Light-cone PDFs from Lattice QCD

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JLab Theory Seminar

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In collaboration with

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4. Temple University
5. DESY Zeuthen

Based on:

- M. Constantinou, H. Panagopoulos, Phys. Rev. D 96 (2017) 054506, [arXiv:1705.11193]
- C. Alexandrou et al., Nucl. Phys. B 923 (2017) 394 (Frontiers Article), [arXiv:1706.00265]
- C. Alexandrou et al., [arXiv:1803.02685]

OUTLINE OF TALK

A. Introduction to quasi-PDFs

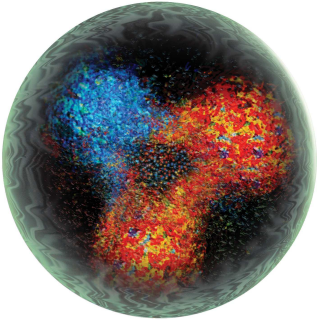
B. quasi-PDFs in Lattice QCD

C. Lattice Matrix Elements

D. Renormalization

E. Towards light-cone PDFs

F. Discussion

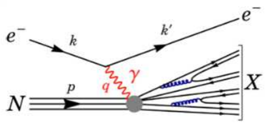


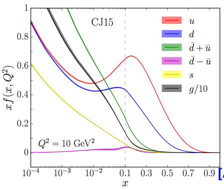
A

Introduction

to quasi-PDFs

Probing Nucleon Structure





CJ15 PDFs

[A. Accardi et al., arXiv:1602.03154]

Parton Distribution Functions

- ★ Universal quantities for the description of the nucleon's structure (non-perturbative nature)
- ★ 1-dimensional picture of nucleon structure
- ★ Distribution functions are necessary for the analysis of Deep inelastic scattering data
- ★ Parametrized in terms of off-forward matrix of light-cone operators
- ★ Not directly accessible in a euclidean lattice

PDFs on the Lattice

★ **Moments of PDFs easily accessible in lattice QCD**

$$f^n = \int_{-1}^{1} dx\, x^n f(x)$$

- **one relies on OPE to reconstruct the PDFs**
- **reconstruction difficult task:**

  $\Rightarrow$ **signal-to-noise is bad for higher moments**
  $\Rightarrow$ **n $>$ 3: operator mixing (unavoidable!)**

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★ **Alternative approaches to access PDFs:**

Purely spatial matrix elements that can be matched to PDFs

- **quasi-PDFs** [X. Ji, arXiv:1305.1539]
- **pseudo-PDFs** [A. Radyushkin, arXiv:1705.01488]
- **good lattice cross-sections** [Y-Q Ma&J. Qiu, arXiv:1709.03018]

B

quasi-PDFs

in Lattice QCD

Access of PDFs on a Euclidean Lattice

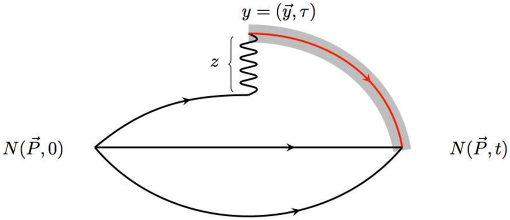
Novel direct approach: [X.Ji, Phys. Rev. Lett. 110 (2013) 262002, arXiv:1305.1539]

★ **computation of quasi-PDF:**
matrix elements (ME) of spatial operators

$$\tilde{q}(x,\mu^2,P_3) = \int \frac{dz}{4\pi} e^{-i\,x\,P_3\,z} \langle N(P_3)|\bar{\Psi}(z)\,\gamma^z\,\mathcal{A}(z,0)\Psi(0)|N(P_3)\rangle_{\mu^2}$$

- $\mathcal{A}(z,0)$: **Wilson line** ($0 \to z$)
- z: **distance in any spatial direction**

★ **Nucleon is boosted with momentum in spatial direction** (z)



PDFs on the Lattice

Contact with light-cone PDFs:

- ★ **Difference between quasi-PDFs and light-cone PDFs:**
  $\mathcal{O}\left(\frac{\Lambda^2_{\rm QCD}}{P_3^2}, \frac{m_N^2}{P_3^2}\right)$

- ★ **Matching procedure in large momentum EFT (LaMET) to relate quasi-PDFs to light-cone PDFs**
  (provided that momenta are finite but feasibly large for lattice)

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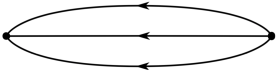
Exploratory studies of various aspects are maturing:

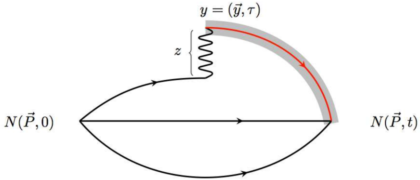
[X. Xiong et al., arXiv:1310.7471], [H-W. Lin et al., arXiv:1402.1462], [Y. Ma et al., arXiv:1404.6860],
[Y.-Q. Ma et al., arXiv:1412.2688], [C. Alexandrou et al., arXiv:1504.07455], [H.-N. Li et al., arXiv:1602.07575],
[J.-W. Chen et al., arXiv:1603.06664], [J.-W. Chen et al., arXiv:1609.08102], [T. Ishikawa et al., arXiv:1609.02018],
[C. Alexandrou et al., arXiv:1610.03689], [C. Monahan et al., arXiv:1612.01584], [A. Radyushkin et al., arXiv:1702.01726],
[C. Carlson et al., arXiv:1702.05775], [R. Briceno et al., arXiv:1703.06072], [M. Constantinou et al., arXiv:1705.11193],
[C. Alexandrou et al., arXiv:1706.00265], [J-W Chen et al., arXiv:1706.01295], [X. Ji et al., arXiv:1706.08962],
[K. Orginos et al., arXiv:1706.05373], [T. Ishikawa et al., arXiv:1707.03107], [J. Green et al., arXiv:1707.07152],
[Y-Q Ma et al., arXiv:1709.03018], [I. Stewart et al, arXiv:1709.04933], [J. Karpie et al., arXiv:1710.08288,
[J-W Chen et al., arXiv:1711.07858], [C.Alexandrou et al., arXiv:1710.06408], [T. Izubuchi et al., arXiv:1801.03917],
[C.Alexandrou et al., arXiv:1803.02685], [J-W Chen et al, arXiv:1803.04393], · · ·

Calculation of nucleon matrix elements

A multi-component task:

1. **Calculation of 2pt- and 3pt-correlators (C^{2pt}, C^{3pt}) dependence on : length of Wilson line z and nucleon momentum P**





$$C^{3pt} : \langle N | \overline{\psi}(z) \Gamma \mathcal{A}(z, 0) \psi(0) | N \rangle$$

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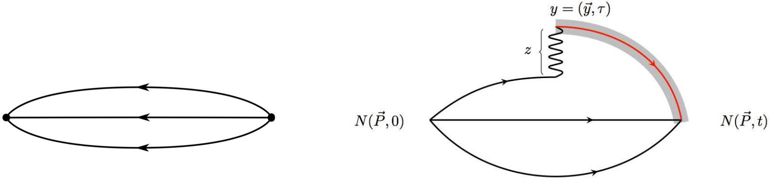

$$C^{3pt} : \langle N|\overline{\psi}(z)\Gamma\mathcal{A}(z,0)\psi(0)|N\rangle$$

2. **Construction of ratios at zero momentum transfer**

$$\frac{C^{3pt}(t,\tau,0,\vec{P})}{C^{2pt}(t,0,\vec{P})} \overset{0<<\tau<<t}{=} h_0(P_3, z)$$

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4. **Fourier transform to momentum space (x)**

$$\tilde{q}(x,\mu^2,P_3) = \int \frac{dz}{4\pi}\, e^{ixP_3 z} \langle N|\overline{\psi}(z)\Gamma\mathcal{A}(z,0)\psi(0)|N\rangle$$

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5. **Matching to light-cone PDFs (LaMET)**

$$q(x,\mu) = \int_{-\infty}^{\infty} \frac{d\xi}{|\xi|}\, C\left(\xi, \frac{\mu}{xP_3}\right) \tilde{q}\left(\frac{x}{\xi}, \mu, P_3\right)$$

C: **matching kernel**

Calculation of nucleon matrix elements

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C**: matching kernel**

6. **Target mass corrections**
   elimination of residual m_N/P_3 dependence

C

Lattice

Matrix Elements

Parameters of Calculation

[C. Alexandrou et al. (ETMC), arXiv:1803.02685]

- ★ $N_f=2$ **Twister Mass fermion action with clover term**
- ★ **Ensemble parameters:**

$\beta=2.10$,	$c_{\rm SW}=1.57751$,	$a=0.0938(3)(2)$ fm
$48^3 \times 96$	$a\mu = 0.0009$	$m_N = 0.932(4)$ GeV
$L = 4.5$ fm	$m_\pi = 0.1304(4)$ GeV	$m_\pi L = 2.98(1)$

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★ **Nucleon Momentum & Measurements**

$P = \frac{6\pi}{L}$ (0.83 GeV)			$P = \frac{8\pi}{L}$ (1.11 GeV)			$P = \frac{10\pi}{L}$ (1.38 GeV)		
Ins.	$N_{\rm conf}$	$N_{\rm meas}$	Ins.	$N_{\rm conf}$	$N_{\rm meas}$	Ins.	$N_{\rm conf}$	$N_{\rm meas}$
γ_3	100	9600	γ_3	425	38250	γ_3	655	58950
γ_0	50	4800	γ_0	425	38250	γ_0	655	58950
$\gamma_5\gamma_3$	65	6240	$\gamma_5\gamma_3$	425	38250	$\gamma_5\gamma_3$	655	58950

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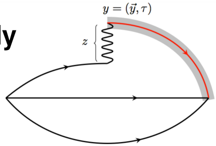
- ★ **Excited states investigation:**

$T_{\rm sink}/a = 8,\ 10,\ 12\ (T_{\rm sink} = 0.75,\ 0.094,\ 1.13{\rm fm})$

Set up of calculation

Signal-to-noise problem must be tamed to reliably investigate systematic uncertainties



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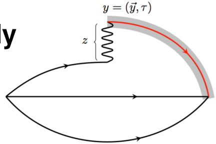

★ Statistics:

- **6 directions of Wilson line:** $\pm x, \pm y, \pm z$
  with momentum boosted in same direction
- **16 source positions using (CAA):**
  $\Rightarrow$ **1 high precision (HP) inversion**
  $\Rightarrow$ **16 low precision (LP) inversions**
  [E. Shintani et al., Phys. Rev. D91, 114511 (2015)]

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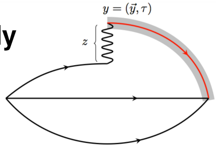

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★ Signal improvement:

- **Stout Smearing: 0, 5, 10, 15, 20 steps**
- **Momentum smearing: tuning for each momentum** P

Systematic uncertainties

Laborious effort to eliminate uncertainties

- Cut-off Effects due to finite lattice spacing
- Finite Volume Effects
- Contamination from other hadron states
- Chiral extrapolation for unphysical pion mass
- Renormalization and mixing

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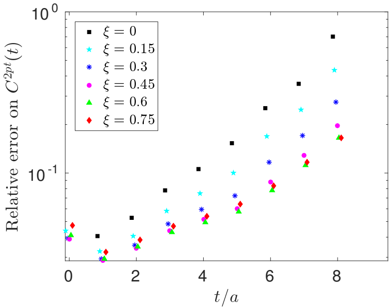
Discussed in this talk

Reduction of Noise-to-signal ratio

$$S_{\rm mom}[\psi(x)] = \frac{1}{1+6\alpha}\left(\psi(x) + \alpha\sum_{j=\pm1}^{\pm3} U_j(x)e^{i\xi\hat{j}}\psi(x+\hat{j})\right)$$

[G. Bali et al., Phys. Rev. D93, 094515 (2016)]



★ **Momentum smearing helps reach higher momenta**

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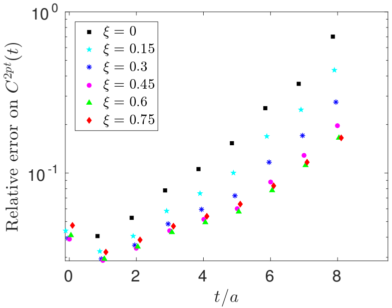

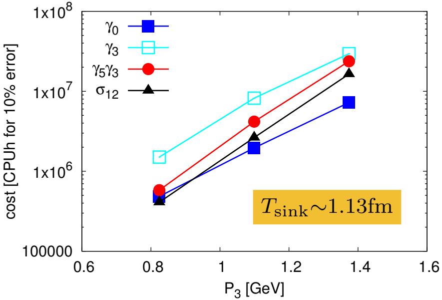

- ★ **Momentum smearing helps reach higher momenta**
- ★ **BUT: limitations in max momentum due to comput. cost**

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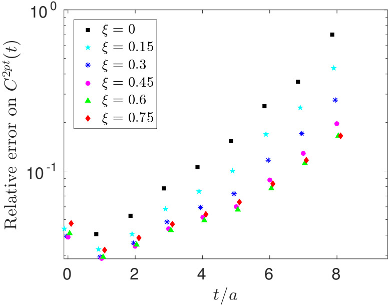
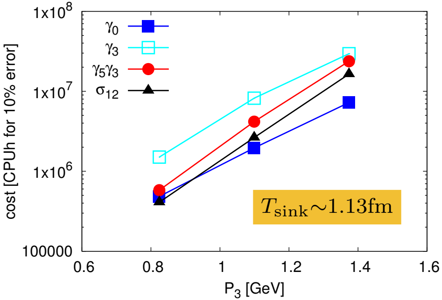




★ **Momentum smearing helps reach higher momenta**

★ **BUT: limitations in max momentum due to comput. cost**

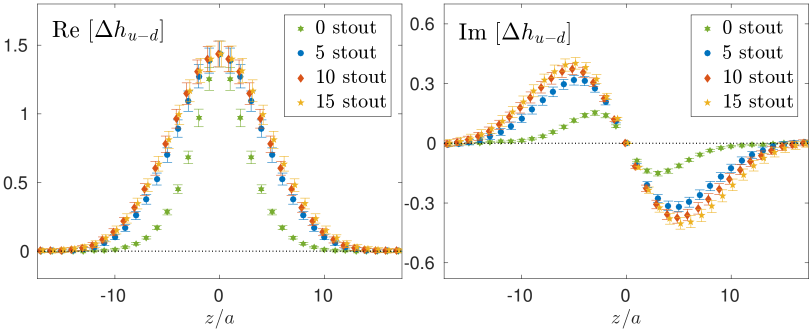
Conclusion:

Reliable results ($T_{\rm sink}$>1fm) limit the momentum we can reach

Reduction of Noise-to-signal ratio

★ **Smearing improves the signal-to-noise ratio**

$P=6\pi/L$



★ **Smearing suppresses linear divergence**

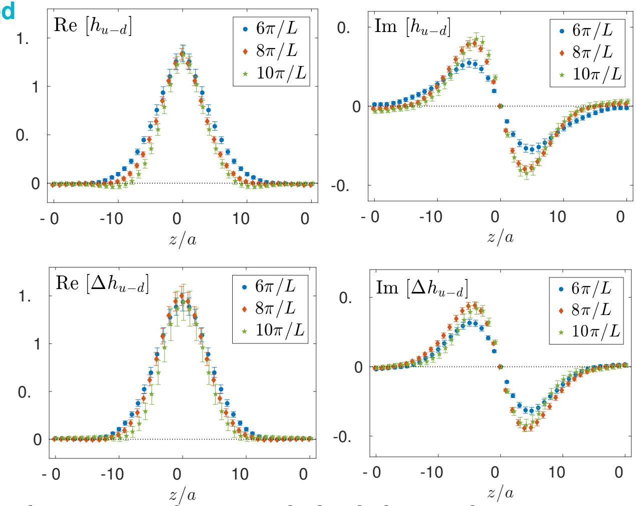
★ **Application of stout smearing with 0, 5, 10, 15, 20 steps**

Bare Nucleon Matrix Elements

[C. Alexandrou et al. (ETMC), arXiv:1710.06408] [C. Alexandrou et al. (ETMC), arXiv:1803.02685]

Unpolarized

Polarized

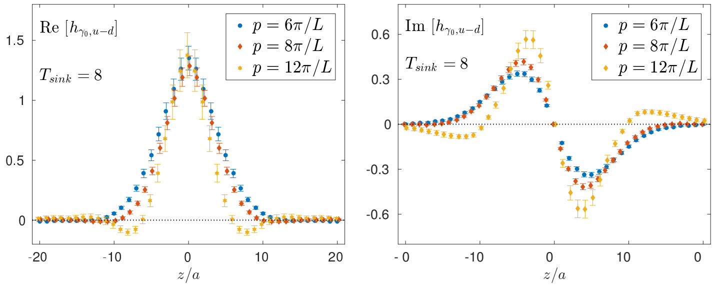


★ **Addressing systematic uncertainties is imperative**

Challenges of Calculation

Excited States

$T_{\text{sink}} \sim 0.75\text{fm}$

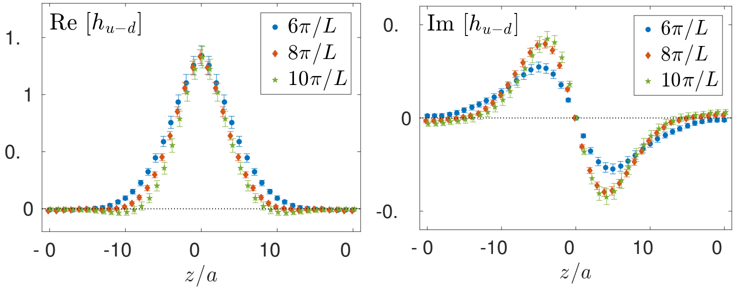


★ **Excited states contamination are worse for large momenta**

Challenges of Calculation

Excited States

$T_{\text{sink}} \sim 1.13\text{fm}$



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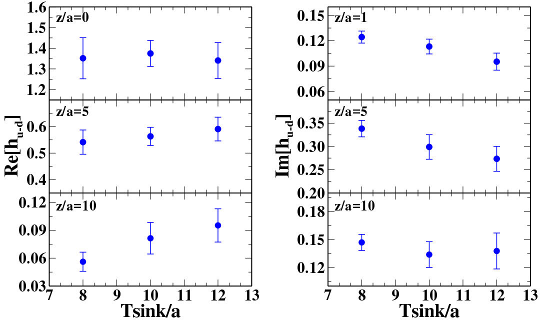
Challenges of Calculation

Excited States

$P = 0.83\text{GeV}$



- ★ Non-predictable behavior for all regions of z
- ★ Real and imaginary part of ME affected differently

Challenges of Calculation

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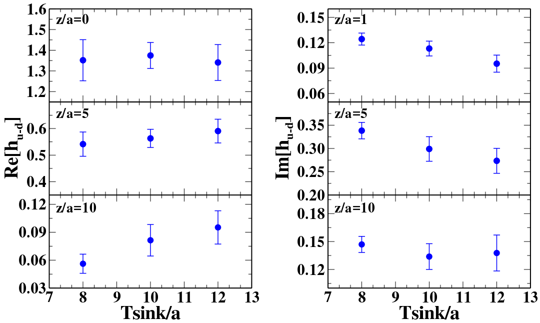


★ **Non-predictable behavior for all regions of z**

★ **Real and imaginary part of ME affected differently**

Conclusion: **Reliable results require $T_{\text{sink}} > 1\text{fm}$**

Systematic uncertainties in a nutshell

- ★ **Excited states uncontrolled for source-sink separations below 1fm**
- ★ **Excited states contamination worse for large momenta**
- ★ **Exponential signal-to-noise problem difficult to tackle**
- ★ **2-state fit and summation method: alternative analysis techniques**

similar accuracy between different source-sink separations vital to eliminate bias from the small $T_{\rm sink}$ values

D

Renormalization of quasi-PDFs

Renormalization

Critical part of calculation

- ★ elimination of power and logarithmic divergences and dependence on regulator
- ★ identification and elimination of mixing
- ★ Comparison with phenomenology becomes a real possibility

M. Constantinou, H. Panagopoulos, Phys. Rev. D 96 (2017) 054506, [arXiv:1705.11193]

C. Alexandrou, et al., Nucl. Phys. B 923 (2017) 394 (Frontier Article), [arXiv:1706.00265]

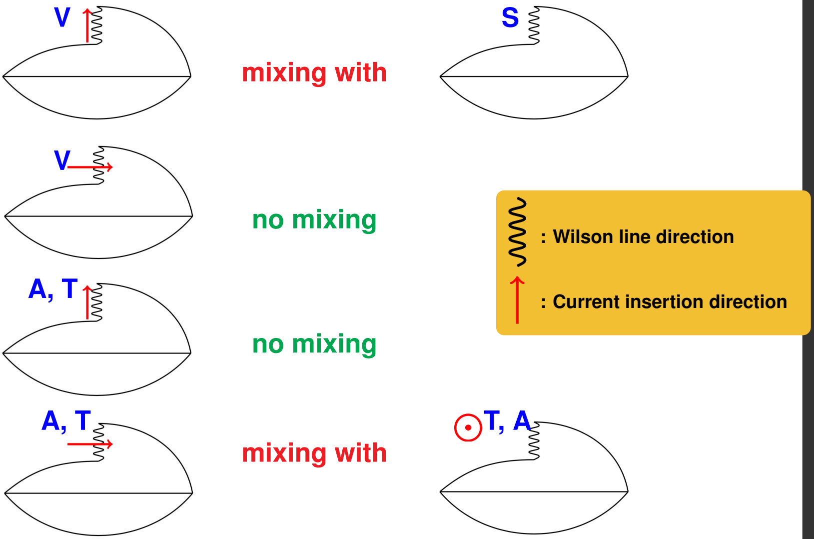
J. Chen, et al., Phys. Rev. D 97, (2018) 014505, [arXiv:1706.01295]

Renormalization scheme

- ★ RI′-type
- ★ Use 1-loop conversion factor to convert to the $\overline{\text{MS}}$ at 2 GeV
- ★ Also applicable for cases of mixing

Mixing pattern (based on PT)

Depends on the relation between the current & Wilson line direction



Non-perturbative Renormalization

★ **same divergence in vertex function and nucleon ME**

No mixing: helicity, transversity, unpolarized (γ_0)

$$Z_{\mathcal{O}}(z) = \frac{Z_q}{\mathcal{V}_{\mathcal{O}}(z)}\,, \qquad \mathcal{V}_{\mathcal{O}} = \frac{\mathrm{Tr}}{12}\left[\mathcal{V}(p)\left(\mathcal{V}^{\mathrm{Born}}(p)\right)^{-1}\right]\Big|_{p=\bar{\mu}}$$

★ Z_q: **fermion field renormalization**

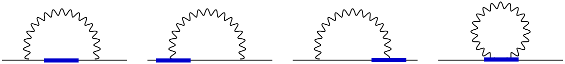
★ $Z_{\mathcal{O}}$ **includes the linear divergence**

Mixing: Unpolarized (γ_3)

$$\begin{pmatrix} \mathcal{O}_V^R(P_3,z) \\ \mathcal{O}_S^R(P_3,z) \end{pmatrix} = \hat{Z}(z)\cdot\begin{pmatrix} \mathcal{O}_V(P_3,z) \\ \mathcal{O}_S(P_3,z) \end{pmatrix}, \qquad Z_q^{-1}\,\hat{Z}(z)\,\hat{\mathcal{V}}(p,z)\Big|_{p=\bar{\mu}} = \hat{1}$$

$$h_V^R(P_3,z) = Z_{VV}(z)\; h_V(P_3,z) + Z_{VS}(z)\; h_S(P_3,z)$$

Conversion to $\overline{\text{MS}}$ - Evolution to 2GeV

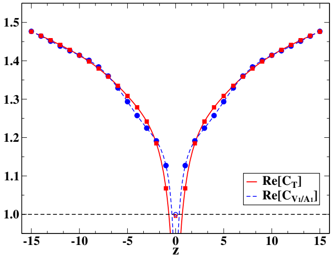


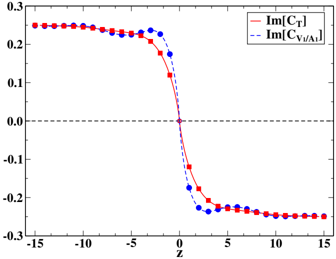
★ **1-loop perturbative calculation in Dimensional Regularization**

★ **Evaluation of conversion factor to $\overline{\text{MS}}$**

★ **Conversion factor: a complex function**

★ **Necessary ingredient for non-perturbative renormalization**

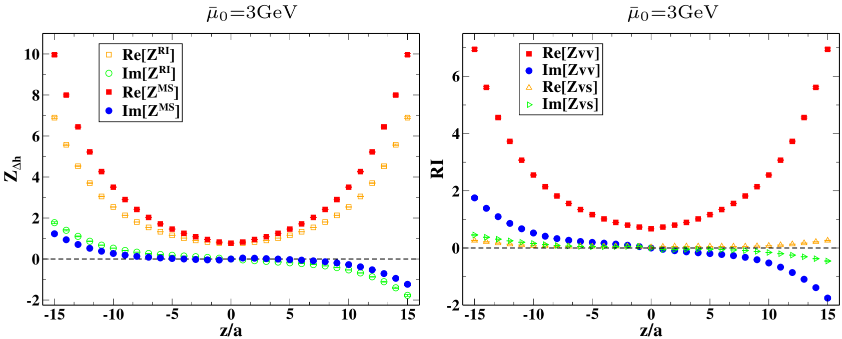




Numerical Results

★ **Twisted Mass fermions,** m_π=375MeV, $32^3 \times 64$, **HYP smearing**

★ **Conversion** $\&$**Evolution to** $\overline{\text{MS}}$**(2GeV)** **(Perturbatively)**



Plot from [C. Alexandrou at al., arXiv:1706.00265]

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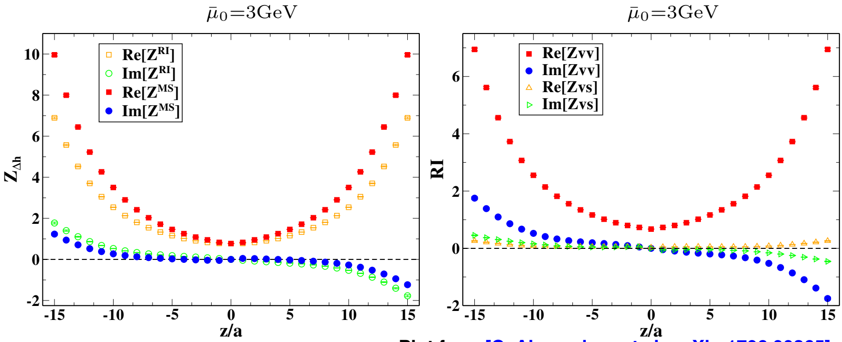

Plot from [C. Alexandrou at al., arXiv:1706.00265]

- ★ **Z-factors are complex functions**
- ★ $Im[Z_{\mathcal{O}}^{\overline{\mathrm{MS}}}] < Im[Z_{\mathcal{O}}^{RI'}]$ **(expected from pert. theory)**

Systematic uncertainties

Ultimate goal: Reliability in final estimates

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Systematic uncertainties need to be addresses

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Systematic uncertainties need to be addresses

- ★ **Upper bounds estimated in** [C. Alexandrou et al., arXiv:1706.00265]
- ★ **Both the ME and Z-factors are complex functions,**
  in absence of mixing, e.g. unpolarized with γ_0 ($h \equiv h_{u-d}$):

$$
\begin{aligned}
h^{ren} = Z_h\, h \quad &= \quad Re[Z_h]\, Re[h] - Im[Z_h]\, Im[h] \\
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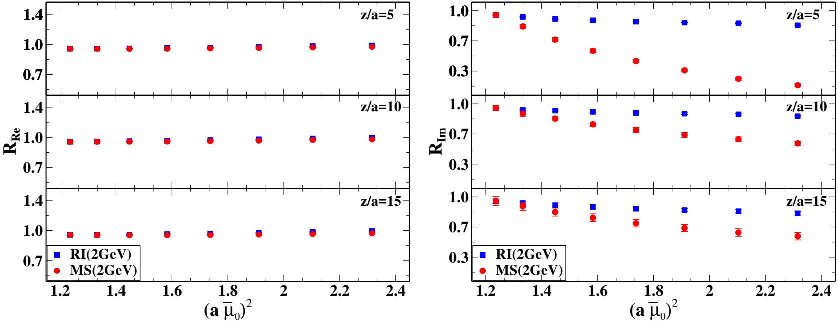
★ **Uncertainties in Z-factors may have important implications on the final estimates for PDFs**

Systematic uncertainties

Truncation effects in C:

$$R^{\mathrm{RI}'\,(\overline{\mathrm{MS}})}_{\mathrm{Re}\,(\mathrm{Im})}(z,\bar\mu_0,\bar\mu_0';\bar\mu) \equiv \frac{Z^{\mathrm{RI}'\,(\overline{\mathrm{MS}})}_{\mathrm{Re}\,(\mathrm{Im})}(z,\bar\mu_0;\bar\mu)}{Z^{\mathrm{RI}'\,(\overline{\mathrm{MS}})}_{\mathrm{Re}\,(\mathrm{Im})}(z,\bar\mu_0';\bar\mu)}, \qquad (\bar\mu_0'=2.67\mathrm{GeV})$$

Evolution to 2 GeV in RI' and $\overline{\mathrm{MS}}$ schemes:
slope in R reveals truncation effect in conversion factor



★ **Effect in Real part:** $\sim 2\%$

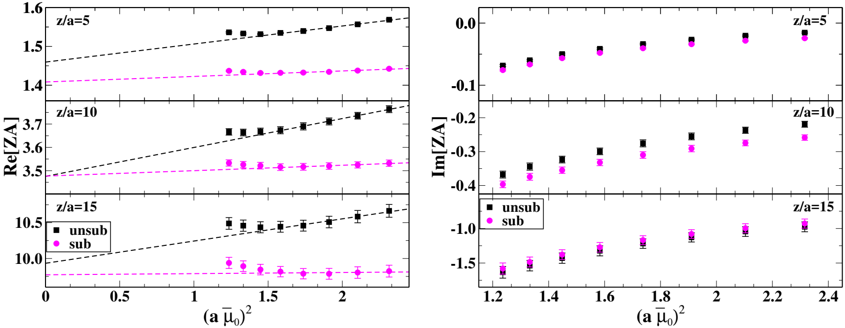
★ **Effect in Imaginary part:** $\sim 100\%$ $\quad$ ($Im[Z^{\overline{\mathrm{MS}}}]=0$ in Dim. Regul.)

Refining Renormalization

★ **Improvement Technique:**

- Computation of 1-loop lattice artifacts to $\mathcal{O}(g^2\, a^\infty)$
- Subtraction of lattice artifacts from non-perturbative estimated

★ **Application to the quasi-PDFs:** PRELIMINARY



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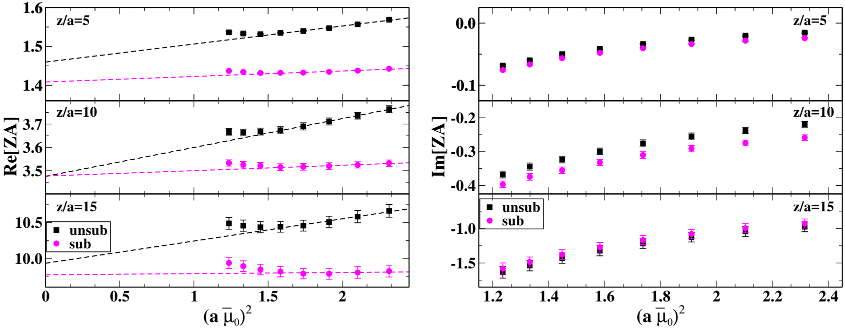


★ **Real part significantly improved**

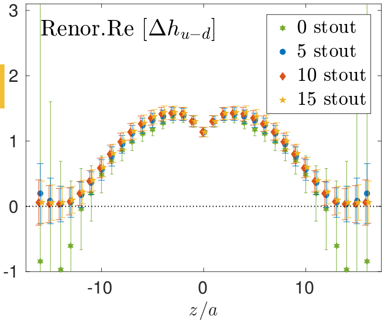
★ **Mild change in imaginary part (expected to change with smearing)**

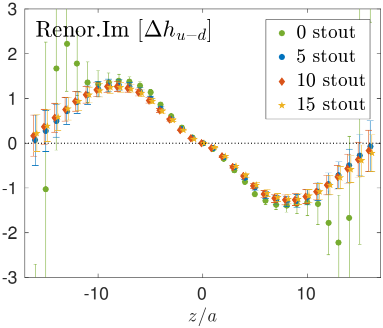
- Behavior might be a consequence of absence of smearing in pert. calculation

Renormalized Matrix Elements

★ **Renormalized ME must be independent of stouts steps**

$P_3 = \frac{6\pi}{L}$





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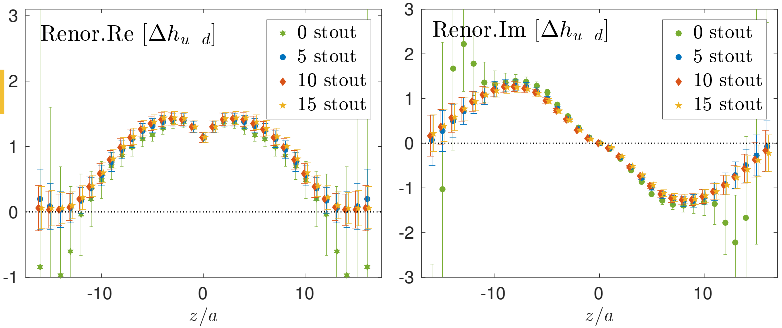


★ **Renormalized ME with and without smearing are compatible**

★ **Absence of stout smearing leads to increased noise**

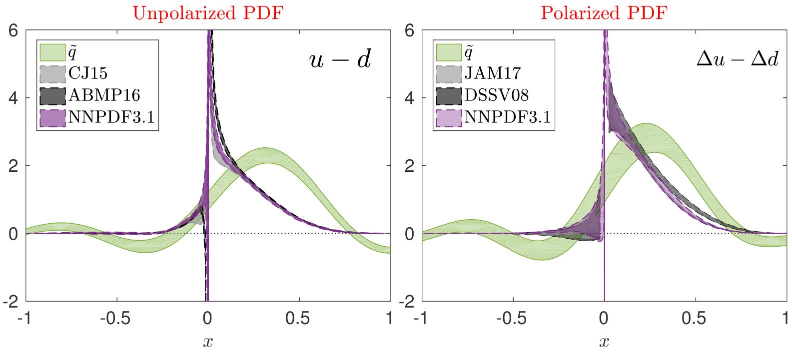
E

Towards light-cone PDFs

Towards light-cone PDFs

Upon Fourier Transform of renormarmalized matrix elements

$P_3 = 1.4\text{GeV}$



Towards light-cone PDFs

Upon matching of quasi-PDFs

$$q(x,\mu) = \int_{-\infty}^{\infty} \frac{d\xi}{|\xi|}\, C\left(\xi, \frac{\mu}{xP_3}\right) \tilde{q}\left(\frac{x}{\xi}, \mu, P_3\right)$$

$$C\left(\xi, \frac{\xi\mu}{xP_3}\right) = \delta(1-\xi) + \frac{\alpha_s}{2\pi} C_F \begin{cases} \left[\frac{1+\xi^2}{1-\xi} \ln \frac{\xi}{\xi-1} + 1 + \frac{3}{2\xi}\right]_+ & \xi>1, \\ \left[\frac{1+\xi^2}{1-\xi} \ln \frac{x^2 P_3^2}{\xi^2\mu^2} \left(4\xi(1-\xi)\right) - \frac{\xi(1+\xi)}{1-\xi} + 2\iota(1-\xi)\right]_+ & 0<\xi<1, \\ \left[-\frac{1+\xi^2}{1-\xi} \ln \frac{\xi}{\xi-1} - 1 + \frac{3}{2(1-\xi)}\right]_+ & \xi<0, \end{cases}$$

[J.W. Chen et al., Nucl. Phys. B911 (2016) 246, arXiv:1603.06664]

$\gamma_0 : \iota=0, \quad \gamma_3/\gamma_5\gamma_3 : \iota=1$

Prescription at $\xi=1$:

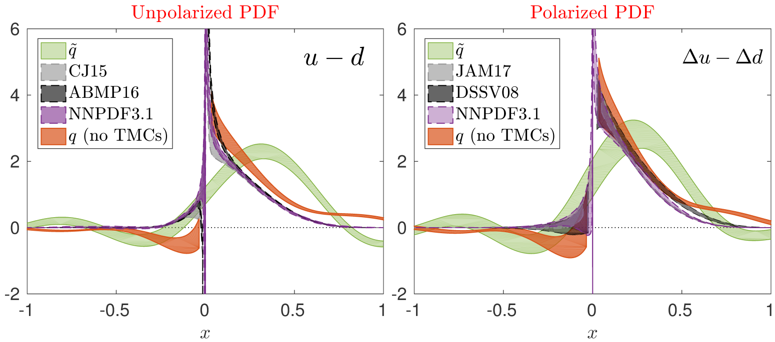
$$\int \frac{d\xi}{|\xi|} \left[C\left(\xi, \frac{\xi\mu}{xP_3}\right)\right]_+ \tilde{q}\left(\frac{x}{\xi}\right) = \int \frac{d\xi}{|\xi|} C\left(\xi, \frac{\xi\mu}{xP_3}\right) \tilde{q}\left(\frac{x}{\xi}\right) - \tilde{q}\left(x\right) \int d\xi C\left(\xi, \frac{\mu}{xP_3}\right)$$

[C. Alexandrou et al. (ETMC), arXiv:1803.02685]

Towards light-cone PDFs

Upon matching of quasi-PDFs

$P_3 = 1.4\text{GeV}$



- ★ **Matched quasi-PDFs have similar behavior with the phenomenological curves**
- ★ **Last piece missing: target mass corrections (TMC)**

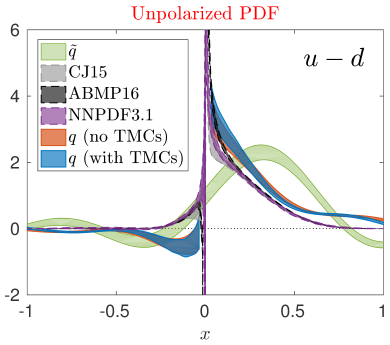
Towards light-cone PDFs

Upon target mass corrections

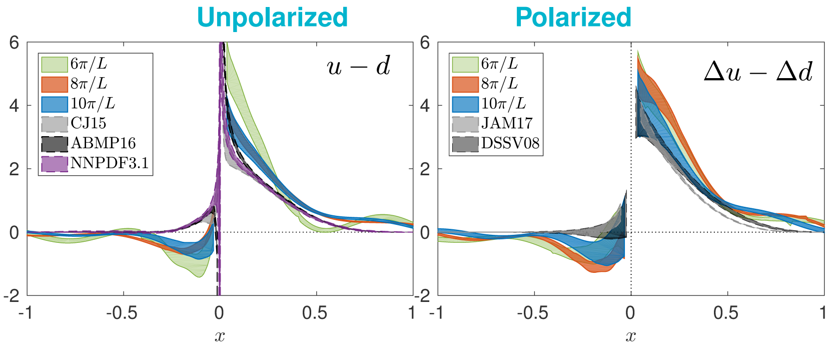
- ★ **Finite nucleon momentum** $\Rightarrow$
- ★ **Correction is necessary for** $m_N/P_3 \neq 0$ **(particle number is conserved)**

[J.W. Chen et al., Nucl. Phys. B911 (2016) 246, arXiv:1603.06664]

$P_3 = 1.4\text{GeV}$



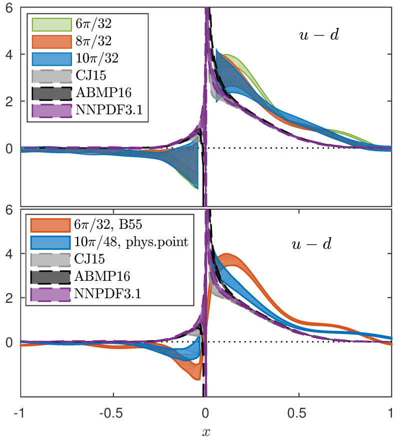
Towards light-cone PDFs



- ★ **Increasing momentum approaches the phenomenological fits a saturation of PDFs for** $p{=}8\pi/L$ **and** $p{=}10\pi/L$
- ★ $0<x<0.5$: **Lattice polarized PDF overlap with phenomenology**
- ★ **Negative** x **region: anti-quark contribution**

Pion Mass Dependence for quasi-PDFs

★ **Simulations at physical m_π crucial for above conclusions**



[C. Alexandrou et al. (ETMC), arXiv:1803.02685]

★ **Large pion mass ensembles: Lattice data saturate away from phenomenological curves**

F

Discussion

DISCUSSION

Great progress over the last years:

- ★ **Simulations at the physical point**
- ★ **unpolarized operator that avoid mixing (γ_0)**
- ★ **Development of non-perturbative renormalization**
- ★ **Improving matching to light-cone PDFs**

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Further investigations:

Careful assessment of systematic uncertainties

- ★ Volume effects
- ★ quenching effect (strange and charm)
- ★ continuum limit

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Dedicated effort from community

Lattice PDF Workshop, 6–8 April 2018

BROOKHAVEN NATIONAL LABORATORY · DESY · MARYLAND CENTER for Fundamental Physics · NIC · UNIVERSITY OF MARYLAND DEPARTMENT OF PHYSICS

THANK YOU

U.S. DEPARTMENT OF ENERGY | Office of Science

TMD Topical Collaboration

NSF

Grant No. PHY-1714407

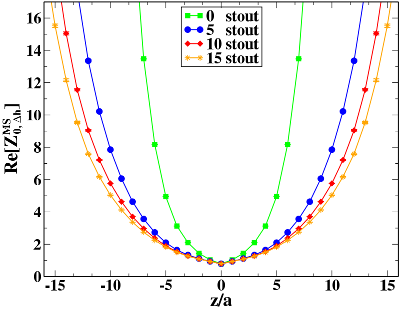
BACKUP SLIDES

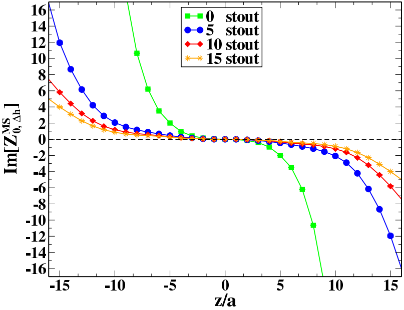
Numerical Results

★ **Computation on a variety of scales**

★ **Conversion &Evolution to $\overline{\mathrm{MS}}$(2GeV)** **(Perturbatively)**

★ **Extrapolation to eliminate residual dependence on initial scale**





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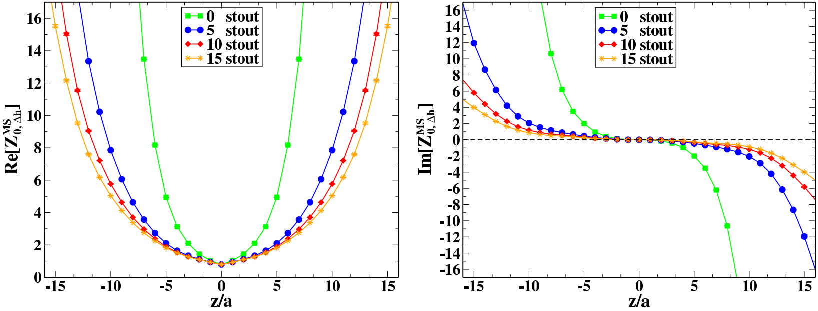


- ★ **Z-factors are complex functions**
- ★ **Increasing stout steps reduces renormalization**

Standard vs. derivative Fourier transform

Standard Fourier transform defining qPDFs:

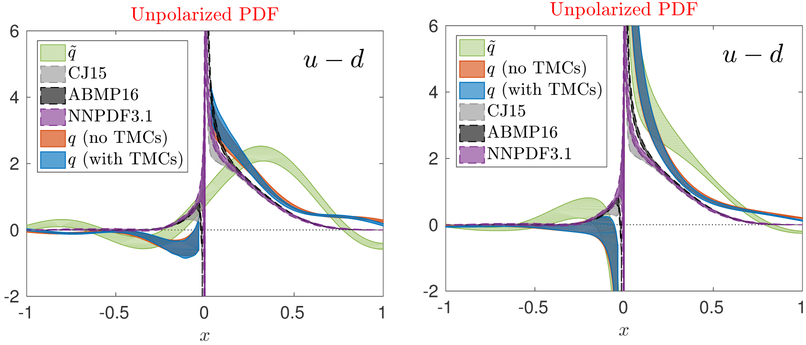
$$\tilde{q}(x) = 2P_3 \int_{-z_{\max}}^{z_{\max}} \frac{dz}{4\pi} e^{ixzP_3} h(z)$$

can be rewritten using integration by parts as:

$$\tilde{q}(x) = h(z) \frac{e^{ixzP_3}}{2\pi i x}\Big|_{-z_{\max}}^{z_{\max}} - \int_{-z_{\max}}^{z_{\max}} \frac{dz}{2\pi} \frac{e^{ixzP_3}}{ix} h'(z)$$

[H.W. Lin et al., arXiv:1708.05301]

Truncation $h(|z| \geq z_{\max})=0$: **equivalent to neglecting surface term**



Oscillations reduced, but small-x not well-behaved