Resummation 1

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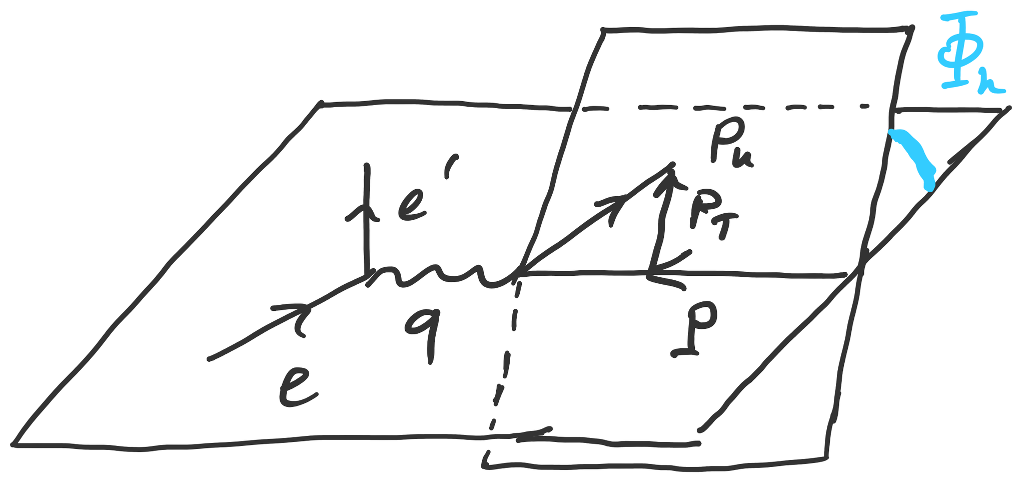
Resummation of azimuthal modulations in unpolarized SIDIS and Drell-Yan ( $\cos\phi$ and $\cos 2\phi$ ) in collaboration with A. Bacchetta, G. Bozzi, C. Pisano, M. Radici

Motivation:

Let us consider Semi Inclusive Deep Inelastic Scattering in $\gamma^* P$ center of mass frame

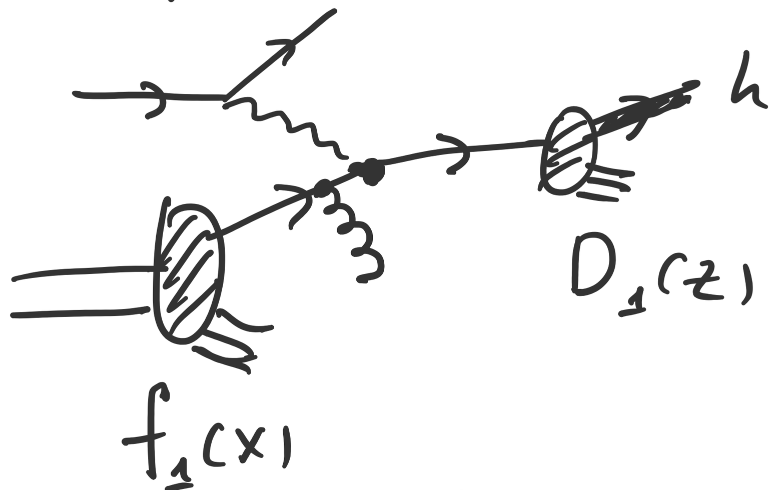



$$e P(S) \to e' h X$$

$$\frac{d^5\sigma}{dx\, dy\, dz\, d\phi_h\, dP_T^2} = \frac{2\pi\alpha^2}{x y Q^2}\left(1 - y + \tfrac{1}{2}y^2\right) F_{UU}\left\{1 + \cos\phi_h \frac{(2-y)\sqrt{1-y}}{1-y+\frac{1}{2}y^2} A_{UU}^{\cos\phi} + \cos 2\phi_h \frac{(1-y)}{1-y+\frac{1}{2}y^2} A_{UU}^{\cos 2\phi}\right\}$$

Here $x, y, z$ are usual SIDIS variables, $F_{UU}$ is the unpolarized structure function, and $\underbrace{A_{UU}^{\cos\phi} = \frac{F_{UU}^{\cos\phi}}{F_{UU}},\ A_{UU}^{\cos 2\phi} = \frac{F_{UU}^{\cos 2\phi}}{F_{UU}}}_{\text{azimuthal modulations}}$

Structure functions can be calculated in large $P_T$ limit by using collinear approximation. Usually $hP$ center of mass frame is used, so that virtual photon will have transverse momentum $\vec{q}_T \simeq -\vec{P}_T/z$




$\vec{q}_T$ is generated by recoil off the gluon

To the order $\alpha_s$ one has $F_{UU} = \frac{1}{q_T^2}\,\frac{\alpha_s}{2\pi^2 z^2}\sum_q x e_q^2\left(f_1(x) D_1(z)\, L\!\left(\frac{Q^2}{q_T^2}\right) + \text{„ } f_1 \otimes D_1 \text{ convolutions"}\right)$

here $L(Q^2/q_T^2) = \underbrace{2C_F \ln Q^2/q_T^2 - 3C_F}_{\text{diverges at small } q_T \to 0}$, $C_F = \frac{N_c^2 - 1}{2N_c}$, $N_c = 3$

in addition, at each higher order $\alpha_s^n$, these $L(Q^2/q_T^2)$ terms are present and thus must be dealt with.

The technique of taking care of those terms is well known and is called resummation. Resummation sums all these logs up to all orders in $\alpha_s$.

$$F_{UU}^{\cos\phi} = -\frac{1}{Q\, q_T}\,\frac{\alpha_s}{2\pi^2 z^2}\sum x e_q^2\left(f_1(x) D_1(z)\, L\!\left(\frac{Q^2}{q_T^2}\right) + \text{„ } f_1 \otimes D_1 \text{ convolutions "}\right)$$

$$F_{UU}^{\cos 2\phi} = \frac{1}{Q^2}\,\frac{\alpha_s}{2\pi^2 z^2}\sum x e_q^2\left(f_1(x) D_1(z)\, L\!\left(\frac{Q^2}{q_T^2}\right) + \text{„ } f_1 \otimes D_1 \text{ convolutions "}\right)$$

To simplify my discussion, I will limit to „next-to-leading-log" accuracy and not discuss „convolution terms".

Resummation is performed in Fourier conjugate (to $q_T$) space and uses results of an important theorem proven by Anatoly Radyushkin and Gregory Korchemsky that states that all soft gluon radiation exponentiates and has a definite form at all orders. Technically resummation was developed by many people including Collins, Soper, Sterman.

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Using
$$\int d^2b \; e^{-i\vec{b}\vec{q}_T} \ln^2 \frac{b^2Q^2}{b_0^2} = -\frac{8\pi}{q_T^2} \ln \frac{Q^2}{q_T^2}, \quad \text{where } b_0 = 2e^{-\gamma_E}$$
$$\int d^2b \; e^{-i\vec{b}\vec{q}_T} \ln \frac{b^2Q^2}{b_0^2} = -\frac{4\pi}{q_T^2}$$

We obtain
$$F_{UU} = +\frac{1}{z^2}\sum_q x e_q^2 f_1(x) D_1(z) \int \frac{d^2b}{(2\pi)^2} e^{-i\vec{b}\vec{q}_T} \underbrace{e^{-S_p(b)}}_{\text{Sudakov form factor}}, \quad \text{where } S_p(b) = \frac{\alpha_s}{2\pi}\Big(\underbrace{C_F \ln^2 \frac{b^2Q^2}{b_0^2}}_{\text{the famous double log}} - 3C_F \ln \frac{b^2Q^2}{b_0^2}\Big)$$

Such that expending $e^{-S_p}$ we recover perturbative result

Let us rewrite it using $\int_0^{2\pi} d\varphi \, e^{-i\vec{b}\vec{q}_T} = J_0(q_T b)\frac{1}{2\pi}$
$$F_{UU} = \frac{1}{z^2}\sum_q x e_q^2 \int \frac{b\,db}{2\pi} J_0(bq_T) f_1(x) D_1(z) e^{-S(b)}$$

The natural question is:

How to resum $F_{UU}^{\cos\phi}$ and $F_{UU}^{\cos 2\phi}$ ?

Notice different analytical structure for $F_{UU}$, $F_{UU}^{\cos\phi}$, $F_{UU}^{\cos 2\phi}$ in terms of $\frac{1}{q_T^2}$ .

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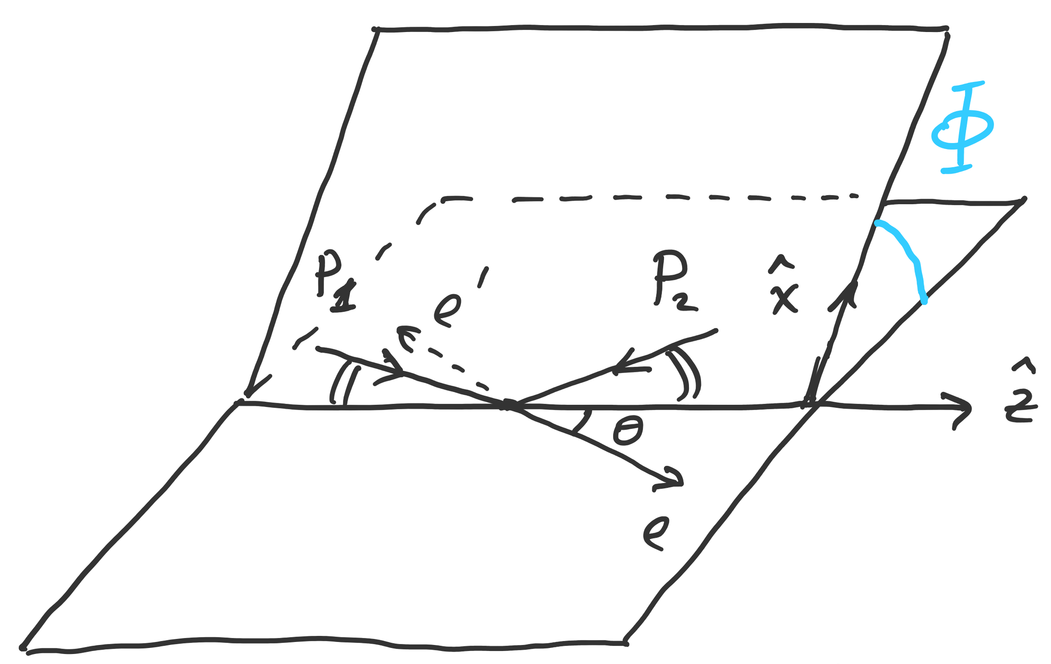
Drell-Yan $\quad P_1 P_2 \to e\bar{e} X$

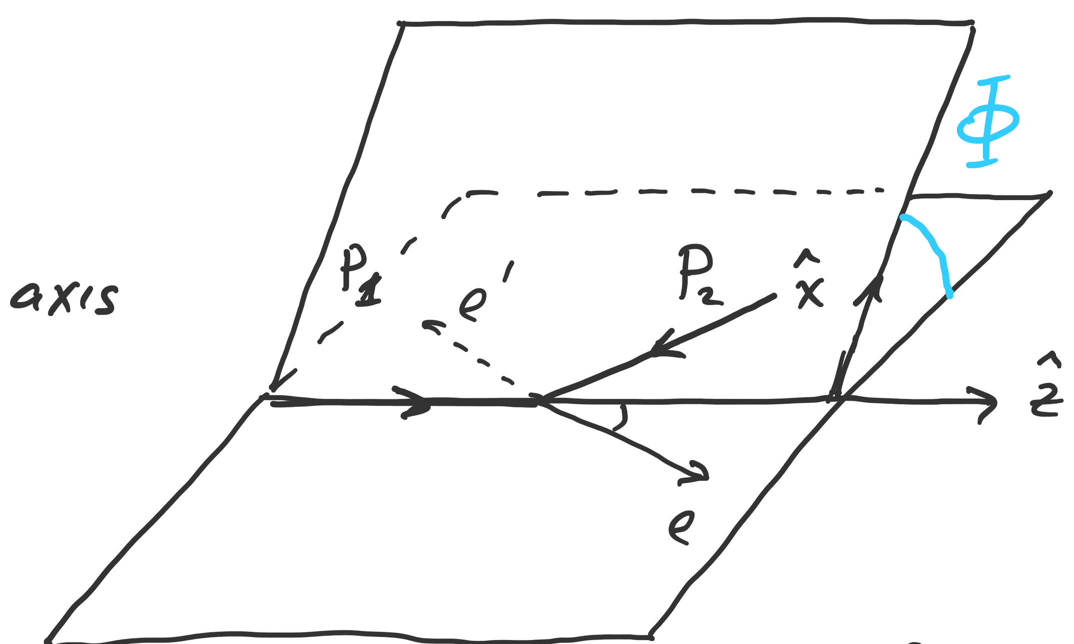
The angular distribution of the Drell-Yan cross-section is most conveniently considered in dilepton rest frame like the Collins-Soper frame or Gottfried-Jackson frame




Collins-Soper (CS)

Connected by rotation around y axis

$\longleftrightarrow$




Gottfried-Jackson (GJ)

The cross section reads

$$\frac{d\sigma}{\underbrace{d\Omega}_{\text{orientation of leptons}} d^4 q} = \frac{\alpha^2}{2(2\pi)^4 Q^2 s^2} \left\{ W_T (1+\cos^2\theta) + W_L (1-\cos^2\theta) + W_\Delta \sin 2\theta \cos\phi + W_{\Delta\Delta} \sin^2\theta \cos 2\phi \right\}$$

Structure functions in two frames are related by a matrix transformation. See e.g. Boer, Vogelsang 2006

Again one can calculate structure functions at large $q_T$ using collinear approximation

$$W_{T,CS} = \frac{\alpha_s}{2\pi} \frac{Q^2}{q_T^2} L\left(\frac{Q^2}{q_T^2}\right) \sum_q e_q^2 q(x_1) \bar{q}(x_2) + \text{„convolutions"}$$

$$W_{L,CS} = 2 W_{\Delta\Delta,CS} = \frac{\alpha_s}{2\pi} L\left(\frac{Q^2}{q_T^2}\right) \sum_q e_q^2 q(x_1) \bar{q}(x_2) + \text{„convolutions"}$$

$$W_{\Delta,CS} = \frac{\alpha_s}{2\pi} \frac{Q}{q_T} \left( \text{no } \log\left(\frac{Q^2}{q_T^2}\right) \text{terms} + \text{„convolutions"} \right)$$

and

$$W_{T,GJ} = W_{T,CS}$$

$$W_{L,GJ} = 2 W_{\Delta\Delta,GJ} = 2 \frac{\alpha_s}{2\pi} L\left(\frac{Q^2}{q_T^2}\right) \sum_q e_q^2 q(x_1) \bar{q}(x_2) + \text{„convolutions"}$$

$$W_{\Delta,GJ} = \frac{\alpha_s}{2\pi} \frac{Q}{q_T} \left( L\left(\frac{Q^2}{q_T^2}\right) \sum_q e_q^2 q(x_1) \bar{q}(x_2) + \text{„convolutions"} \right)$$

Strikingly $W_\Delta$ are very much different in CS and GJ frames

# Resummation4

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Boer, Vogelsang: „Resummation is different from CSS one. Not only splitting functions differ, but also $W_\Delta$ even does not have Sudakov form factor"

„We do not address full NLL resummation in this work, but note that techniques that go beyond collinear factorization should prove useful."

Berger, Qiu, Rodriguez-Pedraza: „Is it possible that a different kind of resummation would handle the non-physical divergence at $q_T=0$ in $W_\Delta$? We do not have an answer to this question in collinear QCD factorization formalism. However, we might gain insight from another prospective - starting from transverse momentum dependent quark-antiquark annihilation."

Boer, Vogelsang: „We finally emphasize again that we hope that our study will provide motivation for a development of the full NLL resummation for the structure functions $W_\perp$, $W_\Delta$, $W_{\Delta\Delta}$."

Indeed it is interesting from both theoretical and phenomenological points of view.

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Let me introduce TMD formalism in SIDIS

Structure functions are convolutions of Transverse Momentum Dependent distribution and fragmentation functions that obey Collins-Soper evolution equations

Convolution:

$$C \equiv \mathbb{C}(wfD) = x\sum_q e_q^2 \int \underbrace{d^2k_\perp\, d^2p_\perp}_{\text{integration over unobserved quark momenta}}\, \delta^{(2)}(\underbrace{z\vec{k}_\perp + \vec{p}_\perp}_{\text{Final hadron } \vec{P}_T \text{ results from incoming } (\vec{k}_\perp) \text{ and outgoing } (\vec{p}_\perp) \text{ quark momenta.}} - \vec{P}_T)\, w(k_\perp, p_\perp)\, f(x, k_\perp^2)\, D(z, p_\perp^2)$$

For instance

$$F_{uu} = C(f_1 D_1) = x\sum_q e_q^2 \int d^2k_\perp\, d^2p_\perp\, \delta^{(2)}(z\vec{k}_\perp + \vec{p}_\perp - \vec{P}_T)\, f_1(x, k_\perp^2)\, D_1(z, p_\perp^2) =$$
$$= x\sum_q e_q^2 \int d^2k_\perp \frac{d^2p_\perp}{z^2}\frac{d^2b}{(2\pi)^2}\, e^{i\vec{b}(\vec{k}_\perp + \frac{\vec{p}_\perp}{z} - \frac{\vec{P}_T}{z})}\, f_1(x, k_\perp^2)\, D_1(z, p_\perp^2) =$$
$$= 2\pi x \sum_q e_q^2 \int db\, b\, J_0\left(\frac{b P_T}{z}\right) \tilde{f}_1(x, b^2)\, \tilde{D}_1(z, b^2)$$

Relevant solutions of CS equations valid at small $b$ (large-$q_T$) are

$$\tilde{f}_1(x, b^2) = \frac{1}{2\pi}\underbrace{f_1(x)}_{\text{collinear pdf}}\, \underbrace{e^{-S(b)/2}}_{\text{Sudakov form factor}} \qquad \tilde{D}_1(z, b^2) = \frac{1}{2\pi}\frac{D_1(z)}{z^2}\, e^{-S(b)/2}$$

Check with $\mu_b$ and calculate $S(b)$ explicitely

And thus we obtain

$$F_{uu} = \sum_q e_q^2 \frac{x}{z^2} f_1(x)\, D_1(z) \int \frac{db\, b}{2\pi} J_0\left(\frac{P_T b}{z}\right) e^{-S(b)}$$

the same result as in perturbative QCD

Now let us examine $F_{uu}^{\cos\phi}$:

$$F_{uu}^{\cos\phi} = \frac{2M}{Q}\, C\left[\frac{\hat{h}\cdot\vec{p}_\perp}{z\, m_h}\left(x h H_1^\perp + \frac{m_h}{M} f_1 \frac{\tilde{D}_1}{z}\right) - \frac{\hat{h}\cdot\vec{k}_\perp}{M}\left(x f^\perp D_1 + \frac{m_h}{M} h_1^\perp \frac{\tilde{H}}{z}\right)\right]$$

here $\hat{h} = \frac{\vec{P}_T}{P_T}$ and $xh$, $\tilde{D}/z$, $xf^\perp$, $\tilde{H}/z$ are twist-3 functions, $f_1, D_1, H_1^\perp, h_1^\perp$ are twist-2 functions

We can use QCD equation of motion and Wandzura-Wilczek approximation and derive

$$F_{uu}^{\cos\phi} \simeq \underbrace{\frac{2M}{Q}\, \mathbb{C}\left(-\frac{\hat{h}\vec{k}_\perp}{M} f_1 D_1\right)}_{\text{Also known as Cahn contribution}}$$

We obtain

$$F_{uu}^{\cos\phi} = -2\sum_q e_q^2 x \int d^2k_\perp\, d^2p_\perp\, \frac{\hat{h}\vec{k}_\perp}{Q} f_1(x, k_\perp^2)\, D_1(z, p_\perp^2)\, \delta^{(2)}(z\vec{k}_\perp + \vec{p}_\perp - \vec{P}_T) =$$
$$= -4\pi \frac{M^2}{Q}\sum_q e_q^2 x \int db\, b^2 J_1\left(\frac{bP_T}{z}\right) \tilde{f}_1^{(1)}(x, b^2)\, \tilde{D}_1(z, b^2)$$

where $\tilde{f}_1^{(1)}(x, b^2) \equiv -\frac{1}{M^2}\frac{\partial}{b\,\partial b}\tilde{f}_1(x, b^2)$ (see Gamberg, Boer, Musch, Prokudin 2010)

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Expanding at large $q_T$ we obtain

$$F_{UU}^{\cos\phi} = 4\pi \frac{1}{Q}\sum_q e_q^2\, x \int db\, b^2 J_1\left(\frac{bP_T}{z}\right)\frac{f_1(x)D_1(z)}{(2\pi)^2 z^2}\, e^{-S(b)}\left(\frac{\partial}{2b\partial b}S(b)\right) = -\frac{1}{Q q_T}\frac{\alpha_s}{2\pi^2 z^2}\sum_q x\, e_q^2 f_1(x) D_1(z)\, L\left(\frac{Q^2}{q_T^2}\right)$$

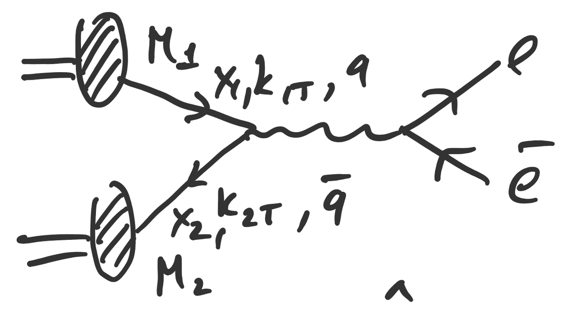
at first order in $\alpha_s$ — the same as perturbative result

Similar procedure reveals correct result for $F_{UU}^{\cos 2\phi}$

Let us return now to Drell-Yan and consider $W_\Delta$ structure function ( $\cos\phi$ )

Using Lu, Schmidt 2011 we can write

$$W_\Delta \equiv F_{UU}^{\cos\phi} = \frac{2}{Q}\,\mathcal{C}\left[(\hat h\cdot \vec k_{1T})\left(\hat f_1^\perp \bar f_1 - \frac{M_2}{M_1} h_1^\perp \hat{\bar h}\right) - (\hat h\cdot\vec k_{2T})\left(f_1 \hat{\bar f}^\perp - \frac{M_1}{M_2}\hat h\, \bar h_1^\perp\right)\right]$$




$f_1^\perp$, $h$ are twist-3 functions, $f_1$, $h_1^\perp$ are twist-2 functions

and we have

$$\left.\begin{aligned}\hat f &\equiv x_1\big((1-c)f + c\,\underbrace{\tilde f}_{\text{twist-3 function}}\big)\\ \hat{\bar f} &\equiv x_2\big(c\bar f + (1-c)\tilde{\bar f}\big)\end{aligned}\right\}\ \text{where}$$

$c=0$ in Gottfried-Jackson frame

$c=1/2$ in Collins-Soper frame

Again using EOM & WW we obtain

$$F_{UU}^{\cos\phi} = \frac{2}{Q}\,\mathcal{C}\left[(\hat h\cdot\vec k_{1T})(1-c)f_1\bar f_1 - (\hat h\cdot \vec k_{2T})\,c\, f_1\bar f_1\right]$$

Here in case of Drell-Yan we have

$$\mathcal{C}(\omega f g) \equiv \sum_a e_a^2\int d^2k_{1T}\, d^2k_{2T}\,\delta^{(2)}(\vec q_T - \vec k_{1T} - \vec k_{2T})\,\omega(k_{1T},k_{2T})(f\bar g + \bar f g)$$

So that

$$F_{UU}^{\cos\phi} = \frac{2}{Q}\sum_q e_q^2\int d^2k_T\, d^2k_{2T}\frac{d^2b}{(2\pi)^2}\, e^{i\vec b(\vec q_T - \vec k_{1T}-\vec k_{2T})}\left((\hat h\cdot\vec k_{1T})(1-c)\, f_1(x_1,k_{1T}^2)\bar f_1(x_2,k_{2T}^2) - (\hat h\cdot \vec k_{2T})\, c\, f_1(x_1,k_{1T}^2)\bar f_1(x_2,k_{2T}^2)\right) =$$

$$= -\frac{4\pi M^2}{Q}\sum_q e_q^2\int db\, b^2 J_1(bq_T)\left[(1-c)\tilde f_1^{(1)}\tilde{\bar f}_1 - c\,\tilde f_1\overset{\text{F.T.}}{\tilde{\bar f}_1^{(1)}}\right]$$

and again we have

$$\tilde f_1 \equiv \int\frac{d^2b}{2\pi}e^{-i\vec b\vec k_T} f_1(x,k_T^2)$$

$$\tilde f_1^{(1)} \equiv -\frac{1}{M^2}\frac{\partial}{b\partial b}\tilde f_1(x,b^2)$$

Now using $\tilde f_1(x,b) = \frac{f_1(x)}{2\pi}e^{-S(b)/2}$ we obtain

$$F_{UU}^{\cos\phi} = -\frac{4\pi M^2}{Q}\sum_q e_q^2 f_1(x_1)\bar f_1(x_2)\,(1-2c)\,\frac{\alpha_s}{2\pi q_T}\frac{1}{M^2}\frac{1}{(2\pi)^2}L\left(\frac{Q^2}{q_T^2}\right) =$$

$$= -\frac{\alpha_s}{2\pi^2 Q q_T}L\left(\frac{Q^2}{q_T^2}\right)(1-2c)\sum_q e_q^2 f_1(x_1)\bar f_1(x_2)$$

when $c=1/2$, CS we have no log terms
$c=0$, GJ we have log terms

} In accordance with perturbative calculations

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We conclude that resummation formulas in TMD formalism we obtained, in particular

SIDIS

$$F_{UU}^{\cos\phi} = -4\pi \frac{M^2}{Q} \sum_q e_q^2 \, x \int db \; b^2 J_1\left(\frac{b P_T}{z}\right) \tilde{f}_1^{(1)}(x, b^2) \, \tilde{D}_1(z, b^2)$$

$$F_{UU}^{\cos 2\phi} = 2\pi \frac{M^4}{Q^2} \sum_q e_q^2 \, x \int db \; b^3 J_2\left(\frac{b P_T}{z}\right) \underbrace{\tilde{f}_1^{(2)}(x, b^2)}_{\text{second derivative}} \, \tilde{D}_1(z, b^2)$$

Drell-Yan

$$F_{UU}^{\cos\phi} = -\frac{4\pi M^2}{Q} \sum_q e_q^2 \int db \; b^2 J_1(b q_T) \left[ (1-c) \tilde{f}_1^{(1)}(x_1, b^2) \tilde{\bar{f}}_1(x_2, b^2) - c \tilde{f}_1(x_1, b^2) \tilde{\bar{f}}_1^{(1)}(x_2, b^2) \right]$$

$c = 1/2$ Collins Soper

$c = 0$ Gottfried-Jackson