

Quarkonium Production in Jets at LHCb

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Review of Quarkonium Production Theory

Heavy Quarkonium Fragmenting Jet Functions

New Tests of NRQCD Using Jet Observables

Cross sections for e^+e^- , pp collisions

LHCb measurement of J/ψ in jets

Other Observables, Future Work

Color-Singlet Model (pre-1995)

$$\sigma(pp \rightarrow J/\psi + X) = f_{g/p} \otimes f_{g/p} \otimes \sigma[gg \rightarrow c\bar{c}({}^3S_1^{(1)}) + X] |\psi_{c\bar{c}}(0)|^2$$

$c\bar{c}$ pair produced with same quantum numbers as J/ψ

Predictive Formalism

$$\sigma[gg \rightarrow c\bar{c}({}^3S_1^{(1)}) + X] \text{ calculable in QCD perturbation theory}$$
$$|\psi_{c\bar{c}}(0)|^2 \text{ fixed by } \Gamma[J/\psi \rightarrow \ell^+ \ell^-]$$

Suffers from theoretical inconsistencies when applied to χ_{cJ}

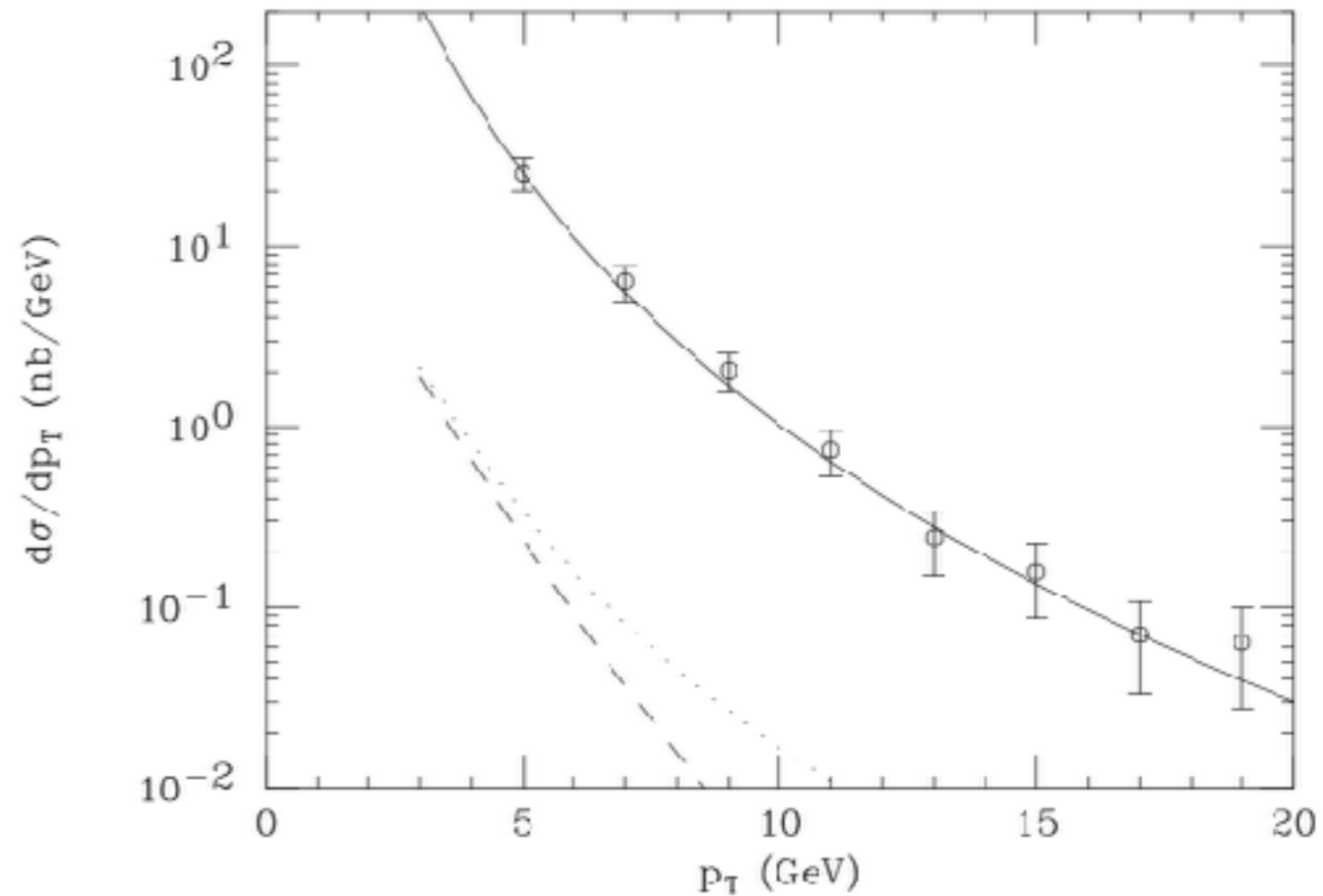
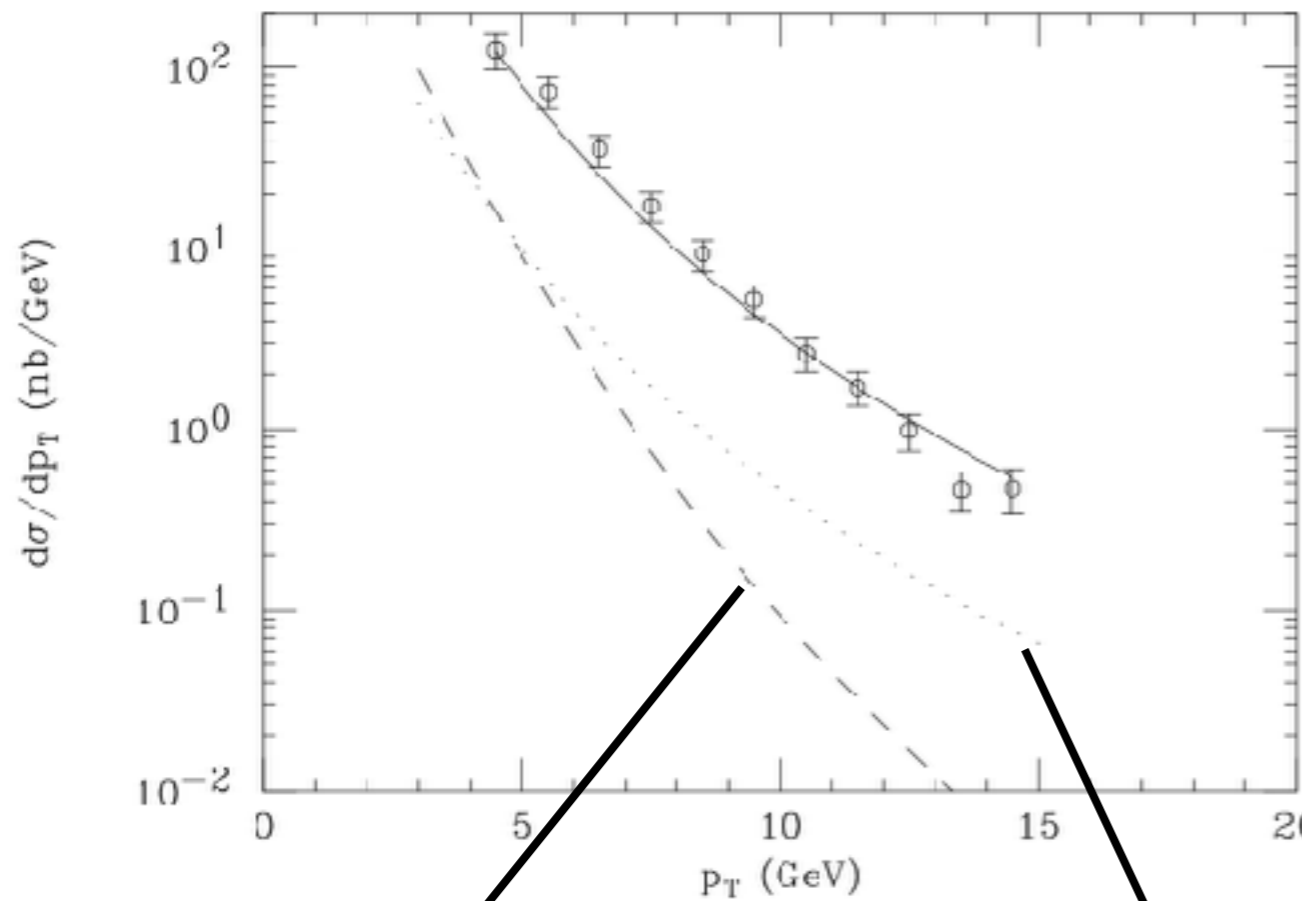
$$\Gamma[\chi_{cJ} \rightarrow \text{hadrons}] = |\psi'_{c\bar{c}}(0)|^2 \left(\sigma(c\bar{c}({}^3P_J^{(1)}) \rightarrow gg) \right) \longleftarrow \text{Not IR Safe}$$

J/ψ production at Tevatron (1996)

CSM badly underpredicts J/ψ and ψ' production at large p_T

J/ψ

ψ'



CSM (LO)

CSM
(Fragmentation)

Non-Relativistic QCD (NRQCD) Factorization Formalism

(Bodwin, Braaten, Lepage)

$$\sigma(gg \rightarrow J/\psi + X) = \sum_n \sigma(gg \rightarrow c\bar{c}(n) + X) \langle \mathcal{O}^{J/\psi}(n) \rangle$$

$n = {}^{2S+1}L_J^{(1,8)}$

double expansion in α_s, v

NRQCD long-distance matrix element (LDME)

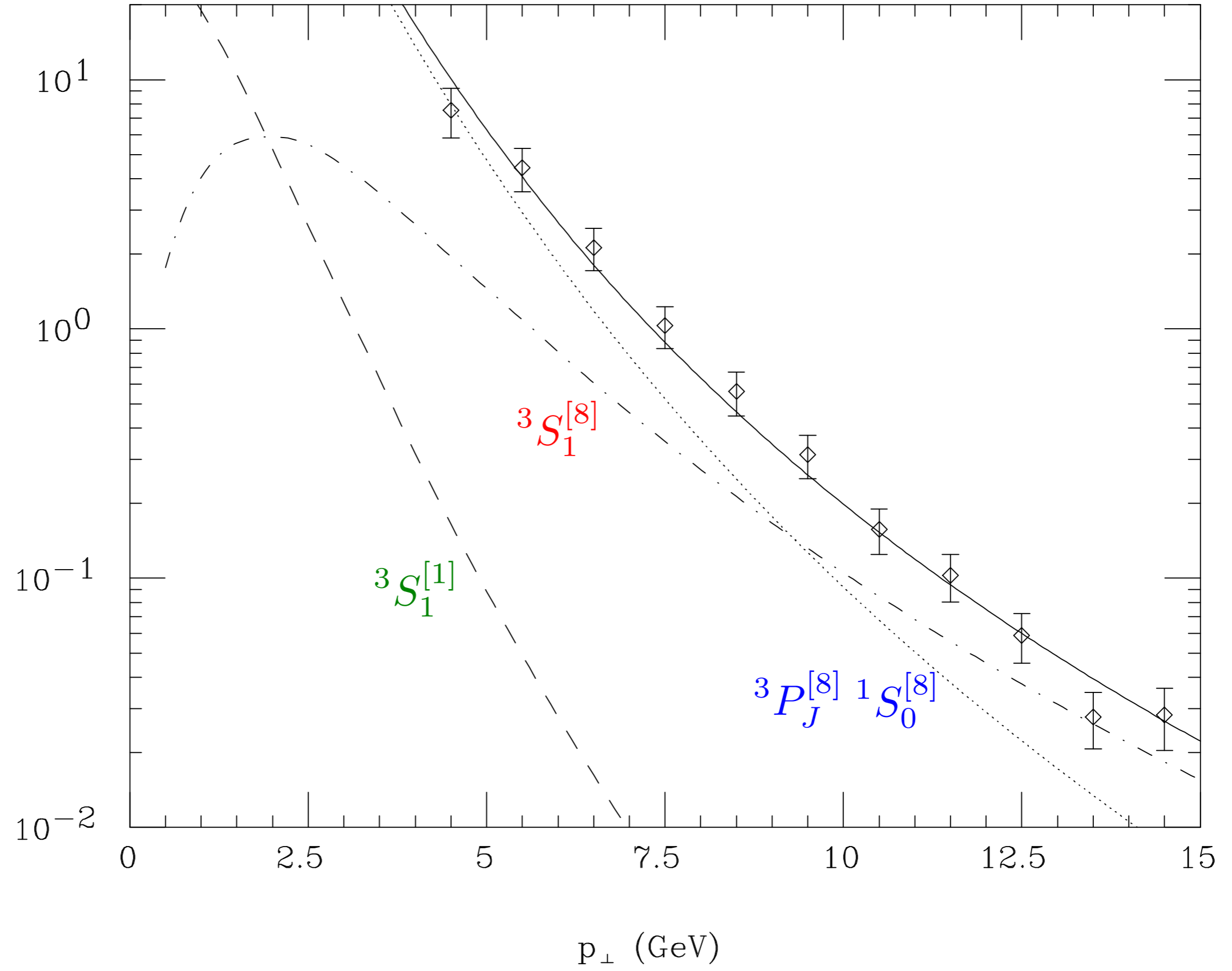
$$\langle \mathcal{O}^{J/\psi}({}^3S_1^{[1]}) \rangle \sim v^3$$

CSM - lowest order in v

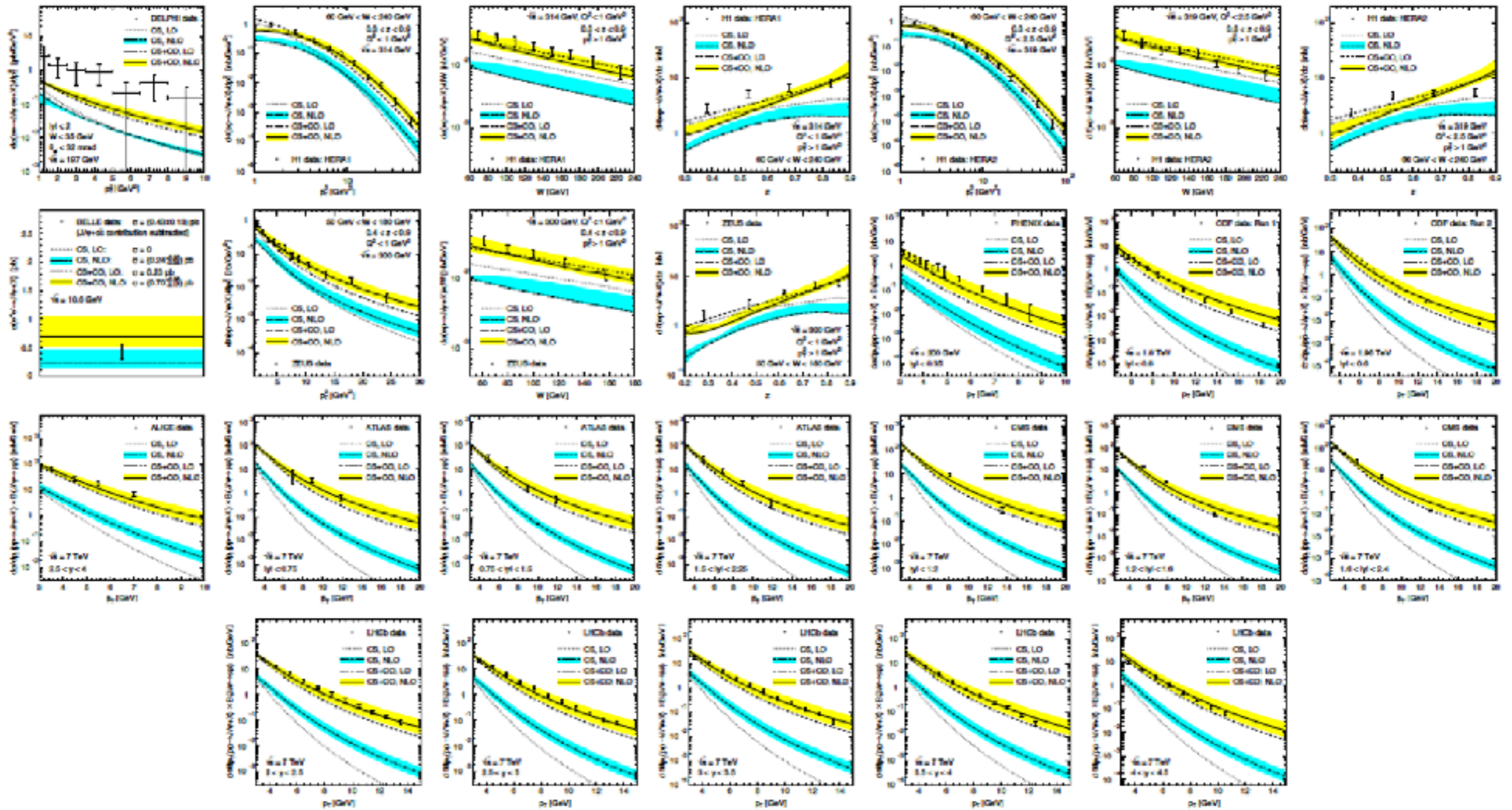
$$\langle \mathcal{O}^{J/\psi}({}^3S_1^{[8]}) \rangle, \langle \mathcal{O}^{J/\psi}({}^1S_0^{[8]}) \rangle, \langle \mathcal{O}^{J/\psi}({}^3P_J^{[8]}) \rangle \sim v^7$$

color-octet mechanisms

$\text{Br}(J/\psi \rightarrow \mu^+ \mu^-) \text{d}\sigma(\text{p}\bar{\text{p}} \rightarrow J/\psi + X) / \text{d}p_{\perp}$ (nb/GeV)



Global Fits with NLO CSM + COM

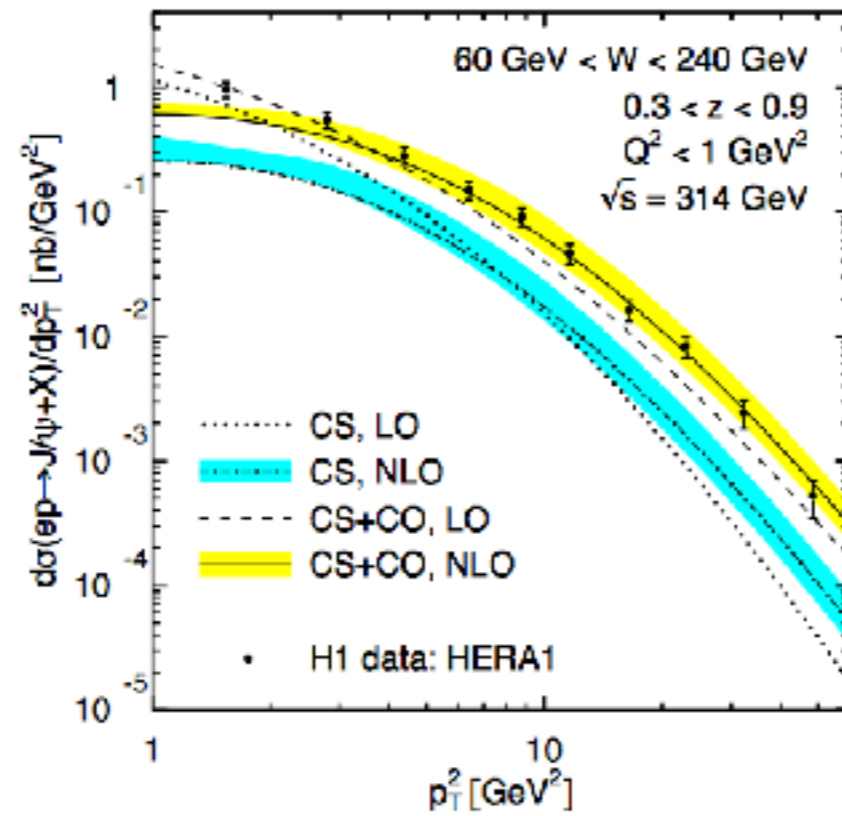
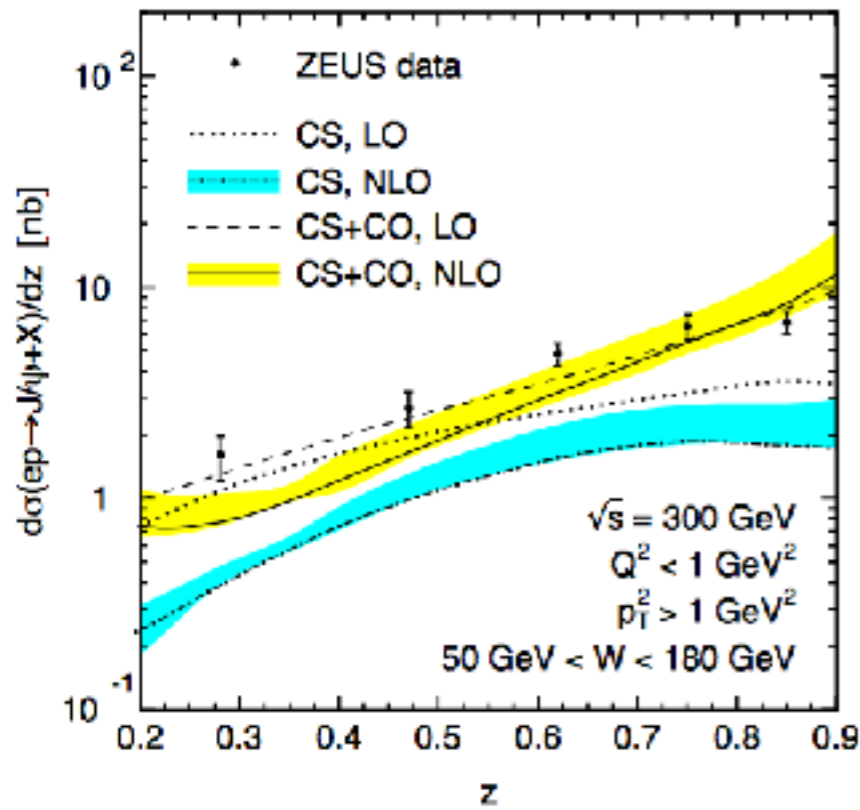


$$e^+e^-, \gamma\gamma, \gamma p, p\bar{p}, pp \rightarrow J/\psi + X$$

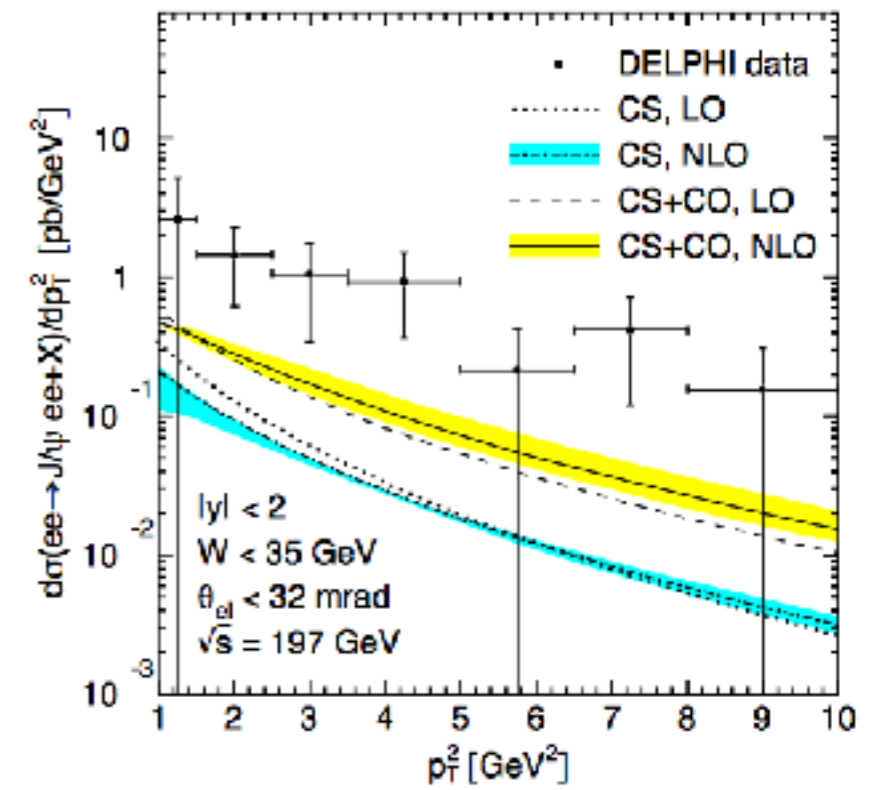
fit to 194 data points, 26 data sets,

Butenschoen and Kniehl, PRD 84 (2011) 051501

NLO: CSM + COM Required to Fit Data



$$ep \rightarrow J/\psi + X$$



$$\gamma^* \gamma^* \rightarrow J/\psi + X$$

Status of NRQCD approach to J/ψ Production

NLO: COM + CSM required for most processes

extracted LDME satisfy NRQCD v-scaling

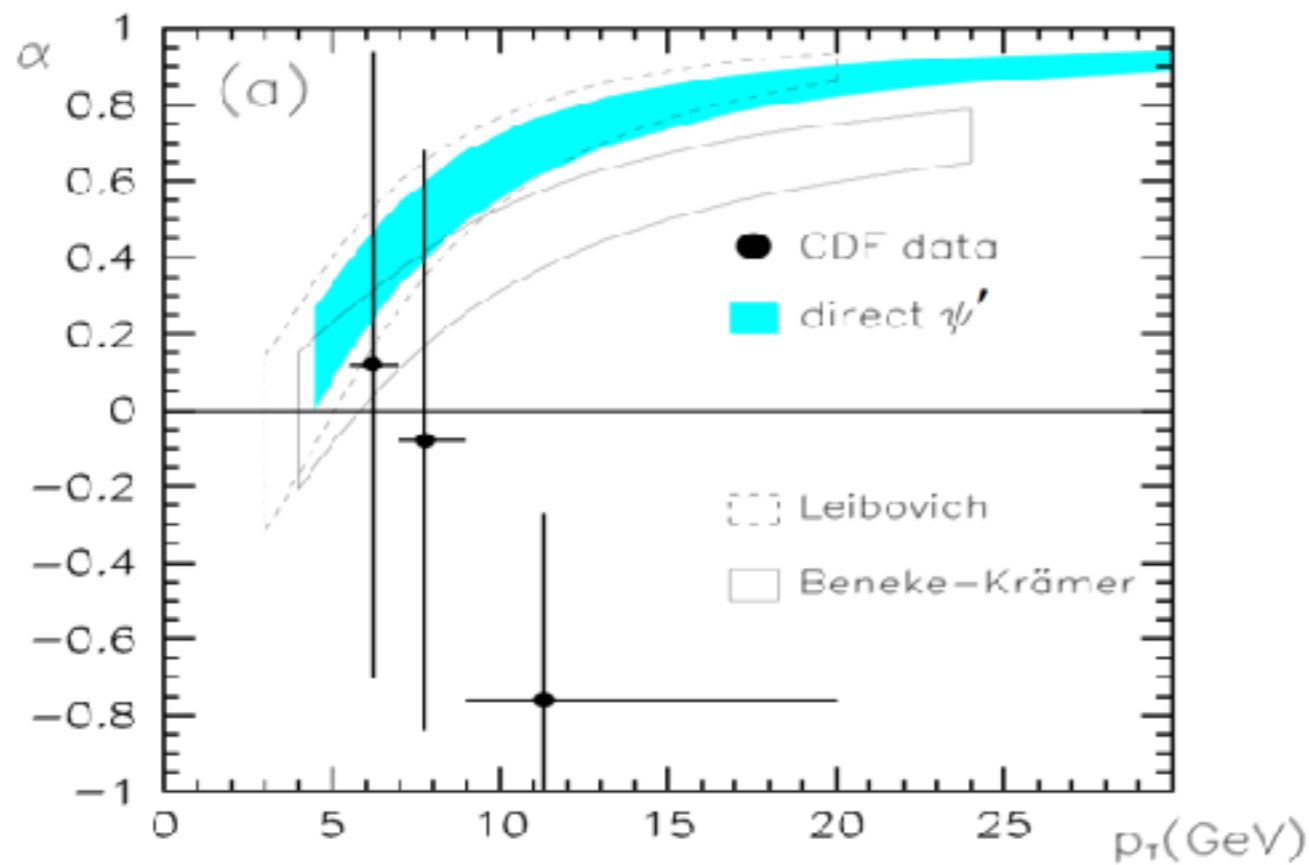
$$\langle \mathcal{O}^{J/\psi}(^3S_1^{[1]}) \rangle = 1.32 \text{ GeV}^3$$

| | |
|-----------------------------------------------------|-------------------------------------------------|
| $\langle \mathcal{O}^{J/\psi}(^1S_0^{[8]}) \rangle$ | $(4.97 \pm 0.44) \times 10^{-2} \text{ GeV}^3$ |
| $\langle \mathcal{O}^{J/\psi}(^3S_1^{[8]}) \rangle$ | $(2.24 \pm 0.59) \times 10^{-3} \text{ GeV}^3$ |
| $\langle \mathcal{O}^{J/\psi}(^3P_0^{[8]}) \rangle$ | $(-1.61 \pm 0.20) \times 10^{-2} \text{ GeV}^5$ |

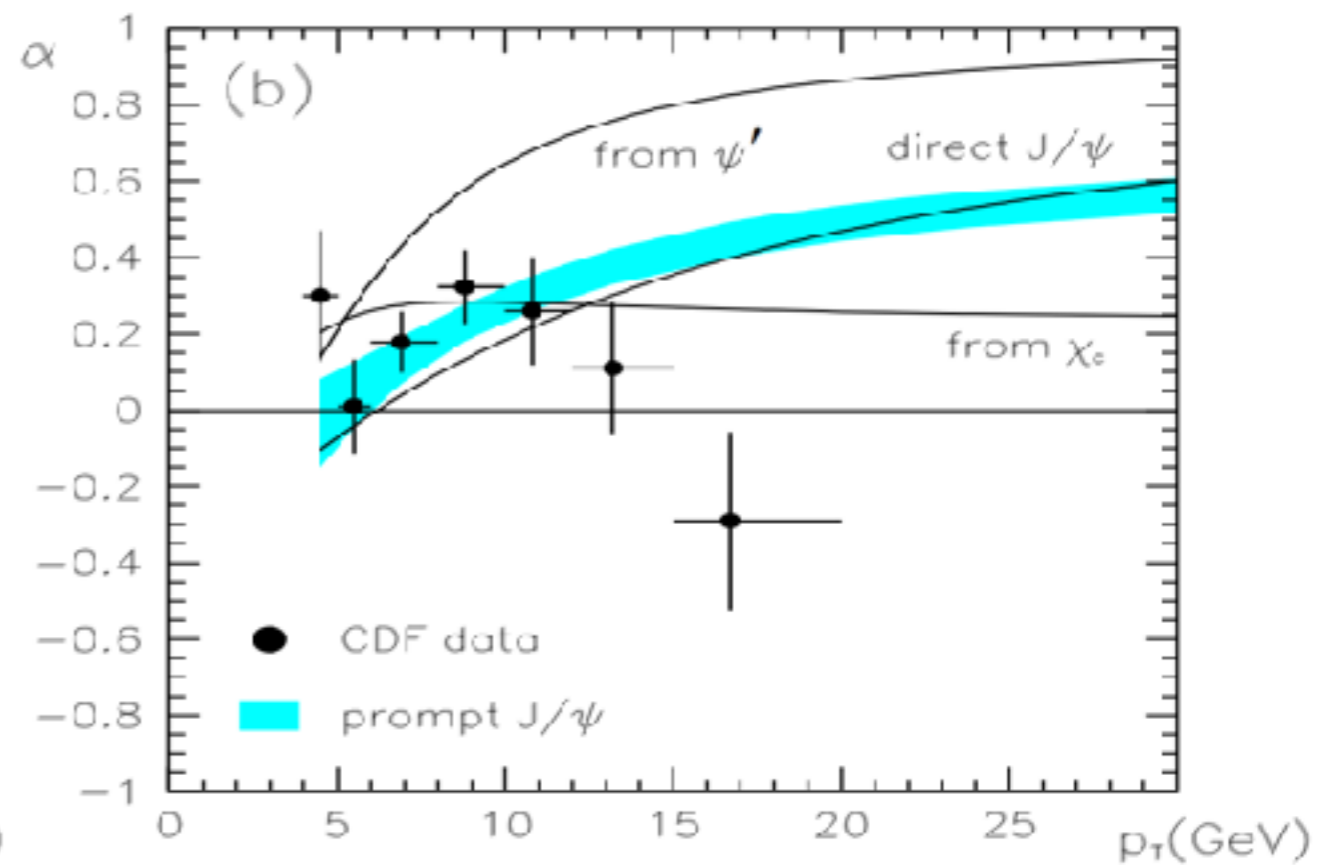
$$\chi_{\text{d.o.f.}}^2 = 857/194 = 4.42$$

Polarization Puzzle

$^3S_1^{[8]}$ fragmentation at large p_T predicts transversely polarized J/ψ , ψ'

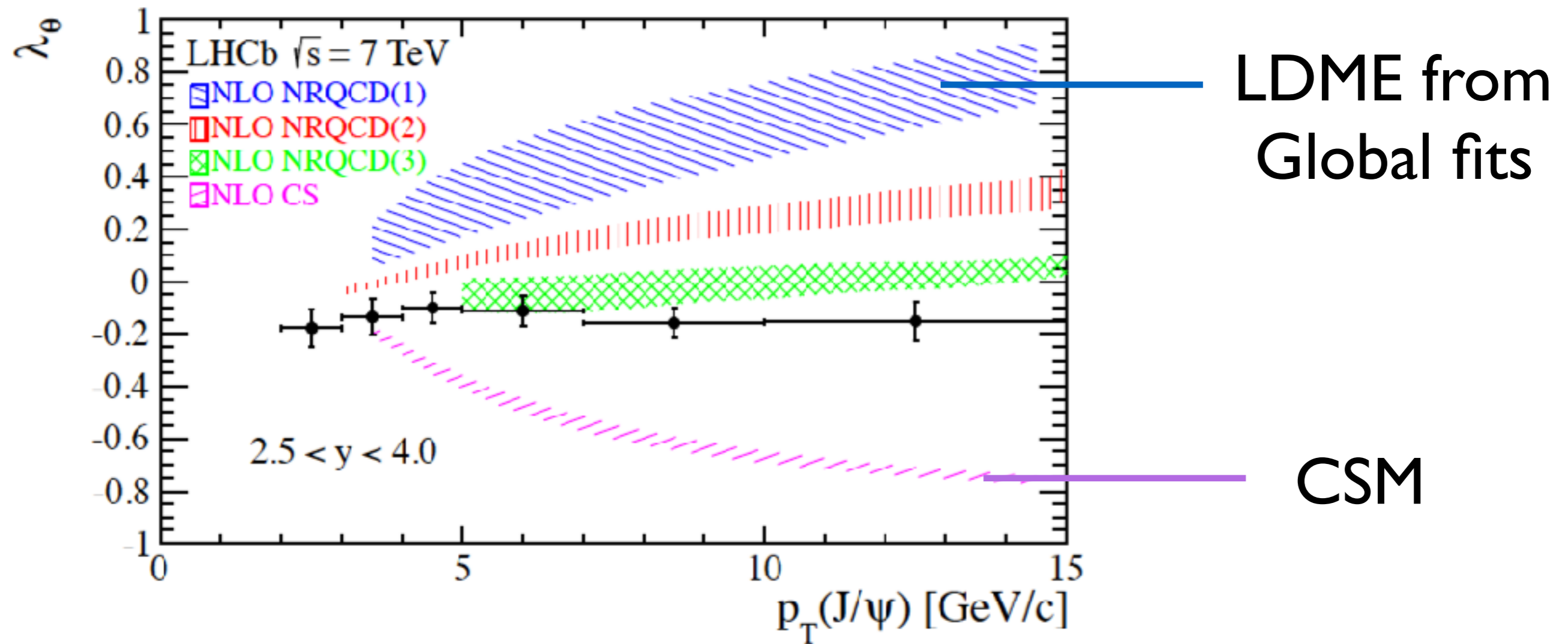


ψ'

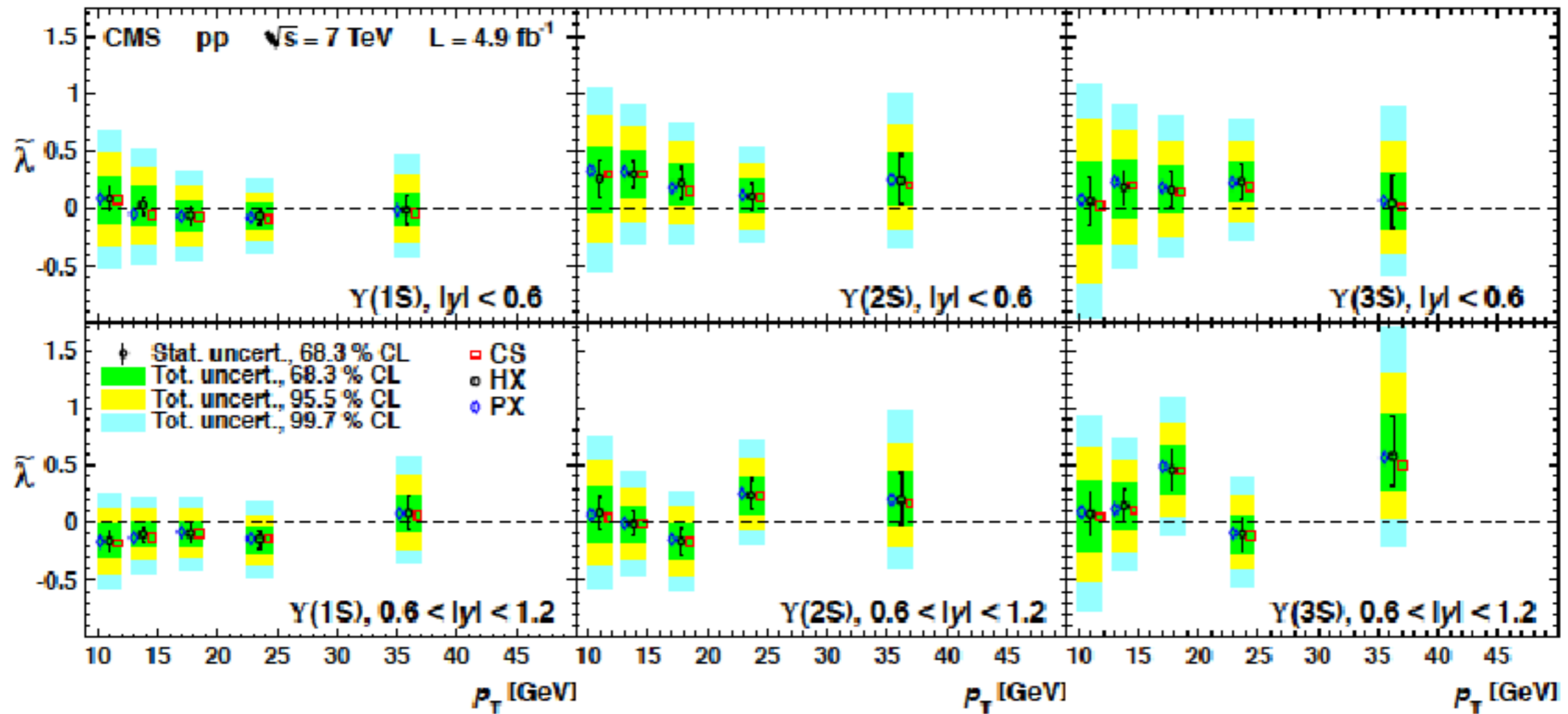


J/ψ

Polarization of J/ψ at LHCb



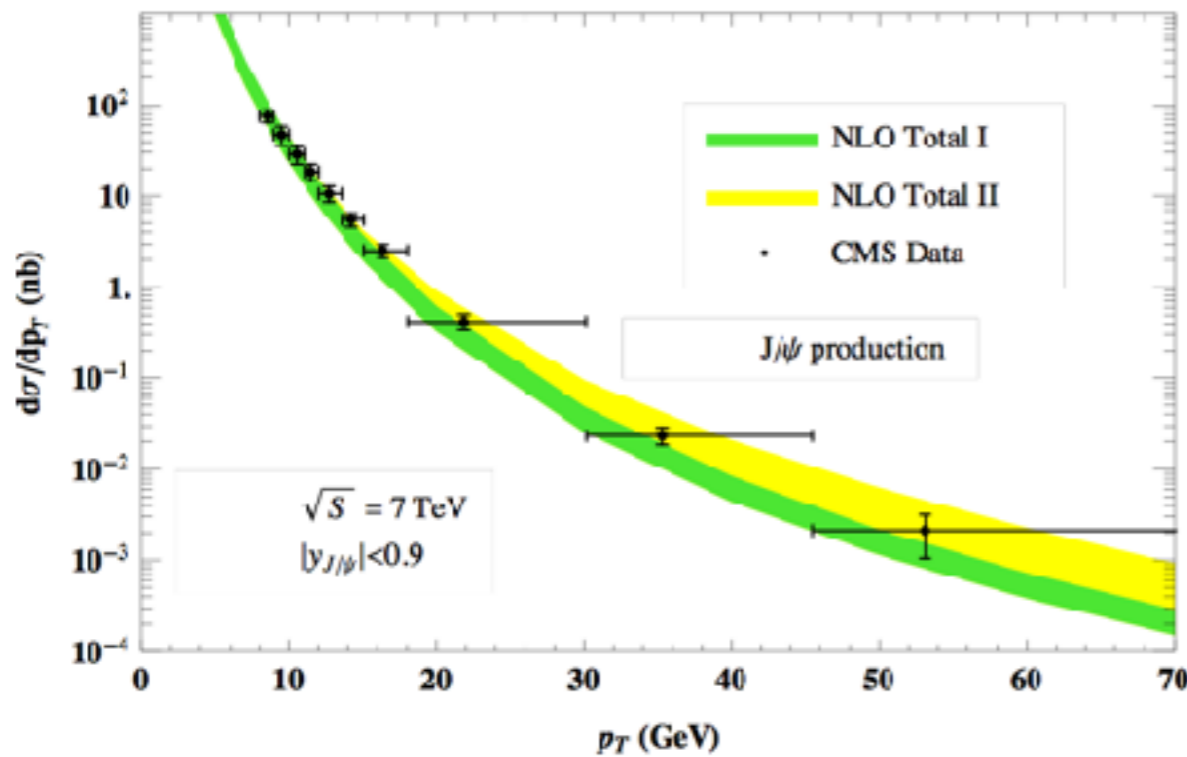
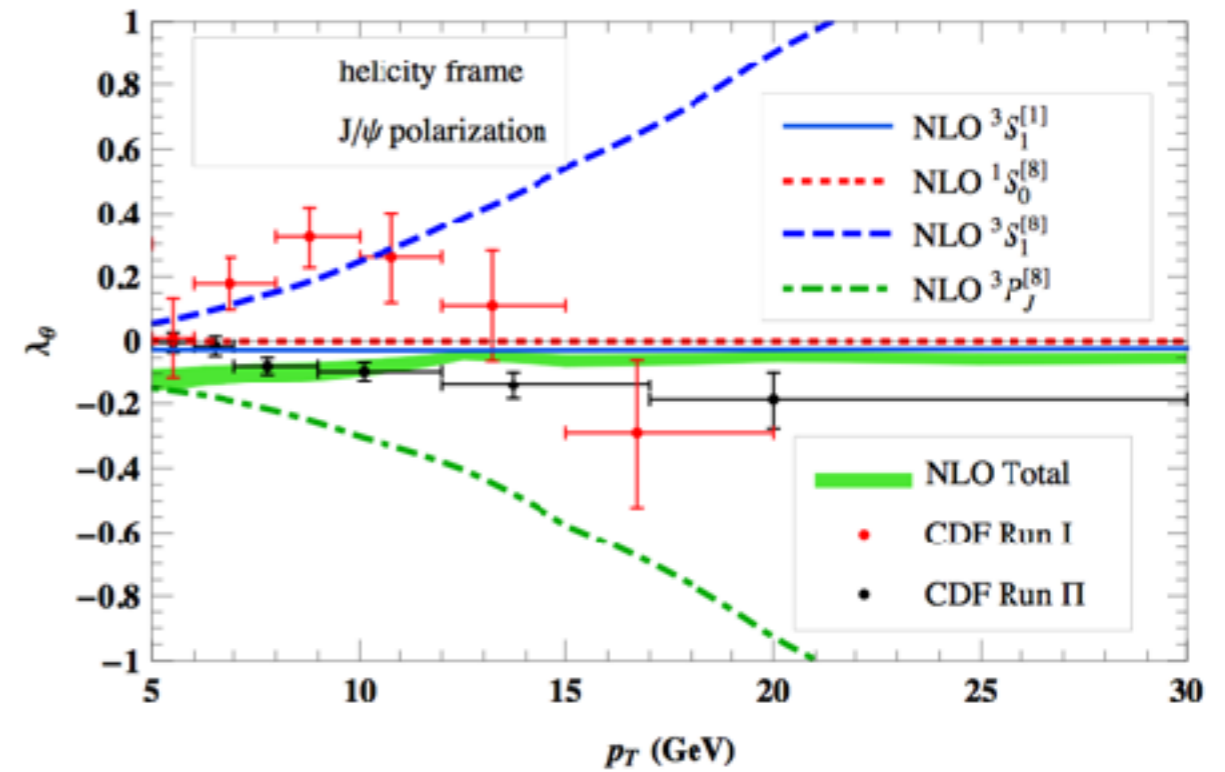
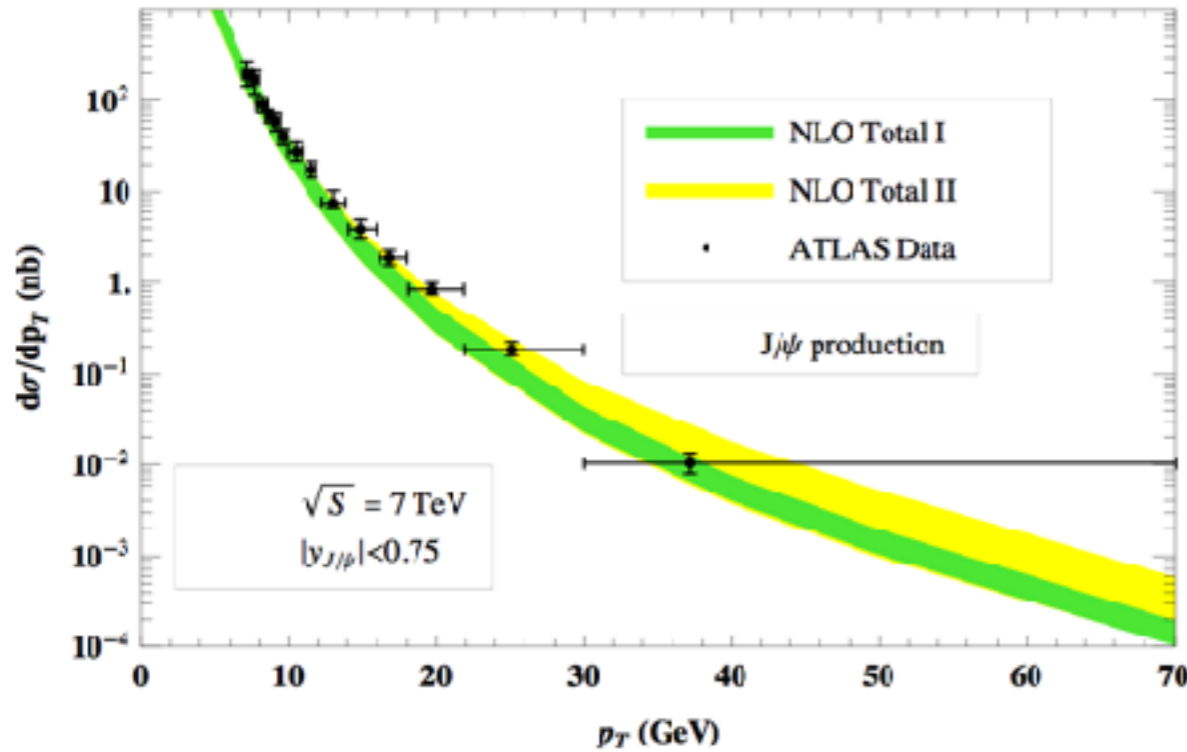
Polarization of $\Upsilon(nS)$ at CMS



Recent Attempts to Resolve J/ψ Polarization Puzzle

simultaneous NLO fit to CMS, ATLAS high p_T production, polarization

Chao, et. al. PRL 108, 242004 (2012)



| $\langle \mathcal{O}(^3S_1^{[1]}) \rangle$ GeV ³ | $\langle \mathcal{O}(^1S_0^{[8]}) \rangle$ 10 ⁻² GeV ³ | $\langle \mathcal{O}(^3S_1^{[8]}) \rangle$ 10 ⁻² GeV ³ | $\langle \mathcal{O}(^3P_0^{[8]}) \rangle / m_c^2$ 10 ⁻² GeV ³ |
|----------------------------------------------------------------|---------------------------------------------------------------------------------|---------------------------------------------------------------------------------|-----------------------------------------------------------------------------------------|
| 1.16 | 8.9 ± 0.98 | 0.30 ± 0.12 | 0.56 ± 0.21 |
| 1.16 | 0 | 1.4 | 2.4 |
| 1.16 | 11 | 0 | 0 |

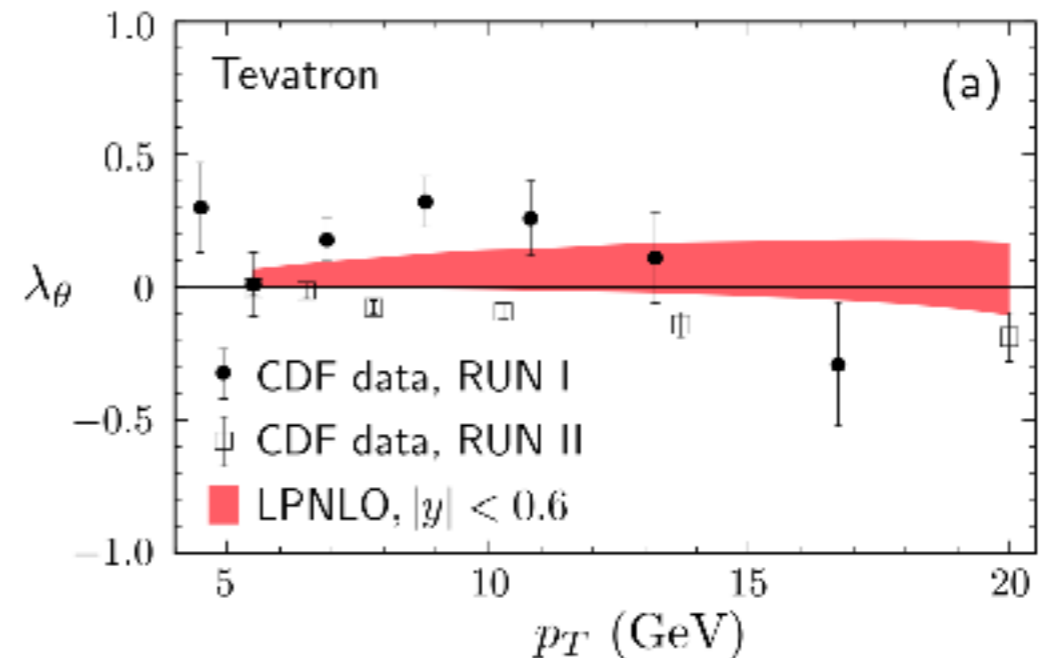
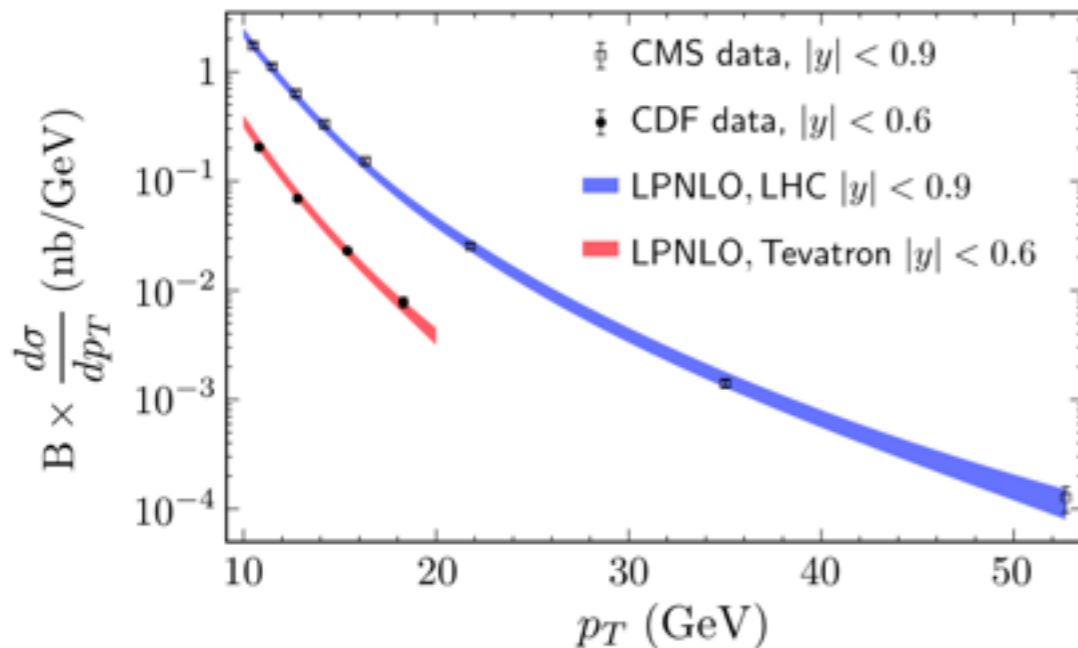
Recent Attempts to Resolve J/ψ Polarization Puzzle

i) large p_t production at CDF

Bodwin, et. al., PRL 113, 022001 (2014)

ii) resum logs of p_t/m_c using DGLAP evolution

iii) fit COME to p_t spectrum, predict basically no polarization



Extracted COME **inconsistent** with global fits

$$\langle \mathcal{O}^{J/\psi} (^1S_0^{(8)}) \rangle = 0.099 \pm 0.022 \text{ GeV}^3$$

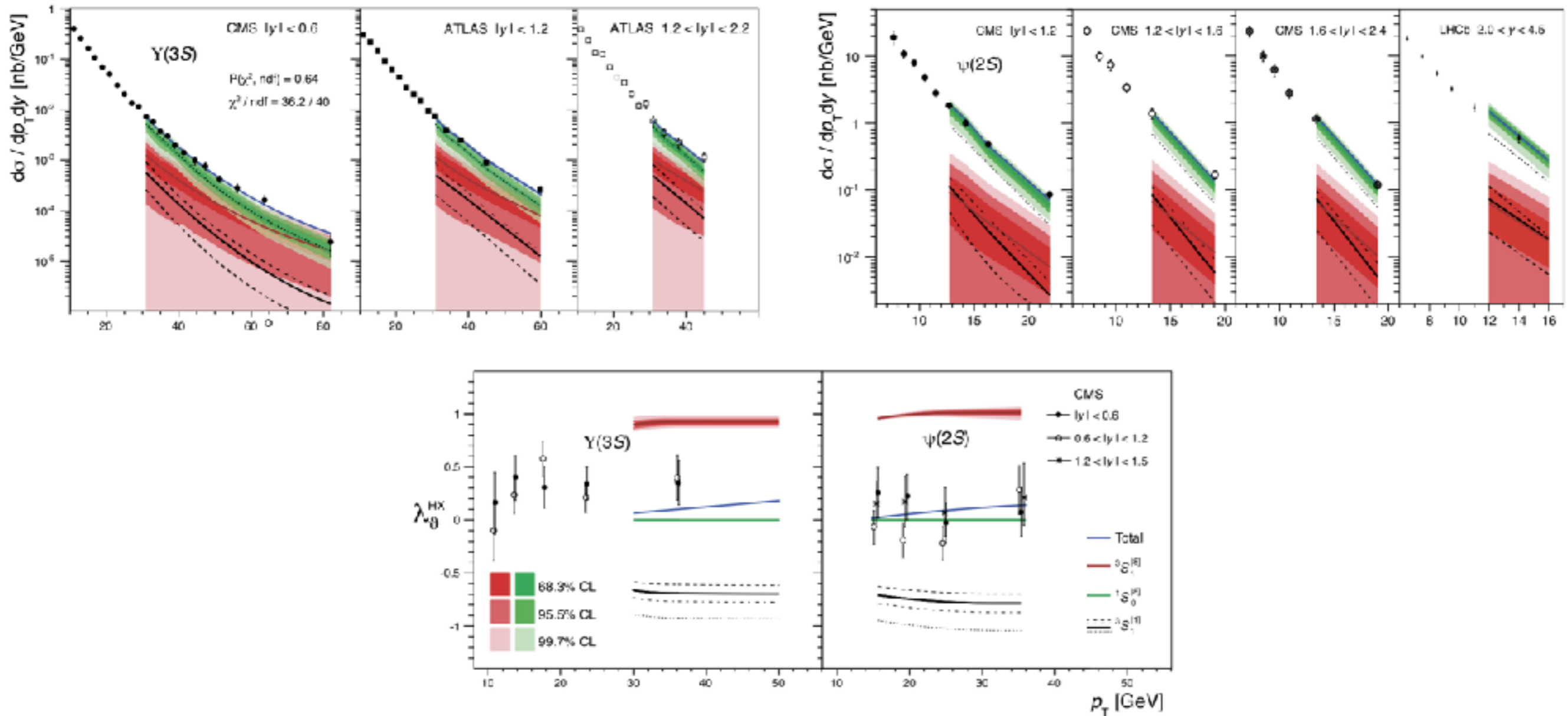
$$\langle \mathcal{O}^{J/\psi} (^3S_1^{(8)}) \rangle = 0.011 \pm 0.010 \text{ GeV}^3$$

$$\langle \mathcal{O}^{J/\psi} (^3P_0^{(8)}) \rangle = 0.011 \pm 0.010 \text{ GeV}^5$$

Recent Attempts to Resolve J/ψ Polarization Puzzle

Faccioli, et. al. PLB736 (2014) 98

Lourenco, et. al., NPA, in press



argue for $1S_0^{(8)}$ dominance in both $\psi(2S)$ & $Y(3S)$ production

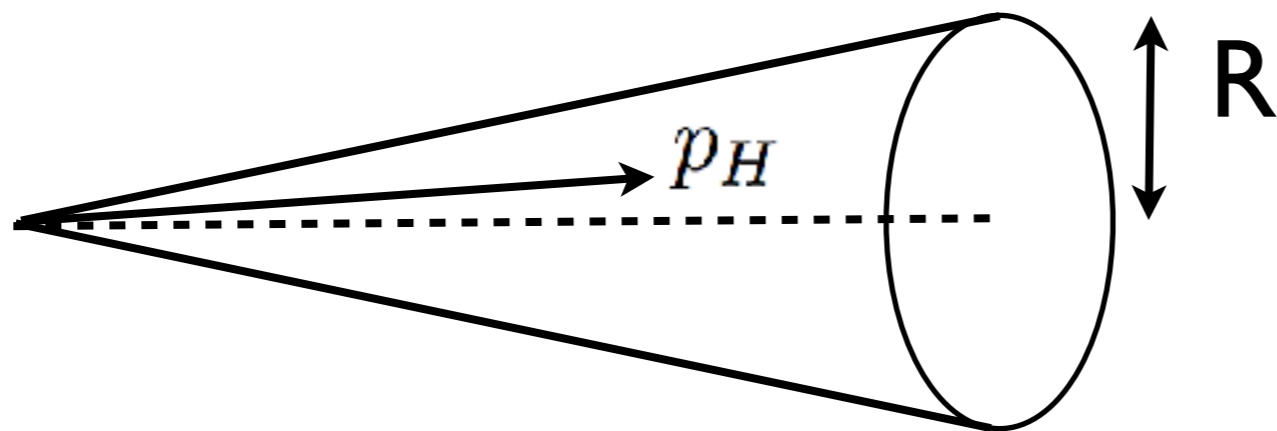
Fragmenting Jet Functions

M. Procura, I. Stewart, PRD 81 (2010) 074009

A. Jain, M. Procura, W. Waalewijn, JHEP 1105 (2011) 035

A. Procura, W. Waalewijn, PRD 85 (2012) 114041

jets with identified hadrons



Jet Energy: E

$$p_H^+ = z p_{\text{jet}}^+$$

cross sections determined by **fragmenting jet function (FJF)**:

$$\mathcal{G}_g^h(E, R, \mu, z)$$

inclusive hadron production: fragmentation functions

$$\frac{1}{\sigma_0} \frac{d\sigma^h}{dz}(e^+e^- \rightarrow h X) = \sum_i \int_z^1 \frac{dx}{x} C_i(E_{\text{cm}}, x, \mu) D_i^h(z/x, \mu)$$

jet cross sections: jet functions

$$d\sigma(E, R) = \int d\Phi_N \text{tr}[H_N S_N] \prod_{\ell} J_{\ell}$$

$$\mathcal{G}_g^h(E, R, \mu, z) \longrightarrow D_i^h(z/x, \mu), J_{\ell}$$

relationship to jet function:

$$\sum_h \int_0^1 dz z D_j^h(z, \mu) = 1$$

$$\longrightarrow J_i(E, R, z, \mu) = \frac{1}{2} \sum_h \int \frac{dz}{(2\pi)^3} z \mathcal{G}_i^h(E, R, z, \mu)$$

cross section for jet w/ identified hadron from jet cross section

$$\frac{d\sigma}{dE} = \int d\Phi_N \text{tr}[H_N S_N] \prod_{\ell} J_{\ell} J_i(E, R, \mu)$$

$$\longrightarrow \frac{d\sigma}{dE dz} = \int d\Phi_N \text{tr}[H_N S_N] \prod_{\ell} J_{\ell} \mathcal{G}_i^h(E, R, z, \mu)$$

relationship to fragmentation functions

$$\mathcal{G}_i^h(E, R, z, \mu) = \sum_i \int_z^1 \frac{dz'}{z'} \mathcal{J}_{ij}(E, R, z', \mu) D_j^h\left(\frac{z}{z'}, \mu\right) \left[1 + \mathcal{O}\left(\frac{\Lambda_{\text{QCD}}^2}{4E^2 \tan^2(R/2)}\right)\right]$$

matching coefficients calculable in perturbation theory

$$\frac{\mathcal{J}_{gg}(E, R, z, \mu)}{2(2\pi)^3} = \delta(1-z) + \frac{\alpha_s(\mu) C_A}{\pi} \left[\left(L^2 - \frac{\pi^2}{24} \right) \delta(1-z) + \hat{P}_{gg}(z)L + \hat{\mathcal{J}}_{gg}(z) \right]$$

$$\hat{\mathcal{J}}_{gg}(z) = \begin{cases} \hat{P}_{gg}(z) \ln z & z \leq 1/2 \\ \frac{2(1-z+z^2)^2}{z} \left(\frac{\ln(1-z)}{1-z} \right)_+ & z \geq 1/2. \end{cases}$$

$$L = \ln[2E \tan(R/2)/\mu].$$

scale for $\mathcal{J}_{ij}(E, R, z, \mu)$

sum rule for matching coefficients

$$\sum_j \int_0^1 dz z \mathcal{J}_{ij}(R, z, \mu) = 2(2\pi)^3 J_i(R, \mu)$$

NRQCD fragmentation functions

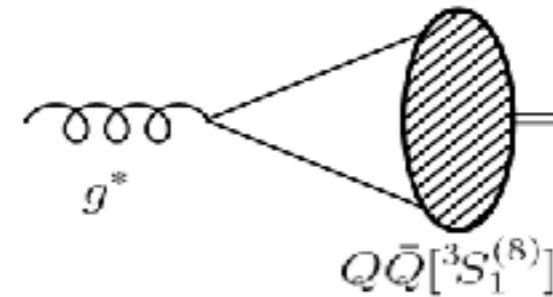
Braaten, Yuan, PRD 48 (1993) 4230

Braaten, Chen, PRD 54 (1996) 3216

Braaten, Fleming, PRL 74 (1995) 3327

Perturbatively calculable **at the scale $2m_c$**

$$D_g^{\psi(8)}(z, 2m_c) = \frac{\pi\alpha_s(2m_c)}{3M_\psi^3} \langle O^\psi(^3S_1^{(8)}) \rangle \delta(1-z).$$



$$D_g^{\psi(1)}(z, 2m_c) = \frac{5\alpha_s^3(2m_c)}{648\pi^2} \frac{\langle O^\psi(^3S_1^{(1)}) \rangle}{M_\psi^3} \int_0^z dr \int_{(r+z^2)/2z}^{(1+r)/2} dy \frac{1}{(1-y)^2(y-r)^2(y^2-r)^2} \sum_{i=0}^2 z^i \left(f_i(r, y) + g_i(r, y) \frac{1+r-2y}{2(y-r)\sqrt{y^2-r}} \ln \frac{y-r+\sqrt{y^2-r}}{y-r-\sqrt{y^2-r}} \right),$$

DGLAP evolution: $2m_c$ to $2E \tan(R/2)$

FJF in terms of fragmentation function

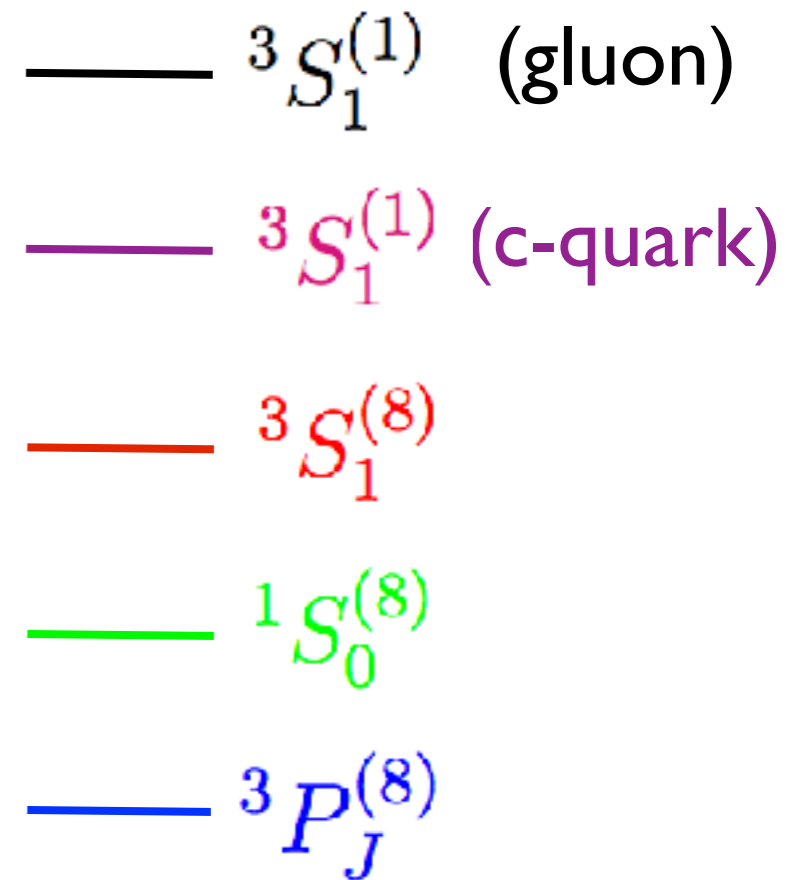
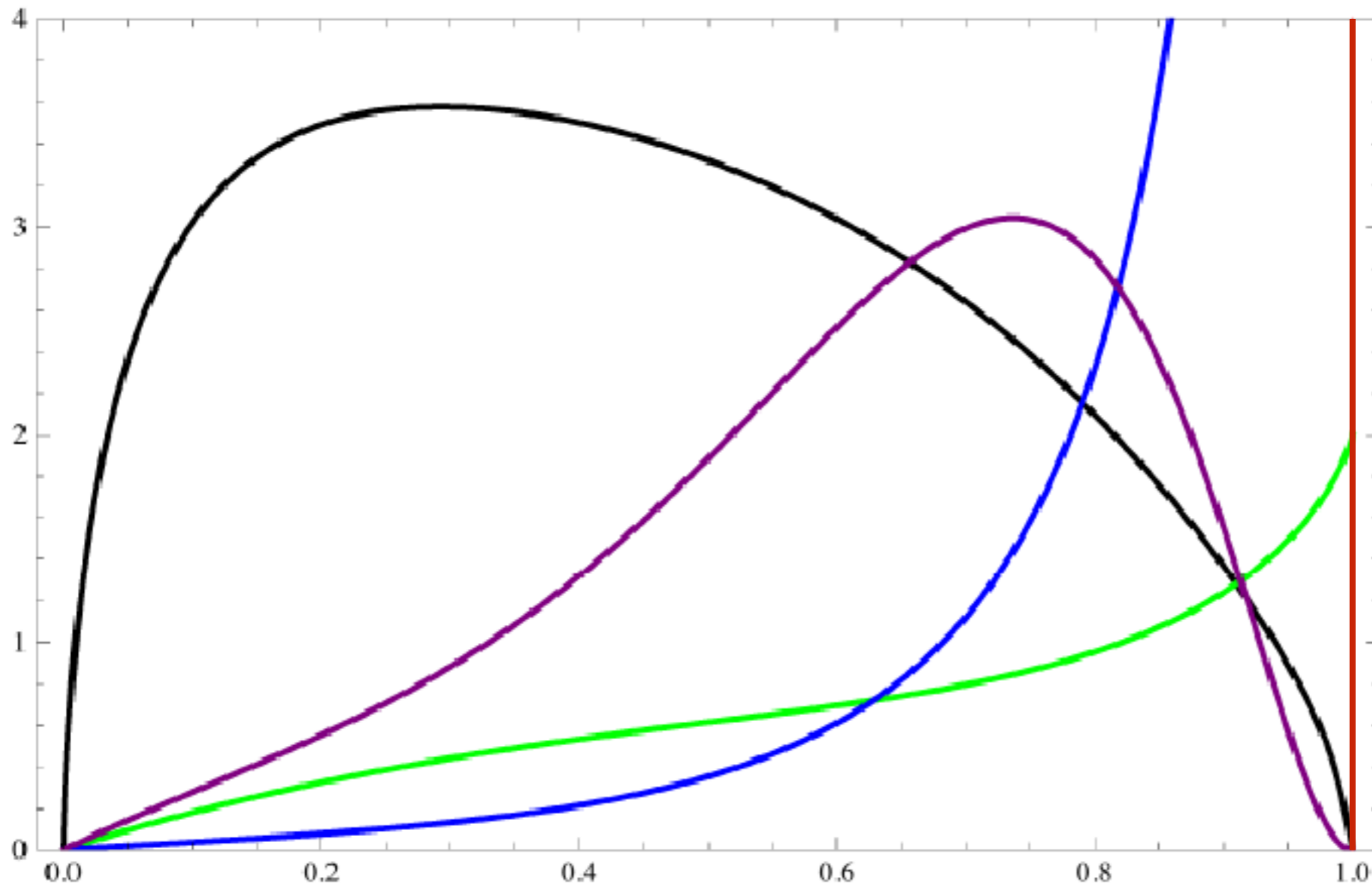
$$\begin{aligned}
 \mathcal{G}_g^\psi(E, R, z, \mu) = & D_{g \rightarrow \psi}(z, \mu) \left(1 + \frac{C_A \alpha_s}{\pi} \left(L_{1-z}^2 - \frac{\pi^2}{24} \right) \right) \\
 & + \frac{C_A \alpha_s}{\pi} \left[\int_z^1 \frac{dy}{y} \tilde{P}_{gg}(y) L_{1-y} D_{g \rightarrow \psi} \left(\frac{z}{y}, \mu \right) \right. \\
 & + 2 \int_z^1 dy \frac{D_{g \rightarrow \psi}(z/y, \mu) - D_{g \rightarrow \psi}(z, \mu)}{1-y} L_{1-y} \\
 & \left. + \theta \left(\frac{1}{2} - z \right) \int_z^{1/2} \frac{dy}{y} \hat{P}_{gg}(y) \ln \left(\frac{y}{1-y} \right) D_{g \rightarrow \psi} \left(\frac{z}{y}, \mu \right) \right]
 \end{aligned}$$

$$L_{1-z} = \ln \left(\frac{2E \tan(R/2)(1-z)}{\mu} \right)$$

For large E, FJF \sim NRQCD frag. function (at scale $2E \tan(R/2)$)

$$\mathcal{G}_g^h(E, R, \mu = 2E \tan(R/2), z) \rightarrow D_g^\psi(z, 2E \tan(R/2)) + O(\alpha_s)$$

NRQCD FF's (at scale $2m_c$)



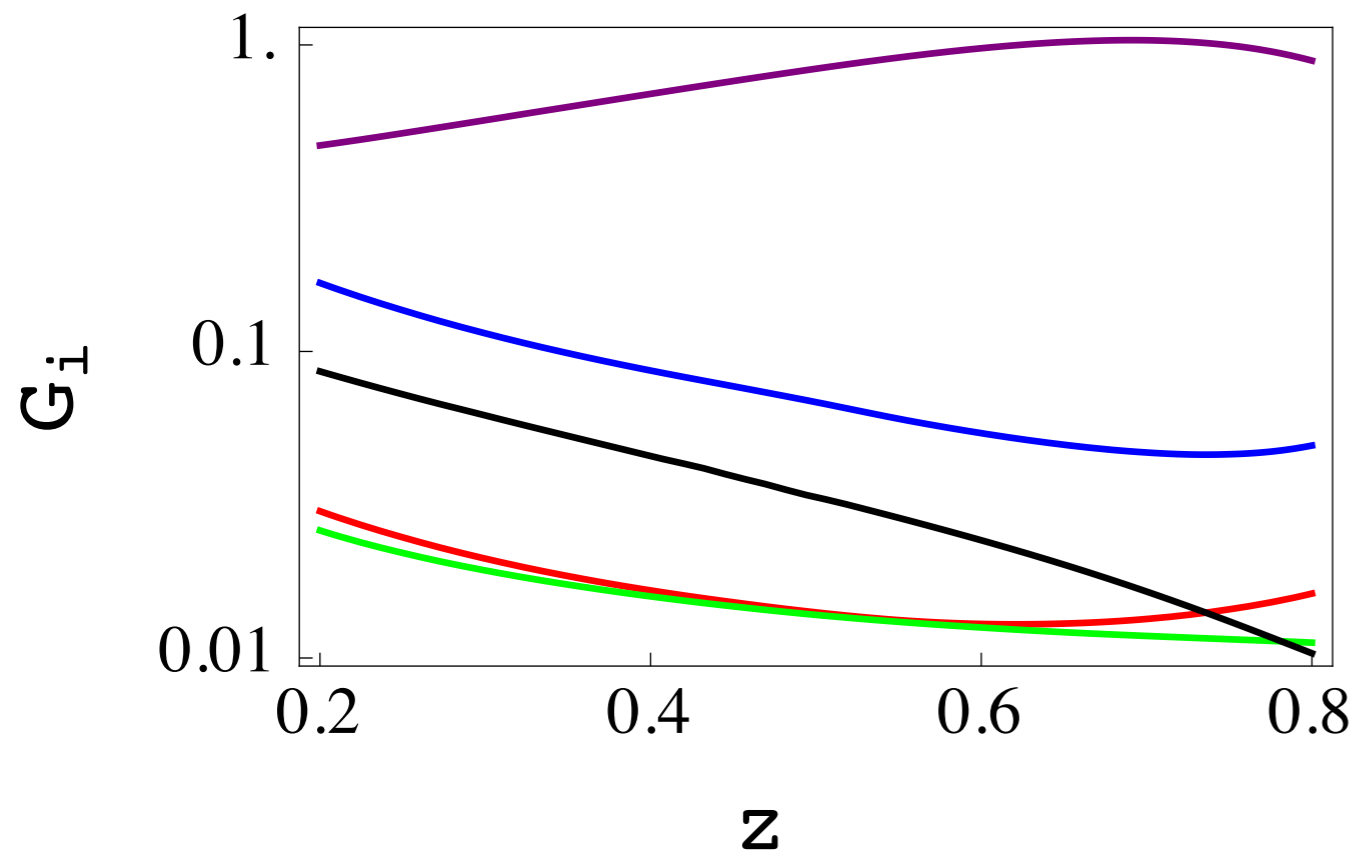
(normalization arbitrary)

Evolution to $2E \tan(R/2)$ will soften discrepancies

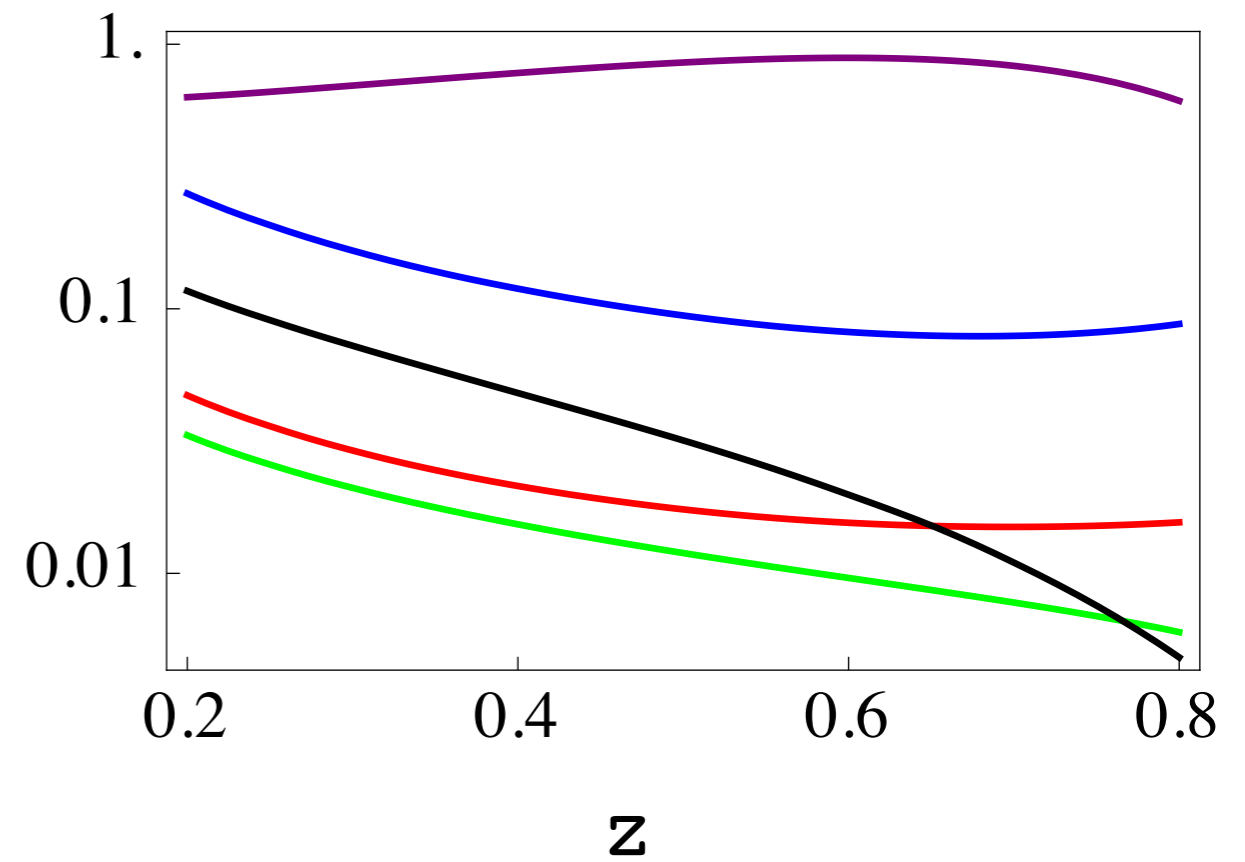
FJF's at Fixed Energy vs. z

M. Baumgart, A. Leibovich, T.M., I. Z. Rothstein, JHEP 1411 (2014) 003

$E = 50 \text{ GeV}$

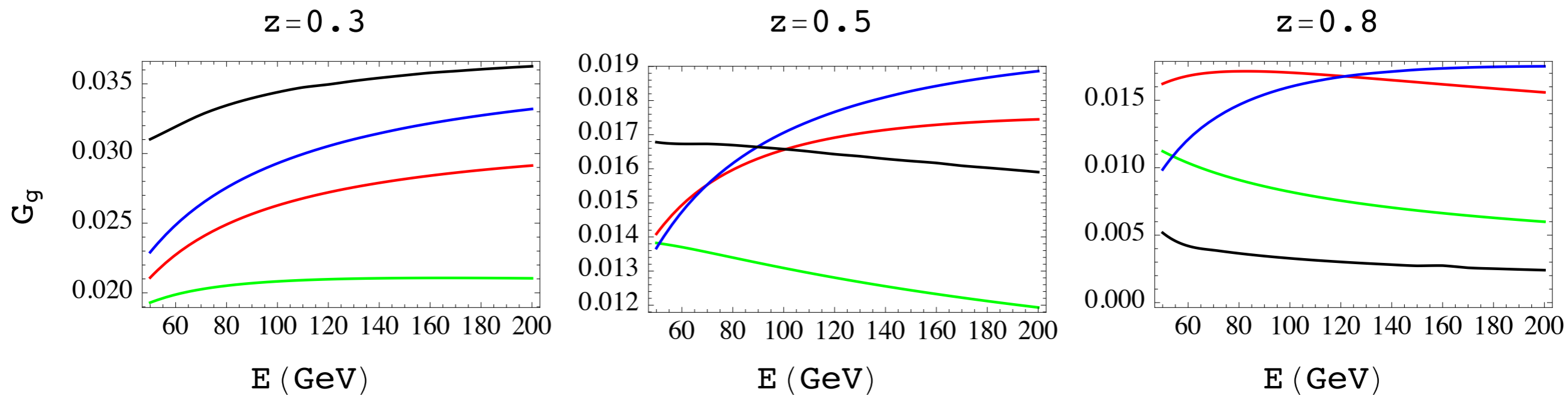


$E = 200 \text{ GeV}$



FJF's at Fixed z vs. Energy

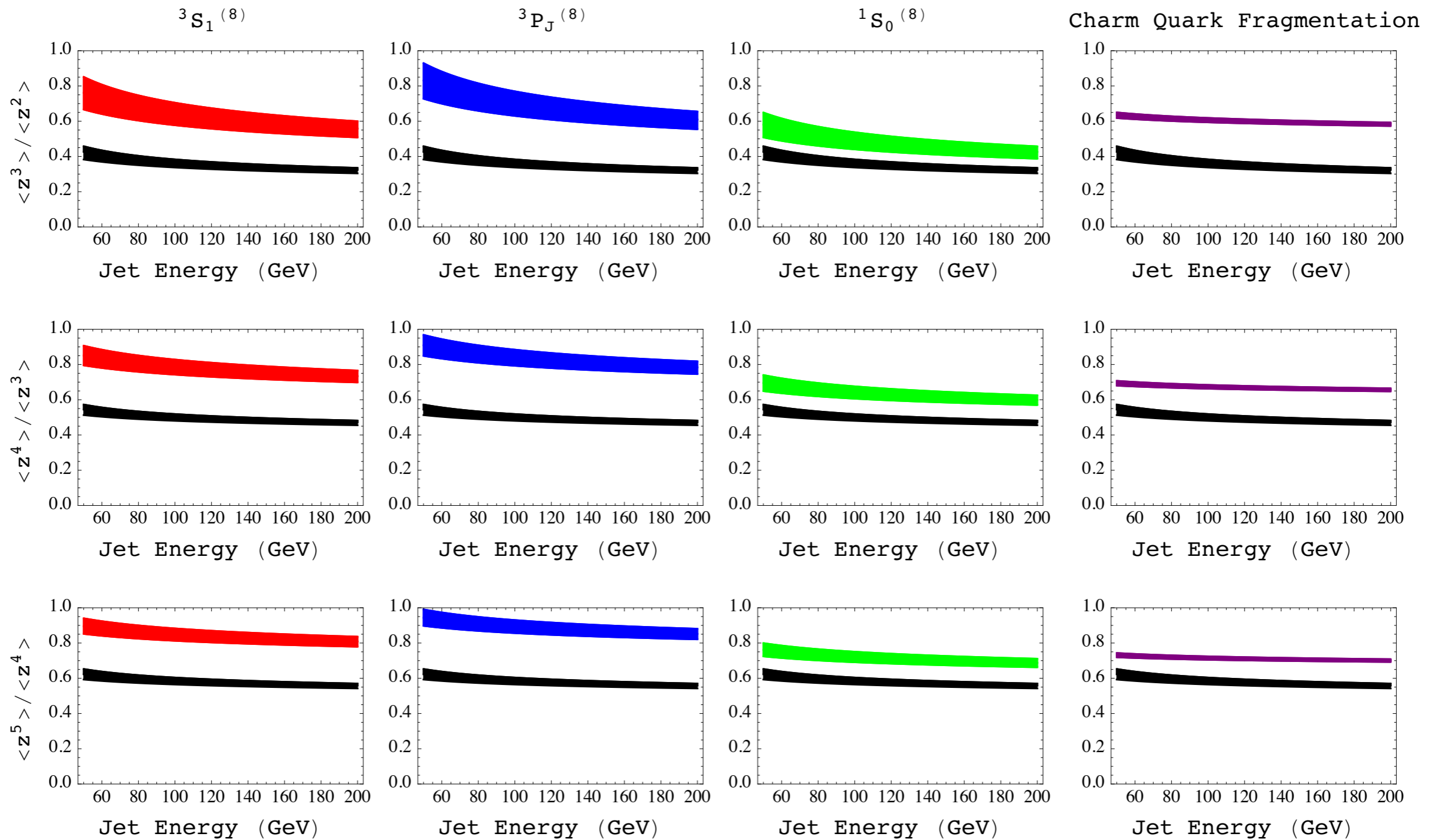
M. Baumgart, A. Leibovich, T.M., I. Z. Rothstein, JHEP 1411 (2014) 003



$^1S_0^{(8)}$ dominance predicts negative slope for z vs. E if $z > 0.5$

Ratios of Moments

$$E \tan(R/2) < \mu < 4E \tan(R/2)$$



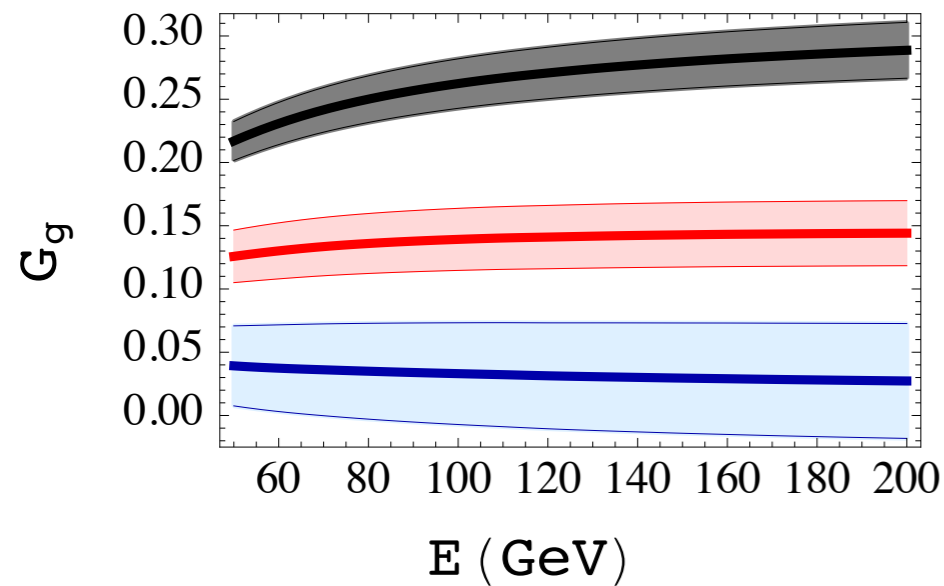
Ratios of Moments

$$\frac{\langle z^{n+1} \rangle}{\langle z^n \rangle} \Big|_{3P_J^{(8)}} \approx \frac{\langle z^{n+1} \rangle}{\langle z^n \rangle} \Big|_{3S_1^{(8)}} > \frac{\langle z^{n+1} \rangle}{\langle z^n \rangle} \Big|_{1S_0^{(8)}} \approx \frac{\langle z^{n+1} \rangle}{\langle z^n \rangle} \Big|_{\text{c-quark}} > \frac{\langle z^{n+1} \rangle}{\langle z^n \rangle} \Big|_{3S_1^{(1)}}$$

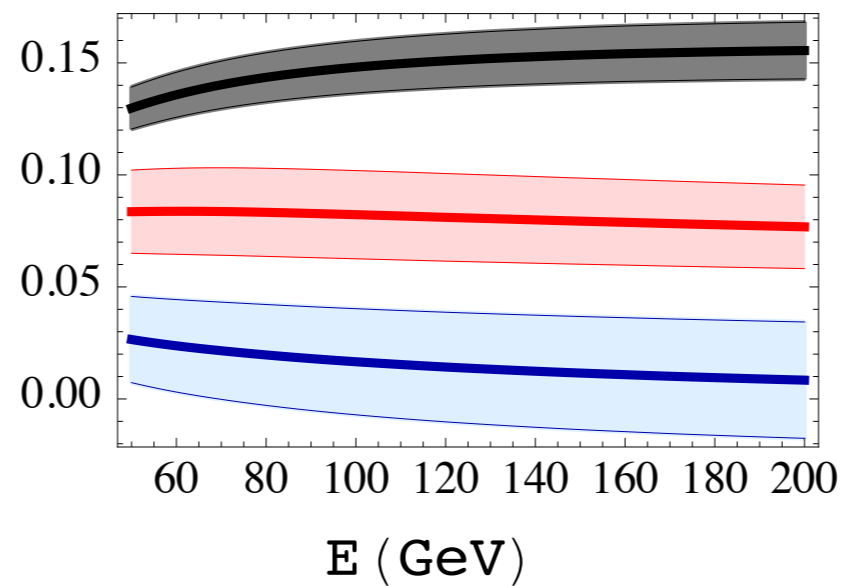
Gluon FJF for different extractions of LDME

fix z , vary energy

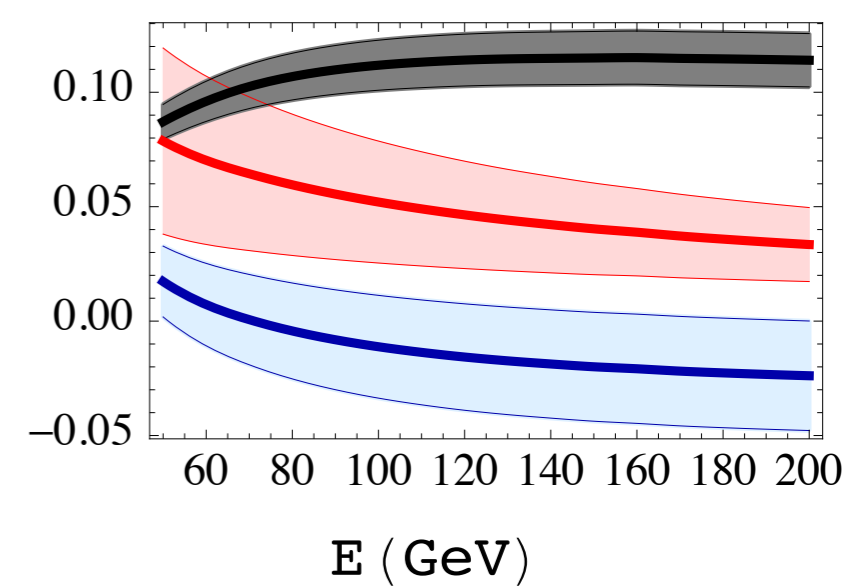
$z=0.3$



$z=0.5$



$z=0.8$

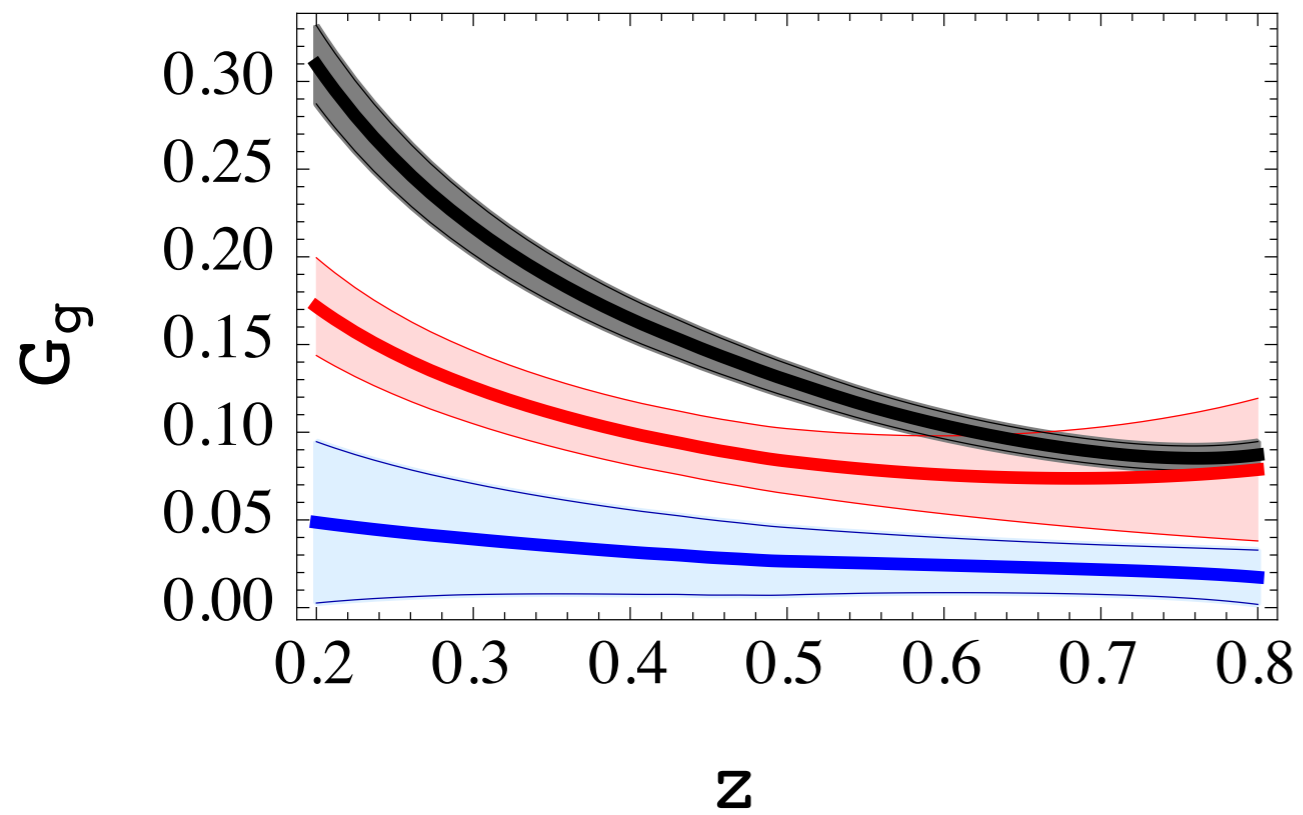


- Butenschoen and Kniehl, PRD 84 (2011) 051501, arXiv:1105.0822
- Bodwin, et. al. arXiv:1403.3612
- Chao, et. al. PRL 108, 242004 (2012)

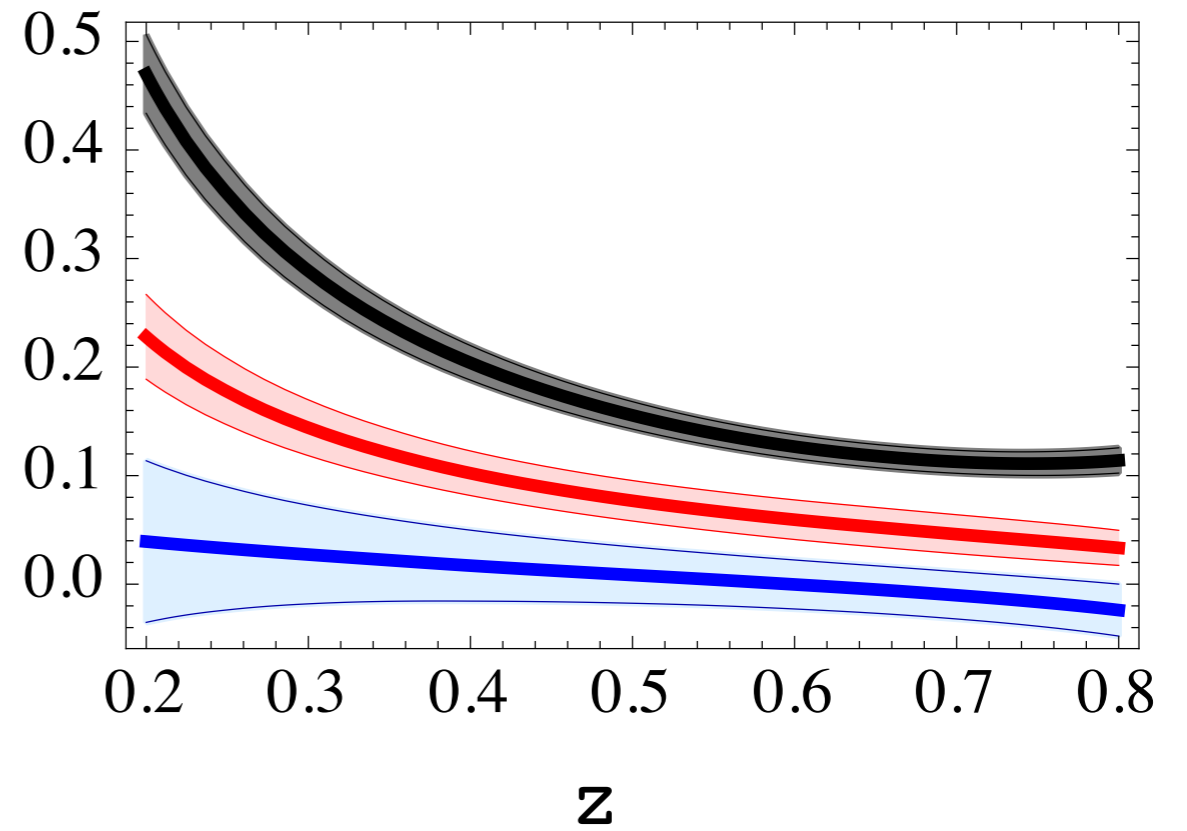
Gluon FJF for different extractions of LDME

fix energy, vary z

$E = 50 \text{ GeV}$

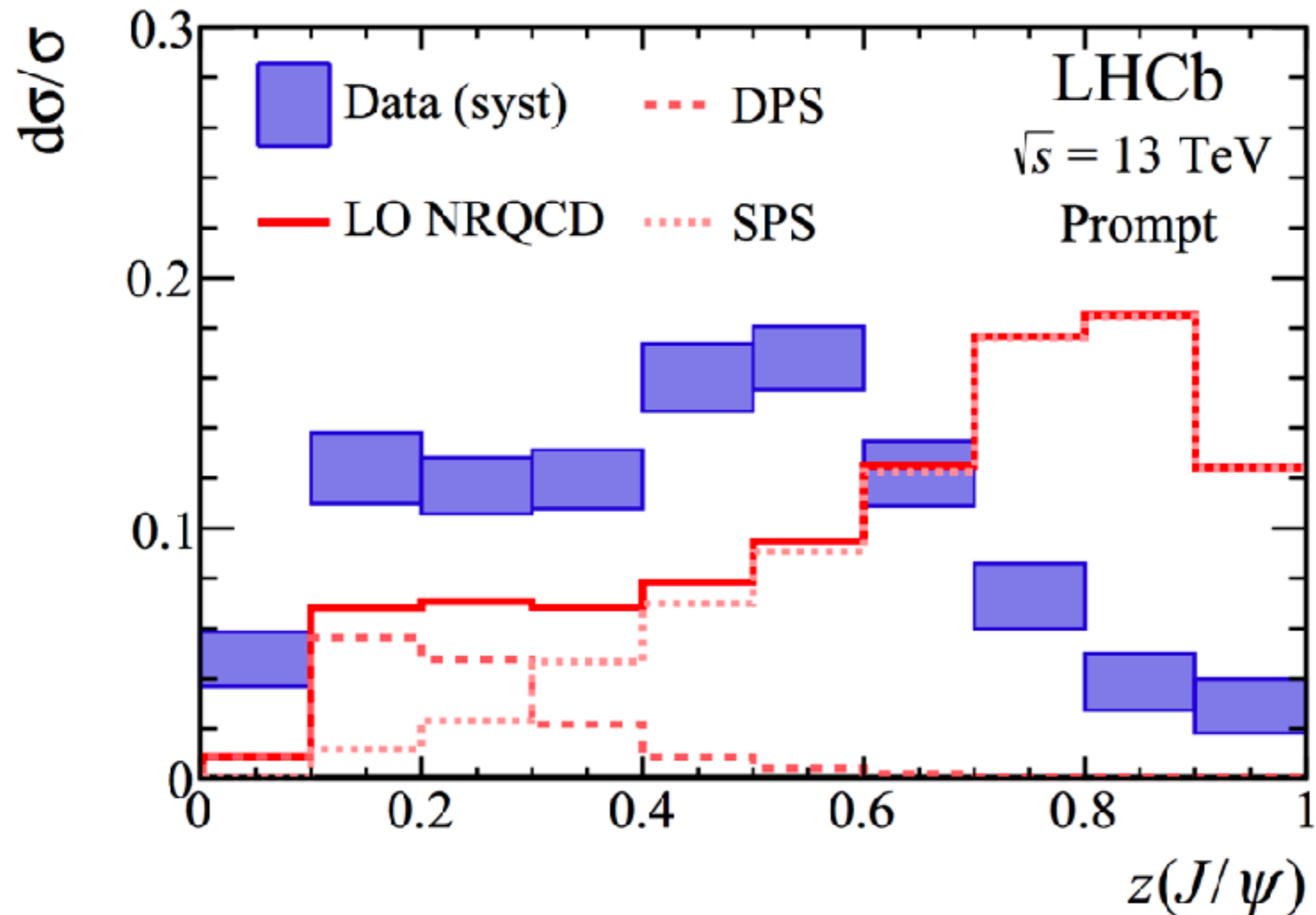


$E = 200 \text{ GeV}$



Recent Observations of Quarkonia within Jets

LHCb collaboration, Phys. Rev. Lett. 118 (2017) no.19, 192001



cuts: $2.5 < \eta_{jet} < 4.0$ $p_{T,jet} > 20$ GeV $p(\mu) > 5$ GeV

This result was anticipated in:

Jets w/ Heavy Mesons: NLL' vs. Monte Carlo

R. Bain, L. Dai, A. Hornig, A. Leibovich, T.M.Y. Makris JHEP 1606 (2016) 121

$$e^+e^- \rightarrow b\bar{b}$$

\hookrightarrow B jet

$$e^+e^- \rightarrow q\bar{q}g$$

\hookrightarrow J/ψ jet

$e^+e^- \rightarrow$ Jets in SCET

S.D. Ellis, et.al., JHEP1011(2010)101

$$d\sigma = H \times J_q \otimes J_{\bar{q}} \otimes J_g \otimes S$$

\longrightarrow $d\sigma = H \times J_q \otimes J_{\bar{q}} \otimes \mathcal{G}_g^{J/\psi} \otimes S$

unmeasured jets:

E, R

measured jets:

angularity: $\tau_a = \frac{1}{\omega} \sum_i (p_i^+)^{1-a/2} (p_i^-)^{a/2}$

$$\omega = \sum_i p_i^- \quad s = \omega^2 \tau_0$$

$e^+e^- \rightarrow$ Jets Formula (NLL')

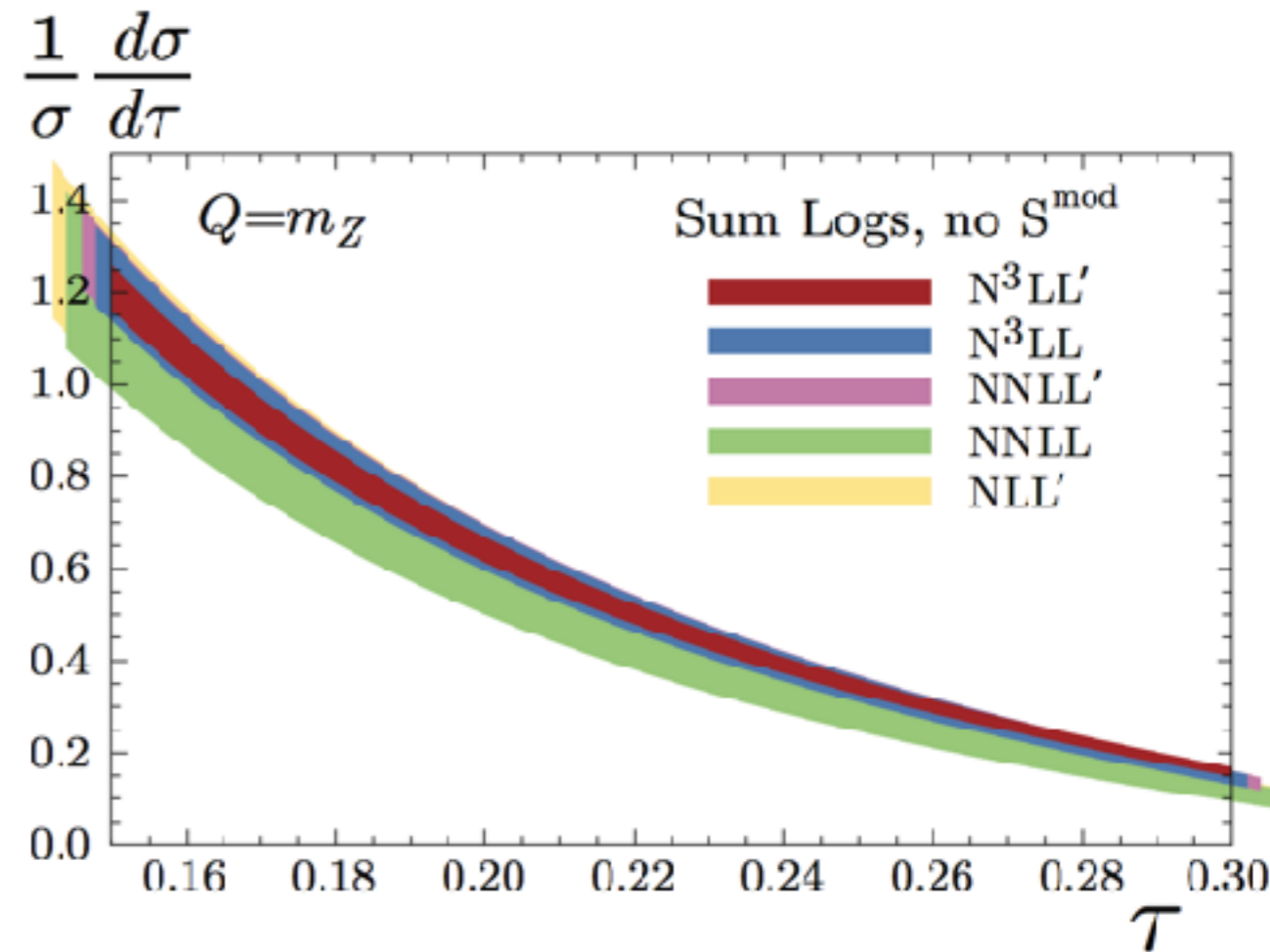
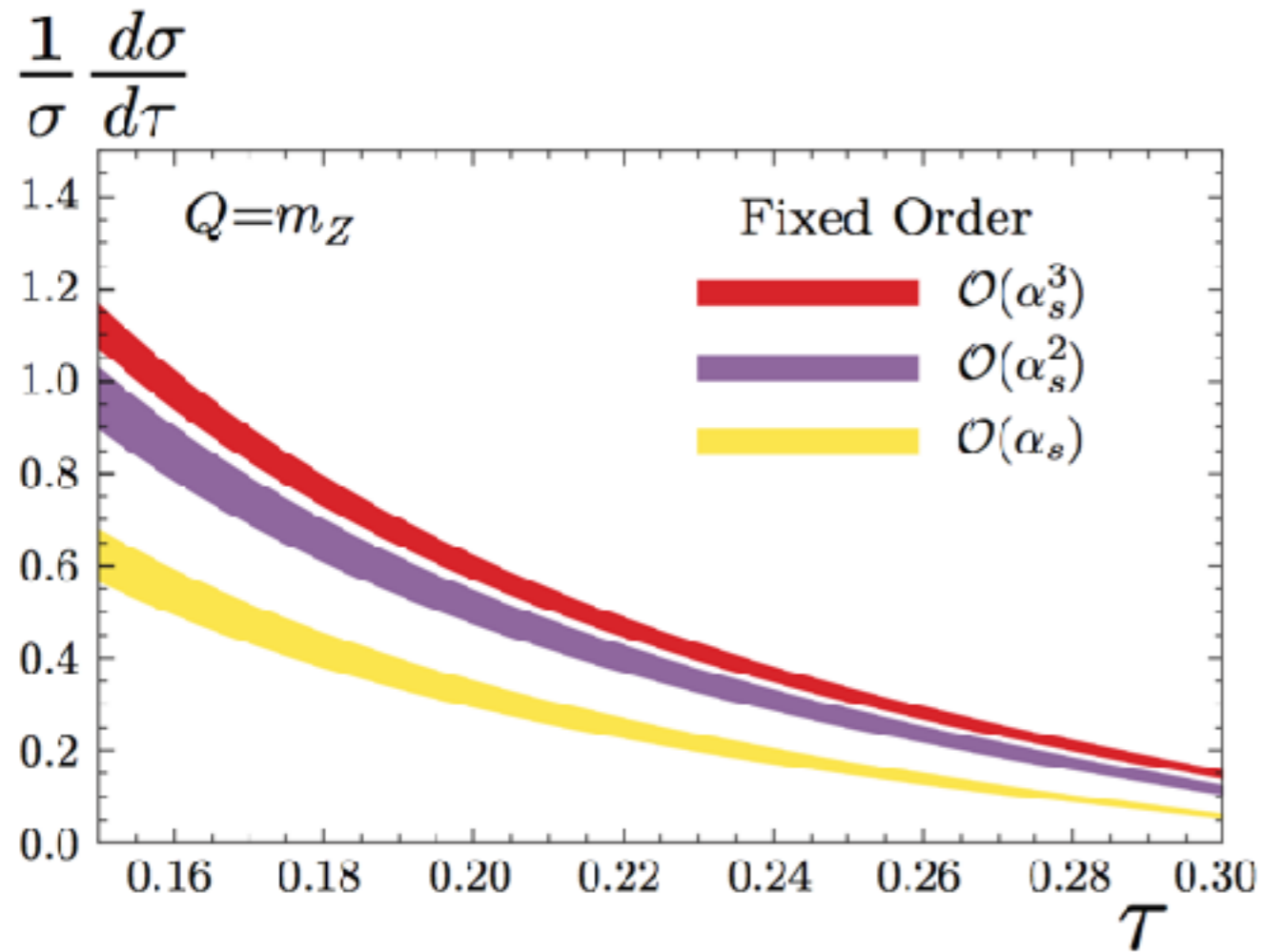
$$\begin{aligned}
 \frac{1}{\sigma^{(0)}} \frac{d\sigma^{(i)}}{dzd\tau_a} &= \sum_j \int_z^1 \frac{dx}{x} D_j(x; \mu_J) H_2(\mu_H) \left(\frac{\mu_H}{\omega}\right)^{\omega_H(\mu, \mu_H)} S^{\text{unmeas}}(\mu_\Lambda) J_\omega(\mu_R) \left(\frac{\mu_R}{\omega \tan \frac{R}{2}}\right)^{\omega_R(\mu, \mu_R)} \\
 &\times \left\{ \left[\delta_{ij} \delta(1 - z/x) (1 + f_S(\tau_a, \mu_S)) + f_J^{ij}(\tau_z, z/x; \mu_J) \right] \left(\frac{\mu_S \tan^{1-a} \frac{R}{2}}{\omega_1}\right)^{\omega_S(\mu, \mu_S)} \right. \\
 &\times \left. \left(\frac{\mu_J}{\omega}\right)^{(2-a)\omega_J(\mu, \mu_J)} \frac{1}{\Gamma[-\omega_J(\mu, \mu_J) - \omega_S(\mu, \mu_S)]} \frac{1}{\tau_a^{1+\omega_J(\mu, \mu_J)+\omega_S(\mu, \mu_S)}} \right\}_+ \\
 &\times \exp [\mathcal{K}(\mu; \mu_H, \mu_R, \mu_J, \mu_S, \mu_\Lambda) + \gamma_E \Omega(\mu; \mu_J, \mu_S)].
 \end{aligned}$$

$e^+e^- \rightarrow$ Jets Formula (NLL')

$$\begin{aligned}
 \frac{1}{\sigma^{(0)}} \frac{d\sigma^{(i)}}{dzd\tau_a} &= \sum_j \int_z^1 \frac{dx}{x} D_j(x; \mu_J) H_2(\mu_H) \left(\frac{\mu_H}{\omega}\right)^{\omega_H(\mu, \mu_H)} S^{\text{unmeas}}(\mu_\Lambda) J_\omega(\mu_R) \left(\frac{\mu_R}{\omega \tan \frac{R}{2}}\right)^{\omega_R(\mu, \mu_R)} \\
 &\times \left\{ \left[\delta_{ij} \delta(1 - z/x) (1 + f_S(\tau_a, \mu_S)) + f_J^{ij}(\tau_z, z/x; \mu_J) \right] \left(\frac{\mu_S \tan^{1-a} \frac{R}{2}}{\omega_1}\right)^{\omega_S(\mu, \mu_S)} \right. \\
 &\times \left. \left(\frac{\mu_J}{\omega}\right)^{(2-a)\omega_J(\mu, \mu_J)} \frac{1}{\Gamma[-\omega_J(\mu, \mu_J) - \omega_S(\mu, \mu_S)]} \frac{1}{\tau_a^{1+\omega_J(\mu, \mu_J)+\omega_S(\mu, \mu_S)}} \right\} + \\
 &\times \exp[\mathcal{K}(\mu; \mu_H, \mu_R, \mu_J, \mu_S, \mu_\Lambda) + \gamma_E \Omega(\mu; \mu_J, \mu_S)].
 \end{aligned}$$

RGE evolution

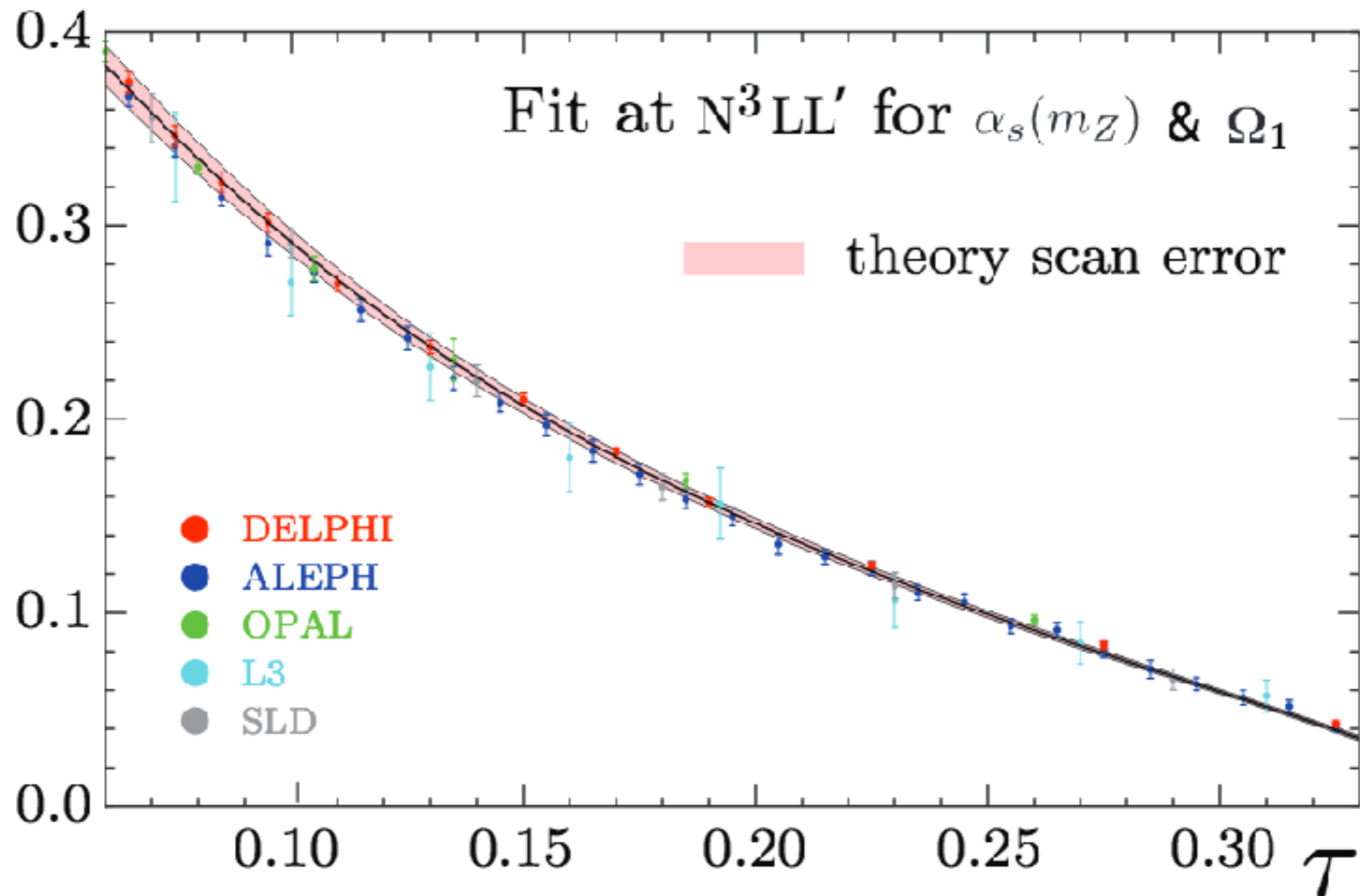
Importance of Resummation of Logarithms in Thrust



R. Abbate, et.al., Phys.Rev. D83 (2011) 074021

$$\frac{\tau}{\sigma} \frac{d\sigma}{d\tau}$$

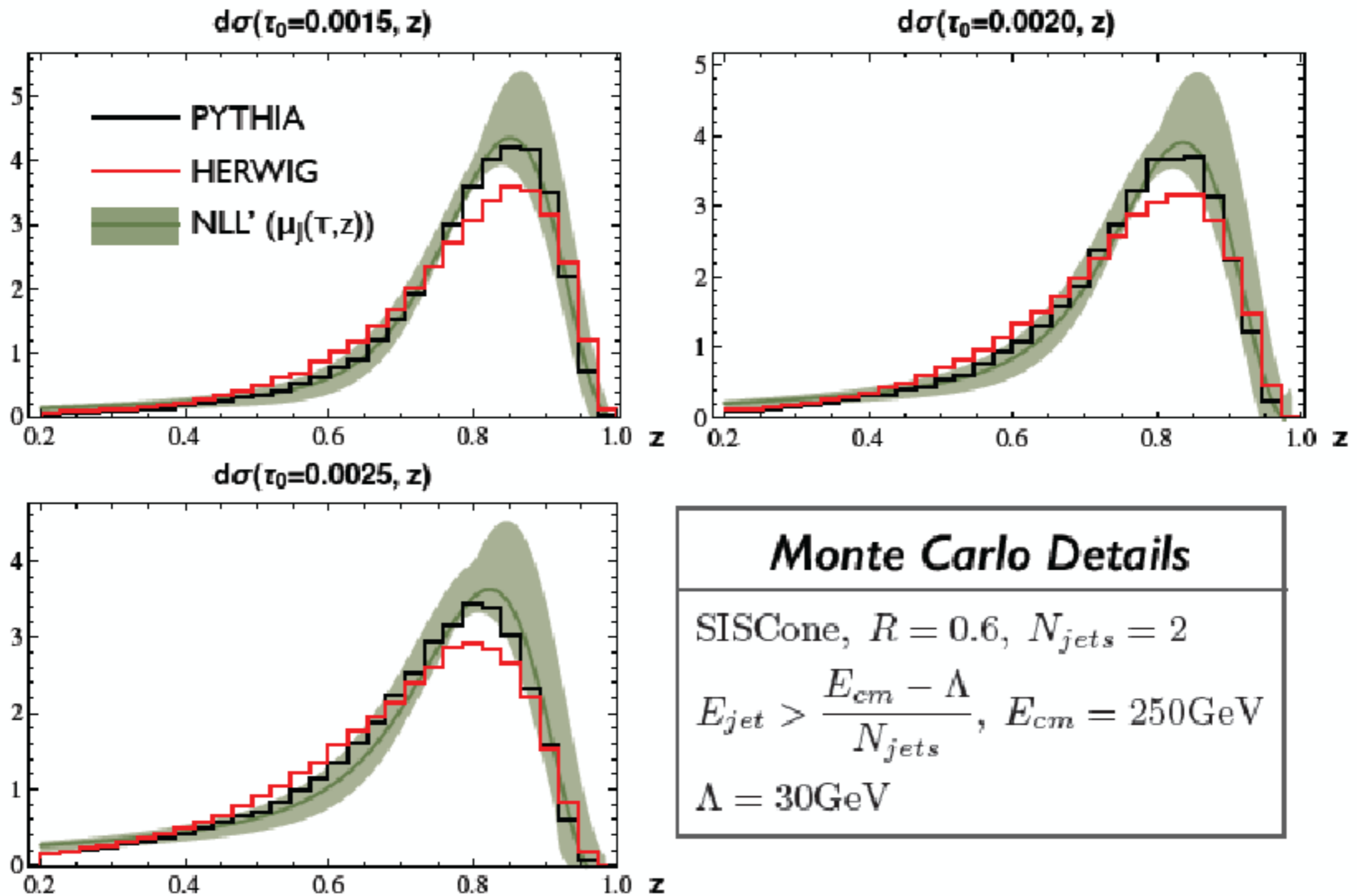
Comparison with Data



$$\alpha_s(M_Z) = 0.1135 \pm 0.0022$$

NLL vs. Monte Carlo (B mesons)

fixed τ_0 , variable z

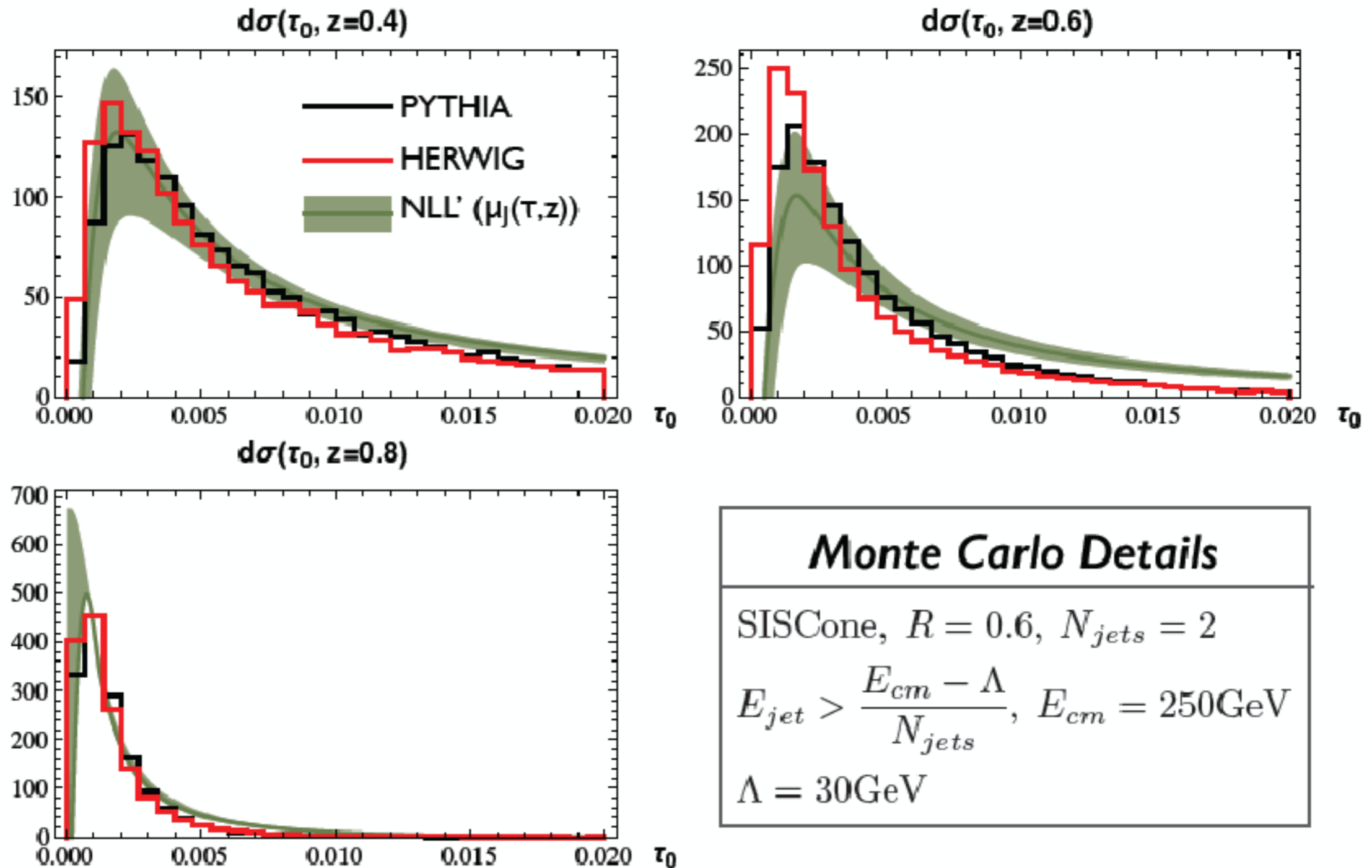


(HQ FF from LEP data)

Kniesl, et. al. Phys.Rev. D77 (2008) 014011

NLL vs. Monte Carlo (B mesons)

fixed z , variable τ_0

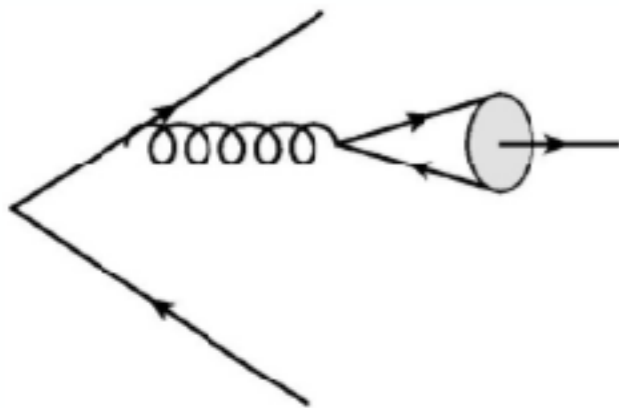


(HQ FF from LEP data)

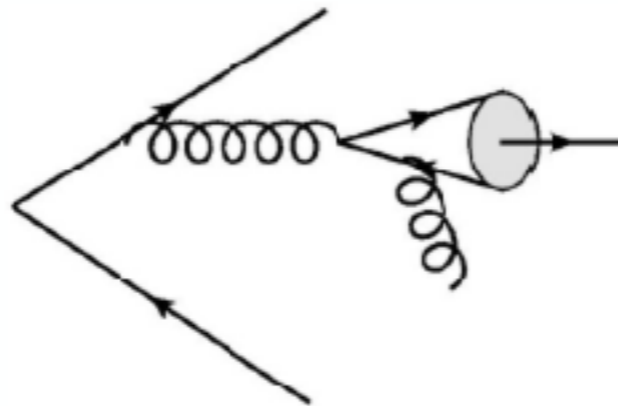
Kniesl, et. al. Phys.Rev. D77 (2008) 014011

Madgraph + PYTHIA

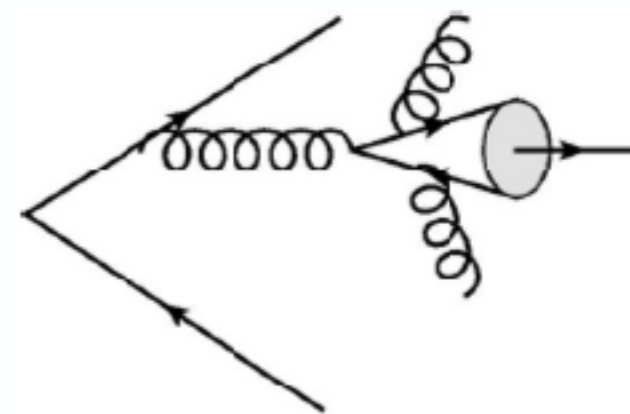
$$e^+e^- \rightarrow b\bar{b}c\bar{c} \left[{}^3S_1^{(8)} \right]$$



$$e^+e^- \rightarrow b\bar{b}g c\bar{c} \left[{}^1S_0^{(8)} \right]$$



$$e^+e^- \rightarrow b\bar{b}gg c\bar{c} \left[{}^3S_1^{(1)} \right]$$

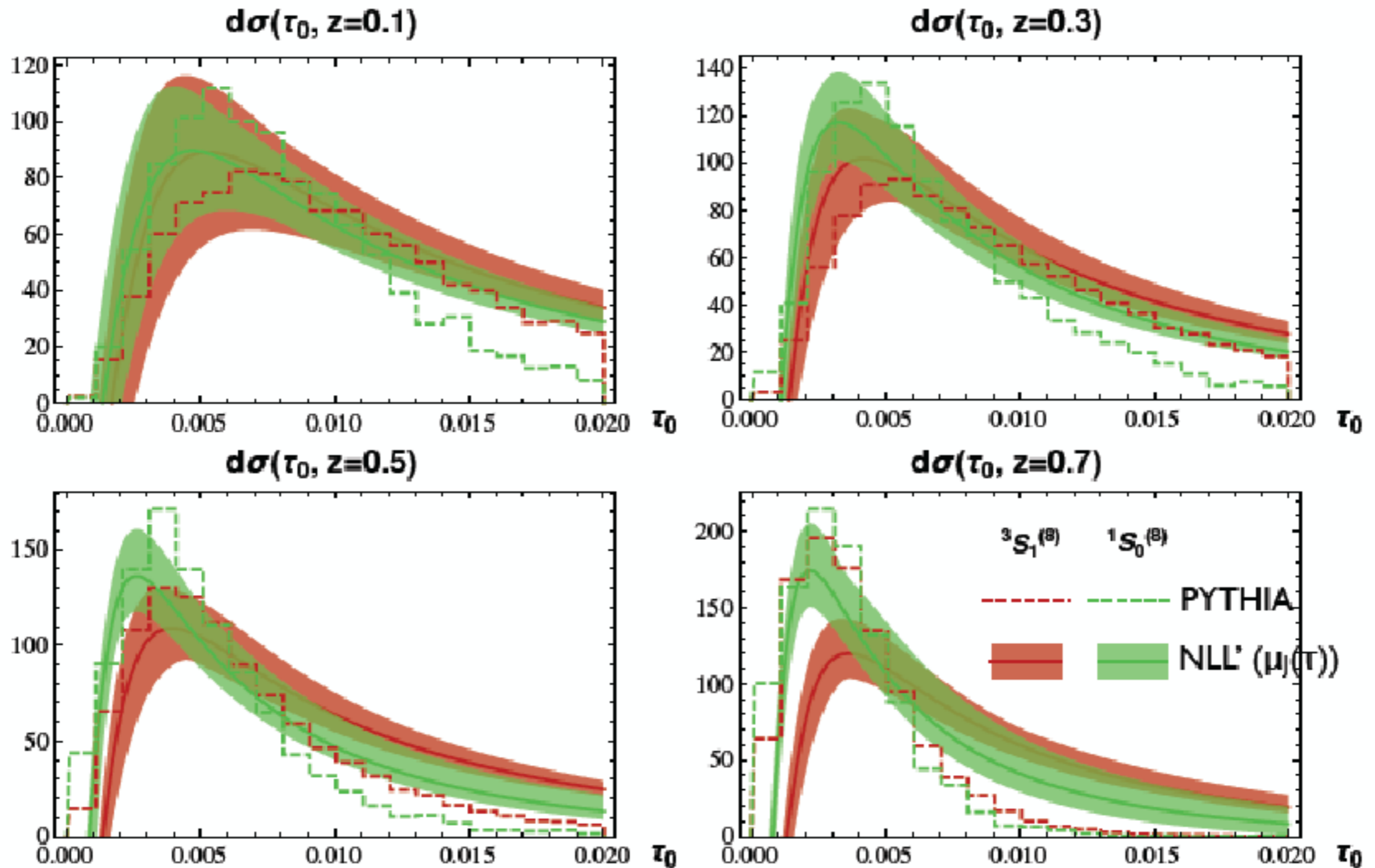


Force Madgraph to create J/ψ from gluon initiated jet

PYTHIA: parton shower, hadronization

NLL' vs. Monte Carlo

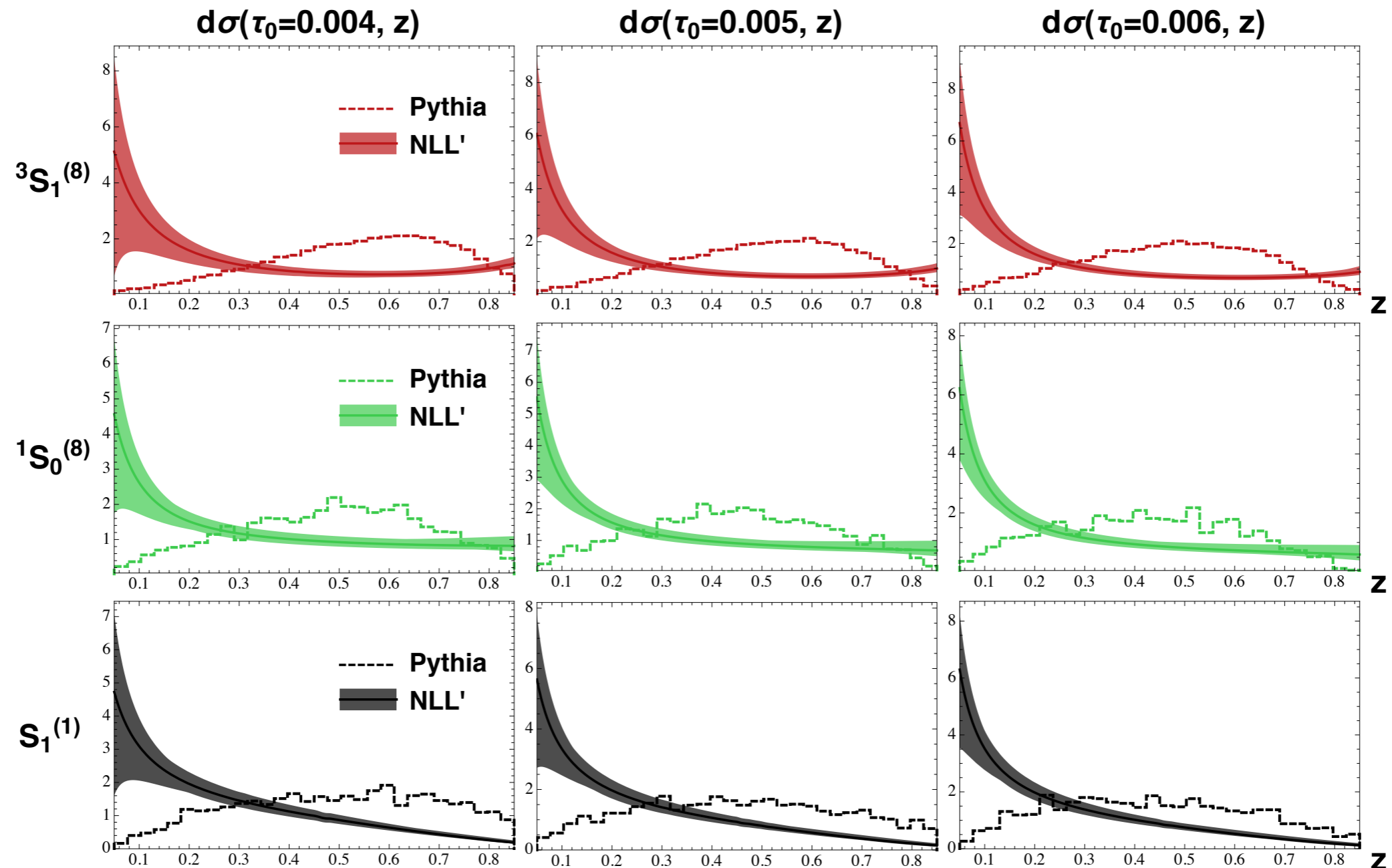
fixed z , variable τ_0



good agreement, some discrimination for large z

NLL' FJF vs. Pythia

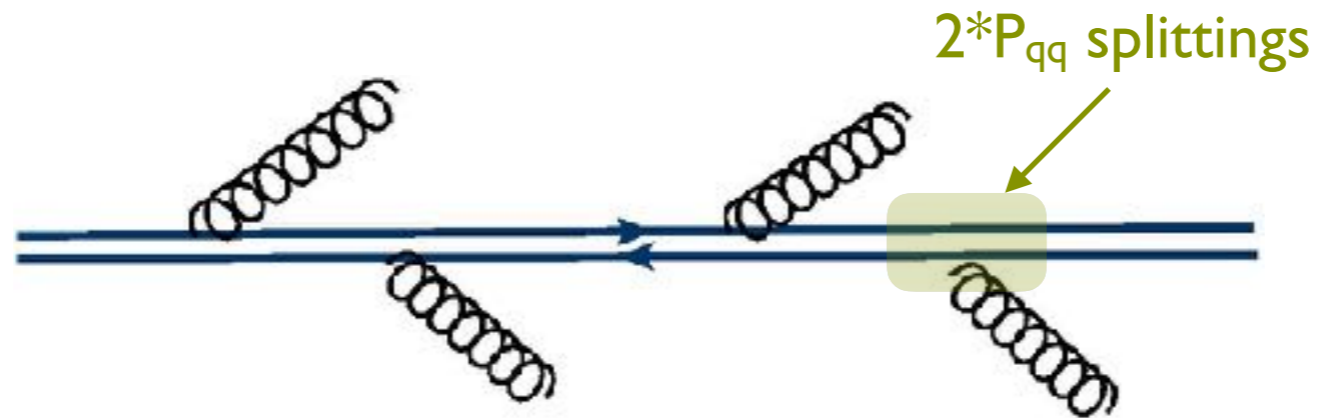
R. Bain, L. Dai, A. Hornig, A. K. Leibovich, Y. Makris, T.M., JHEP 1606 (2016) 121



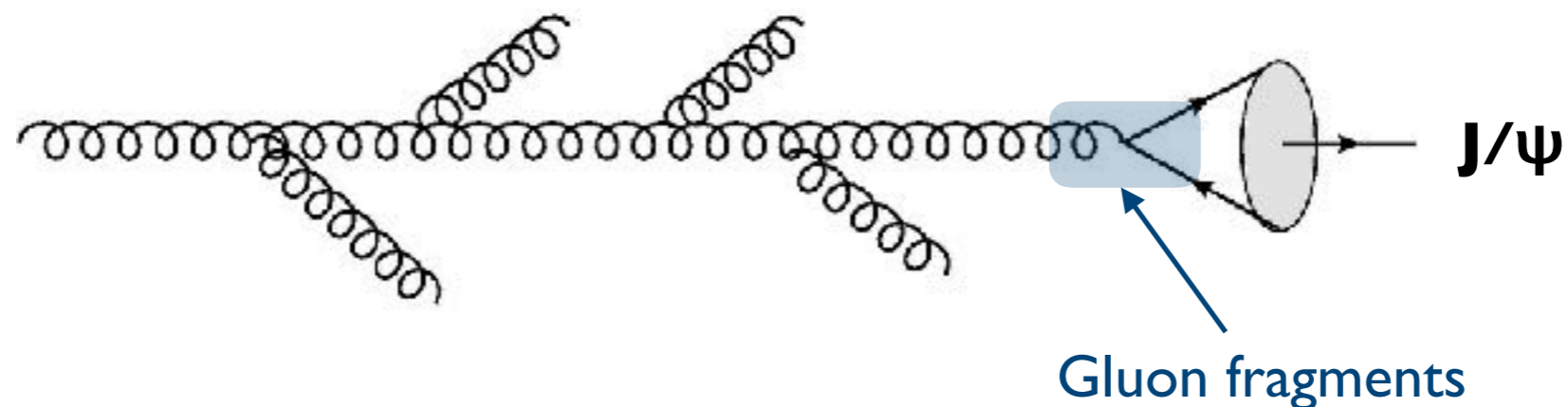
$e^+e^- \rightarrow \bar{q}qq$ $E_{CM} = 250 \text{ GeV}$ $\tau_0 = s/\omega^2$
 \hookrightarrow jet w/ J/ψ

Explaining difference between NLL' vs Pythia

PYTHIA's model for showering color-octet $c\bar{c}$ pairs:



Physical picture of analytical calculation

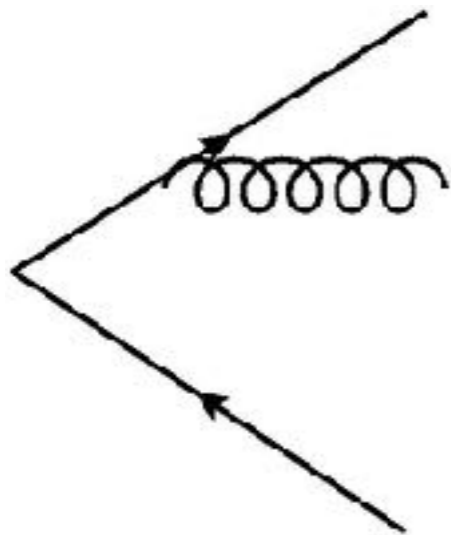


Pythia z distributions much harder than NLL' calculations

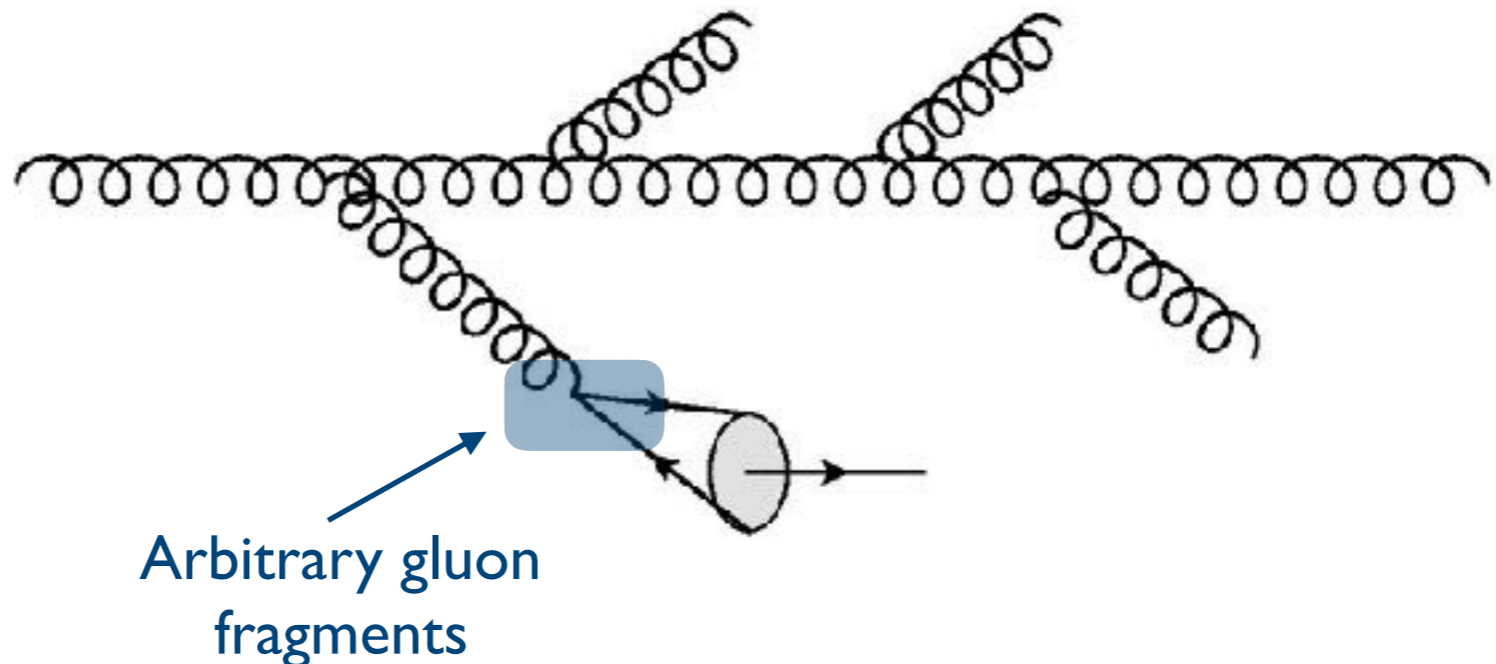
Gluon Fragmentation Improved PYTHIA (GFIP)

Madgraph 5

$$e^+ e^- \rightarrow b \bar{b} g$$

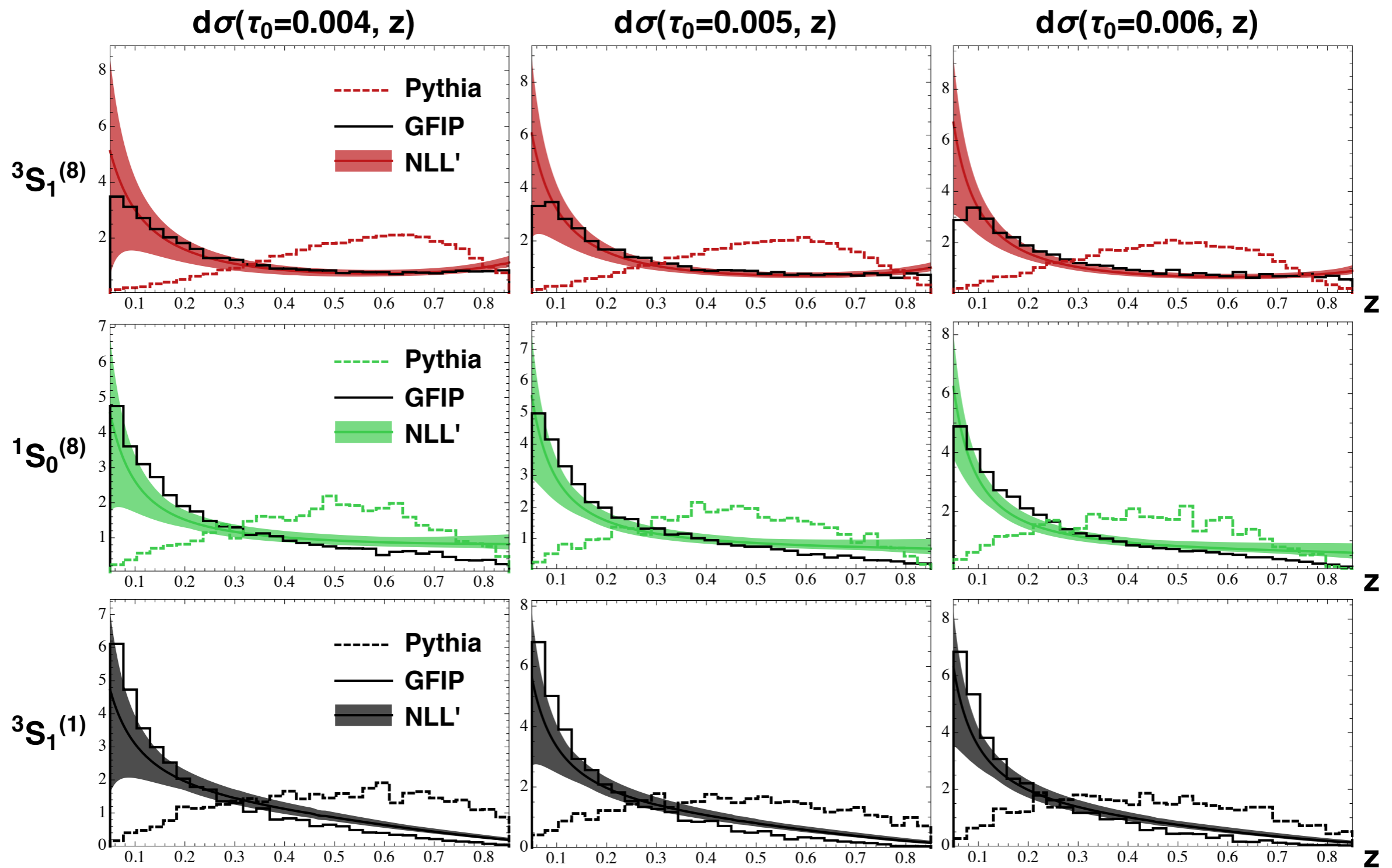


PYTHIA + Convolution



shower gluon with PYTHIA down to scale $\sim 2m_c$, no hadronization
convolve final state gluon distribution w/ NRQCD FFs

NLL', PYTHIA, and GFIP

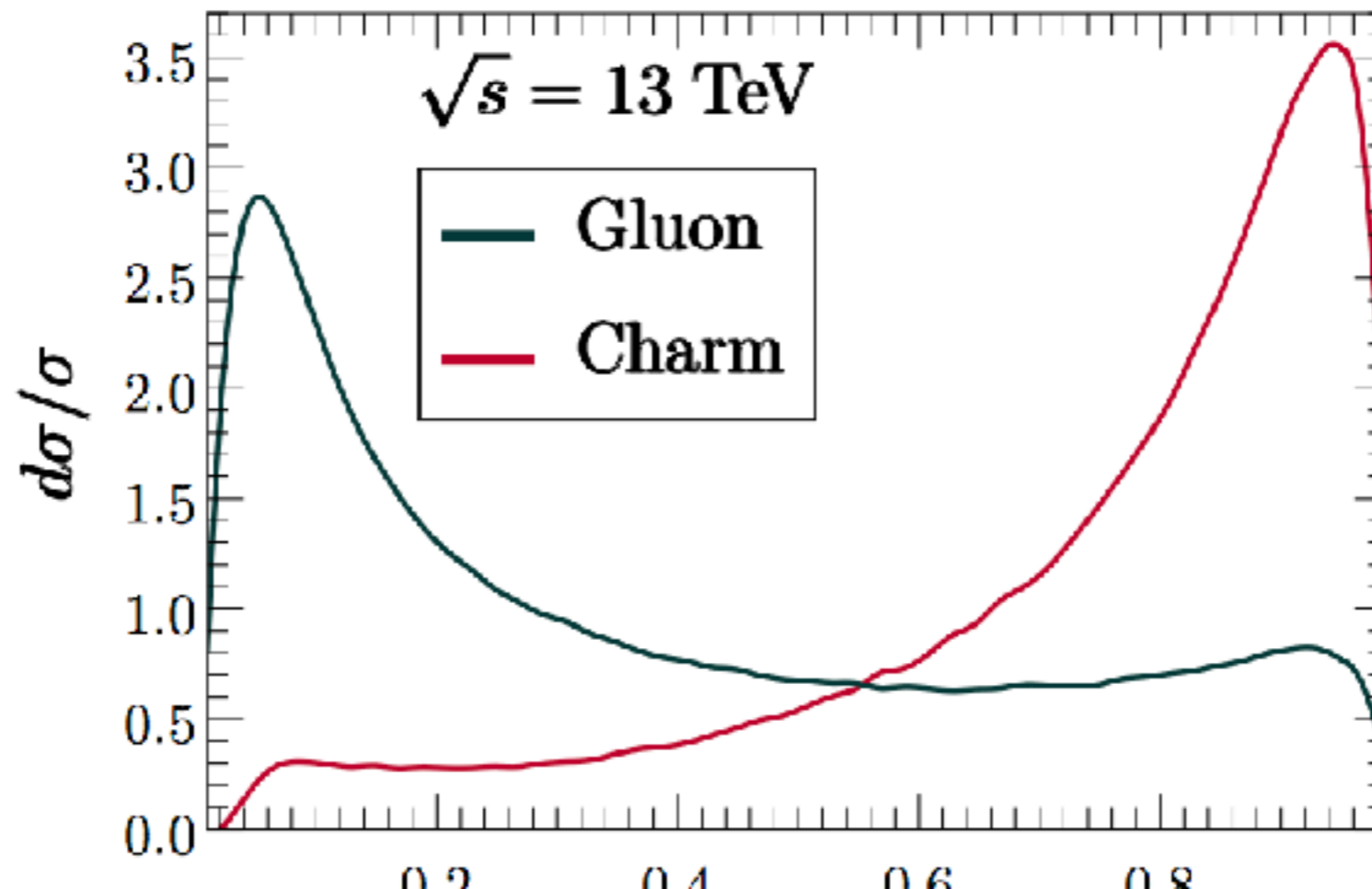


GFIP and Recent LHCb Observations

R. Bain, L. Dai, A. K. Leibovich, Y. Makris, T. M., PRL 119 (2017) 032002

generate events with hard c-quark , gluons

LHCb: pp collisions $\sqrt{s} = 13 \text{ TeV}$ cuts: $2 < \eta < 4.5$
 $R = 0.5$
 $p_{T, \text{JET}} < 20 \text{ GeV}$
evolve shower to scale $\sim 2m_c$ $p_\mu < 5 \text{ GeV}$



convolve w/ NRQCD FF for c quarks, gluons $\sim 2m_c$

LHCb data is normalized so $\sum_i \Delta z \left(\frac{d\sigma}{\sigma} \right)_i = \Delta z$

compare $0.1 < z < 0.9$

Use following three sets of LDMEs

| | $\langle \mathcal{O}^{J/\psi}({}^3S_1^{[1]}) \rangle$ $\times \text{GeV}^3$ | $\langle \mathcal{O}^{J/\psi}({}^3S_1^{[8]}) \rangle$ $\times 10^{-2} \text{GeV}^3$ | $\langle \mathcal{O}^{J/\psi}({}^1S_0^{[8]}) \rangle$ $\times 10^{-2} \text{GeV}^3$ | $\langle \mathcal{O}^{J/\psi}({}^3P_0^{[8]}) \rangle / m_c^2$ $\times 10^{-2} \text{GeV}^3$ |
|--------------------|--------------------------------------------------------------------------------|----------------------------------------------------------------------------------------|----------------------------------------------------------------------------------------|------------------------------------------------------------------------------------------------|
| B & K [5, 6] | 1.32 ± 0.20 | 0.224 ± 0.59 | 4.97 ± 0.44 | -0.72 ± 0.88 |
| Chao, et al. [12] | 1.16 ± 0.20 | 0.30 ± 0.12 | 8.9 ± 0.98 | 0.56 ± 0.21 |
| Bodwin et al. [13] | 1.32 ± 0.20 | 1.1 ± 1.0 | 9.9 ± 2.2 | 0.49 ± 0.44 |

Butenschoen and Kniehl, PRD 84 (2011) 051501

global fits to world's data

Chao, et. al. PRL 108, 242004 (2012)

fits to high p_T hadron collider data

Bodwin, et. al., PRL 113, 022001 (2014)

FJF and Recent LHCb Observations

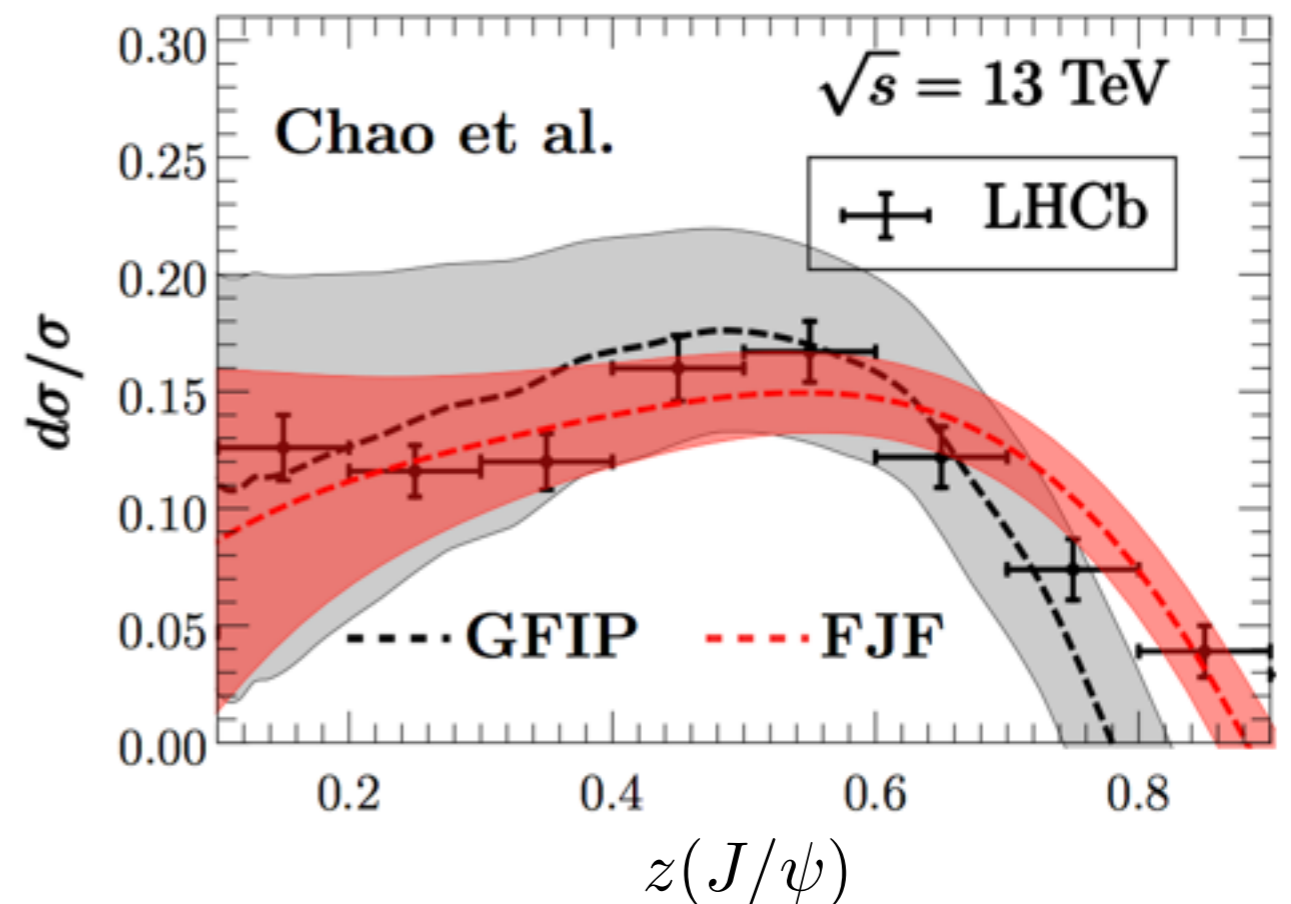
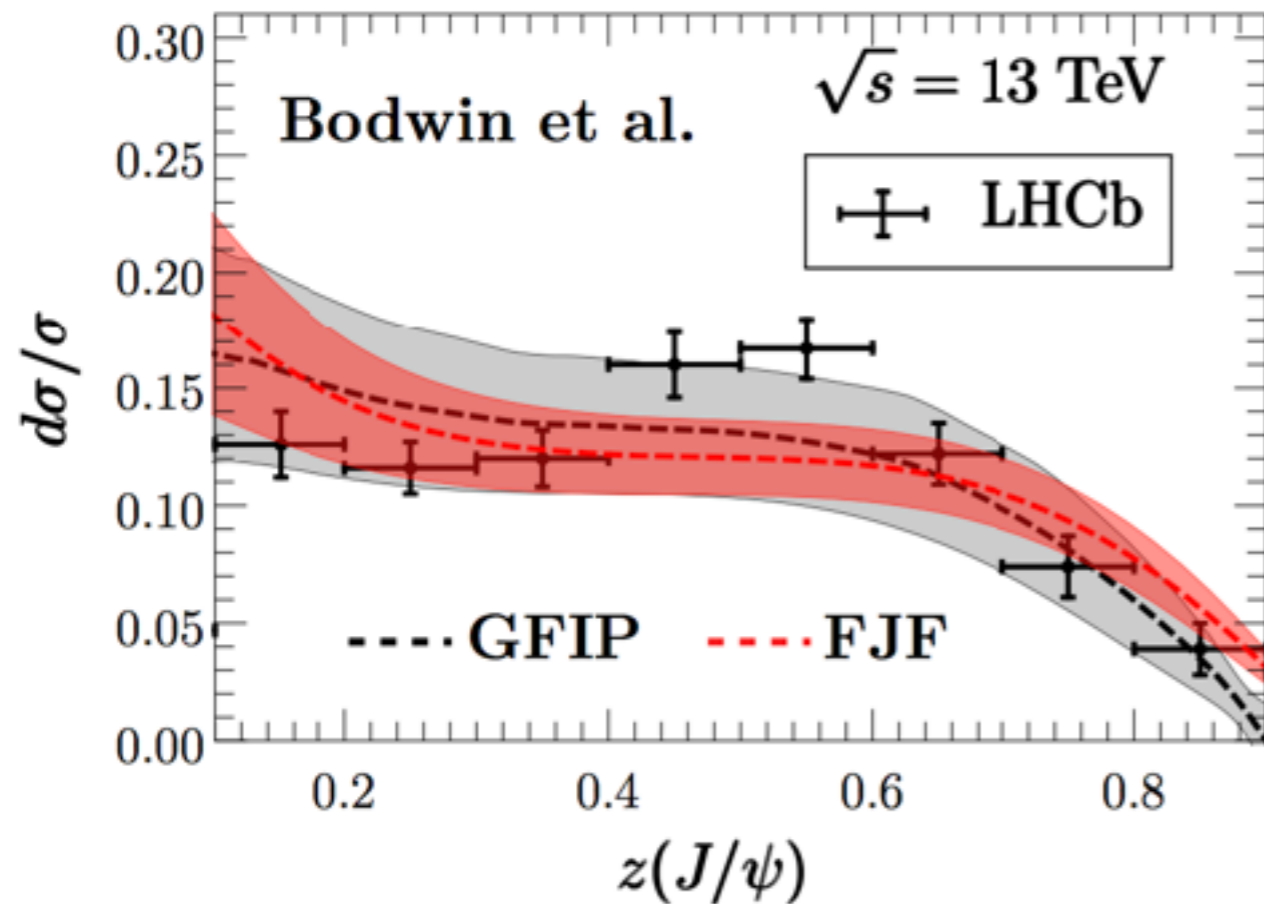
combine FJFs with hard events generated by Madgraph

NRQCD FFs evolved from $2m_c$ to jet energy scale using DGLAP

factorization theorem with tree level hard function,
trivial soft function, no NLL' resummation

FJF is only term in factorization dependent on $z(J/\psi)$

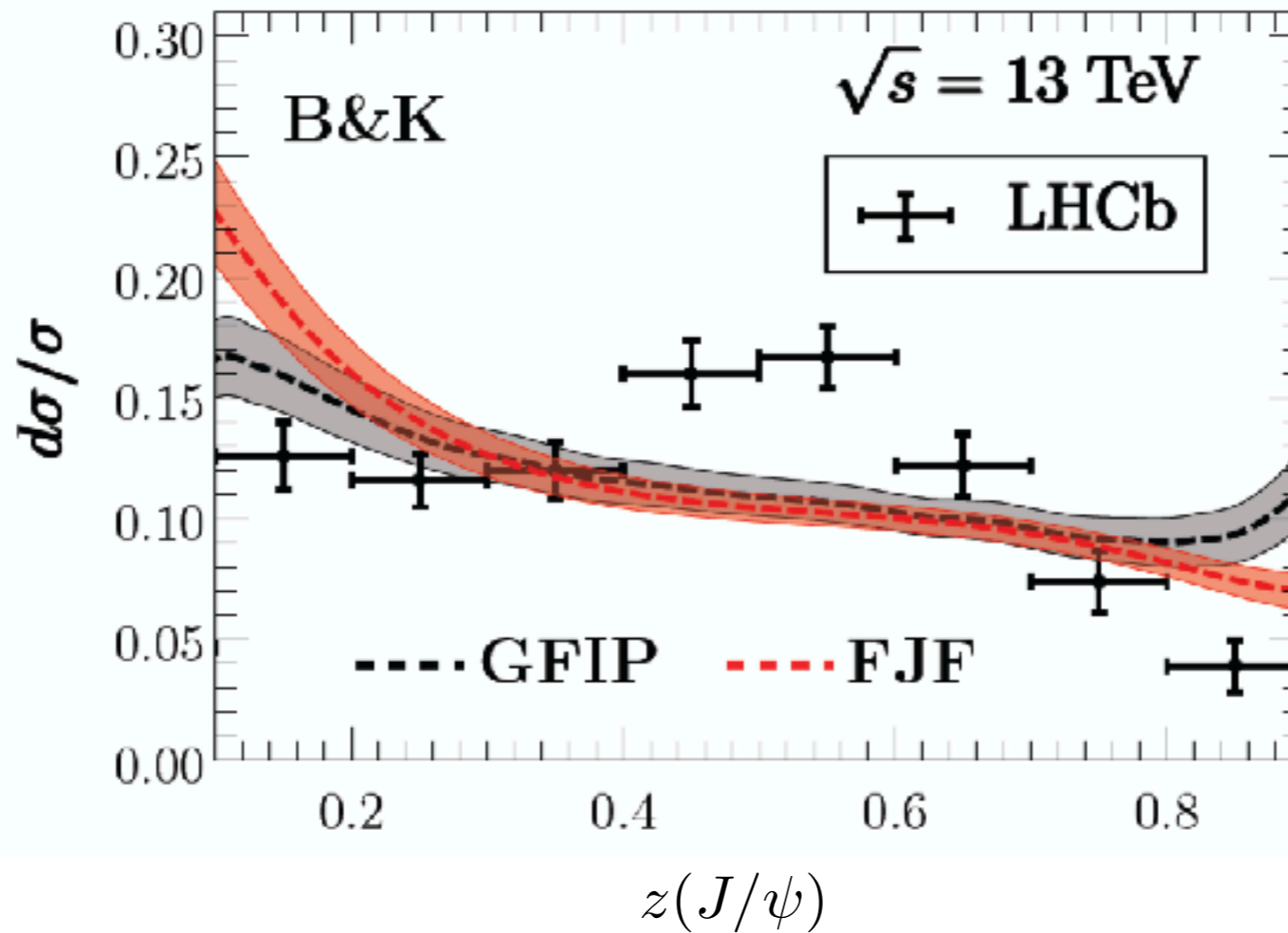
Results



FJFs, GFIP consistent

LDME from fits high p_T agree with LHCb

Results



LDME from global fits:
poorer agreement with LHCb, better than PYTHIA

Future Measurements

polarization of J/ψ in jets

Z.-B. Kang, J.-W. Qiu, F. Ringer, H. Xing, H. Zhang, PRL 119 (2017) 032001

absolute cross sections

alternative jet definitions, e.g., soft drop

p_T dependent FJFs

Jet Shapes in Dijet Events at the LHC

A. Hornig, Y. Makris, T.M, JHEP 1604 (2016) 097

A. Hornig, D. Kang, Y. Makris, T.M, JHEP 1712 (2017) 043

boost invariant angularity

$$\begin{aligned}\tau_a^{e^+e^-} &= \frac{1}{2E_J} \sum_i |p_T^{iJ}| e^{-(1-a)|y_{iJ}|} \\ &= (2E_J)^{-(2-a)} (p_T)^{1-a} \sum_i |p_T^i| \left(\frac{\theta_{iJ}}{\sin \theta_J} \right)^{2-a} (1 + \mathcal{O}(\theta_{iJ}^2)) \\ \tau_a^{pp} &= \left(\frac{2E_J}{p_T} \right)^{2-a} \tau_a^{e^+e^-} + \mathcal{O}(\tau_a^2)\end{aligned}$$

modified jet function

$$J_i(\tau_a) = \left(\frac{p_T}{2E_J} \right)^{2-a} J_i^{e^+e^-} \left(\left(\frac{p_T}{2E_J} \right)^{2-a} \tau_a \right)$$

Analogous Formulae for pp collisions

Soft Function in e^+e^-

rotationally invariant cuts: $E < E_{\min}$

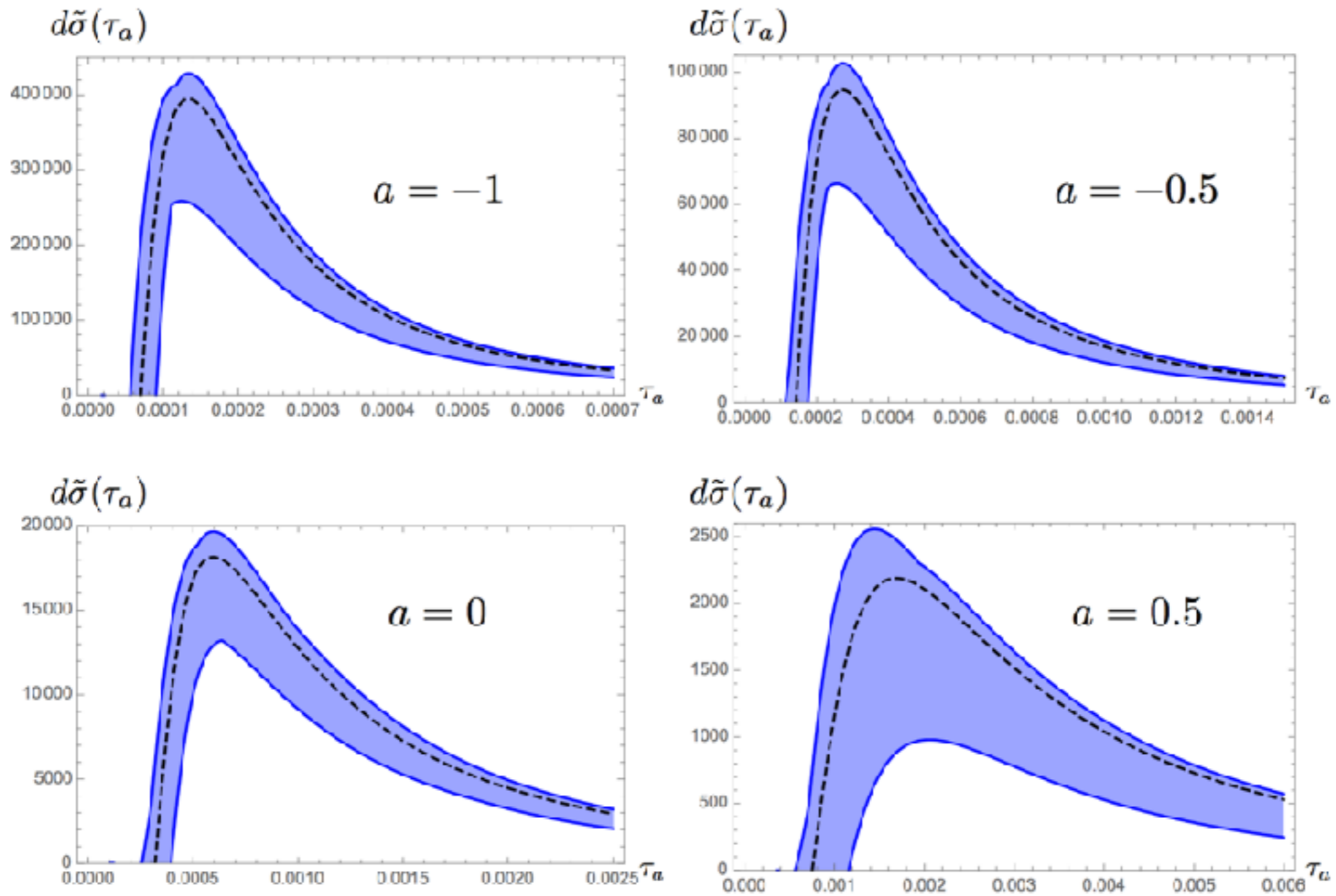
Soft Function in pp

boost invariant cuts, observables: p_T , rapidity

hard, soft functions are matrices in color space

$$d\sigma(\tau_a^1, \tau_a^2) = \frac{p_T x_1 x_2}{8\pi E_{\text{cm}}^4} \frac{1}{N} B(x_1; \mu) \bar{B}(x_2; \mu) \text{Tr}\{\mathbf{H}(\mu) \mathbf{S}(\tau_a^1, \tau_a^2; \mu)\} \otimes [J_1(\tau_a^1; \mu) J_2(\tau_a^2; \mu)]$$

$pp \rightarrow 2$ jets with boost invariant soft function



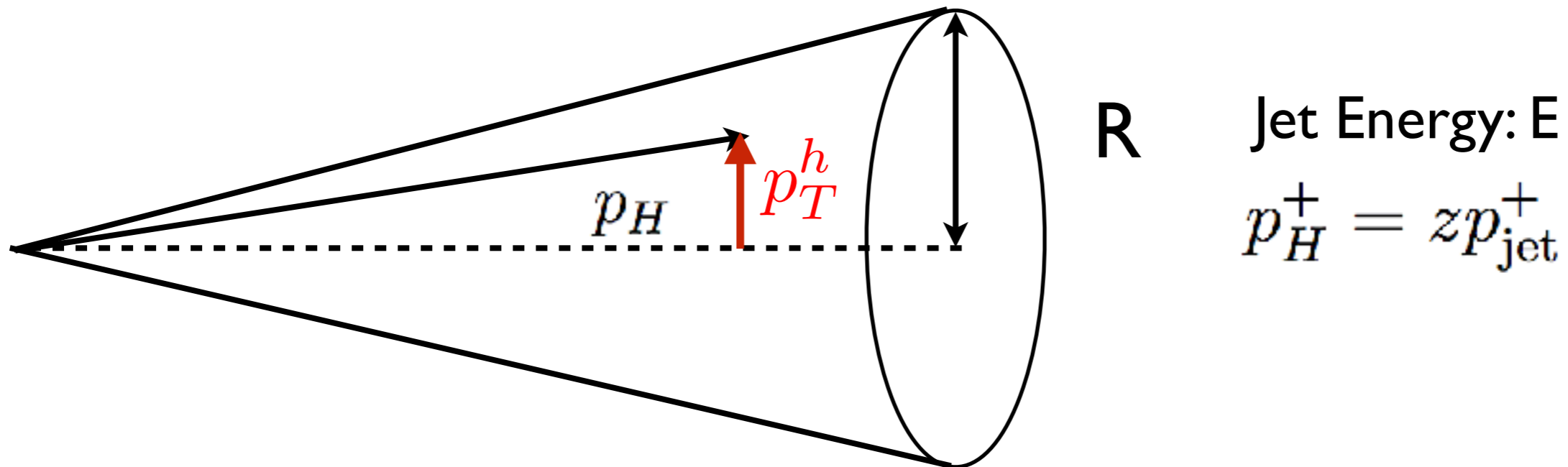
$q\bar{q}$ channel only

Study dependence on: a , R , p_T cut, scale variation

Transverse Momentum Dependent FJFs

R. Bain, Y. Makris, TM, JHEP 1611 (2016) 144

jets with identified hadron: hadron z , p_T are both measured



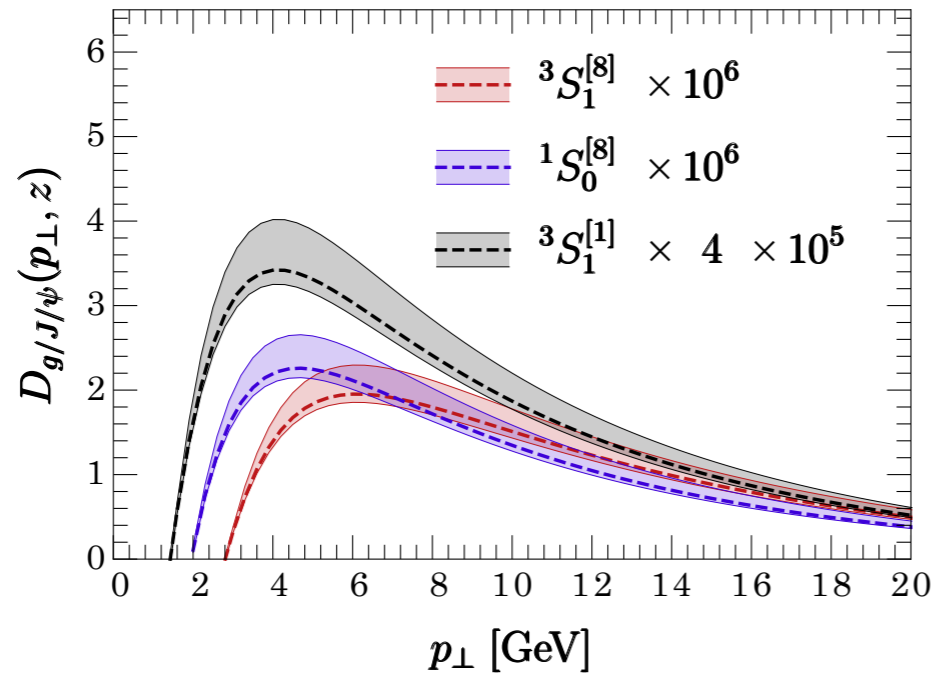
transverse momentum measured w/ rspt. to jet axis

jet axis \sim parton initiating jet if out of jet radiation is ultrasoft

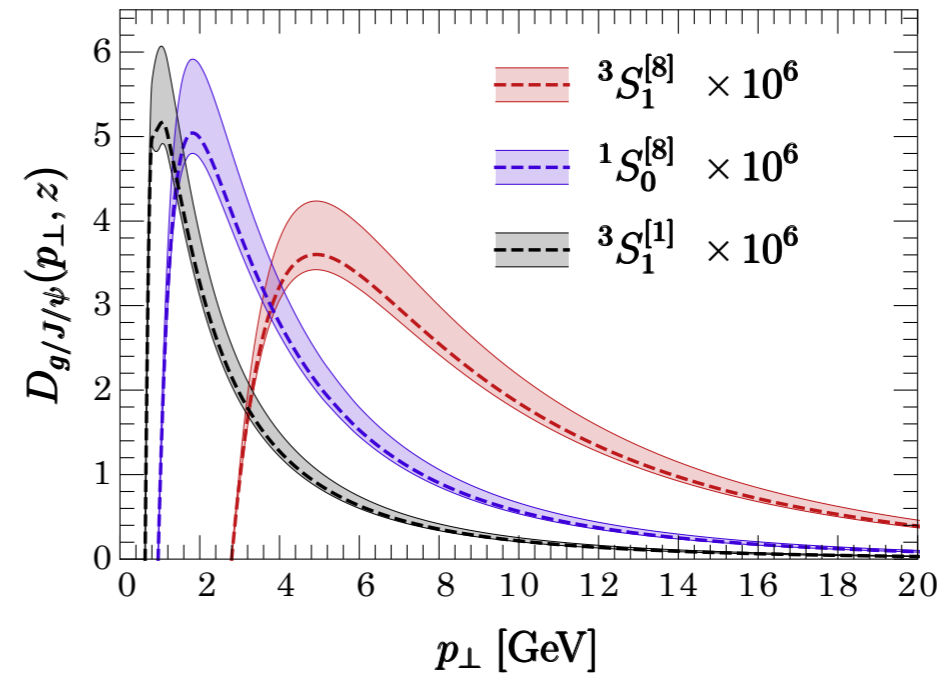
$$\omega \gg p_T^h \gg \Lambda \gg \Lambda_{\text{QCD}}$$

Application to Quarkonium Production

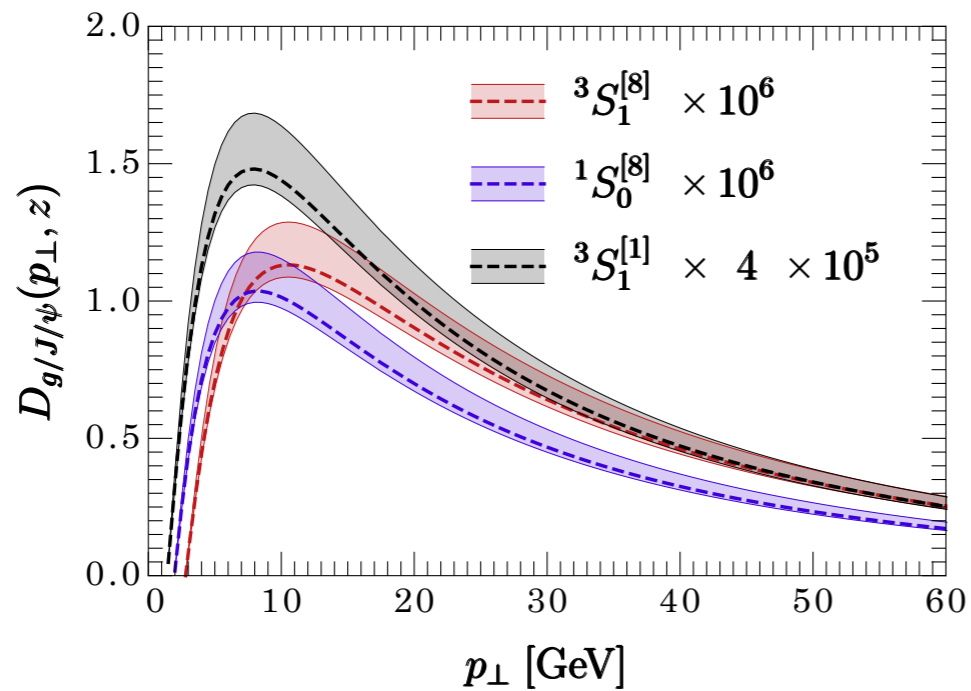
$E_J = 100$ [GeV], $z = 0.3$



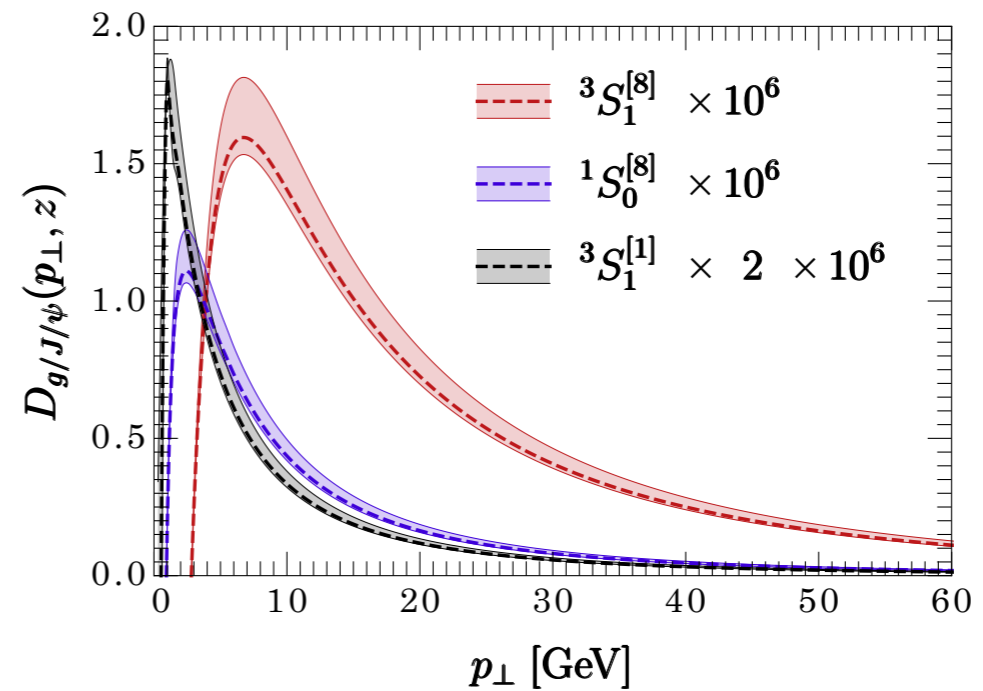
$E_J = 100$ [GeV], $z = 0.9$



$E_J = 500$ GeV, $z = 0.3$

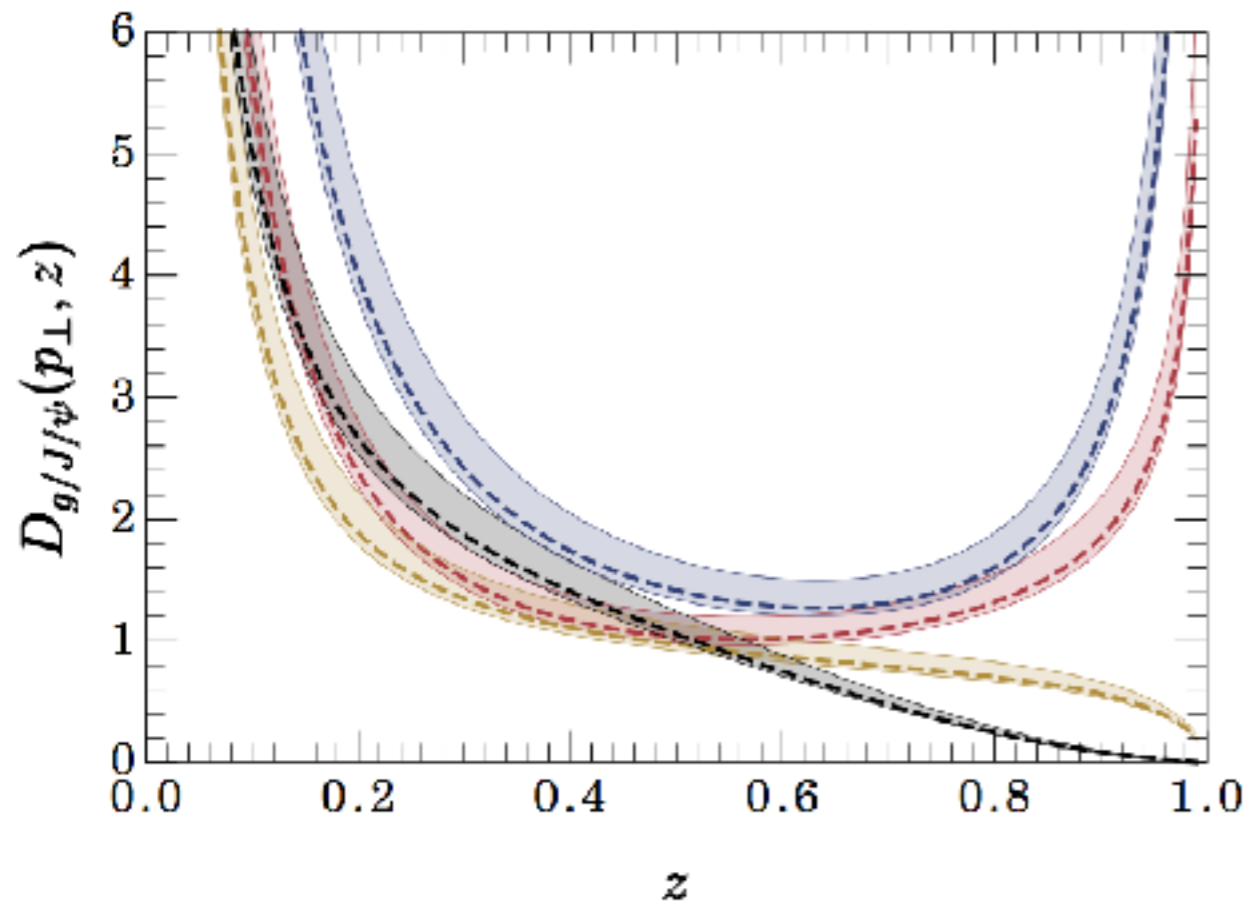


$E_J = 500$ GeV, $z = 0.9$

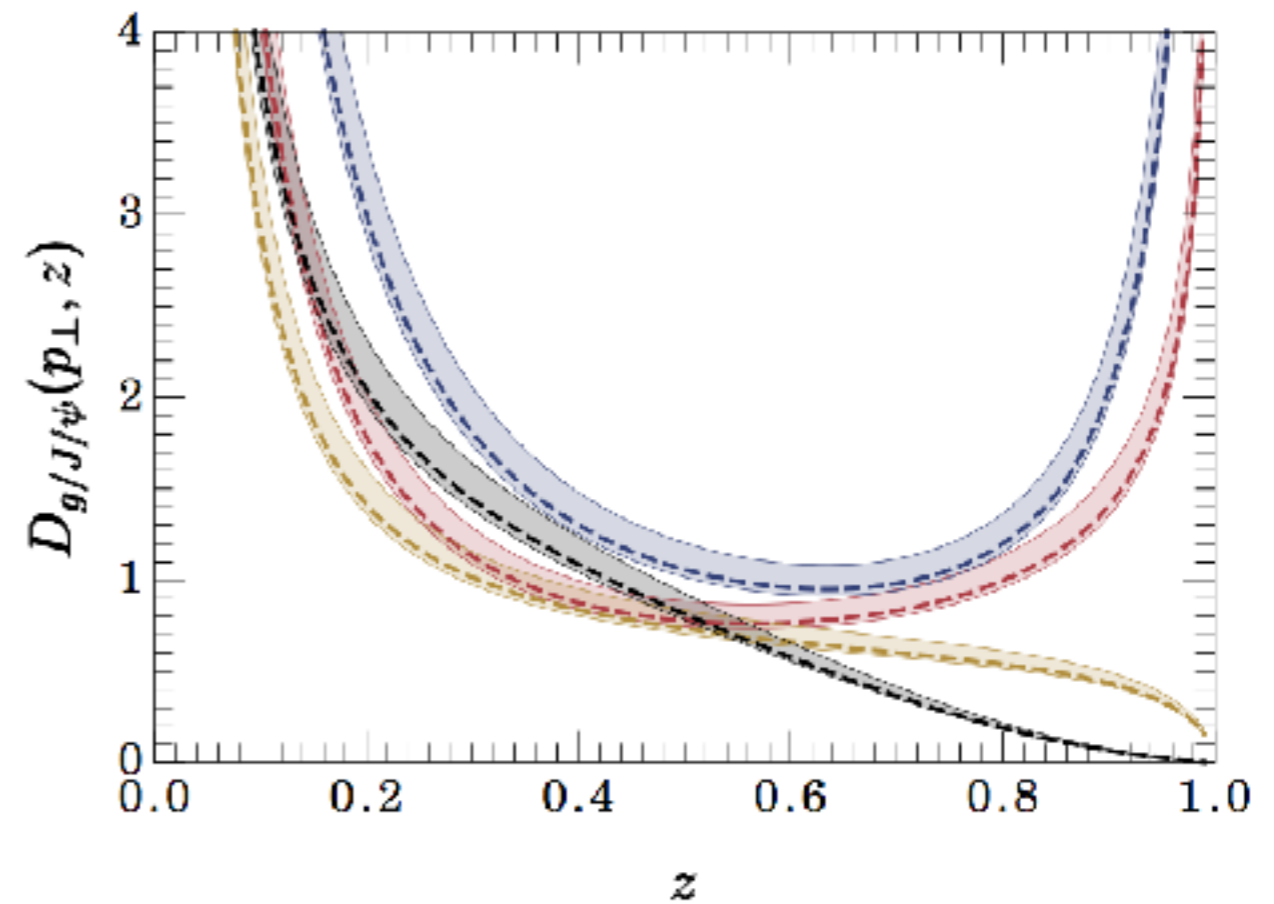


Application to Quarkonium Production

$E_J = 100 \text{ GeV}, p_{\perp} = 10 \text{ GeV}$



$E_J = 500 \text{ GeV}, p_{\perp} = 10 \text{ GeV}$



$\text{---} \quad {}^3S_1^{[8]} \times 10^6$

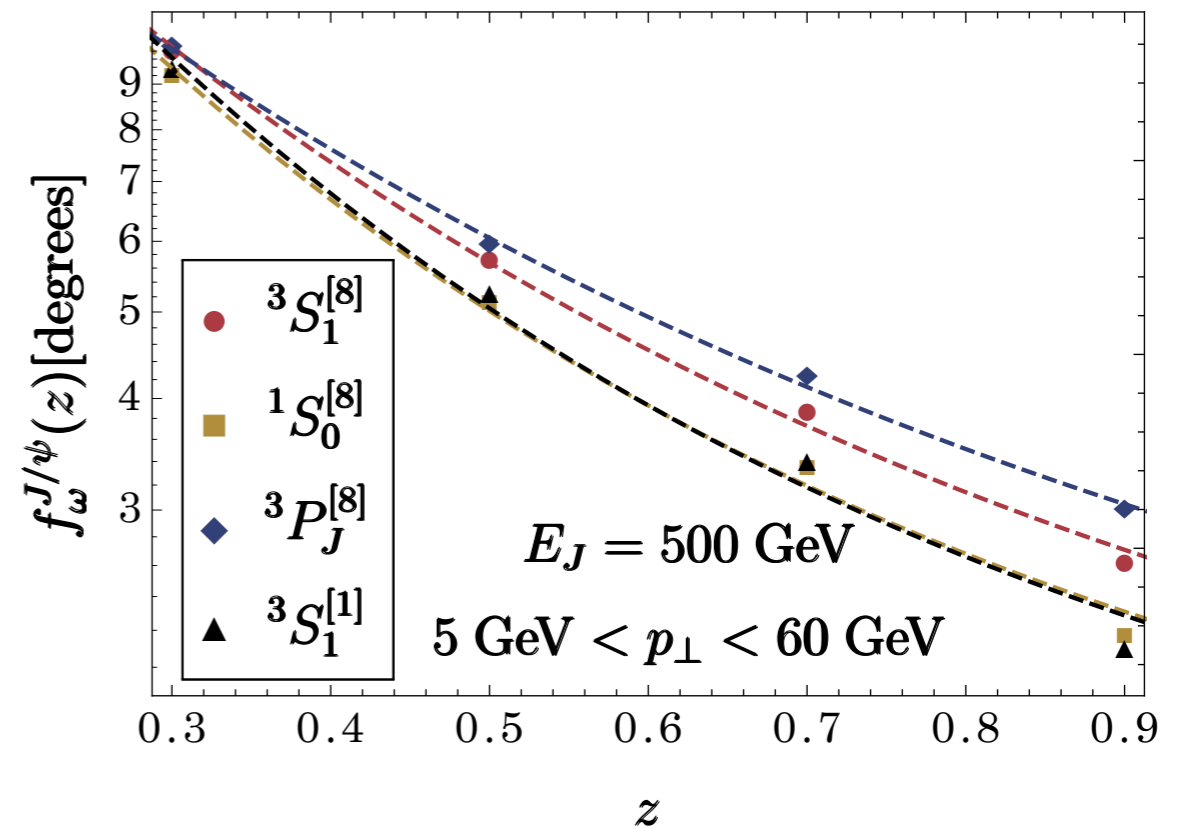
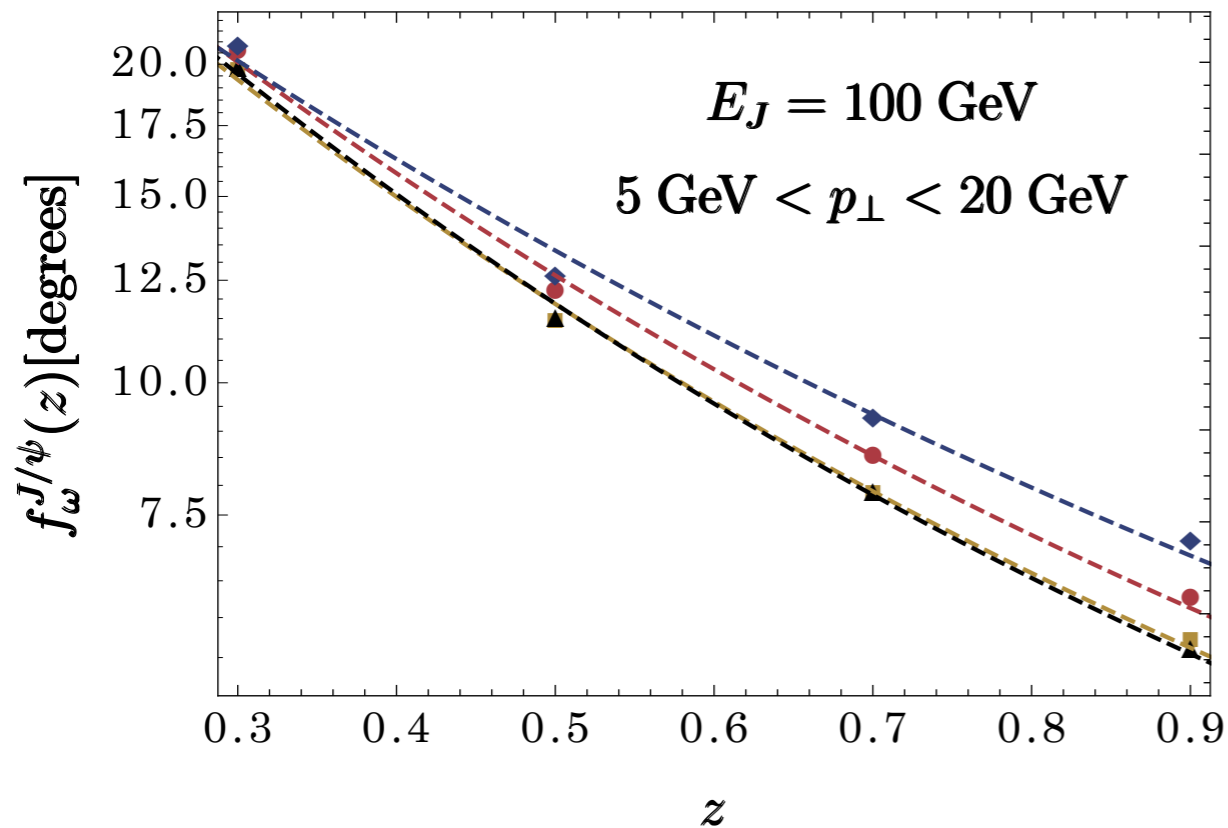
$\text{---} \quad {}^1S_0^{[8]} \times 10^6$

$\text{---} \quad {}^3P_J^{[8]} \times 3 \cdot 10^5$

$\text{---} \quad {}^3S_1^{[1]} \times 4 \cdot 10^5$

Application to Quarkonium Production

$$\langle \theta \rangle(z) \sim \frac{2 \int dp_{\perp} p_{\perp} D_{g/h}(p_{\perp}, z, \mu)}{z\omega \int dp_{\perp} D_{g/h}(p_{\perp}, z, \mu)} \equiv f_{\omega}^h(z)$$



| $E_J = 100 \text{ GeV}$ | | |
|-------------------------|-------|-------|
| $2S+1 L_J^{[1,8]}$ | C_0 | C_1 |
| $3S_1^{[1]}$ | 3.92 | 0.92 |
| $3S_1^{[8]}$ | 3.86 | 0.84 |
| $1S_0^{[8]}$ | 3.88 | 0.90 |
| $3P_J^{[8]}$ | 3.75 | 0.74 |

| $E_J = 500 \text{ GeV}$ | | |
|-------------------------|-------|-------|
| $2S+1 L_J^{[1,8]}$ | C_0 | C_1 |
| $3S_1^{[1]}$ | 3.75 | 1.68 |
| $3S_1^{[8]}$ | 3.48 | 1.39 |
| $1S_0^{[8]}$ | 3.66 | 1.64 |
| $3P_J^{[8]}$ | 3.28 | 1.20 |

$$\ln(f(x)) = g(x; C_0, C_1) \text{ s.t. } g(x=0) = C_0$$

$$g_2(x) = C_0 \exp(-C_1 x)$$

Conclusions

measuring quarkonia within jets and using jet observables should provide insights into quarkonia production

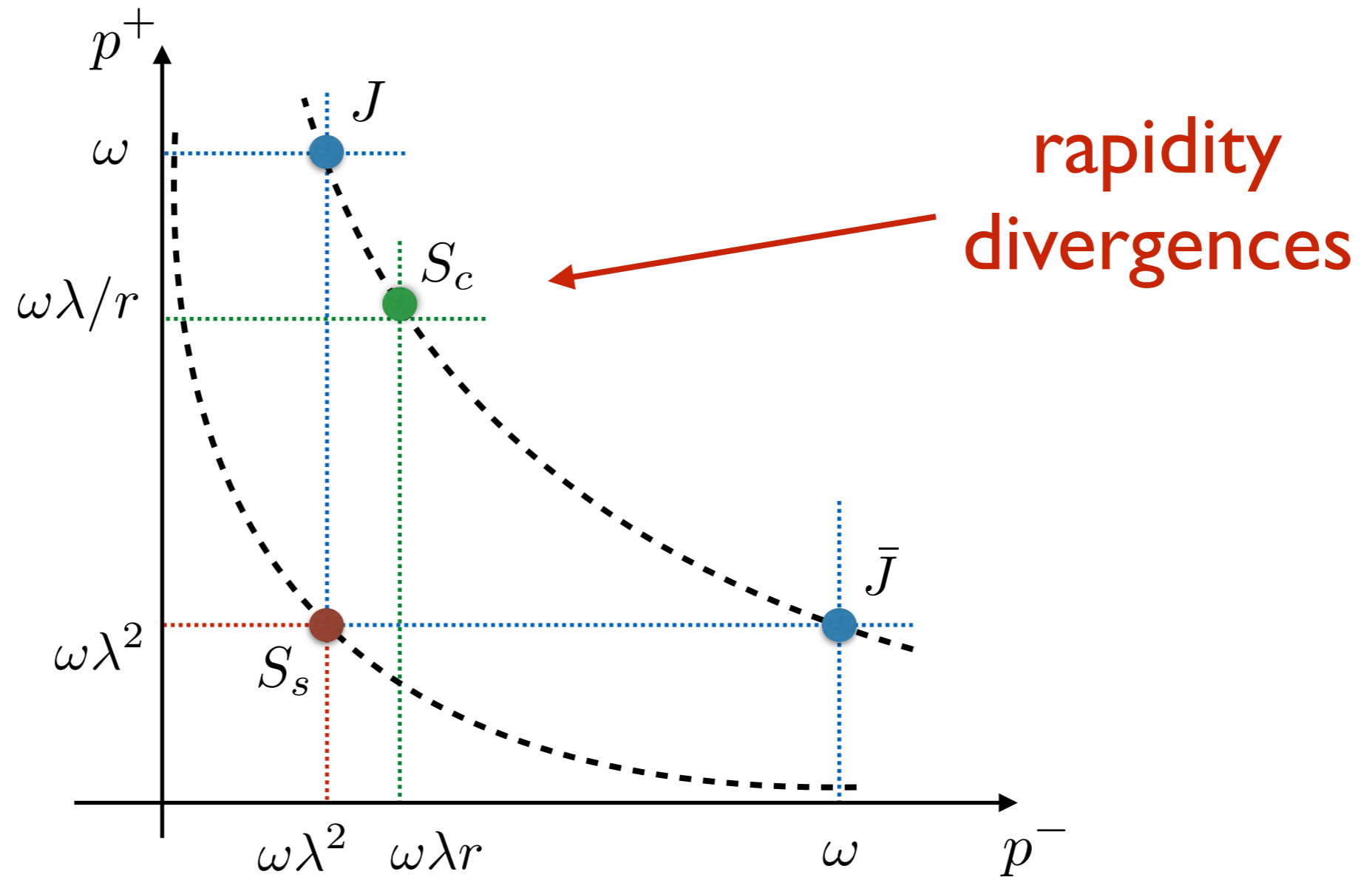
If $^1S_0^{(8)}$ mechanism dominates high p_T production FJF should have negative slope for $z(E)$, for $z > 0.5$

LHCb data on $z(J/\psi)$ well-described by FJF, GFIP improvement over default PYTHIA, consistent w/ NLL' calculations LDME extracted from high p_T slightly preferred

TMD FJFs: p_T^h, θ discriminate between NRQCD mechanisms

Extra Slides

Scales in TMDFFJF



$$p_c \sim \omega(\lambda^2, 1, \lambda) \quad p_{cs} \sim p_h^\perp(r, 1/r, 1) \quad p_{us} \sim \Lambda(1, 1, 1)$$

$$\lambda = p_h^\perp / \omega$$

Factorization Theorem

$$D_{q/h}(\mathbf{p}_\perp, z, \mu) = H_+(\mu) \times \left[\mathcal{D}_{q/h} \otimes_\perp S_C \right](\mathbf{p}_\perp, z, \mu)$$

$$H_+(\mu) = (2\pi)^2 N_c C_+^\dagger(\mu) C_+(\mu)$$

$$\mathcal{D}_{q/h}(\mathbf{p}_\perp^D, z) \equiv \frac{1}{z} \sum_{X_n} \frac{1}{2N_c} \delta(p_{X_n^-, r}^-) \delta^{(2)}(p_{X_n^-, r}^\perp) \text{Tr} \left[\frac{\not{p}}{2} \langle 0 | \delta_{\omega, \bar{P}} \chi_n(0) \delta^{(2)}(\mathcal{P}_\perp^{X_n} + \mathbf{p}_\perp^D) | X_n h \rangle \right. \\ \left. \times \langle X_n h | \bar{\chi}_n(0) | 0 \rangle \right]$$

$$D_{i/h}(\mathbf{p}_\perp, z, \mu, \nu) = \int_z^1 \frac{dx}{x} \mathcal{J}_{i/j}(\mathbf{p}_\perp, x, \mu, \nu) D_{j/h} \left(\frac{z}{x}, \mu \right) + \mathcal{O} \left(\frac{\Lambda_{QCD}^2}{|\mathbf{p}_\perp|^2} \right)$$

$$S_C(\mathbf{p}_\perp^S) \equiv \frac{1}{N_c} \sum_{X_{cs}} \text{Tr} \left[\langle 0 | V_n^\dagger(0) U_n(0) \delta^{(2)}(\mathcal{P}_\perp + \mathbf{p}_\perp^S) | X_{cs} \rangle \langle X_{cs} | U_n^\dagger(0) V_n(0) | 0 \rangle \right]$$

Anomalous Dimensions for RGE, RRGE

RGE

$$\gamma_{\mu}^{SC}(\nu) = \frac{\alpha_s C_i}{\pi} \ln \left(\frac{\mu^2}{r^2 \nu^2} \right)$$

$$\gamma_{\mu}^D(\nu) + \gamma_{\mu}^{SC}(\nu) = \gamma_{\mu}^J = \frac{\alpha_s C_i}{\pi} \left(\ln \left(\frac{\mu^2}{r^2 \omega^2} \right) + \bar{\gamma}_i \right)$$

$$\gamma_{\mu}^D(\nu) = \frac{\alpha_s C_i}{\pi} \left(\ln \left(\frac{\nu^2}{\omega^2} \right) + \bar{\gamma}_i \right)$$

Rapidity Renormalization Group

$$\gamma_{\nu}^{SC}(p_{\perp}, \mu) = +(8\pi)\alpha_s C_i \mathcal{L}_0(\mathbf{p}_{\perp}, \mu^2)$$

$$\gamma_{\nu}^D(\mathbf{p}_{\perp}, \mu) + \gamma_{\nu}^S(\mathbf{p}_{\perp}, \mu) = 0$$

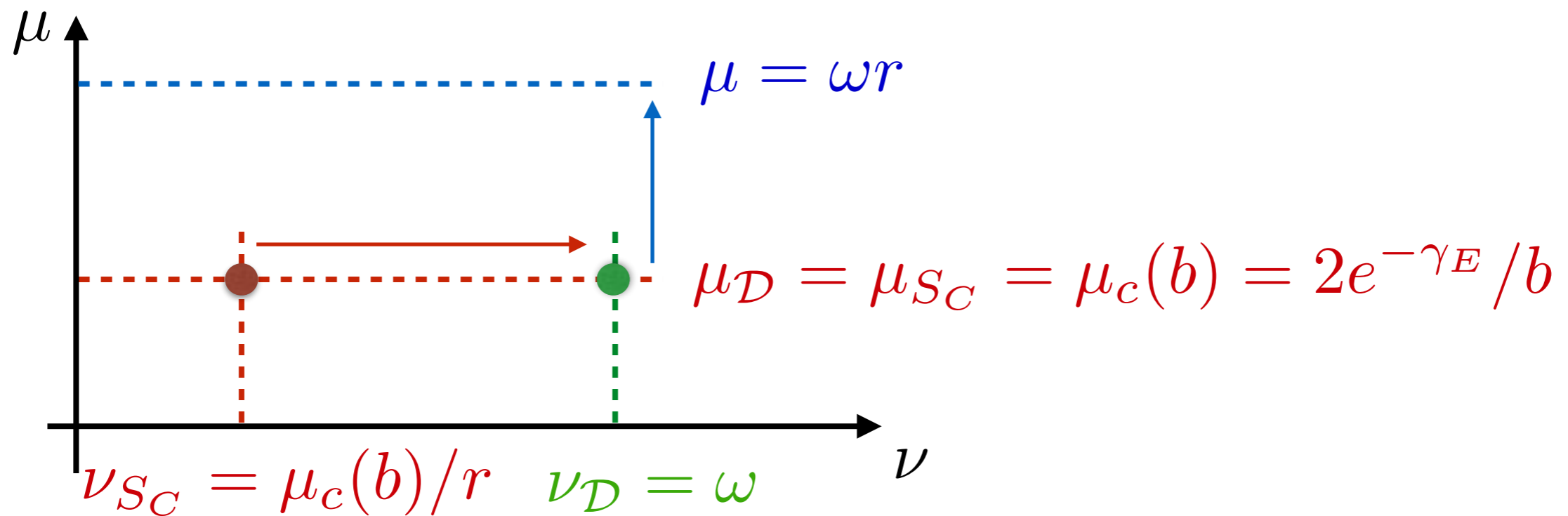
$$\gamma_{\nu}^D(p_{\perp}, \mu) = -(8\pi)\alpha_s C_i \mathcal{L}_0(\mathbf{p}_{\perp}, \mu^2)$$

J-y. Chiu, A. Jain, D. Neill, I.Z. Rothstein, PRL108 (2012) 151601

J-y. Chiu, A. Jain, D. Neill, I.Z. Rothstein, JHEP1205 (2012) 084

Solution to Evolution Eqs. in Fourier Space

$$D_{i/h}(p_{\perp}, z, \mu) = (2\pi)^2 p_{\perp} \int_0^{\infty} db b J_0(bp_{\perp}) \mathcal{U}_{S_C}(\mu, \mu_{S_C}, m_{S_C}) \mathcal{U}_{\mathcal{D}}(\mu, \mu_{\mathcal{D}}, 1) \\ \times \mathcal{V}_{S_C}(b, \mu_{S_C}, \nu_{\mathcal{D}}, \nu_{S_C}) \mathcal{FT} \left[\mathcal{D}_{i/h}(\mathbf{p}_{\perp}, z, \mu_{\mathcal{D}}, \nu_{\mathcal{D}}) \otimes_{\perp} S_C^i(\mathbf{p}_{\perp}, \mu_{S_C}, \nu_{S_C}) \right]$$



fragmentation function (QCD)

$$D_q^h(z) = z \int \frac{dx^+}{4\pi} e^{ik^- x^+ / 2} \frac{1}{4N_c} \text{Tr} \sum_X \langle 0 | \not{n} \Psi(x^+, 0, 0_\perp) | Xh \rangle \langle Xh | \bar{\Psi}(0) | 0 \rangle \Big|_{p_h^\perp=0}$$

fragmentation function (SCET)

$$D_q^h\left(\frac{p_h^-}{\omega}, \mu\right) = \pi\omega \int dp_h^+ \frac{1}{4N_c} \text{Tr} \sum_X \not{n} \langle 0 | [\delta_{\omega, \bar{P}} \delta_{0, P_\perp} \chi_n(0)] | Xh \rangle \langle Xh | \bar{\chi}_n(0) | 0 \rangle$$

Jet function (SCET)

$$J_u(k^+ \omega) = -\frac{1}{\pi\omega} \text{Im} \int d^4x e^{ik \cdot x} i \langle 0 | \text{T} \bar{\chi}_{n, \omega, 0_\perp}(0) \frac{\not{n}}{4N_c} \chi_n(x) | 0 \rangle$$

fragmentation jet function (SCET)

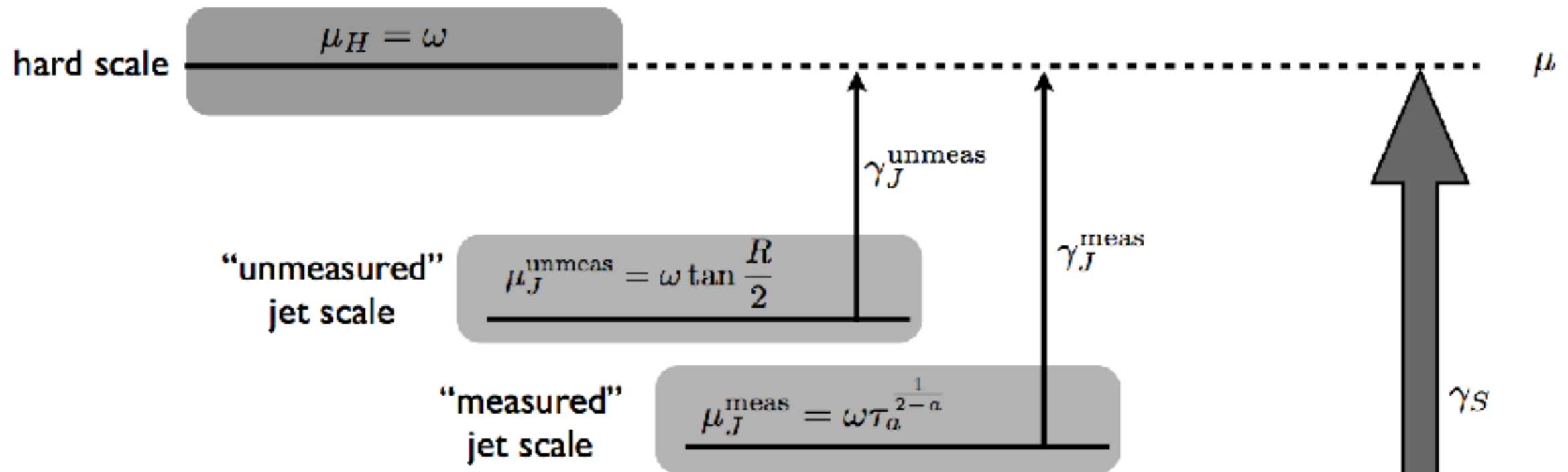
$$\mathcal{G}_{q, \text{bare}}^h(s, z) = \int d^4y e^{ik^+ y^- / 2} \int dp_h^+ \sum_X \frac{1}{4N_c} \text{tr} \left[\frac{\not{n}}{2} \langle 0 | [\delta_{\omega, \bar{P}} \delta_{0, P_\perp} \chi_n(y)] | Xh \rangle \langle Xh | \bar{\chi}_n(0) | 0 \rangle \right]$$

$$\delta(p^+ / z - P_H^+) \rightarrow \delta(p^+ / z - P_H^+) \delta(p^- - s / p^+)$$

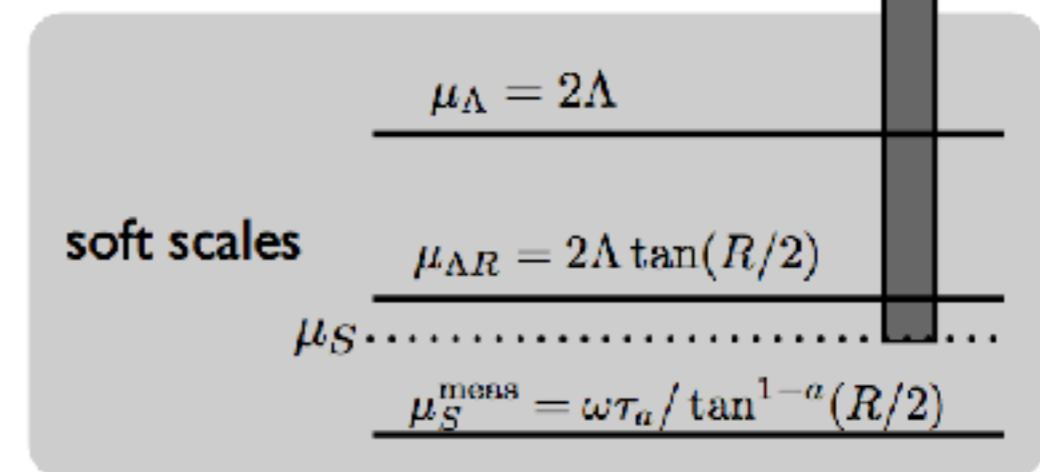
FF

FJF

Scales in Jet Cross section

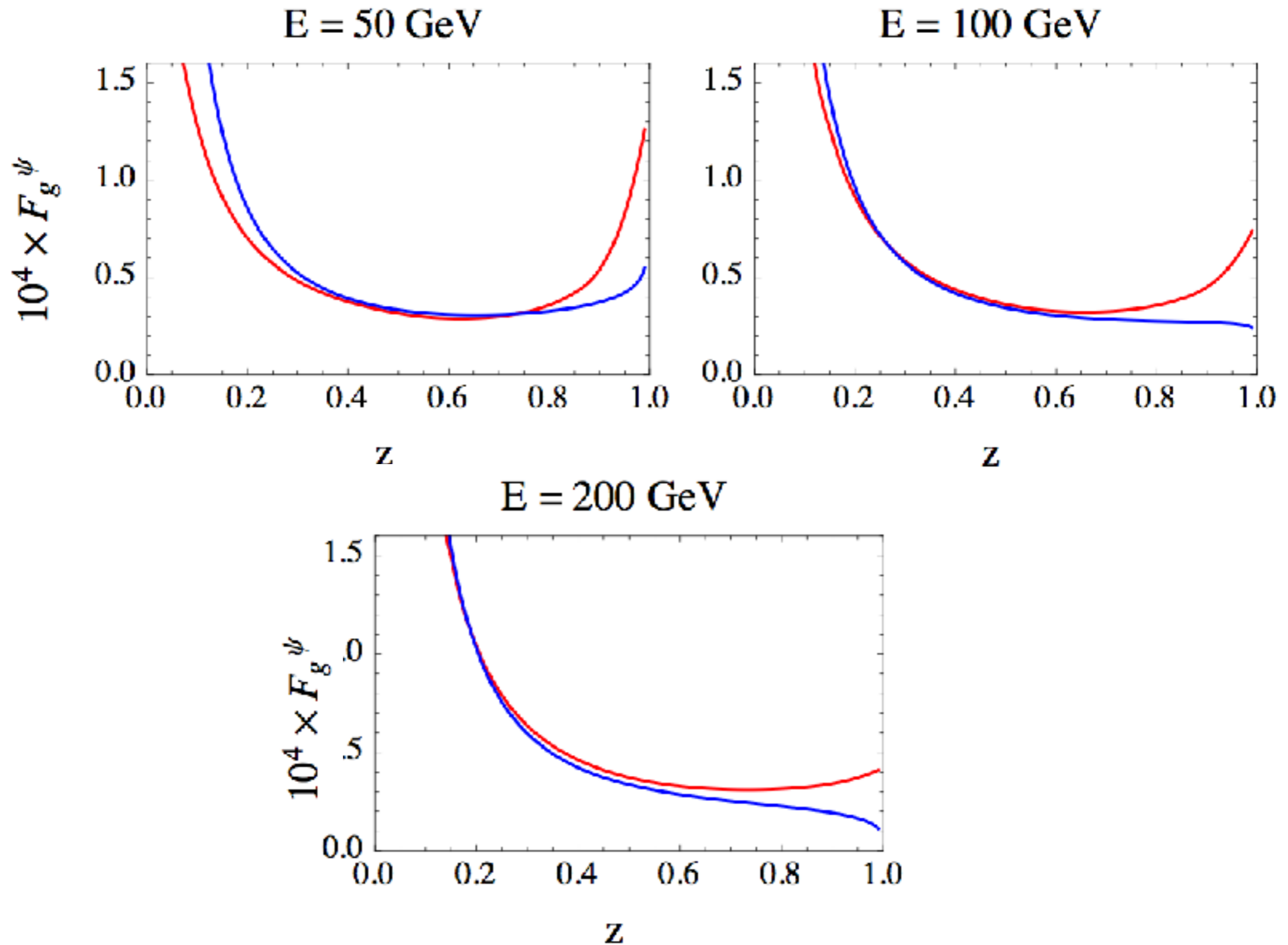


| EFT counting | matching/ matrix element | Γ_{cusp} | $\gamma_{H,J,S}$ | $\beta[\alpha_s]$ |
|--------------|-----------------------------|------------------------|------------------|-------------------|
| LL | tree | 1-loop | tree | 1-loop |
| NLL | tree | 2-loop | 1-loop | 2-loop |
| NNLL | 1-loop | 3-loop | 2-loop | 3-loop |



Color-Octet 3S_1 fragmentation function, FJF

M. Baumgart, A. Leibovich, T.M., I. Z. Rothstein, JHEP 1411 (2014) 003



— fragmentation function

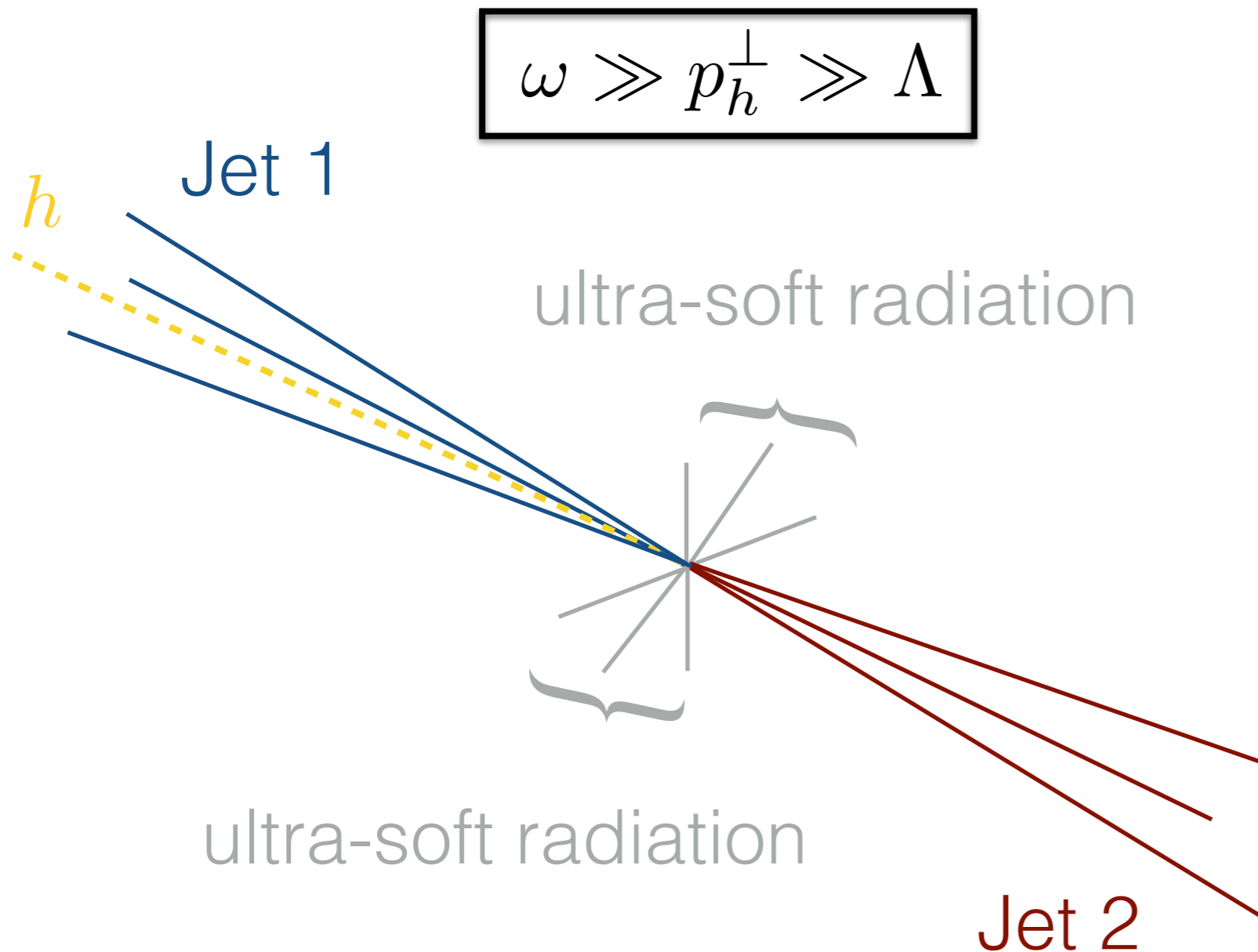
— fragmenting jet function

$$D_{q/h}(\mathbf{p}_\perp, z, \mu) = \frac{1}{z} \sum_X \frac{1}{2N_c} \delta(p_{Xh;r}^-) \delta^{(2)}(\mathbf{p}_\perp + \mathbf{p}_\perp^X) \text{Tr} \left[\frac{\not{n}}{2} \langle \mathbf{0} | \delta_{\omega, \bar{p}} \chi_n^{(0)}(0) | Xh \rangle \right. \\ \left. \langle Xh | \bar{\chi}_n^{(0)}(0) | \mathbf{0} \rangle \right]$$

$$\int d^2 \mathbf{p}_\perp^h D_{q/h}(\mathbf{p}_\perp^h, z, \mu) = D_{q/h}(z, \mu)$$

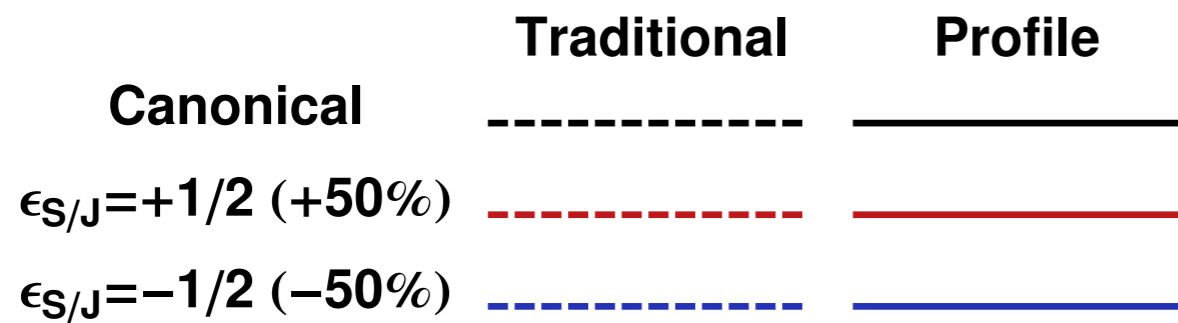
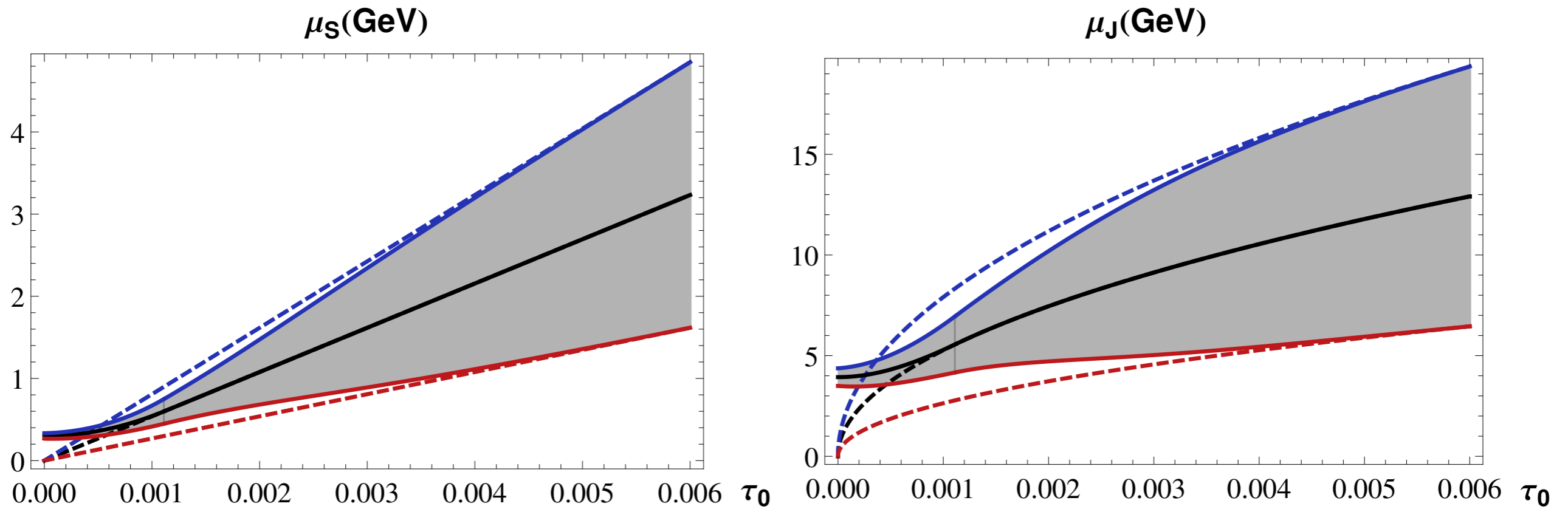
Transverse Momentum Dependent FJFs

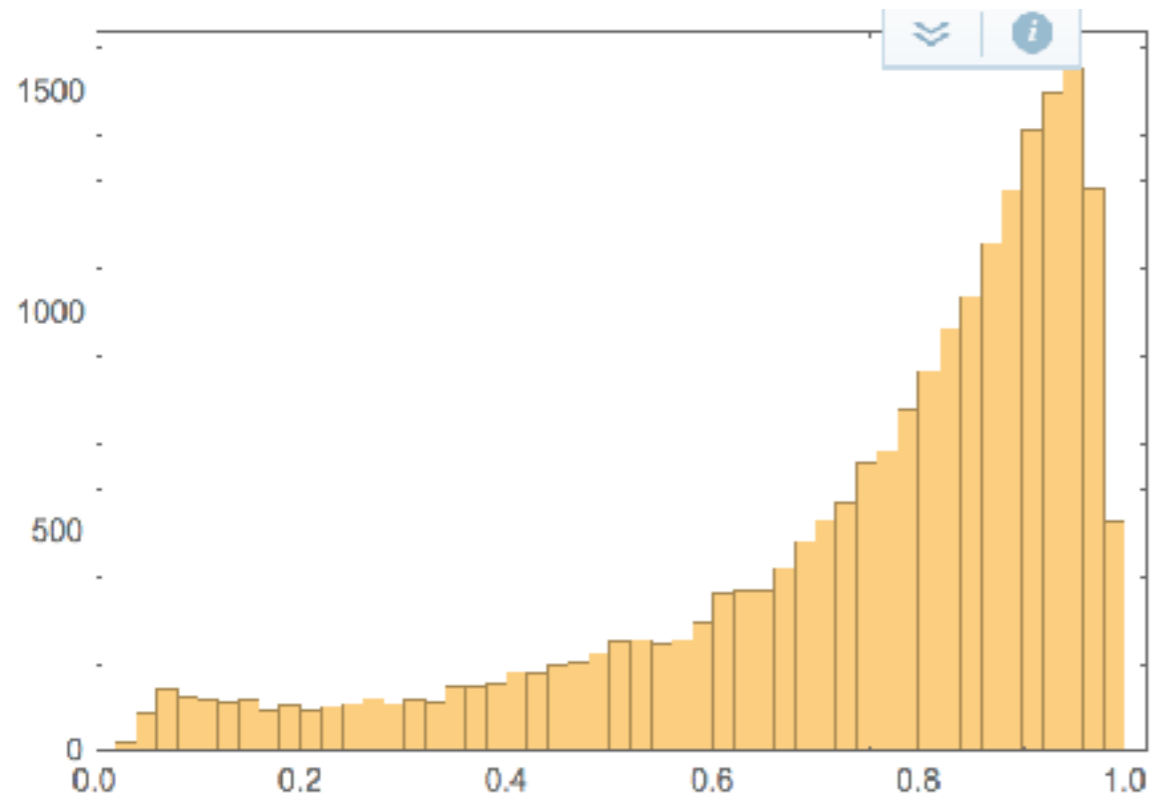
R. Bain, Y. Makris, TM, JHEP 1611 (2016) 144



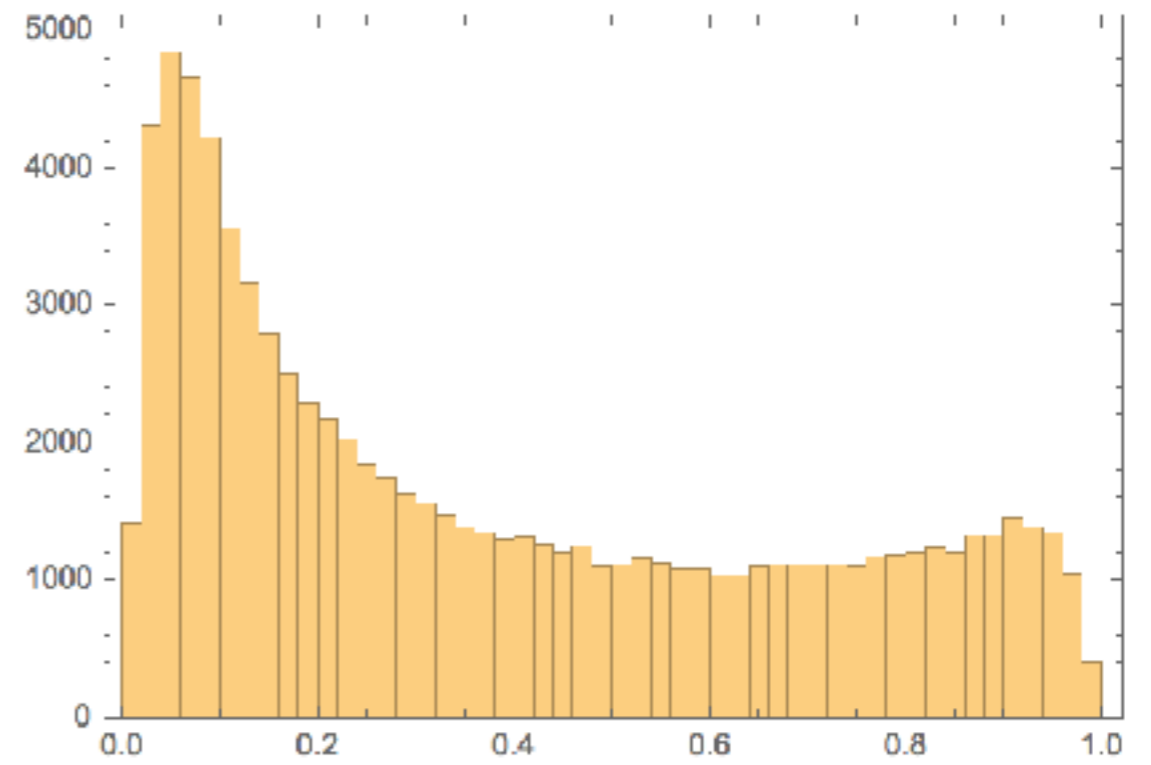
| |
|------------------------------------------|
| $D_{i/h} (z, p_h^\perp, \mu)$ |
| $p_c \sim \omega(\lambda^2, 1, \lambda)$ |
| $p_{cs} \sim p_h^\perp (r, 1/r, 1)$ |
| $p_{us} \sim \Lambda(1, 1, 1)$ |
| $\lambda = p_h^\perp / \omega$ |

Profile Functions





c distribution



g distribution