Twist-3 GPDs
in Deeply-Virtual Compton Scattering

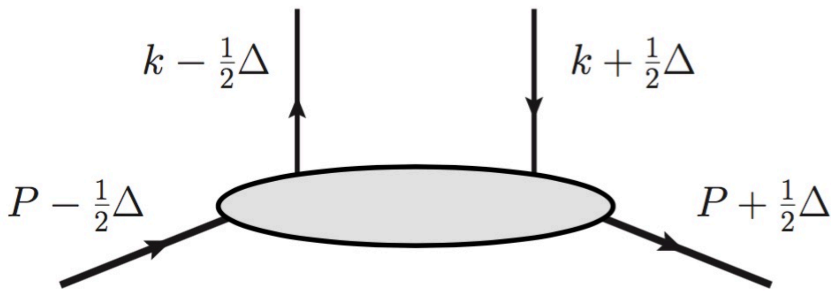
(A. Metz, Temple University)

- Introduction and Motivation
- DVCS amplitude at twist-3 accuracy
- Wandzura-Wilczek approximation
  - discontinuities of twist-3 GPDs
  - factorization of DVCS amplitude
- Quark-target model (QTM)
  - discontinuities of twist-3 GPDs
  - factorization of DVCS amplitude
- Summary and Outlook

based on: F. Aslan, M. Burkardt, C. Lorcé, A. Metz., B. Pasquini, arXiv:1802.06243

Definition of Quark GPDs

- GPD correlator: graphical representation



$$P = \frac{p + p'}{2} \qquad \Delta = p' - p$$

- GPD correlator: vector and axial-vector

$$F_q^\mu = \int_{-\infty}^{\infty} \frac{d\lambda}{2\pi} e^{-i\lambda x} \langle p' | \, \bar{q}(\tfrac{\lambda}{2}n) \, \gamma^\mu \, \mathcal{W}(\tfrac{\lambda}{2}n, -\tfrac{\lambda}{2}n) \, q(-\tfrac{\lambda}{2}n) \, | p \rangle$$

$$\widetilde{F}_q^\mu = \int_{-\infty}^{\infty} \frac{d\lambda}{2\pi} e^{-i\lambda x} \langle p' | \, \bar{q}(\tfrac{\lambda}{2}n) \, \gamma^\mu \gamma_5 \, \mathcal{W}(\tfrac{\lambda}{2}n, -\tfrac{\lambda}{2}n) \, q(-\tfrac{\lambda}{2}n) \, | p \rangle$$

  - F_q^μ and F_q^μ parameterized through GPDs $X^q(x, \xi, t; \mu)$

$$x = \frac{k^+}{P^+} \qquad \xi = \frac{p^+ - p'^+}{p^+ + p'^+} = -\frac{\Delta^+}{2P^+} \qquad t = \Delta^2$$

- Parameterization of GPDs (notation of Kiptily, Polyakov, 2002)

$$F^\mu = \frac{P^\mu}{P^+}\,\bar{u}(p')\left[\gamma^+\,H + \frac{i}{2m}\sigma^{+\nu}\Delta_\nu\,E\right]u(p)$$

$$+\,\bar{u}(p')\left[\frac{\Delta_\perp^\mu}{2m}\,G_1 + \gamma_\perp^\mu\,(H + E + G_2) + \Delta_\perp^\mu\frac{\gamma^+}{P^+}\,G_3 + \tilde{\Delta}_\perp^\mu\frac{\gamma^+\gamma_5}{P^+}\,G_4\right]u(p)$$

$$\widetilde{F}^\mu = \frac{P^\mu}{P^+}\,\bar{u}(p')\left[\gamma^+\gamma_5\,\widetilde{H} + \frac{\Delta^+}{2m}\gamma_5\,\widetilde{E}\right]u(p)$$

$$+\,\bar{u}(p')\left[\frac{\Delta_\perp^\mu}{2m}\gamma_5\,(\widetilde{E} + \widetilde{G}_1) + \gamma_\perp^\mu\gamma_5\,(\widetilde{H} + \widetilde{G}_2) + \Delta_\perp^\mu\frac{\gamma^+\gamma_5}{P^+}\,\widetilde{G}_3 + \tilde{\Delta}_\perp^\mu\frac{\gamma^+}{P^+}\,\widetilde{G}_4\right]u(p)$$

$$\Delta_\perp^\mu = g_\perp^{\mu\nu}\Delta_\nu \qquad \tilde{\Delta}_\perp^\mu = i\varepsilon_\perp^{\mu\nu}\Delta_\nu$$

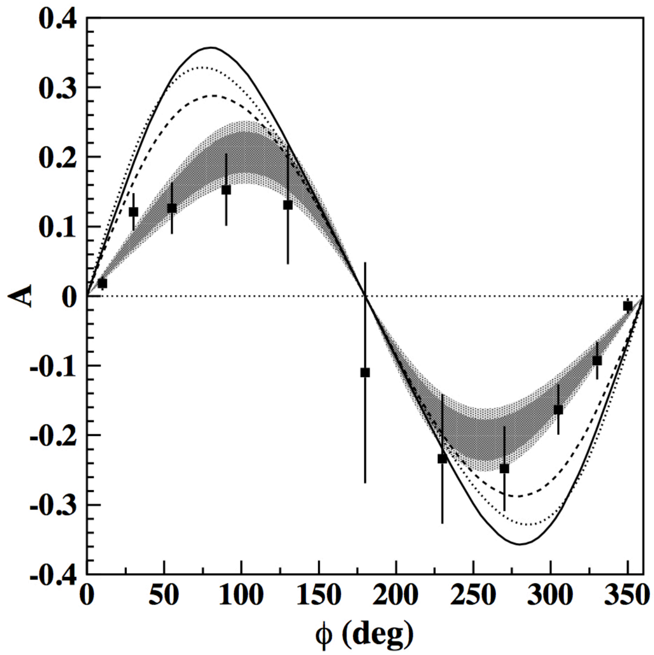
  - other parameterizations of twist-3 GPDs exist
    (Belitsky, Müller, 2000 / Meissner, A.M., Schlegel, 2009)
  - forward limit: only one chiral-even twist-3 GPD survives in correlators

$$\widetilde{G}_2(x,0,0) = g_T(x)$$

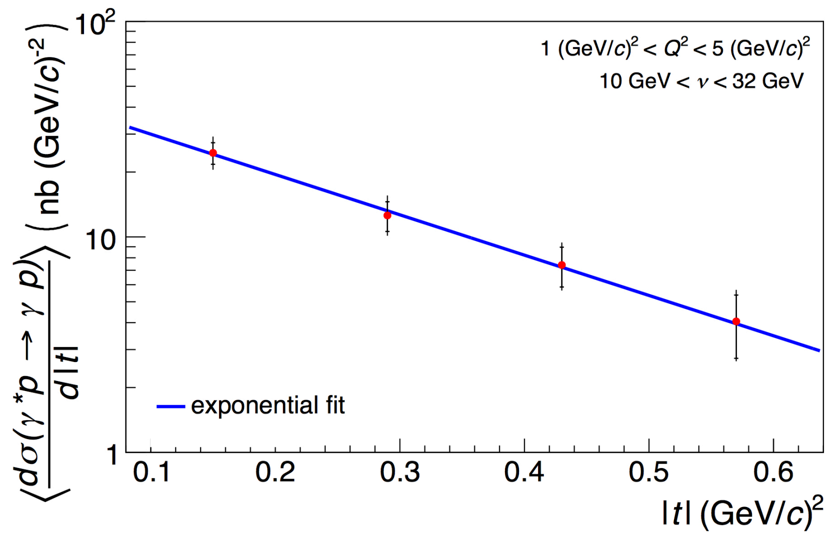
Motivation to study Twist-3 GPDs

- DVCS data in region of low Q^2
  - HERMES, JLab (Hall A, Hall B), COMPASS

Hall B, 2001 $\langle Q^2 \rangle = 1.25\,\mathrm{GeV}^2$



COMPASS, 2018 $\langle Q^2 \rangle = 1.8\,\mathrm{GeV}^2$



  - estimate of twist-3 effects may be important for precise extraction of twist-2 GPDs

- (Kinetic) orbital angular momentum (OAM) of quarks
  - Ji's method: subtract spin from total angular momentum (Ji, 1996)

$$L^q_{\mathrm{kin}} = \frac{1}{2} \int_{-1}^{1} dx\, x\, (H^q + E^q)(x,0,0) - \frac{1}{2} \Delta q$$

  - OAM via twist-3 GPD (Penttinen, Polyakov, Shuvaev, Strikman, 2000 / Hatta, Yoshida, 2012)

$$L^q_{\mathrm{kin}} = - \int_{-1}^{1} dx\, x\, G^q_2(x,0,0)$$

  - several papers speculate/argue that G_2 accessible in DVCS
    Penttinen, Polyakov, Shuvaev, Strikman, 2000 / Kivel, Polyakov, 2000 /
    Courtoy, Goldstein, Gonzalez, Liuti, Rajan, 2013, 2014 / ...

    (polarization) observables in DVCS ? $\rightarrow$ no explicit equations available

$$A_{UL}^{\sin\phi} \quad (?) \qquad\qquad A_{UL}^{\sin 2\phi} \quad (?)$$

  - OAM density $L^q_{\mathrm{kin}}(x)$ via $G_2(x)$? $\rightarrow$ most likely not

- Relation to generalized TMDs
  (Rajan, Courtoy, Engelhardt, Liuti, 2016 / Rajan, Engelhardt, Liuti, 2017)

$$G_2 \;\longleftrightarrow\; F_{1,4} \qquad\qquad -\widetilde{G}_2 + 2(\xi \widetilde{G}_3 - \widetilde{G}_4) \;\longleftrightarrow\; G_{1,1}$$

  – may allow one to get information on generalized TMDs (with straight gauge link)

- Transverse force on quarks in transversely polarized nucleon
  – relation between average transverse momentum and Qiu-Sterman function T_F
    (Boer Mulders, Teryaev, 1997 / Burkardt, 2004 / Meissner, A.M., Goeke, 2007 / Gamberg, A.M., Pitonyak, Prokudin, 2017)

$$\langle k_\perp(x) \rangle \;\longleftrightarrow\; T_F(x,x)$$

  – relation between average transverse force and matrix element d_2 (related to $g_T(x)$)
    (Burkardt, 2008)

$$\langle F_\perp \rangle \;\longleftrightarrow\; d_2 \sim \langle p\uparrow |\, \bar{q}(0)\, \gamma^+ \, F^{+\perp}(0)\, q(0) \,|\, p\uparrow \rangle$$

  – impact-parameter dependence of transverse force (tomography)
    (Burkardt, talk at INT, 2017)

$$\langle F_\perp(b_\perp) \rangle \;\longleftrightarrow\; \widetilde{G}_2(t)$$

DVCS Amplitude at Twist-3 Accuracy

- Process

$$\gamma^*(q) + N(p) \to \gamma(q') + N(p')$$

- Compton tensor

$$T^{\mu\nu} = -\,i \int d^4x \; e^{-i\,q\,\cdot\,x} \langle p'|\, T \left[J^{\mu}_{\rm e.m.}(x) J^{\nu}_{\rm e.m.}(0) \right] |p\rangle$$

- Factorization in generalized Bjorken limit (Ji, 1996 / Radyushkin, 1996 / Collins, Freund, 1998)

$$Q^2 \equiv -q^2 \to \infty \qquad 2p\cdot q \to \infty \qquad x_B = \frac{Q^2}{2p\cdot q} = \text{constant} \qquad |t| \ll Q^2$$

- Some pioneering work for twist-3 DVCS
  - spin-0: Anikin, Pire, Teryaev, 2000 / Kivel, Polyakov, Schäfer, Teryaev, 2000 / Radyushkin, Weiss, 2000 / ...
  - spin-$\frac{1}{2}$: Penttinen, Polyakov, Shuvaev, Strikman, 2000 / Belitsky, Müller, 2000 / Kivel, Polyakov, 2000 / Kivel, Polyakov, Vanderhaeghen, 2000 / ...

- Twist-3 amplitude
  - twist-3 amplitude for transverse polarization of γ^* given by twist-2 GPDs
  - twist-3 amplitude for longitudinal polarization of γ^* given by twist-3 GPDs
  - longitudinal amplitude in terms of GPD correlators

$$\varepsilon_{L\mu} T^{\mu\nu} = \frac{2\xi}{Q} \int_{-1}^{1} dx \left[F_{\perp}^{\nu} C^{+}(x,\xi) - i\varepsilon_{\perp\alpha}^{\nu} \widetilde{F}_{\perp}^{\alpha} C^{-}(x,\xi) \right]$$

$$C^{\pm}(x,\xi) = \frac{1}{x-\xi+i\varepsilon} \pm \frac{1}{x+\xi-i\varepsilon}$$

  - linear combination of vector and axial-vector correlator
  - aim: explicit expression for $\varepsilon_{L\mu} T^{\mu\nu}$ in terms of twist-3 GPDs
    * use independent Dirac bilinears only
    * one can express Dirac bilinears of $i\varepsilon_{\perp\alpha}^{\nu} \widetilde{F}_{\perp}^{\alpha}$ through the ones of $F_{\perp}^{\nu}$ (and of course vice versa)
    * nice exercise with Gordon-type identities

- Longitudinal twist-3 amplitude in terms of GPDs

$$\int_{-1}^{1} dx \left[F_\perp^\nu \, C^+(x,\xi) - i\varepsilon^\nu_{\perp\alpha} \, \widetilde{F}_\perp^\alpha \, C^-(x,\xi) \right]$$

$$= \int_{-1}^{1} dx \Bigg[\Delta_\perp^\nu \frac{b}{2m} \Big(G_1 \, C^+ + (\widetilde{E} + \widetilde{G}_1) \, C^- \Big)$$

$$+ h_\perp^\nu \Big((H + E + G_2) \, C^+ - \frac{\Delta_\perp^2}{4\xi m^2} (\widetilde{E} + \widetilde{G}_1) \, C^- - \frac{1}{\xi} (\widetilde{H} + \widetilde{G}_2) \, C^- \Big)$$

$$+ \Delta_\perp^\nu \frac{h^+}{P^+} \Big(G_3 \, C^+ - \frac{\bar{m}^2}{2m^2} (\widetilde{E} + \widetilde{G}_1) \, C^- - \widetilde{G}_4 \, C^- \Big)$$

$$+ \tilde{\Delta}_\perp^\nu \frac{\tilde{h}^+}{P^+} \Big(G_4 \, C^+ + \frac{t}{8\xi m^2} (\widetilde{E} + \widetilde{G}_1) \, C^- + \frac{1}{2\xi} (\widetilde{H} + \widetilde{G}_2) \, C^- - \widetilde{G}_3 \, C^- \Big) \Bigg]$$

  - four independent structures only
  - in each case, linear combination (LC) of vector and axial vector twist-3 GPDs
  - no individual twist-3 GPD can be addressed
  - result independent of (polarization) observable and parameterization of GPDs
  - situation different from twist-2 case

Wandzura-Wilczek Approximation

- Allows to express twist-3 two-parton PDFs through twist-2 PDFs plus matrix elements of quark-gluon-quark operators

- Classic example (Wandzura, Wilczek, 1977 / Accardi, Bacchetta, Melnitchouk, Schlegel, 2009 / ...)

$$g_T(x) = g_T^{WW}(x) + \tilde{g}_T(x) = \int_x^1 \frac{dy}{y}\, g_1(y) + \tilde{g}_T(x)$$

$$\int_0^1 dx\, x^2\, \tilde{g}_T(x) = \frac{1}{3}\, d_2 \sim \langle p\uparrow |\, \bar{q}(0)\, \gamma^+\, F^{+\perp}(0)\, q(0)\, | p\uparrow\rangle$$

  - theoretical estimates provide small d_2
    - instanton vacuum (Balla, Polyakov, Weiss, 1998)
    - lattice QCD (Göckeler et al, 2005)

- Some pioneering work on DVCS in WW approximation
  Belitsky, Müller, 2000 / Kivel, Polyakov, Schäfer, Teryaev, 2000 / Radyushkin, Weiss, 2000 / Kivel, Polyakov, 2000 / Kivel, Polyakov, Vanderhaeghen, 2000

- Small x^2-moment of genuine twist-3 contribution to twist-3 GPDs in instanton vacuum (Kiptily, Polyakov, 2002)

- WW approximation for twist-3 GPDs: example

$$G_1^{\mathrm{WW}}(x,\xi) = \frac{1}{\xi}\,E(x,\xi) + \frac{1}{\xi}\int_{-1}^{1} du\, W_+(x,u,\xi)\left(u\frac{\partial}{\partial u} + \xi\frac{\partial}{\partial \xi}\right)\left[E(u,\xi)\right]$$

$$- \frac{1}{\xi}\int_{-1}^{1} du\, W_-(x,u,\xi)\left(u\frac{\partial}{\partial u} + \xi\frac{\partial}{\partial \xi}\right)\left[\xi\,\widetilde{E}(u,\xi)\right]$$

  - $W_{\pm}(x,u,\xi)$ are WW kernels
  - similar structure for remaining twist-3 GPDs

- Discontinuities of twist-3 GPDs in WW approximation

$$f_{W_{\pm}}(x,\xi) \equiv \int_{-1}^{1} du\, W_{\pm}(x,u,\xi)\, f(u,\xi)$$

$$\lim_{\delta\to 0}\left[f_{W_{\pm}}(\xi+\delta,\xi) - f_{W_{\pm}}(\xi-\delta,\xi)\right] = \frac{1}{2}\,\mathrm{PV}\int_{-1}^{1} du\,\frac{f(u,\xi)}{u-\xi}$$

$$\lim_{\delta\to 0}\left[f_{W_{\pm}}(-\xi+\delta,\xi) - f_{W_{\pm}}(-\xi-\delta,\xi)\right] = \pm\frac{1}{2}\,\mathrm{PV}\int_{-1}^{1} du\,\frac{f(u,\xi)}{u+\xi}$$

  - all (chiral-even) twist-3 GPDs are discontinuous at $x = \pm\,\xi$

- Factorization of longitudinal amplitude: discussion at level of GPD correlators
(Kivel, Polyakov, Schäfer, Teryaev, 2000 / Radyushkin, Weiss, 2000 / Kivel, Polyakov, 2000)

$$
\begin{aligned}
\varepsilon_{L\mu}\, T^{\mu\nu}_{WW} &= \frac{2\xi}{Q}\int_{-1}^{1} dx\, \Big[F^{\nu}_{\perp,WW}\, C^{+}(x,\xi) - i\varepsilon^{\nu}_{\perp\alpha}\, \widetilde{F}^{\alpha}_{\perp,WW}\, C^{-}(x,\xi)\Big] \\
&= \frac{2\xi}{Q}\int_{-1}^{1} dx\, \Big[\big(F^{\nu}_{\perp,WW} - i\varepsilon^{\nu}_{\perp\alpha}\, \widetilde{F}^{\alpha}_{\perp,WW}\big)\, \frac{1}{x-\xi+i\varepsilon} \\
&\qquad + \big(F^{\nu}_{\perp,WW} + i\varepsilon^{\nu}_{\perp\alpha}\, \widetilde{F}^{\alpha}_{\perp,WW}\big)\, \frac{1}{x+\xi-i\varepsilon}\Big]
\end{aligned}
$$

  - $F^{\nu}_{\perp,WW}$ and $\widetilde{F}^{\nu}_{\perp,WW}$ are discontinuous at $x = \pm\,\xi$
  - convolution integral potentially undefined due to divergence
  - $\big(F^{\nu}_{\perp,WW} - i\varepsilon^{\nu}_{\perp\alpha}\, \widetilde{F}^{\alpha}_{\perp,WW}\big)$ is continuous at $x = +\,\xi$
    $\big(F^{\nu}_{\perp,WW} + i\varepsilon^{\nu}_{\perp\alpha}\, \widetilde{F}^{\alpha}_{\perp,WW}\big)$ is continuous at $x = -\,\xi$
  - LO twist-3 amplitude factorizes
  - NLO (non-singlet) amplitude factorizes in WW approximation
    (Kivel, Mankiewicz, 2003)

- Factorization of longitudinal amplitude: representation in terms of twist-3 GPDs
  - in LCs of GPDs only two combinations of Wilson coefficients:

$$\int_{-1}^{1} dx \int_{-1}^{1} du \left[W_{\pm}(x,u,\xi)\, C^{+}(x,\xi) - W_{\mp}(x,u,\xi)\, C^{-}(x,\xi)\right] f(u,\xi)$$
$$= \pm \int_{-1}^{1} dx \frac{1}{x-\xi+i\varepsilon} \int_{-1}^{1} du \left[W_{+}(x,u,\xi) - W_{-}(x,u,\xi)\right] f(u,\xi)$$
$$+ \int_{-1}^{1} dx \frac{1}{x+\xi-i\varepsilon} \int_{-1}^{1} du \left[W_{+}(x,u,\xi) + W_{-}(x,u,\xi)\right] f(u,\xi)$$

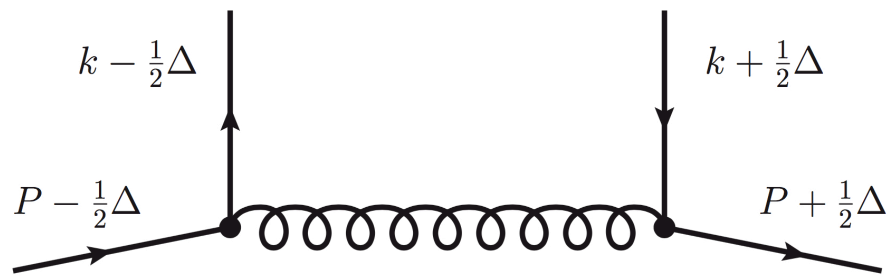
$$\int_{-1}^{1} du \left[W_{+}(x,u,\xi) - W_{-}(x,u,\xi)\right] f(u,\xi) \ \text{continuous at}\ \ x = +\xi$$
$$\int_{-1}^{1} du \left[W_{+}(x,u,\xi) + W_{-}(x,u,\xi)\right] f(u,\xi) \ \text{continuous at}\ \ x = -\xi$$

  - consistency check of amplitude and of WW approximation for twist-3 GPDs
  - in WW approximation one can hardly
    (1) study impact individual twist-3 GPD on observables
    (2) extract individual twist-3 GPD from DVCS data

- Are discontinuities of twist-3 GPDs an artifact of the WW approximation ?

Twist-3 GPDs in QTM

- Diagram (light-cone gauge $A^+ = 0$)



- Expression for correlators (use $\gamma_\perp^\mu \gamma_5$ for $\widetilde{F}_\perp^\mu$)

$$F_\perp^\mu = -i \frac{C_F\, g^2}{(2\pi)^4} P^+ \int_{-\infty}^{\infty} dk^- \, d^2\vec{k}_\perp \, \frac{N_\perp^\mu}{D}$$

$$N_\perp^\mu = -\bar{u}(p')\,\gamma^\alpha \left(\not{k} + \frac{\not{\Delta}}{2} + m \right) \gamma_\perp^\mu \left(\not{k} - \frac{\not{\Delta}}{2} + m \right) \gamma^\beta \, u(p)\, D_{\alpha\beta}(P-k)$$

$$D = \left[\left(k - \frac{\Delta}{2} \right)^2 - m^2 + i\varepsilon \right] \left[\left(k + \frac{\Delta}{2} \right)^2 - m^2 + i\varepsilon \right] \left[(P-k)^2 + i\varepsilon \right]$$

$$D^{\mu\nu}(k) = -g^{\mu\nu} + \frac{k^\mu\, n^\nu + k^\nu\, n^\mu}{k \cdot n}$$

- Integral without k^- in numerator

$$I = \int_{-\infty}^{\infty} dk^- \frac{1}{D} = \frac{1}{C} \int_{-\infty}^{\infty} dk^- \frac{1}{(k^- - k_1^-)(k^- - k_2^-)(k^- - k_3^-)}$$

$$C = -8\,(x+\xi)(x-\xi)(1-x)(P^+)^3$$

$$\begin{aligned} k_1^- &= \frac{\Delta^-}{2} + \frac{(\vec{k}_\perp - \frac{\vec{\Delta}_\perp}{2})^2 + m^2 - i\varepsilon}{2(x+\xi)P^+} \\ k_2^- &= -\frac{\Delta^-}{2} + \frac{(\vec{k}_\perp + \frac{\vec{\Delta}_\perp}{2})^2 + m^2 - i\varepsilon}{2(x-\xi)P^+} \\ k_3^- &= P^- - \frac{\vec{k}_\perp^2 - i\varepsilon}{2(1-x)P^+} \end{aligned}$$

  – position of poles depends on x
  – expect different functional form for $x > \xi$, $-\xi \le x \le \xi$, $x < -\xi$

– calculation provides

$$I = \begin{cases} I_1 = \frac{2\pi i}{C}\frac{1}{(k_1^- - k_3^-)(k_2^- - k_3^-)}\,, \text{ for } \; x > \xi \\ I_2 = -\frac{2\pi i}{C}\frac{1}{(k_1^- - k_2^-)(k_1^- - k_3^-)}\,, \text{ for } \; -\xi \le x \le \xi \\ I_3 = 0\,, \text{ for } \; x < -\xi \end{cases}$$

– indeed different functional form for three regions

– what happens for $x = \pm\,\xi$?

$$I_1 - I_2 \;=\; \frac{2\pi i}{C}\frac{1}{(k_1^- - k_2^-)(k_2^- - k_3^-)} \sim x - \xi$$

$$I_2 \;\sim\; x + \xi$$

– I is continuous at $x = \pm\,\xi$

- Integral with k^- in numerator

$$I^k = \int_{-\infty}^{\infty} dk^- \frac{k^-}{D} = \frac{1}{C} \int_{-\infty}^{\infty} dk^- \frac{k^-}{(k^- - k_1^-)(k^- - k_2^-)(k^- - k_3^-)}$$

  - calculation provides

$$I^k = \begin{cases} I_1^k = \frac{2\pi i}{C} \frac{k_3^-}{(k_1^- - k_3^-)(k_2^- - k_3^-)}\,, & \text{for } x > \xi \\ I_2^k = -\frac{2\pi i}{C} \frac{k_1^-}{(k_1^- - k_2^-)(k_1^- - k_3^-)}\,, & \text{for } -\xi \leq x \leq \xi \\ I_3^k = 0\,, & \text{for } x < -\xi \end{cases}$$

  - different functional form for three regions

- what happens for $x = \pm\xi$?

$$
\begin{aligned}
I_1^k - I_2^k &= \frac{2\pi i}{C}\left[\frac{k_3^-}{(k_1^- - k_2^-)(k_2^- - k_3^-)} + \frac{1}{k_1^- - k_2^-}\right] \\
I_2^k &= -\frac{2\pi i}{C}\left[\frac{k_3^-}{(k_1^- - k_2^-)(k_1^- - k_3^-)} + \frac{1}{k_1^- - k_2^-}\right]
\end{aligned}
$$

- 1st term in square brackets vanishes at relevant crossover point
- but $C(k_1^- - k_2^-)$ is finite at $x = \pm\xi$
- I^k is not continuous at $x = \pm\xi$
- GPDs discontinuous if non-vanishing k^- dependence in numerator

- Result for twist-3 GPDs (k^- dependence only)

$$N_\perp^\mu = \frac{2P^+ k^-}{1-x} \Bigg[4(1-\xi^2)\, h_\perp^\mu - 2(1-2x)\, \frac{\vec{k}_\perp \cdot \vec{\Delta}_\perp}{\vec{\Delta}_\perp^2}\, \Delta_\perp^\mu \frac{h^+}{P^+}$$
$$- \left(1 - x - 2\xi\, \frac{\vec{k}_\perp \cdot \vec{\Delta}_\perp}{\vec{\Delta}_\perp^2} \right) \tilde{\Delta}_\perp^\mu \frac{\tilde{h}^+}{P^+} \Bigg] + \ldots$$

$$\widetilde{N}_\perp^\mu = \frac{2P^+ k^-}{1-x} \Bigg[4x(1-\xi^2)\, \tilde{h}_\perp^\mu + 2\left(\xi(1-x) - (1-2x)\, \frac{\vec{k}_\perp \cdot \vec{\Delta}_\perp}{\vec{\Delta}_\perp^2} \right) \Delta_\perp^\mu \frac{\tilde{h}^+}{P^+}$$
$$+ \left(1 - x + 2\xi\, \frac{\vec{k}_\perp \cdot \vec{\Delta}_\perp}{\vec{\Delta}_\perp^2} \right) \tilde{\Delta}_\perp^\mu \frac{h^+}{P^+} \Bigg] + \ldots$$

  - chiral-even twist-3 GPDs discontinuous at $x = \pm\,\xi$, except G_1 and $\widetilde{G}_1$
  - continuous G_1 and $\widetilde{G}_1$ artifact of the model ?
  - for twist-2 GPDs, k^- dependence proportional to $x^2 - \xi^2$
  - k^- dependence for forward twist-3 PDFs $e(x)$ and $h_L(x)$ gives rise to $\delta(x)$ (Burkardt, 1995 / Burkardt, Koike, 2001)
  - do results in QTM break factorization of the twist-3 DVCS amplitude ?

- Factorization of twist-3 DVCS amplitude in QTM
  - analysis for A_2 (2nd LC of GPDs in $\varepsilon_{L\mu}\, T^{\mu\nu}$)

$$
\begin{aligned}
A_2 &= G_2\, C^+ - \frac{1}{\xi}\, \widetilde{G}_2\, C^- \\
&= -i\, \frac{C_F\, g^2}{(2\pi)^4}\, P^+ \int_{-\infty}^{\infty} dk^-\, d^2\vec{k}_\perp\, 8P^+ k^-\, \frac{1-\xi^2}{1-x}\, \underbrace{\left(C^+ - \frac{x}{\xi}\, C^-\right)}_{=\,0}\, \frac{1}{D} \;=\; 0
\end{aligned}
$$

  - analysis for A_3 (3rd LC of GPDs in $\varepsilon_{L\mu}\, T^{\mu\nu}$)

$$
\begin{aligned}
A_3 &= G_3\, C^+ - \widetilde{G}_4\, C^- \\
&= i\, \frac{C_F\, g^2}{(2\pi)^4}\, P^+ \int_{-\infty}^{\infty} dk^-\, d^2\vec{k}_\perp\, \frac{2P^+ k^-}{1-x} \Bigg[2(1-2x)\, \frac{\vec{k}_\perp \cdot \vec{\Delta}_\perp}{\vec{\Delta}_\perp^2}\, C^+ \\
&\quad + \left(1 - x + 2\xi\, \frac{\vec{k}_\perp \cdot \vec{\Delta}_\perp}{\vec{\Delta}_\perp^2}\right) C^- \Bigg] \frac{1}{D} \\
&= \frac{1}{x - \xi + i\varepsilon}\, \mathcal{A}_{3,+\xi} + \frac{1}{x + \xi - i\varepsilon}\, \mathcal{A}_{3,-\xi}
\end{aligned}
$$

x integration in $\varepsilon_{L\mu}\, T^{\mu\nu}$ can be performed if $\mathcal{A}_{3,+\xi}$ continuous at $x = +\,\xi$ and $\mathcal{A}_{3,-\xi}$ continuous at $x = -\,\xi$

$$\lim_{\delta\to 0}\left[\mathcal{A}_{3,+\xi}(x=\xi+\delta)-\mathcal{A}_{3,+\xi}(x=\xi-\delta)\right]$$

$$= i\,\frac{C_F\,g^2}{(2\pi)^4}\,2(P^+)^2\int_{-\infty}^{\infty} d^2\vec{k}_\perp\left(1+2\,\frac{\vec{k}_\perp\cdot\vec{\Delta}_\perp}{\vec{\Delta}_\perp^2}\right)(I_1^k-I_2^k)\big|_{x=+\xi}$$

$$= -\,\frac{C_F\,g^2}{(2\pi)^3}\,\frac{1}{4\xi(1-\xi)}\,\frac{1}{\vec{\Delta}_\perp^2}\underbrace{\int_{-\infty}^{\infty} d^2\vec{l}_\perp\,\frac{2\,\vec{l}_\perp\cdot\vec{\Delta}_\perp}{\vec{l}_\perp^2+m^2}}_{=0} = 0$$

also $\mathcal{A}_{3,-\xi}$ continuous at $x=-\xi$

- corresponding analysis holds for A_4
- results for GPDs in QTM compatible with factorization of twist-3 DVCS amplitude
- further support of factorization hypothesis
- in that regard, model calculation complimentary to analysis in WW approximation (Kivel, Mankiewicz, 2003)

Summary and Outlook

- (Several) twist-3 GPDs contain interesting physics
- In DVCS, only linear combinations of vector and axial-vector GPDs accessible
- Wandzura-Wilczek approximation
  - all (chiral-even) twist-3 GPDs are discontinuous at $x = \pm\,\xi$
  - nevertheless, DVCS amplitude factorizes
- Quark-target model (LO approximation)
  - most (chiral-even) twist-3 GPDs are discontinuous at $x = \pm\xi$
  - nevertheless, DVCS amplitude factorizes
- Some lessons
  - discontinuities of twist-3 GPDs apparently general feature of QCD dynamics
  - impact of individual twist-3 GPDs on DVCS observables can hardly be studied
  - support of factorization hypothesis for twist-3 DVCS amplitude
- Open questions
  - do accessible linear combinations of twist-3 GPDs have physical interpretation ?
  - other processes which allow one to study twist-3 GPDs ?