Fragmentation Functions of light charged hadrons

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Outline

1. Fragmentation Functions: the basics
   - factorisation, evolution
   - higher-order corrections, theoretical constraints
   - statistical representation of uncertainties
   - FFs: why should we bother? Are current FF sets good?
2. The NNFF1.0 analysis
   - data set and fit settings
   - the NNPDF methodology: parametrisation and uncertainty representation
   - results: fit quality, perturbative stability, dependence on the data set/kinematic cuts
   - is the NNFF1.0 set better than currently available sets?
3. Summary and outlook
   - global fits, simultaneous fits

DISCLAIMER

This is not a review talk on Fragmentation Functions

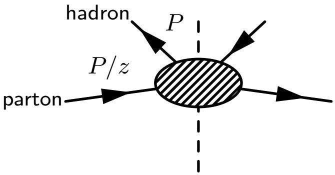
Focus on topics which I've worked on recently

[JHEP 1503 (2015) 046; EPJ C77 (2017) 516; PoS DIS2017 (2018) 231]

1. Fragmentation Functions: the basics

Hadrons in the final state: Fragmentation Functions

FFs allow for a proper field-theoretic definition as matrix elements of bilocal operators



collinear transition
of a parton i into a hadron h
with fractional momentum z
no local OPE $\Longrightarrow$ no lattice formulation
[Rev.Mod.Phys. 67 (1995) 157]

$$D_i^h(z) = \frac{1}{12\pi}\sum_X \int dy^- e^{-i\frac{P^+}{z}y^-}\,\mathrm{Tr}\,\left[\gamma^+\langle 0|\psi(y)\mathcal{P}|h(P)\,X\rangle\langle h(P)\,X|\mathcal{P}'\bar{\psi}(0)|0\rangle\right]$$

with light-cone coordinates and appropriate gauge links $\mathcal{P}$, $\mathcal{P}'$

$$y = (y^+, y^-, \mathbf{y}_\perp)\,, \quad y^+ = (y^0 + y^z)/\sqrt{2}\,, \quad y^- = (y^0 - y^z)/\sqrt{2}\,, \quad \mathbf{y}_\perp = (v^x, v^y)$$

All these definitions have ultraviolet divergences which must be renormalised
in order to define finite FFs to be used in the factorisation formulas
(FFs, like PDFs, are scheme dependent)

The definition above can be generalised to include longitudinal/transverse polarisations

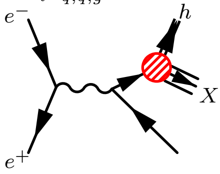
Factorisation of physical observables [Adv.Ser.Direct.HEP 5 (1988) 1]

1. A variety of sufficiently inclusive processes allow for a factorised description

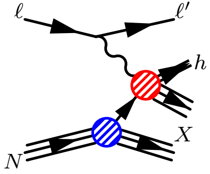
short-distance part		long-distance part
hard interaction of partons	$\xleftrightarrow[\text{scheme \& scale } \mu]{\text{factorisation}}$	nucleon structure
process-dependent kernels		universal PDFs/FFs

2. Physical observables are written as a convolution of coefficient functions and FFs

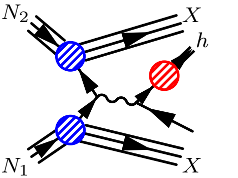
$$\mathcal{O}_I = \sum_{i=q,\bar{q},g} C_{Ii}(y,\alpha_s(\mu^2)) \otimes D_i(y,\mu^2) + \text{p.s. corrections} \qquad f \otimes g = \int_x^1 \frac{dy}{y} f\left(\frac{x}{y}\right) g(y)$$



$e^+ + e^- \to h + X$
single-inclusive annihilation (SIA)



$\ell + N \to \ell' + h + X$
semi-inclusive deep-inelastic scattering (SIDIS)



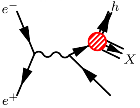
$N_1 + N_2 \to h + X$
high-p_T hadron production in proton-proton collisions (PP)

3. Coefficient functions allow for a perturbative expansion

$$C_{If}(y,\alpha_s) = \sum_{k=0} a_s^k C_{If}^{(k)}(y), \qquad a_s = \alpha_s/(4\pi)$$

4. After factorisation, all quantities (including FFs) depend on μ

Factorisation of physical observables

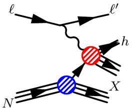


$e^+ + e^- \to h + X$
single-inclusive annihilation (SIA)

$$\frac{d\sigma^h}{dz} = F_T^h(z,Q^2) + F_L^h(z,Q^2) = F_2^h(x,Q^2)$$

$$F^h_{k=T,L,2} = \frac{4\pi\alpha^2_{\rm em}}{Q^2}\langle e^2\rangle \left\{ D^h_\Sigma \otimes C^{\rm S}_{k,q} + n_f D^h_g \otimes C^{\rm S}_{k,g} + D^h_{\rm NS} \otimes C^{\rm NS}_{k,q} \right\}$$

up to NNLO [PLB 386 (1996) 422; NPB 487 (1997) 233; PLB 392 (1997) 207]



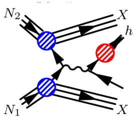
$\ell + N \to \ell' + h + X$
semi-inclusive deep-inelastic scattering (SIDIS)

$$\frac{d\sigma^h}{dx\,dy\,dz} = \frac{2\pi\alpha^2_{\rm em}}{Q^2}\left[\frac{1+(1-y)^2}{y}2F_1^h + \frac{2(1-y)}{y}F_L^h\right]$$

$$2F_1^h = e_q^2 \left\{ q \otimes D_q^h + \frac{\alpha_s}{2\pi}\left[q \otimes C^1_{qq} \otimes D^h_q + q \otimes C^1_{gq} \otimes D^h_g + g \otimes C^1_{qg} \otimes D^h_q \right]\right\}$$

$$F_L^h = \frac{\alpha_s}{2\pi}\sum_{q,\bar q} e_q^2 \left[q \otimes C^L_{qq} \otimes D^h_q + q \otimes C^L_{gq} \otimes D^h_g + g \otimes C^L_{qg} \otimes D^h_q \right]$$

up to NLO [NPB 160 (1979) 301; PRD 57 (1998) 5811]
partial NNLO [PRD 95 (2017) 034027]



$N_1 + N_2 \to h + X$
high-p_T hadron production in pp collisions (PP)

$$E_h\frac{d^3\sigma}{dp_h^3} = \sum_{a,b,c} f_a \otimes f_b \otimes \hat\sigma^c_{ab} \otimes D^h_c$$
$$=$$
$$\sum_{i,j,k}\int\frac{dx_a}{x_a}\int\frac{dx_b}{x_b}\int\frac{dz}{z^2} f^{i/p_a}(x_a) f^{j/p_b}(x_b) D^{h/k}(z)\hat\sigma^{ij\to k}\delta(\hat s + \hat t + \hat u)$$

up to NLO [PRD 67 (2003) 054004; PRD 67 (2003) 054005]

Evolution of FFs: DGLAP equations [NPB 126 (1977) 298]

1. A set of $(2n_f+1)$ integro-differential equations (n_f=number of active flavours)

$$\frac{\partial}{\partial \ln \mu^2} D_i(x,\mu^2) = \sum_j^{n_f} \int_x^1 \frac{dz}{z} P_{ji}\left(z, \alpha_s(\mu^2)\right) D_j\left(\frac{x}{z}, \mu^2\right)$$

2. Often written in a convenient basis of FFs

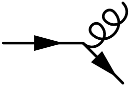
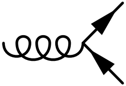
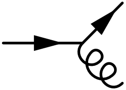
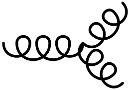
$$D_{\mathrm{NS};\pm} = (D_q \pm D_{\bar q}) - (D_{q'} \pm D_{\bar q'}) \qquad D_{\mathrm{NS;v}} = \sum_q^{n_f}(D_q - D_{\bar q}) \qquad D_\Sigma = \sum_q^{n_f}(D_q + D_{\bar q})$$

$$\frac{\partial}{\partial \ln \mu^2} D_{\mathrm{NS};\pm,v}(x,\mu^2) = P^{\pm,\mathrm{v}}(x,\mu_F^2) \otimes D_{\mathrm{NS};\pm,v}(x,\mu^2)$$

$$\frac{\partial}{\partial \ln \mu^2} \begin{pmatrix} D_\Sigma(x,\mu^2) \\ D_g(x,\mu^2) \end{pmatrix} = \begin{pmatrix} P^{qq} & 2n_f P^{gq} \\ \frac{1}{2n_f} P^{qg} & P^{gg} \end{pmatrix} \otimes \begin{pmatrix} D_\Sigma(x,\mu^2) \\ D_g(x,\mu^2) \end{pmatrix}$$

3. With perturbative computable (time-like) splitting functions

$$P_{ji}(z,\alpha_s) = \sum_{k=0} a_s^{k+1} P_{ji}^{(k)}(z), \qquad a_s = \alpha_s/(4\pi)$$

$P_{qq}^{(0)}$  $P_{gq}^{(0)}$  $P_{qg}^{(0)}$  $P_{gg}^{(0)}$ 

Splitting functions: LO and NLO

$$
\begin{aligned}
P^{(0)}_{\rm ns}(x) &= C_F\,(2p_{\rm qq}(x)+3\delta(1-x)) \\
P^{(0)}_{\rm ps}(x) &= 0 \\
P^{(0)}_{\rm qg}(x) &= 2n_f\,p_{\rm qg}(x) \\
P^{(0)}_{\rm gq}(x) &= 2C_F\,p_{\rm gq}(x) \\
P^{(0)}_{\rm gg}(x) &= C_A\Big(4p_{\rm gg}(x)+\frac{11}{3}\delta(1-x)\Big)-\frac{2}{3}n_f\,\delta(1-x)
\end{aligned}
$$

LO: 1973

$$
\begin{aligned}
P^{(1)+}_{\rm ns}(x) = {} & 4C_AC_F\Big(p_{\rm qq}(x)\Big[\frac{67}{18}-\zeta_2+\frac{11}{6}{\rm H}_0+{\rm H}_{0,0}\Big]+p_{\rm qq}(-x)\Big[\zeta_2+2{\rm H}_{-1,0}-{\rm H}_{0,0}\Big] \\
& +\frac{14}{3}(1-x)+\delta(1-x)\Big[\frac{17}{24}+\frac{11}{3}\zeta_2-3\zeta_3\Big]\Big)-4C_Fn_f\Big(p_{\rm qq}(x)\Big[\frac{5}{9}+\frac{1}{3}{\rm H}_0\Big]+\frac{2}{3}(1-x) \\
& +\delta(1-x)\Big[\frac{1}{12}+\frac{2}{3}\zeta_2\Big]\Big)+4C_F^{\,2}\Big(2p_{\rm qq}(x)\Big[{\rm H}_{1,0}-\frac{3}{4}{\rm H}_0+{\rm H}_2\Big]-2p_{\rm qq}(-x)\Big[\zeta_2+2{\rm H}_{-1,0} \\
& -{\rm H}_{0,0}\Big]-(1-x)\Big[1-\frac{3}{2}{\rm H}_0\Big]-{\rm H}_0-(1+x){\rm H}_{0,0}+\delta(1-x)\Big[\frac{3}{8}-3\zeta_2+6\zeta_3\Big]\Big) \\
P^{(1)-}_{\rm ns}(x) = {} & P^{(1)+}_{\rm ns}(x)+16C_F\Big(C_F-\frac{C_A}{2}\Big)\Big(p_{\rm qq}(-x)\Big[\zeta_2+2{\rm H}_{-1,0}-{\rm H}_{0,0}\Big]-2(1-x) \\
& -(1+x){\rm H}_0\Big)
\end{aligned}
$$

$$
P^{(1)}_{\rm ps}(x) = 4C_Fn_f\Big(\frac{20}{9}\frac{1}{x}-2+6x-4{\rm H}_0+x^2\Big[\frac{8}{3}{\rm H}_0-\frac{56}{9}\Big]+(1+x)\Big[5{\rm H}_0-2{\rm H}_{0,0}\Big]\Big)
$$

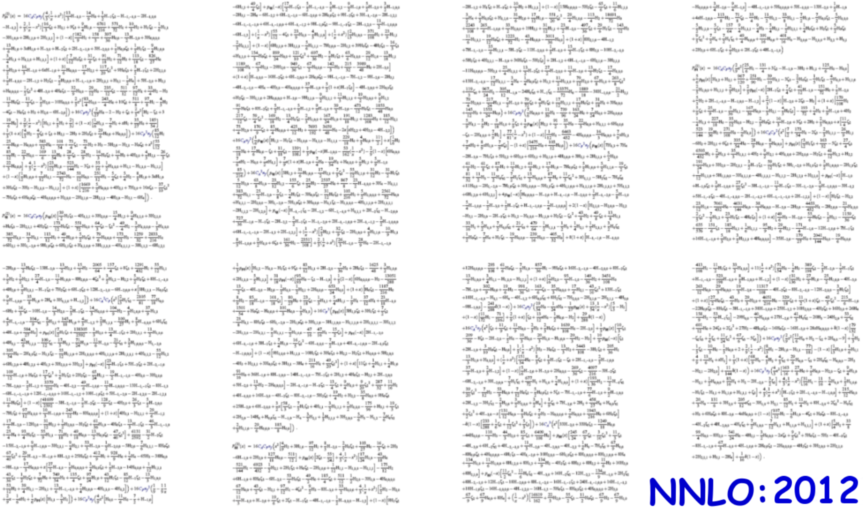
$$
\begin{aligned}
P^{(1)}_{\rm qg}(x) = {} & 4C_An_f\Big(\frac{20}{9}\frac{1}{x}-2+25x-2p_{\rm qg}(-x){\rm H}_{-1,0}-2p_{\rm qg}(x){\rm H}_{1,1}+x^2\Big[\frac{44}{3}{\rm H}_0-\frac{218}{9}\Big] \\
& +4(1-x)\Big[{\rm H}_{0,0}-2{\rm H}_0+x{\rm H}_1\Big]-4\zeta_2x-6{\rm H}_{0,0}+9{\rm H}_0\Big)+4C_Fn_f\Big(2p_{\rm qg}(x)\Big[{\rm H}_{1,0}+{\rm H}_{1,1}+{\rm H}_2 \\
& -\zeta_2\Big]+4x^2\Big[{\rm H}_0+{\rm H}_{0,0}+\frac{5}{2}\Big]+2(1-x)\Big[{\rm H}_0+{\rm H}_{0,0}-2x{\rm H}_1+\frac{29}{4}\Big]-\frac{15}{2}-{\rm H}_{0,0}-\frac{1}{2}{\rm H}_0\Big)
\end{aligned}
$$

$$
\begin{aligned}
P^{(1)}_{\rm gq}(x) = {} & 4C_AC_F\Big(\frac{1}{x}+2p_{\rm gq}(x)\Big[{\rm H}_{1,0}+{\rm H}_{1,1}+{\rm H}_2-\frac{11}{6}{\rm H}_1\Big]-x^2\Big[\frac{8}{3}{\rm H}_0-\frac{44}{9}\Big]+4\zeta_2-2 \\
& -7{\rm H}_0+2{\rm H}_{0,0}-2{\rm H}_1x+(1+x)\Big[2{\rm H}_{0,0}-5{\rm H}_0+\frac{37}{9}\Big]-2p_{\rm gq}(-x){\rm H}_{-1,0}\Big)-4C_Fn_f\Big(\frac{2}{3}x \\
& -p_{\rm gq}(x)\Big[\frac{2}{3}{\rm H}_1-\frac{10}{9}\Big]\Big)+4C_F^{\,2}\Big(p_{\rm gq}(x)\Big[3{\rm H}_1-2{\rm H}_{1,1}\Big]+(1+x)\Big[{\rm H}_{0,0}-\frac{7}{2}+\frac{7}{2}{\rm H}_0\Big]-3{\rm H}_{0,0} \\
& +1-\frac{3}{2}{\rm H}_0+2{\rm H}_1x\Big)
\end{aligned}
$$

$$
\begin{aligned}
P^{(1)}_{\rm gg}(x) = {} & 4C_An_f\Big(1-x-\frac{10}{9}p_{\rm gg}(x)-\frac{13}{9}\Big(\frac{1}{x}-x^2\Big)-\frac{2}{3}(1+x){\rm H}_0-\frac{2}{3}\delta(1-x)\Big)+4C_A^{\,2}\Big(27 \\
& +(1+x)\Big[\frac{11}{3}{\rm H}_0+8{\rm H}_{0,0}-\frac{27}{2}\Big]+2p_{\rm gg}(-x)\Big[{\rm H}_{0,0}-2{\rm H}_{-1,0}-\zeta_2\Big]-\frac{67}{9}\Big(\frac{1}{x}-x^2\Big)-12{\rm H}_0 \\
& -\frac{44}{3}x^2{\rm H}_0+2p_{\rm gg}(x)\Big[\frac{67}{18}-\zeta_2+{\rm H}_{0,0}+2{\rm H}_{1,0}+2{\rm H}_2\Big]+\delta(1-x)\Big[\frac{8}{3}+3\zeta_3\Big]\Big)+4C_Fn_f\Big(2{\rm H}_0 \\
& +\frac{2}{3}\frac{1}{x}+\frac{10}{3}x^2-12+(1+x)\Big[4-5{\rm H}_0-2{\rm H}_{0,0}\Big]
\end{aligned}
$$

NLO: 1980

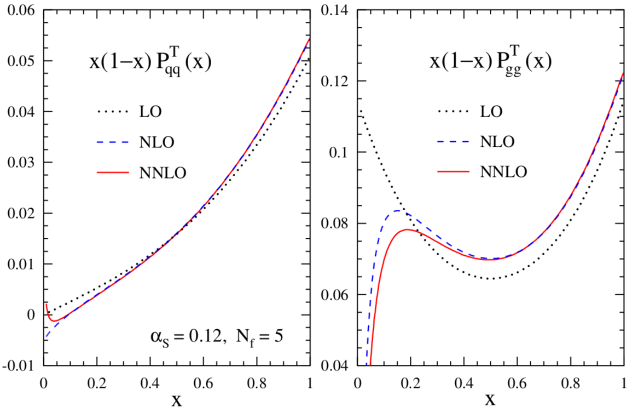
Splitting functions: NNLO



NNLO: 2012

Properties of splitting functions

1. At LO [Sov. J. Nucl. Phys. 15 (1973) 438; NPB 126 (1977) 298; NPB 136 (1978) 445]
   time-like and space-like splitting functions are equal, provided $P_{qg}^{S,(0)} \leftrightarrow P_{gq}^{T,(0)}$
2. At NLO [NPB 175 (1980) 27, PLB 97 (1980) 497, PRD 48 (1993) 116]
   time-like and space-like splitting functions are related by analytic continuation
3. at NNLO [PLB 638 (2006) 61, PLB 659 (2008) 290, NPB 854 (2012) 133]
   an uncertainty still remains on the exact form of $P_{\rm qg}^{(2)}$ (it does not affect its log behavior)



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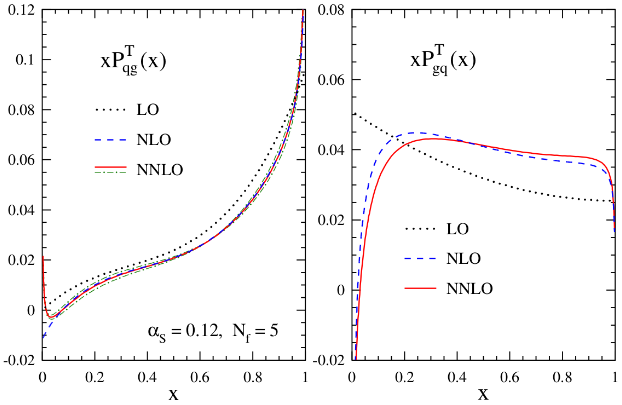


Properties of splitting functions

Must be careful with fixed-order splitting functions as $z \to 0$ ($m = 1, \ldots, 2k+1$)

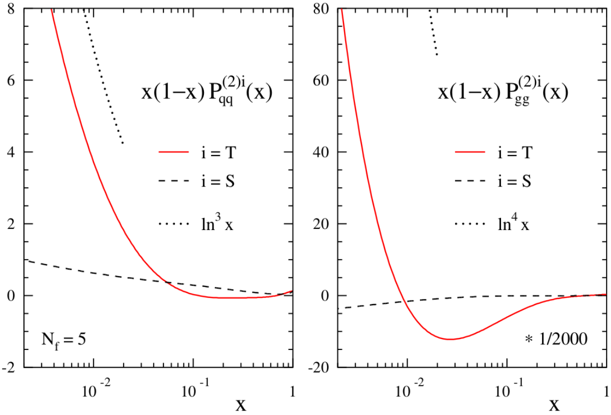
SPACE-LIKE CASE

$$P_{ji} \propto \frac{a_s^{k+1}}{x} \log^{k+1-m} \frac{1}{x}$$

TIME-LIKE CASE

$$P_{ji} \propto \frac{a_s^{k+1}}{z} \log^{2(k+1)-m-1} z$$

Soft gluon logarithms diverge more rapidly in the TL case than in the SL case: as z decreases, the unresummed SGLs spoil the convergence of the FO series for $P(z, a_s)$ if $\log \frac{1}{z} \geq \mathcal{O}\left(a_s^{-1/2}\right)$



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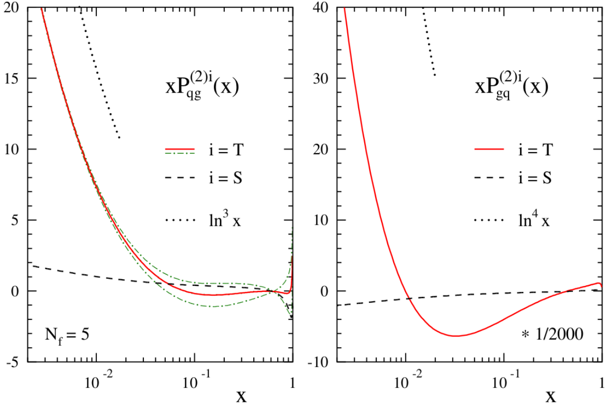


Theoretical constraints

1. Momentum sum rule

$$\sum_h \int_0^1 dz z\, D_i^h(z,\mu^2) = 1 \;\forall \text{ parton } i$$

2. Charge sum rule

$$\sum_h \int_0^1 dz\, e_h\, D_i^h(z,\mu^2) = e_i \;\forall \text{ parton } i$$

where $e_{h(i)}$ is the electric charge of the hadron h (parton i)

3. Charge conjugation symmetry

$$D_{q\,(\bar{q})}^{h^+} = D_{\bar{q}\,(q)}^{h^-} \qquad D_g^{h^+} = D_g^{h^-} \;\forall \text{ hadron species } h^\pm$$

4. Isospin symmetry of the strong interaction

$$D_u^{\pi^+} = D_d^{\pi^-} \qquad D_d^{\pi^+} = D_u^{\pi^-}$$

approximate, as $m_u \sim m_d$, but no phenomenological evidences of violation

5. Positivity of cross sections
implies that FFs should be positive-definite at LO

Determining FFs from data: a global QCD analysis

Determine the probability density $\mathcal{P}[D]$ in the space of FFs $[D]$

$$\mathcal{P}(D|data) = \mathcal{L}(data|D)\pi(D)$$

Project the infinite-dim. space of FFs $[D]$ onto the finite-dim. space of parameters $\{\mathbf{a}\}$

$$\mathcal{P}(D|data) = \mathcal{L}(data|D)\pi(D) \longleftrightarrow \mathcal{P}(\mathbf{a}|data) = \mathcal{L}(data|\mathbf{a})\pi(\mathbf{a})$$

$$\mathcal{L}(data|\mathbf{a}) = \exp\left[-\frac{1}{2}\chi^2\right]$$ Gaussian likelihood

$$\pi(\mathbf{a}) = \prod_i \theta(a_i - a_i^{\min})\theta(a_i^{\max} - a_i)$$ uninformative flat prior

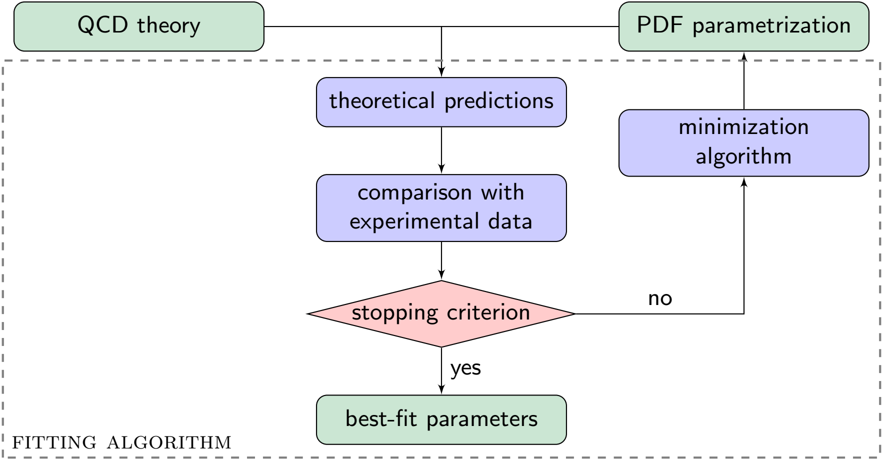
For any observable $\mathcal{O}$ depending on a set of FFs $[D]$

its expectation value and uncertainty are functional integrals over the space of FFs

$$E[\mathcal{O}] = \langle \mathcal{O}[D] \rangle = \int d^n a \mathcal{P}(\mathbf{a}|data)\mathcal{O}[\mathbf{a}]$$ expectation value

$$V[\mathcal{O}] = \sigma^2_{\mathcal{O}}[D] = \int d^n a \mathcal{P}(\mathbf{a}|data)\left(\mathcal{O}[\mathbf{a}] - E[\mathcal{O}]\right)^2$$ variance

A global FF determination: the underlying strategy



Assume a reasonable PDF parametrisation

Obtain theoretical predictions for various processes and compare predictions to data

Determine the best-fit parameters via minimization (maximum likelihood) of a proper figure of merit (usually the log-likelihood χ^2)

Maximum likelihood: Hessian method

1. Expand the χ^2 about its global minimum at first (nontrivial) order

$$\chi^2\{\mathbf{a}\} \approx \chi^2\{\mathbf{a_0}\} + \delta a^i H_{ij} \delta a^j, \qquad H_{ij} = \frac{1}{2} \left. \frac{\partial^2 \chi^2\{\mathbf{a}\}}{\partial a_i \partial a_j} \right|_{\{\mathbf{a}\}=\{\mathbf{a_0}\}}$$

2. Assume linear error propagation for any observable $\mathcal{O}$ depending on $\{\mathbf{a}\}$

$$\langle \mathcal{O}\{\mathbf{a}\} \rangle \approx \mathcal{O}\{\mathbf{a_0}\} + a_i \left. \frac{\partial \mathcal{O}\{\mathbf{a}\}}{\partial a_i} \right|_{\{\mathbf{a}\}=\{\mathbf{a_0}\}} \qquad \sigma_{\mathcal{O}\{\mathbf{a}\}} \approx \sigma_{ij} \frac{\partial \mathcal{O}\{\mathbf{a}\}}{\partial a_i} \left. \frac{\partial \mathcal{O}\{\mathbf{a}\}}{\partial a_j} \right|_{\{\mathbf{a}\}=\{\mathbf{a_0}\}}$$

3. Determine σ_{ij} from H_{ij} from maximum likelihood (under Gaussian hypothesis)

$$\sigma_{ij}^{-1} = \left. \frac{\partial^2 \chi^2\{\mathbf{a}\}}{\partial a_i \partial a_j} \right|_{\{\mathbf{a}\}=\{\mathbf{a_0}\}} = H_{ij}$$

4. A C.L. about the best fit is obtained as the volume (in parameter space) about $\chi^2\{\mathbf{a_0}\}$ that corresponds to a fixed increase of the χ^2; for Gaussian uncertainties:

$$68\%\ \text{C.L.} \Longleftrightarrow \Delta\chi^2 = \chi^2\{\mathbf{a}\} - \chi^2\{\mathbf{a_0}\} = 1$$

Maximum likelihood: Hessian method

1. Parameters can always be adjusted so that all eigenvalues of H_{ij} are equal to one (diagonalise H_{ij} and rescale the eigenvectors by their eigenvalues)

$$\delta a_i H_{ij} \delta a_j = \sum_{i=1}^{N_{\rm par}} \left[a_i'(a_i)\right]^2 \Longleftrightarrow \sigma_{\mathcal{O}\{\mathbf{a}'\}} = \left|\nabla' \mathcal{O}\{\mathbf{a}'\}\right|$$

2. Compact representation and computation of observables and their uncertainties

$$\langle \mathcal{O}[D(x,Q^2)]\rangle = \mathcal{O}[D_0(x,Q^2)]$$

$$\sigma_{\mathcal{O}}[D(x,Q^2)] = \left[\sum_{i=1}^{N_{\rm par}} \left(\mathcal{O}[D_i(x,Q^2)] - \mathcal{O}[D_0(x,Q^2)]\right)^2\right]^{1/2}$$

3. Uncertainties obtained with $\Delta\chi^2 = 1$ might be unrealistically small (inadequacy of the linear approximation)
4. Rescale to the $\Delta\chi^2 = T$ interval such that correct C.L.s are reproduced (no statistically rigorous interpretation of T (tolerance)
5. Unmanageable Hessian matrix if the numer of parameters is huge

Maximum likelihood: Monte Carlo method

1. Generate (art) replicas of (exp) data according to the distribution

$$\mathcal{O}_i^{(art)(k)} = \mathcal{O}_i^{(exp)} + r_i^{(k)}\sigma_{\mathcal{O}_i}\,, \qquad i = 1, \ldots N_{\rm dat}\,, \qquad k = 1, \ldots, N_{\rm rep}$$

   where $r_i^{(k)}$ are (Gaussianly distributed) random numbers for each k-th replica ($r_i^{(k)}$ can be generated with any distribution, not neccesarily Gaussian)

2. Validate the Monte Carlo sample size against experimental data
3. Perform a fit for each replica $k = 1, \ldots, N_{\rm rep}$
4. Compact computation of observables and their uncertainties (PDF replicas are equally probable members of a statistical ensemble)

$$\langle \mathcal{O}[D(x,Q^2)] \rangle = \frac{1}{N_{\text{rep}}} \sum_{k=1}^{N_{\text{rep}}} \mathcal{O}[D^{(k)}(x,Q^2)]$$

$$\sigma_{\mathcal{O}}[D(x,Q^2)] = \left[\frac{1}{N_{\text{rep}}-1} \sum_{k=1}^{N_{\text{rep}}} \left(\mathcal{O}[D^{(k)}(x,Q^2)] - \langle \mathcal{O}[D(x,Q^2)] \rangle \right)^2 \right]^{1/2}$$

$\Rightarrow$ **no need to rely on linear approximation**

$\Rightarrow$ **computational expensive: need to perform $N_{\rm rep}$ fits instead of one**

Maximum likelihood: parameter optimisation

Optimisation usually performed by means of simple gradient descent:
compute and minimise the gradient of the fit quality with respect to the fit parameters

$$\frac{\partial\chi^2}{\partial a_i}\,, \qquad \text{for } i=1,\dots,N_{\rm par} \qquad \chi^2=\sum_{i,j}^{\rm N_{dat}}\left(T_i[\{\mathbf{a}\}]-D_i\right)\left({\rm cov}^{-1}\right)_{ij}\left(T_j[\{\mathbf{a}\}]-D_j\right)$$

$$({\rm cov})_{ij}=\delta_{ij}s_i^2+\left(\sum_{\alpha}^{N_c}\sigma_{i,\alpha}^{(c)}\sigma_{j,\alpha}^{(c)}+\sum_{\alpha}^{N_{\mathcal{L}}}\sigma_{i,\alpha}^{(\mathcal{L})}\sigma_{j,\alpha}^{(\mathcal{L})}\right)D_iD_j$$

Optimisation should minimise the noise in the χ^2 driven by noisy experimental data

Additional complications in case of a redundant parametrisation (huge parameter space)

1. need to explore the parameter space as uniformly as possible
   (in order to avoid stopping the fit in a local minimum) $\longrightarrow$ genetic algorithhms
2. need for a computationally efficient minimisation
   (non-trivial relationship between FFs and observables) $\longrightarrow$ adaptive algorithms
3. need to define a criterion for minimisation stopping
   (avoid learning statistical fluctuations of the data) $\longrightarrow$ partition and cross-validation

Covariance Matrix Adaption Evolution Strategy [N. Hansen, Springer (2016)]

Nested sampling [Astrophys.J. 638 (2006) L51; Mon.Not.Roy.Astron.Soc. 398 (2009) 1601]

Basic idea: compute

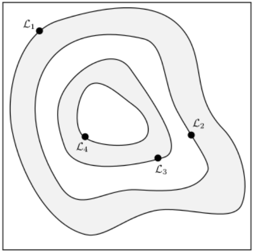
$$Z = \int \mathcal{L}(data|\mathbf{a})\pi(\mathbf{a})d^n a = \int_0^1 \mathcal{L}(X)dX$$

The algorithm traverses ordered isolikelihood contours in the variable X such that X follows the progression $X_i = t_i X_{i-1}$

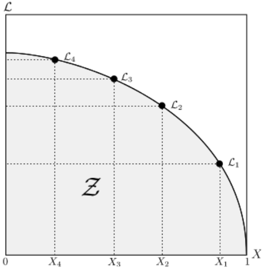
The variable t_i is estimated statistically

The algorithm can be optimised iteration by iteration
One can sample only in the regions where the likelihood is larger (*importance sampling*)

$\mathcal{L}(data|\mathbf{a})$ in $\mathbf{a}$ space

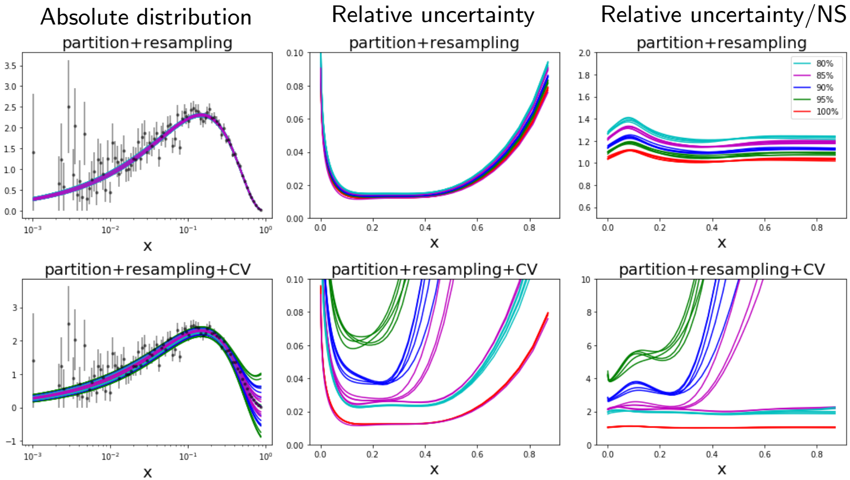


$\mathcal{L}(X)$ in X space



Comparing maximum likelihood and sampling methods

Toy example: $data(x) = N x^{\alpha}(1-x)^{\beta}$



Results are independent from the parametrisation and from the shape of the data
A quantitative explanation of these results require a proper Bayesian formulation

Available FF sets (status 2017)

		DHESS	HKNS	JAM	NNFF
DATA	SIA	☑	☑	☑	☑
	SIDIS	☑	☒	☒	☒
	PP	☑	☒	☒	☒
METH.	statistical treatment	Iterative Hessian 68% - 90%	Hessian $\Delta\chi^2 = 15.94$	Iterative Monte Carlo	Monte Carlo
	parametrisation	standard	standard	standard	neural network
THEORY	pert. order	(N)NLO	NLO	NLO	LO, NLO, NNLO
	HF scheme	ZM(GM)-VFN	ZM-VFN	ZM-VFN	ZM-VFN
	hadron species	$\pi^\pm$, $K^\pm$, $p/\bar{p}$, $h^\pm$	$\pi^\pm$, $K^\pm$, $p/\bar{p}$	$\pi^\pm$, $K^\pm$	$\pi^\pm$, $K^\pm$, $p/\bar{p}$
	latest update	PRD 91 (2015) 014035 PRD 95 (2017) 094019	PTEP 2016 (2016) 113B04	PRD 94 (2016) 114004	EPJ C77 (2017) 516

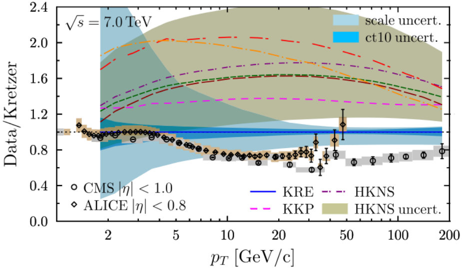
$+$ some others (including analyses for specific hadrons)

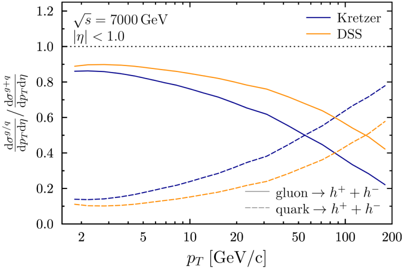
BKK95 [ZPB 65 (1995) 471]	$\pi^\pm, K^\pm$	AESS11 [PRD 83 (2011) 034002]	η
BKK96 [PRD 53 (1996) 3553]	K^0	SKMNA13 [PRD 88 (2013) 054019]	$\pi^\pm, K^\pm$
DSV97 [PRD 57 (1998) 5811]	Λ^0	LSS15 [PRD 96 (2016) 074026]	SIDIS only
BFGW00 [EPJ C19 (2001) 89]	$h^\pm$		

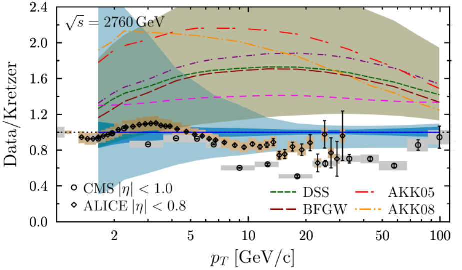
Focus on π and K which constitute the largest fraction in measured yields

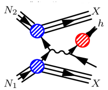
Fragmentation functions: why should we bother?

Example 1: Ratio of the inclusive charged-hadron spectra measured by CMS and ALICE









$$E\frac{d^3\sigma}{dp_T^3} = \sum_{a,b,c} f_a \otimes f_b \otimes \hat{\sigma}_{ab}^c \otimes D_c^h$$

Predictions from all available FF sets are not compatible with CMS and ALICE data, not even within scale and PDF/FF uncertainties $\longrightarrow$ How well do we know the gluon FF?

Figures taken from [NPB 883 (2014) 615]

Fragmentation functions: why should we bother?

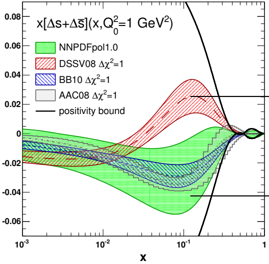
Example 2: The strange polarised parton distribution at $Q^2 = 2.5$ GeV^2 ($\Delta s = \Delta\bar{s}$)

NNPDFpol1.0 [NPB 874 (2013) 36]
$\int_0^1 dx[\Delta s + \Delta\bar{s}] = -0.13 \pm 0.09$

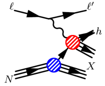
JAM17 [PRL 119 (2017) 132001]
$\int_0^1 dx[\Delta s + \Delta\bar{s}] = -0.03 \pm 0.10$

First moment constrained by
$a_3 = \int_0^1 dx[\Delta u^+ - \Delta d^+]$
$= 1.2701 \pm 0.0025$

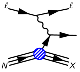
$a_8 = \int_0^1 dx[\Delta u^+ + \Delta d^+ - 2\Delta s^+]$
$= 0.585 \pm 0.176$



directly from SIDIS Kaon data



indirectly from DIS + SU(3)



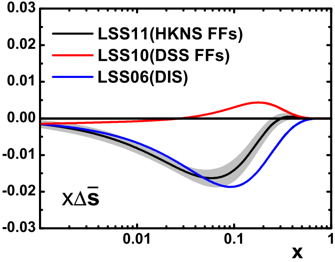


Figure taken from [PRD D84 (2011) 014002]

$$\frac{d\sigma}{dxdy} = \frac{2\pi\alpha_{\rm em}^2}{Q^2}\left[(2-y)g_1\right]$$

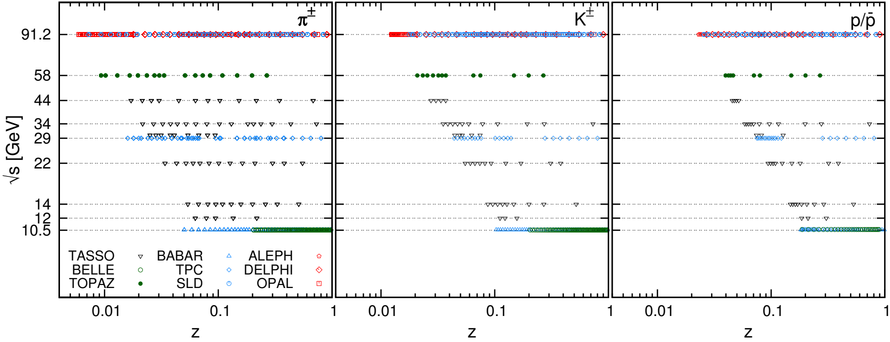
$$\frac{d\sigma^h}{dxdydz} = \frac{2\pi\alpha_{\rm em}^2}{Q^2}\left[\frac{1+(1-y)^2}{y}2g_1^h + \frac{2(1-y)}{y}g_L^h\right]$$

If SIDIS data is used to determine Δs, $K^{\pm}$ FFs for different sets lead to different results Such results may differ significantly among them and w.r.t. the results obtained from DIS
$\longrightarrow$ How well do we know kaon FFs?

2. The NNFF1.0 analysis

[EPJ C77 (2017) 516]

The dataset



CERN-LEP: ALEPH [ZP C66 (1995) 353] DELPHI [EPJ C18 (2000) 203] OPAL [ZP C63 (1994) 181]

KEK: BELLE ($n_f = 4$) [PRL 111 (2013) 062002] TOPAZ [PL B345 (1995) 335]

DESY-PETRA: TASSO [PL B94 (1980) 444, ZP C17 (1983) 5, ZP C42 (1989) 189]

SLAC: BABAR ($n_f = 4$) [PR D88 (2013) 032011] SLD [PR D58 (1999) 052001] TPC [PRL 61 (1988) 1263]

$$\frac{d\sigma^h}{dz} = \frac{4\pi\alpha^2(Q^2)}{Q^2}\mathcal{F}_2^h(z,Q^2) \quad h = \pi^+ + \pi^-,\ K^+ + K^-,\ p + \bar{p} \quad \text{possibly normalised to } \sigma_{\rm tot}$$

$N_{\rm dat}^{\pi^\pm} = 428$ $\quad N_{\rm dat}^{K^\pm} = 385$ $\quad N_{\rm dat}^{p/\bar{p}} = 360$

From observables to fragmentation functions

$$\boxed{\mathcal{F}_2^h = \langle e^2 \rangle \left\{ C_{2,q}^{\rm S} \otimes D_\Sigma^h + n_f \mathcal{C}_{2,g}^{\rm S} \otimes D_g^h + \mathcal{C}_{2,q}^{\rm NS} \otimes D_{\rm NS}^h \right\}}$$

$$\langle e^2 \rangle = \frac{1}{n_f}\sum_{q=1}^{n_f} \hat{e}_q^2 \qquad D_\Sigma^h = \sum_{q=1}^{n_f} D_{q^+}^h \qquad D_{\rm NS}^h = \sum_{q=1}^{n_f}\left(\frac{\hat{e}_q^2}{\langle e^2 \rangle} - 1\right) D_{q^+}^h \qquad D_{q^+}^h = D_q^h + D_{\bar{q}}^h$$

Coefficient functions and splitting functions known up to NNLO

[NPB 751 (2006) 18; NPB 749 (2006) 1; PLB 638 (2006) 61; NPB 845 (2012) 133]

$$\begin{aligned} F_2^{h,n_f=5} =& \frac{1}{5}\left[\left(2\hat{e}_u^2 + 3\hat{e}_d^2\right) C_{2,q}^{\rm S} + 3\left(\hat{e}_u^2 - \hat{e}_d^2\right) C_{2,q}^{\rm NS}\right] \otimes \left(D_{u^+}^h + D_{c^+}^h\right) \\ &+ \frac{1}{5}\left[\left(2\hat{e}_u^2 + 3\hat{e}_d^2\right) C_{2,q}^{\rm S} - 2\left(\hat{e}_u^2 - \hat{e}_d^2\right) C_{2,q}^{\rm NS}\right] \otimes \left(D_{d^+}^h + D_{s^+}^h + D_{b^+}^h\right) \\ &+ \left(2\hat{e}_u^2 + 3\hat{e}_d^2\right) C_{2,g}^{\rm S} \otimes D_g^h \end{aligned}$$

No sensitivity to individual quark and antiquark FFs

Limited sensitivity to flavour separation via the variation of $\hat{e}_q$ with Q^2

$\hat{e}_u^2/\hat{e}_d^2(Q^2 = 10\,\mathrm{GeV}) \sim 4 \Rightarrow D_{u^+}^h,\ D_{d^+}^h + D_{s^+}^h;\quad \hat{e}_u^2/\hat{e}_d^2(Q^2 = M_Z) \sim 0.8 \Rightarrow D_\Sigma^h$

Flavor separation between uds and c, b quarks achieved thanks to tagged data

Direct sensitivity to D_g^h only beyond LO, as $C_{2,g}^{\rm S}$ is $\mathcal{O}(\alpha_s^2)$, and tenous

Indirect sensitivity to D_g^h via scale violations in the time-like DGLAP evolution

The NNPDF methodology: parametrisation

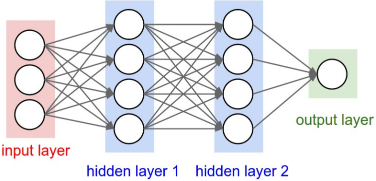
1. Neural network (NN), *i.e.* a generator of random functions in the space of FFs

$$\boxed{zD_i^h(z,Q_0^2) = \mathscr{F}_i^h(z,\{\mathbf{c}\})}$$

$\mathscr{F}_i^h(z,\{\mathbf{c}\})$ is a feed-forward neural network in terms of a huge set of parameters ($\mathcal{O}(200)$ per PDF set)

$$\{\mathbf{c}\} = \{\omega_{ij}^{(L-1),D_i^h}, \theta_i^{(L),D_i^h}\}$$

2. What a feed-forward NN exactly is?



$$\xi_i^{(l)} = g\left(\sum_j^{n_l-1} \omega_{ij}^{(l-1)} \xi_j^{(l-1)} - \theta_i^{(l)}\right)$$

$$g(y) = \frac{1}{1+e^{-y}}$$

- made of neurons grouped into layers (define the architecture)
- each neuron receives input from neurons in the preceding layer (feed-forward NN)
- activation $\xi_i^{(l)}$ determined by a set of parameters (weights and thresholds)
- activation determined according to a non-linear function (except the last layer)

⇒ **potentially non-smooth**

⇒ **bias due to the parametrisation reduced as much as possible**

Fit settings

Physical parameters: consistent with the NNPDF3.1 PDF set [EPJ C77 (2017) 663]

$\alpha_s(M_Z) = 0.118$, $\alpha(M_Z) = 1/127$, $m_c = 1.51$ GeV, $m_b = 4.92$ GeV

Solution of DGLAP equations: numerical solution in z-space as implemented in APFEL

extensive benchmark performed up to NNLO [JHEP 1503 (2015) 046]

Parametrisation: each FF is parametrised with a feed-forward neural network (2-5-3-1)

$$D_i^h(Q_0, z) = \mathrm{NN}(x) - \mathrm{NN}(1), \quad Q_0 = 5 \text{ GeV}$$

$$h = \pi^+ + \pi^-, \; h = K^+ + K^-, \; h = p + \bar{p} \qquad i = u^+, d^+ + s^+, c^+, b^+, g$$

we assume charge conjugation, from which $D_{q^+}^{\pi^+} = D_{q^+}^{\pi^-}$

initial scale above m_b, but below the lowest c.m. energy of the data, avoid threshold crossing

Heavy flavours: heavy-quark FFs are parametrised independently at the initial scale Q_0

Hadron mass corrections: included exactly à la Albino-Kniehl-Kramer [NPB 803 (2008) 42]

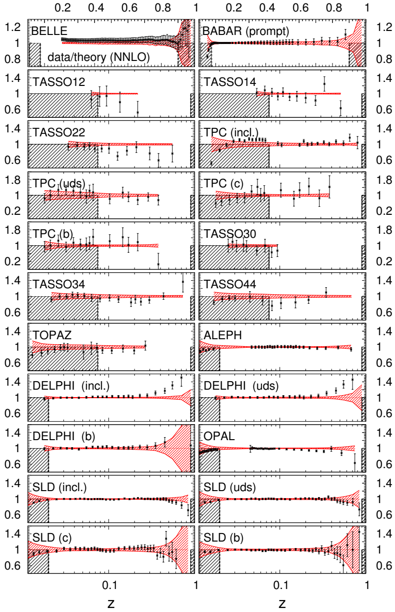
Kinematic cuts: $z \to 0$: contributions $\propto \ln z$; $z \to 1$: contributions $\propto \ln(1-z)$

$$z_{\min} = 0.075, \; z_{\min} = 0.02 \; (\sqrt{s} = M_Z); \; z_{\max} = 0.90$$

Momentum sum rule: check a posteriori that

$$\sum_{h=\pi^\pm, K^\pm, p/\bar{p}} \int_{z_{\min}}^1 dz\, z D_i^h(z, Q) < N \qquad N \begin{cases} = 1 & \text{for } i = g \\ = 2 & \text{for } i = u^+, c^+, b^+ \\ = 4 & \text{for } i = d^+ + s^+ \end{cases}$$

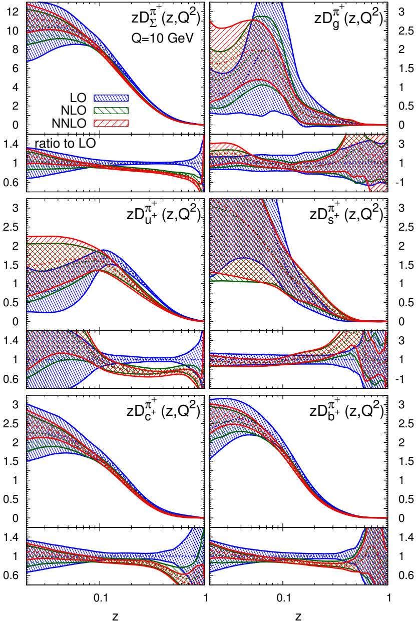
Fit quality: π^+



Exp.	$N_{\rm dat}$	NNLO theory $\chi^2/N_{\rm dat}$	remarks
BELLE	70	0.09	lack of correlations
BABAR	40	0.78	☑
TASSO12	4	0.87	small sample
TASSO14	9	1.70	} data fluctuations
TASSO22	8	1.91	
TPC	13	0.85	☑
TPC-UDS	6	0.49	☑
TPC-C	6	0.52	☑
TPC-B	6	1.43	☑
TASSO34	9	1.00	☑
TASSO44	6	2.34	data fluctuations
TOPAZ	5	0.80	☑
ALEPH	23	0.78	☑
DELPHI	21	1.86	tension with OPAL
DELPHI-UDS	21	1.54	tension with OPAL
DELPHI-B	21	0.95	☑
OPAL	24	1.84	tension with DELPHI
SLD	34	0.83	☑
SLD-UDS	34	0.52	☑
SLD-C	34	1.06	☑
SLD-B	34	0.36	☑
TOTAL	428	0.87	☑

Overall good description of the dataset
Signs of tension OPAL vs DELPHI (inclusive)
Anomalously small $\chi^2/N_{\rm dat}$ for BELLE

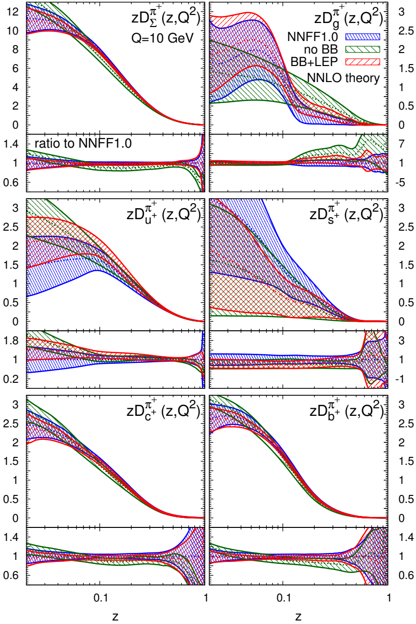
Dependence upon perturbative order: π^+



Exp.	$N_{\rm dat}$	LO $\chi^2/N_{\rm dat}$	NLO $\chi^2/N_{\rm dat}$	NNLO $\chi^2/N_{\rm dat}$
BELLE	70	0.60	0.11	0.09
BABAR	40	1.91	1.77	0.78
TASSO12	4	0.70	0.85	0.87
TASSO14	9	1.55	1.67	1.70
TASSO22	8	1.64	1.91	1.91
TPC	13	0.46	0.65	0.85
TPC-UDS	6	0.78	0.55	0.49
TPC-C	6	0.55	0.53	0.52
TPC-B	6	1.44	1.43	1.43
TASSO34	9	1.16	0.98	1.00
TASSO44	6	2.01	2.24	2.34
TOPAZ	5	1.04	0.82	0.80
ALEPH	23	1.68	0.90	0.78
DELPHI	21	1.44	1.79	1.86
DELPHI-UDS	21	1.30	1.48	1.54
DELPHI-B	21	1.21	0.99	0.95
OPAL	24	2.29	1.88	1.84
SLD	34	2.33	1.14	0.83
SLD-UDS	34	0.95	0.65	0.52
SLD-C	34	3.33	1.33	1.06
SLD-B	34	0.45	0.38	0.36
TOTAL	428	1.44	1.02	0.87

Excellent perturbative convergence
FFs almost stable from NLO to NNLO
LO FF uncertainties larger than HO

Dependence upon the dataset: π^+

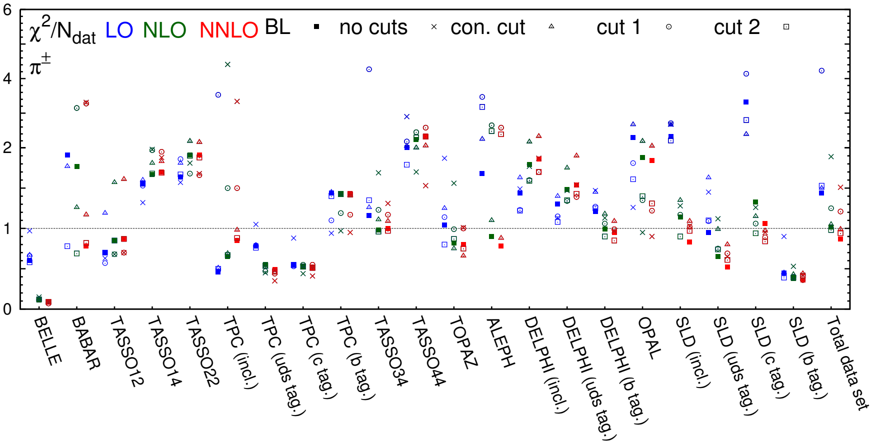


NNLO theory Exp.	$N_{\rm dat}$	NNFF1.0 $\chi^2/N_{\rm dat}$	no BB $\chi^2/N_{\rm dat}$	BB+LEP $\chi^2/N_{\rm dat}$
BELLE	70	0.09	[4.92]	0.09
BABAR	40	0.78	[144]	0.88
TASSO12	4	0.87	0.52	[0.87]
TASSO14	9	1.70	1.38	[1.71]
TASSO22	8	1.91	1.29	[2.15]
TPC	13	0.85	2.12	[2.15]
TPC-UDS	6	0.49	0.54	[0.77]
TPC-C	6	0.52	0.74	[0.58]
TPC-B	6	1.43	1.60	[1.48]
TASSO34	9	1.00	1.17	[1.38]
TASSO44	6	2.34	2.52	[1.97]
TOPAZ	5	0.80	0.92	[1.72]
ALEPH	23	0.78	0.57	0.74
DELPHI	21	1.86	1.97	1.82
DELPHI-UDS	21	1.54	1.56	1.42
DELPHI-B	21	0.95	1.01	0.95
OPAL	24	1.84	1.75	1.92
SLD	34	0.83	0.87	0.95
SLD-UDS	34	0.52	0.53	0.63
SLD-C	34	1.06	0.69	0.96
SLD-B	34	0.36	0.49	0.37
TOTAL		0.87	1.06	0.82

no BB: larger uncertainties; different gluon shape and different light flavour separation
BB+LEP: comparable uncertainties; slightly different size of gluon and light flavoured quarks

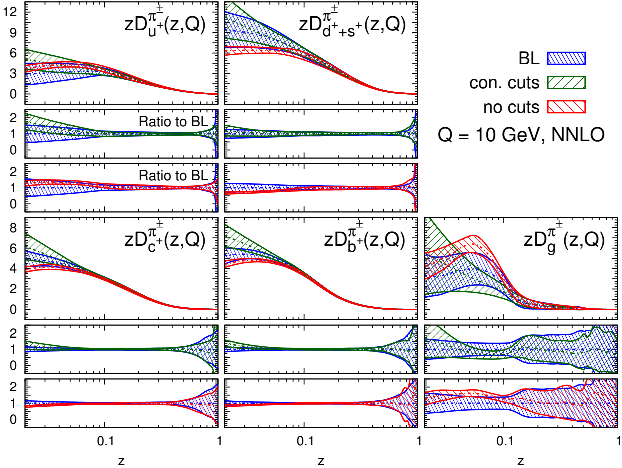
Dependence upon kinematic cuts: π^+

BL		no cuts		con. cut		cut1		cut2	
$z_{\min}^{(m_Z)}$	$z_{\min}$	$z_{\min}^{(m_Z)}$	$z_{\min}$	$z_{\min}^{(m_Z)}$	$z_{\min}$	$z_{\min}^{(m_Z)}$	$z_{\min}$	$z_{\min}^{(m_Z)}$	$z_{\min}$
0.02	0.075	0.00	0.00	0.05	0.10	0.01	0.05	0.01	0.075



Dependence upon kinematic cuts: π^+

BL		no cuts		con. cut		cut1		cut2	
$z_{\rm min}^{(m_Z)}$	$z_{\rm min}$	$z_{\rm min}^{(m_Z)}$	$z_{\rm min}$	$z_{\rm min}^{(m_Z)}$	$z_{\rm min}$	$z_{\rm min}^{(m_Z)}$	$z_{\rm min}$	$z_{\rm min}^{(m_Z)}$	$z_{\rm min}$
0.02	0.075	0.00	0.00	0.05	0.10	0.01	0.05	0.01	0.075

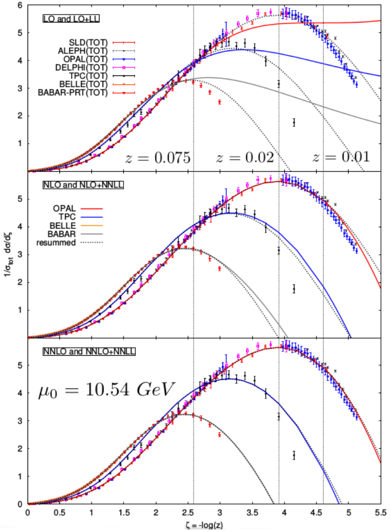


Small-z resummed Fragmentation Functions [PRD 95 (2017) 054003]

436 Total data Points:

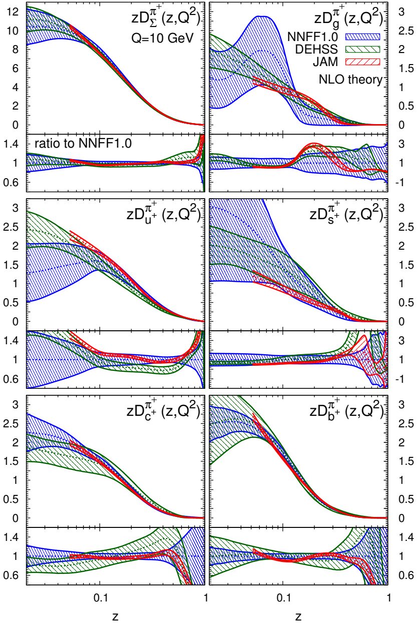
- LEP cut (z = 0.01) due to inconsistency between OPAL and ALEPH
- TPC lower cut (z = 0.02) based on difference of energy fraction $z = 2\,E_h/Q$ and three momentum fraction $x_p = z - 2m_h^2/(zQ^2) + \mathcal{O}(1/Q^4)$ in c.m.s being less than at least 15%

accuracy	χ^2	χ^2/dof
LO	1260.78	2.89
NLO	354.10	0.81
NNLO	330.08	0.76
LO+LL	405.54	0.93
NLO+NNLL	352.28	0.81
NNLO+NNLL	329.96	0.76



Slide: courtesy of D. P. Anderle

Comparison with other FF determinations: π^+



DEHSS [PRD 91 (2015) 014035]
(+SIDIS +PP)

JAM [PRD 94 (2016) 114004]
(almost same dataset as NNFF1.0)

different cuts at small z

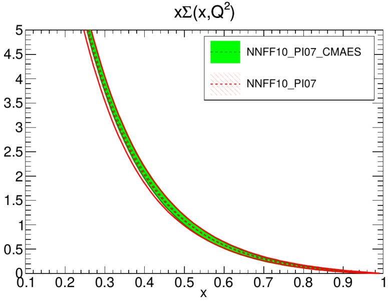
$D_\Sigma^{\pi^+}$: excellent mutual agreement
both c.v. and unc. (bulk of the dataset)

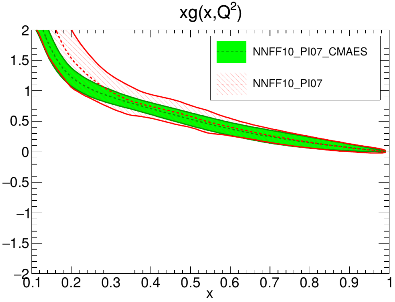
$D_g^{\pi^+}$: slight disagreement
different shapes, larger uncertainties
DEHSS: data; JAM: parametrisation

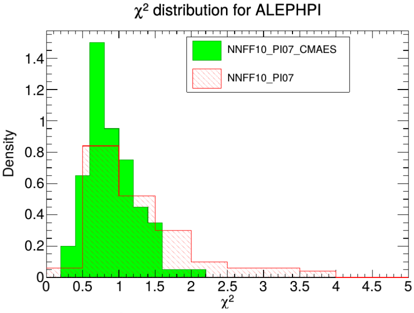
$D_{u^+}^{\pi^+}$, $D_{s^+}^{\pi^+}$: good overall agreement
excellent with JAM, though larger uncertainties
slightly different shape w.r.t. DHESS (dataset)

$D_{c^+}^{\pi^+}$, $D_{b^+}^{\pi^+}$: good overall agreement
excellent with JAM, same uncertainties
slightly different shape w.r.t. DHESS (dataset)

Dependence upon the minimisation algorithm





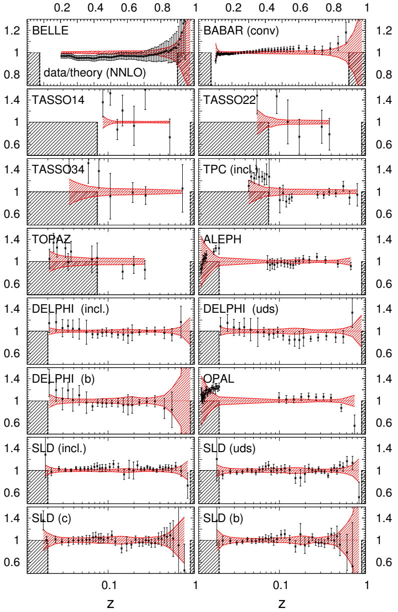


Singlet and Gluon FFs at $Q = 5$ GeV

	NNFF10_PI07_CMAES	NNFF10_PI07
$\chi^2_{\rm tot}/N_{\rm dat}$	0.93	0.92
$\langle E_{\rm tr}\rangle \pm \sigma_{\rm tr}$	1.97±0.28	1.97±0.71
$\langle E_{\rm val}\rangle \pm \sigma_{\rm val}$	2.16±0.37	2.91±1.72
$\langle {\rm TL}\rangle \pm \sigma_{\rm TL}$	3065±1673	5560±9394

Fit quality: K^+

Exp.	$N_{\rm dat}$	NNLO theory $\chi^2/N_{\rm dat}$	remarks
BELLE	70	0.32	lack of correlations
BABAR	43	0.95	☑
TASSO12	3	1.02	small sample
TASSO14	9	2.07	small sample
TASSO22	6	2.62	small sample
TPC	13	1.01	☑
TASSO34	5	0.36	small sample
TOPAZ	3	0.99	small sample
ALEPH	18	0.56	☑
DELPHI	22	0.34	☑
DELPHI-UDS	22	1.32	☑
DELPHI-B	22	0.52	☑
OPAL	10	1.66	tension with other M_Z data
SLD	35	0.57	☑
SLD-UDS	35	0.93	☑
SLD-C	34	0.38	☑
SLD-B	35	0.62	☑
TOTAL	385	0.73	☑



Overall good description of the dataset

Excellent BELLE/BABAR consistency

Signs of tension OPAL vs DELPHI (inclusive)

Anomalously small $\chi^2/N_{\rm dat}$ for BELLE

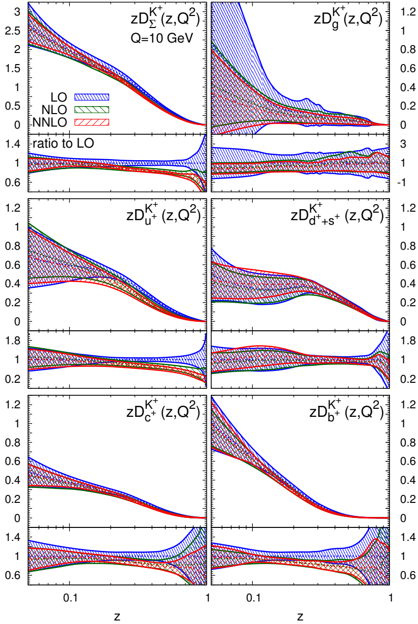
Dependence upon the data set and kin cuts similar to pions

Dependence upon perturbative order: K^+

Exp.	$N_{\rm dat}$	LO $\chi^2/N_{\rm dat}$	NLO $\chi^2/N_{\rm dat}$	NNLO $\chi^2/N_{\rm dat}$
BELLE	70	0.21	0.32	0.32
BABAR	43	2.86	1.11	0.95
TASSO12	3	1.10	1.03	1.02
TASSO14	9	2.17	2.13	2.07
TASSO22	6	2.14	2.77	2.62
TPC	13	0.94	1.09	1.01
TASSO34	5	0.27	0.44	0.36
TOPAZ	3	0.61	1.19	0.99
ALEPH	18	0.47	0.55	0.56
DELPHI	22	0.28	0.33	0.34
DELPHI-UDS	22	1.38	1.49	1.32
DELPHI-B	22	0.58	0.49	0.52
OPAL	10	1.67	1.57	1.66
SLD	35	0.86	0.62	0.57
SLD-UDS	35	1.31	1.02	0.93
SLD-C	34	0.92	0.47	0.38
SLD-B	35	0.59	0.67	0.62
TOTAL	385	1.02	0.78	0.73

Excellent perturbative convergence
FFs almost stable from NLO to NNLO
LO FF uncertainties larger than HO

i	N^{i+1}LO/N^{i}LO	D_g	D_Σ	D_{c^+}	D_{b^+}
0	NLO/LO [%]	95-300	70-80	65-80	70-85
1	NNLO/NLO [%]	70-130	90-100	90-110	95-115



Comparison with other FF determinations: K^+

DEHSS [PRD 95 (2017) 094019]
(+SIDIS +PP)

JAM [PRD 94 (2016) 114004]
(almost same dataset as NNFF1.0)

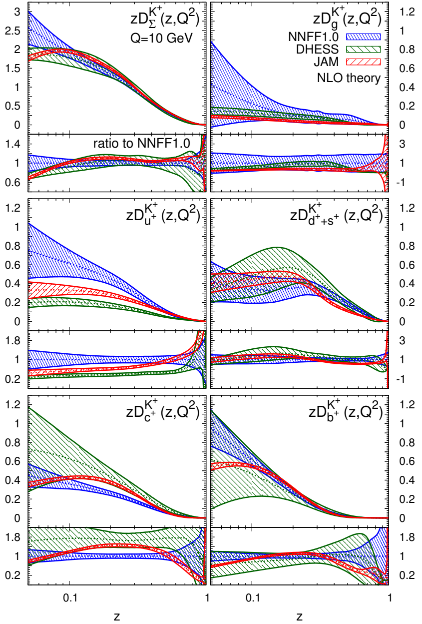
$D_\Sigma^{\pi^+}$: excellent agreement (both c.v. and unc.)
bulk of the dataset

$D_g^{\pi^+}$: good mutual agreement
similar shapes, larger uncertainties
DEHSS: data; JAM: parametrisation

$D_{u^+}^{\pi^+}$: mutual sizable disagreement
differences in dataset and parametrisation
comparable uncertainties in the data region

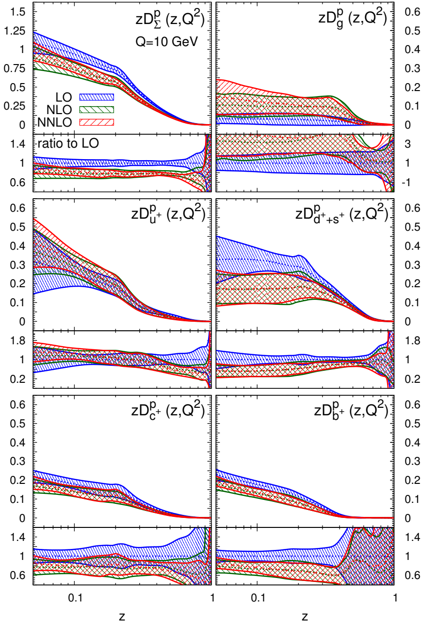
$D_{d^+}^{\pi^+} + D_{s^+}^{\pi^+}$: fair mutual agreement
differences in dataset and parametrisation
comparable uncertainties in the data region

$D_{c^+}^{\pi^+}$, $D_{b^+}^{\pi^+}$: excellent mutual agreement
uncertainties similar to JAM
DHESS shows inflated uncertainties



Dependence upon perturbative order: $p/\bar{p}$

Exp.	$N_{\rm dat}$	LO $\chi^2/N_{\rm dat}$	NLO $\chi^2/N_{\rm dat}$	NNLO $\chi^2/N_{\rm dat}$
BABAR	43	0.10	0.31	0.50
BELLE	29	4.74	2.75	1.25
TASSO12	3	0.69	0.70	0.72
TASSO14	9	1.32	1.25	1.22
TASSO22	9	0.98	0.92	0.93
TPC	20	1.04	1.10	1.08
TASSO30	2	0.25	0.19	0.18
TASSO34	6	0.82	0.81	0.78
TOPAZ	4	0.79	1.21	0.19
ALEPH	26	1.36	1.43	1.28
DELPHI	22	0.48	0.49	0.49
DELPHI-UDS	22	0.47	0.46	0.45
DELPHI-B	22	0.89	0.89	0.91
SLD	36	0.66	0.65	0.64
SLD-UDS	36	0.77	0.76	0.78
SLD-C	36	1.22	1.22	1.21
SLD-B	35	1.12	1.29	1.33
TOTAL	360	1.31	1.13	0.98



Excellent perturbative convergence
FFs almost stable from NLO to NNLO
LO FF uncertainties larger than HO
Dependence upon data set and kin cuts similar to pions and kaons

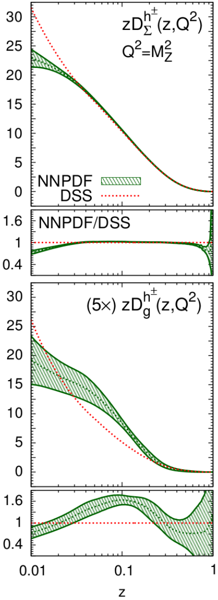
Fragmentation functions of unidentified charged hadrons

Combine the information from identified $\pi^{\pm}$, $K^{\pm}$ and $p/\bar{p}$ FFs with that from residual light charged hadrons [PoS DIS2017 231]

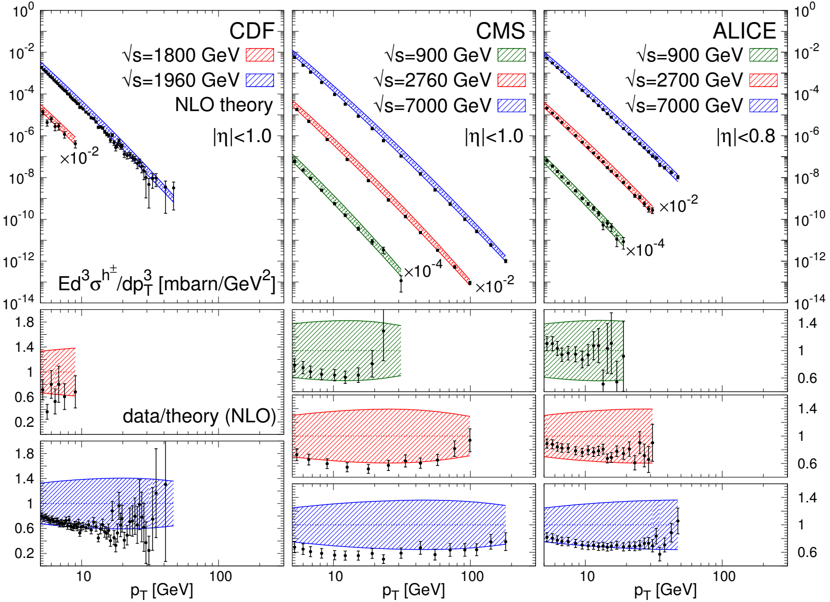
Preliminary analysis done consistently with NNFF1.0 (NLO)

Additional F_L data (non-vanishing $\mathcal{O}(\alpha_s)$ contribution at LO)

Experiment	Observable	$\sqrt{s}$ [GeV]	$N_{\rm dat}$	$\chi^2/N_{\rm dat}$
TASSO14	F_2 (incl.)	14.00	15 (20)	1.23
TASSO22	F_2 (incl.)	22.00	15 (20)	0.51
TPC	F_2 (incl.)	29.00	21 (34)	1.65
TASSO35	F_2 (incl.)	35.00	15 (20)	1.14
TASSO44	F_2 (incl.)	44.00	15 (20)	0.68
ALEPH	F_2 (incl.)	91.20	32 (35)	1.04
	F_L (incl.)	91.20	19 (21)	0.36
DELPHI	F_2	91.20	21 (27)	0.65
	F_2 (uds tagged)	91.20	21 (27)	0.17
	F_2 (b tagged)	91.20	21 (27)	0.82
	F_L (incl.)	91.20	20 (22)	0.72
	F_L (b tagged)	91.20	20 (22)	0.44
OPAL	F_2 (incl.)	91.20	20 (22)	2.41
	F_2 (uds tagged)	91.20	20 (22)	0.90
	F_2 (c tagged)	91.20	20 (22)	0.61
	F_2 (b tagged)	91.20	20 (22)	0.21
	F_L (incl.)	91.20	20 (22)	0.31
SLD	F_2	91.28	34 (40)	0.75
	F_2 (uds tagged)	91.28	34 (40)	1.03
	F_2 (c tagged)	91.28	34 (40)	0.62
	F_2 (b tagged)	91.28	34 (40)	0.97
Total dataset			471 (527)	0.83



Fragmentation functions of unidentified charged hadrons



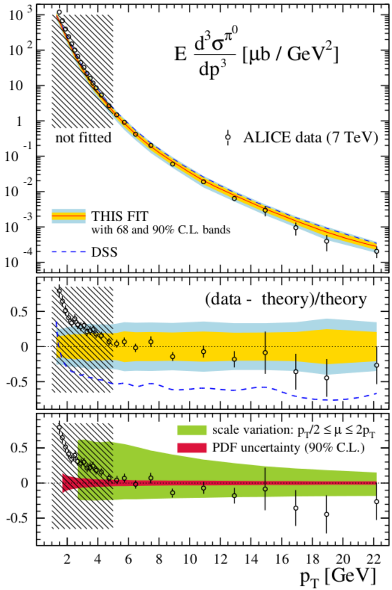
3. Summary and outlook

More global fits

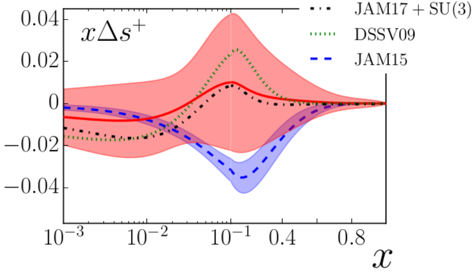
pions: $\chi^2_{\rm tot}/N_{\rm dat} = 1.19$

experiment		data type	norm. N_i	# data in fit	χ^2
TPC [48]		incl.	1.043	17	17.3
		uds tag	1.043	9	2.1
		c tag	1.043	9	5.9
		b tag	1.043	9	9.2
TASSO [49]	34 GeV	incl.	1.043	11	30.2
	44 GeV	incl.	1.043	7	22.2
SLD [19]		incl.	0.986	28	15.3
		uds tag	0.986	17	18.5
		c tag	0.986	17	16.1
		b tag	0.986	17	5.8
ALEPH [16]		incl.	1.020	22	22.9
DELPHI [17]		incl.	1.000	17	28.3
		uds tag	1.000	17	33.3
		b tag	1.000	17	10.6
OPAL [18, 20]		incl.	1.000	21	14.0
		u tag	0.786	5	31.6
		d tag	0.786	5	33.0
		s tag	0.786	5	51.3
		c tag	0.786	5	30.4
		b tag	0.786	5	14.6
BABAR [28]		incl.	1.031	45	46.4
BELLE [29]		incl.	1.044	78	44.0
HERMES [30]		π^+ (p)	0.980	32	27.8
		π^- (p)	0.980	32	47.8
		π^+ (d)	0.981	32	40.3
		π^- (d)	0.981	32	59.1
COMPASS [31] prel.		π^+ (d)	0.946	199	174.2
		π^- (d)	0.946	199	229.0
PHENIX [21]		π^0	1.112	15	15.8
STAR [33–36]	$0 \leq \eta \leq 1$	π^0	1.161	7	5.7
	$0.8 \leq \eta \leq 2.0$	π^0	0.954	7	2.7
	$\lvert\eta\rvert < 0.5$	$\pi^\pm$	1.071	8	4.3
	$\lvert\eta\rvert < 0.5$	$\pi^+,\pi^-/\pi^+$	1.006	16	17.2
ALICE [32]	7 TeV	π^0	0.766	11	27.7
TOTAL:				973	1154.6

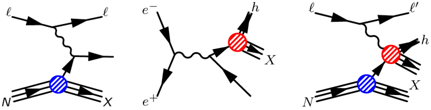
[PRD 91 (2015) 014035; PRD 95 (2017) 094019]

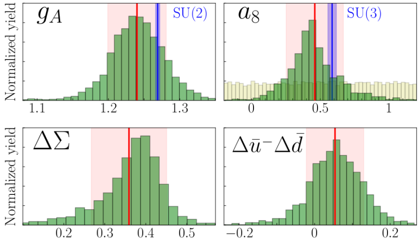


Simultaneous fits [PRL 119 (2017) 132001]



process	target	$N_{\rm dat}$	χ^2
DIS	$p, d, {}^3$He	854	854.8
SIA $(\pi^\pm, K^\pm)$		850	997.1
SIDIS $(\pi^\pm)$			
HERMES	d	18	28.1
HERMES	p	18	14.2
COMPASS	d	20	8.0
COMPASS	p	24	18.2
SIDIS $(K^\pm)$			
HERMES	d	27	18.3
COMPASS	d	20	18.7
COMPASS	p	24	12.3
Total:		**1855**	**1969.7**





$g_A = 1.24 \pm 0.04 \qquad a_8 = 0.46 \pm 0.21$

confirmation of SU(2) symmetry to $\sim 2\%$

$\sim 20\%$ SU(3) breaking $\pm 20\%$

$$\Delta s^+ = -0.03 \pm 0.09$$

$\Delta\Sigma = 0.36 \pm 0.09 \qquad \Delta u - \Delta d = 0.05 \pm 0.08$

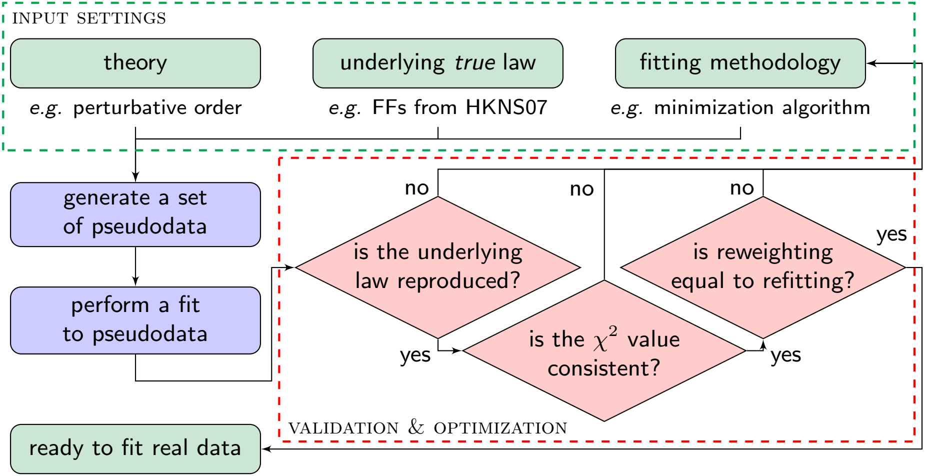
Conclusions

1. A number of hard-scattering processes require an appropriate knowledge of FFs
   - probing nucleon momentum, spin and flavour
   - underlying spatial distributions and the dynamics of nuclear matter
2. FFs are poorly known in comparison to PDFs
   - limited set of available data, observables often require PDFs and FFs simultanously
   - are uncertainties faithfully represented?
   - troubles in describing some observables in pp and SIDIS from current FF sets
3. New analysis based on the NNPDF methodology, NNFF1.0
   - at LO, NLO and NNLO, based on SIA data for $\pi^{\pm}$, $K^{\pm}$ and $p/\bar{p}$
   - detailed study of the stability of the results upon variations of the data set/kin cuts
   - FF uncertainties (gluon) larger than in previous determinations
   - differences in shapes, to be further investigated
   - good description of the hadron spectra at the LHC within uncertainties
   - applicability limited by insensitivity to favoured/unfavoured FFs
4. Future improvements
   - More global fits
   - Simultaneous fits of FFs and PDFs

4. Additional slides

Methodology validation: closure tests [JHEP 1504 (2015) 040]

Validation and optimization of the fitting strategy with known underlying physical law



Full control of procedural uncertainties

Closure tests: levels

1. Level 0: generate pseudodata D_i^0 with zero uncertainty
   (but $(\mathrm{cov})_{ij}$ in the χ^2 is the data covariance matrix)
   → fit quality can be arbitrarily good, if the fitting methodology is efficient: $\chi^2/N_{\mathrm{dat}} \sim 0$
   → validate fitting methodology (parametrisation, minimisation)
   → interpolation and extrapolation uncertainty
2. Level 1: generate pseudodata D_i^1 with stochastic fluctuations (no replicas)

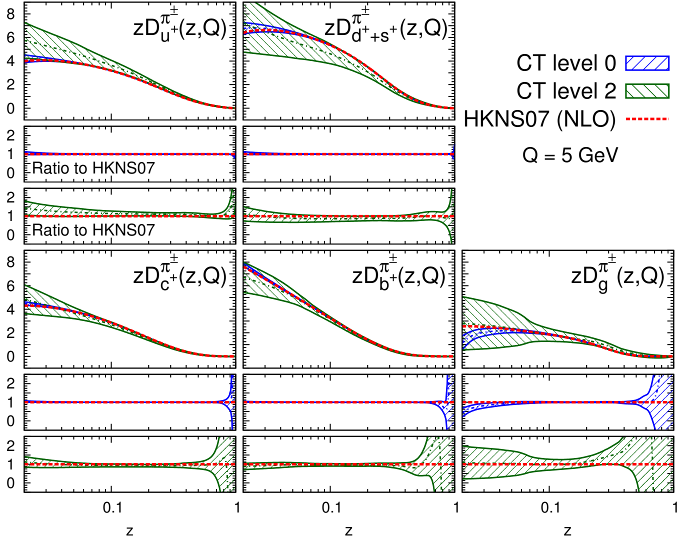
$$D_i^1 = (1 + r_i^{\mathrm{nor}} \sigma_i^{\mathrm{nor}}) \left(D_i^0 + \sum_p^{N_{\mathrm{sys}}} r_{i,p}^{\mathrm{sys}} \sigma_{i,p}^{\mathrm{sys}} + r_i^{\mathrm{stat}} \sigma_i^{\mathrm{stat}} \right)$$

   → experimental uncertainties are not propagated into FFs: $\chi^2/N_{\mathrm{dat}} \sim 1$
   → functional uncertainty (a large number of functional forms with equally good χ^2)
3. Level 2: generate N_{rep} Monte Carlo pseudodata replicas $D_i^{2,k}$ on top of Level 1

$$D_i^{2,k} = (1 + r_i^{\mathrm{nor},k} \sigma_i^{\mathrm{nor}}) \left(D_i^1 + \sum_p^{N_{\mathrm{sys}}} r_{i,p}^{\mathrm{sys},k} \sigma_{i,p}^{\mathrm{sys}} + r_i^{\mathrm{stat},k} \sigma_i^{\mathrm{stat}} \right)$$

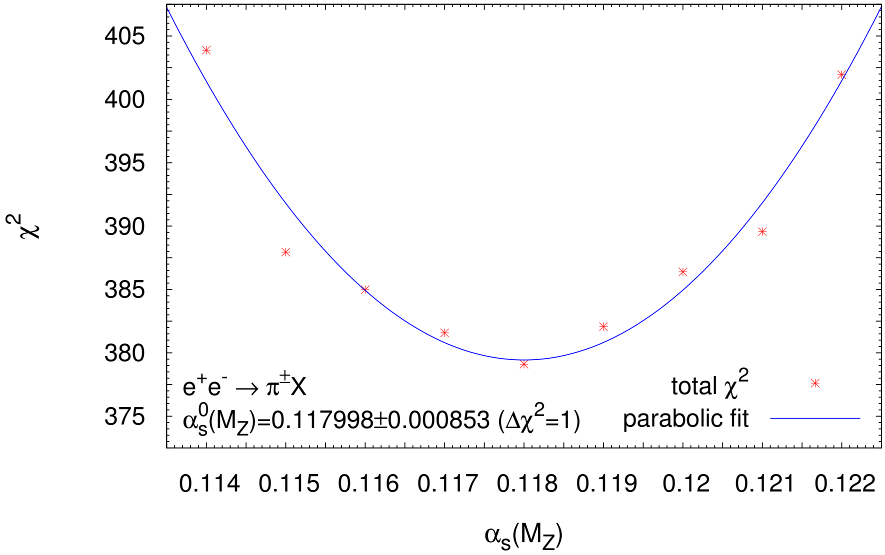
   → propagate the fluctuations due to experimental uncertainties into FFs: $\chi^2/N_{\mathrm{dat}} \sim 1$
   → input FFs lie within the one-sigma band of the fitted FFs with a probability of $\sim$ 68%
   → data uncertainty

Closure testing NNFF1.0



$\chi^2/N_{\rm dat} = 0.0001$ **(L0)** $\chi^2/N_{\rm dat} = 1.0262$ **(L1)**

Dependence on α_s



Minimisation: adaptive algorithms [N. Hansen, Springer (2016)]

The Covariance Matrix Adaption - Evolution Strategy (CMA-ES)

1. Initialisation at the (0)-th generation

$$\mathbf{a}^{(0)} \sim \mathcal{N}(0, \mathbf{C}^{(0)})\,, \qquad \mathbf{C}^{(0)} = \mathbf{I}$$

2. Mutation at the (i)-th generation, λ mutants, step-size $\sigma^{(i-1)}$

$$\mathbf{x}_k^{(i)} \sim \mathbf{a}^{(i-1)} + \sigma^{(i-1)}\mathcal{N}(0, \mathbf{C}^{(i-1)})\,, \qquad \text{for } k = 1, \dots, \lambda$$

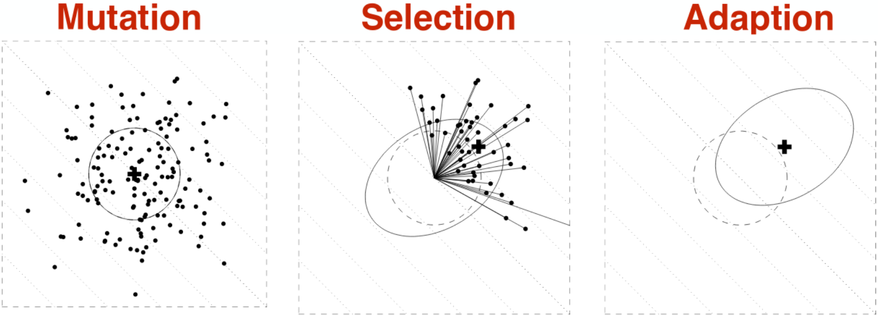
compute the fitness of each mutant and rank them such that $\chi^2(\mathbf{x}_k) < \chi^2(\mathbf{x}_{k+1})$

3. (Non-elitist) recombination
compute the new search centre as a weighted average over the $\mu = \lambda/2$ best mutants

$$\mathbf{a}^{(i)} = \mathbf{a}^{(i-1)} + \sum_{i=1}^{\mu} w_i \left(\mathbf{x}_k^{(i)} - \mathbf{a}^{(i-1)}\right)$$

update $\mathbf{C}$ using information on the parameter space learnt from the mutants
iterate until convergence is reached

Minimisation: the CMA-ES algorithms [N. Hansen, Springer (2016)]



The key features of the CMA-ES family of algorithms are the determination of the search distribution covariance matrix $\mathbf{C}^{(i)}$ (and possibly of the step-size σ^i)

These features are optimised by the fit procedure, making use of the information present in the ensemble of mutants to learn preferred directions in parameter space

Internal parameters ($\sigma^{(0)}$, λ, w_i) tuned by trial and error