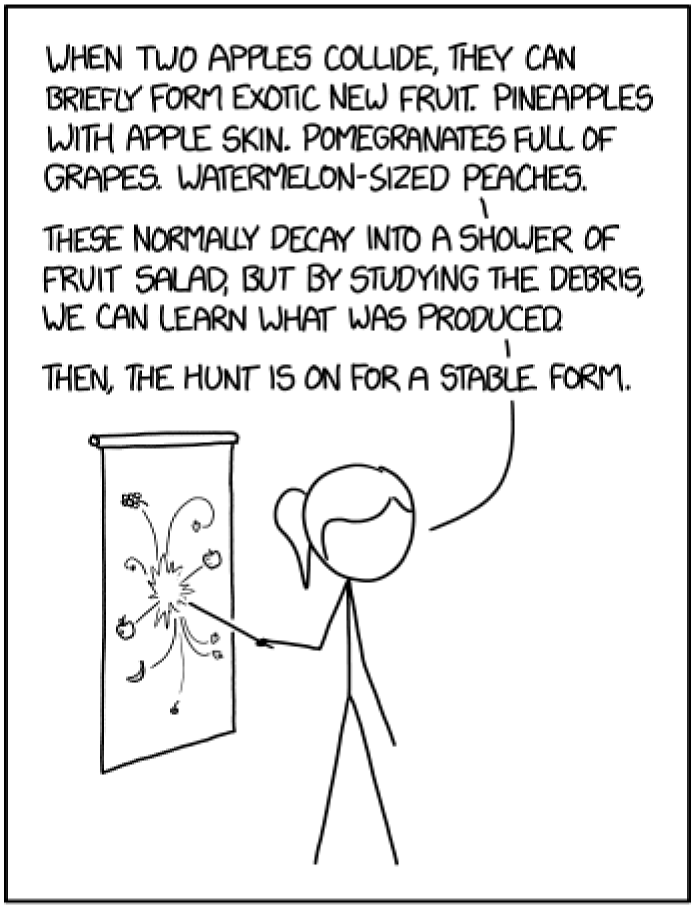
Building the Dynamical Diquark Model for Exotic Hadrons



HOW NEW TYPES OF FRUIT ARE DEVELOPED

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June, 2018

Outline

1) Introduction: The exotics zoo in 2018

2) Diquarks as hadronic components

3) The dynamical diquark picture

4) Extended hadrons in the Born-Oppenheimer approximation

5) Exotics spectroscopy using B-O potentials

6) The future: Building realistic B-O based models

The Exotics Zoo

- Our textbooks still (for the most part) tell us that hadrons only appear in two species: $q\bar{q}$ mesons and qqq baryons
- But so many other types of color-singlet compound hadrons, the so-called exotics, are possible:
- gg, ggg, $\cdots$ (*glueball*)
- $q\bar{q}g$, $q\bar{q}gg$, $\cdots$ (*hybrid meson*)
- $q\bar{q}q\bar{q}$, $q\bar{q}q\bar{q}q\bar{q}$, $\cdots$ (*tetraquark*, *hexaquark*, …)
- $qqqq\bar{q}$, $qqqqqq\bar{q}$, $\cdots$ (*pentaquark*, *octoquark*, …)
- $qqqqqq$, $\cdots$ (*dibaryon*, …)
- Some of these were already suggested by Gell-Mann and Zweig in their original 1964 quark model papers!

Signs and Portents
Where are the light-quark exotics?

- The 0^{++} mesons $f_0(980)$ and $a_0(980)$ are widely (*not* universally) believed to be $s\bar{s}q\bar{q}$ tetraquarks (or, if you like, $K\bar{K}$ molecules)
- The mesons $\pi_1(1400)$ and $\pi_1(1600)$ appear to have non-$q\bar{q}$ $J^{PC} = 1^{-+}$ quantum numbers
- The baryon resonance $\Lambda(1405)$ is suspected to have a large pentaquark (or KN molecular) component
- Other more recent suspects are appearing at the NN threshold, in ϕN processes, *etc*.
- And who can forget the 2002-2005 rise and fall of the $\Theta^+(1535)$ pentaquark?

The Fundamental Problem with Light-Quark Exotics

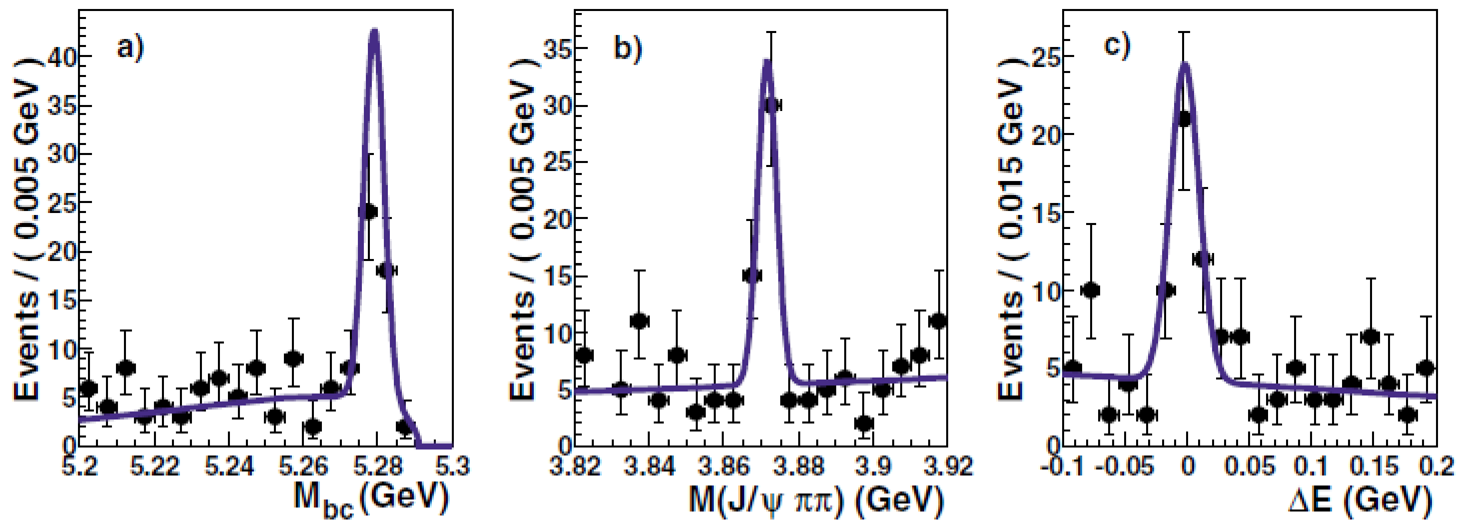
$$\Lambda_{\text{QCD}} \gtrsim m_s \gg m_{u,d}$$

- In other words, it is not always easy to tell whether a $q\bar{q}$ pair ($q = u, d$, even sometimes s) is a sea-quark or valence pair
- This ambiguity is greatly diminished for $c\bar{c}$ or $b\bar{b}$ pairs
- It is the ultimate reason that quark potential models (*e.g.*, the Cornell model) work well in the heavy-quark sector
- To get ironclad evidence for the existence of exotic hadrons, the clearest path is to look for heavy-quark exotics

Modern Exotics Studies Begin in 2003

The Belle Collaboration:
Evidence for a new particle at mass 3872 MeV

S.K. Choi *et al.*, Phys. Rev. Lett. **91** (2003) 262001

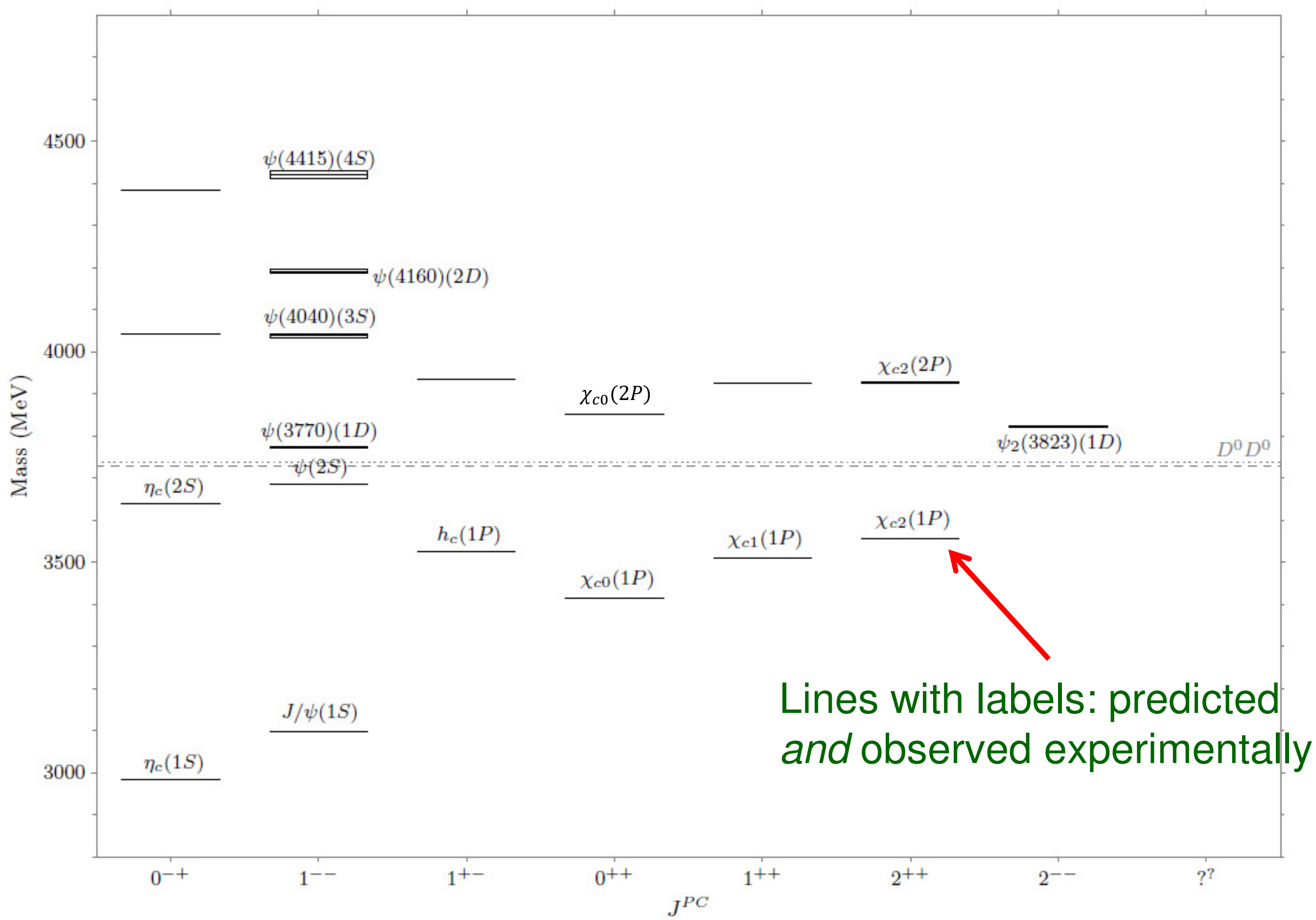


X = Unknown

- Belle found a new **charmoniumlike** resonance appearing in
$$B \to K\,(J/\psi\,\pi^+\pi^-)$$
  - In the same mass range as charmonium,
and it always decays into a final state containing $c\bar{c}$
- Has been confirmed at BABAR, CDF, DØ, LHCb, CMS, COMPASS
- $J^{PC} = 1^{++}$, but not believed to be ordinary $c\bar{c}$:
Mass is many 10's of MeV below the nearest $\bar{c}c$ candidate with these quantum numbers, $\chi_{c1}(2P)$
- Now called $\boldsymbol{X(3872)}$ [and believed to be a $(c\bar{c}u\bar{u})$ state]
  - $m_{X(3872)} = 3871.69 \pm 0.17\ \mathrm{MeV}$
  - Note: $m_{X(3872)} - m_{D^{*0}} - m_{D^0} = -0.01 \pm 0.18\ \mathrm{MeV}$
Leads to endless speculation that *X*(3872) is a $D^0\bar{D}^{*0}$ hadronic molecule
  - Width: $\Gamma_{X(3872)} < 1.2\ \mathrm{MeV}$

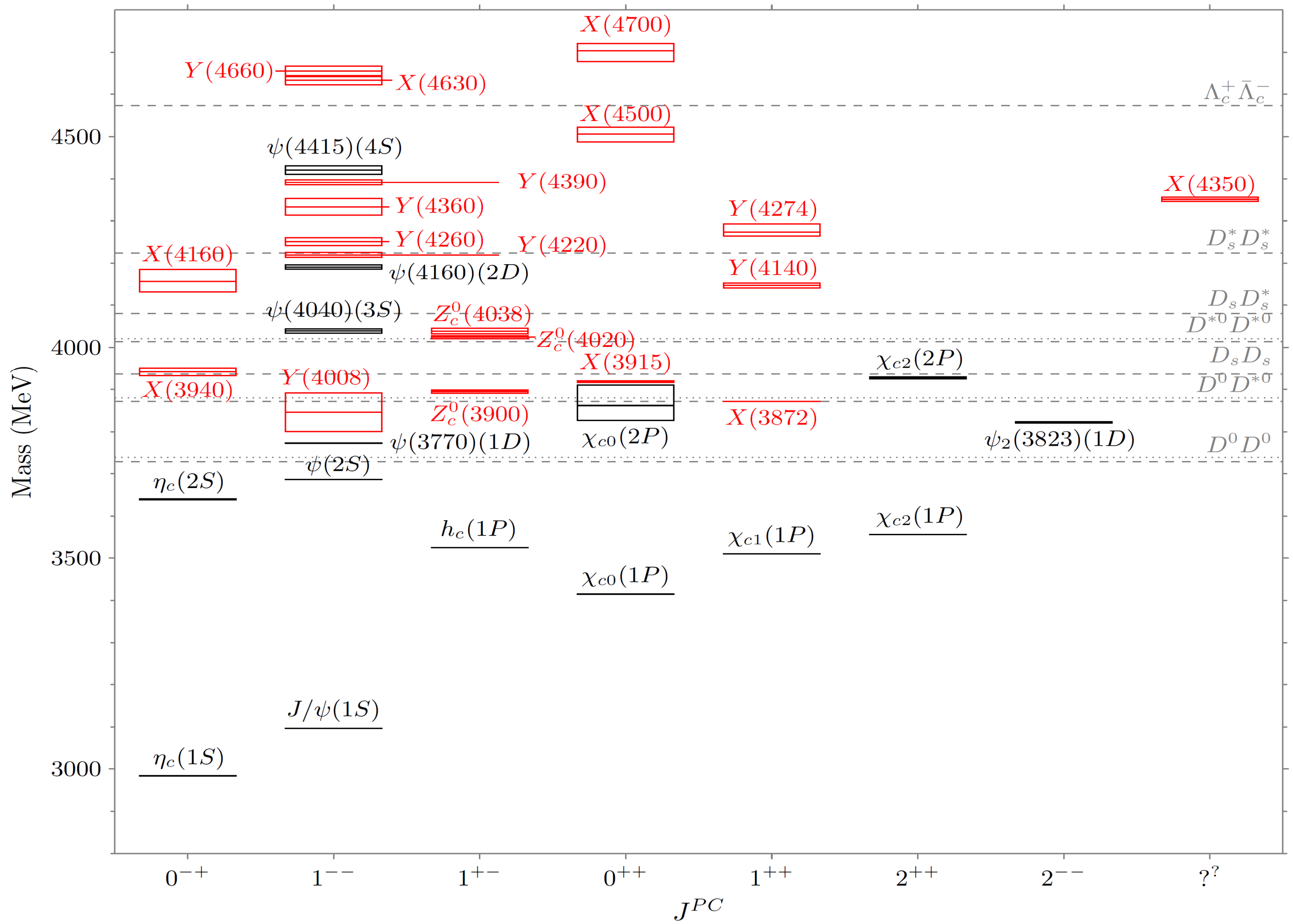
What the Charmonium System Should Look Like

(as predicted from quark potential models)

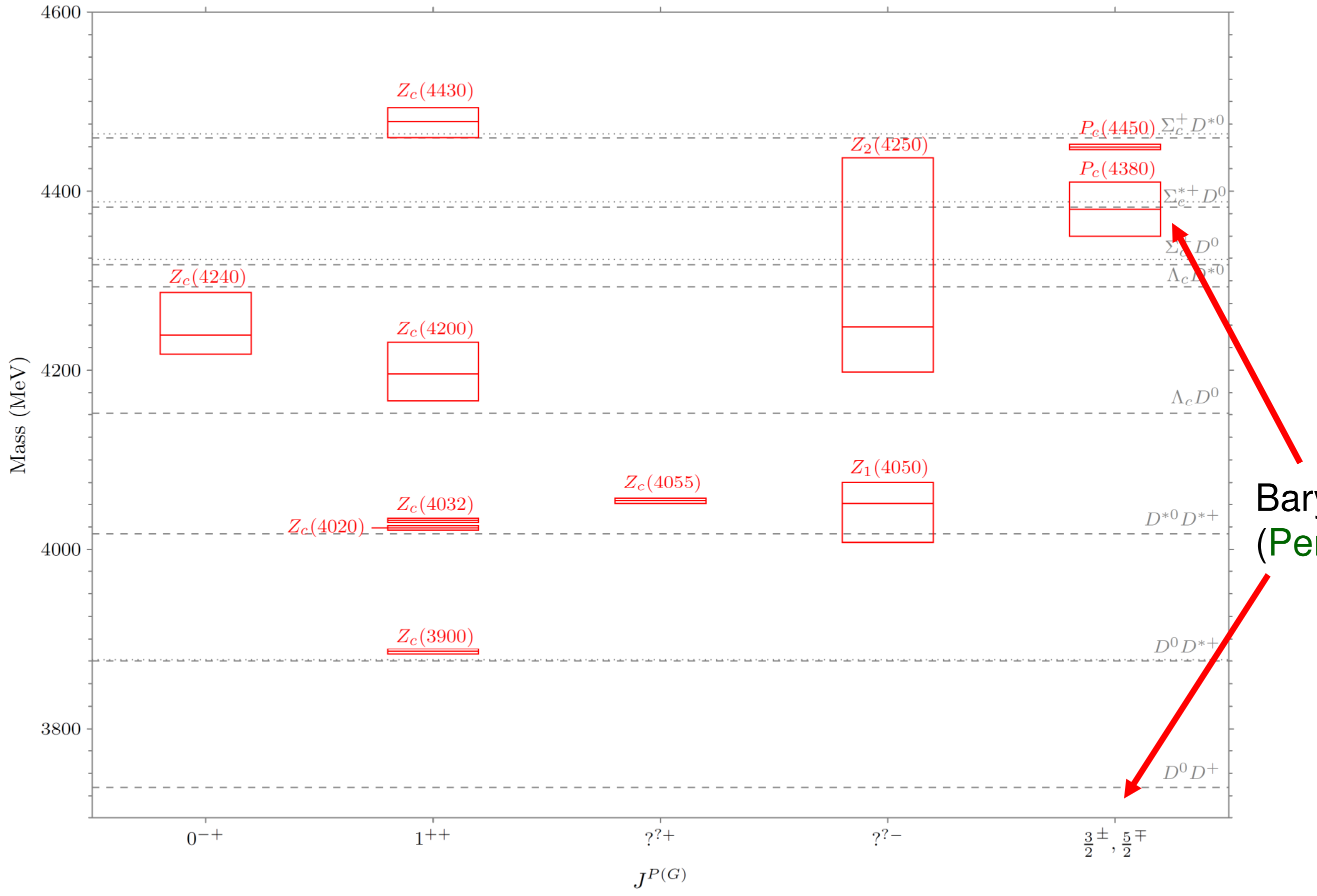


What the Charmonium System Really Looks Like

(June 2018)



Charmonium: June 2018
Charged sector



Baryonic ones too!
(Pentaquarks)

The Exotics Scorecard: June 2018

- **35** observed exotics
  - 30 in the charmonium sector
  - 4 in the (much less explored) bottomonium sector
  - 1 with a single *b* quark (and an *s*, a *u*, and a *d*)
- **15** confirmed (& none of the other 20 disproved)

Shameless Self-Promotion

Prog. Part. Nucl. Phys. **93** (2017) 143; 1610.04528

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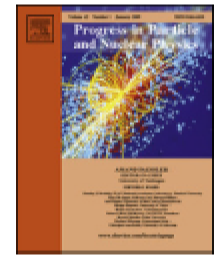
journal homepage: www.elsevier.com/locate/ppnp



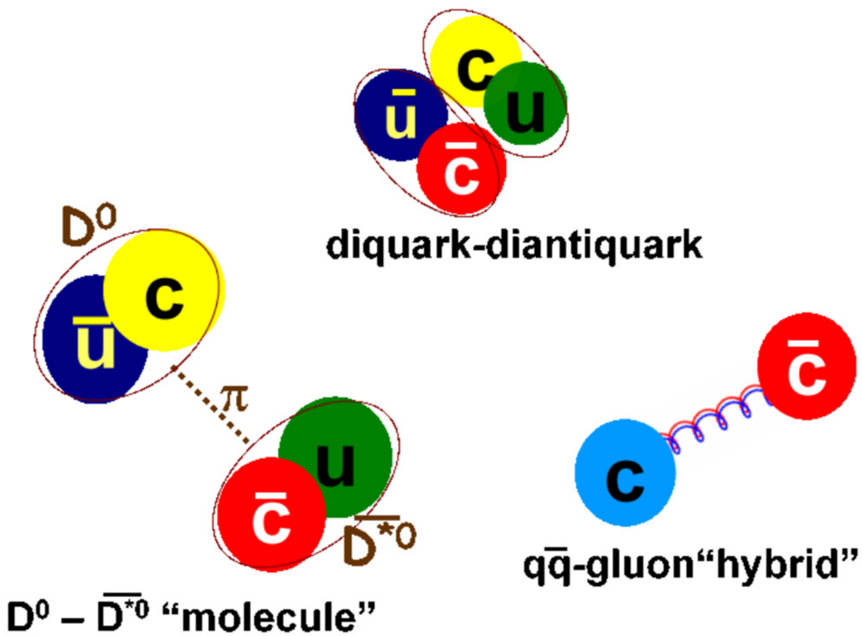
Review

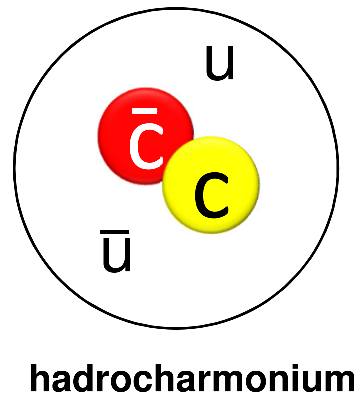
Heavy-quark QCD exotica

Richard F. Lebed [a,*], Ryan E. Mitchell [b], Eric S. Swanson [c]

…to learn in detail about the history of the discoveries
and the various theoretical interpretations attempted

How are Tetraquarks Assembled?





cusp effect:
Resonance created by rapid opening of meson-meson threshold

Image from Godfrey & Olsen,
Ann. Rev. Nucl. Part. Sci. **58** (2008) 51

Diquarks as Hadronic Components

- The short-distance color attraction of combining two color-$\mathbf{3}$ quarks ($\mathbf{3}$ = red, blue, green) into a color-$\overline{\mathbf{3}}$ diquark is *fully half as strong* as that of combining a $\mathbf{3}$ and a $\overline{\mathbf{3}}$ into a color-neutral singlet (*i.e.*, **diquark attraction** is nearly as strong as the **confining attraction**)

- Just as one computes a $SU(2)$ spin-spin coupling,
$$\vec{s}_1 \cdot \vec{s}_2 = \frac{1}{2}\left[(\vec{s}_1 + \vec{s}_2)^2 - \vec{s}_1^{\,2} - \vec{s}_2^{\,2}\right],$$
from two particles
in representations 1 and 2 combined into representation 1+2:

- If s_1, s_2 = spin $\frac{1}{2}$, and $\vec{s}_1 + \vec{s}_2$= spin 0, get $-\frac{3}{4}$; if spin 1, get $+\frac{1}{4}$

- The exact $SU(3)_{\text{color}}$ analogue formula for color charges gives the result stated above

Evidence for Diquarks?

- As formal entities, diquarks have always been with us:
- In any baryon, each quark is a color $\mathbf{3}$, meaning that the other two quarks together must be in a color $\overline{\mathbf{3}}$: technically, a diquark
- In a Λ_Q baryon, one heavier quark $Q = s, c, b$ is singled out, and the ud pair is necessarily isosinglet and spin-singlet
- Jaffe [Phys. Rep. **409**, 1 (2005)] calls this ud a "good" diquark since models predict it to be the most tightly bound combination
- The production of diquarks in fragmentation processes has long been studied [*e.g.*, Fontannaz *et al*., Phys. Lett. **77B** (1979) 315]
- An ideal gas of q and $\bar{q}$ (even including color screening) would produce preferentially diquark attraction $O(10\%)$ of the time [RFL, Phys. Rev. D**94** (2016) 034039]

Diquarks as Quasiparticles

- A diquark composed of a heavy (c or b) quark Q and a light quark q has a better chance of being identified as a localized quasiparticle, because the Q can be localized to a space of dimension $\lambda_C = \frac{1}{m_Q} \lesssim O(0.1\ \mathrm{fm})$
- Since the characteristic dimension of the compound is given by its reduced mass μ, the heavy-light diquark should be about half the size of a light-light diquark or meson, $\lesssim 0.5\ \mathrm{fm}$
- For example, Albertus *et al.* [Nucl. Phys. A **740**, 333 (2004)] compute the matter radius of Λ_c to be $\approx 0.3\ \mathrm{fm}$

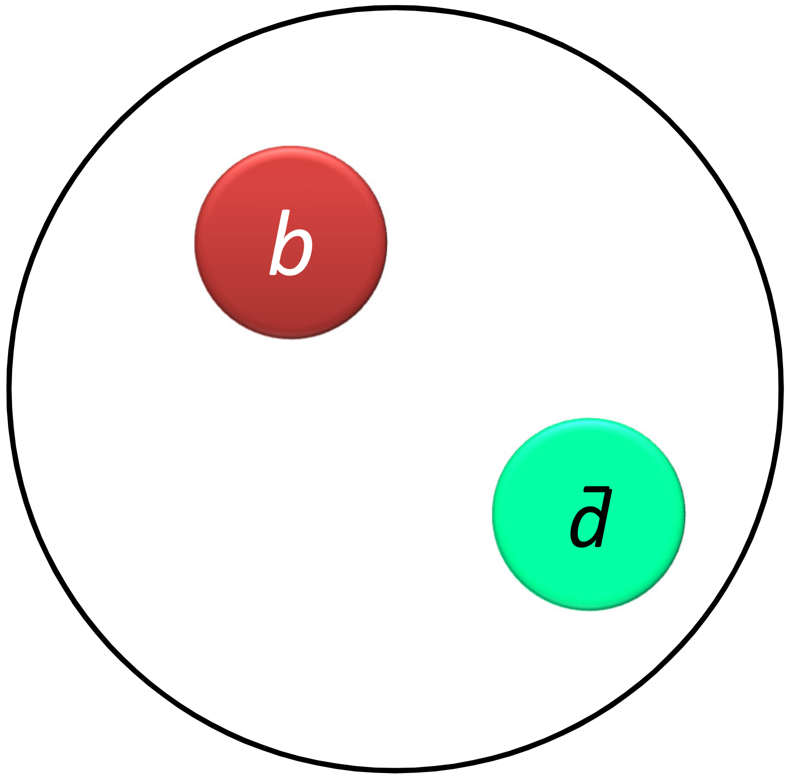
The Dynamical Diquark Picture

Stanley J. Brodsky, Dae Sung Hwang, RFL

Physical Review Letters **113**, 112001 (2014)

- CLAIM: At least some of the observed tetraquark states are bound states of diquark-antidiquark pairs
- Likewise, pentaquark states are bound states of diquark-anti*triquark* pairs
- BUT the pairs are not in a static configuration; they are created with a lot of relative energy, and rapidly separate from each other
- Diquarks are not color neutral! They cannot, by confinement, separate asymptotically far
- They must hadronize via large-*r* tails of mesonic wave functions, which suppresses decay widths to make them observably narrow

Nonleptonic $\overline{B}^0$ meson decay



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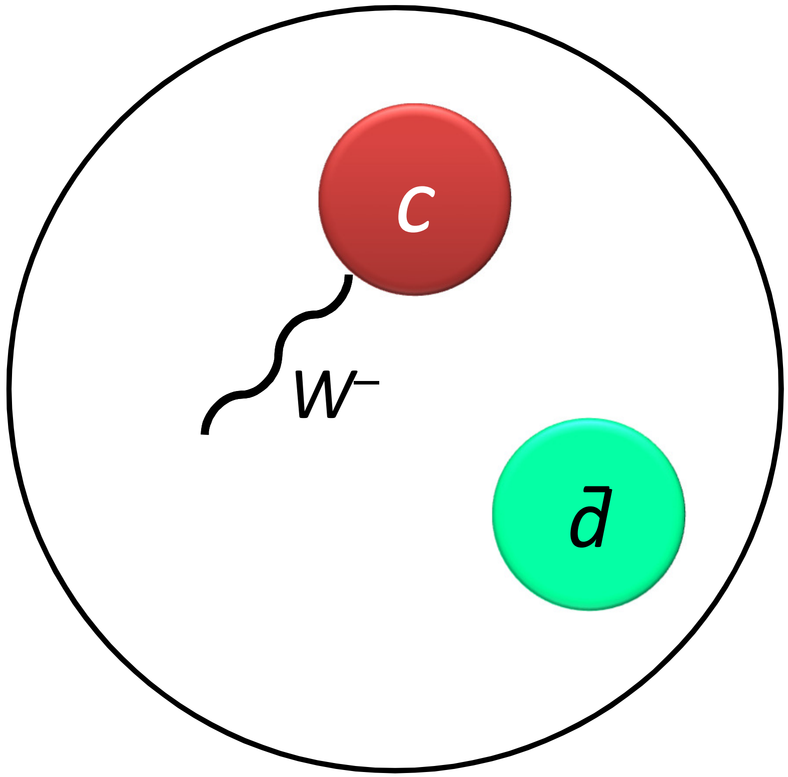
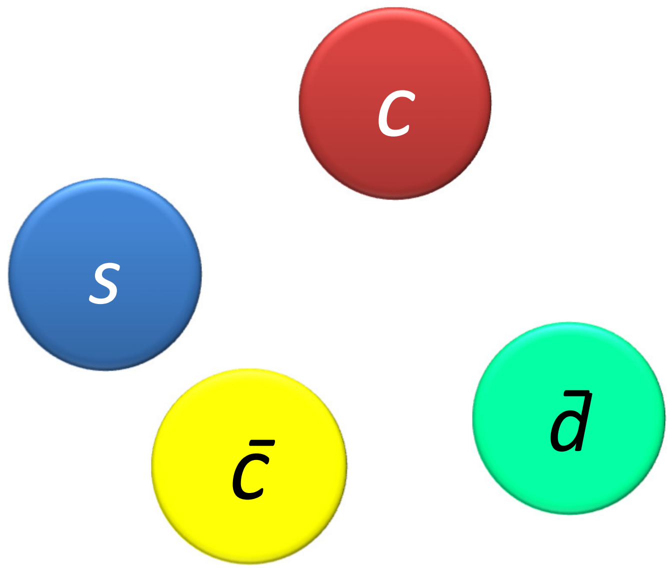


Nonleptonic $\bar{B}^0$ meson decay

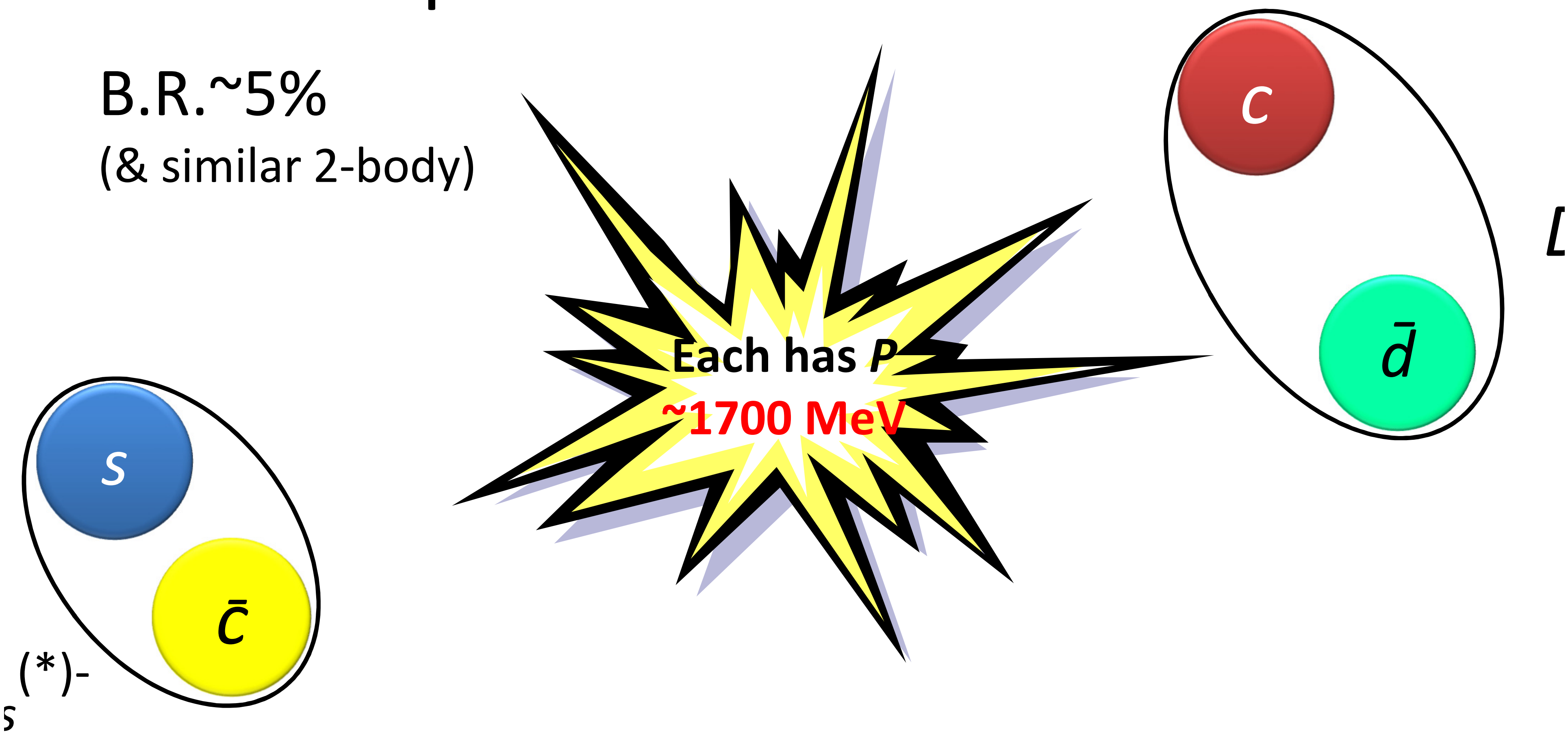
B.R.~22%

(Branching Ratio = probability)

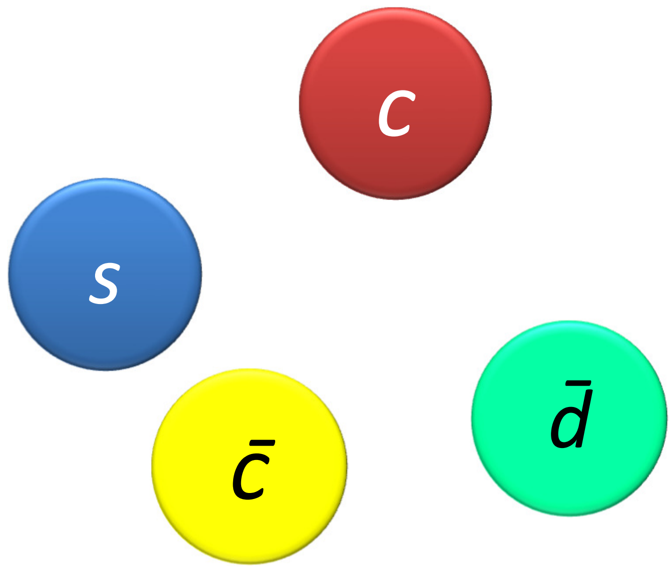


What happens next?
Option 1: Color-allowed

B.R.~5%
(& similar 2-body)



What happens next?
Option 2: Color-suppressed



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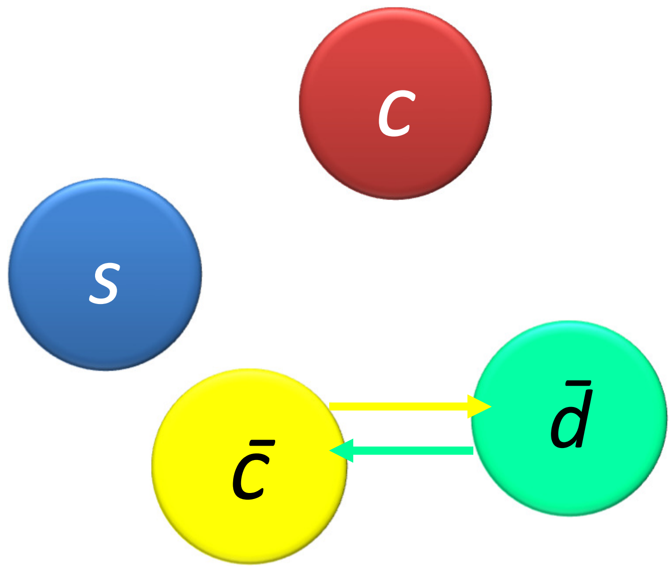

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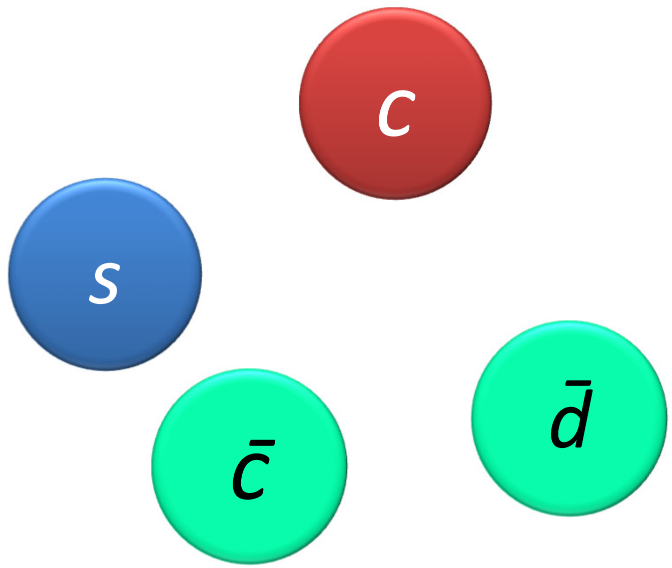

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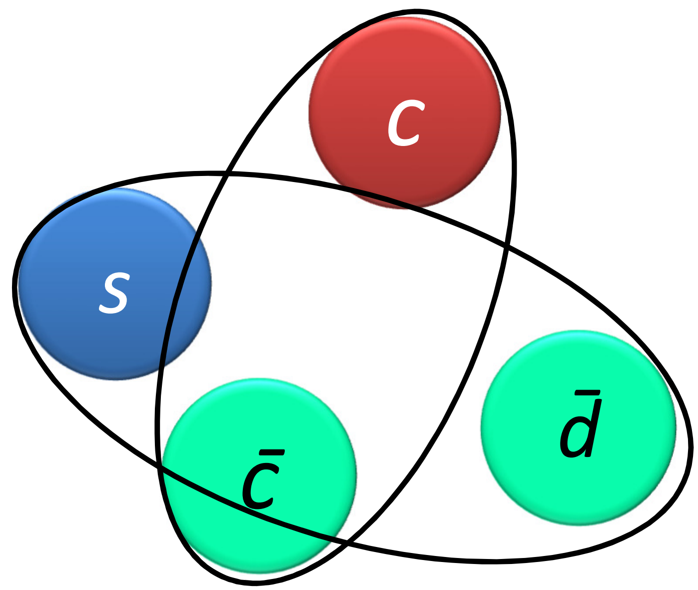
B.R.~2.3%



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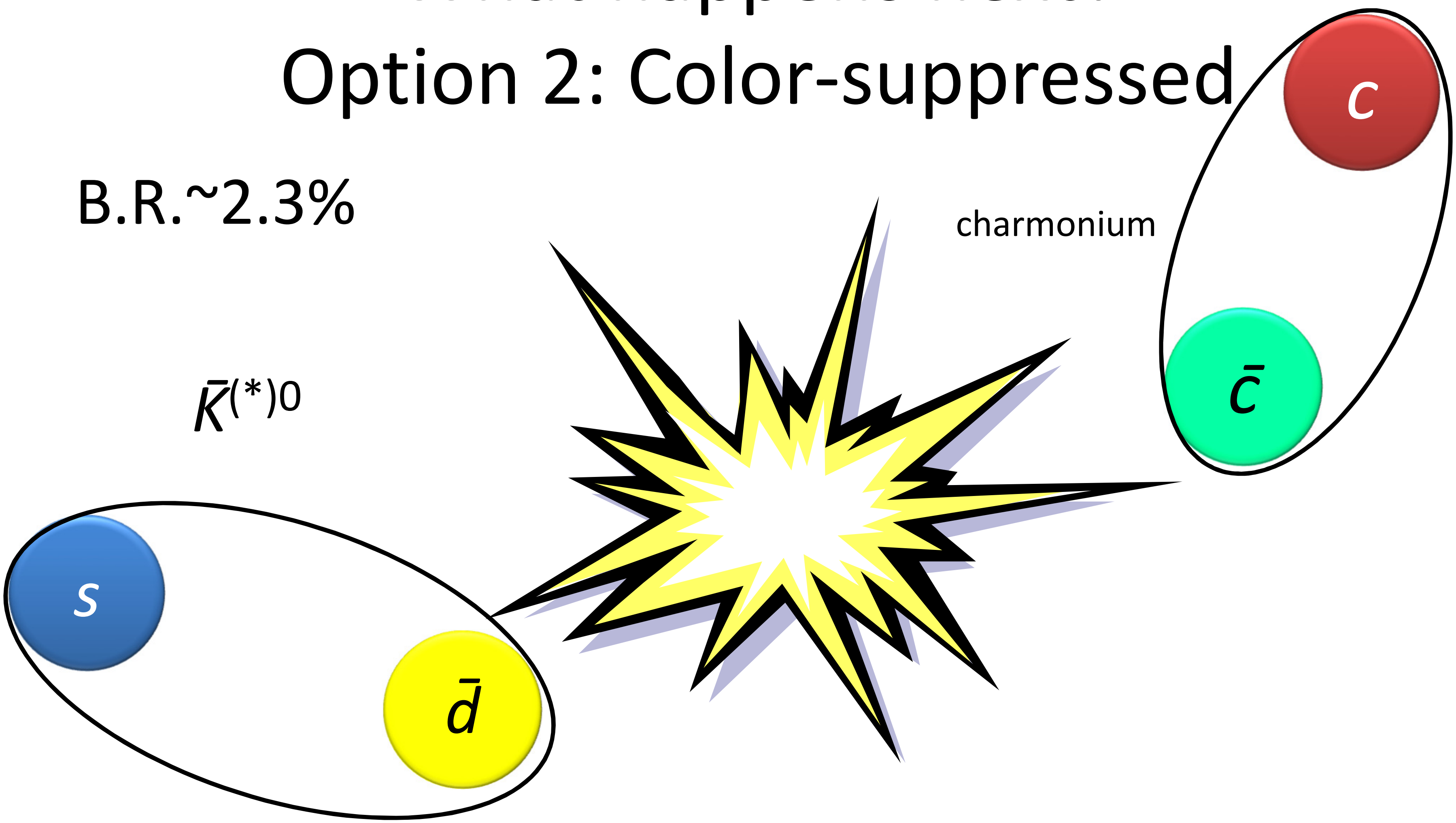
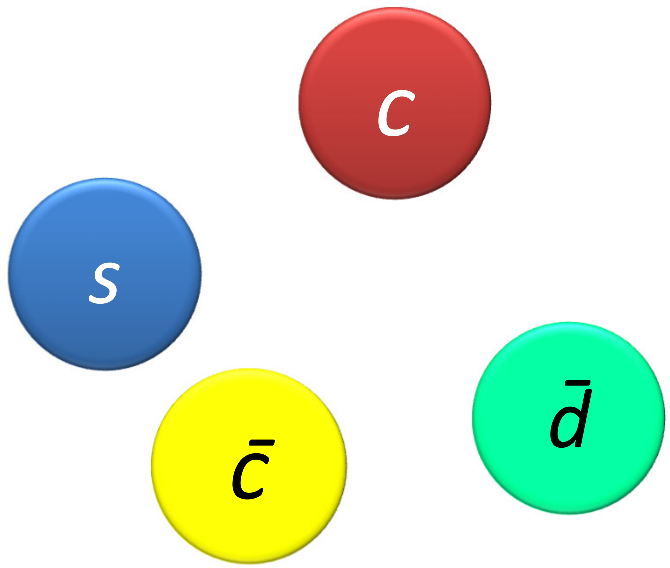


What happens next?
Option 3: Diquark formation



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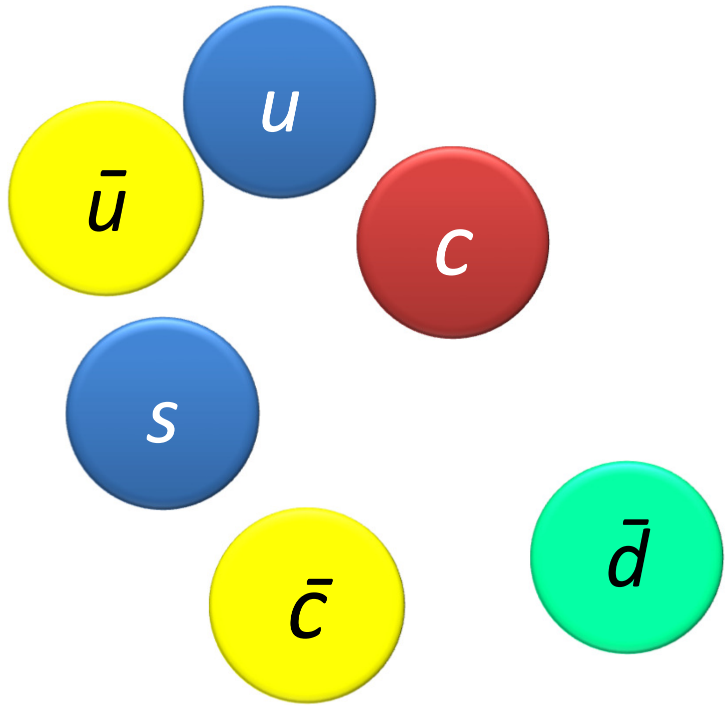

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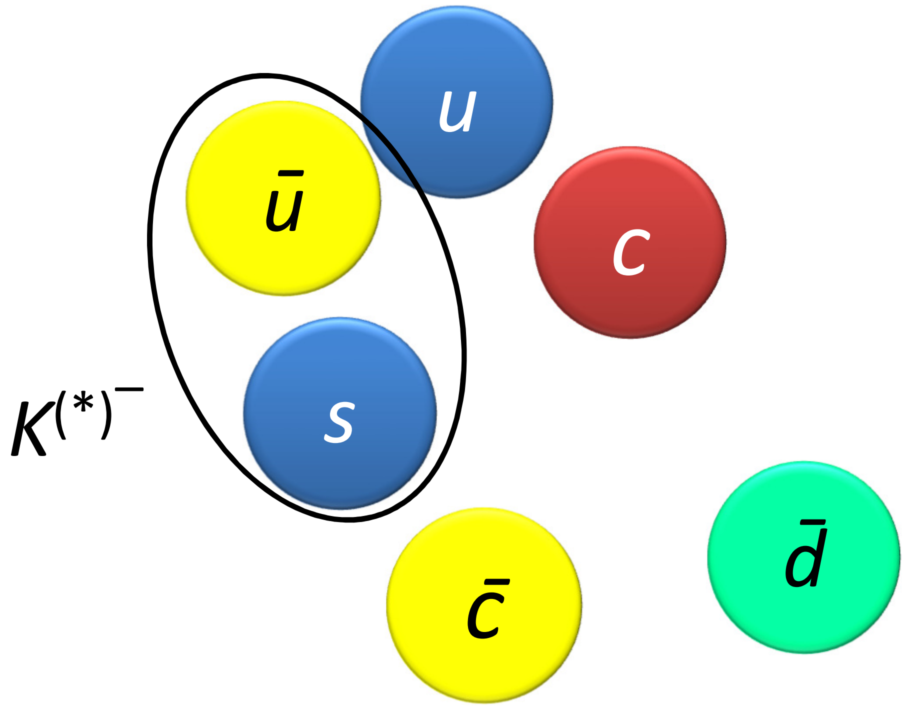

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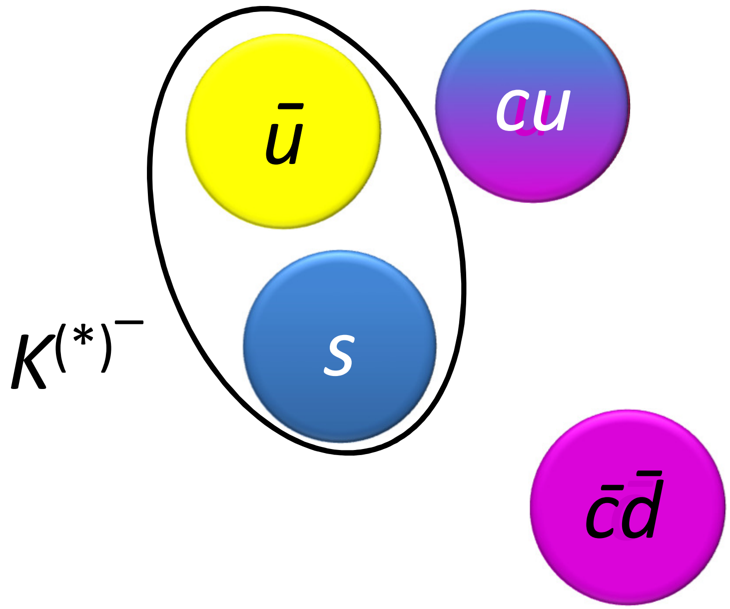

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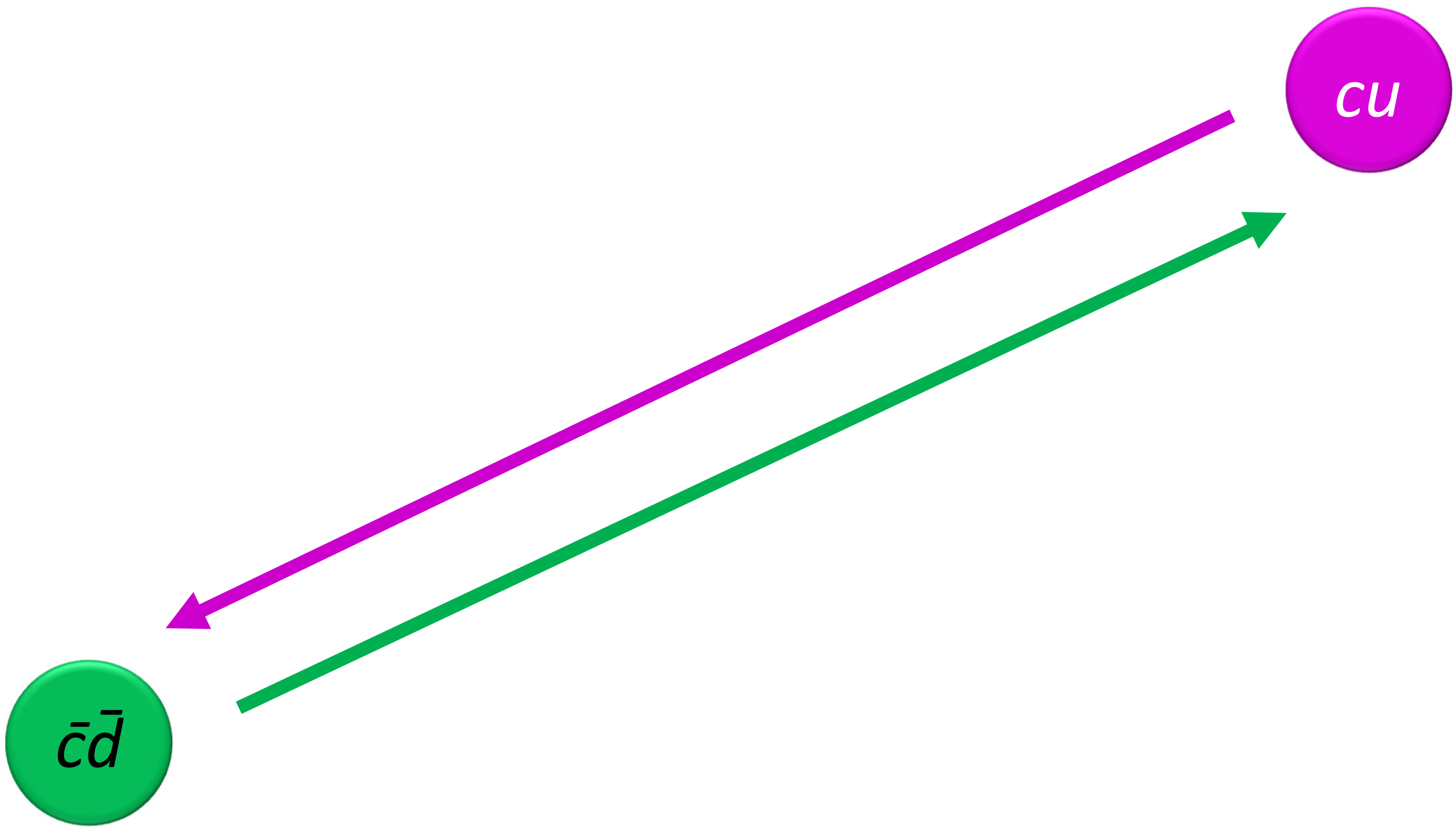
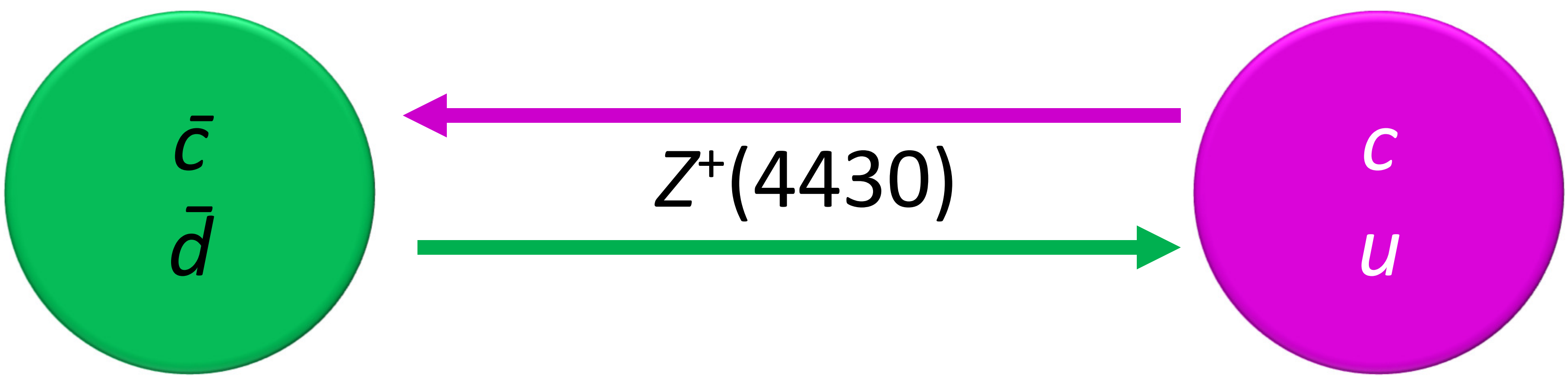


This state, with a quantized glue field, is the proposed nature of the tetraquark



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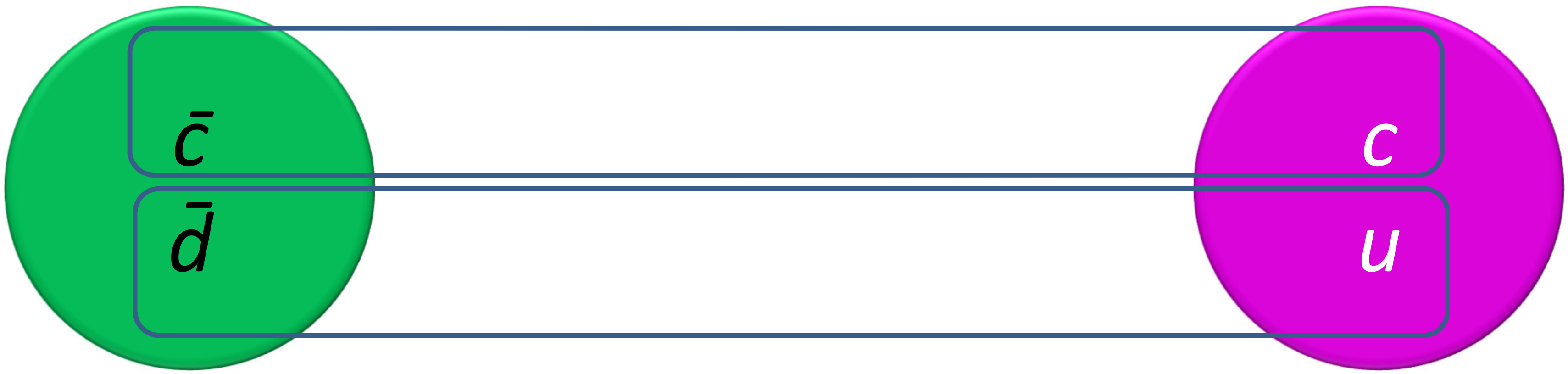


This state, with a quantized glue field, is the proposed nature of the tetraquark

charmonium Ψ(2S)
π^+

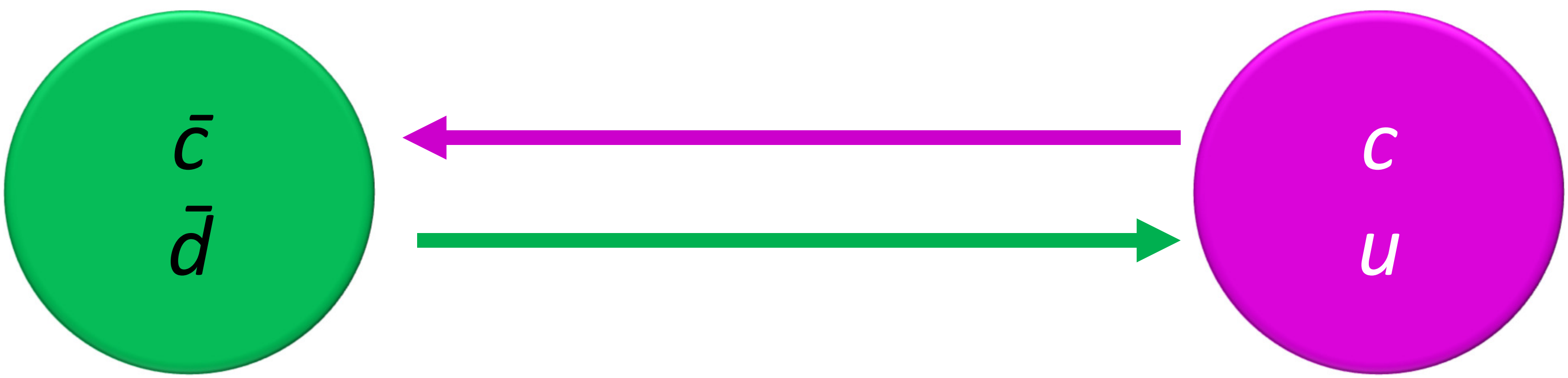
How far apart do the diquarks actually get?

- Since this is still a $\mathbf{3} \leftrightarrow \overline{\mathbf{3}}$ color interaction, just use the Cornell potential:

$$V(r) = -\frac{4}{3}\frac{\alpha_s}{r} + br + \frac{32\pi\alpha_s}{9m_{cq}^2}\left(\frac{\sigma}{\sqrt{\pi}}\right)^3 e^{-\sigma^2 r^2}\mathbf{S}_{cq}\cdot\mathbf{S}_{\overline{cq}},$$

[This variant: Barnes *et al.*, PRD **72**, 054026 (2005)]

- Use that the kinetic energy released in $\overline{B}^0 \longrightarrow K^- + Z^+(4430)$ converts into potential energy until the diquarks come to rest
- Decay transition most effective at this point (WKB turning point)



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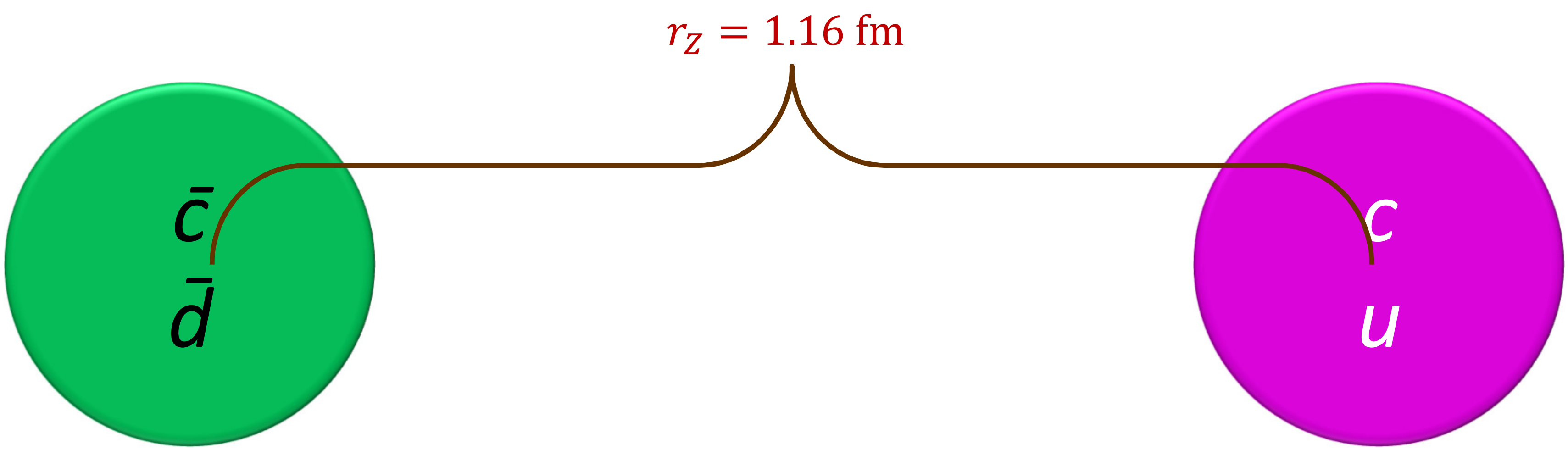
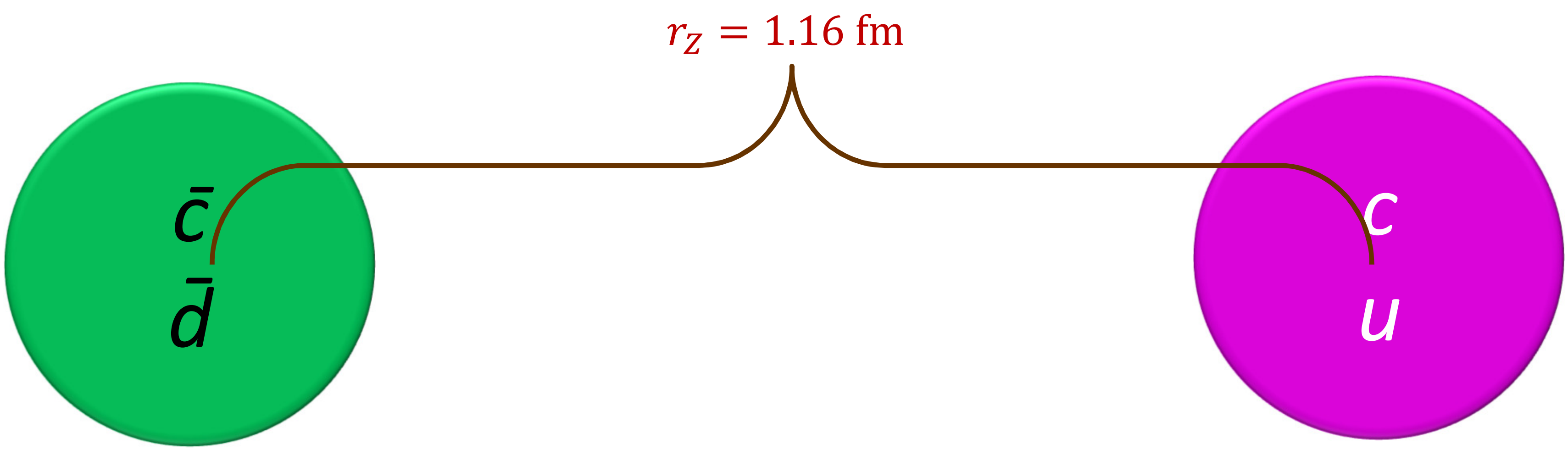


Fascinating $Z(4430)$ fact:

Belle [K. Chilikin *et al.*, PRD **90**, 112009 (2014)] says:

$$\frac{\mathrm{B.R.}[Z^-(4430) \to \psi(2S)\pi^-]}{\mathrm{B.R.}[Z^-(4430) \to J/\psi\pi^-]} > \mathbf{10}$$

and LHCb has not reported seeing the J/ψ $(1S)$ mode



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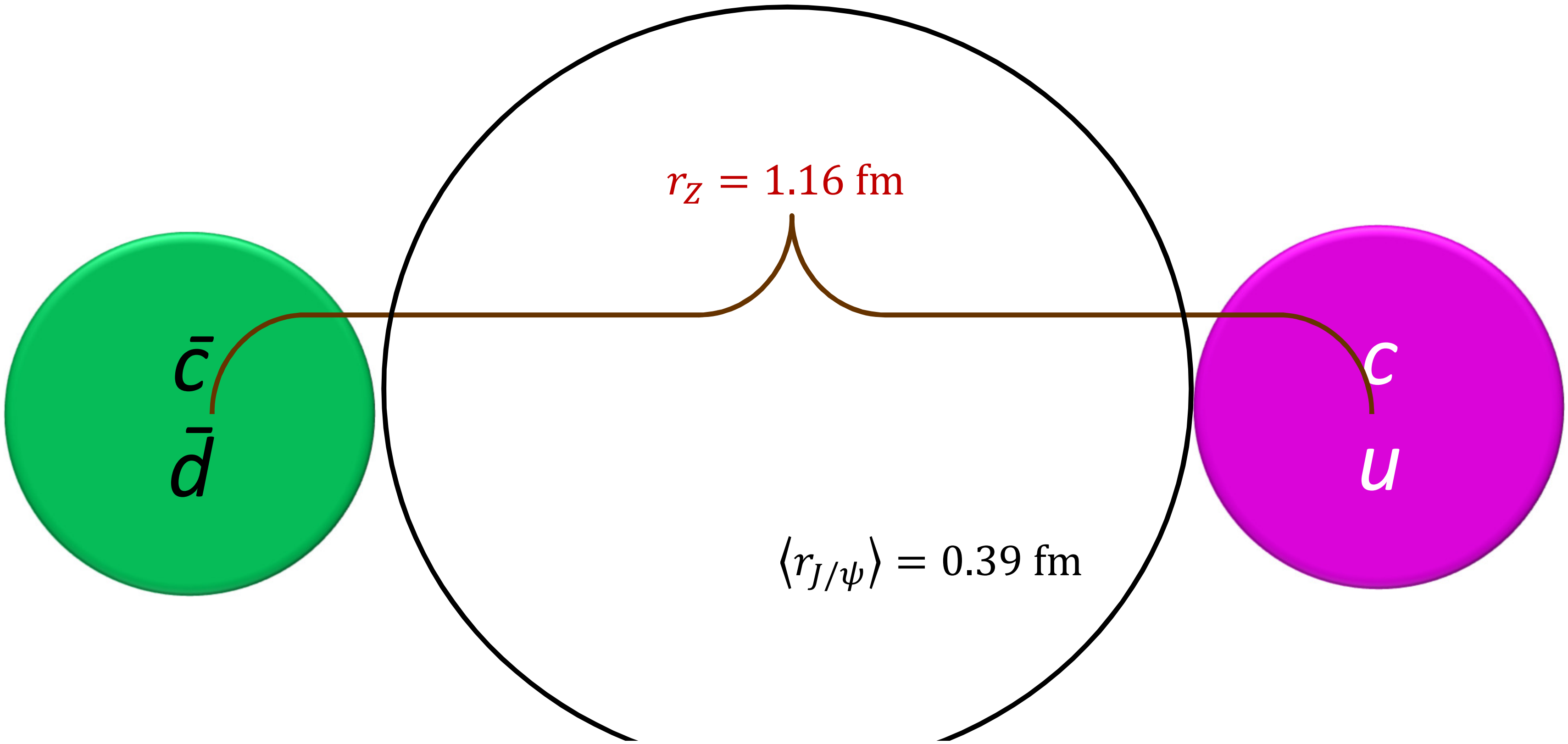

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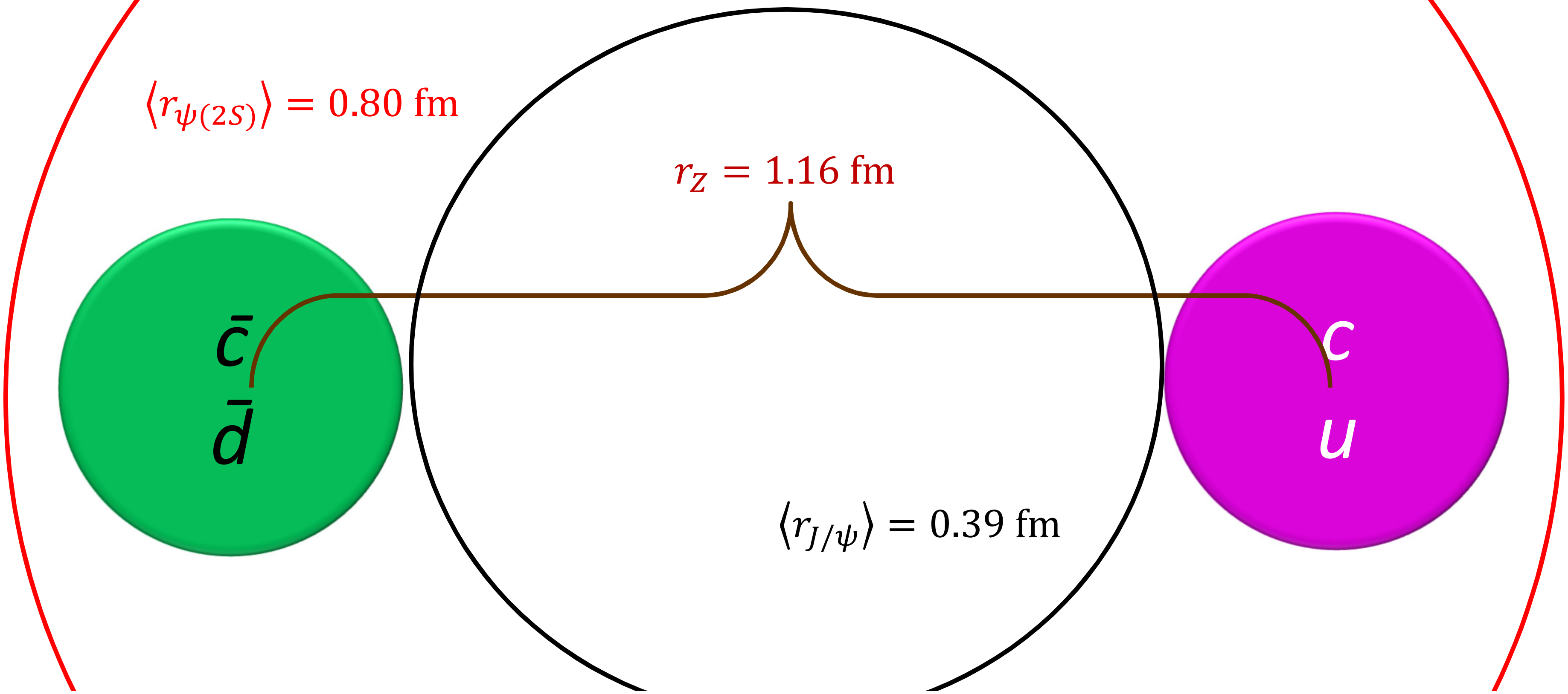
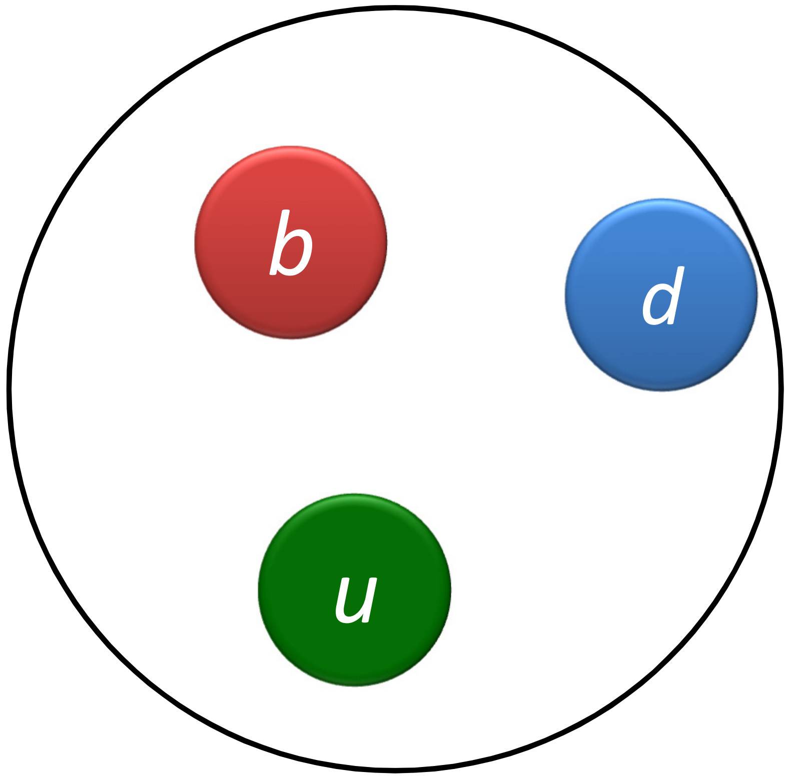


Nonleptonic Λ_b baryon decay



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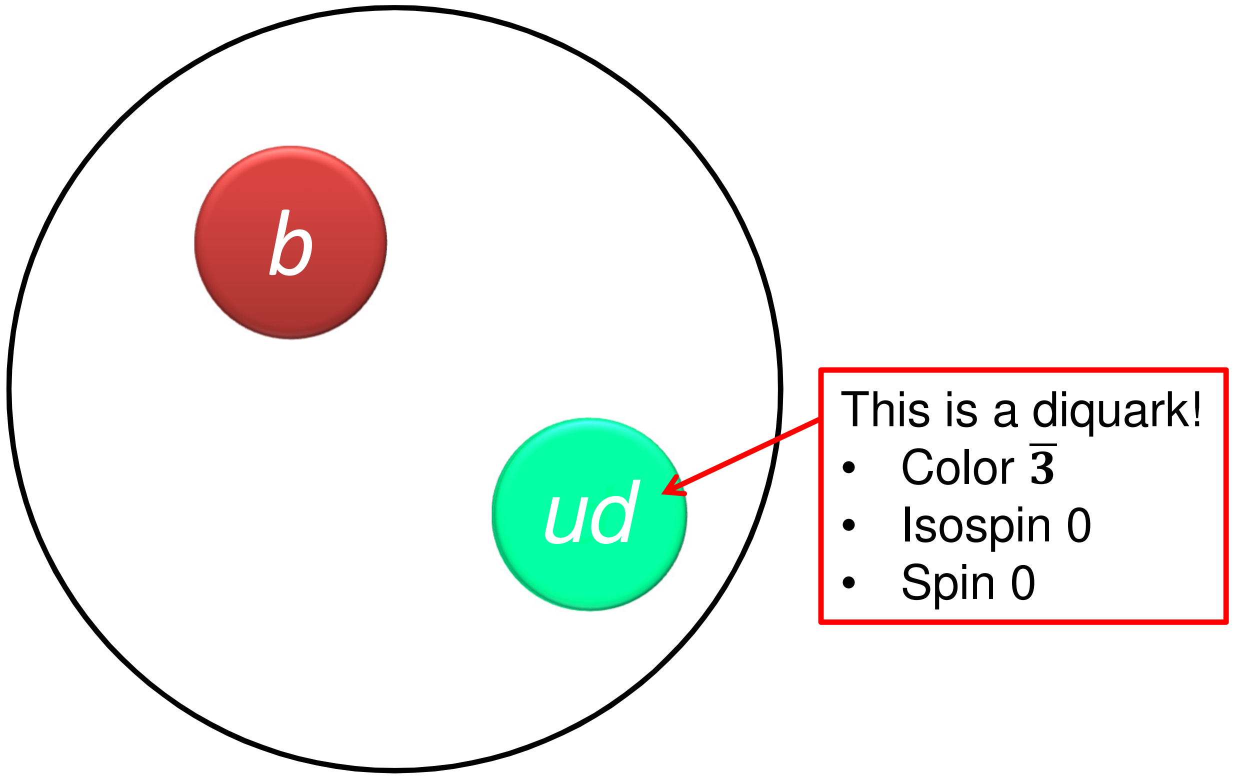

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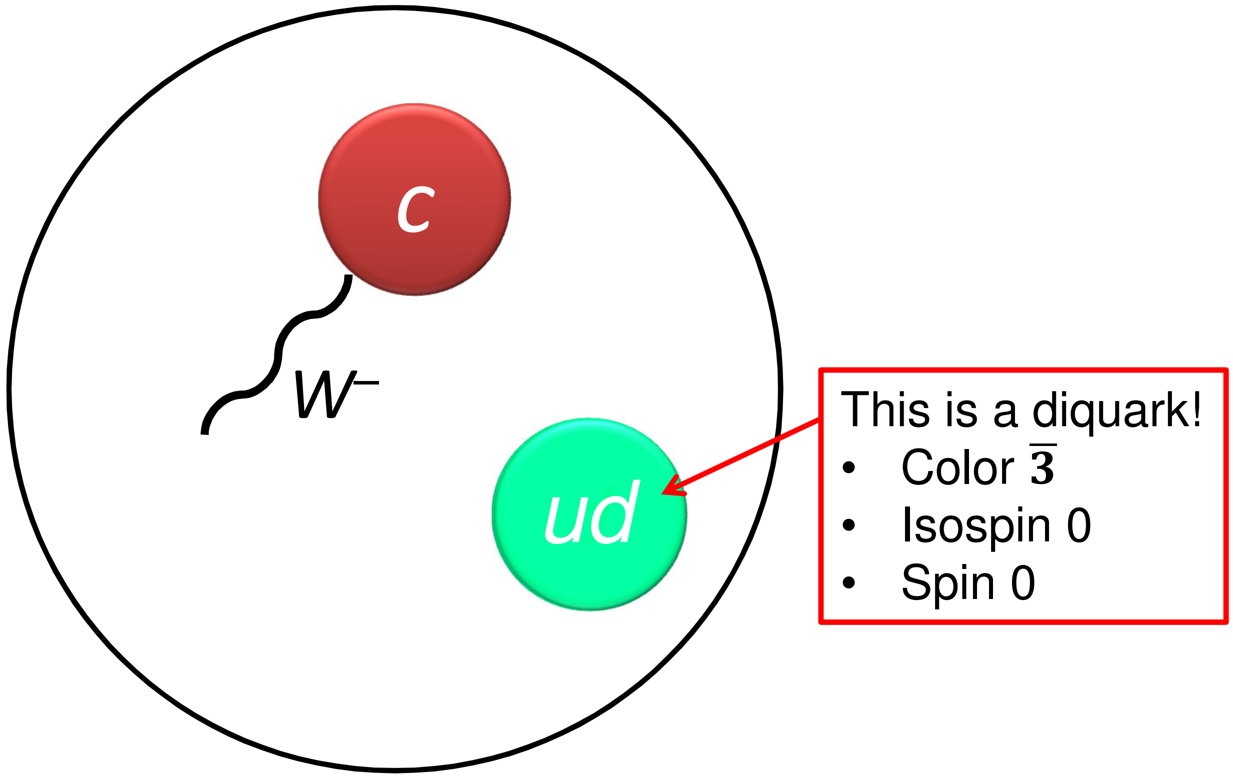

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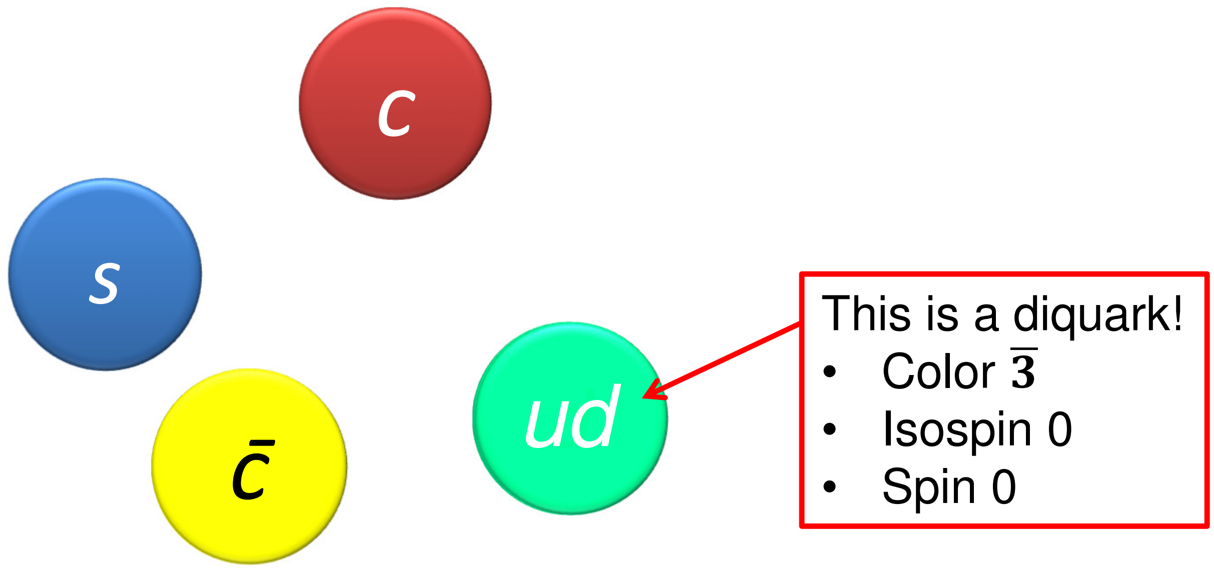
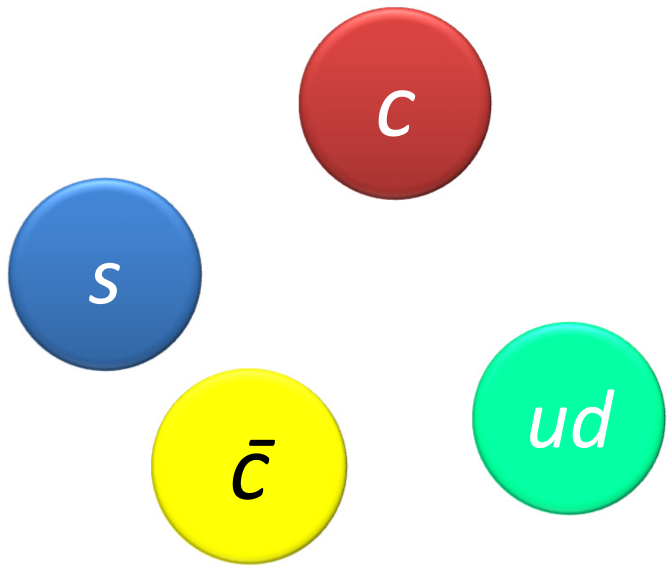


What happens next?
Diquark *and triquark* formation



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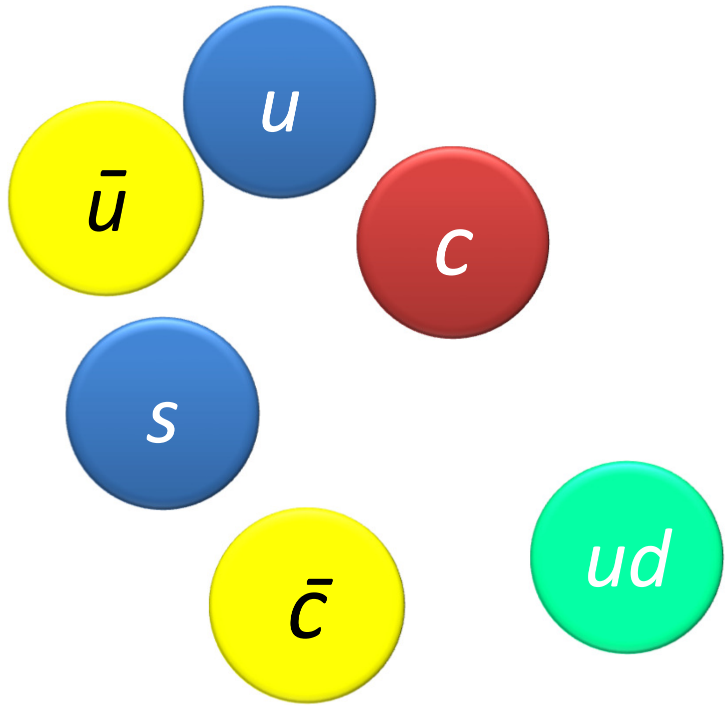

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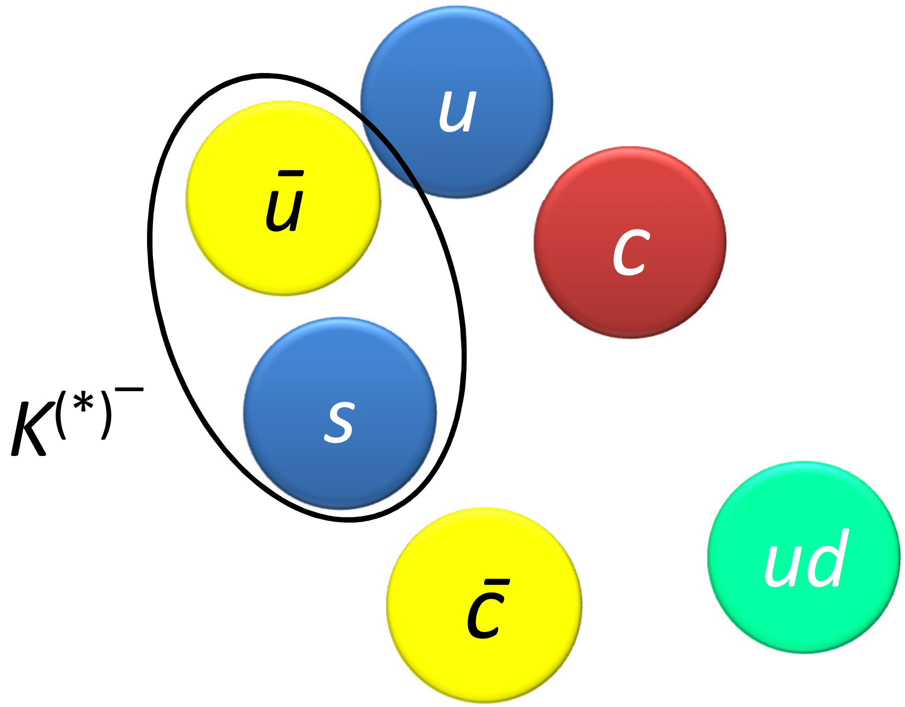

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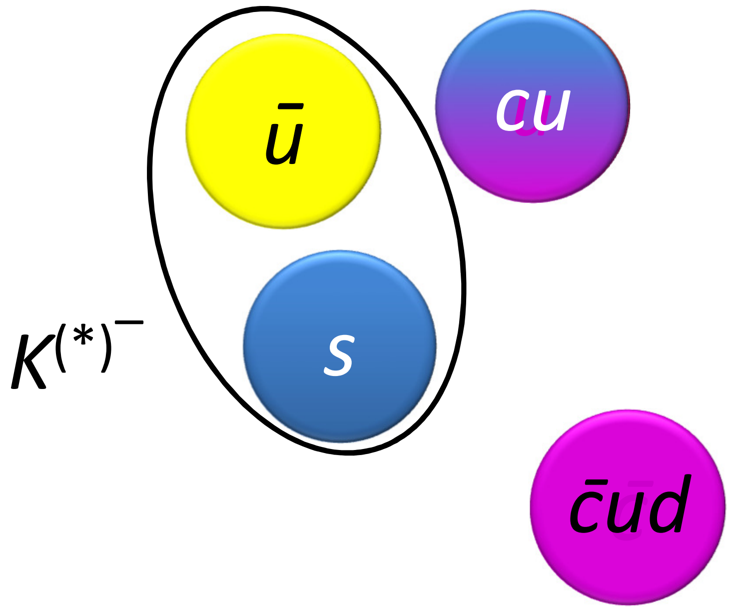

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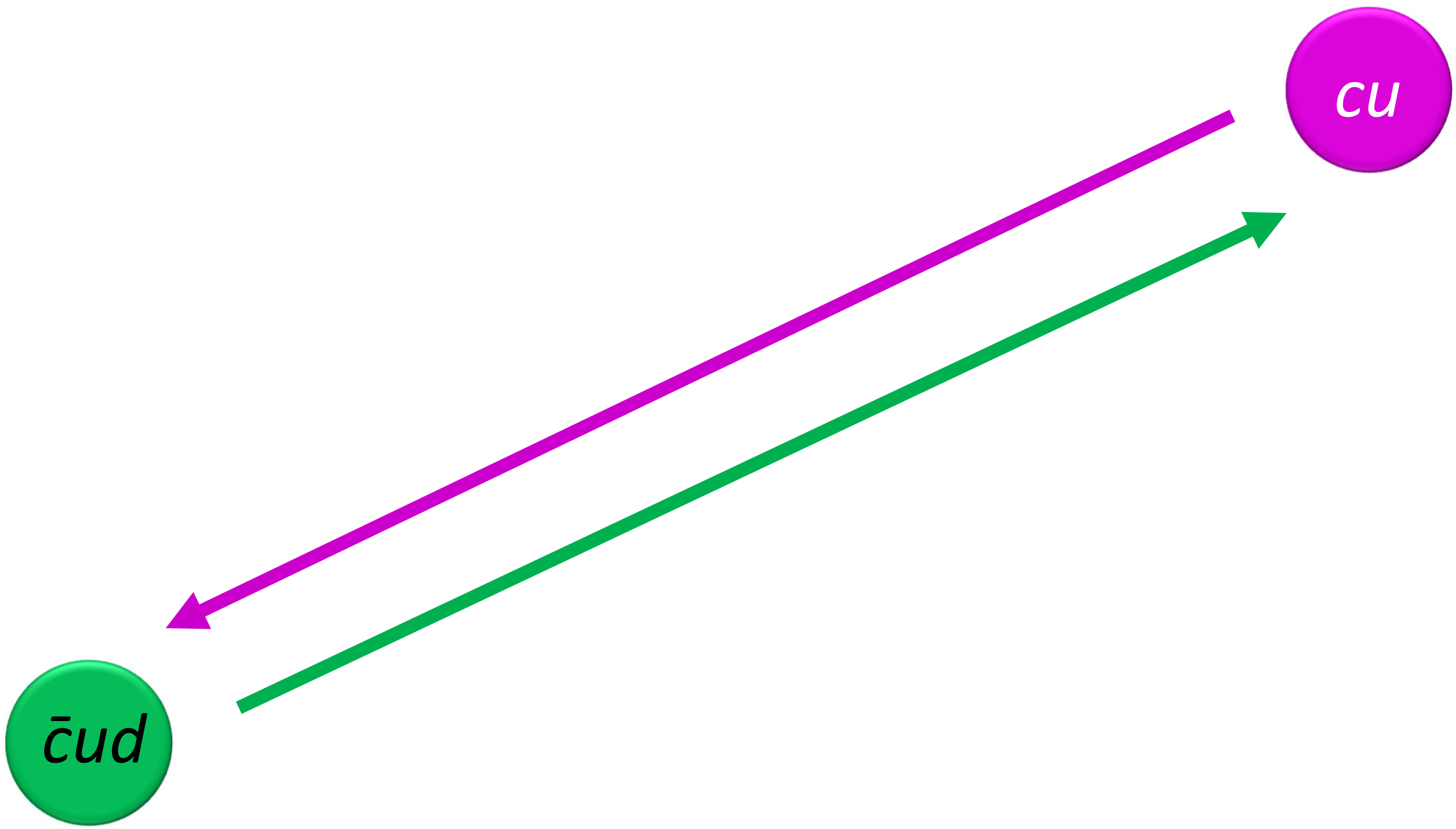
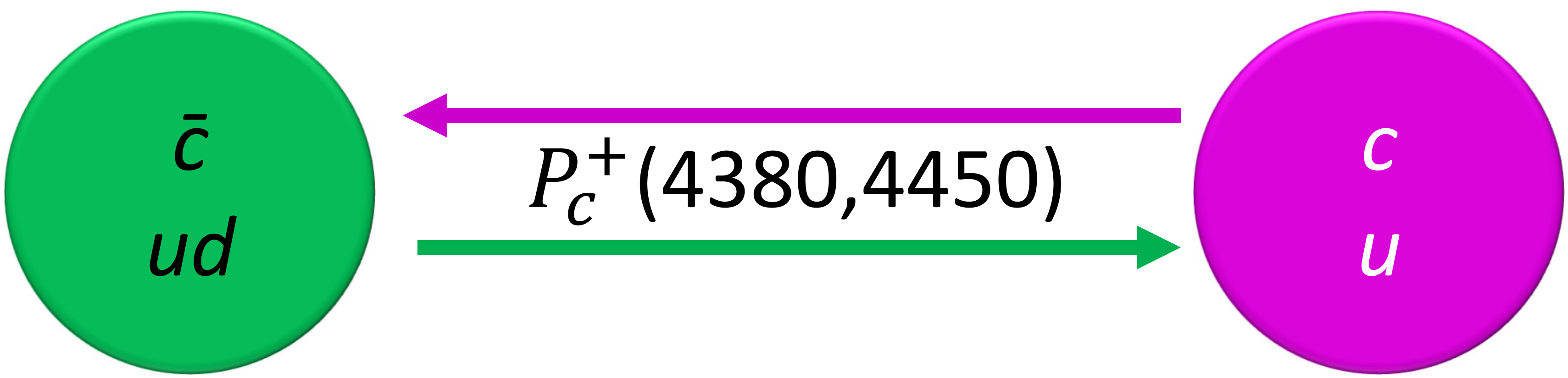


The same color-triplet mechanism, supplemented with the fact that the *ud* in Λ baryons themselves act as diquarks, predicts a rich spectrum of *pentaquarks*



$J^P = \frac{1}{2}^-$ $\quad$ $J^P = L^{(-1)^L}$ $\quad$ $J^P = 0^+, 1^+$

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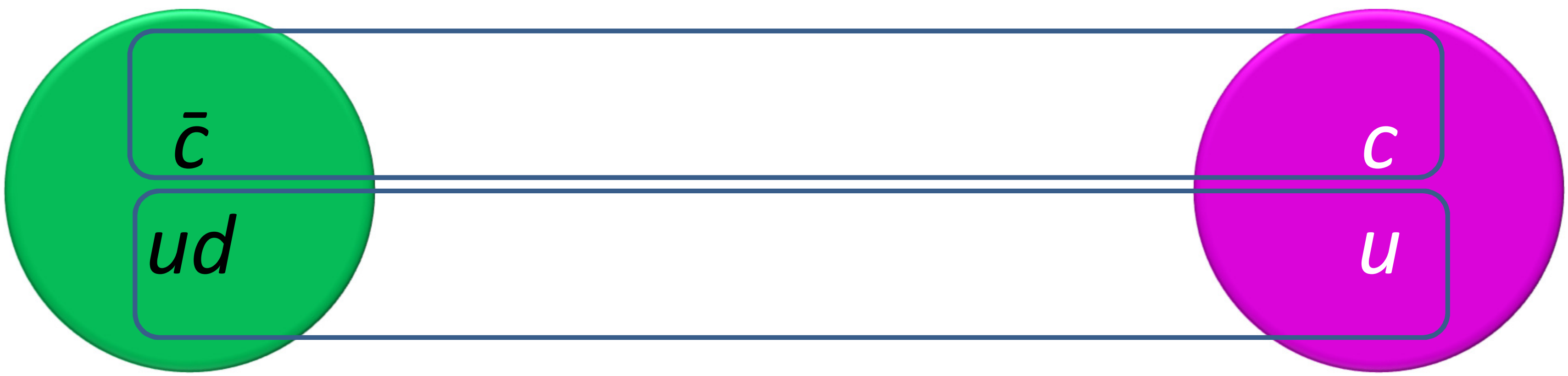

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J/Ψ

p

$J^P = \frac{1}{2}^-$

$J^P = L^{(-1)^L}$

$J^P = 0^+, 1^+$

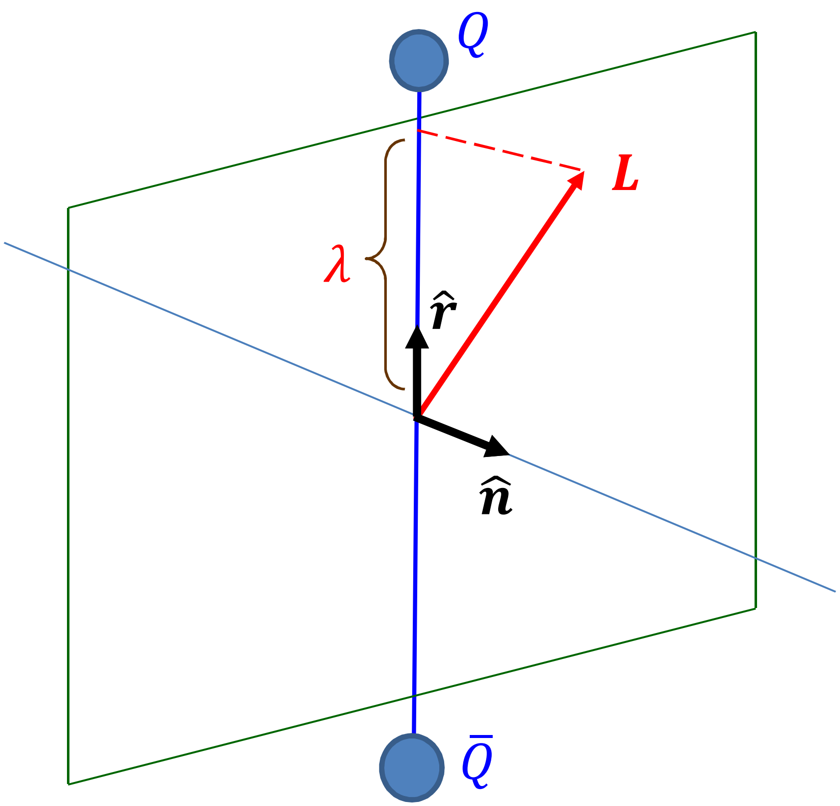
Exotics in the Born-Oppenheimer Approximation

- When studying physical chemistry or atomic physics, as students we encountered a qualitative definition of the Born-Oppenheimer approximation [Ann. Phys. **389** (1927) 457]: "*The light degrees of freedom (the electrons) in an atom or molecule adapt their state rapidly and adiabatically with respect to the much more slowly changing nuclei*"
- This is a true statement, but it can also be recast rigorously into particle-physics language:
  - The dynamics exhibits a scale separation in powers of m_e/m_N
  - The wave functions factor into light-d.o.f. and heavy-d.o.f. parts, with the light d.o.f. acting as potentials [B-O potentials] for the heavy d.o.f.
  - One can build an effective field theory, with m_e/m_N as the expansion parameter [Brambilla *et al.*, PRD **97** (2018) 016016]

When Is the B-O Approximation Needed?

- With only a single heavy source and a single light d.o.f. (*e.g.*, hydrogen or mesons composed of constituent quarks), then the usual trick of using a reduced mass is sufficient
- A system with at least two heavy sources plus light d.o.f. has B-O potentials that depend upon the separation and orientation of the heavy sources
- A simple such system is the H_2^+ ion: 2 protons, 1 electron [Griffiths QM, Sec. 7.3]
- Another is the Ξ_{cc} (ccq) baryon
- Another is the charmoniumlike hybrids $c\bar{c}g$, as well as charmoniumlike tetraquarks $c\bar{c}q_1\bar{q}_2$ and pentaquarks $c\bar{c}q_1q_2q_3$, …

B-O Quantum Numbers for the "Homonuclear Diatomic" $Q\bar{Q}$ System



- Symmetry group is that of a cylinder, $D_{\infty h}$:
- Rotations about the axis $\hat{\boldsymbol{r}}$ (eigenvalues $\lambda \equiv \hat{\boldsymbol{r}} \cdot \boldsymbol{L}$)
- Reflection (R_{light}) through a plane containing the axis $\hat{\boldsymbol{r}}$ (eigenvalues $\epsilon = \pm 1$)
- Reflection through the origin (P_{light}) is *not* a symmetry since Q, $\bar{Q}$ not equivalent, but $(CP)_{\text{light}}$ *is* a symmetry (eigenvalues $\eta = \pm 1$, called g and u, respectively)

B-O Quantum Numbers for the "Homonuclear Diatomic" $Q\bar{Q}$ System

- $\lambda \equiv \hat{\boldsymbol{r}} \cdot \boldsymbol{L}$ is a pseudoscalar:
  Invariant under rotations, odd under reflections
  Reflection R_{light} gives physically equivalent system, but $\lambda \to -\lambda$
- Thus, the energy of the system can only depend upon $\Lambda \equiv |\lambda|$
- The B-O potentials are thus labeled by Λ_η^ϵ
  - $\Lambda = 0,1,2,\cdots$ are labeled, respectively, by the letters $\Sigma, \Pi, \Delta, \cdots$ (analogous to $S, P, D, \cdots$)
  - Can show that the P_{light} eigenvalue equals $\epsilon(-1)^\Lambda$
  - If the light d.o.f. contain explicit spins (e^- for molecules), then its total s is also good quantum number$\Longrightarrow$ ${}^{2s+1}\Lambda_\eta^\epsilon$

Notes on the $D_{\infty h}$ B-O Quantum Numbers

- Only Σ ($\Lambda = 0$) potentials are automatically eigenstates of R_{light} (definite ϵ), but one can make $\Pi, \Delta, \cdots$ into eigenstates of definite ϵ by taking combination of $+\lambda$ and $-\lambda$ states (just as one does to form even/odd functions)
- The term label $\boxed{\Gamma \equiv \Lambda_\eta^\epsilon}$ fully specifies the $D_{\infty h}$ irreducible representations, but it is still possible to specify not only s, but also L, which satisfies the constraint $L \geq |\hat{\boldsymbol{r}} \cdot \boldsymbol{L}| = \Lambda$
- If the heavy sources are not truly "homonuclear" (*e.g.*, $b\bar{c}$), then one loses the $(CP)_{\text{light}}$ eigenvalue η
- If the light d.o.f. carry isospin (*e.g.*, $c\bar{c}u\bar{d}$), then C-parity symmetry is replaced by G-parity symmetry, $G \equiv C(-1)^I$

Exotics spectroscopy using B-O potentials

RFL, Phys. Rev. D **96** (2017), 116003 [1709.06097]

- Given quantum numbers of the light d.o.f., combine with the heavy quantum numbers to find the full spectrum of states
- For hybrid mesons, the light d.o.f. consist of an extended gluon field plus sea $q\bar{q}$ (gluelump)
- For tetraquarks, the light valence quarks can *in principle* be included with the light d.o.f. [Braaten *et al.*, PRD **90** (2014) 014044]
- Diquark model: It is more appropriate to separate out $\boldsymbol{s}_{q\bar{q}}$

$$\boldsymbol{J} = \underbrace{\boldsymbol{J}_{\text{light}} + \boldsymbol{L}_{Q\bar{Q}}}_{\boldsymbol{L}} + \underbrace{\boldsymbol{s}_{q\bar{q}} + \boldsymbol{s}_{Q\bar{Q}}}_{\boldsymbol{S}}$$

- Still have $\lambda \equiv \hat{\boldsymbol{r}} \cdot \boldsymbol{L} = \hat{\boldsymbol{r}} \cdot \boldsymbol{J}_{\text{light}}$ since $\hat{\boldsymbol{r}} \cdot \boldsymbol{L}_{Q\bar{Q}} = 0$

Exotics spectroscopy using B-O potentials

- Hybrid discrete symmetry quantum numbers:
$$P = \epsilon(-1)^{\Lambda+L+1}, \qquad C = \eta\epsilon(-1)^{\Lambda+L+s_{Q\bar{Q}}}$$
- Tetraquark discrete symmetry quantum numbers:
$$P = \epsilon(-1)^{\Lambda+L}, \qquad C = \eta\epsilon(-1)^{\Lambda+L+s_{q\bar{q}}+s_{Q\bar{Q}}}$$
- Pentaquark discrete symmetry quantum numbers:
$$P = \epsilon(-1)^{\Lambda+L+1}, \qquad C \text{ no longer good}$$
- Now work out the multiplets based on the B-O potentials, starting with underlying states classified according to spins $s_{q\bar{q}}$, $s_{Q\bar{Q}}$, S [Maiani *et al.*, PRD **89** (2014) 114010], *e.g.*,
$$\tilde{Z}' \equiv \left|0_{s_{q\bar{q}}}, 1_{s_{Q\bar{Q}}}\right\rangle_{S=1}$$

Exotics spectroscopy using B-O potentials:

Tetraquarks

Boldface = exotic quantum numbers for $q\bar{q}$

BO potential	State notation State J^{PC}		
$\Sigma_g^+(1S)$	$\tilde{X}_{0\,S}^{(0)++}$ 0^{++}	$\tilde{Z}_S^{(1)++},\ \tilde{Z}_S'^{(1)++}$ $2\times 1^{+-}$	$\tilde{X}_{0\,S}'^{(0)++},\ X_{1\,S}^{(1)++},\ X_{2\,S}^{(2)++}$ $[0,1,2]^{++}$
$\Sigma_g^+(1P)$	$\tilde{X}_{0\,P}^{(1)++}$ 1^{--}	$[\tilde{Z}_P^{(0),(\mathbf{1}),(2)}]^{++},\ [\tilde{Z}_P'^{(0),(\mathbf{1}),(2)}]^{++}$ $2\times(0,\mathbf{1},2)^{-+}$	$\tilde{X}_{0\,P}'^{(1)++},\ [X_{1\,P}^{(\mathbf{0}),(1),(2)}]^{++},\ [X_{2\,P}^{(1),(2),(3)}]^{++}$ $[1,\ (\mathbf{0},1,2),\ (1,2,3)]^{--}$
$\Sigma_g^+(1D)$	$\tilde{X}_{0\,D}^{(2)++}$ 2^{++}	$[\tilde{Z}_D^{(1),(\mathbf{2}),(3)}]^{++},\ [\tilde{Z}_D'^{(1),(\mathbf{2}),(3)}]^{++}$ $2\times(1,\mathbf{2},3)^{+-}$	$\tilde{X}_{0\,D}'^{(2)++},\ [X_{1\,D}^{(1),(2),(3)}]^{++},\ [X_{2\,D}^{(0),(1),(2),(3),(4)}]^{++}$ $[2,\ (1,2,3),\ (0,1,2,3,4)]^{++}$
$\Pi_u^+(1P)$ & $\Sigma_u^-(1P)$	$\tilde{X}_{0\,P}^{(1)-+}$ 1^{+-}	$[\tilde{Z}_P^{(0),(1),(2)}]^{-+},\ [\tilde{Z}_P'^{(0),(1),(2)}]^{-+}$ $2\times(0,1,2)^{++}$	$\tilde{X}_{0\,P}'^{(1)-+},\ [X_{1\,P}^{(\mathbf{0}),(1),(\mathbf{2})}]^{-+},\ [X_{2\,P}^{(1),(\mathbf{2}),(3)}]^{-+}$ $[1,\ (\mathbf{0},1,\mathbf{2}),\ (1,\mathbf{2},3)]^{+-}$
$\Pi_u^-(1P)$	$\tilde{X}_{0\,P}^{(\mathbf{1})+-}$ $\mathbf{1}^{-+}$	$[\tilde{Z}_P^{(\mathbf{0}),(1),(2)}]^{+-},\ [\tilde{Z}_P'^{(\mathbf{0}),(1),(2)}]^{+-}$ $2\times(\mathbf{0},1,2)^{--}$	$\tilde{X}_{0\,P}'^{(\mathbf{1})+-},\ [X_{1\,P}^{(0),(\mathbf{1}),(2)}]^{+-},\ [X_{2\,P}^{(\mathbf{1}),(2),(\mathbf{3})}]^{+-}$ $[\mathbf{1},\ (0,\mathbf{1},2),\ (\mathbf{1},2,\mathbf{3})]^{-+}$
$\Sigma_u^-(1S)$	$\tilde{X}_{0\,S}^{(0)-+}$ 0^{-+}	$\tilde{Z}_S^{(1)-+},\ \tilde{Z}_S'^{(1)-+}$ $2\times 1^{--}$	$\tilde{X}_{0\,S}'^{(0)-+},\ X_{1\,S}^{(\mathbf{1})-+},\ X_{2\,S}^{(2)-+}$ $[0,\mathbf{1},2]^{-+}$
$\Pi_u^+(1D)$	$\tilde{X}_{0\,D}^{(2)-+}$ 2^{-+}	$[\tilde{Z}_D^{(1),(2),(3)}]^{-+},\ [\tilde{Z}_D'^{(1),(2),(3)}]^{-+}$ $2\times(1,2,3)^{--}$	$\tilde{X}_{0\,D}'^{(2)-+},\ [X_{1\,D}^{(\mathbf{1}),(2),(\mathbf{3})}]^{-+},\ [X_{2\,D}^{(0),(\mathbf{1}),(2),(\mathbf{3}),(4)}]^{-+}$ $[2,\ (\mathbf{1},2,\mathbf{3}),\ (0,\mathbf{1},2,\mathbf{3},4)]^{-+}$

Exotics spectroscopy using B-O potentials:

Pentaquarks

BO potential	State notation State J^P	
$\Sigma^+(1S)$	$\tilde{P}^{(\frac{1}{2})+}_{\frac{1}{2}S}, \tilde{P}'^{(\frac{1}{2})+}_{\frac{1}{2}S}$ $2\times\frac{1}{2}^-$	$P^{(\frac{3}{2})+}_{\frac{3}{2}S}$ $\frac{3}{2}^-$
$\Sigma^+(1P)$	$\left[\tilde{P}^{(\frac{1}{2}),(\frac{3}{2})}_{\frac{1}{2}P}\right]^+, \left[\tilde{P}'^{(\frac{1}{2}),(\frac{3}{2})}_{\frac{1}{2}P}\right]^+$ $2\times\left(\frac{1}{2},\frac{3}{2}\right)^+$	$\left[P^{(\frac{1}{2}),(\frac{3}{2}),(\frac{5}{2})}_{\frac{3}{2}P}\right]^+$ $\left(\frac{1}{2},\frac{3}{2},\frac{5}{2}\right)^+$
$\Sigma^+(1D)$	$\left[\tilde{P}^{(\frac{3}{2}),(\frac{5}{2})}_{\frac{1}{2}D}\right]^+, \left[\tilde{P}'^{(\frac{3}{2}),(\frac{5}{2})}_{\frac{1}{2}D}\right]^+$ $2\times\left(\frac{3}{2},\frac{5}{2}\right)^-$	$\left[P^{(\frac{1}{2}),(\frac{3}{2}),(\frac{5}{2}),(\frac{7}{2})}_{\frac{3}{2}D}\right]^+$ $\left(\frac{1}{2},\frac{3}{2},\frac{5}{2},\frac{7}{2}\right)^-$
$\Pi^+(1P)$ & $\Sigma^-(1P)$	$\left[\tilde{P}^{(\frac{1}{2}),(\frac{3}{2})}_{\frac{1}{2}P}\right]^-, \left[\tilde{P}'^{(\frac{1}{2}),(\frac{3}{2})}_{\frac{1}{2}P}\right]^-$ $2\times\left(\frac{1}{2},\frac{3}{2}\right)^-$	$\left[P^{(\frac{1}{2}),(\frac{3}{2}),(\frac{5}{2})}_{\frac{3}{2}P}\right]^-$ $\left(\frac{1}{2},\frac{3}{2},\frac{5}{2}\right)^-$
$\Pi^-(1P)$	Same as $\Sigma^+(1P)$	
$\Sigma^-(1S)$	$\tilde{P}^{(\frac{1}{2})-}_{\frac{1}{2}S}, \tilde{P}'^{(\frac{1}{2})-}_{\frac{1}{2}S}$ $2\times\frac{1}{2}^+$	$P^{(\frac{3}{2})-}_{\frac{3}{2}S}$ $\frac{3}{2}^+$
$\Pi^+(1D)$	$\left[\tilde{P}^{(\frac{3}{2}),(\frac{5}{2})}_{\frac{1}{2}D}\right]^-, \left[\tilde{P}'^{(\frac{3}{2}),(\frac{5}{2})}_{\frac{1}{2}D}\right]^-$ $2\times\left(\frac{3}{2},\frac{5}{2}\right)^+$	$\left[P^{(\frac{1}{2}),(\frac{3}{2}),(\frac{5}{2}),(\frac{7}{2})}_{\frac{3}{2}D}\right]^-$ $\left(\frac{1}{2},\frac{3}{2},\frac{5}{2},\frac{7}{2}\right)^+$

e.g.,

$$\tilde{P}_{\frac{1}{2}} \equiv \left| \frac{1}{2}_{s_{qqq}}, 0_{s_{Q\bar{Q}}} \right\rangle_{S=\frac{1}{2}}$$

Exotics with Known J^{PC}

- Can these multiplets accommodate the states with known (or favored values of) $J^{P(C)}$?
- No problem:

0^{++}	$X(3915)$, $X(4500)$, $X(4700)$
0^{--}	$Z_c^0(4240)$
1^{--}	$Y(4008)$, $Y(4220)$, $Y(4260)$, $Y(4360)$, $Y(4390)$, $X(4630)$, $Y(4660)$, $Y_b(10888)$
1^{++}	$X(3872)$, $Y(4140)$, $Y(4274)$
1^{+-}	$Z_c^0(3900)$, $Z_c^0(4200)$, $Z_c^0(4430)$, $Z_b^0(10610)$, $Z_b^0(10650)$
$\frac{3}{2}^{\pm}, \frac{5}{2}^{\mp}$	$P_c(4380)$, $P_c(4450)$

- Well, what about *all the other* predicted ones?
  Only a few production modes have been used to date, which prefer certain J^{PC}, such as 1^{--} for initial-state γ radiation

Ordering of the B-O Potentials

- How do we know what are lowest, next lowest, *etc.* B-O potentials? That's nonperturbative QCD!
- In the case of hybrids and pure-glue configurations, that information comes from numerous lattice QCD simulations
- State-of-the-art results:
  Hadron Spectrum Collaboration, JHEP **1207** (2012) 126; **1612** (2016) 089
- But it has a very long history:
  Griffiths, Michael, Rakow: PLB **129B** (1983) 351
  Juge, Kuti, Morningstar: Nucl. Phys. Proc. Suppl. **63** (1998) 326;
  PRL **82**, (1999) 4400; PRL **90** (2003) 161601
  Bali *et al*.: PRD **62** (2000) 054503
  Bali, Pineda: PRD **69** (2004) 094001
  Foster *et al*.: PRD **59** (1999) 094509
  Marsh, Lewis: PRD **89** (2014) 014502

Ordering of the B-O Potentials

- But all pure-glue simulations agree:
  - Ground-state potential: Σ_g^+
  - 1st excited potential: Π_u; 2nd excited potential: Σ_u^-
- Additionally, in the small-size limit, some potentials become degenerate gluelumps and mix, *e.g.*, $\Pi_u^+(1P)$ and $\Sigma_u^-(1P)$
  [Λ doubling: Berwein *et al.*, PRD **92** (2015) 114019]
- Great for hybrids! What about tetra/pentaquarks?
- Here, the only relevant lattice results use flavor-nonsinglet potentials for color-adjoint mesons:
  Foster, Michael: PRD **59** (1999) 094509
- What we really need for the diquark model is simulations with heavy sources that *also* carry isospin

Selection Rules

- Heavy-quark spin symmetry: $s_{Q\bar{Q}}$ should be conserved in a decay of a $Q\bar{Q}q_1\bar{q}_2$ (or $Q\bar{Q}q_1q_2q_3$) to $Q\bar{Q} + \text{light hadrons}$
- Exotics with $s_{Q\bar{Q}} = 1$ should decay to ψ (Υ) or χ
- Exotics with $s_{Q\bar{Q}} = 0$ should decay to η or h
- The evidence is mixed: For example,
  - The $c\bar{c}u\bar{d}$ states $Z_c^+(3900) \to J/\psi$, while $Z_c^+(4020) \to h_c$
  - The $b\bar{b}u\bar{d}$ states $Z_b^+(10610), Z_b^+(10650) \to$ both Υ, h_b
- The latter case suggests a mixture of $s_{Q\bar{Q}}$ eigenstates
  One way for this to occur is molecular states (good $s_{Q\bar{q}}$, $s_{\bar{Q}q}$)
  Or, good diquark-spin quantum numbers (good s_{Qq}, $s_{\bar{Q}\bar{q}}$)

Selection Rules

- B-O potential quantum numbers:
  Separate conservation of light d.o.f. quantum numbers (since they undergo more rapid transitions than heavy d.o.f.)
- Example: Consider $Q\bar{Q}q_1\bar{q}_2\ (\Lambda_\eta^\epsilon) \rightarrow Q\bar{Q}\ \left(\Sigma_g^+\right) + \rho/\omega$ (s-wave)
- Then J^{PC} conservation forbids this decay unless:
  $$\Lambda \leq 1 + s_{q\bar{q}}, \qquad \epsilon = (-1)^{\Lambda+1}, \qquad \eta = +$$
- But in comparing to the known decays, these rules only work if some Λ_η^ϵ potentials besides the ones seen for pure glue are among those of lowest energy
- Again, lattice simulations with heavy diquark sources would completely resolve this question

Summary

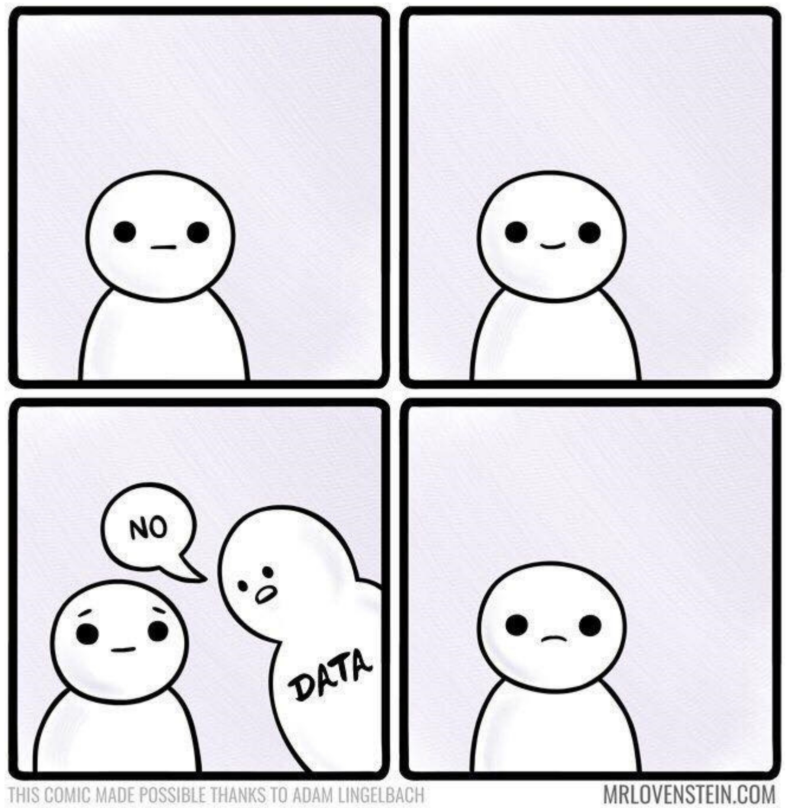
- We now appear to live in an age of at least four known hadron species: mesons, baryons, tetraquarks, and pentaquarks
- This talk focused on the construction of multiquark exotics composed of colored diquark (and triquark) components
- The dynamical diquark picture says that several properties of the exotics can be explained if the colored diquark components achieve a substantial spatial separation
- The most convenient framework for describing such states is the Born-Oppenheimer approximation
  We studied the relevant quantum numbers,
  built the particle spectrum, and examined decay selection rules

So What Next?

- Choose particular forms for $V_{\Lambda_\eta^\epsilon}(r)$, feed into Schrödinger equations, solve for the spectrum and decay amplitudes
- Issue: Need high-quality lattice results including heavy diquark sources to know the correct forms of $V_{\Lambda_\eta^\epsilon}(r)$
- Are there isospin-dependent forces analogous to π exchange? One lesson from dense QCD color-flavor-locking [Alford, Rajagopal, Wilczek, PLB **422** (1998) 247]: Isospin-carrying Goldstone bosons exist even inside glue fields
- Genuine hadronic (*e.g.*, meson-meson) thresholds mix with (*e.g.*, diquark-antidiquark) resonances and can lead to nontrivial level-crossing behavior in the spectrum and decays

Will It All Work?

Ask me again in a couple of years!



Thank you!