cross section is related to the hadron cross section by weighting it by the hadron's p.d.f.s,

$$\mathrm{d}\sigma^{(\ell \mathrm{h})} = \sum_{f=q,\bar{q},g} \int_0^1 \mathrm{d}y \, f_{\mathrm{h}}(y) \mathrm{d}\hat{\sigma}^{(\ell f)} \left(\frac{x}{y} \right) . \tag{3.227}$$

Notes:

- No reason given.