Extracting Resonances in Eta-Pi Diffractive Production

Vincent MATHIEU

Jefferson Lab
Joint Physics Analysis Center

Cake Seminar
JLab, May 2018

Jefferson Lab
Thomas Jefferson National Accelerator Facility

JPAC
Joint Physics Analysis Center

Outline

Introduction

- **Joint Physics Analysis Center (JPAC)**
- **Exotic mesons**
- **Reaction** $\pi^- p \to \eta\pi^- p$

Past:

- **Extracted a2(1320) and a2(1700) pole position**

  Jackura et al (JPAC), PLB774, arXiv:1707.02848

Present:

- **Extraction of exotic meson pole position**

  A. Rodas, A. Pilloni et al (JPAC) in preparation

Future:

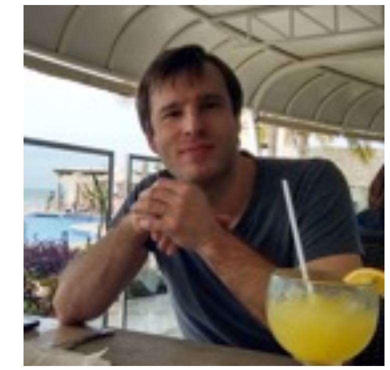
- **Implementation of DR constraining model**
- **Transposition to GlueX/CLAS12 data**

  VM et al (JPAC), work in progress

Joint Physics Analysis Center

Emilie Passemar Indiana U.
Andrew Jackura Indiana U.
Nathan Sherrill Indiana U.
Tim Londergan Indiana U.

Astrid Hiller Blin Mainz U.
Misha Mikhasenko Bonn U.
VM JLab
Jannes Nys Ghent U.
Adam Szczepaniak Indiana U.



Cesar Fernández-Ramírez
UNAM

Alessandro Pilloni
JLab

Łukasz Bibrzycki
Cracow P. U.

Arkaitz Rodas Bilbao
Madrid U.

Viktor Mokeev
JLab

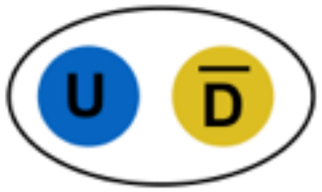
Miguel Albaladejo
Murcia U.

Hybrid Mesons

0^{--}	0^{-+}	0^{+-}	0^{++}
1^{--}	1^{-+}	1^{+-}	1^{++}
2^{--}	2^{-+}	2^{+-}	2^{++}
3^{--}	3^{-+}	3^{+-}	3^{++}
4^{--}	4^{-+}	4^{+-}	4^{++}

J^{PC} $q\bar{q}$ **allowed**

Ordinary



J^{PC} $q\bar{q}$ **not allowed**

Hybrid

coupled to $\eta\pi$ **and** $\eta'\pi$
(isospin 1)

Quantum numbers filter ordinary mesons

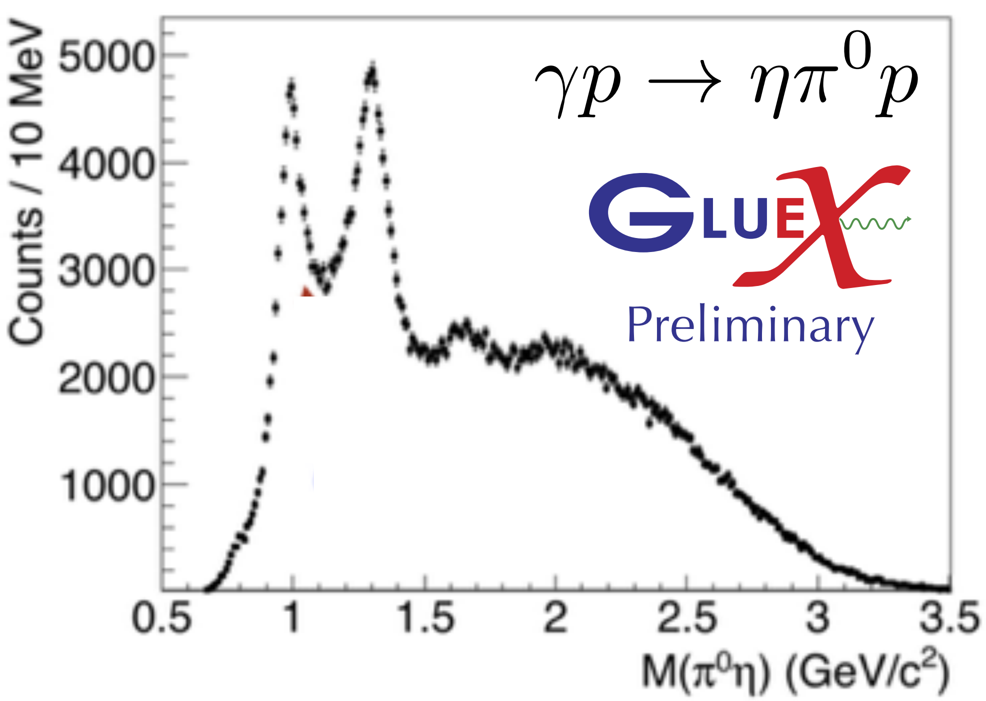
Easier identification of hybrid mesons with exotic quantum numbers

Light Meson Spectroscopy

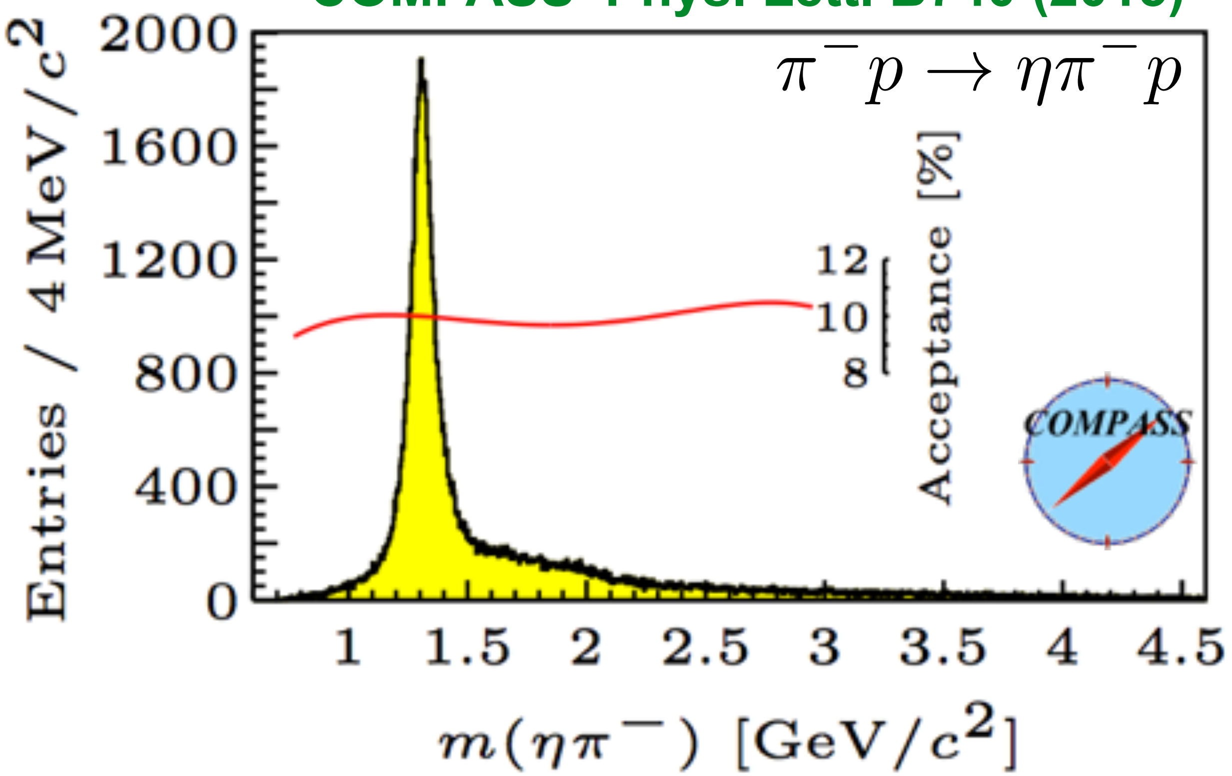
$$E_{\text{beam}} = 9\ \text{GeV}$$

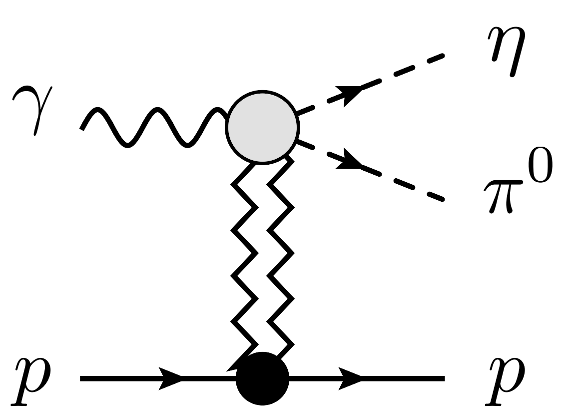
$$E_{\text{beam}} = 190\ \text{GeV}$$

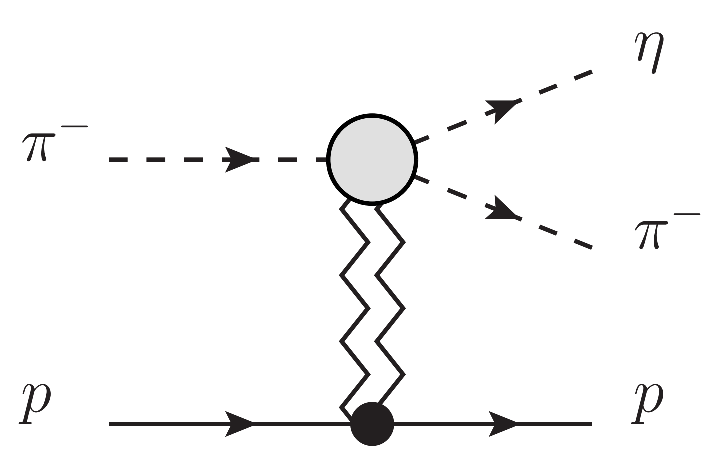
GlueX



COMPASS Phys. Lett. B740 (2015)

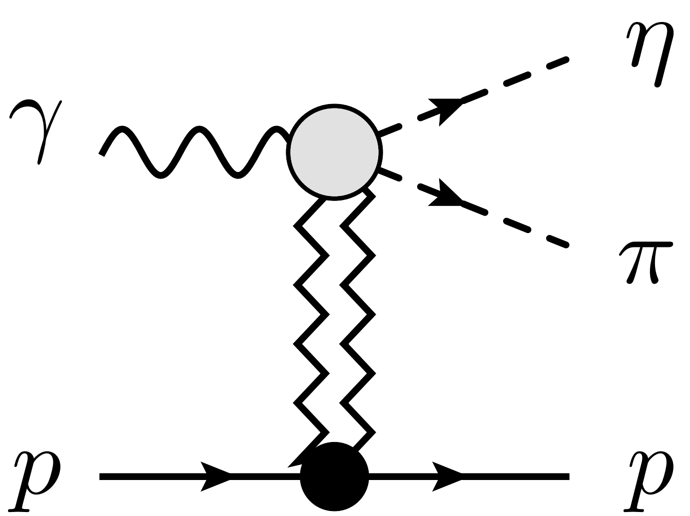


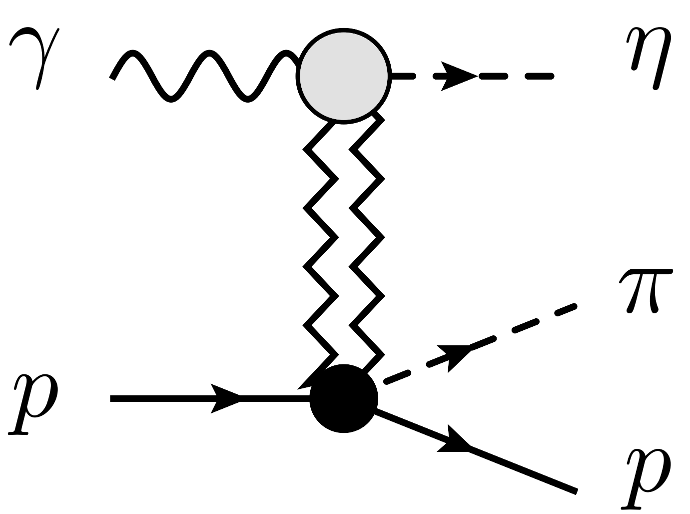


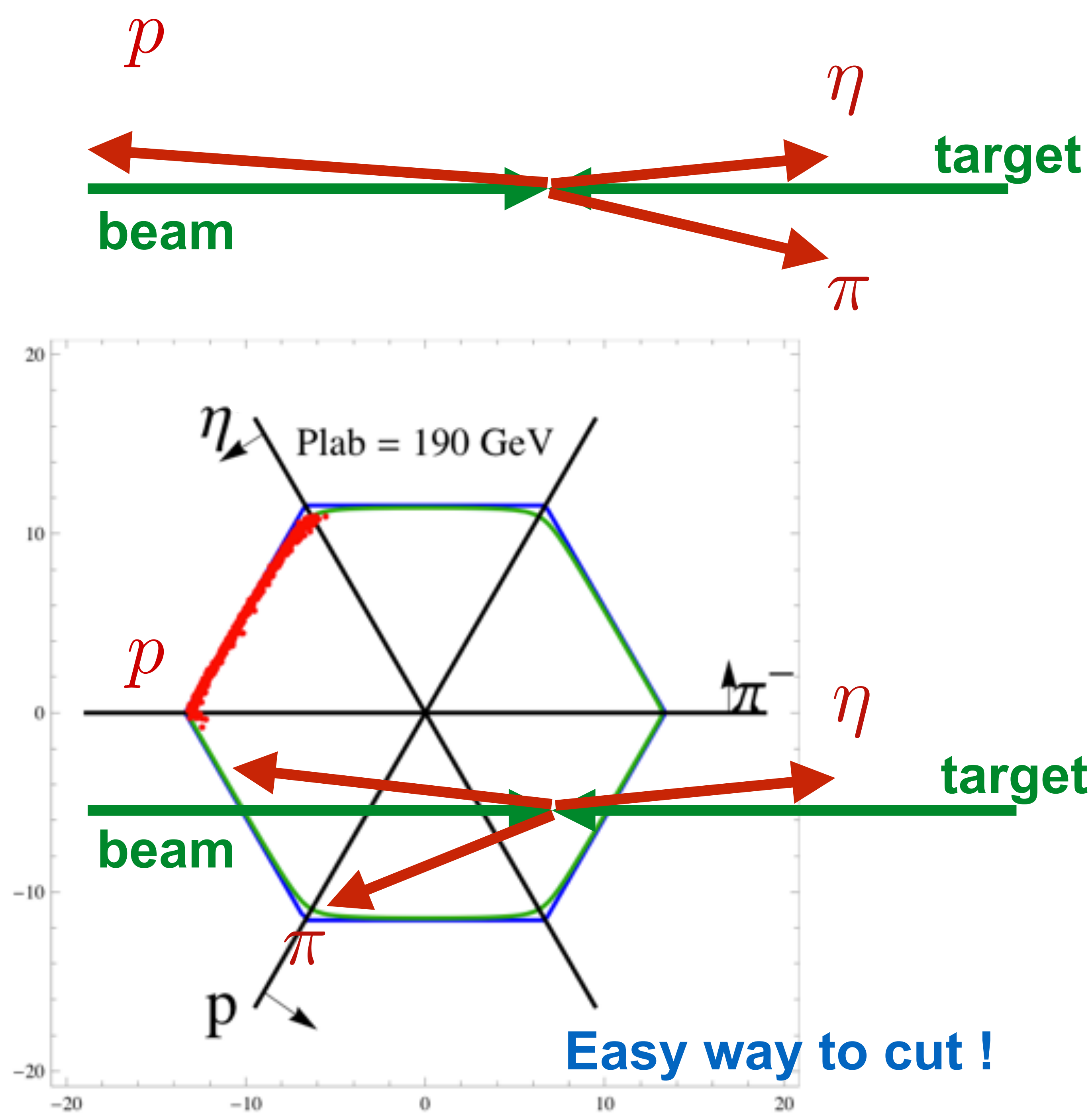


Contamination by Target Fragmentation

How do we select beam fragmentation ? → **Boost in the rest frame**





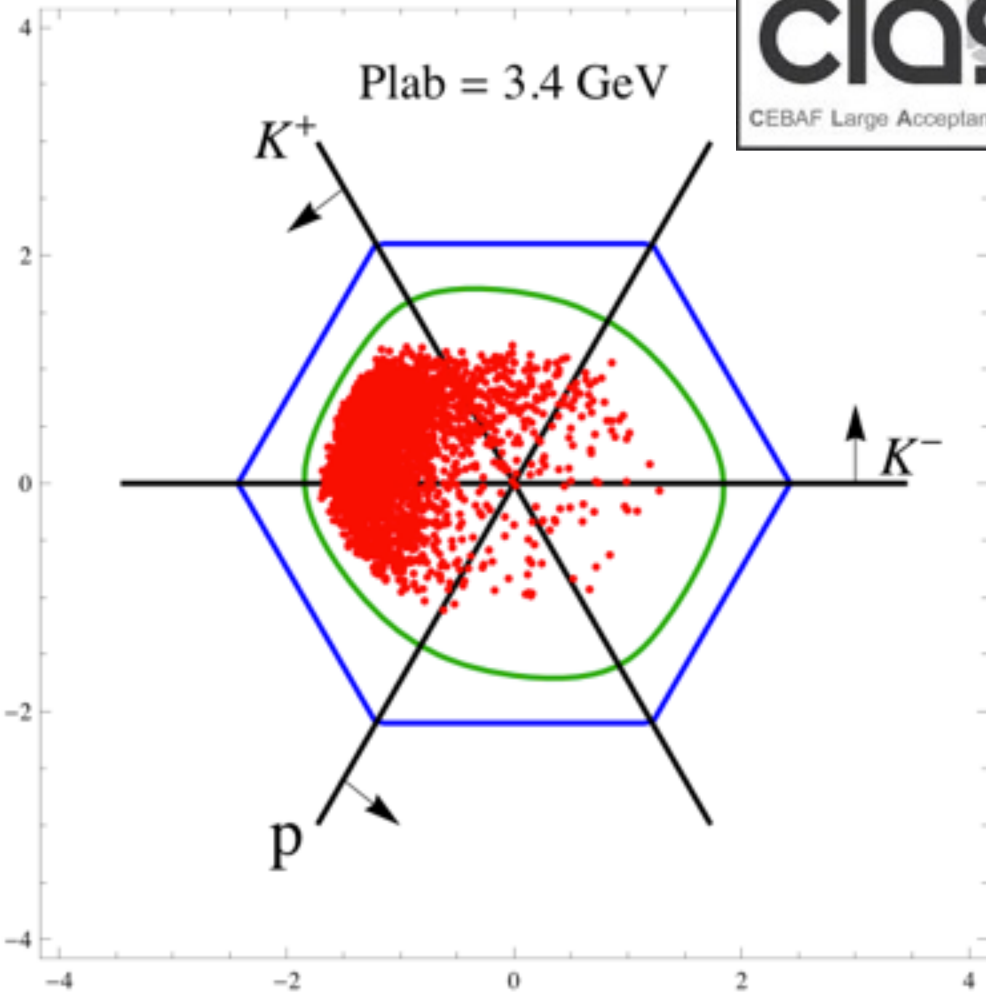


Easy way to cut !

Van Hove NPB9 (1969) 331

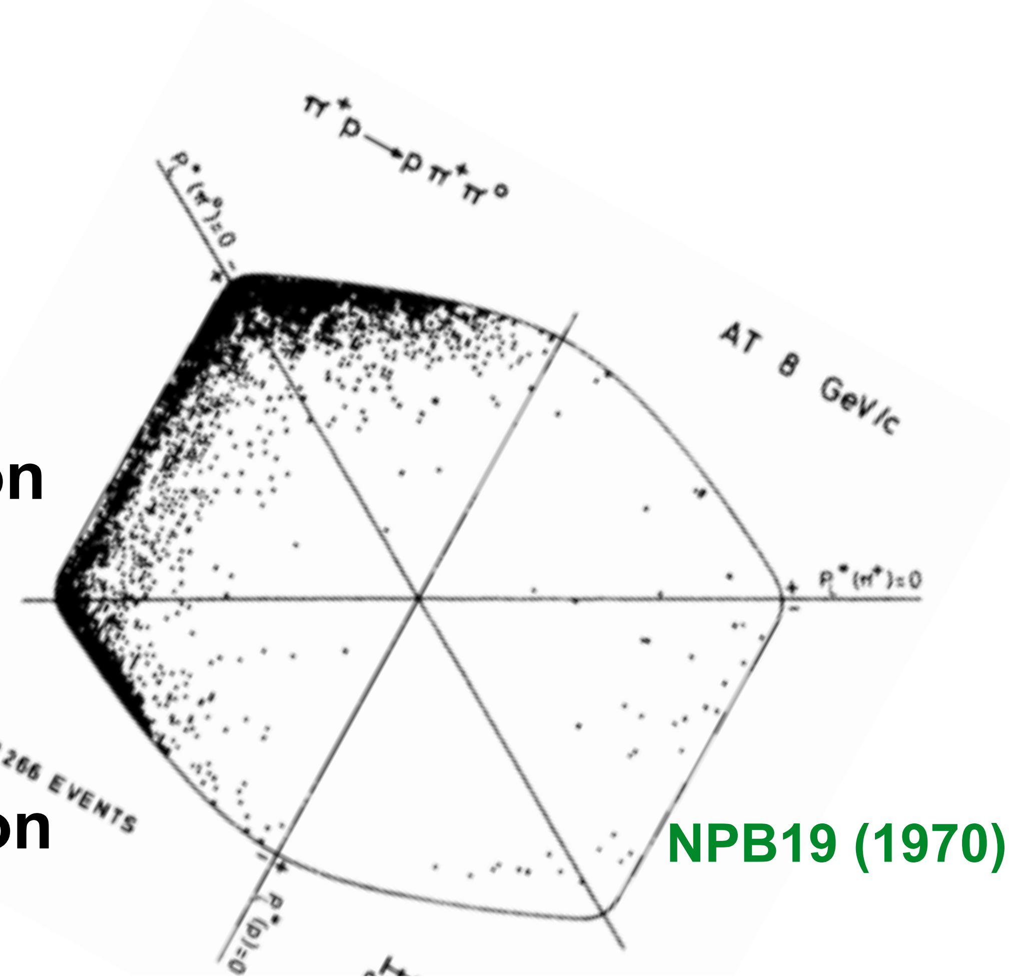
M. Shi et al (JPAC) PRD91 (2015) 034007

$$\gamma p \to K^+ K^- p$$

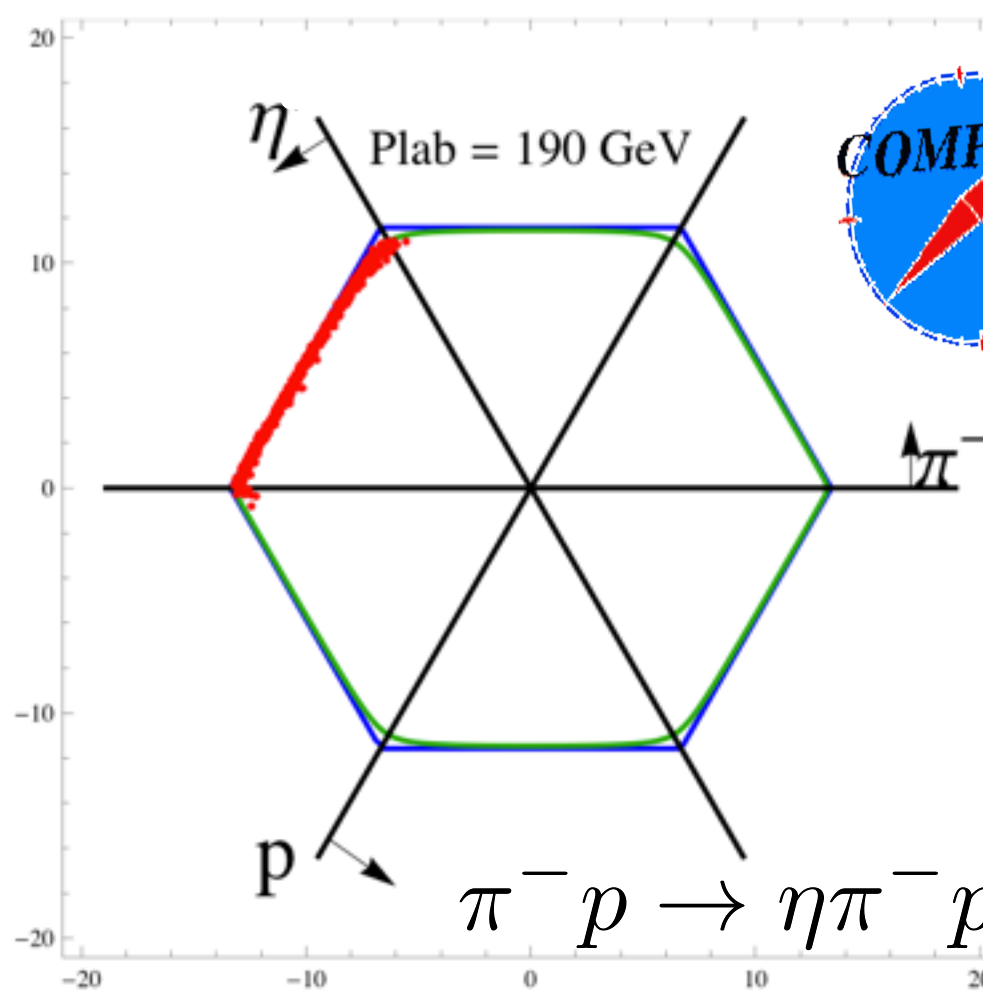


meson

baryon

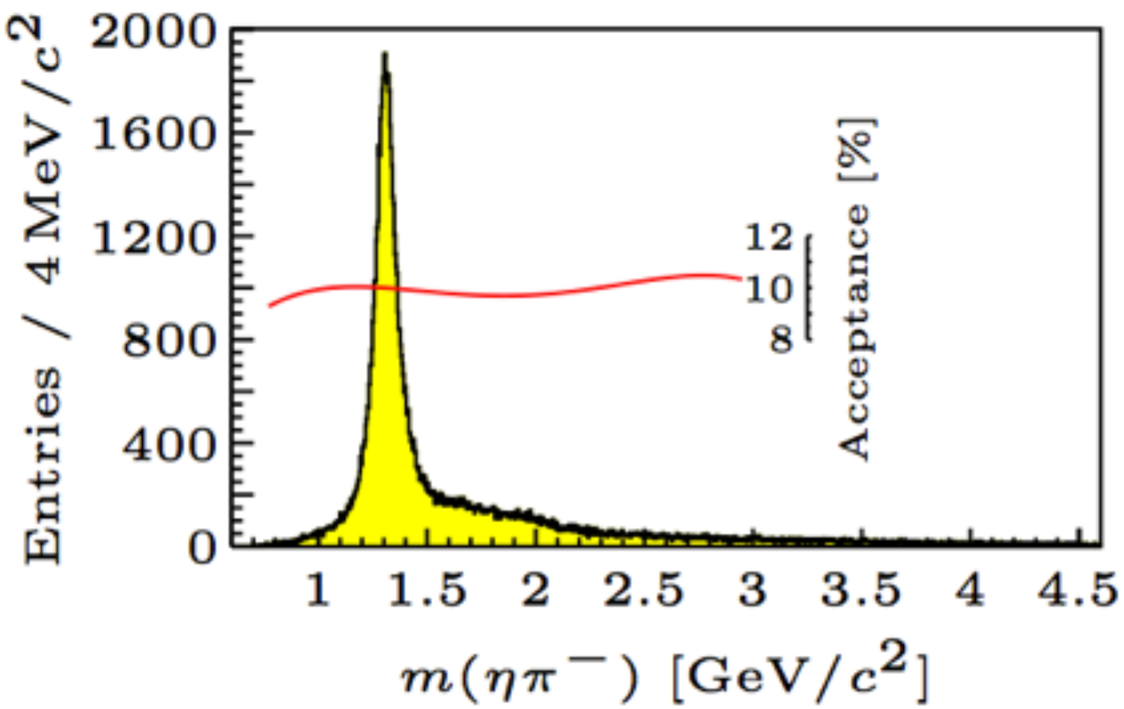


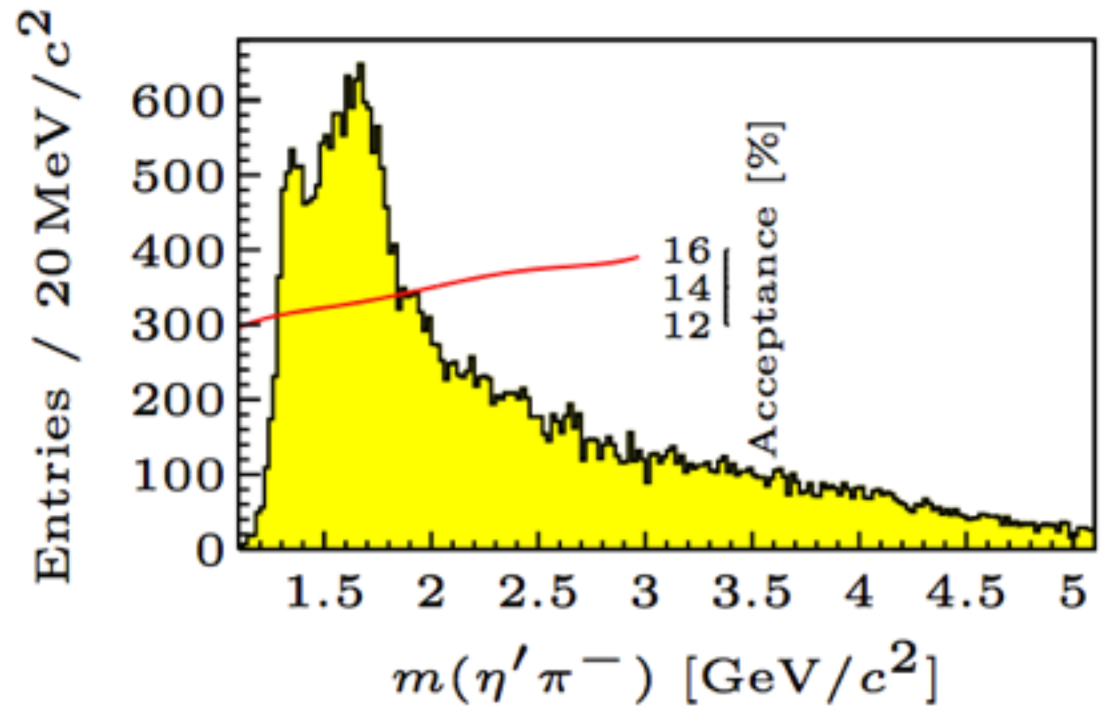
NPB19 (1970) 381

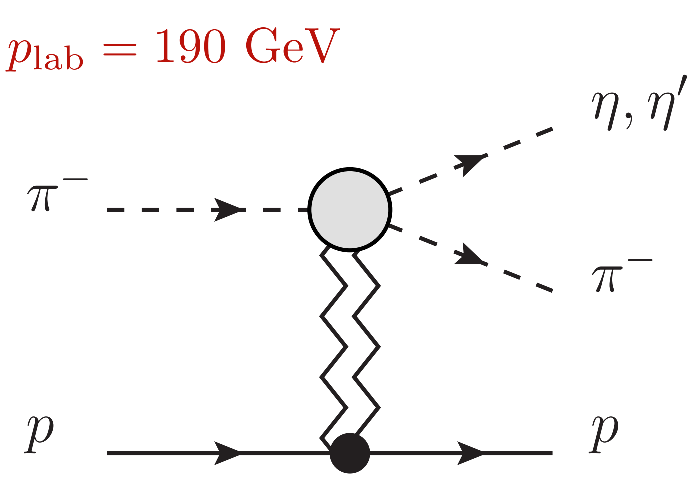


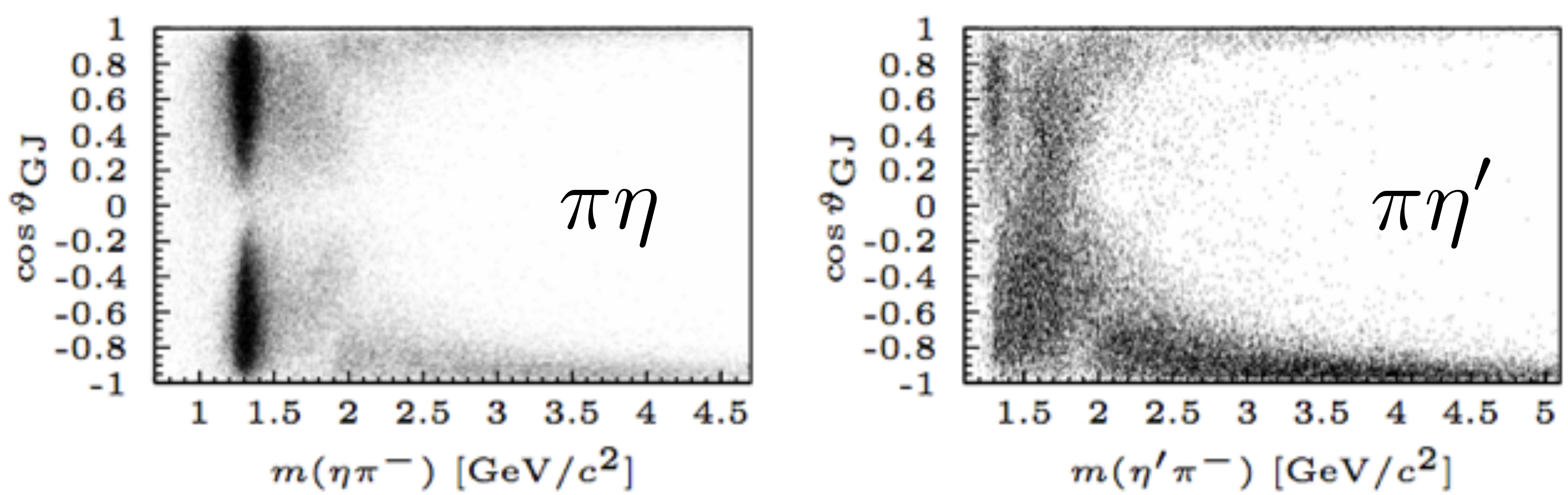
$$\pi^- p \to \eta \pi^- p$$

Eta-Pi @COMPASS









$$(s, t_{\mathbb{P}}, s_{\eta\pi}, \theta, \phi)$$

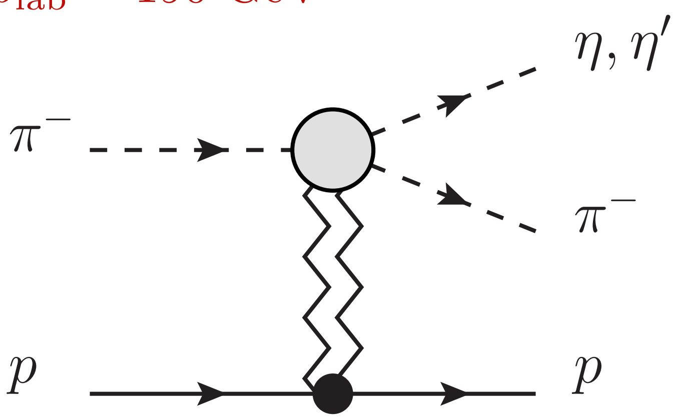
Gottfried-Jackson frame

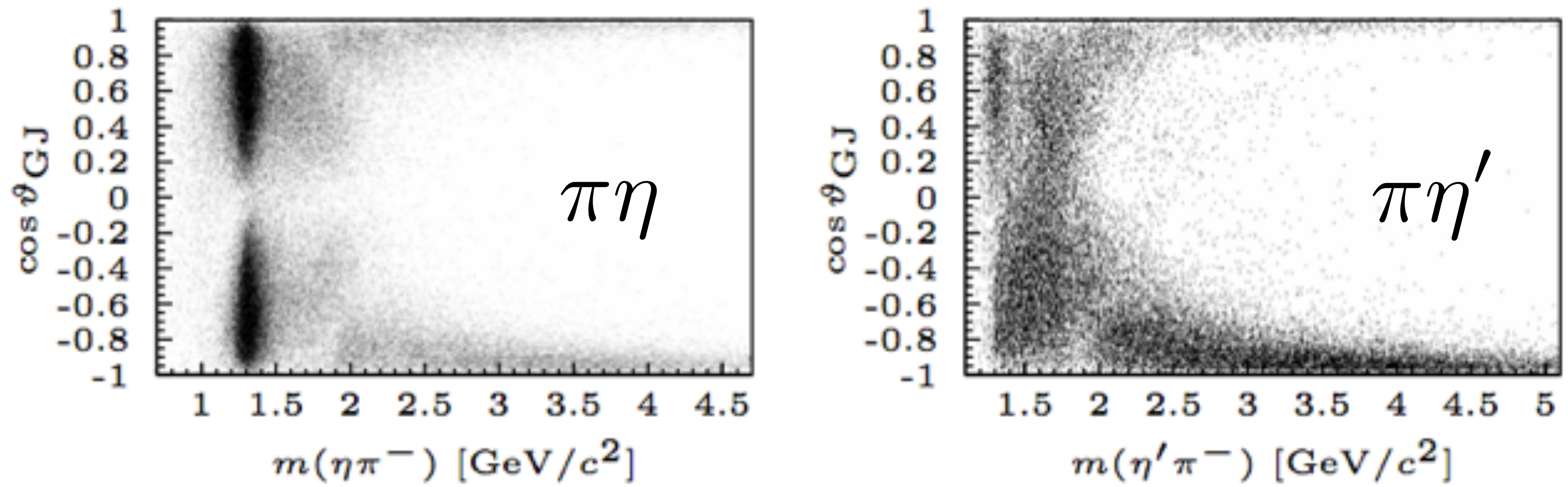
COMPASS Phys. Lett. B740 (2015)

Partial Waves

COMPASS Phys. Lett. B740 (2015)

$p_{\text{lab}} = 190$ GeV



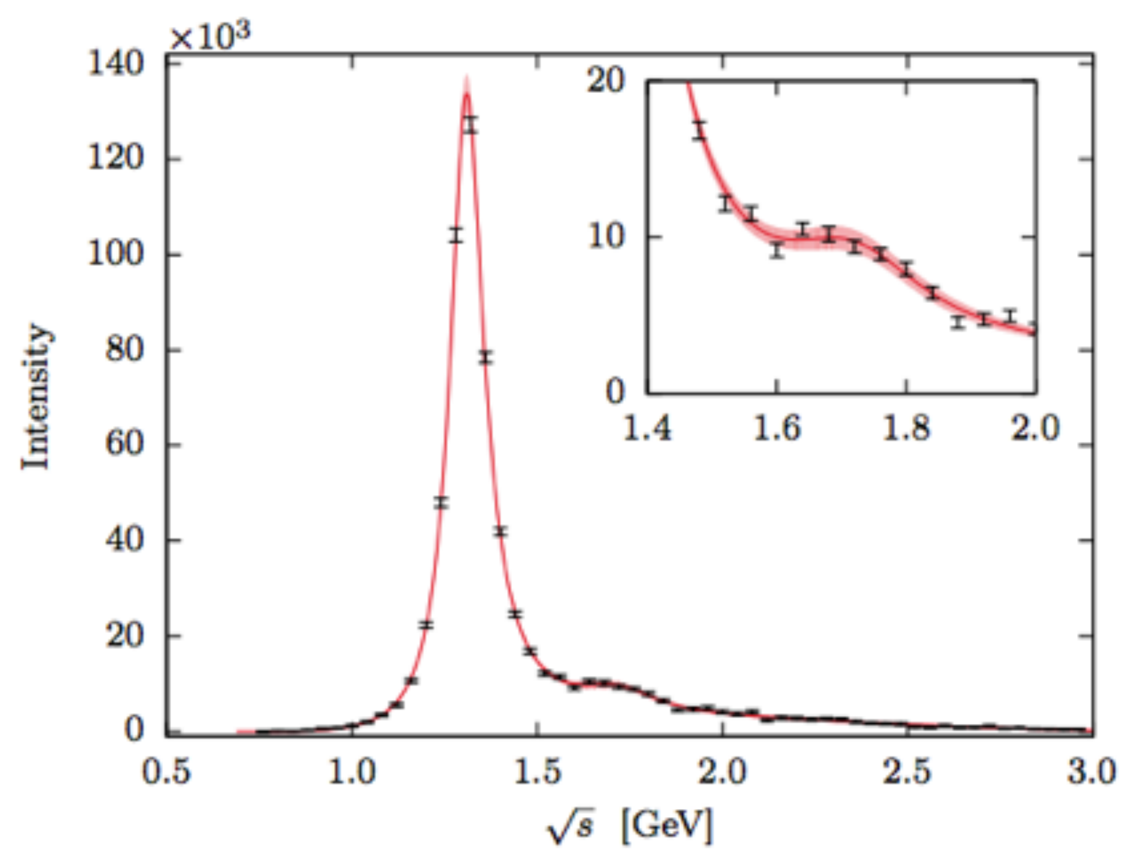


Quantum numbers determined by angular momentum

Which partial wave can yield the '8' in $\eta\pi$?

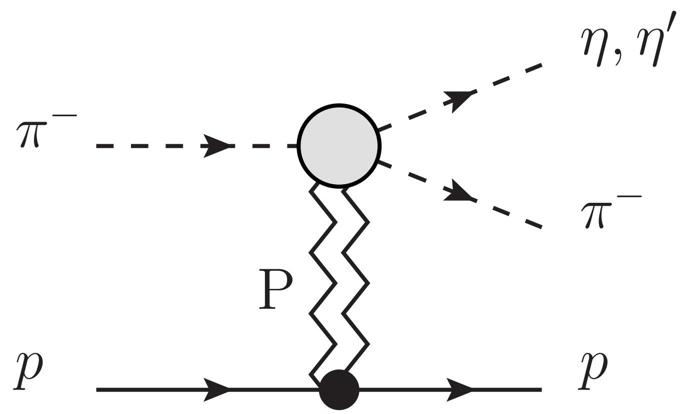
$$a_2(1320) : I^G J^{PC} = 1^- 2^{++}$$

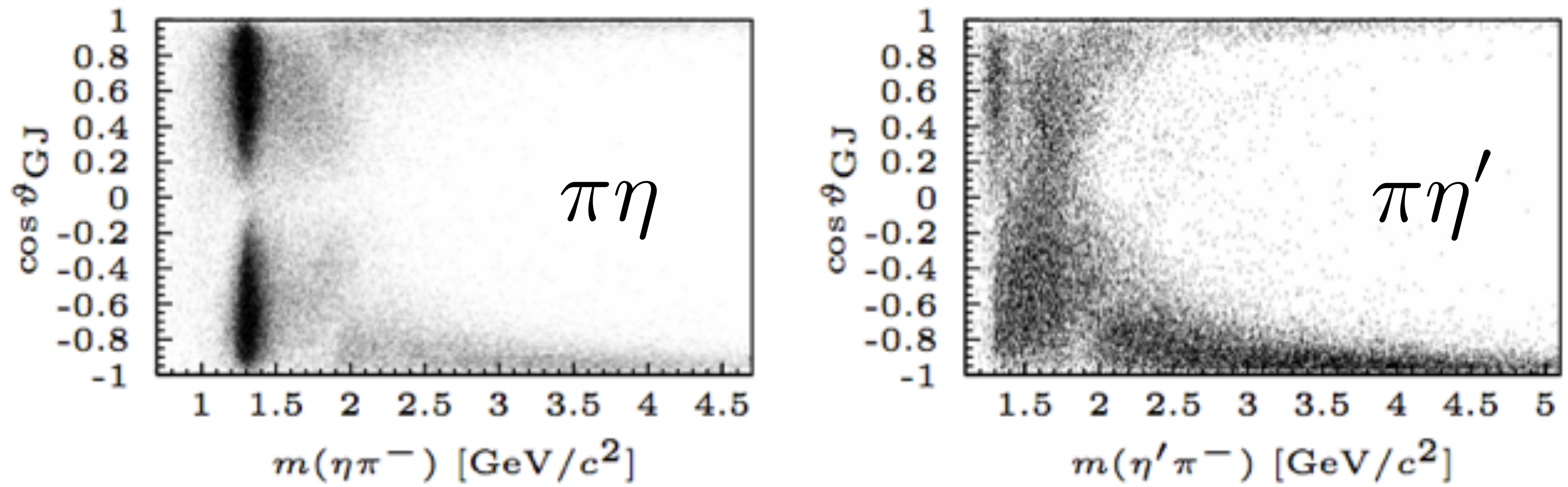
$$d^2_{1,0}(\theta) \propto Y^1_2(\theta, 0) \propto \sin\theta\cos\theta$$

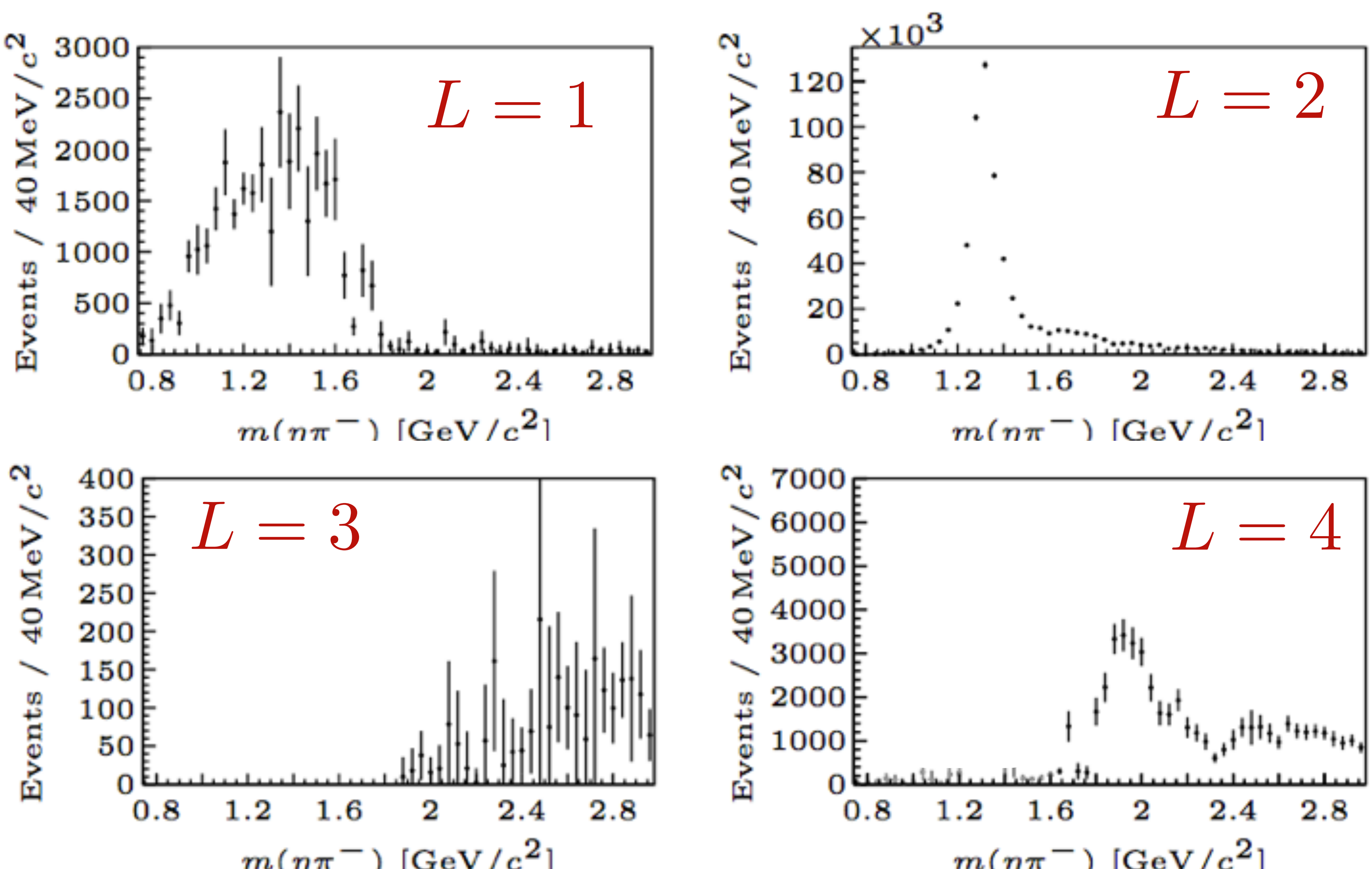


Partial Waves

COMPASS Phys. Lett. B740 (2015)

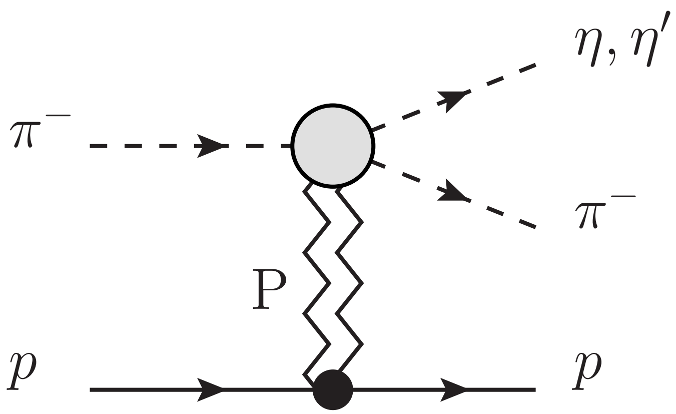


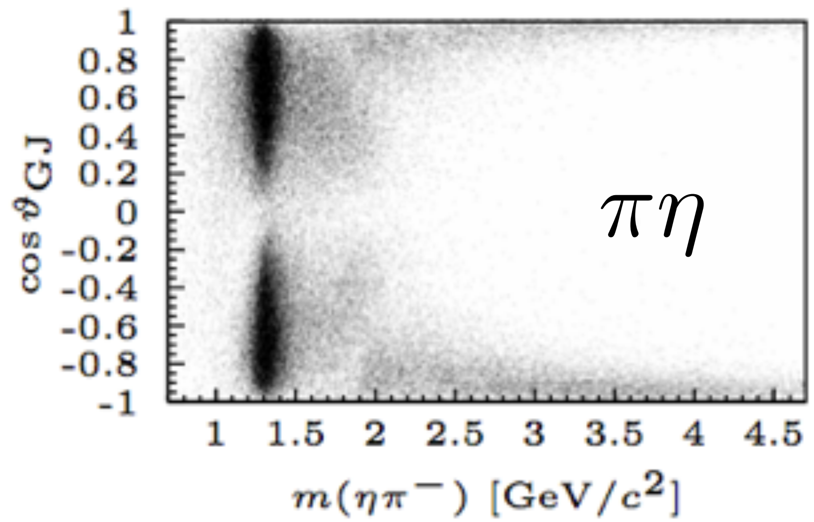


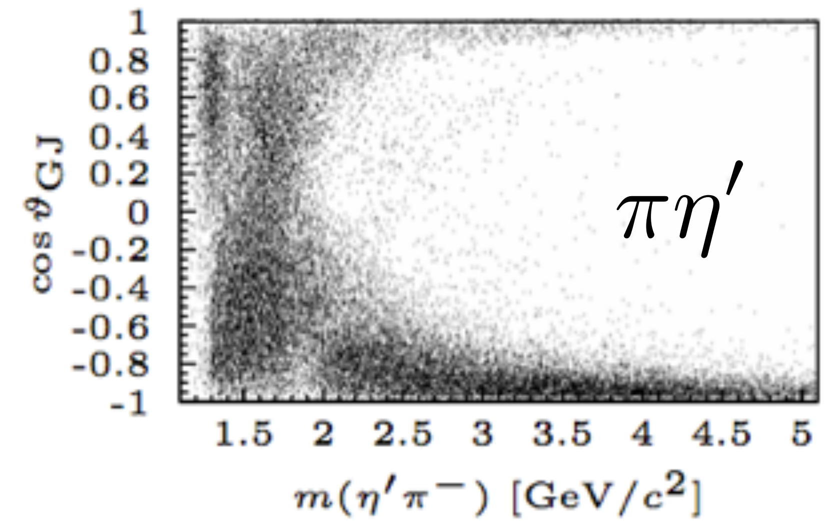


Partial Waves

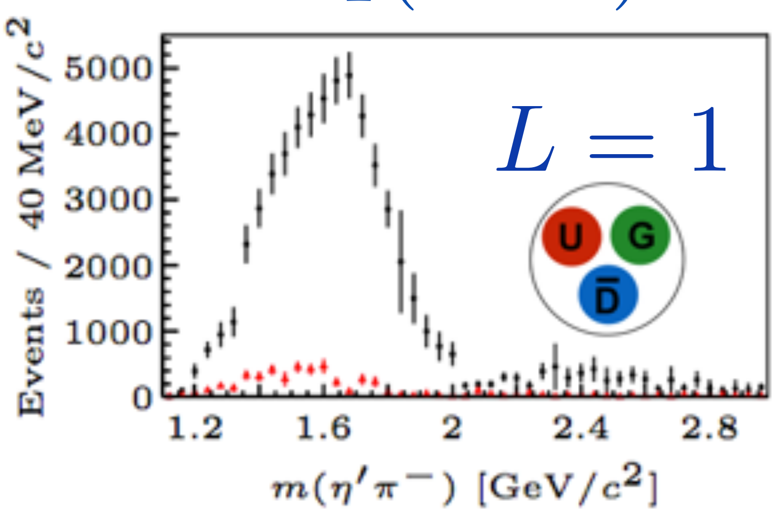
COMPASS Phys. Lett. B740 (2015)



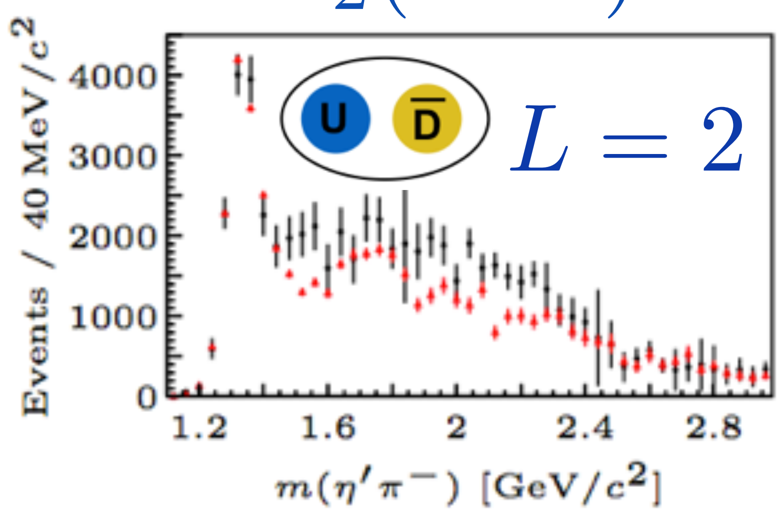




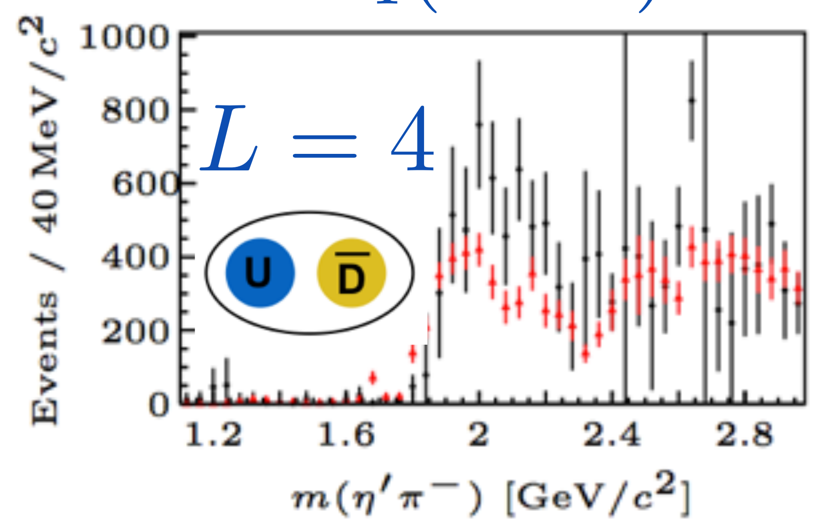
$\pi_1(1600)$?

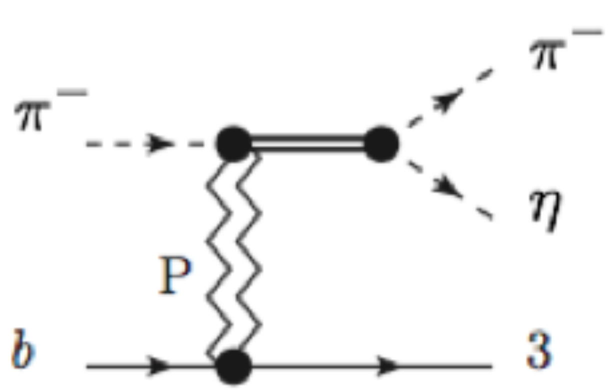


$a_2(1320)$



$a_4(2040)$



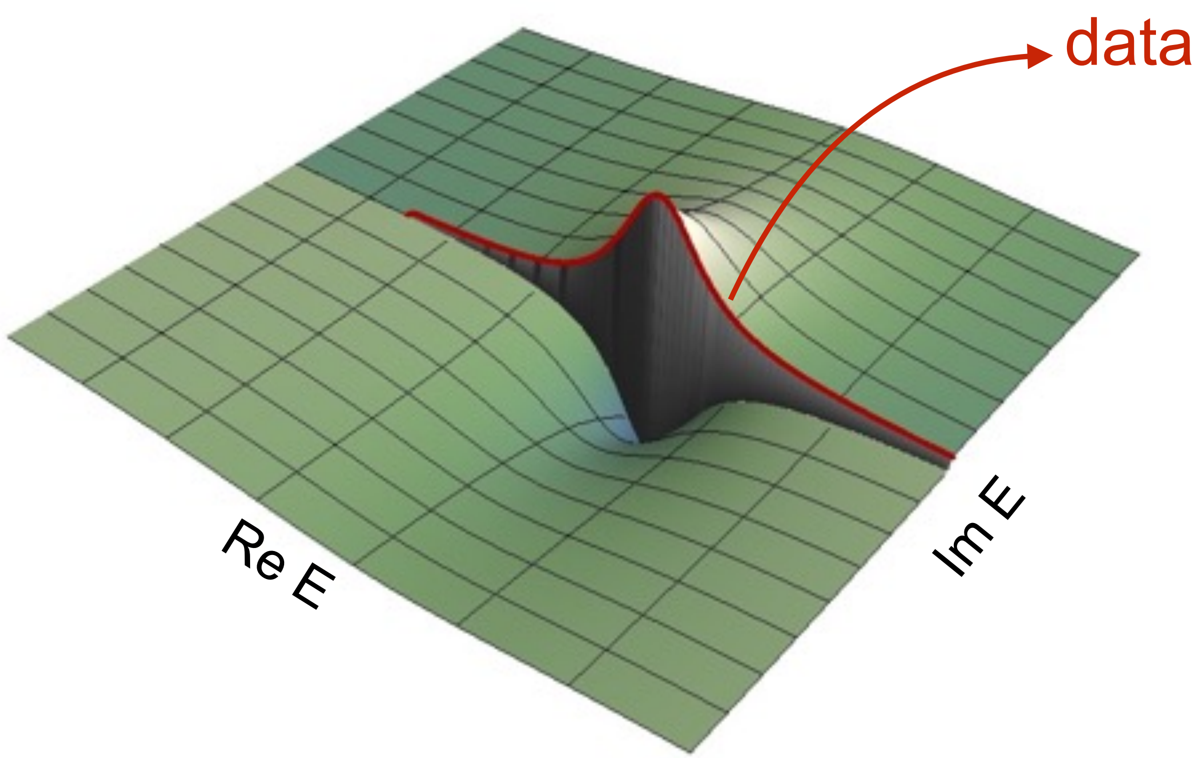


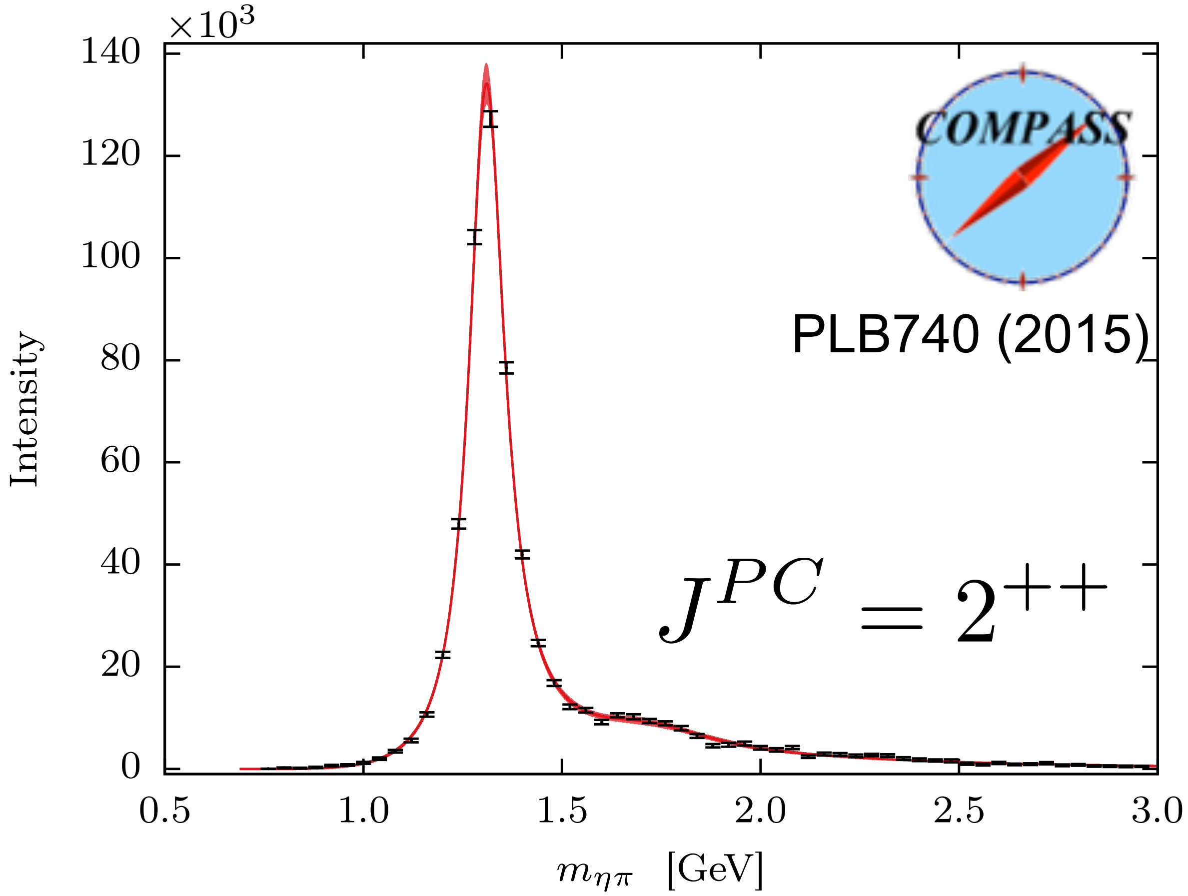
black: $\pi\eta'$

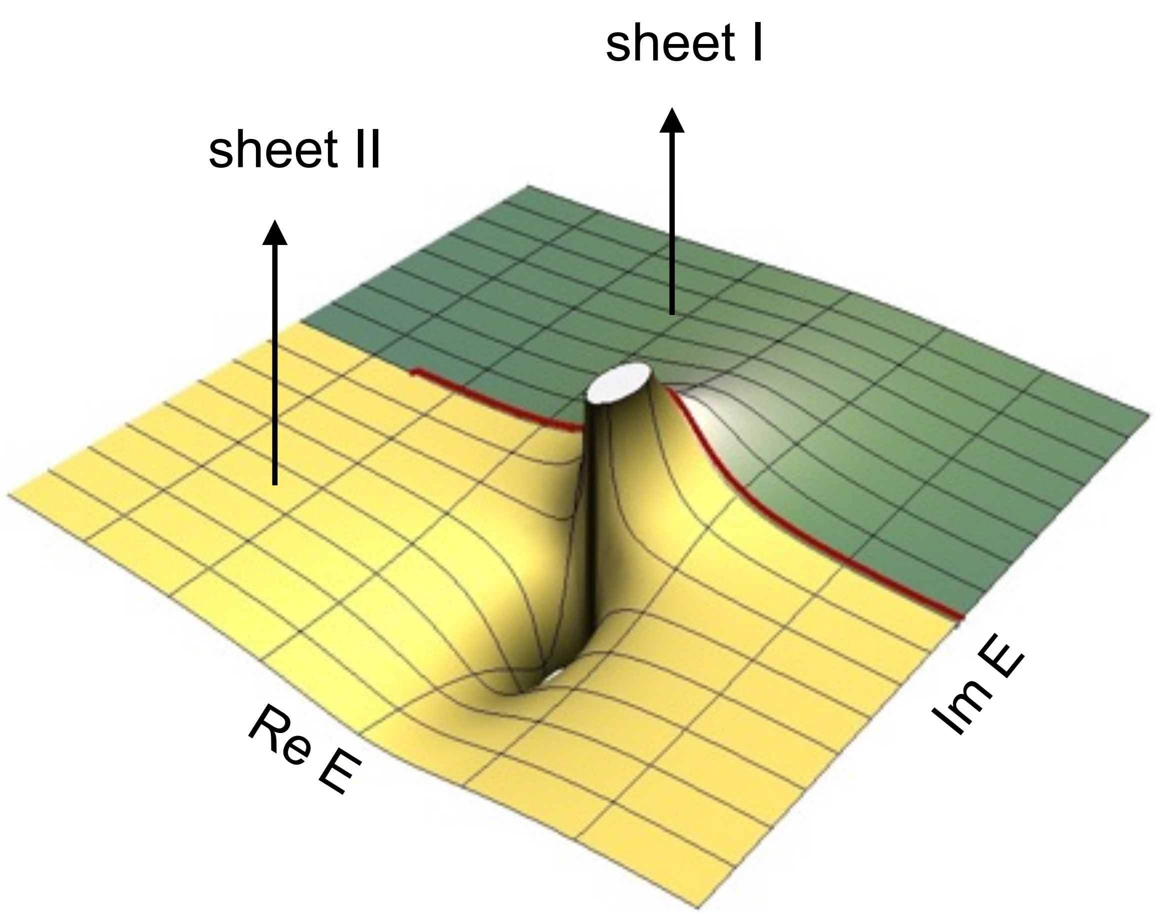
red: $\pi\eta$ **(scaled)**

Resonance in angular mom. L = 1 ?

Resonances as poles



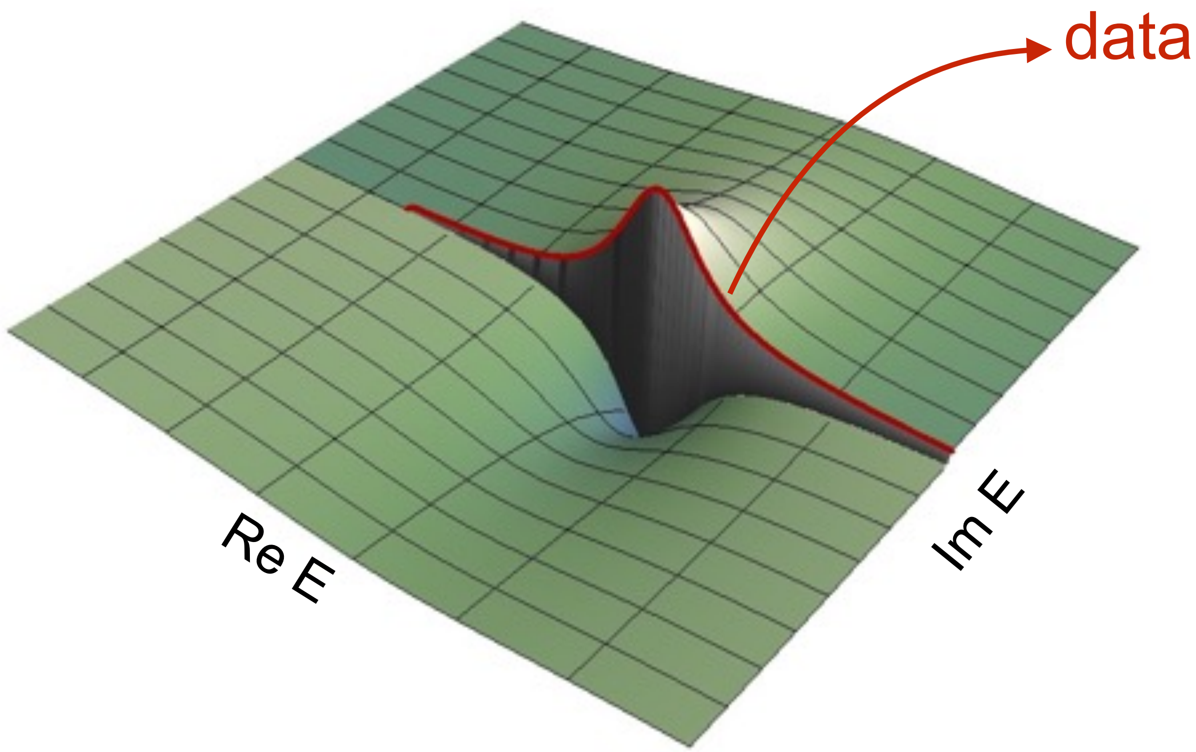




Poles in the complex energy plane:
- **Real part ~ mass**
- **Imaginary part ~ width**
- **Residue ~ coupling**

Poles or resonances are the universal building blocks of reactions

Two-body unitarity

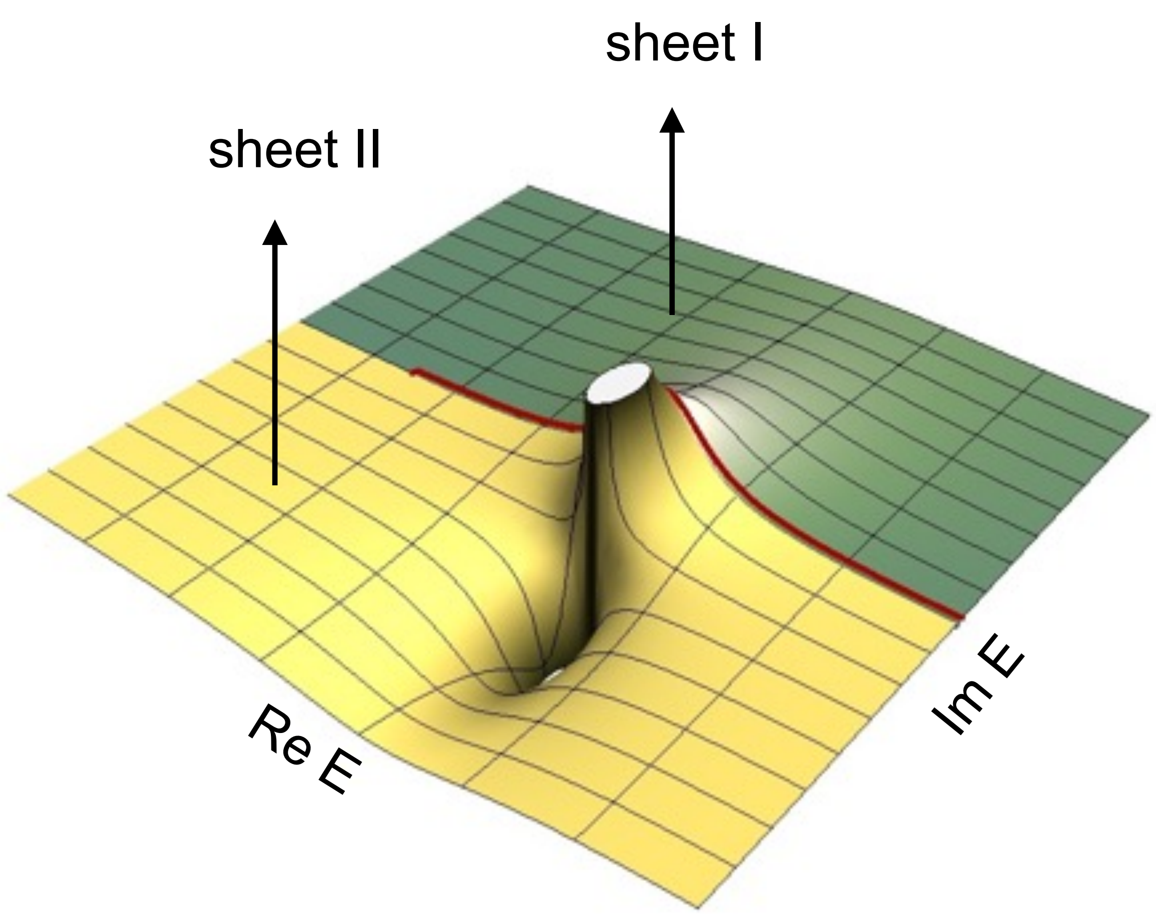


partial wave ← → phase space

$$\mathrm{Im}\ t_\ell^{-1}(s) = -\rho(s)$$

$$t_\ell(s \pm i\epsilon) = \frac{1}{K(s) \mp i\rho(s)}$$

satisfies causality
(regular outside the real axis)



define function on sheet II
on the lower half plane

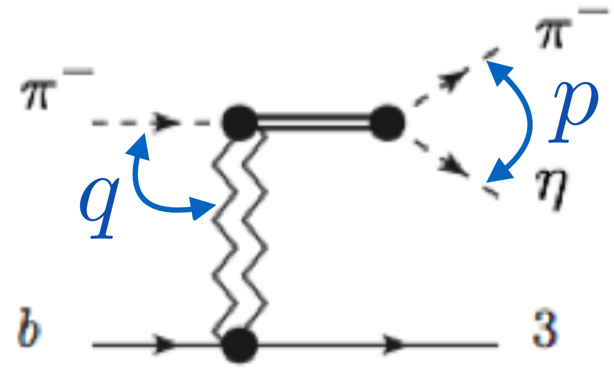
$$t_\ell^{II}(s) = \frac{1}{K(s) - i\rho(s)}$$

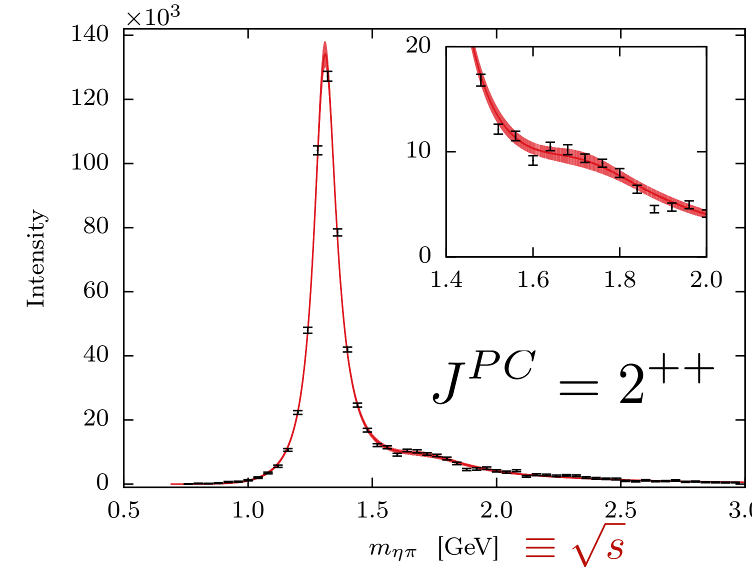
example $= \dfrac{m\Gamma}{m^2 - s - i\rho(s)m\Gamma}$

Eta-Pi@COMPASS

normalization

$$\frac{d\sigma}{d\sqrt{s}} = N p |a(s)|^2$$





Chebyshev polynomials

production

$$n(s) = \frac{q}{c_3 - s} \sum_n a_n T_n(\omega(s))$$

$$\omega(s) = \frac{s}{s + \Lambda}$$

$$a(s) = p^2 \frac{n(s)}{D(s)}$$

dynamics (poles)

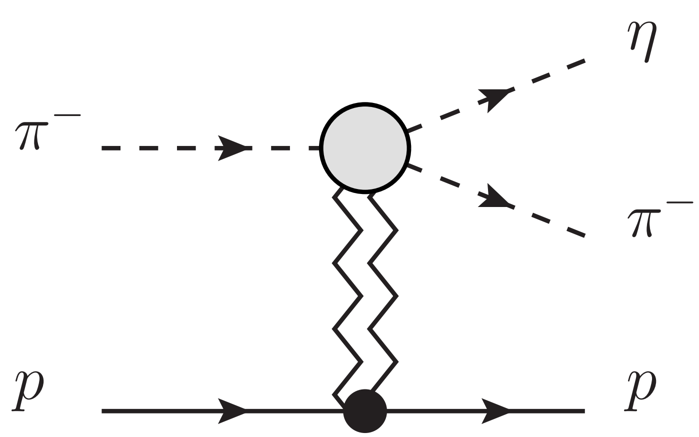
$$D(s) = D_0(s) - \frac{s}{\pi} \int_{s_{\rm th}}^{\infty} \frac{ds'}{s'} \frac{\rho(s')N(s')}{s' - s}$$

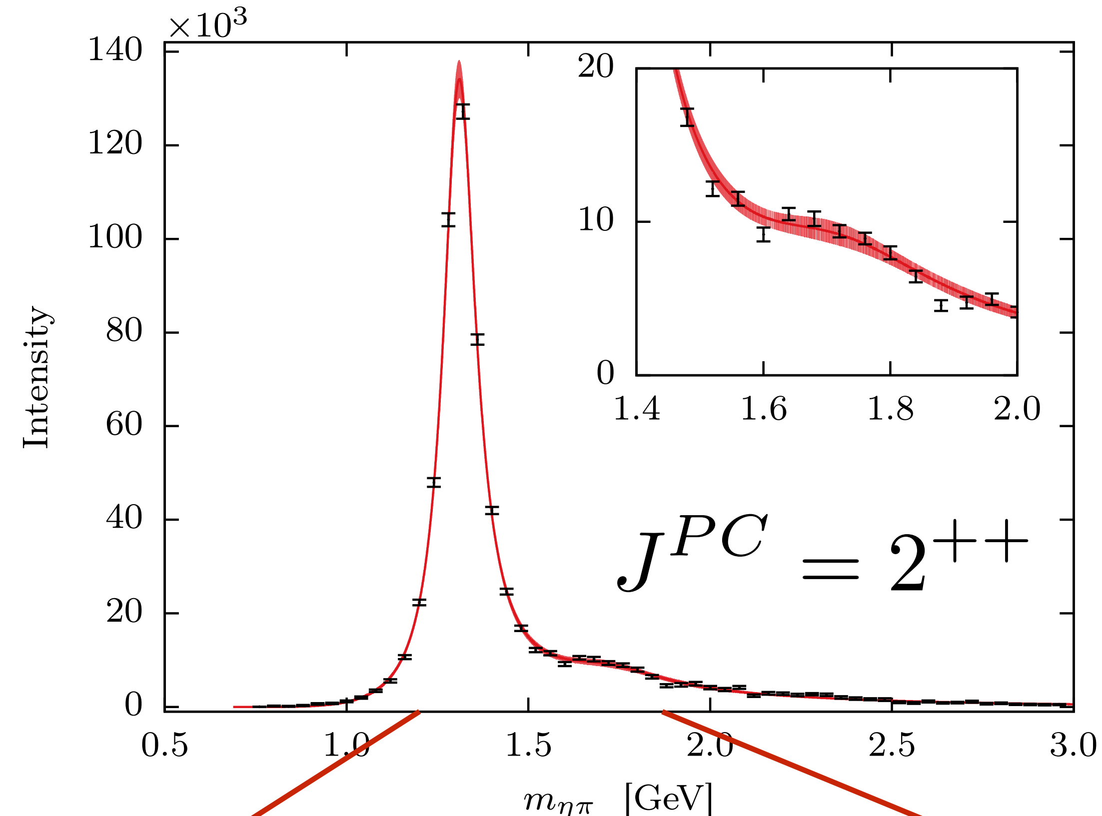
real (masses)

imaginary (widths)

$$D_0(s) = c_0 - c_1 s$$

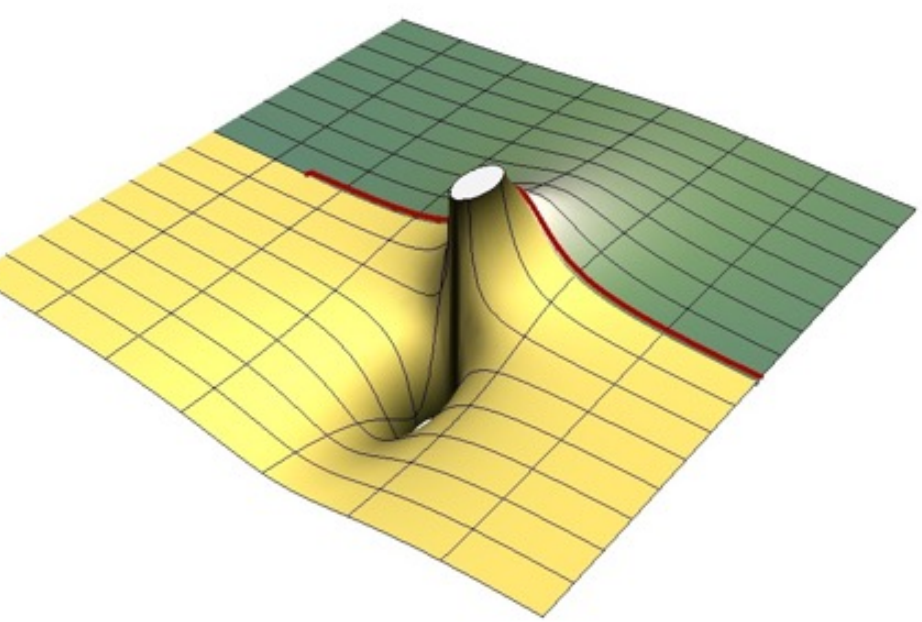
$$\rho(s)N(s) = g \frac{\lambda^{J+\frac{1}{2}}(s, m_\eta^2, m_\pi^2)}{(s + s_R)^{2J+3}}$$

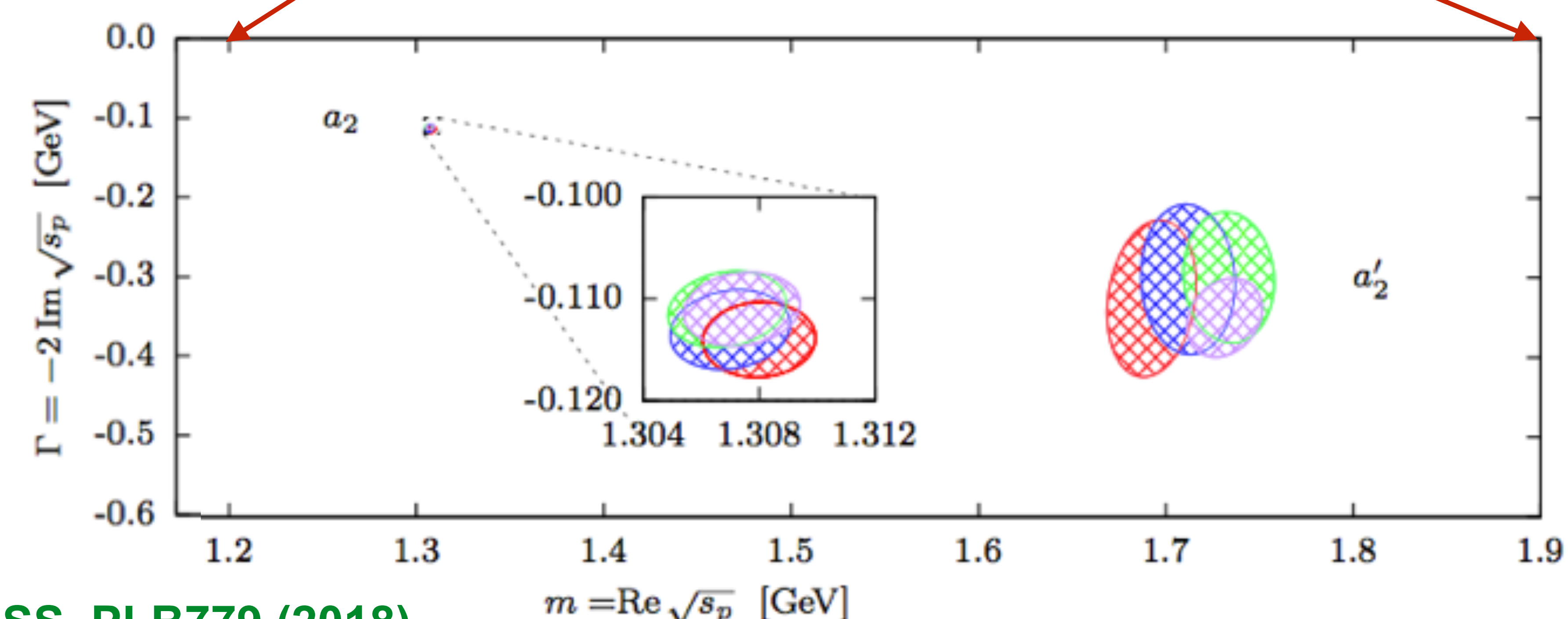




first precise determination of

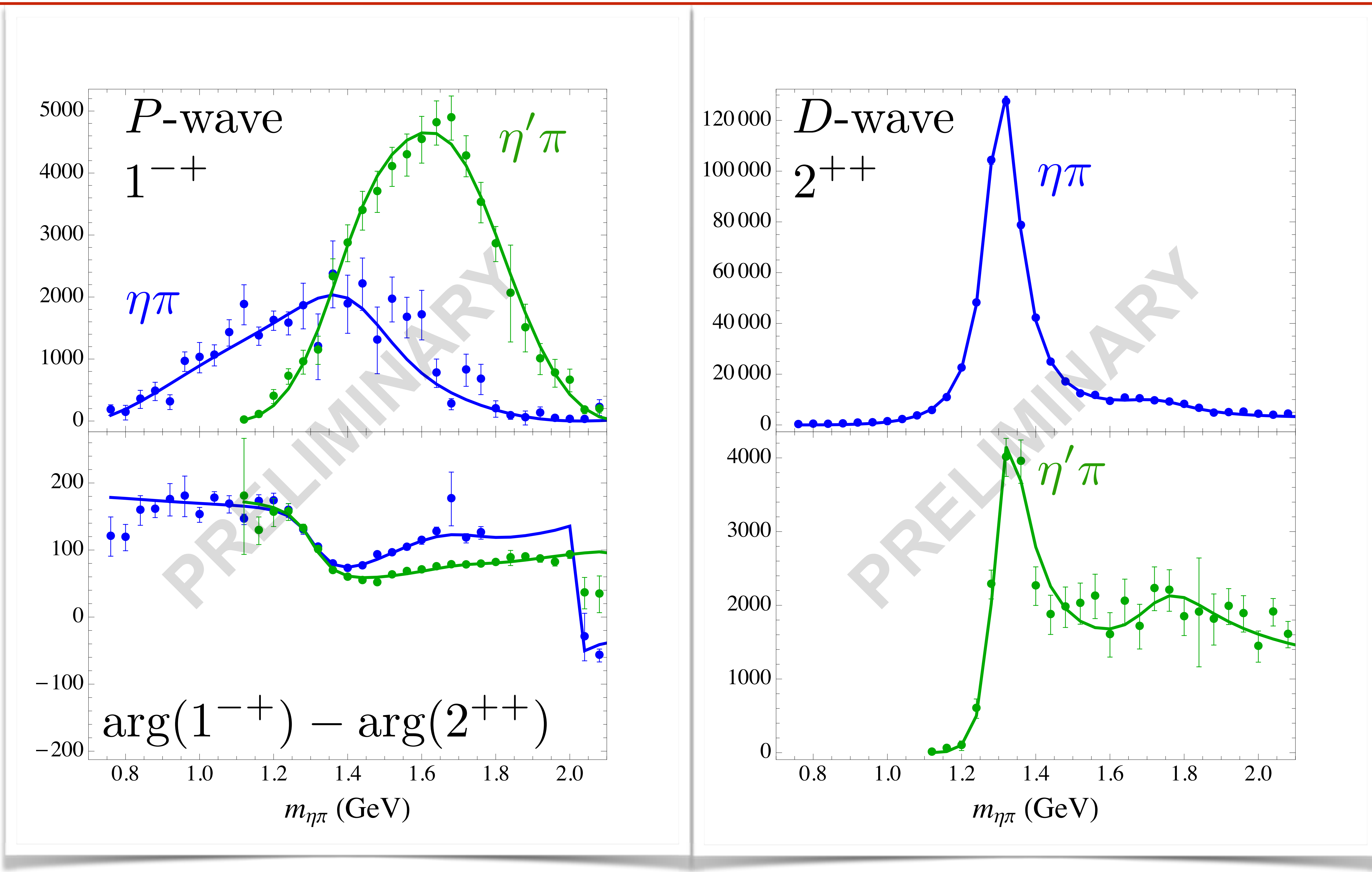
$a_2(1700)$ **pole location**





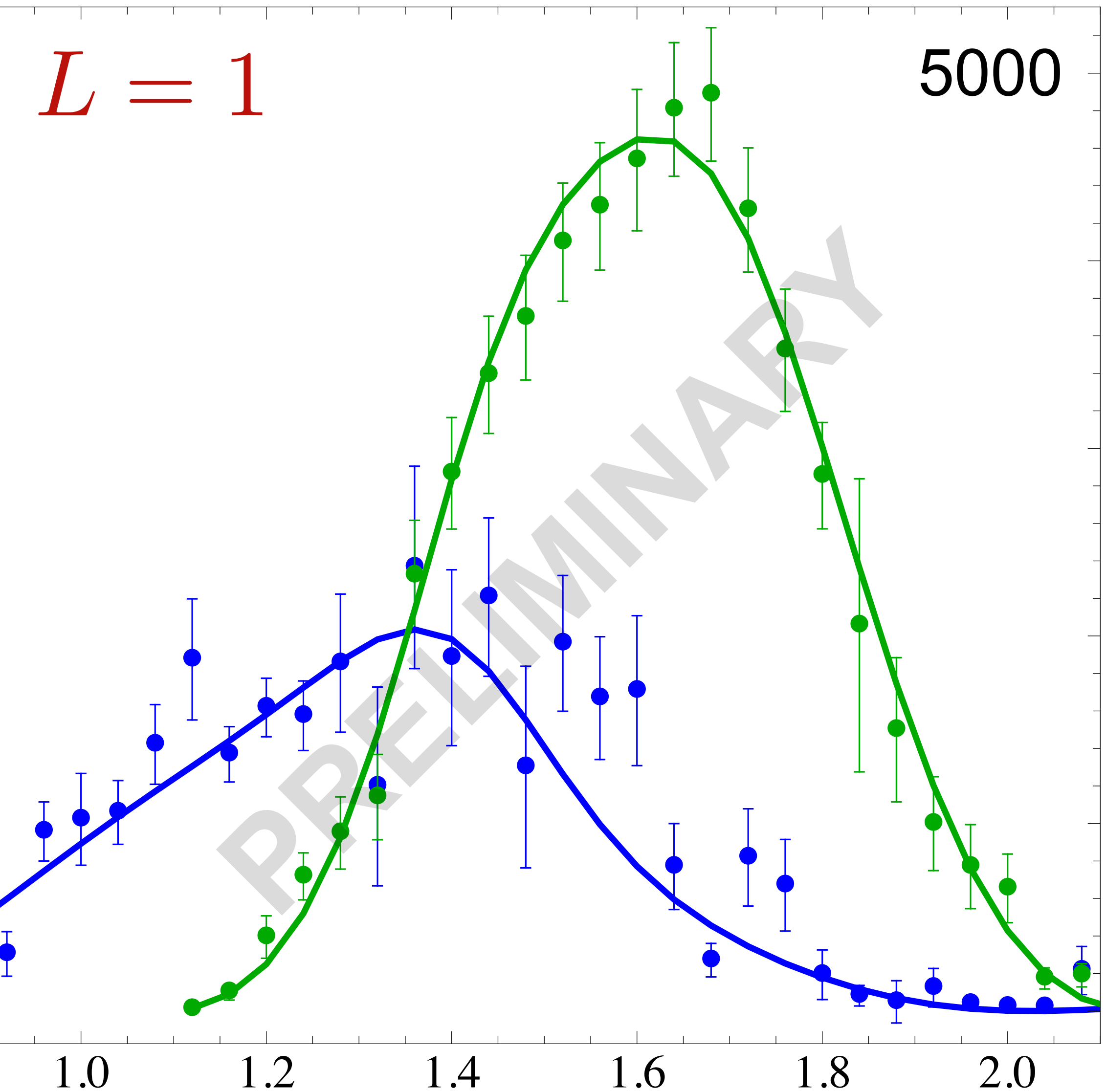
A. Jackura et al (JPAC) and COMPASS, PLB779 (2018)

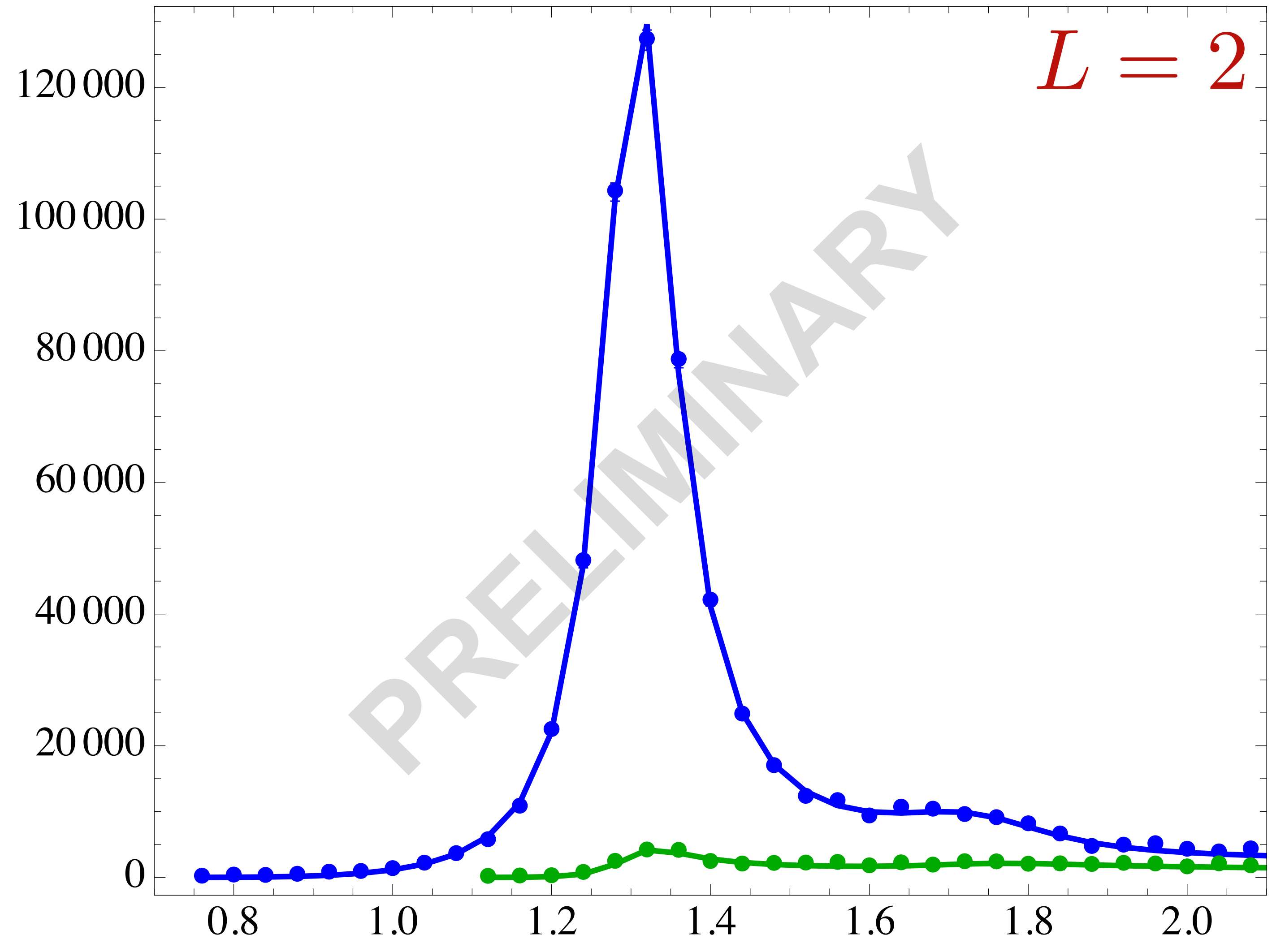
Exotic wave @COMPASS

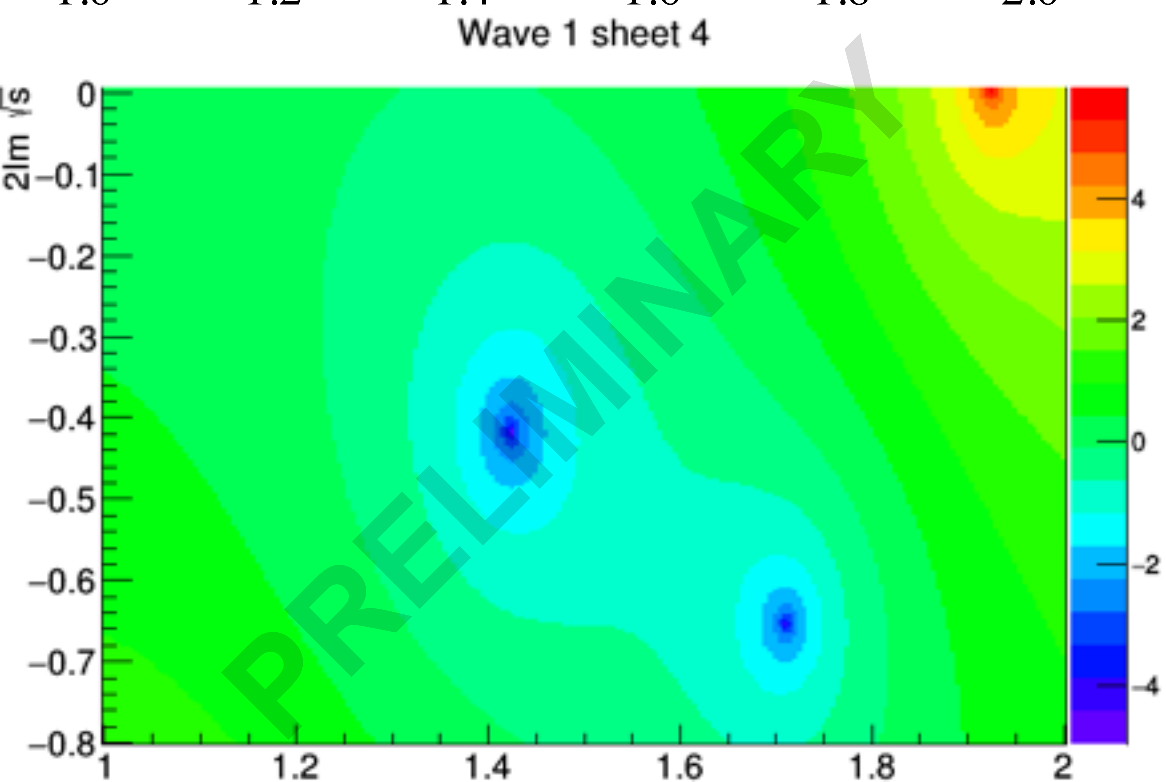


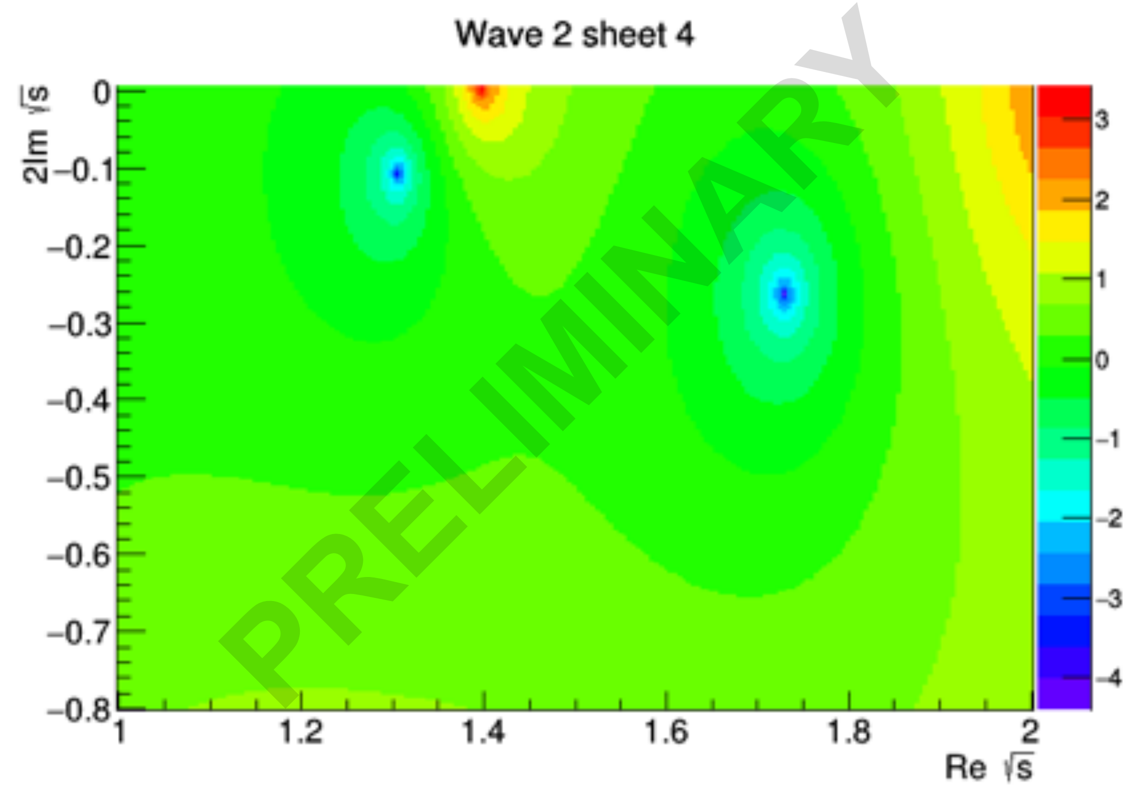
On-going analysis (Arkaitz and Alessandro):
Systematic studies and exploration of the complex plane

Complex Planes

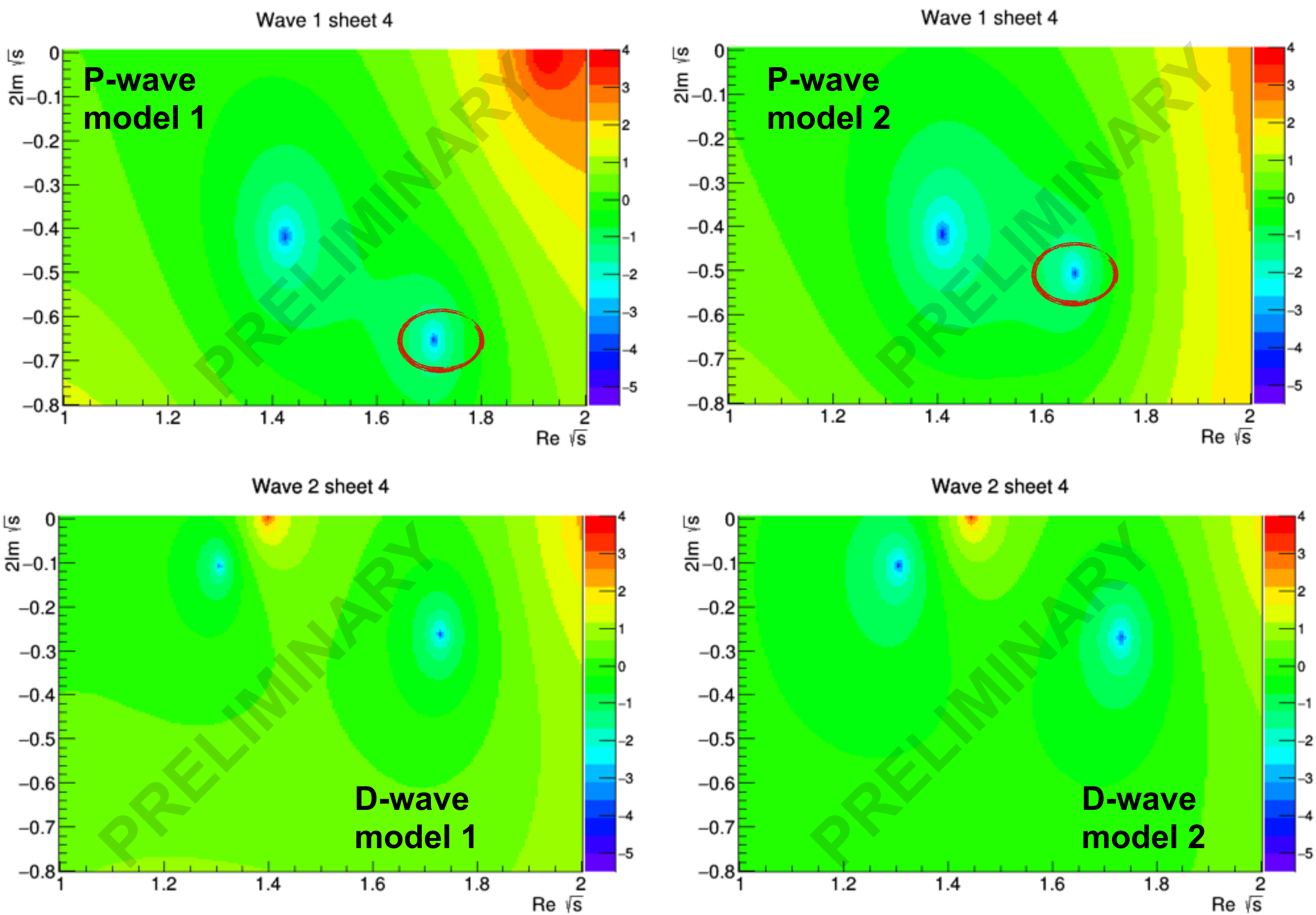




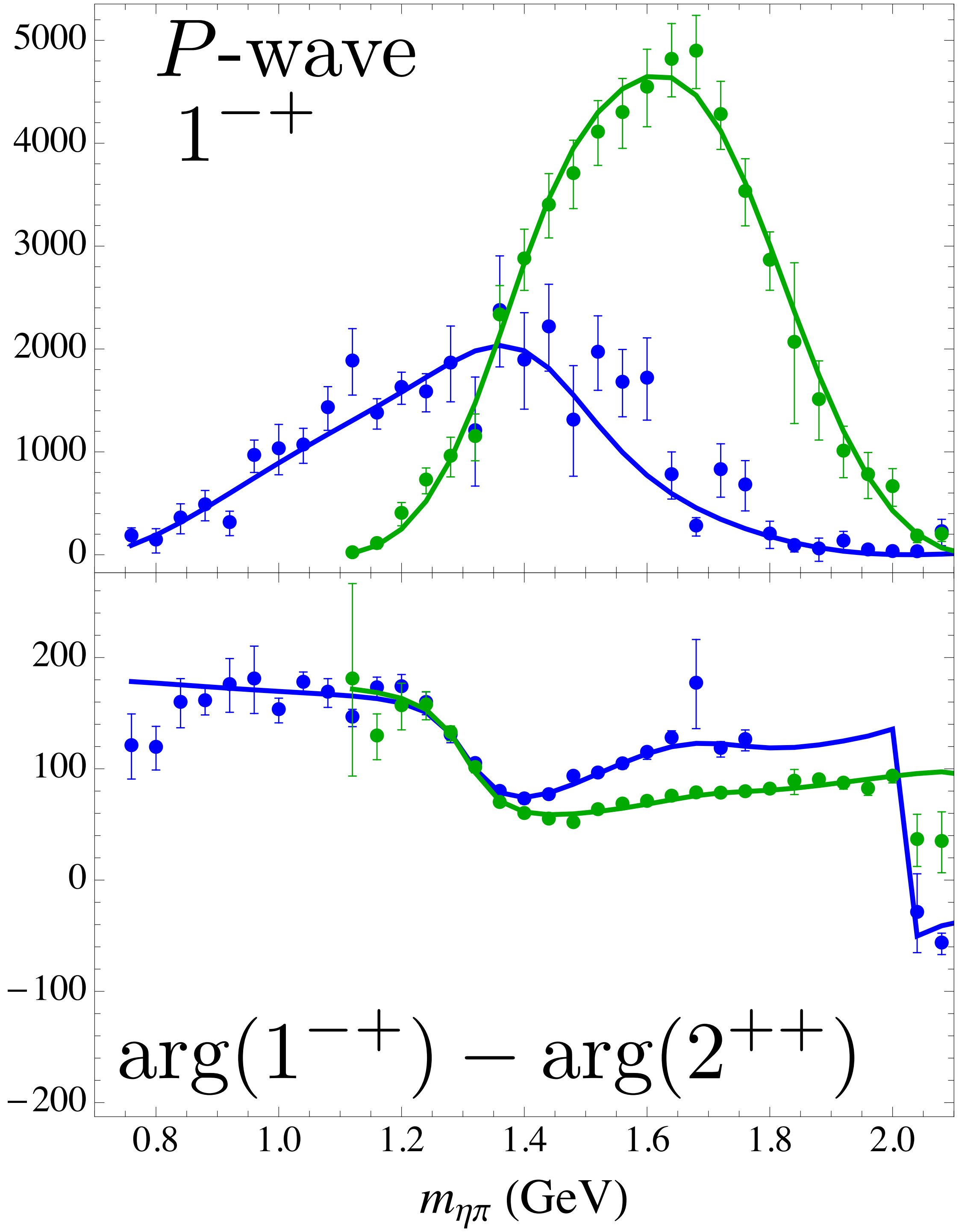




Complex Planes: systematic

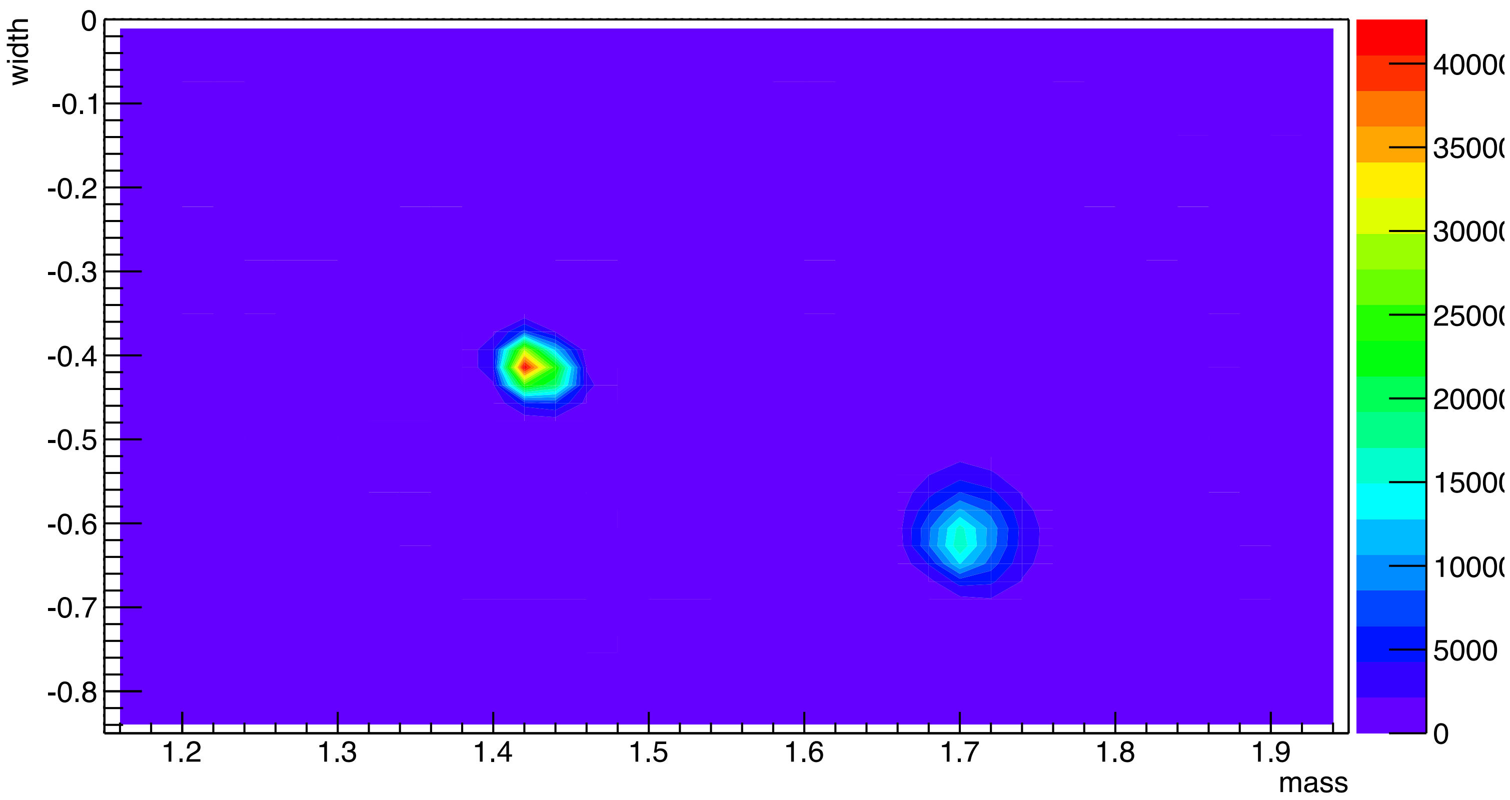


Complex Planes: Boostrapping



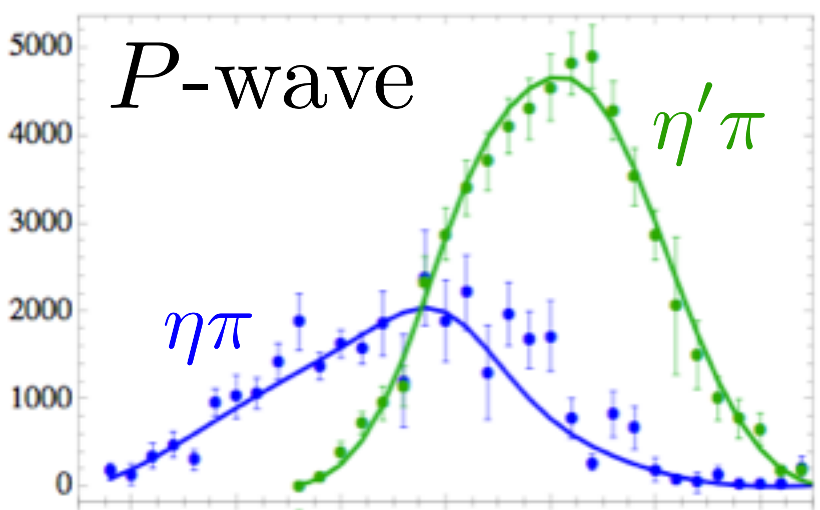
Generate 250k 'new data'
Fit with previous best fit as starting point

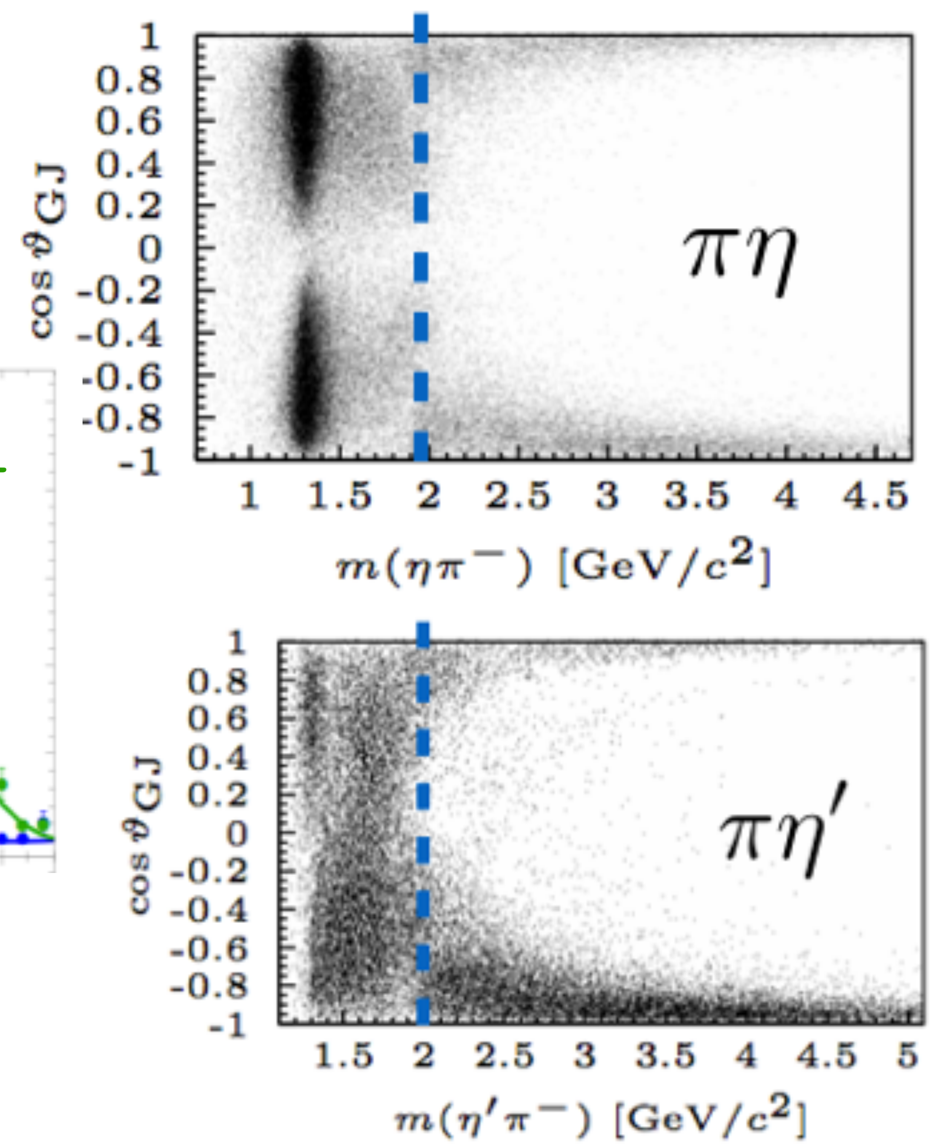
Display the 250k x2 P-wave poles:

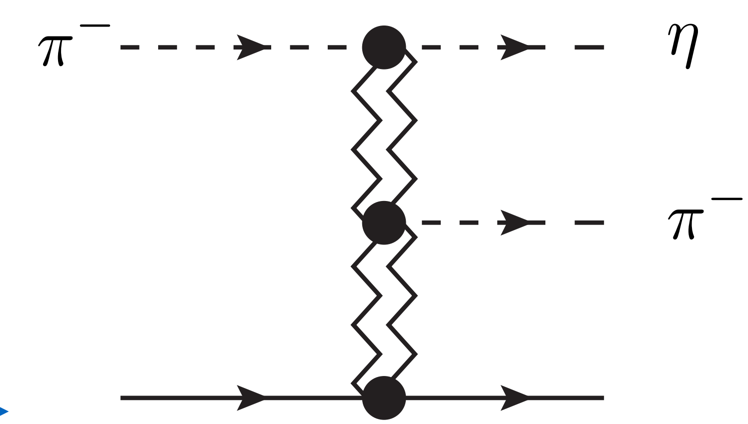


Exotic wave @COMPASS

Dispersion relation relates the high (exchanges) and the low (resonances) regions

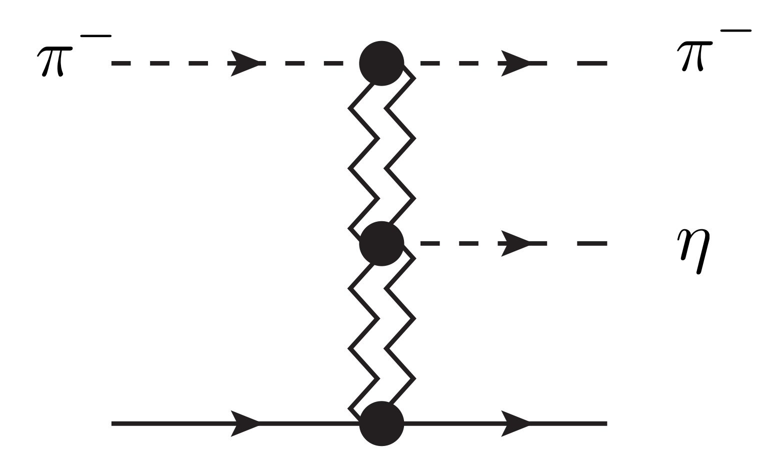






$\cos\theta_{GF} \sim 1 \to \eta$ forward

$\cos\theta_{GF} \sim -1 \to \eta$ backward

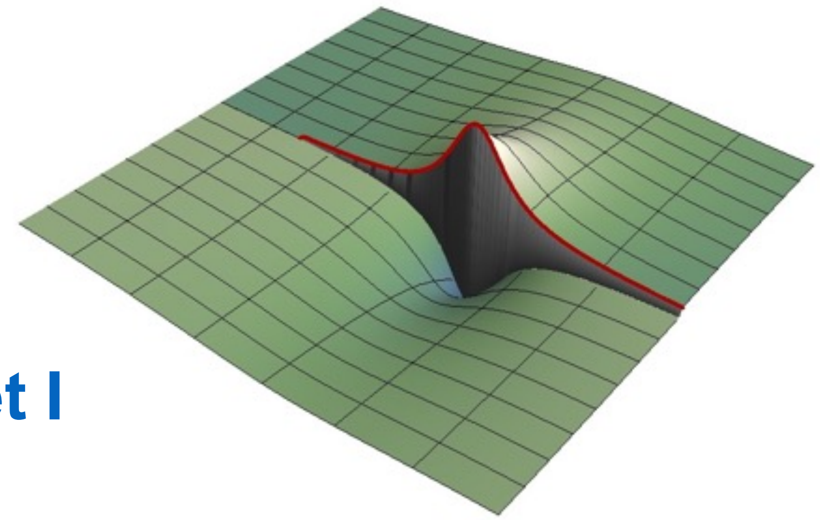


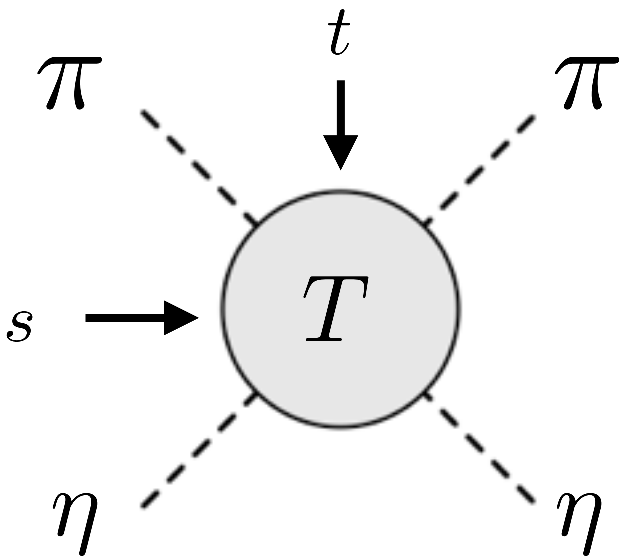
Dispersion Relations

$T(s,t)$

has a right- and a left-hand cuts

and has no other singularity on sheet I



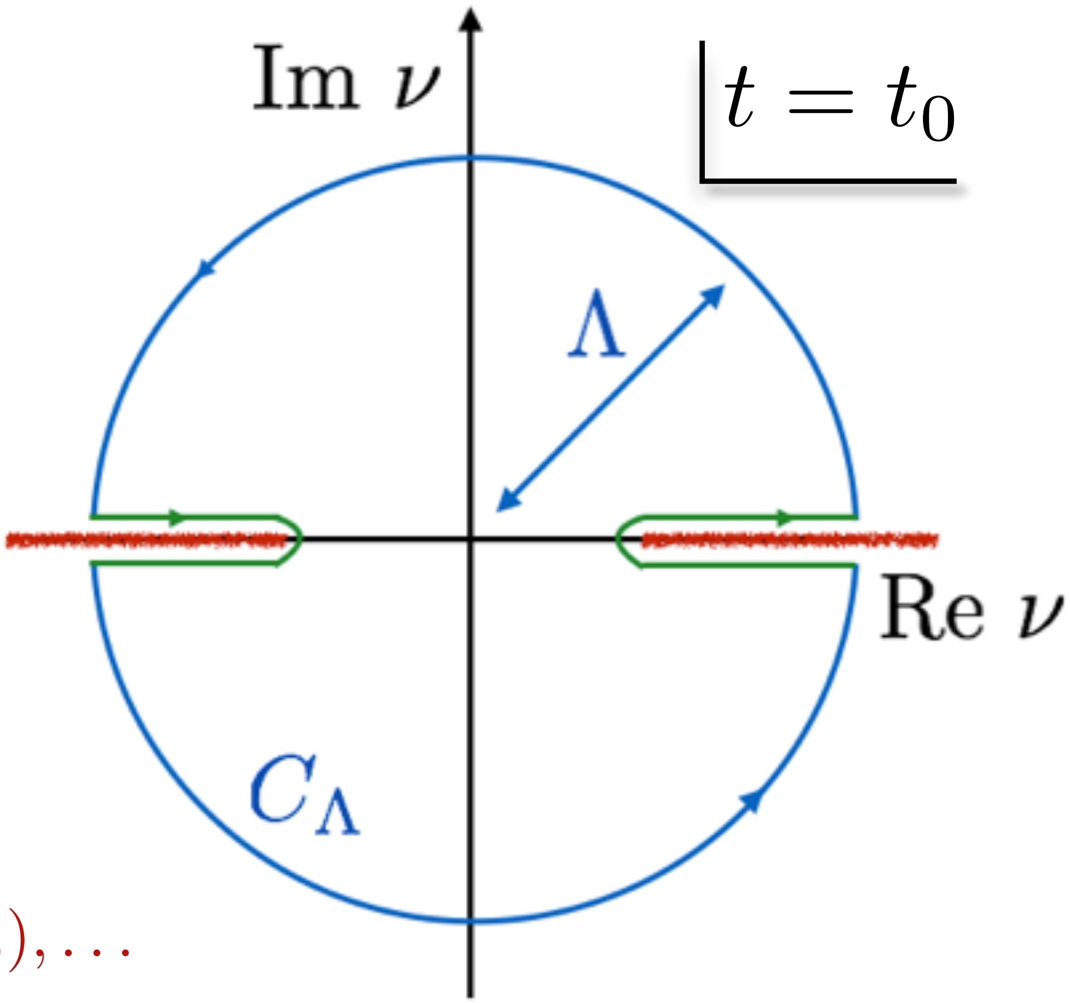


$\nu = (s-u)/2$

$$f(\nu) = \int_{\nu_0}^{\Lambda} \left(\frac{\text{Im } f(\nu')}{\nu' - \nu} + \frac{\text{Im } f(-\nu')}{\nu' + \nu} \right) \frac{d\nu'}{\pi}$$

$$+ \oint_{C_\Lambda} \frac{f(\nu')}{\nu' - \nu} \frac{d\nu'}{2i\pi} \qquad (+\text{sub.}) \qquad (I)$$

$$\int_{\nu_0}^{\Lambda} \text{Im } f(\nu') + \text{Im } f(-\nu') d\nu' = \oint_{C_\Lambda} f(\nu') \frac{d\nu'}{2i\pi} \; (II)$$

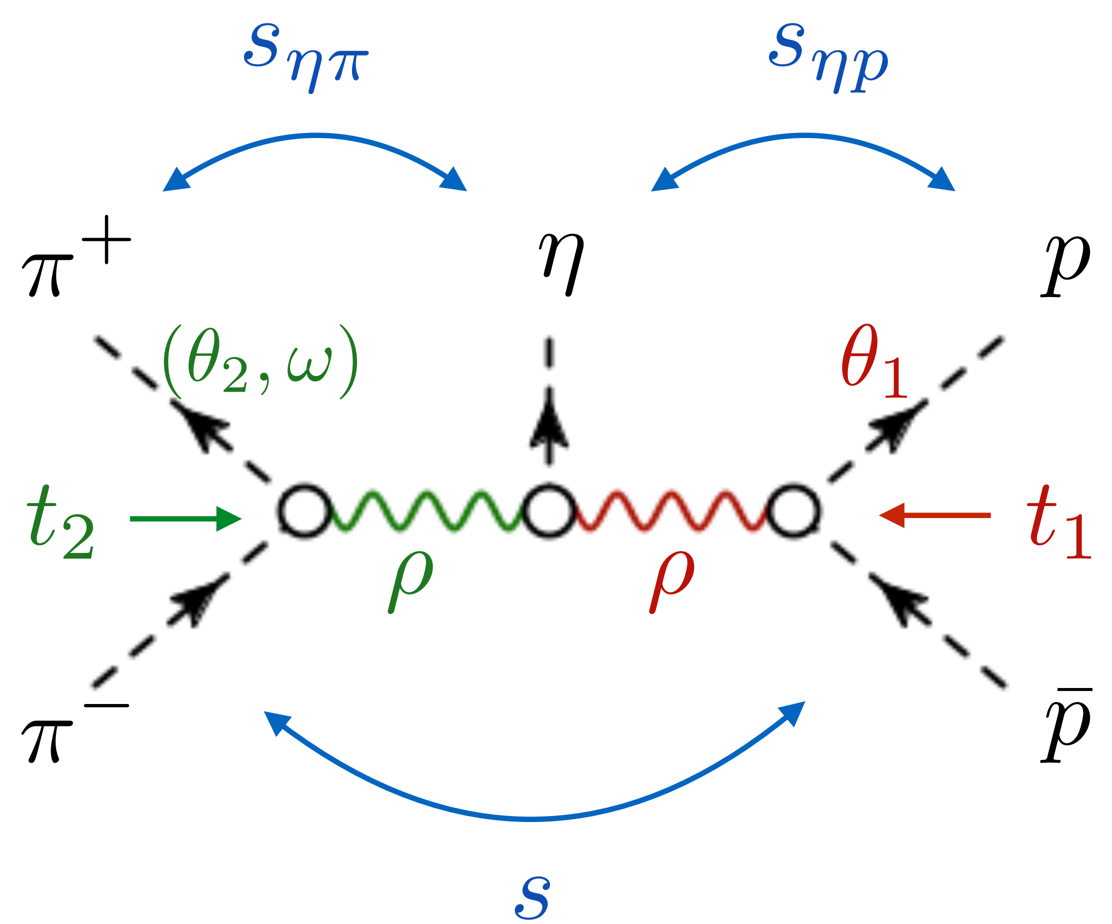


$$f(\nu) = \ldots, \frac{T(\nu,t_0)}{\nu^2}, \frac{T(\nu,t_0)}{\nu}, T(\nu,t_0), \nu T(\nu,t_0), \nu^2 T(\nu,t_0), \ldots$$

Dispersion Relation for 2-to-3

case with ‘scalar protons’

work at fixed t1 and t2



Partial wave expansion

$$A(\theta_1, \theta_2, \omega, t_1, t_2) = \sum_{J_1=0}^{\infty} \sum_{J_2=0}^{\infty} \sum_{\lambda=-M}^{M} d^{J_1}_{0\lambda}(\theta_1) e^{i\lambda\omega} d^{J_2}_{\lambda 0}(\theta_2) a^{J_1 J_2}_{\lambda}(t_1, t_2)$$

Example

$$A^{\rho\rho}(\theta_1, \theta_2, \omega, t_, t_2) \propto \sin\theta_1 \sin\theta_2 \sin\omega$$

Mellin representation

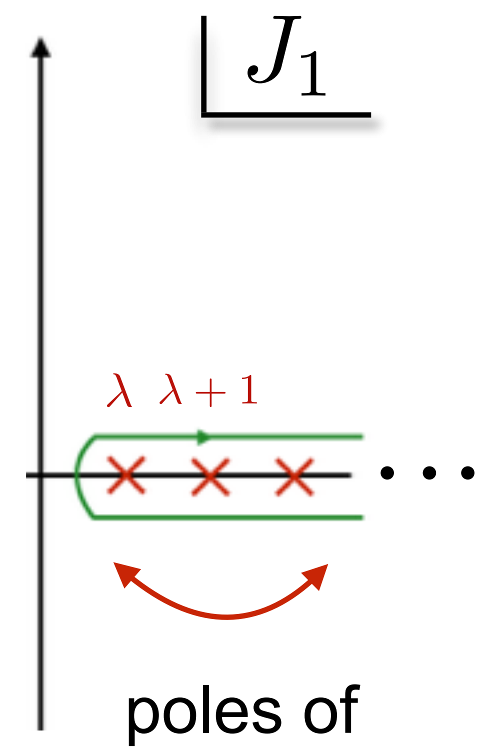
$$A(s, s_{\eta\pi}, s_{\eta p}, t_1, t_2) = \oint \frac{d\lambda}{2\pi i} \oint \frac{dJ_1}{2\pi i} \oint \frac{dJ_2}{2\pi i} \Gamma(1-\lambda)\Gamma(\lambda - J_1)\Gamma(\lambda - J_2)(-s)^{\lambda}(-s_{\eta p})^{J_1-\lambda}(-s_{\eta\pi})^{J_2-\lambda} a(J_1, J_2, \lambda, t_1, t_2)$$

Dispersion Relation for 2-to-3

Brower, DeTar, Weis, Phys. Rep. 14 (1974) 257

Mellin representation:

$$A(s, s_{\eta\pi}, s_{\eta p}, t_1, t_2) = \oint \frac{d\lambda}{2\pi i} \oint \frac{dJ_1}{2\pi i} \oint \frac{dJ_2}{2\pi i} \Gamma(1-\lambda)\Gamma(\lambda - J_1)\Gamma(\lambda - J_2)(-s)^{\lambda}(-s_{\eta p})^{J_1-\lambda}(-s_{\eta\pi})^{J_2-\lambda} a(J_1, J_2, \lambda, t_1, t_2)$$



poles of $\Gamma(\lambda - J_1)$

Do the J_1 integration:

$$A = (-s_{\eta p})^{\alpha_1} \oint \frac{d\lambda}{2\pi i} \oint \frac{dJ_2}{2\pi i} \Gamma(1-\lambda)\Gamma(\lambda-\alpha_1)\Gamma(\lambda - J_2)(s/s_{\eta p})^{\lambda}(-s_{\eta\pi})^{J_2-\lambda} R(J_2, \lambda, t_1, t_2)$$

Do the λ integration:

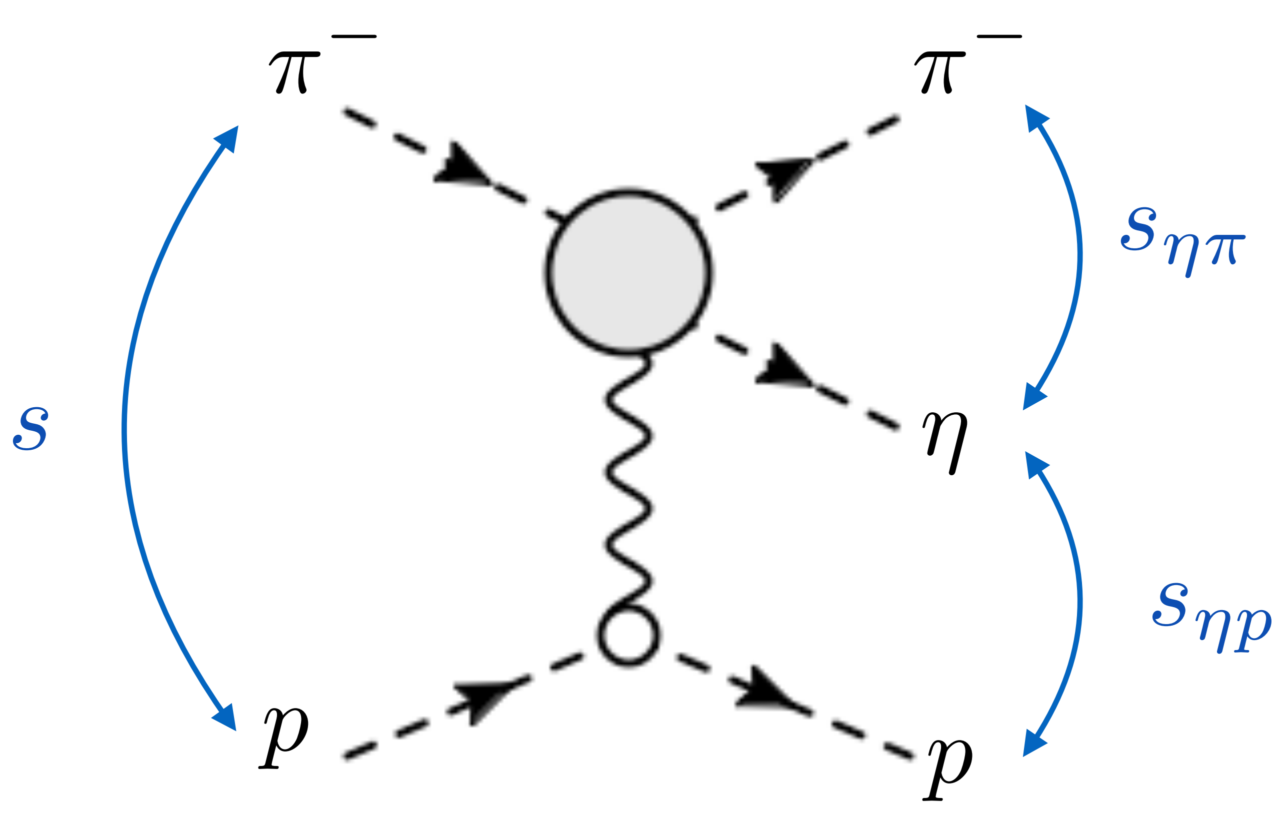
$$A = (-s)^{\alpha_1}\left\{ \sum_{i=0}^{\infty} \frac{\Gamma(1+i-\alpha_1)}{\Gamma(i+1)} \left(-\frac{s_{\eta p}}{s}\right)^i \left[\oint \frac{dJ_2}{2\pi i}\Gamma(\alpha_1 - J_2 - i)(-s_{\eta\pi})^{J_2-\alpha_1+i} R(J_2, \alpha_1 - i, t_1, t_2)\right] \right.$$
$$\left. + \left(\frac{s_{\eta p}}{s}\right)^{\alpha_1-\alpha_2} \sum_{i=0}^{\infty} \frac{\Gamma(1+i-\alpha_1)}{\Gamma(i+1)} \Gamma(\alpha_2 - \alpha_1 - i)\beta(\alpha_2 - i, t_1, t_2)\left(\frac{s_{\eta\pi}s_{\eta p}}{s}\right)^i \right\}$$

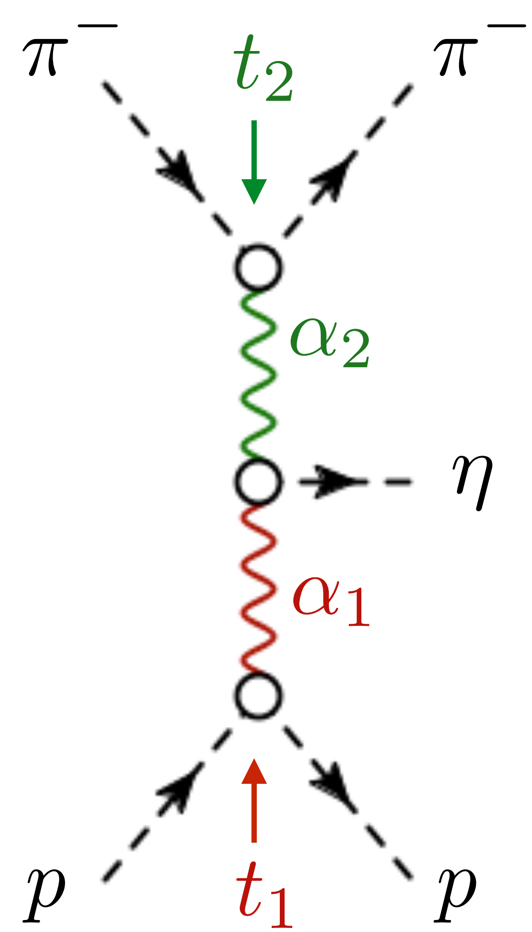
Dispersion Relation for 2-to-3

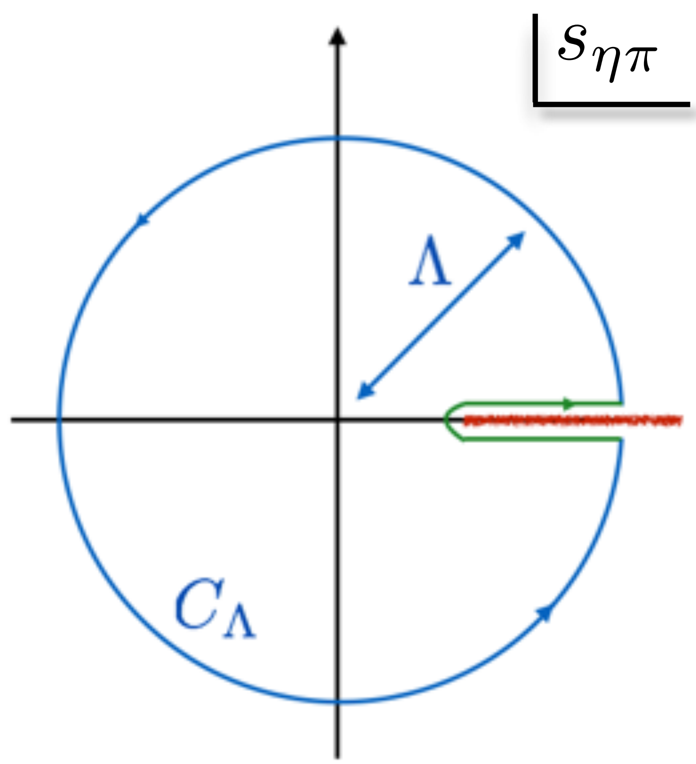
$$A = (-s)^{\alpha_1}\left\{ \sum_{i=0}^{\infty} \frac{\Gamma(1+i-\alpha_1)}{\Gamma(i+1)} \left(-\frac{s_{\eta p}}{s}\right)^i \left[\oint \frac{dJ_2}{2\pi i} \Gamma(\alpha_1 - J_2 - i)(-s_{\eta\pi})^{J_2-\alpha_1+i} R(J_2, \alpha_1 - i, t_1, t_2) \right] \right.$$
$$\left. + \left(\frac{s_{\eta p}}{s}\right)^{\alpha_1-\alpha_2} \sum_{i=0}^{\infty} \frac{\Gamma(1+i-\alpha_1)}{\Gamma(i+1)} \Gamma(\alpha_2 - \alpha_1 - i)\beta(\alpha_2 - i, t_1, t_2) \left(\frac{s_{\eta\pi} s_{\eta p}}{s}\right)^i \right\}$$

infinite number of subtractions

Reggeon-particle amplitude







Consider $t_1,\ t_2,\ s_{\eta p}/s$ **fixed**

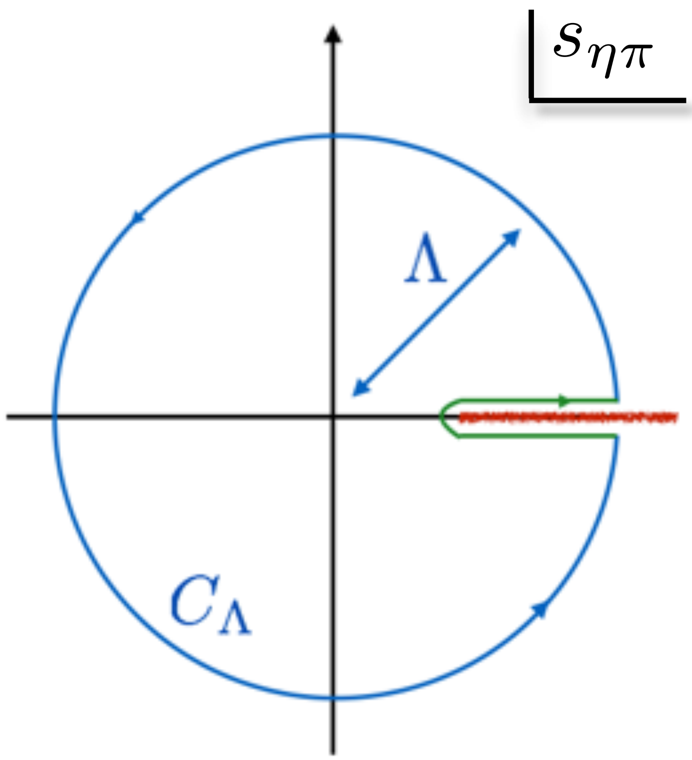
Dispersion Relation for 2-to-3

$$A = (-s)^{\alpha_1} \left\{ \sum_{i=0}^{\infty} \frac{\Gamma(1+i-\alpha_1)}{\Gamma(i+1)} \left(-\frac{s_{\eta p}}{s}\right)^i \left[\oint \frac{dJ_2}{2\pi i} \Gamma(\alpha_1 - J_2 - i)(-s_{\eta\pi})^{J_2-\alpha_1+i} R(J_2, \alpha_1 - i, t_1, t_2) \right] \right.$$
$$\left. + \left(\frac{s_{\eta p}}{s}\right)^{\alpha_1-\alpha_2} \sum_{i=0}^{\infty} \frac{\Gamma(1+i-\alpha_1)}{\Gamma(i+1)} \Gamma(\alpha_2 - \alpha_1 - i) \beta(\alpha_2 - i, t_1, t_2) \left(\frac{s_{\eta\pi} s_{\eta p}}{s}\right)^i \right\}$$

Consider $t_1, \ t_2, \ s_{\eta p}/s$ fixed

DR with infinite number of subtractions

Close the contour at finite Λ



and obtain sum rules:

$$\int_{s_{\rm th}}^{\Lambda} \mathrm{Im}\ A(s, s_{\eta p}, s'_{\eta\pi}, t_1, t_2) ds'_{\eta\pi} = (-s)^{\alpha_1} \Lambda^{\alpha_2-\alpha_1+1} \sum_{i=0}^{\infty} \frac{\Gamma(1+i-\alpha_1)}{\Gamma(i+1)} \frac{\beta(\alpha_1 - i, t_1, t_2)}{\Gamma(2+i+\alpha_2-\alpha_1)} \left(\frac{\Lambda s_{\eta p}}{s}\right)^i$$

Sum Rules for 2-to-3

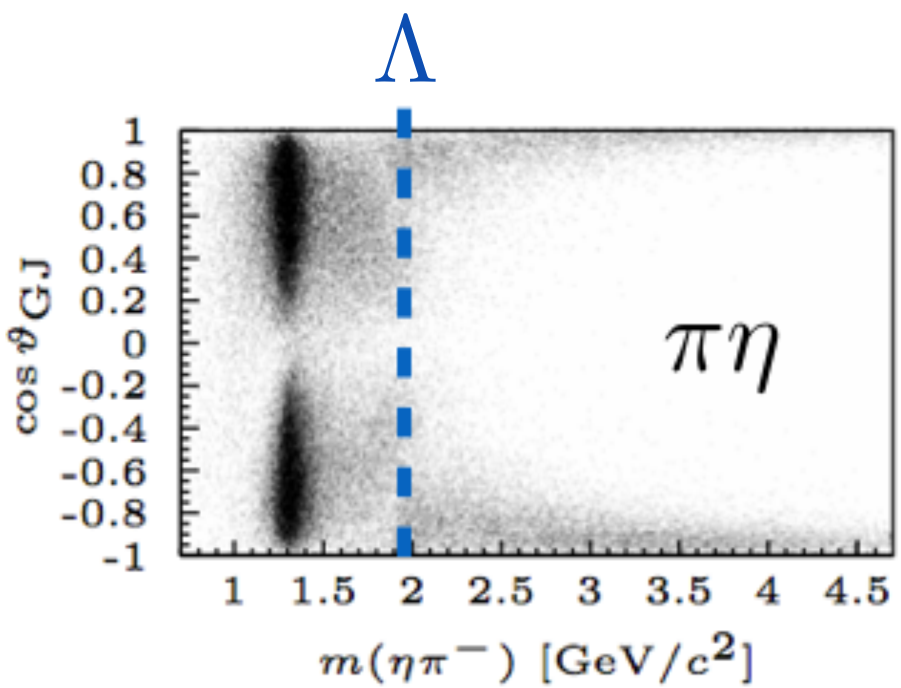
Sum rules at $t_1,\ \ t_2,\ \ s_{\eta p}/s$ fixed

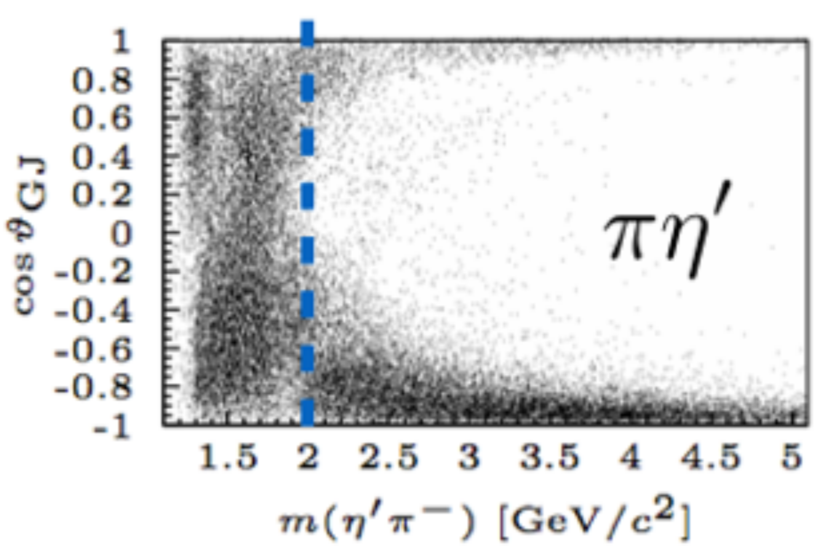
$$\int_{s_{\rm th}}^{\Lambda} \text{Im}\ A(s, s_{\eta p}, s'_{\eta\pi}, t_1, t_2) ds'_{\eta\pi} = (-s)^{\alpha_1}\Lambda^{\alpha_2-\alpha_1+1}\sum_{i=0}^{\infty}\frac{\Gamma(1+i-\alpha_1)}{\Gamma(i+1)}\frac{\beta(\alpha_1-i,t_1,t_2)}{\Gamma(2+i+\alpha_2-\alpha_1)}\left(\frac{\Lambda s_{\eta p}}{s}\right)^i$$

constraint on low mass model

fixed ; determined by high mass data

$t_2 \equiv$

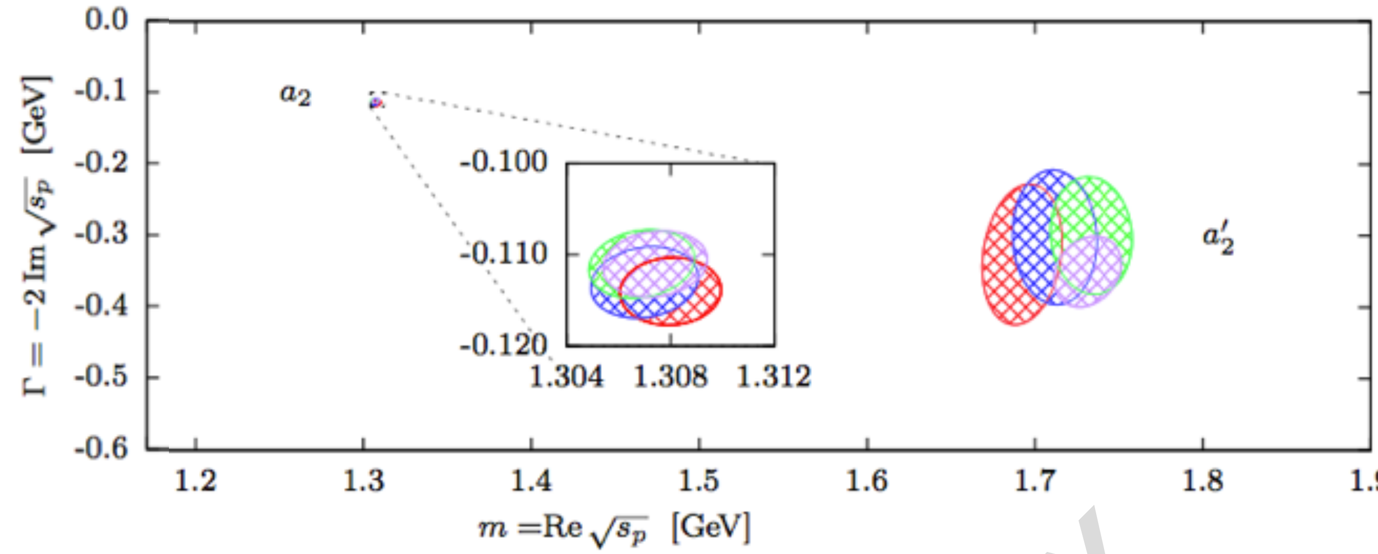




Summary

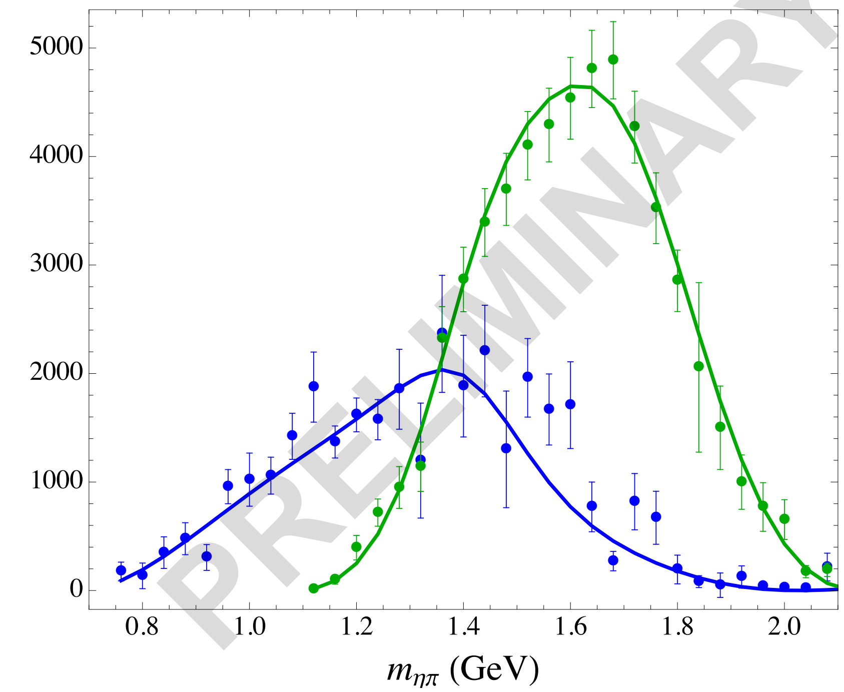
Analyticity and unitarity constrain amplitude construction

A. Jackura et al (JPAC) and COMPASS, PLB779 (2018)



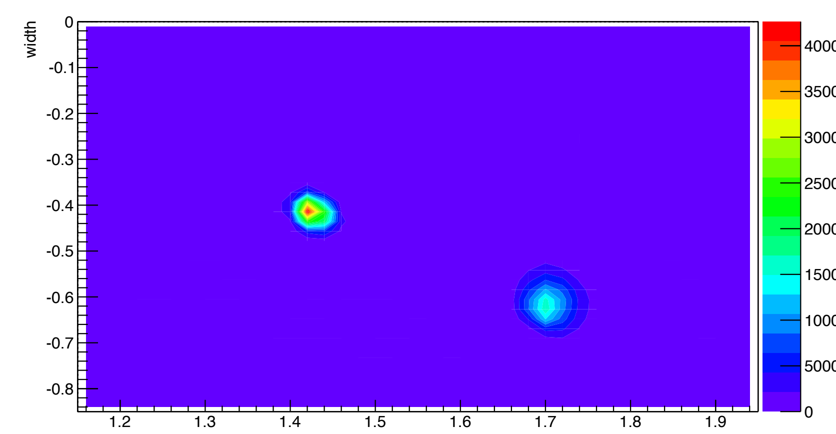
Past:

Extracted a2(1320) and a2(1700) pole position



Present:

Extraction of exotic meson pole position



Future:

Implementation of DR constraining model

Transposition to GlueX/CLAS12 data

Interactive webpage: http://www.indiana.edu/~jpac/

INDIANA UNIVERSITY BLOOMINGTON

Jefferson Lab — Thomas Jefferson National Accelerator Facility

THE GEORGE WASHINGTON UNIVERSITY WASHINGTON, DC

Joint Physics Analysis Center

HOME   PROJECTS   PUBLICATIONS   LINKS

Resources

- **Publication:** [Mat15a]
- **Fortran:** Fortran file, Input file, Output file
- **C/C++:** AmpTools class, C/C++ file, AmpTools class header
- **Mathematica:** notebook , converted in text
- **Data:** Anderson, All data
- **Contact person:** Vincent Mathieu
- **Last update:** November 2015

Description of the Fortran code: [show/hide]
Description of the C/C++ code: [show/hide]

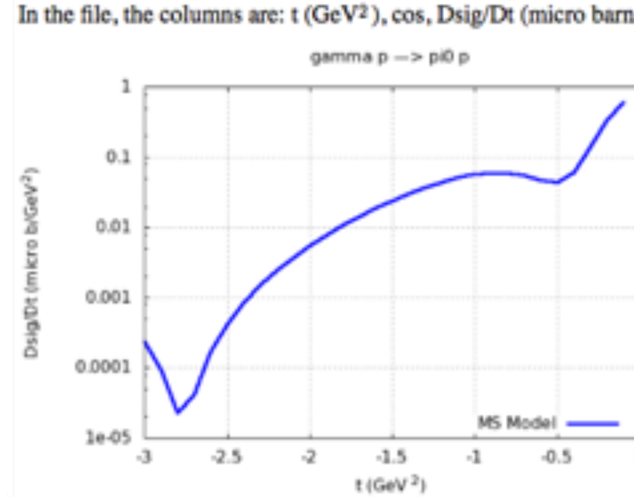
Run the code

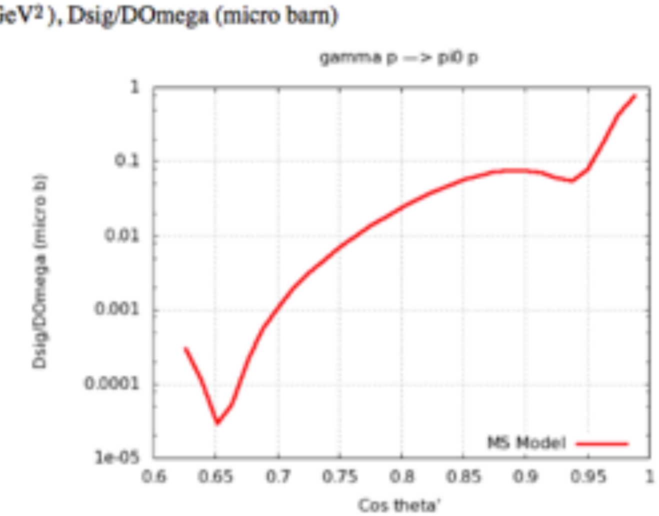
Choose the beam energy in the lab frame E_γ , the other variable (t or $\cos\theta$) and its minimal, maximal, and increment values.
If you choose t (cos) only the min, max and step values of t ($\cos\theta$) are read.

E_γ in GeV 9
t / cos
t in GeV² (min max step) -3 -0.1 0.1
$\cos\theta$ (min max step) 0.5 0.95 0.01
Start reset

beam energy: 9 GeV
Observable: differential cross section
X variable: t with interval -3:0.1:-0.1

Download the output file, the plot with Ox=t , the plot with Ox=cos .
In the file, the columns are: t (GeV²), cos, Dsig/Dt (micro barn/GeV²), Dsig/DOmega (micro barn)



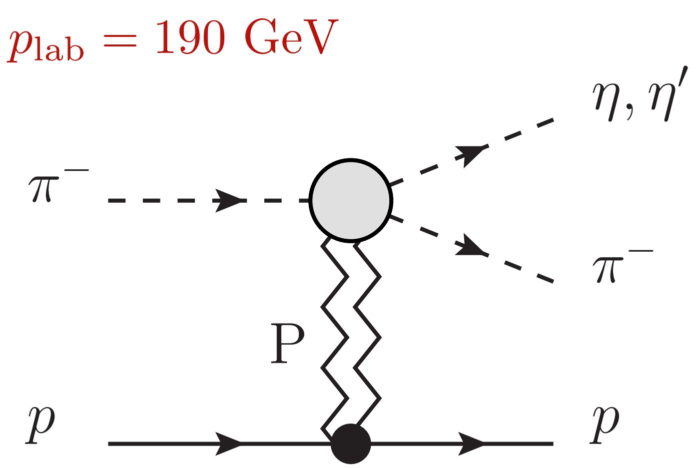


Interactive webpage: http://www.indiana.edu/~jpac/

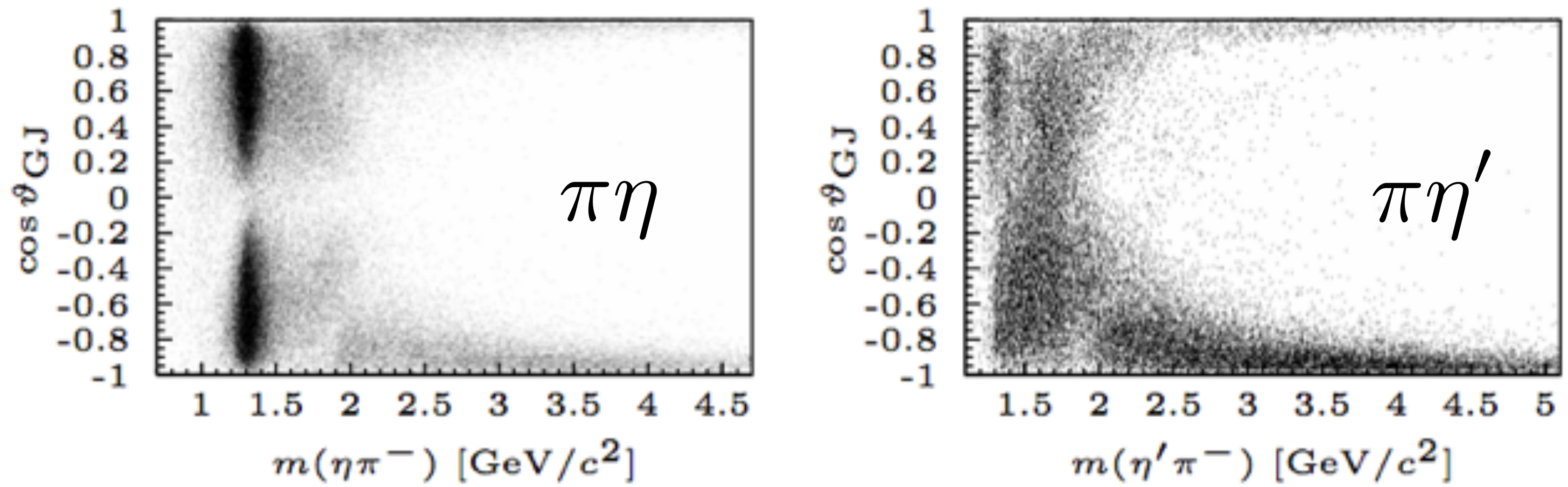
1.	United States	493	(32.18%)
2.	Germany	169	(11.03%)
3.	United Kingdom	150	(9.79%)
4.	Mexico	103	(6.72%)
5.	Italy	57	(3.72%)
6.	Spain	52	(3.39%)
7.	Belgium	38	(2.48%)
8.	China	37	(2.42%)
9.	(not set)	37	(2.42%)
10.	Brazil	35	(2.28%)
11.	Poland	31	(2.02%)
12.	Austria	30	(1.96%)
13.	Japan	26	(1.70%)
14.	Switzerland	25	(1.63%)
15.	Russia	20	(1.31%)
16.	Saudi Arabia	20	(1.31%)
17.	Iraq	17	(1.11%)
18.	India	16	(1.04%)
19.	Canada	15	(0.98%)
20.	Netherlands	15	(0.98%)
21.	France	12	(0.78%)
22.	Bosnia & Herzegovina	10	(0.65%)
23.	Greece	10	(0.65%)
24.	Portugal	9	(0.59%)
25.	South Korea	7	(0.46%)

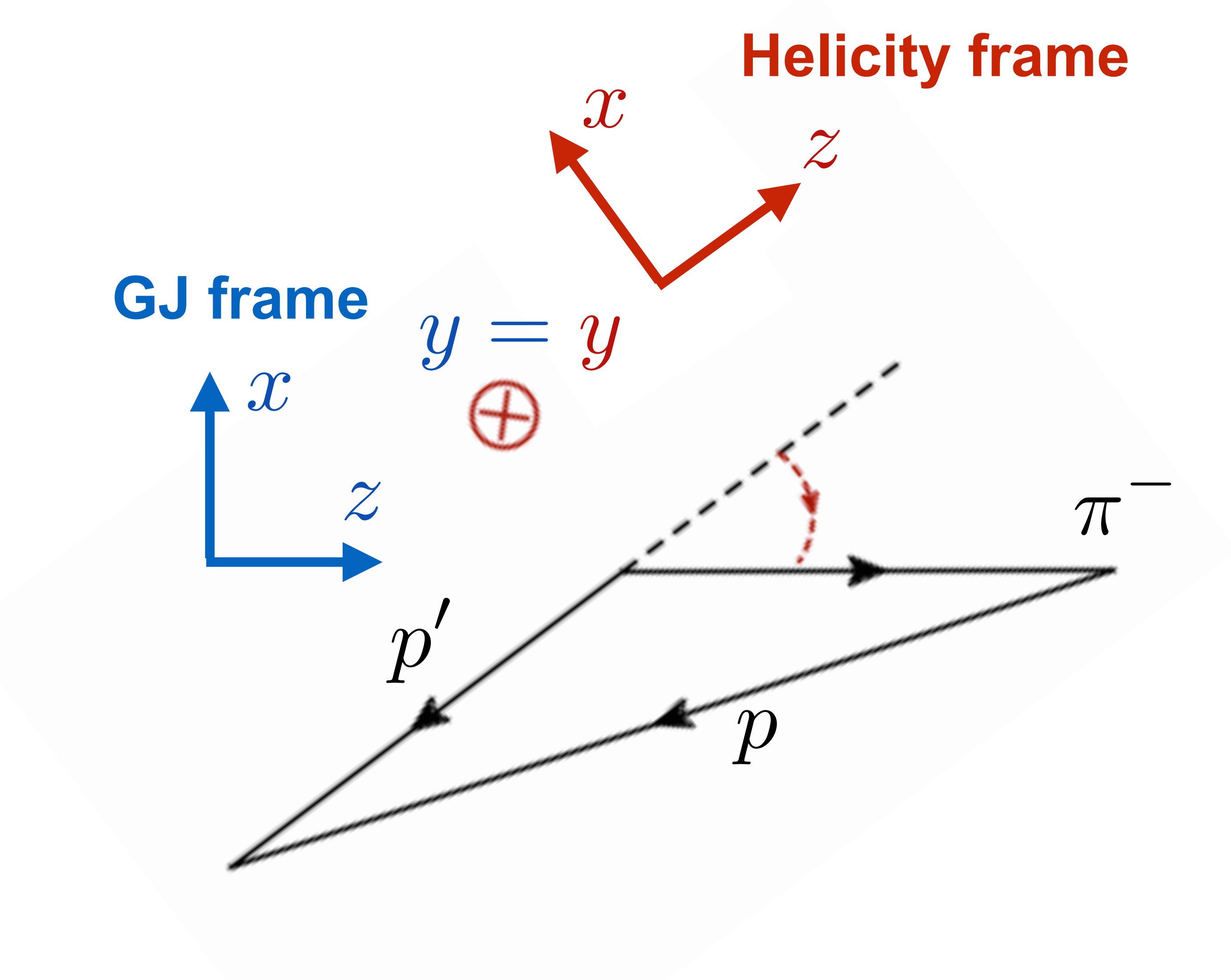
Backup Slides

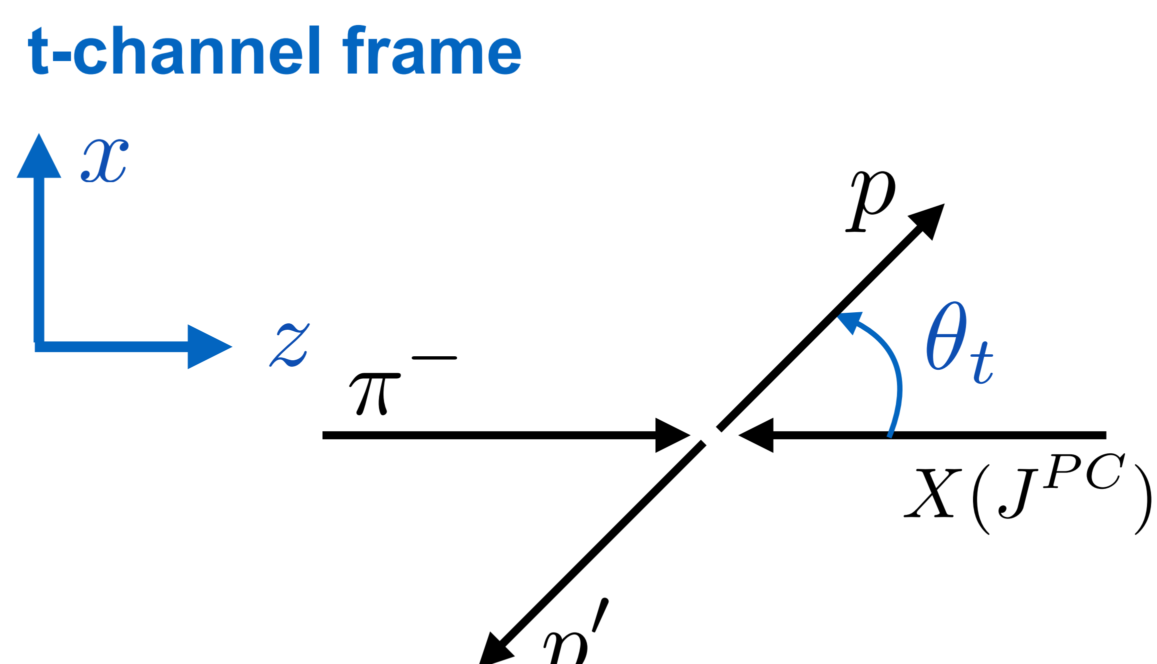
Gottfried-Jackson Frame



COMPASS Phys. Lett. B740 (2015)



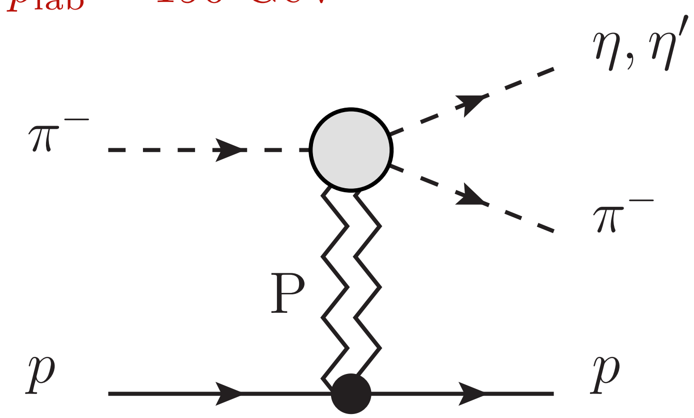


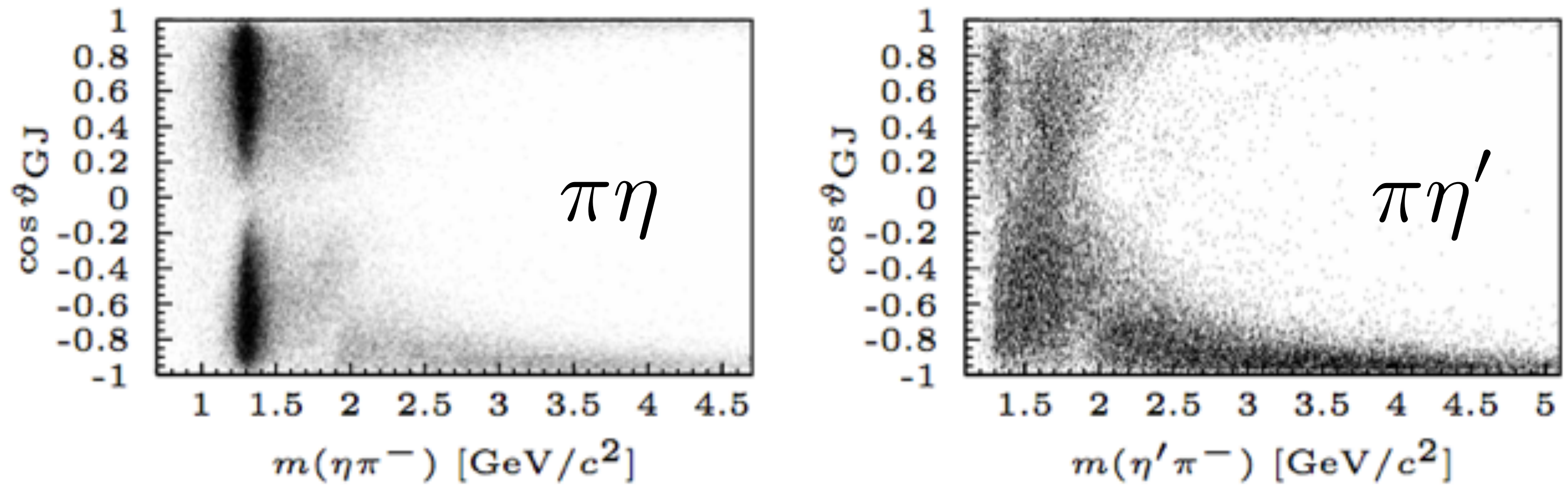


Reflectivity Basis

COMPASS Phys. Lett. B740 (2015)

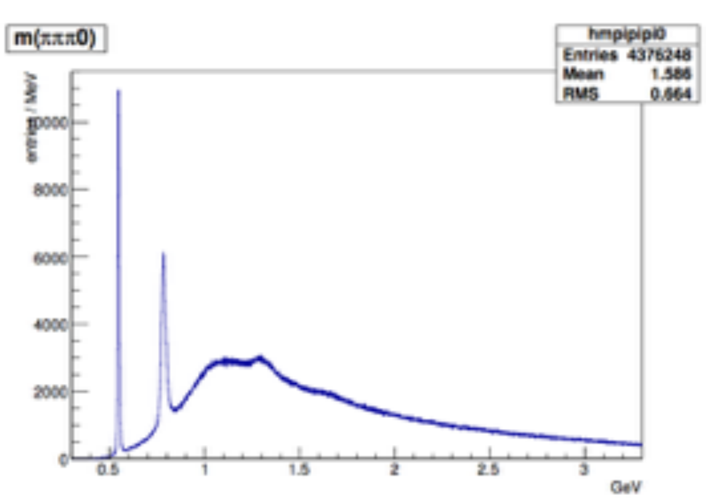
$p_{\rm lab} = 190$ GeV





$$I(\tau) = \sum_{\epsilon} \left| \sum_{L,M} A^{\epsilon}_{LM} \psi^{\epsilon}_{LM}(\tau) \right|^2 + \text{non-}\eta^{(\prime)} \text{ background}$$

$$\psi^{\epsilon}_{LM}(\tau) = f_{\eta}(p_{\pi^-}, p_{\pi^+}, p_{\pi^0}) \times Y^M_L(\vartheta_{\rm GJ}, 0) \times \begin{cases} \sin M\varphi_{\rm GJ} & \text{for } \epsilon = +1 \\ \cos M\varphi_{\rm GJ} & \text{for } \epsilon = -1 \end{cases}$$

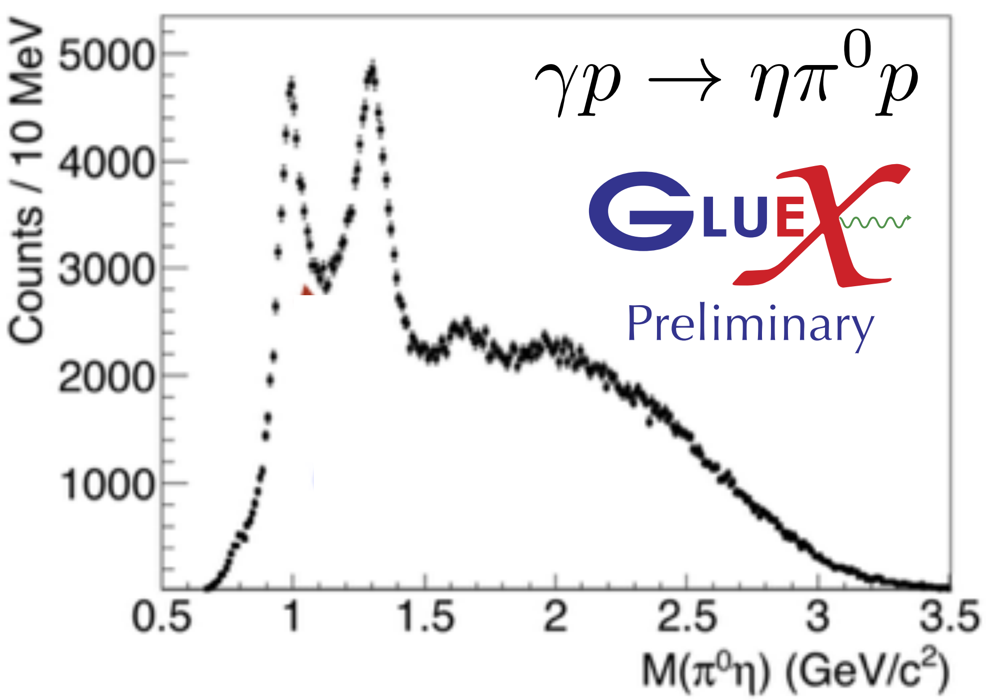


Light Meson Spectroscopy

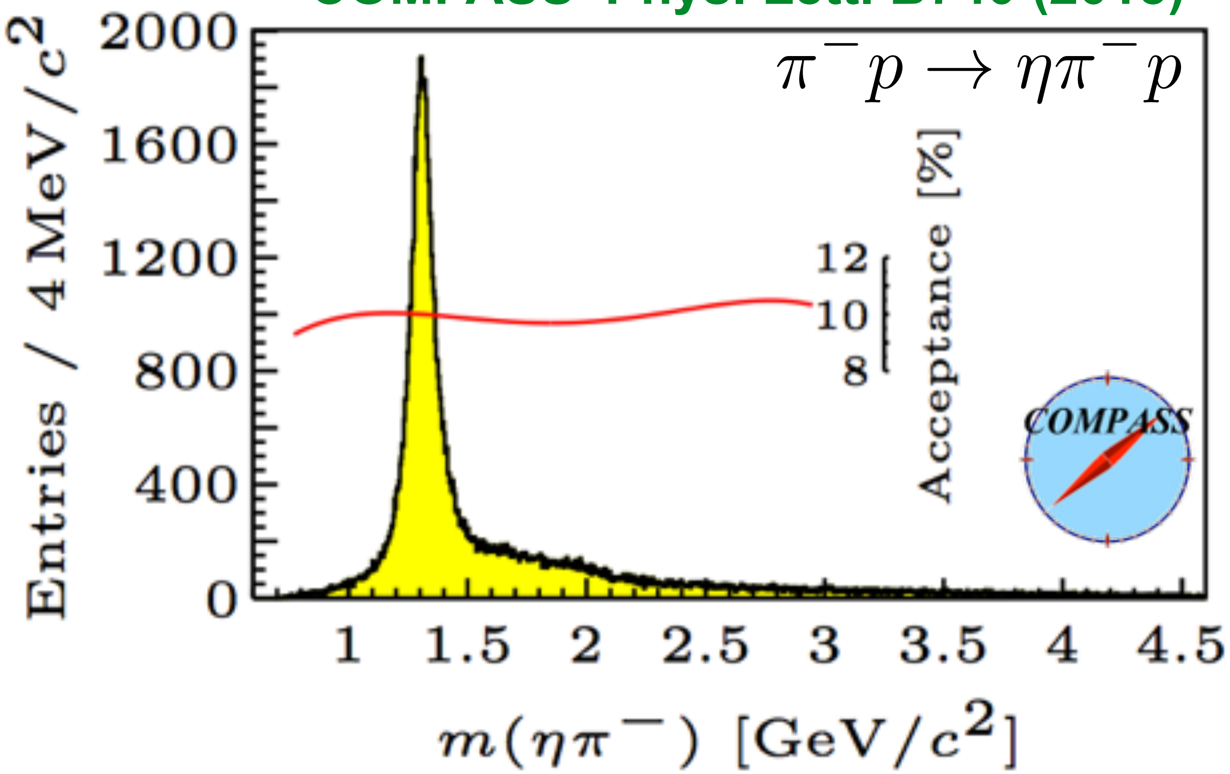
$E_{\text{beam}} = 9\ \text{GeV}$

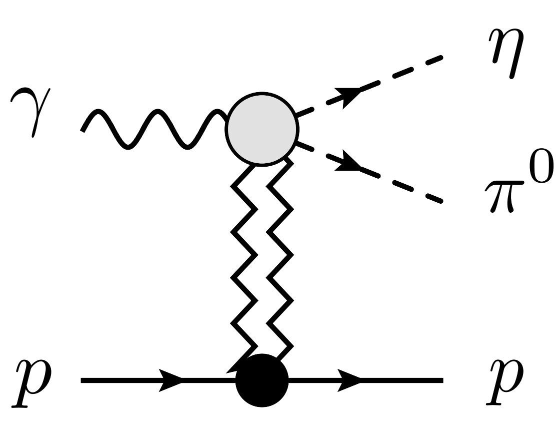
$E_{\text{beam}} = 190\ \text{GeV}$

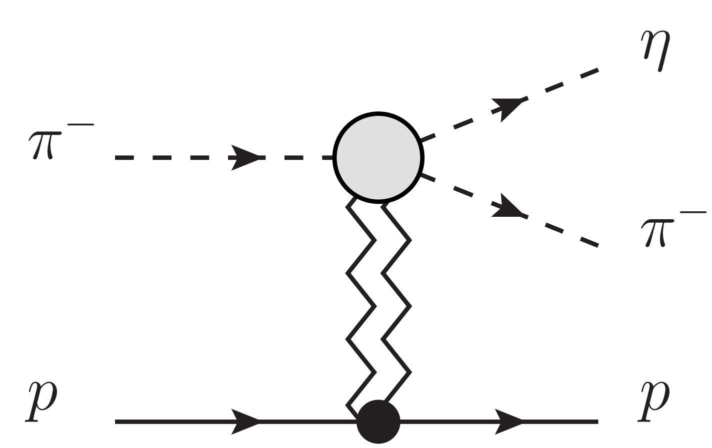
GlueX



COMPASS Phys. Lett. B740 (2015)







$a_0(980)$ **is produced by unnatural exchange** $P(-1)^J = -1$

suppressed at high energy