# Two topics in hadronic physics

The axial anomaly and flavor mixing for mesons with higher spin

F. Giacosa (Kielce), A. Koenigstein (Frankfurt) & RDP, 1709.07454

Tetraquarks and (perhaps) two chiral phase transitions?

RDP & V. V. Skokov (RBRC -> NCSU), 1606.04111

# The axial anomaly and flavor mixing

Axial anomaly & the usual Goldstone bosons

$\eta$ & $\eta'$ mixing: close to pure $SU(3)_V$ states, octet & singlet

Mixing for vector mesons, $J^P = 1-$: $\varrho$, $\omega$ & $\phi$ : flavor eigenstates:

$\varrho$ & $\omega$ mainly light quarks, $\phi$ mainly strange

How does this generalize to mesons with $J^P = 1+, 2, 3...$?

Today: make one strong assumption, many testable predictions

Generally: how does the axial anomaly affect mesons with *higher* spin?

# Chiral symmetry

Quarks in QCD:

$$\overline{q}(\not{D} + m)q = \overline{q}_L \not{D} q_L + \overline{q}_R \not{D} q_R + m_{qk}(\overline{q}_L q_R + \overline{q}_R q_L)$$

Chiral projectors:

$$\mathcal{P}_{L,R} = (1 \mp \gamma_5)/2 \; , \; \mathcal{P}^2_{L,R} = \mathcal{P}_{L,R} \; , \; \mathcal{P}_L \mathcal{P}_R = 0$$

$$q_{L,R} = \mathcal{P}_{L,R}\, q \; , \; \overline{q}_{L,R} = q^\dagger\, \mathcal{P}_{L,R}\, \gamma_0 = \overline{q}\, \mathcal{P}_{R,L}$$

For three flavors, quarks are invariant under

$$q_{\mathrm{L,R}} \longrightarrow \mathrm{e}^{\pm \mathrm{i}\alpha/2}\, U_{\mathrm{L,R}}\, q_{\mathrm{L,R}}$$

Classically, global symmetry of $SU(3)_L \times SU(3)_R \times U(1)_A = G_{fl} \times U(1)_A$ .

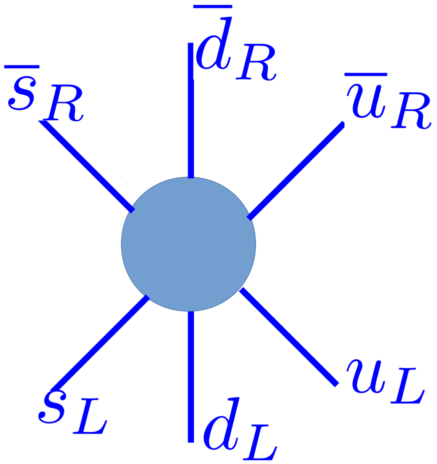
# Axial anomaly

Quantum mechanically ($m_{qk} = 0$):

$$\partial^\mu \, \overline{q}^a \gamma_\mu \, \gamma_5 q^a = \frac{3g^2}{16\pi^2} \, \varepsilon^{\mu\nu\rho\sigma} \mathrm{tr} \, G_{\mu\nu} \, G_{\rho\sigma}$$

Quantum symmetry is $G_{fl}$ x $Z(3)_A$ .

With dynamically generated mass, breaks to $SU(3)_V$.




Since $m_{strange} \gg m_{up} \approx m_{down}$, $SU(3)_V$ -> $SU(2)_V$.

Eigenstates of (softly broken) $SU(3)_V$ , or flavor?

In vacuum, instantons have dominant size, $\langle r_{inst} \rangle \sim 1/3$ fm (Shuryak).

But instantons come in all sizes....

# Scalar multiplet

Simplest is to pair

$$\Phi^{ij} = \overline{q}^j_R \, q^i_L = \overline{q}^j \, \mathcal{P}_L \, q^i \; , \; \Phi \to \mathrm{e}^{-i\alpha} \, U_L \, \Phi \, U_R^\dagger$$

$J^P = 0-$: π, K, η, η´ ;$J^P = 0+$: σ(600), $a_0$(980) + ....; *or* $a_0$(1450)$f_0$(1370), $f_0$(1710)?

All terms must be invariant under $G_{fl}$. Invariant under $U(1)_A$ :

$$\mathcal{V} = m^2 \, \mathrm{tr}\Phi^\dagger \Phi + \lambda_1 (\mathrm{tr}\Phi^\dagger \Phi)^2 + \lambda_2 \mathrm{tr}(\Phi^\dagger \Phi)^2 + ...$$

Under $Z(3)_A$:

$$\mathcal{V}^{Z(3)} = -a_1 (\det(\Phi) + \mathrm{c.c.}) - a_2 \mathrm{tr}(\Phi^\dagger \Phi)(\det(\Phi) + \mathrm{c.c.})$$

$$-a_3 ((\det(\Phi))^2 + \mathrm{c.c.}) + ...$$

$a_1$, $a_2$ generated by zero modes of single instanton; $a_3$ by two instantons

# (pseudo-) Goldstone Bosons

Add

$$\mathcal{L}_{mass} = \mathrm{tr} H(\Phi + \Phi^\dagger) \qquad H = \# \begin{pmatrix} m_{up} & 0 & \\ 0 & m_{down} & 0 \\ 0 & 0 & m_{strange} \end{pmatrix}$$

When $\langle \Phi \rangle = \phi_0 \neq 0$, $\quad m_\pi^2 \sim m_u + m_d \;,\; m_K^2 \sim m_{u,d} + m_s$

*With* the anomaly, GB's eigenstates of $SU(3)_V$ : η -> 3 π, η' -> η + 2 π

η´ mainly singlet, η mainly octet. Mix in calculable manner, θ ~ - 42°.

With*out* the anomaly, GB's eigenstates of flavor, *not* $SU(3)_V$:

Gross, Wilczek & Treiman '78; RDP & Wilczek, '82

$$\pi^0 \sim \overline{u}u \;,\; \eta \sim \overline{d}d \;,\; \eta' \sim \overline{s}s$$

The anomaly prevents massive isospin violation in the GB's.

# Vector multiplet

Insert $\gamma_\mu$ between quarks: as $\gamma_\mu$ flips chirality, can only pair LL and RR

$$L_\mu^{ij} = \overline{q}_{\rm L}^j \, \gamma_\mu \, q_{\rm L}^i \ , \ R_\mu^{ij} = \overline{q}_{\rm R}^j \, \gamma_\mu \, q_{\rm R}^i$$

*Neutral* under $U(1)_A$:

$$L_\mu \longrightarrow U_{\rm L} \, L_\mu \, U_{\rm L}^\dagger \ , \ R_\mu \longrightarrow U_{\rm R} \, R_\mu \, U_{\rm R}^\dagger$$

Obvious mixing and mass terms,

$$\beta \, {\rm tr}(L_\mu \Phi^\dagger \partial_\mu \Phi + R_\mu \Phi \partial_\mu \Phi^\dagger) + m_V^2 \, {\rm tr}(L_\mu^2 + R_\mu^2) + \kappa \, {\rm tr} H (L_\mu^2 + R_\mu^2)$$

# More vectors

Effects of anomaly indirect.

Start with 3rd order in $\partial$'s,   Wess-Zumino-Novikov-Witten:

$$\varepsilon^{\mu\nu\alpha\beta}\text{tr}[L_\mu\Phi(\partial_\nu\Phi^\dagger)\Phi(\partial_\alpha\Phi^\dagger)\Phi(\partial_\beta\Phi^\dagger) + R_\mu\Phi^\dagger(\partial_\nu\Phi)\Phi^\dagger(\partial_\alpha\Phi)\Phi^\dagger(\partial_\beta\Phi)]\}$$

$J^P = 1^-$: $V_\mu = L_\mu + R_\mu$ , $\varrho(770)$, $\omega(782)$, $K^*(892)$ & $\phi(1020)$.  WZNW: $\omega \to 3\pi$

*No* mass terms from the anomaly, so $\varrho$, $\omega$, & $\phi$ are *flavor* eigenstates:

$$\rho_\mu\ ,\ \omega_\mu \sim \bar{l}\gamma_\mu l\ ,\ l = u, d\ ;\ \phi_\mu \sim \bar{s}\gamma_\mu s$$

Obvious from decays: $\rho \to 2\pi$ ,  $\omega \to 3\pi$ , $\phi \to KK$

$J^P = 1^+$: $A_\mu = L_\mu - R_\mu$ , $a_1(1260)$, $K_{1,A}(?)$,$f_1(1285)$, $f_1(1420)$

# What happens with higher spins?

*Assume* that we can classify multiplets according to the *un*broken $G_{fl}$ x $Z(3)_A$ .

Form mesons by inserting some Γ, ~ γ's and D's, between q and q-bar.

Heterochiral:

$$[\Gamma_{\mu\nu\ldots}, \gamma_5] = 0 \;:\; \Phi_{\mu\nu\ldots} = \overline{q}_R \Gamma_{\mu\nu\ldots} q_L \;,\; \Phi_{\mu\nu\ldots} \to \mathrm{e}^{-i\alpha} U_L \Phi_{\mu\nu\ldots} U_R^\dagger$$

*Numerous* anomaly terms: masses close to $SU(3)_V$ eigenstates.

Do effects from anomaly decrease as the mass increases?

Homochiral:

$$\{\Gamma_{\mu\nu\ldots}, \gamma_5\} = 0 \;:\; L_{\mu\nu\ldots} = \overline{q}_L \Gamma_{\mu\nu\ldots} q_L \;;\; R_{\mu\nu\ldots} = \overline{q}_R \Gamma_{\mu\nu\ldots} q_R$$

$$L_{\mu\nu\ldots} \to U_L^\dagger L_{\mu\nu\ldots} U_L \;;\; R_{\mu\nu\ldots} = U_R^\dagger R_{\mu\nu\ldots} U_R$$

Masses close to eigenstates of flavor, as in the usual quark model.

# Heterochiral spin-one

$\Gamma_\mu = \overleftrightarrow{D}_\mu \;, \quad \Phi^{ij}_\mu = \overline{q}^i_R \overleftrightarrow{D}_\mu q^j_L$ *Many* anomaly terms, mixing $\Phi$ and $\Phi_\mu$

$$- \varepsilon^{ijk} \varepsilon^{i'j'k'} (b^{(1)}_A \Phi^{ii'} \Phi^{jj'}_\mu \Phi^{kk'}_\mu + b^{(2)}_A \Phi^{ii'} \partial^\mu \Phi^{jj'} \Phi^{kk'}_\mu + \text{c.c.})$$

$$-b^{(3)}_A (\varepsilon^{ijk} \varepsilon^{i'j'k'} \Phi^{ii'} \Phi^{jj'} \Phi^{kk'}_\mu - \text{c.c.})^2 + \ldots$$

$\Phi_\mu = S_\mu + i\, P_\mu$: obvious candidates:

$P_\mu$ : $J^{PC}$ = 1+- : $b_1(1235)$, $K_{1,B}$, $h_1(1170)$, $h_1(1380)$ $\;{}^{2S+1}L_J = {}^1S_1$.
$S_\mu$: $J^{PC}$ = 1--, $\varrho(1700)$, K*(1680), $\omega(1650)$, $\phi(1680?)$ ${}^{2S+1}L_J = {}^1P_1$.

But $h_1$ (1170), $\omega(1650)$ -> $\varrho\,\pi$; $h_1(1380)$, $\phi(1680?)$ -> KKbar*: *homo*chiral

Where is the heterochiral spin one multiplet, with the spin one $\eta_1$ & $\eta_1$' - ?

# Heterochiral spin-two

$$\Gamma_{\mu\nu} = \overleftrightarrow{D}_\mu \overleftrightarrow{D}_\nu + \overleftrightarrow{D}_\nu \overleftrightarrow{D}_\mu - (\mathrm{tr} = 0) \;\; , \;\; \Phi^{ij}_{\mu\nu} = \overline{q}^i_{\mathrm{R}} \Gamma_{\mu\nu} q^j_{\mathrm{L}}$$

*Many* anomaly terms, mixing $\Phi$ and $\Phi_{\mu\nu}$ (and others with $\Phi_\mu$)

$$-c_A^{(1)} \, \varepsilon^{ijk} \varepsilon^{i'j'k'} \Phi^{ii'} \Phi^{jj'}_{\mu\nu} \Phi^{kk'}_{\mu\nu} + c_A^{(2)} \varepsilon^{ijk} \varepsilon^{i'j'k'} \partial^\mu \Phi^{ii'} \; \partial^\nu \Phi^{jj'} \; \Phi^{kk'}_{\mu\nu} + \mathrm{c.c.}$$

$$-c_A^{(3)} \left( \varepsilon^{ijk} \varepsilon^{i'j'k'} \Phi^{ii'} \Phi^{jj'} \Phi^{kk'}_{\mu\nu} - \mathrm{c.c.} \right)^2 + \ldots$$

$\Phi_{\mu\nu} = S_{\mu\nu} + iP_{\mu\nu}$ :    $S_{\mu\nu}$: $J^{PC}$ = 2++, $a_2$, $K_2$*, $f_2$, $f_2$' ; exp.'y murky

$P_{\mu\nu}$ : $J^{PC}$ = 2-+ : $\pi_2$(1670), $K_2$ (1670), $\eta_2$(1645) , $\eta_2$(1870); $^{2S+1}L_J = {}^1D_2$.

$\eta_2$(1645) -> $a_2$(1320) $\pi$, , $\eta_2$(1870) -> $\eta$ + 2 $\pi$ (like $\eta$'): *hetero*chiral

$\eta_2$(1645) & $\eta_2$(1870) mix like $\eta$ & $\eta$' , $\theta \sim -42^o$   Koenigstein & Giacosa 1608.08777

But: $\eta_2$(1870) *not* in PDG summary table, needs confirmation

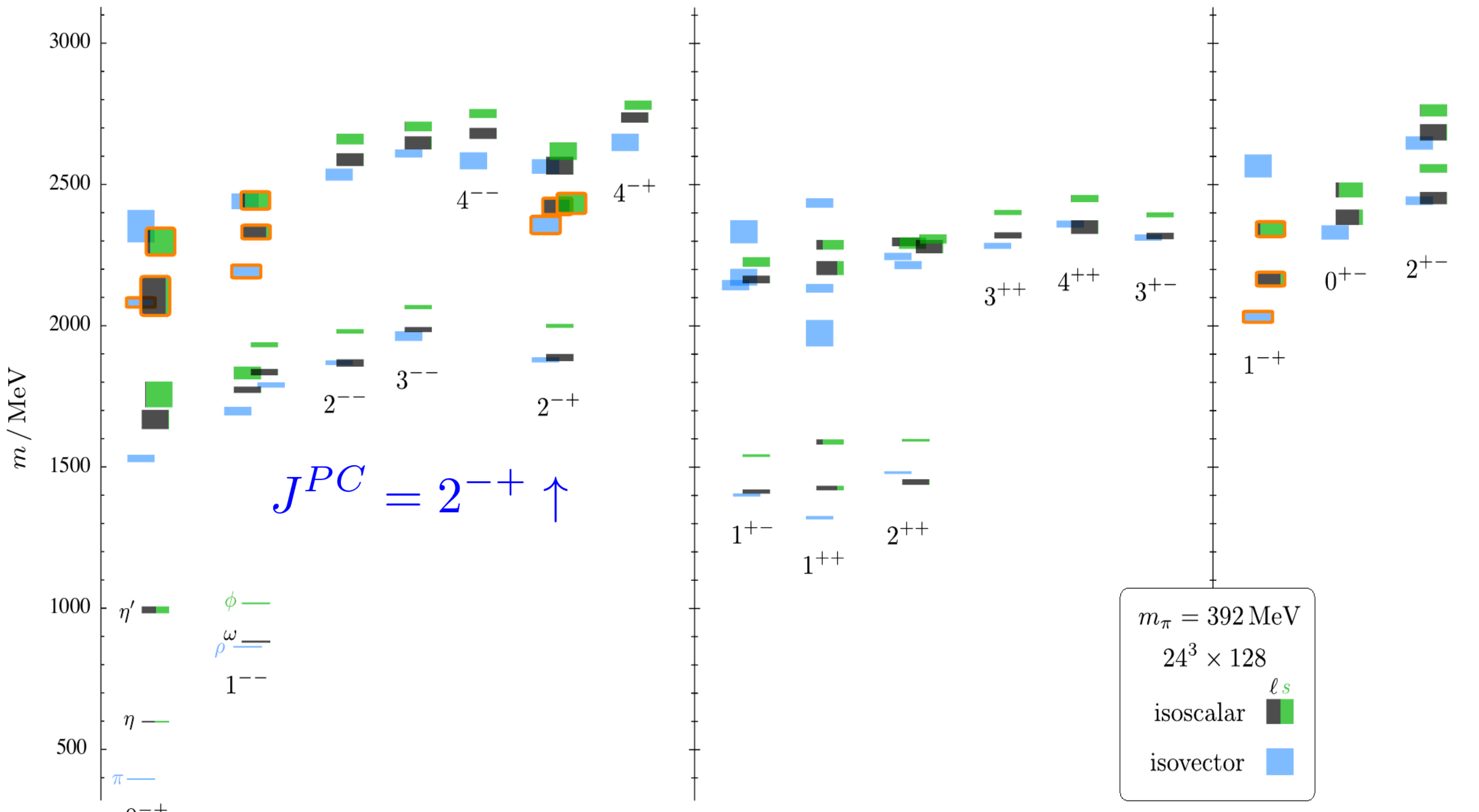
# Lattice: *no* flavor mixing?

Dudek, Edwards, Guo & Thomas, 1309.2609, $m_\pi$ = 392 MeV

"...*little* mixing...in most $J^{PC}$ channels...except η & η' "

$J^{PC}$ = 2-+ : Exp.'y, *no* strange decays of $\eta_2(1870)$:

PDG: $\eta_2(1870)$ -> ηππ , $a_2(1320)\pi$, $f_2(1270)\eta$, $a_0(980)\pi$ KKbar+... *not* seen



# Homochiral spin-one multiplet

$\gamma_\mu \gamma_\nu = g_{\mu\nu} - i\sigma_{\mu\nu}$ , $\Gamma_{\mu\nu} = \sigma_{\mu\nu}$ , $L_{\mu\nu} = \bar{q}_L \sigma_{\mu\nu} q_L$ , $R_{\mu\nu} = \bar{q}_R \sigma_{\mu\nu} q_R$

Anti-symmetric 2-index = spin-1 field: Ecker, Gasser, Pich & de Rafael, '89,

$$L_\mu = \partial_\mu L_{\mu\nu}/M \;,\; R_\mu = \partial_\mu R_{\mu\nu}/M$$

Included by lattice: Dudek, Edwards, Pearson, Richards, & Thomas 1004.4930

$V_\mu = L_\mu + R_\mu$: $J^{PC} = 1^{--}$, $\varrho(1700)$, $K^*(1680)$, $\omega(1650)$, $\phi(1680?)$ $^{2S+1}L_J = {}^1P_1$.

$A_\mu = L_\mu - R_\mu$: $J^{PC} = 1^{+-}$ : $b_1(1235)$, $K_{1,B}$, $h_1(1170)$, $h_1(1380)$ $^{2S+1}L_J = {}^1S_1$.

$h_1(1170)$, $\omega(1650)$ -> $\varrho\,\pi$; $h_1(1380)$, $\phi(1680?)$ -> KKbar* .

*Where is the heterochiral spin one multiplet, with the $\eta_1$ & $\eta_1'$ ?*

# Homochiral spin-two multiplet

$$\Gamma_{\mu\nu} = \gamma_\mu \overleftrightarrow{D}_\nu + \gamma_\nu \overleftrightarrow{D}_\mu + (\mathrm{tr} = 0) \ \ , \ \ L_{\mu\nu} = \overline{q}_L \Gamma_{\mu\nu} q_L \ , \ R_{\mu\nu} = \overline{q}_R \Gamma_{\mu\nu} q_R$$

Obvious mixing and mass terms:

$$\beta\,\mathrm{tr}(L_{\mu\nu}\partial_\mu \Phi^\dagger \partial_\nu \Phi + R_\mu \partial_\mu \Phi \partial_\nu \Phi^\dagger) + m_V^2\,\mathrm{tr}(L_{\mu\nu}^2 + R_{\mu\nu}^2) + \kappa\,\mathrm{tr} H(L_{\mu\nu}^2 + R_{\mu\nu}^2)$$

Only anomaly terms from higher derivatives: *Wess-Zumino-Witten for spin-2?*

$J^{PC}$ = 2++: $V_{\mu\nu}=L_{\mu\nu}+R_{\mu\nu}$ , $a_2(1320)$, $K_2(1430)$, $f_2(1270)$ & $f_2$'(1525); $^{2S+1}L_J = {}^3P_2$.

Anomaly doesn't contribute to mass terms, so eigenstates of flavor, homochiral.

Decays of $f_2(1270)$ & $f_2$'(1525) like $\omega$ & $\phi$

$J^{PC}$ = 2--: $V_{\mu\nu} = L_{\mu\nu} - R_{\mu\nu}$ , $K_2(1820)$ + ?

# $f_2$'s on the lattice

Briceno, Dudek, Edwards & Wilson, 1708.06667

Luscher + coupled channel analysis with two mesons, $m_\pi$ = 391 MeV

$f_2^a(1470)$ -> $2\pi$ , $f_2^b(1602)$ -> KKbar

just like

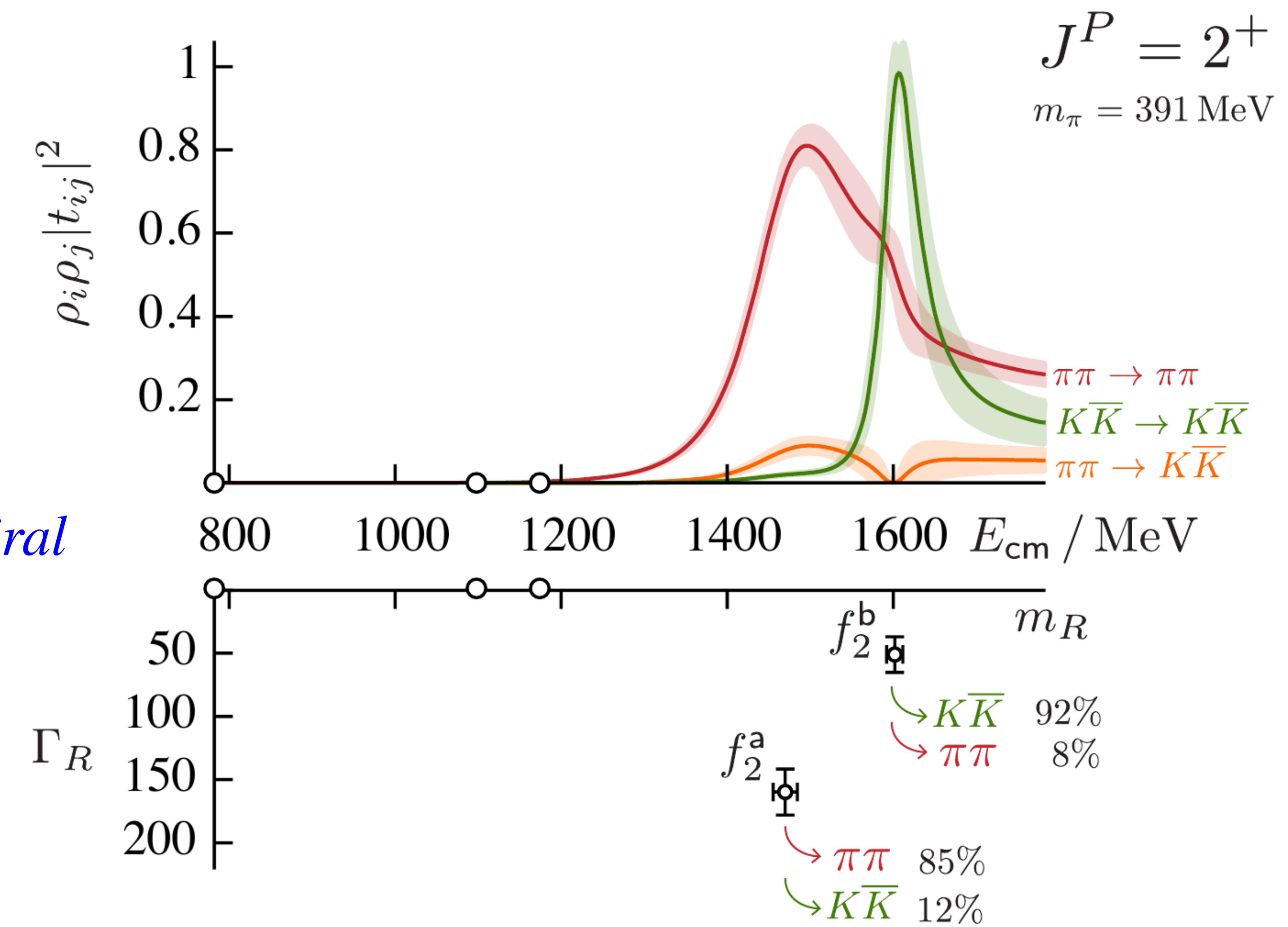
$f_2(1270)$ ~ $f_2^a(1470)$

$f_2'(1525)$ ~ $f_2^b(1602)$

and the ω & ϕ: *homochiral*

N.B.: C = - 1 for ω & ϕ

C = + 1 for $f_2$'s



# Homochiral spin-three multiplet

$$\Gamma_{\mu\nu\rho} = \gamma_\mu \overleftrightarrow{D}_\nu \overleftrightarrow{D}_\rho + \mathrm{sym}, \mathrm{tr} = 0 \ , \ L_{\mu\nu\rho} = \bar{q}_L \Gamma_{\mu\nu\rho} q_L \ , \ R_{\mu\nu\rho} = \bar{q}_R \Gamma_{\mu\nu\rho} q_R$$

$J^{PC}$ = 3--: $V_{\mu\nu\rho} = L_{\mu\nu\rho} + R_{\mu\nu\rho}$: $\rho_3(1690)$, $K_3^*(1780)$, $\omega_3(1670)$ & $\varphi_3(1850)$ $^{2S+1}L_J = {}^3P_2$.

$\omega_3(1670)$ -> $\rho\pi$, $\omega\pi\pi$ ; $\varphi_3(1850)$ -> K Kbar, K Kbar*

PDG: mixing angle ~ $3^o$, homochiral

| $J^{PC}$, ${}^{2S+1}L_J$ | $\begin{cases} I=1(\bar{u}d, \bar{d}u, \frac{\bar{d}d-\bar{u}u}{\sqrt{2}}) \\ I=1(-\bar{u}s, \bar{s}u, \bar{d}s, \bar{s}d) \\ I=0(\frac{\bar{u}u+\bar{d}d}{\sqrt{2}}, \bar{s}s)^{\star\star} \end{cases}$ | Microscopic currents | Chiral multiplet | Transformation under $SU(3)_{\mathrm{L}} \times SU(3)_{\mathrm{R}} \times \times U(1)_{\mathrm{A}}$ |
|---|---|---|---|---|
| $0^{-+}$, ${}^1S_0$ | $\pi$, $K$, $\eta, \eta'(958)$ | $P^{ij} = \frac{1}{2}\bar{q}^j \mathrm{i}\gamma^5 q^i$ | $\Phi = S + \mathrm{i}P$ $(\Phi^{ij} = \bar{q}^j_{\mathrm{R}} q^i_{\mathrm{L}})$ | $\Phi \to \mathrm{e}^{-2\mathrm{i}\alpha} U_{\mathrm{L}} \Phi U_{\mathrm{R}}^{\dagger}$ |
| $0^{++}$, ${}^3P_0$ | $a_0(1450)$, $K_0^*(1430)$, $f_0(1370), f_0(1710)^{\star}$ | $S^{ij} = \frac{1}{2}\bar{q}^j q^i$ | | |
| $1^{--}$, ${}^1S_1$ | $\rho(770)$, $K^*(892)$, $\omega(782), \phi(1020)$ | $V_\mu^{ij} = \frac{1}{2}\bar{q}^j \gamma_\mu q^i$ | $L_\mu = V_\mu + A_\mu$ $(L_\mu^{ij} = \bar{q}^j_{\mathrm{L}} \gamma_\mu q^i_{\mathrm{L}})$ | $L_\mu \to U_{\mathrm{L}} L_\mu U_{\mathrm{L}}^{\dagger}$ |
| $1^{++}$, ${}^3P_1$ | $a_1(1260)$, $K_{1,A}$, $f_1(1285), f_1(1420)$ | $A_\mu^{ij} = \frac{1}{2}\bar{q}^j \gamma^5 \gamma_\mu q^i$ | $R_\mu = V_\mu - A_\mu$ $(R_\mu^{ij} = \bar{q}^j_{\mathrm{R}} \gamma_\mu q^i_{\mathrm{R}})$ | $R_\mu \to U_{\mathrm{R}} R_\mu U_{\mathrm{R}}^{\dagger}$ |
| $1^{+-}$, ${}^1P_1$ | $b_1(1235)$, $K_{1,B}$, $h_1(1170), h_1(1380)$ | $P_\mu^{ij} = -\frac{1}{2}\bar{q}^j \gamma^5 \overleftrightarrow{D}_\mu q^i$ | $\Phi_\mu = S_\mu + \mathrm{i}P_\mu$ $(\Phi_\mu^{ij} = \bar{q}^j_{\mathrm{R}} \mathrm{i}\overleftrightarrow{D}_\mu q^i_{\mathrm{L}})$ | $\Phi_\mu \to \mathrm{e}^{-2\mathrm{i}\alpha} U_{\mathrm{L}} \Phi_\mu U_{\mathrm{R}}^{\dagger}$ |
| $1^{--}$, ${}^3D_1$ | $\rho(1700)$, $K^*(1680)$, $\omega(1650), \phi(?)$ | $S_\mu^{ij} = \frac{1}{2}\bar{q}^j \mathrm{i}\overleftrightarrow{D}_\mu q^i$ | | |
| $2^{++}$, ${}^3P_2$ | $a_2(1320)$, $K_2^*(1430)$, $f_2(1270), f_2'(1525)$ | $V_{\mu\nu}^{ij} = \frac{1}{2}\bar{q}^j (\gamma_\mu \mathrm{i}\overleftrightarrow{D}_\mu + \cdots) q^i$ | $L_{\mu\nu} = V_{\mu\nu} + A_{\mu\nu}$ $(L_{\mu\nu}^{ij} = \bar{q}^j_{\mathrm{L}} (\gamma_\mu \mathrm{i}\overleftrightarrow{D}_\nu + \cdots) q^i_{\mathrm{L}})$ | $L_{\mu\nu} \to U_{\mathrm{L}} L_{\mu\nu} U_{\mathrm{L}}^{\dagger}$ |
| $2^{--}$, ${}^3D_2$ | $\rho_2(?)$, $K_2(1820)$, $\omega_2(?), \phi_2(?)$ | $A_{\mu\nu}^{ij} = \frac{1}{2}\bar{q}^j (\gamma^5 \gamma_\mu \mathrm{i}\overleftrightarrow{D}_\nu + \cdots) q^i$ | $R_{\mu\nu} = V_{\mu\nu} - A_{\mu\nu}$ $(R_{\mu\nu}^{ij} = \bar{q}^j_{\mathrm{R}} (\gamma_\mu \overleftrightarrow{D}_\nu + \cdots) q^i_{\mathrm{R}})$ | $R_{\mu\nu} \to U_{\mathrm{R}} R_{\mu\nu} U_{\mathrm{R}}^{\dagger}$ |
| $2^{-+}$, ${}^1D_2$ | $\pi_2(1670)$, $K_2(1770)$, $\eta_2(1645), \eta_2(1870)$ | $P_{\mu\nu}^{ij} = -\frac{1}{2}\bar{q}^j (\mathrm{i}\gamma^5 \overleftrightarrow{D}_\mu \overleftrightarrow{D}_\nu + \cdots) q^i$ | $\Phi_{\mu\nu} = S_{\mu\nu} + \mathrm{i}P_{\mu\nu}$ $(\Phi_{\mu\nu}^{ij} = \bar{q}^j_{\mathrm{R}} (\overleftrightarrow{D}_\mu \overleftrightarrow{D}_\nu + \cdots) q^i_{\mathrm{L}})$ | $\Phi_{\mu\nu} \to \mathrm{e}^{-2\mathrm{i}\alpha} U_{\mathrm{L}} \Phi_{\mu\nu} U_{\mathrm{R}}^{\dagger}$ |
| $2^{++}$, ${}^3F_2$ | $a_2(?)$, $K_2^*(?)$, $f_2(?), f_2'(?)$ | $S_{\mu\nu}^{ij} = -\frac{1}{2}\bar{q}^j (\overleftrightarrow{D}_\mu \overleftrightarrow{D}_\nu + \cdots) q^i$ | | |
| $3^{--}$, ${}^3D_3$ | $\rho_3(1690)$, $K_3^*(1780)$, $\omega_3(1670), \phi_3(1850)$ | $\vdots$ | $\vdots$ | $\vdots$ |

# Summary

Hadronic states between 1 and 2 GeV are *very* interesting.

Measurable at BESII, GlueX, CLAS12....PANDA at FAIR.

Classification of higher spin states by *un*broken chiral symmetry?

Heterochiral: where is the spin one multiplet with the $\eta_1$ & $\eta_1$' ?

Spin two multiplet: $J^{PC}$ = 2-+ , esp. $\eta_2(1645)$ , confirm $\eta_2(1870)$ ?

Change in the magnitude of anomalous couplings for heavier states?

Homochiral: like usual quark model.

Analogy of WZNW terms for spin-2?

# Light tetraquarks and the chiral phase transition

V. V. Skokov (RBRC → NCSU) & RDP: 1606.04111

XYZ, $P_c$ states: *strong* evidence for tetraquark & pentaquark states,
composed of both light and *heavy* quarks

*Why* do we need heavy quarks to see tetraquark states?
Jaffe '79…Schechter…Close…Tornqvist…Maiani…Giacosa….Peleaz '15:
"the" $\sigma$ meson is "a" tetraquark (diquark anti-diquark). But situation murky….

Tetraquarks *alter* the phase diagram of QCD
(versus quark mass, plane of temperature T and baryon chemical potential $\mu$)

Tetraquarks: for three (*not* two) flavors of *very* light quarks,
tetraquarks *may* generate a second chiral phase transition

# Are there tetraquarks?

Briceno, Dudek, Edwards, & Wilson, 1607.05900 Lattice: Luscher, distillation...

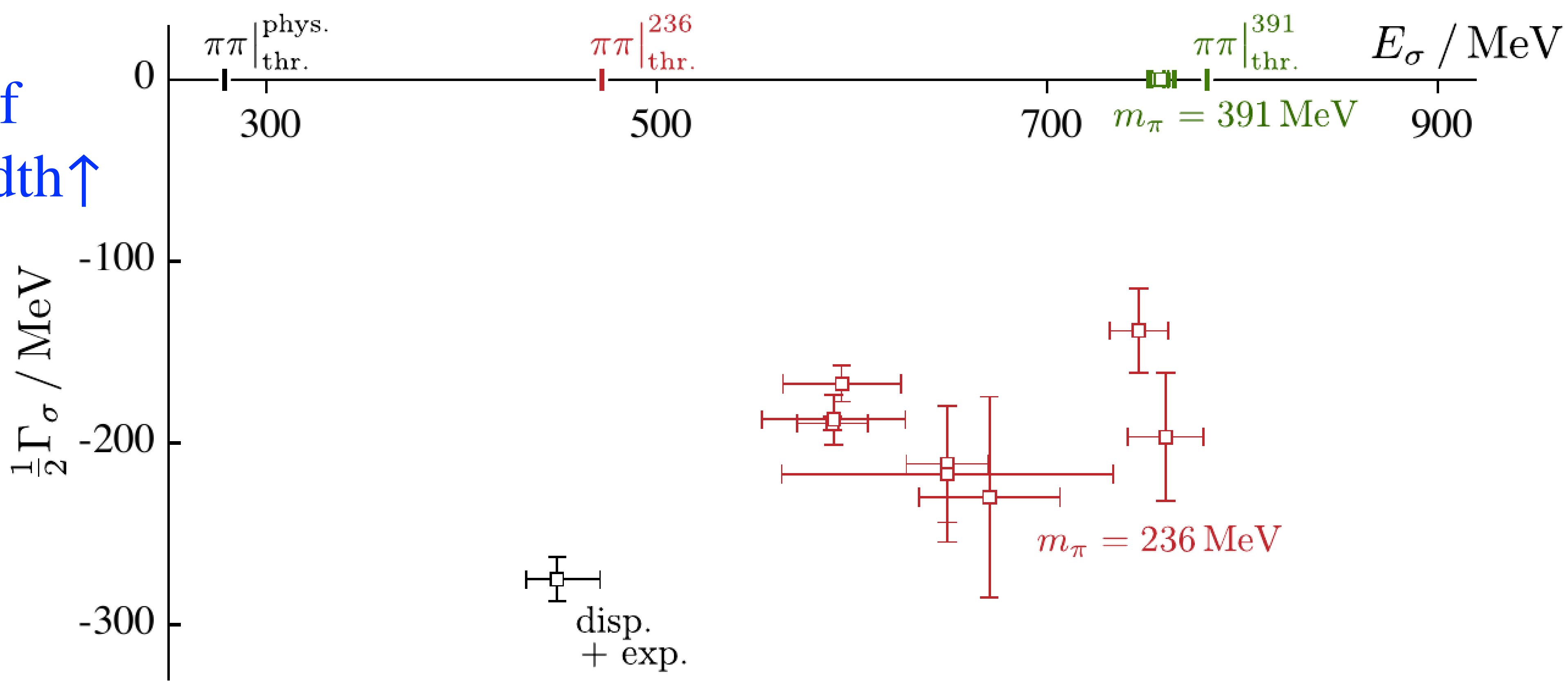
$m_\pi$ = 391 MeV: σ bound state resonance just below ππ threshold

$m_\pi$ = 236 MeV: σ broad resonance, well above ππ threshold

Will assume there are tetraquarks independent of the usual fields

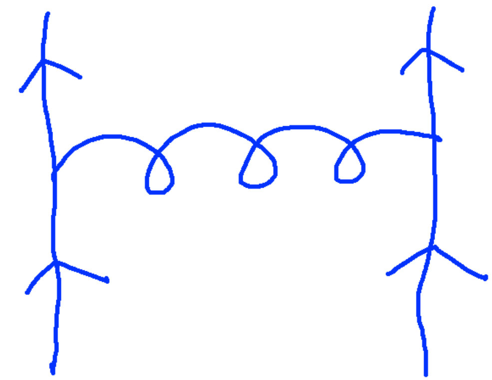
N.B.: near $T_\chi$, $m_\sigma$ may fall below 2 $m_\pi$….

half width↑



# Diquarks and tetraquarks for two flavors

Jaffe '79: most attractive channel for quark-quark scattering is antisym. in *both* flavor and color.



Color: 3 x 3 = <u>3</u> (antisym) + 6 (sym)

Two flavors: 2 x 2 = 1 (antisym) + 3 (sym)

For two flavors diquark is a color triplet, flavor *singlet*,

$$\chi_L^A = \epsilon^{ABC} \; \epsilon^{ab} (q_L^{aB})^T \mathcal{C}^{-1} q_L^{bC}$$

(A, B, C = color; a, b = flavor) Also $\chi_R$. $\chi_L$ and $\chi_R$ *singlets* under $Z(2)_A$.

*One* complex valued tetraquark field:

$$\zeta = (\chi_R^A)^* \chi_L^A$$

# Sigma models and tetraquarks for two flavors

The tetraquark field $\zeta$ is a *singlet* under flavor and $Z(2)_A$.

Split complex $\zeta$ into its real and imaginary parts, $\zeta_r$ and $\zeta_i$.

QCD is even under parity, so only even powers of $\zeta_i$ can appears, forget $\zeta_i$.

But *any* powers of $\zeta_r$ can!

$$\mathcal{V}^A_{\zeta_r} = h_r \zeta_r + m_r^2 \, \zeta_r^2 + \kappa_r \zeta_r^3 + \lambda_r \, \zeta_r^4$$

Hence $\langle \zeta_r \rangle$ is *always* nonzero!

Couplings to φ start with $U(1)_A$ inv.: $\mathcal{V}_{\zeta\Phi} = \kappa \zeta_r \, \mathrm{tr}\, \Phi^\dagger \Phi$

The tetraquark $\zeta_r$ is just a massive field with a v.e.v. . Should *not* affect the chiral phase transition c/o exceptional tuning.

# Tetraquarks for three flavors

Three flavors: 3 x 3 = <u>3</u> + 6 .
Diquark field flavor *anti-triplet*, <u>3</u>

$$\chi_L^{aA} = \epsilon^{abc}\,\epsilon^{ABC}\,(q_L^{bB})^T \mathcal{C}^{-1} q_L^{cC}$$

LR tetraquark field ζ transforms *identically* to Φ under $G_{fl} = SU(3)_L \times SU(3)_R$

$$\zeta^{ab} = (\chi_R^{aA})^* \chi_L^{bA}$$

Under $U(1)_A$, Φ has charge +1, ζ charge -2.

Since ζ & Φ in *same* representation of $G_{fl}$, *direct* mixing term. $Z(3)_A$ invariant:
Black, Fariborz, Schechter ph/9808415 + ….;
’t Hooft, Isidori, Maiani, Polosa 0801.2288 + ….

$$\mathcal{V}^{A}_{\zeta\Phi} = \widetilde{m}^2 \mathrm{tr}\left(\zeta^\dagger \Phi + \Phi^\dagger \zeta\right)$$

An extra *dozen* couplings.
e.g., $U(1)_A$ inv. cubic term:

$$\mathcal{V}^{\infty}_{\zeta\Phi} = \kappa\,\epsilon^{abc}\epsilon^{a'b'c'}\left(\zeta^{aa'}\Phi^{bb'}\Phi^{cc'} + \mathrm{c.c.}\right)$$

# “Mirror” model, T = 0

$\text{Spectrum}: \Phi = \pi, K, \eta, \eta'; a_0, \kappa, \sigma_8, \sigma_0 \quad , \quad \zeta = \widetilde{\pi}, \widetilde{K}, \widetilde{\eta}, \widetilde{\eta}'; \widetilde{a}_0, \widetilde{\kappa}, \widetilde{\sigma}_8, \widetilde{\sigma}_0$

General model has 20 couplings

Fariborz, Jora, & Schechter: ph/0506170; 0707.0843; 0801.2552. Pelaez, 1510.00653

“Mirror” model. $SU(3)_V$ symmetric, so spectrum degenerate octet + singlets:
$\pi = K = \eta \neq \eta'$ etc. $\Phi$ and $\zeta$ start with *identical* couplings

$$\mathcal{V}_\Phi = m^2 \, \mathrm{tr}\Phi^\dagger \Phi - \kappa \, (\det\Phi + \mathrm{c.c.}) + \lambda \, \mathrm{tr}(\Phi^\dagger \Phi)^2 + \ldots$$

$$\mathcal{V}_\zeta = m^2 \, \mathrm{tr}\zeta^\dagger \zeta - \kappa \, (\det\zeta + \mathrm{c.c.}) + \lambda \, \mathrm{tr}(\zeta^\dagger \zeta)^2 + \ldots$$

Assume *only* $\zeta\Phi$ coupling is mass term: $\mathcal{V}^A_{\zeta\Phi} = -\widetilde{m}^2 \, \mathrm{tr}(\zeta^\dagger \Phi + \Phi^\dagger \zeta)$

Simple, because $\zeta\Phi$ coupling mixes: $\pi \leftrightarrow \widetilde{\pi} \quad , \quad \eta' \leftrightarrow \widetilde{\eta}' + \ldots$

# Spectrum of the mirror model

In the chiral limit, the mass eigenstates:

$$\pi, \widetilde{\pi} = 0\,,\; 2\widetilde{m}^2 \;,\; \eta', \widetilde{\eta}' = 3\kappa\phi\,,\; 3\kappa\phi + 2\widetilde{m}^2$$

$$a_0, \widetilde{a}_0 = m^2 + \kappa\phi + 6\lambda\phi^2 \pm \widetilde{m}^2 \;,\; \sigma_0, \widetilde{\sigma}_0 = m^2 - 2\kappa\phi + 6\lambda\phi^2 \pm \widetilde{m}^2$$

*All* states are mixtures of Φ and ζ. Of course 8 Goldstone bosons.
Satisfy *generalized* 't Hooft relation ($SU(3)_V$ symmetric)

$$m^2_{\eta'} + m^2_{\widetilde{\eta}'} - m^2_{\pi} - m^2_{\widetilde{\pi}} = m^2_{a_0} + m^2_{\widetilde{a}_0} - m^2_{\sigma} - m^2_{\widetilde{\sigma}}$$

*Even* with same couplings, all masses are *split* by the mixing term.

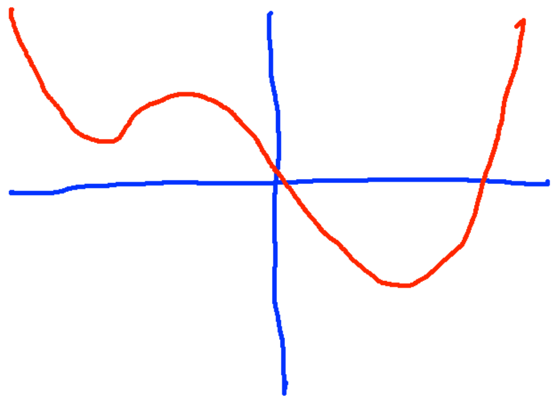
At nonzero T, the thermal masses of the Φ and ζ can*not* be equal!

# Chiral transition for three flavors, no tetraquark

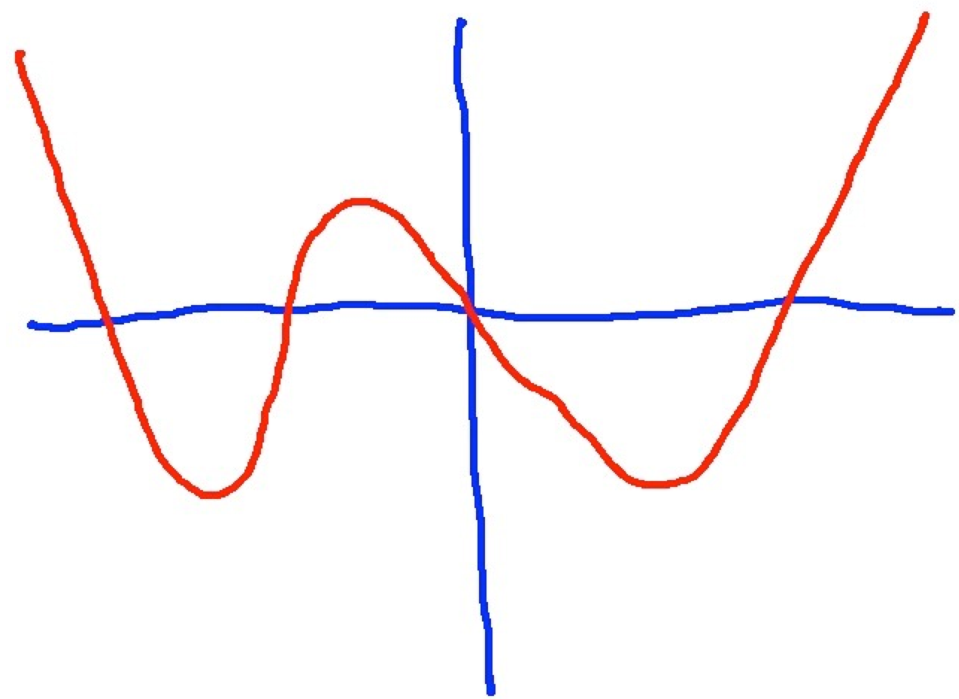
Cubic terms *always* generate *first* order transitions.

$$\mathcal{V} = \frac{m^2}{2}\phi^2 - \kappa\phi^3 + \lambda\phi^4$$

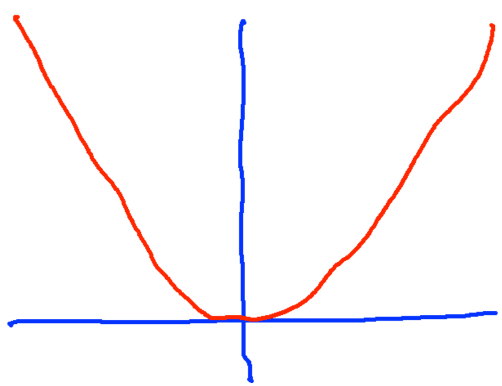
$T = 0$: $m^2 < 0$, $\langle\varphi\rangle \neq 0$ =>



$T = T_\chi$: can*not* flatten the potential =>



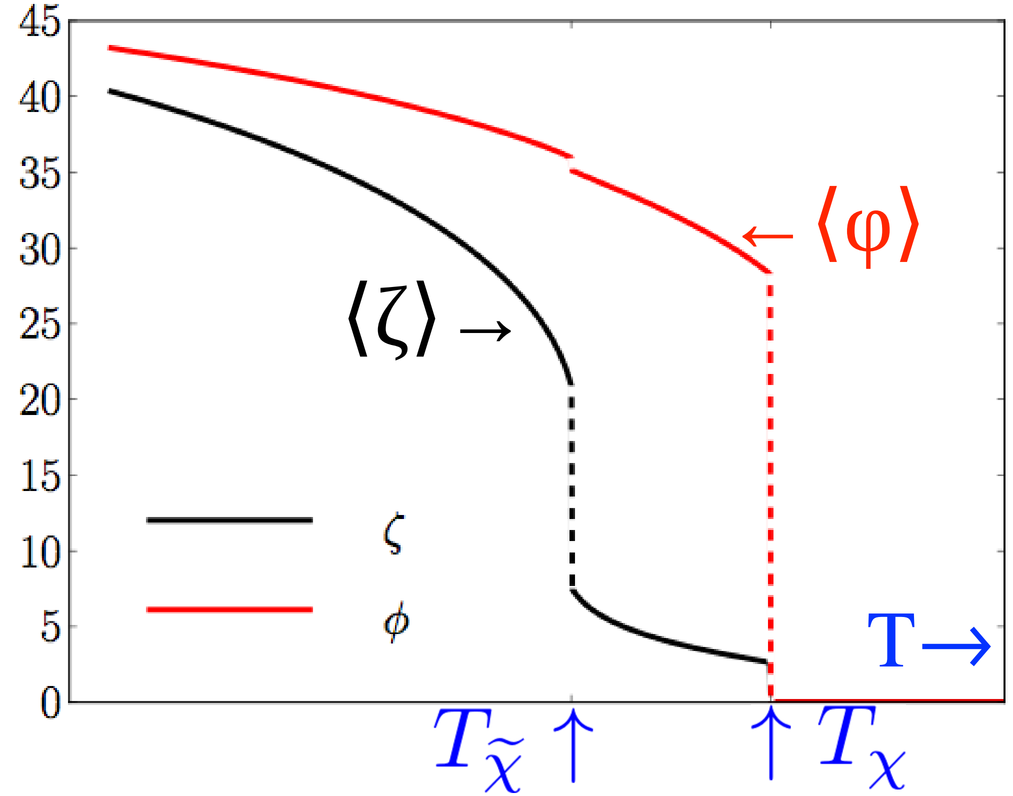
$T \gg f_\pi$: $m^2 > 0$, $\langle\varphi\rangle = 0$ =>



At $T_\chi$, two degenerate minima, with barrier between them. Transition is first order.

# With tetraquarks, *maybe* two chiral transitions

In chiral limit, *may* have have *two* chiral phase transitions. =>

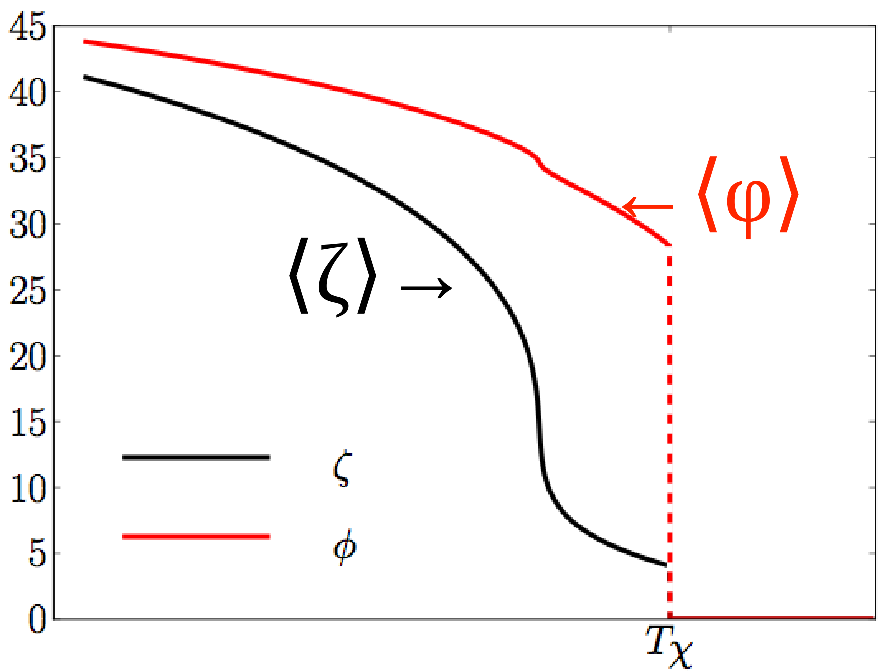



At first, both jump, remain nonzero.
At second, both jump to zero.

$$m_\Phi^2(T) = 3T^2 + m^2$$

$$m_\zeta^2(T) = 5T^2 + m^2$$

$$\widetilde{m}^2 = (100)^2$$




<= Also possible to have single chiral phase transition, tetraquark crossover

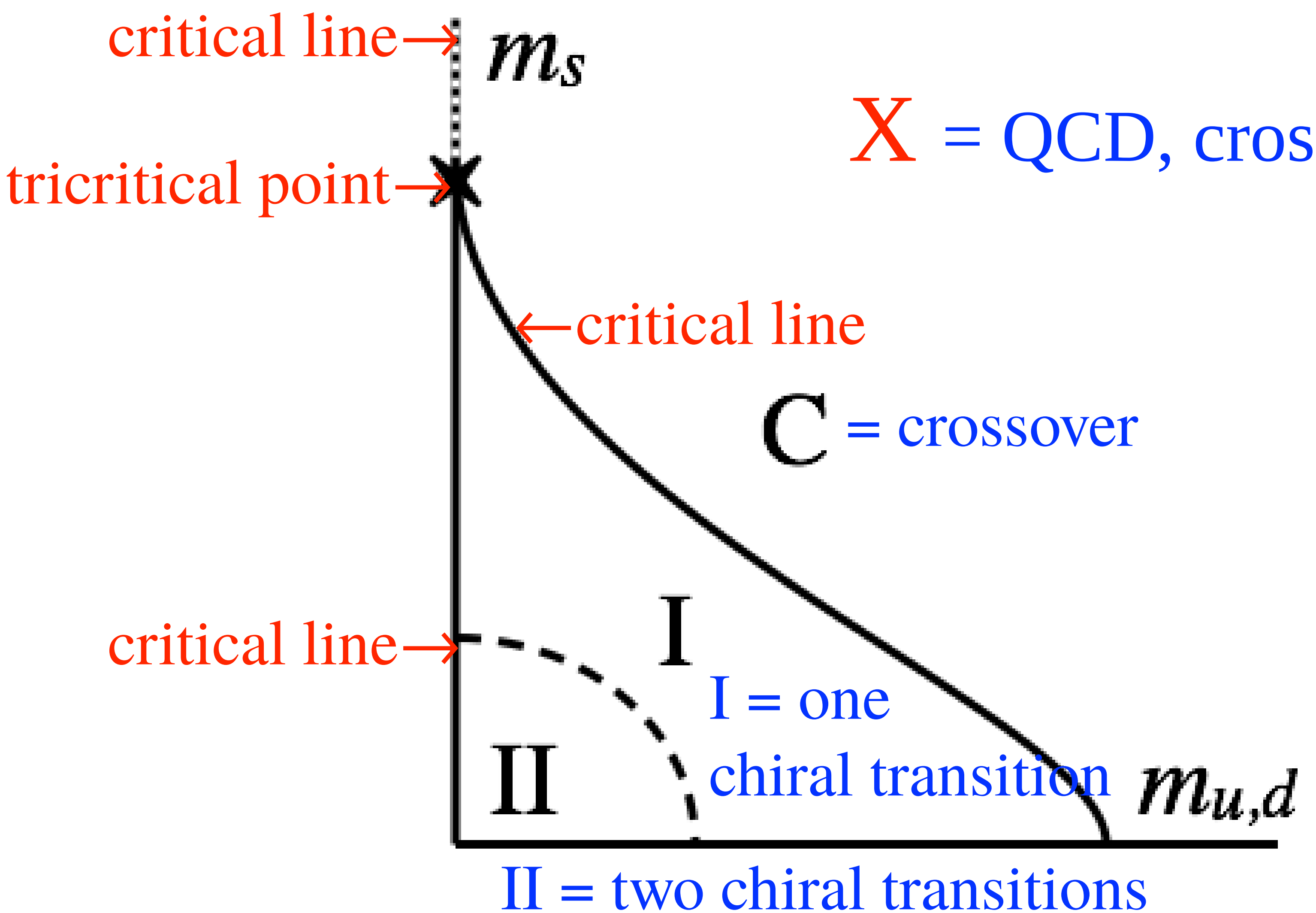
$$\widetilde{m}^2 = (120)^2$$

# "Columbia" phase diagram for light quarks

Lattice: chiral transition crossover in QCD

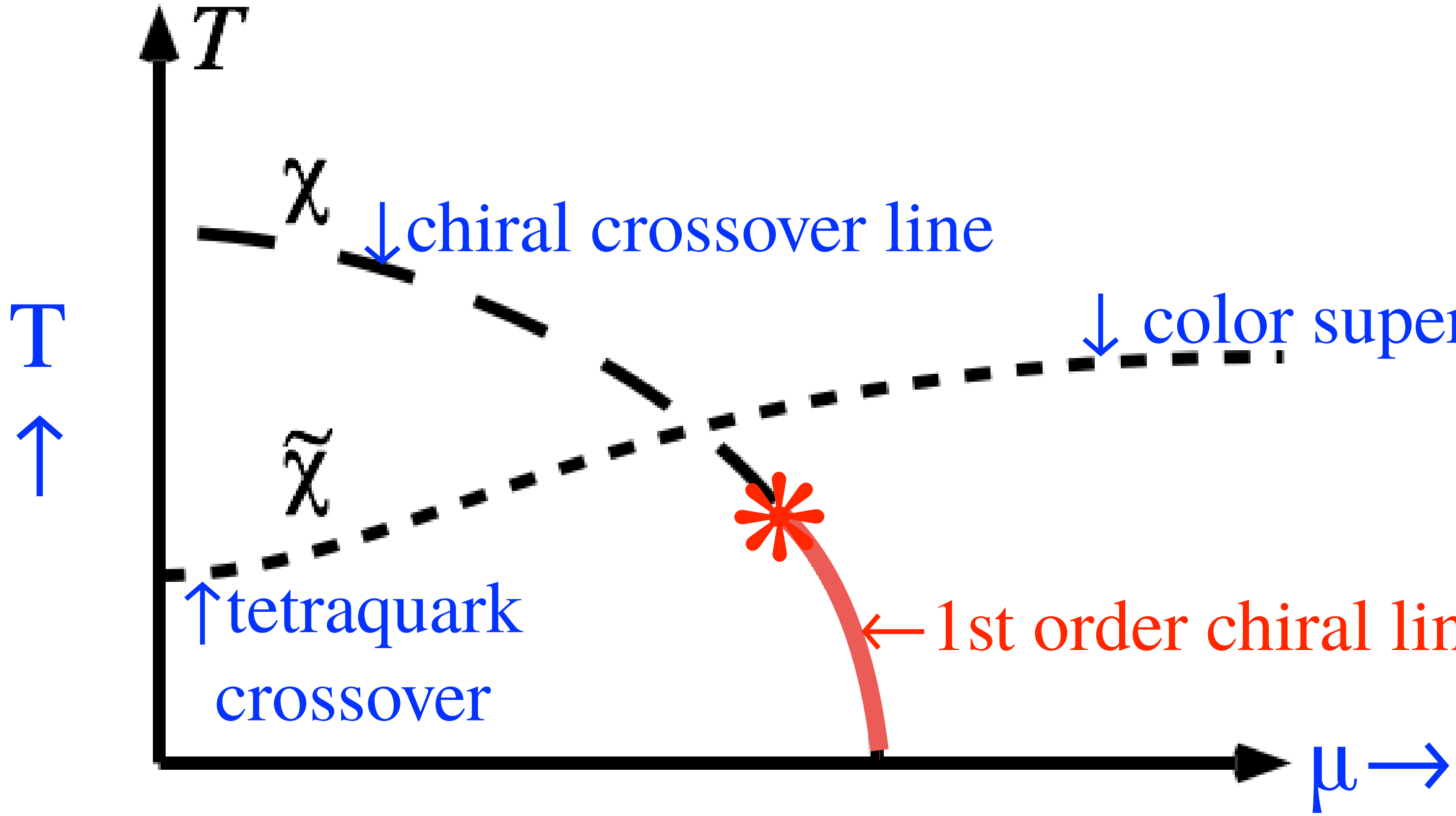
If two chiral phase transitions for three massless flavors, persists for nonzero mass

Implies new phase diagram in the plane of $m_u = m_d$ versus $m_s$:



# Tetraquarks in the plane of T and μ

Diquark fields are *identical* to the order parameters for color superconductivity.
Tetraquark condensate = gauge invariant square of CS condensate. Suggests:




Line for chiral crossover *might* end, meet line for *first* order chiral transition at *Critical EndPoint* (CEP). Massless σ at CEP. Rajagopal, Shuryak & Stephanov, '99

In *effective* models, to find the CEP, *must* include tetraquarks: need the *right* σ!

# *Four* flavors, three colors: *hexa*quarks

Diquark 2-index antisymmetric tensor:

$$\chi_L^{(ab)A} = \epsilon^{abcd}\epsilon^{ABC}\ (q_L^{cB})^T \mathcal{C}^{-1} q_L^{dC}$$

So LR tetraquark is same:

$$\zeta^{(ab),(cd)} = (\chi_R^{(ab)A})^\dagger \chi_L^{(cd)A}$$

Tetraquark couples to usual Φ through cubic, quadratic terms, *so what*.

Instead, consider *tri*quark field:

$$\chi_L^a = \epsilon^{abcd}\epsilon^{ABC} q_L^{bA} (q_L^{cB})^T \mathcal{C}^{-1} q_l^{dC}$$

Triquark is a color singlet, fundamental rep. in flavor.
Hence a LR *hexa*quark field is just like the usual Φ,
and mixes *directly* with it.

$$\xi^{ab} = (\chi_R^a)^\dagger \chi_L^b$$

Analysis for general numbers of flavors and colors is *not* trivial.
Like color superconductivity.