Theory Center Cake Seminar, Jefferson Lab
January 31, 2018

Explore hadron's structure using ab initio lattice QCD calculations

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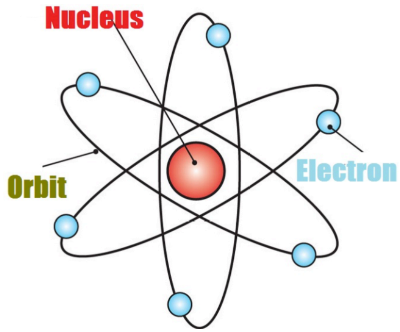
Theory Center, Jefferson Lab

Based on work done with
T. Ishikawa, Y.-Q. Ma, S. Yoshida, …
and work by many others, …

From QED to QCD, …

❑ From Atomic Structure to Nano-Science:

Atom:

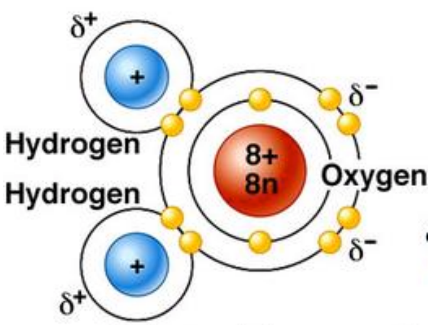


✧ **Tiny, but heavy, nucleus:**
less than 1 trillionth in volume of an atom
slow-moving mass and charge centers

✧ **Light, but fast, electrons:**
quantum probability, …

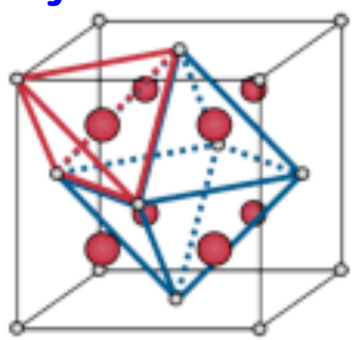
✧ **Massless, charge neutral photons:**
localized charges, …

Molecule:



"Water"

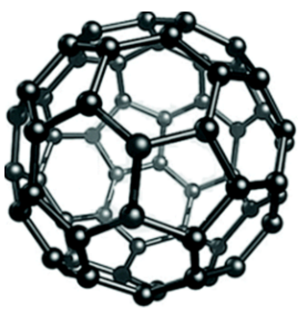
Crystal:



Rare-Earth metal

Nanomaterial:

Carbon-based



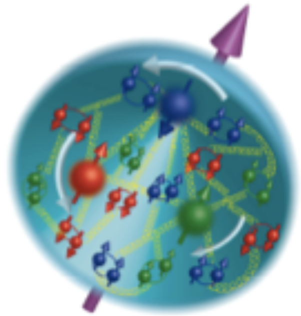
Fullerene

Infinite opportunities to create & improve … !

From QED to QCD, …

❑ From Hadron Structure to Femto-Science:

Proton:

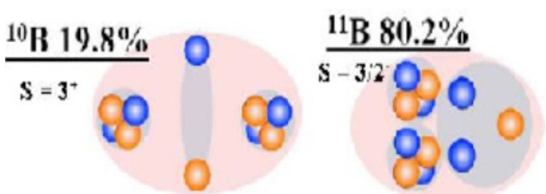


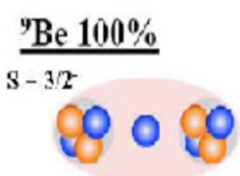
✧ **Extremely light and fast quarks:**
No still picture of the structure, …
fluctuation, quantum probability, …

✧ **Massless, but charged gluons:**
non-local charge, …

✧ **Heavy quarks:**
"localized" charges, …

Nuclei – "Molecule"

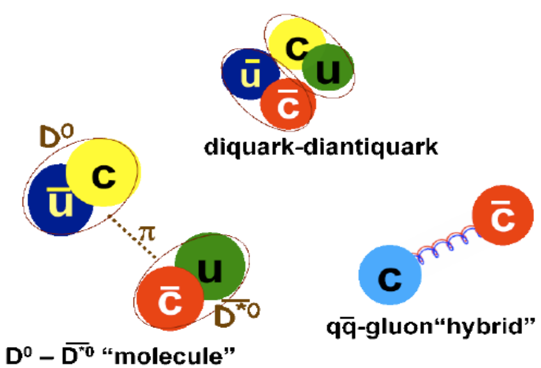




Short-range correlation

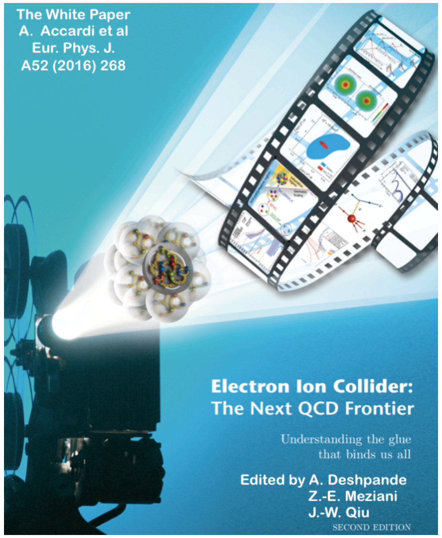
"Light-flavor"

XYZ – "Nuclei"



"Heavy-flavor"

"Femtography"



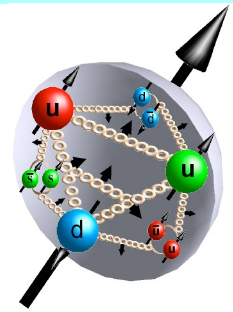
New frontier of hadron physics … !

Outline of the rest of my talk

- ❑ How to quantify hadron structure in QCD?
- ❑ How to "see" hadron structure in experiment?
- ❑ How to calculate hadron structure in QCD?
- ❑ How to explore hadron structure using lattice QCD calculations?
- ❑ Summary and outlook

Hadron structure in QCD

❑ What do we need to know for the structure?



✧ **In theory:** $\langle P,S|\mathcal{O}(\overline{\psi},\psi,A^\mu)|P,S\rangle$ – Hadronic matrix elements

with all possible operators: $\mathcal{O}(\overline{\psi},\psi,A^\mu)$

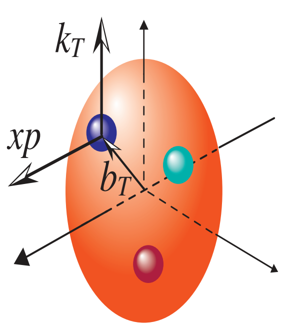
✧ **In fact:** *None of these matrix elements is a direct physical observable in QCD – color confinement!*

✧ **In practice:** Accessible hadron structure
= hadron matrix elements of quarks and gluons, which

1) can be related to physical cross sections of hadrons and leptons with controllable approximation; and/or
2) can be calculated in lattice QCD

❑ Single-parton structure "seen" by a short-distance probe:

✧ **5D structure:**

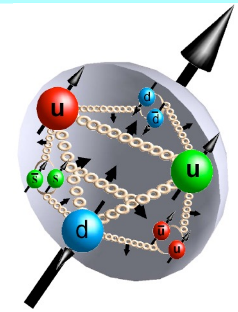


1) $\int d^2b_T$ ⟹ $f(x,k_T,\mu)$ – **TMDs:** *2D confined motion!*

2) $\int d^2k_T$ ⟹ $F(x,b_T,\mu)$ – **GPDs:** *2D spatial imaging!*

3) $\int d^2k_T d^2b_T$ ⟹ $f(x,\mu)$ – **PDFs:** *Number density!*

Hadron structure in QCD



❑ What do we need to know for the structure?

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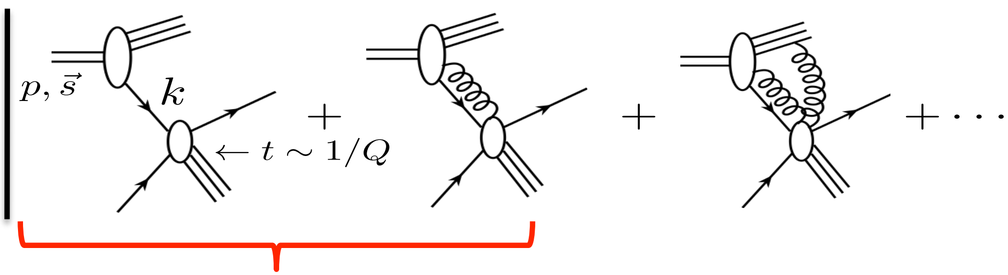
❑ Multi-parton correlations:

$\sigma(Q,\vec{s}) \propto \Big|$



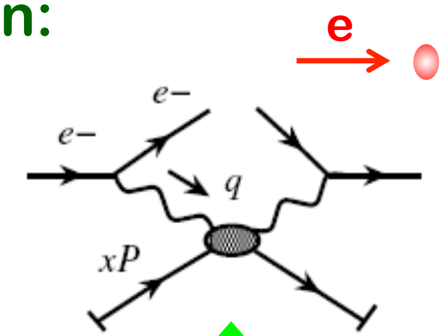
$\Big|^2 \left(\frac{\langle k_\perp\rangle}{Q}\right)^n$ – Expansion

Quantum interference ➡ 3-parton matrix element – not a probability!

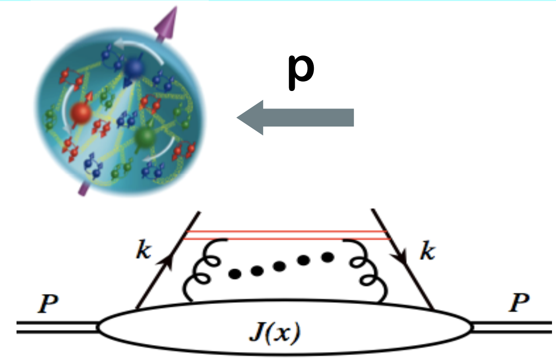
“See” hadron structure in experiments

- **One hadron:**

$\sigma_{\text{tot}}^{\text{DIS}}$:



$\otimes$



$+ O\left(\frac{1}{QR}\right)$

Factorization

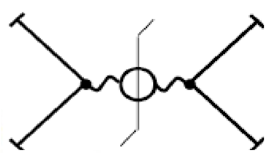
Hard-part
Probe

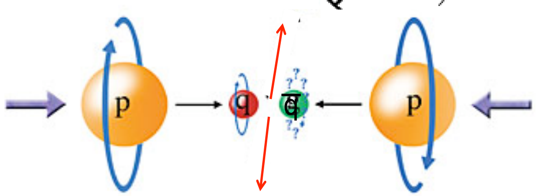
Parton-distribution
Structure

Power corrections
Approximation

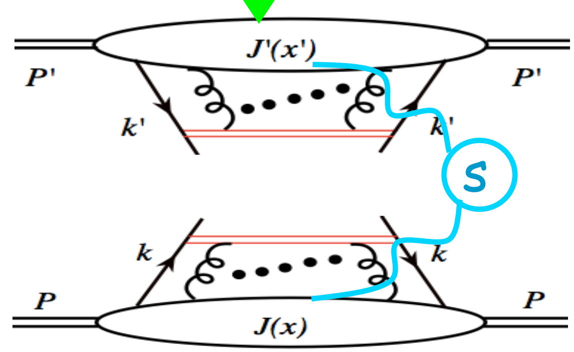
- **Two hadrons:**

$\sigma_{\text{tot}}^{\text{DY}}$:





$\otimes$



$+ O\left(\frac{1}{QR}\right)$

Predictive power:

Ability to calculate the “probes” + Universal Parton Distributions, …

Global QCD analyses – a successful story

- ❑ **World data with "Q" > 2 GeV**

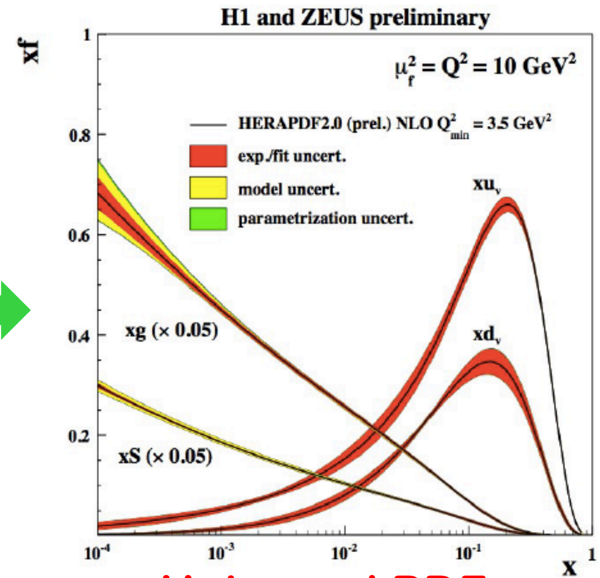
  + Factorization:

DIS: $F_2(x_B, Q^2) = \Sigma_f C_f(x_B/x, \mu^2/Q^2) \otimes f(x, \mu^2)$

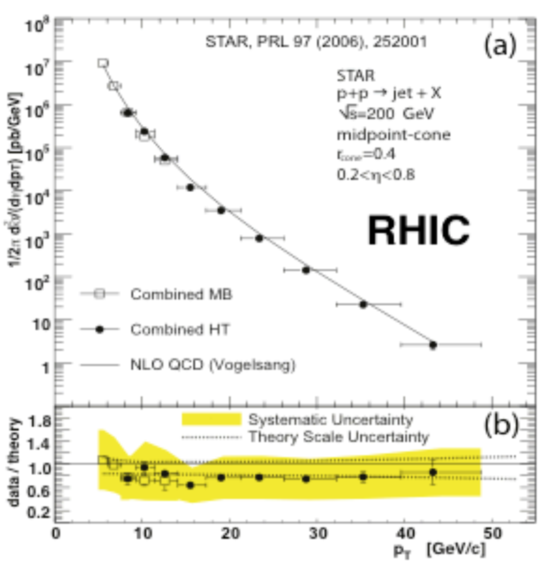
H-H: $\dfrac{d\sigma}{dy dp_T^2} = \Sigma_{ff'} f(x) \otimes \dfrac{d\hat{\sigma}_{ff'}}{dy dp_T^2} \otimes f'(x')$

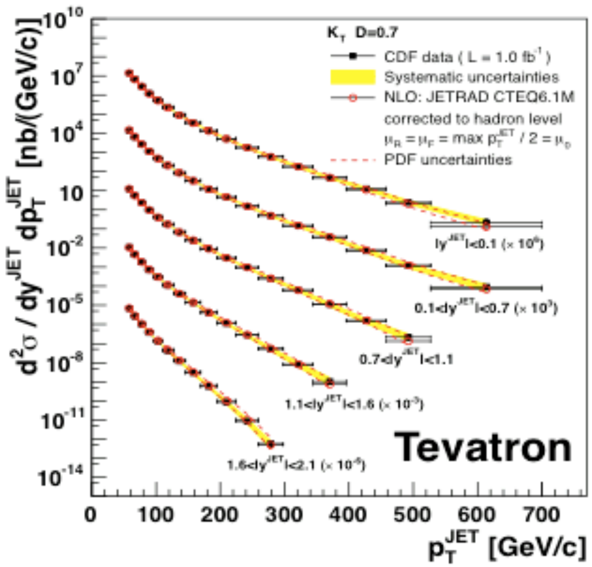
+ DGLAP Evolution:

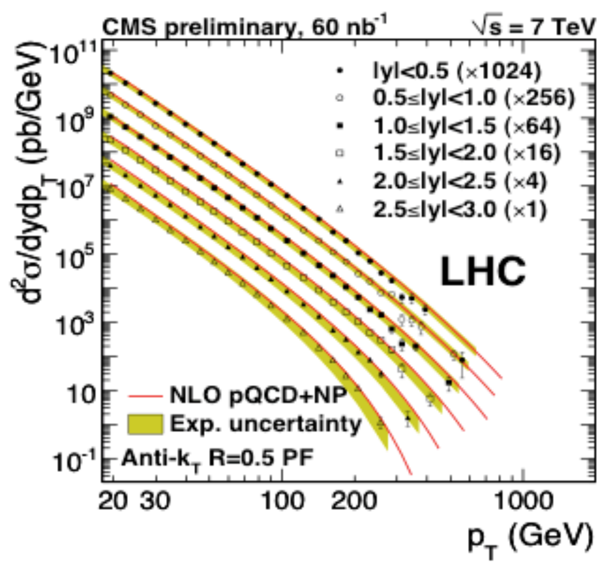
$$\frac{\partial f(x,\mu^2)}{\partial \ln \mu^2} = \Sigma_{f'} P_{ff'}(x/x') \otimes f'(x', \mu^2)$$



Universal PDFs







Global QCD analyses – a successful story

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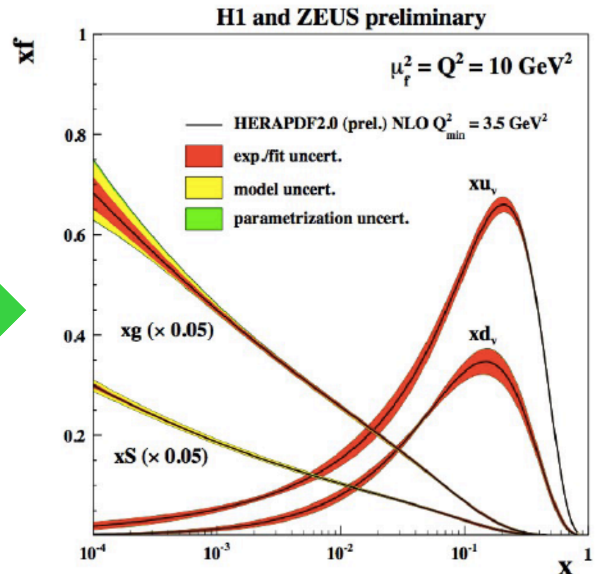


Universal PDFs

- The "BIG" question(s)

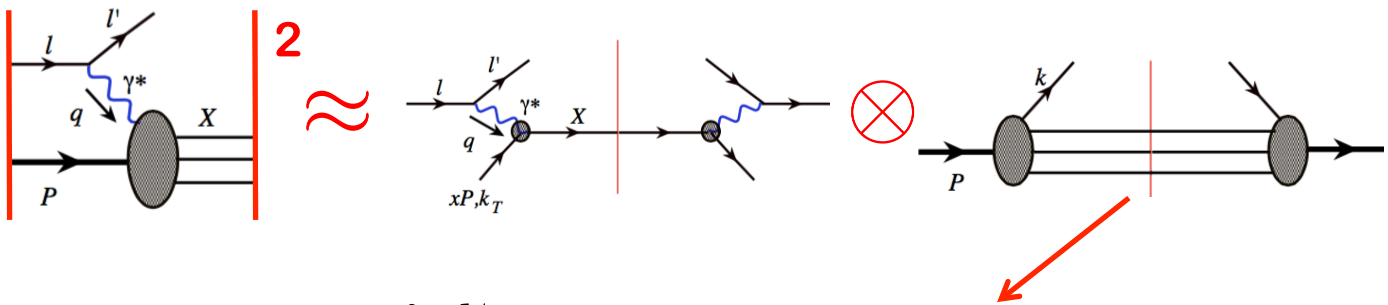
  Why these PDFs behave as what have been extracted from the fits?

  What have been tested is the evolution from μ_1 to μ_2
  But, does not explain why they have the shape to start with!

  Can QCD calculate and predict the shape of PDFs at the input scale, and other parton correlation functions?

Operator definition of PDFs, …

❑ Definition – from QCD factorization:

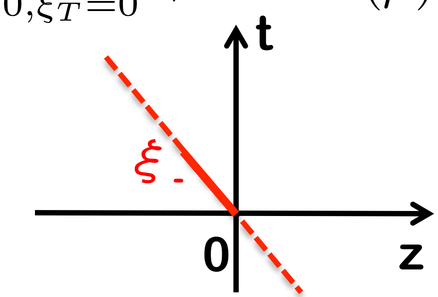


$$\approx \ldots \otimes \ldots + \mathcal{O}\left(\frac{\langle k^2\rangle}{Q^2}\right)$$

$$\Phi^{[U]}(x;P,\mu) = \int \frac{d\xi^-}{(2\pi)}\, e^{i\,k\cdot\xi}\, \langle P|\overline{\psi}(0)U(0,\xi)\psi(\xi)|P\rangle_{\xi^+=0,\vec{\xi}_T=0} + \mathrm{UVCT}(\mu)$$

✧ **Depends on the choice of the gauge link:**

$$U(0,\xi) = e^{-ig\int_0^\xi ds^\mu A_\mu}$$

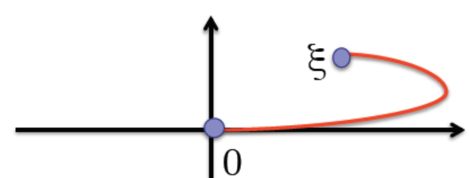


PDFs are well defined in QCD, but, can't be calculated perturbatively

❑ Transverse momentum dependent PDFs (TMDs):

$$\Phi^{[U]}(x,k_T;P,\mu) = \int \frac{d\xi^- d^2\xi_T}{(2\pi)^3}\, e^{i\,k\cdot\xi}\, \langle P|\overline{\psi}(0)U(0,\xi)\psi(\xi)|P\rangle_{\xi^+=0} + \mathrm{UVCT}(\mu)$$

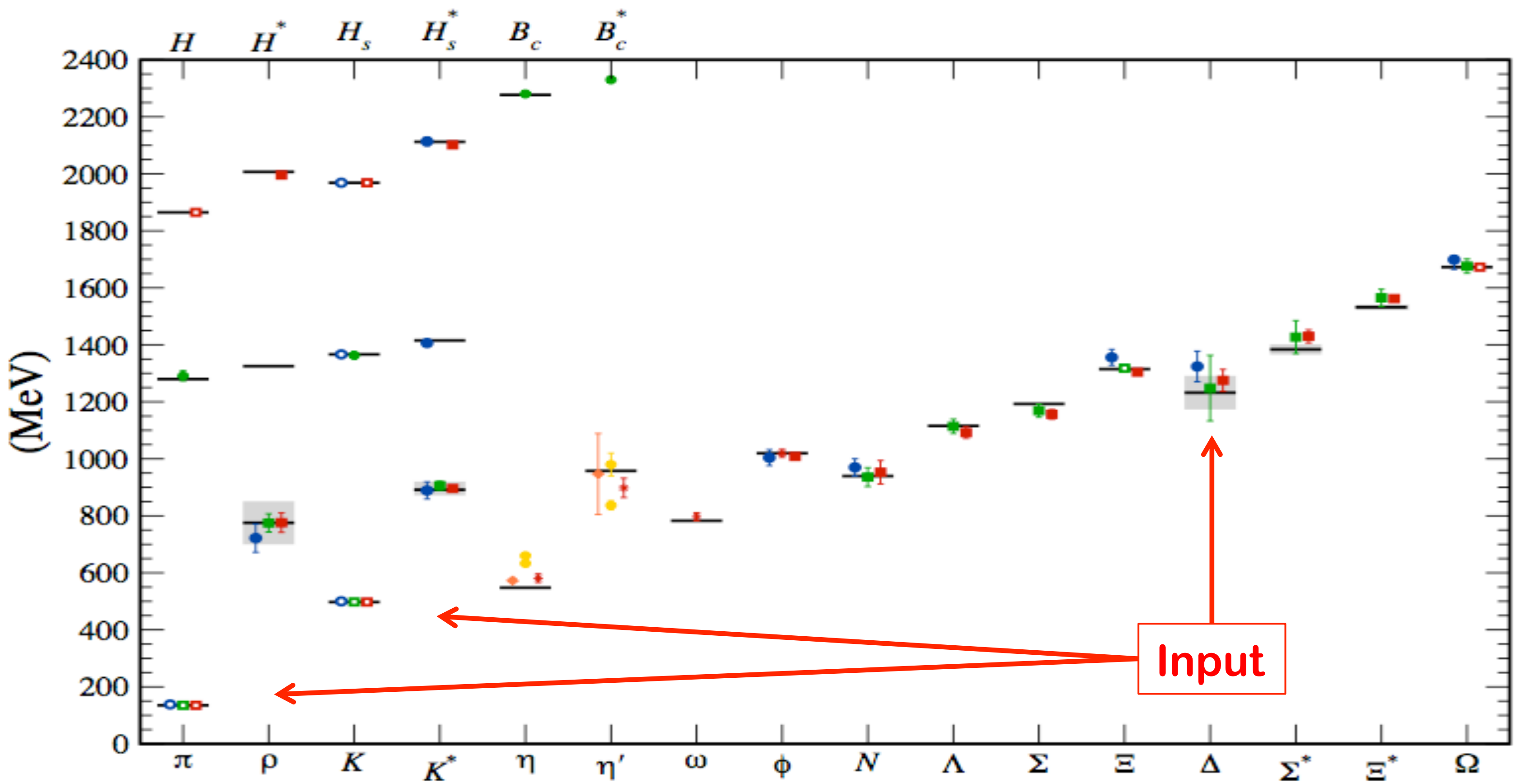
✧ **General gauge link:**



Lattice QCD

- ❑ **Hadron masses:**

Predictions with limited inputs



- ❑ **Lattice "time" is Euclidean:** $\tau = i\,t$

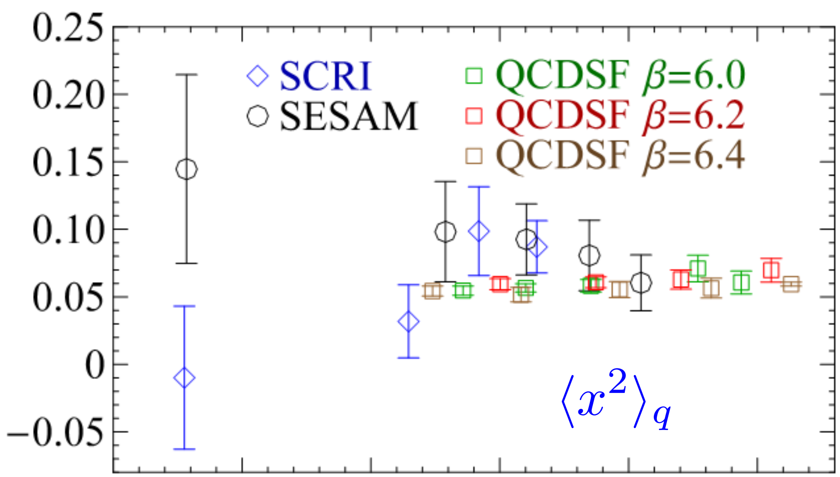
Cannot calculate PDFs, TMDs, …, directly, whose operators are time-dependent

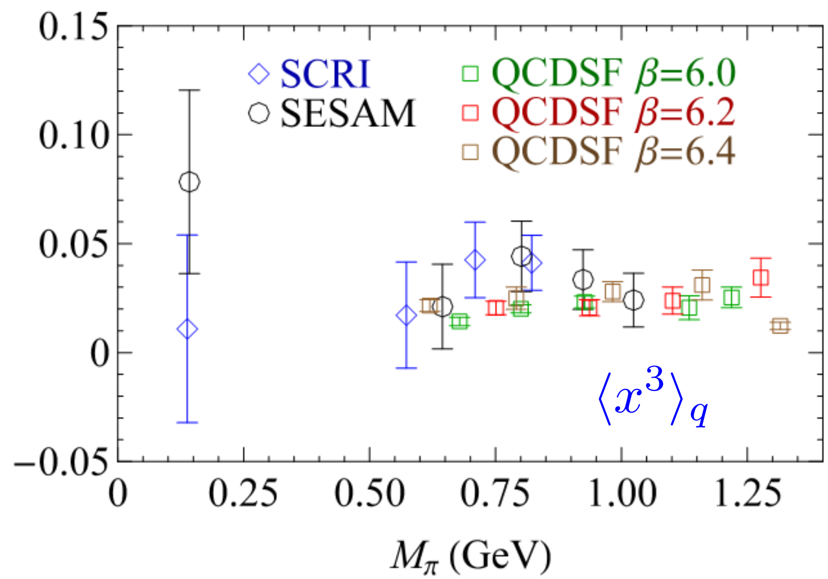
PDFs from lattice QCD

- Moments of PDFs – matrix elements of local operators

$$\langle x^n(\mu^2)\rangle_q \equiv \int_0^1 dx\, x^n\, q(x,\mu^2)$$

- Works, but, hard and limited moments:





Dolgov et al., hep-lat/0201021

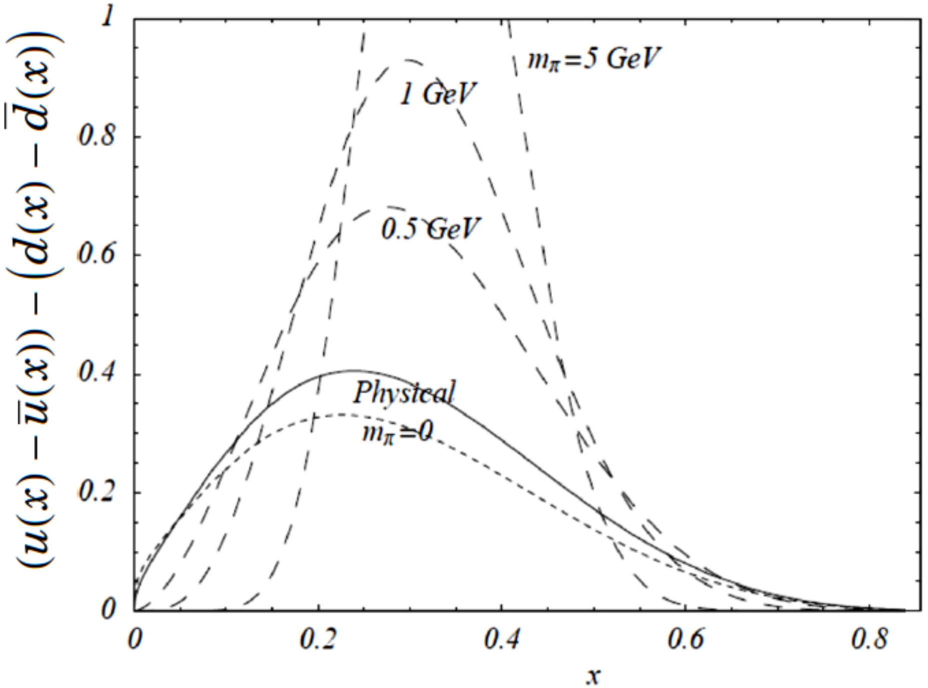
Gockeler et al., hep-ph/0410187

Limited moments – hard to get the full x-dependent distributions!

PDFs from lattice QCD

- How to get x-dependent PDFs with a limited moments?
  - ✧ Assume a smooth functional form with some parameters
  - ✧ Fix the parameters with the lattice calculated moments

$$xq(x) = a\,x^b(1-x)^c(1+\epsilon\sqrt{x}+\gamma\,x)$$



W. Dermold et al., Eur.Phys.J.direct C3 (2001) 1-15

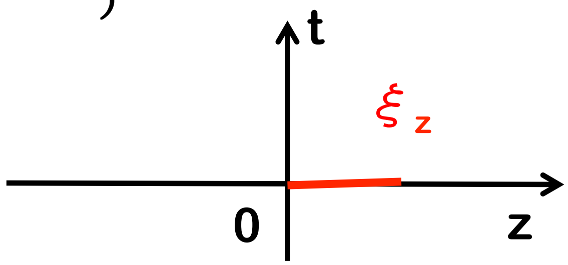
Cannot distinguish valence quark contribution from sea quarks

From quasi-PDFs to PDFs

Ji, arXiv:1305.1539

- "Quasi" quark distribution (spin-averaged):

$$\tilde{q}(x,\mu^2,P_z) \equiv \int \frac{d\xi_z}{4\pi} e^{-ixP_z\xi_z} \langle P|\overline{\psi}(\xi_z)\gamma_z \exp\left\{-ig\int_0^{\xi_z} d\eta_z A_z(\eta_z)\right\}\psi(0)|P\rangle + \mathrm{UVCT}(\mu^2)$$



Quasi-PDFs =\= PDFs

- Proposed matching:

$$\tilde{q}(x,\mu^2,P_z) = \int_x^1 \frac{dy}{y} Z\left(\frac{x}{y},\frac{\mu}{P_z}\right) q(y,\mu^2) + \mathcal{O}\left(\frac{\Lambda^2}{P_z^2},\frac{M^2}{P_z^2}\right)$$

Quasi-PDFs ➔ *Normal PDFs* *when P_z ➔ ∞ ?*

- Excellent idea and great potential:

IDEA: Calculate something =\= PDFs, but, carry all the information of PDFs

CHALLENGES:

- ✧ Quasi-PDFs could be calculated using the lattice QCD method
- ✧ Extract PDFs from what you can calculate, …

"Quasi-PDFs" have no parton interpretation

- **Normal PDFs conserve parton momentum:**

$$
\begin{aligned}
M &= \sum_q \left[\int_0^1 dx\, x f_q(x) + \int_0^1 dx\, x f_{\bar{q}}(x)\right] + \int_0^1 dx\, x f_g(x) \\
&= \sum_q \int_{-\infty}^{\infty} dx\, x f_q(x) + \frac{1}{2}\int_{-\infty}^{\infty} dx\, x f_g(x) \\
&= \frac{1}{2(P^+)^2}\langle P|T^{++}(0)|P\rangle = \text{constant}
\end{aligned}
$$

$T^{\mu\nu}$

Energy-momentum tensor

- **"Quasi-PDFs" do not conserve "parton" momentum:**

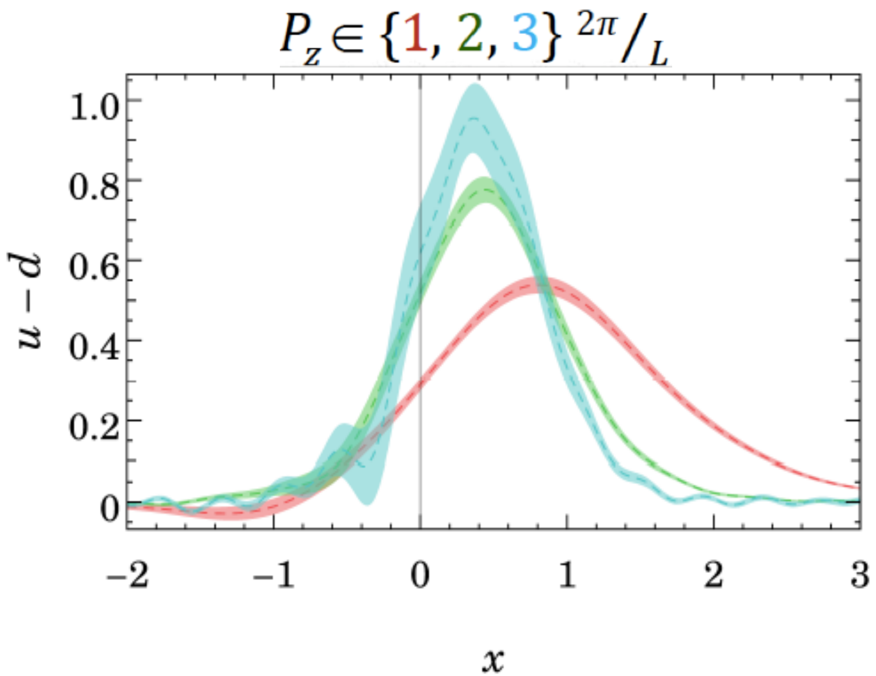
$$
\begin{aligned}
\widetilde{\mathcal{M}} &= \sum_q \left[\int_0^\infty d\tilde{x}\, \tilde{x} \tilde{f}_q(\tilde{x}) + \int_0^\infty d\tilde{x}\, \tilde{x} \tilde{f}_{\bar{q}}(\tilde{x})\right] + \int_0^\infty d\tilde{x}\, \tilde{x} \tilde{f}_g(\tilde{x}) \\
&= \sum_q \int_{-\infty}^{\infty} d\tilde{x}\, \tilde{x} \tilde{f}_q(\tilde{x}) + \frac{1}{2}\int_{-\infty}^{\infty} d\tilde{x}\, \tilde{x} \tilde{f}_g(\tilde{x}) \\
&= \frac{1}{2(P_z)^2}\langle P|\left[T^{zz}(0) - g^{zz}(...)\right]|P\rangle \neq \text{constant}
\end{aligned}
$$

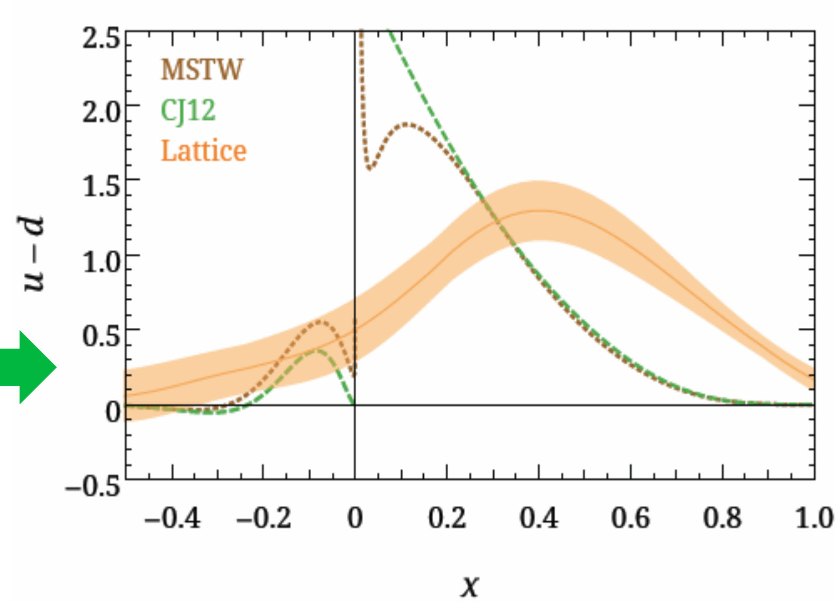
Note: "Quasi-PDFs" are not boost invariant

Lattice calculation of quasi-PDFs

Lin *et al.*, arXiv:1402.1462

- Exploratory study:





Quasi-Quark Distribution with different P_z → Predicted quark distribution along with global fitted one

Matching – taking into account:

Target mass: $(M_N/P_z)^2$

High twist: $a+b/P_z^2$

Pseudo-PDFs

Radyushkin, 2017

❑ Pseudo-PDFs = generalization of PDFs:

✧ **Definition:** $\xi^2 < 0$

$$\mathcal{M}^\alpha(\nu = p\cdot\xi, \xi^2) \equiv \langle p|\overline{\psi}(0)\gamma^\alpha\Phi_v(0,\xi,v\cdot A)\psi(\xi)|p\rangle$$

$$\equiv 2p^\alpha\mathcal{M}_p(\nu,\xi^2) + \xi^\alpha(p^2/\nu)\mathcal{M}_\xi(\nu,\xi^2) \approx 2p^\alpha\mathcal{M}_p(\nu,\xi^2)$$

$$\mathcal{P}(x,\xi^2) \equiv \int\frac{d\nu}{2\pi}\,e^{ix\,\nu}\,\frac{1}{2p^+}\mathcal{M}^+(\nu,\xi^2)$$

with $\xi^\mu = (0^+,\xi^-,0_\perp)$

✧ **Interpretation:**

Off-light-cone extension of PDFs: $f(x) = \mathcal{P}(x,\xi^2=0)$

❑ Quasi-PDFs:

$$\xi^\mu = (0,0_\perp,\xi_z)$$

No longer Lorentz invariant

$$\tilde{q}(x,\mu^2,p_z) = \int\frac{d\nu}{2\pi}\,e^{ix\nu}\,\frac{1}{2p_z}\mathcal{M}^z(\nu = p_z\xi_z, -\xi_z^2)$$

❑ TMDs:

$$\xi^\mu = (0^+,\xi^-,\xi_\perp)$$

$$\mathcal{P}(x,-\xi_\perp^2) \equiv \int d^2k_\perp\,e^{i\vec{k}_\perp\cdot\vec{\xi}_\perp}\mathcal{F}(x,k_\perp^2)$$

← **TMDs with a straight gauge link**

Pseudo-PDFs

Orginos, et al, 2017
1706.05373

❑ Pseudo-PDFs:

✧ **Lattice calculation with** $\alpha = 0$:

$$\mathcal{M}^\alpha(\nu = p\cdot\xi, \xi^2) \equiv \langle p|\overline{\psi}(0)\gamma^\alpha \Phi_v(0,\xi, v\cdot A)\psi(\xi)|p\rangle$$

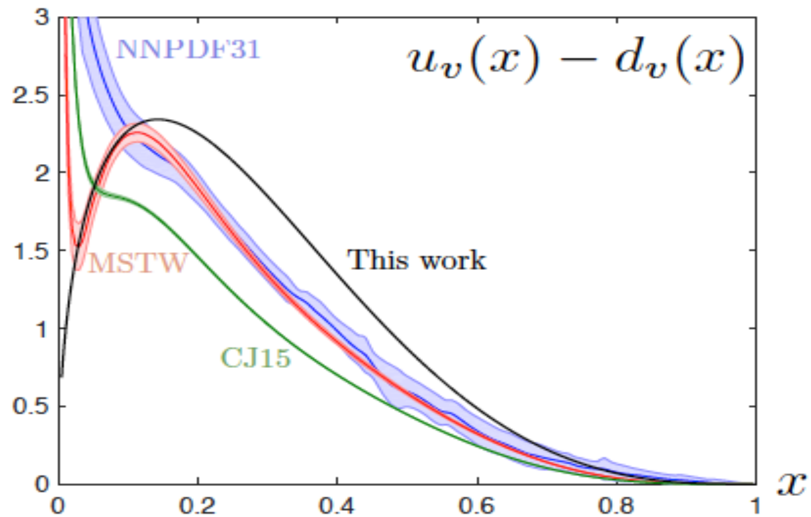
$$\equiv 2p^\alpha \mathcal{M}_p(\nu,\xi^2) + \xi^\alpha (p^2/\nu)\mathcal{M}_\xi(\nu,\xi^2) \approx 2p^\alpha \mathcal{M}_p(\nu,\xi^2)$$

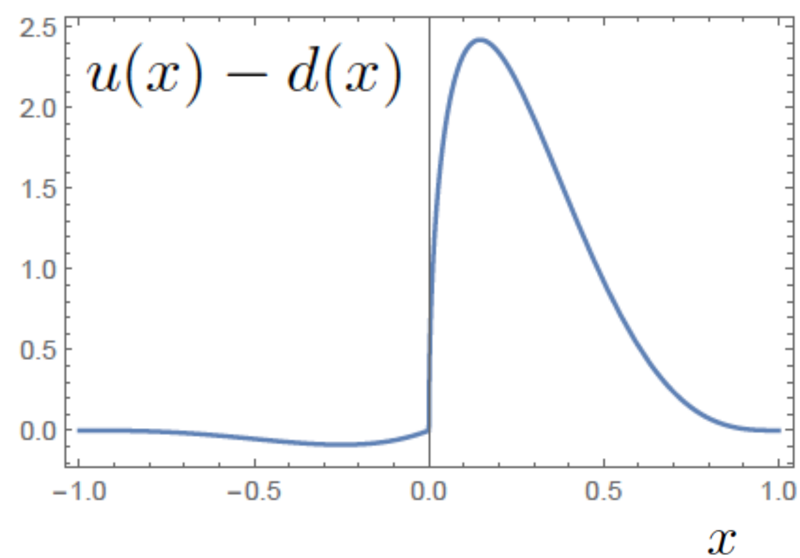
$$\mathcal{P}(x,\xi^2) \equiv \int \frac{d\nu}{2\pi} e^{ix\,\nu} \mathcal{M}_{p=p^0}(\nu,\xi^2)/\mathcal{M}_{p=p^0}(0,\xi^2)$$

Remove UV!

✧ **Model quasi-PDFs:** with $\xi^\mu = (0, 0_\perp, \xi_z)$

❑ Numerical results:





Go beyond quasi- and pseudo-PDFs?

Ma and Qiu, arXiv:1404.6860
arXiv:1709.03018

❑ A pQCD factorization approach:

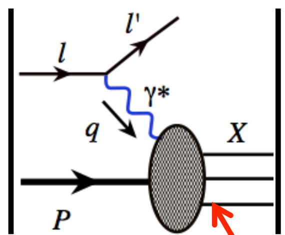
✧ Recall: Collinear factorization of DIS cross section – single hadron

Momentum transfer

CO Factorization

PDFs

$$\sigma^{\rm DIS}(x, Q^2; \sqrt{s}) \propto \left| \quad \right|^2 \approx \sum_f \mathcal{C}_f(x, \frac{Q^2}{\mu^2}, \sqrt{s}) \otimes f_i(x, \mu^2) + \mathcal{O}\left[\frac{1}{Q^2}\right]$$



Collision energy

Single hadron Matrix elements

Perturbative coefficients

Power corrections

✧ Renormalizable + factorizable + lattice calculable "cross section":

PDFs

$$\widetilde{\sigma}_{\rm Lattice}(\tilde{x}, Q, P \sim \sqrt{s}) \approx \sum_f \int \frac{dx}{x} \mathcal{C}_f \left(\frac{x}{\tilde{x}}, \frac{\mu^2}{Q^2}, \alpha_s(\mu); P\right) f(x, \mu^2) + \mathcal{O}\left[\frac{1}{Q^2}\right]$$

Key: controllable hard scale!

DGLAP-Evolution

Lattice "cross section"

Ma and Qiu, arXiv:1404.6860
arXiv:1709.03018

❑ What is lattice "cross section"?

Single hadron matrix elements, with the following properties:

- ✧ **Lattice calculable:**

  Calculable using lattice QCD with an Euclidean time

- ✧ **UV Renormalizable:**

  Ensure a well-defined continuum limit, UV & IR finite!

- ✧ **CO Factorizable:**

  Share the same perturbative collinear divergences with PDFs

  Factorizable to PDFs with IR-safe hard coefficients

  with controllable power corrections

❑ Key requirement:

A controllable large "momentum" scale – conjugate to hadron momentum

to define the "collision" dynamics of the "cross section"
to ensure the necessary condition for the factorization

An example, …

Ma and Qiu, arXiv:1404.6860
arXiv:1709.03018

❑ Current correlators:

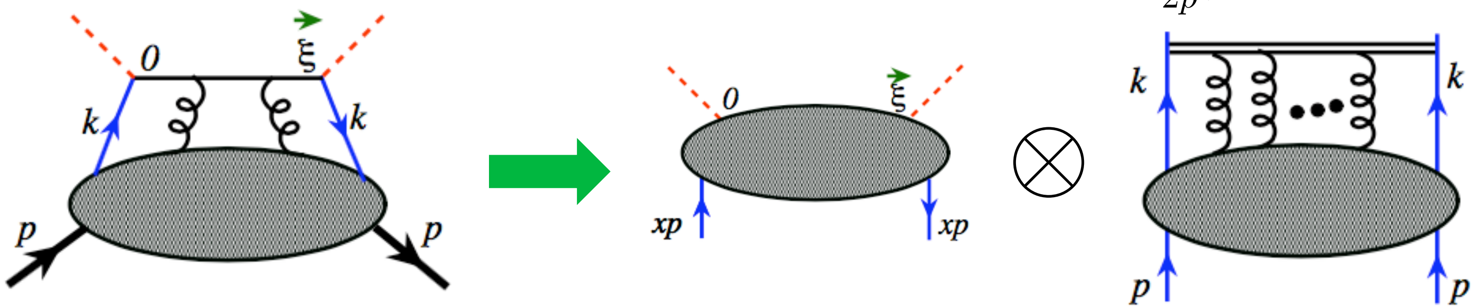
✧ **Coordinate space:**

$$\mathcal{T}_{jj}(p,s,\xi) = \lim_{\xi^0\to 0^+} \langle p,s|T\{\,j_\Gamma(\xi^0,\vec{\xi})\,j_\Gamma(0)\}|p,s\rangle$$

p and ξ define collision kinematics

✧ **Factorization:**

$$\frac{\gamma^+}{2p^+}\delta(x-k^+/p^+)$$



$+$ Corrections

❑ Complementarity and advantages:

✧ **Complementary to existing approaches for extracting PDFs,**

✧ **Quasi-PDFs and pseudo-PDFs are special cases,**

✧ **Have tremendous potentials:**

Neutron PDFs, … (no free neutron target!)
Meson PDFs, such as pion, …
More direct access to gluons – gluonic current, …

A little bit more details, …

Ma and Qiu, arXiv:1404.6860
arXiv:1709.03018

❑ Lattice cross sections – definition:

$$\sigma_n(\xi^2, \omega, P^2) = \langle P|T\{\mathcal{O}_n(\xi)\}|P\rangle \qquad \omega = P\cdot\xi$$

where the operator is defined as

$$\mathcal{O}_{j_1 j_2}(\xi) \equiv \xi^{d_{j_1}+d_{j_2}-2}\, Z_{j_1}^{-1}\, Z_{j_2}^{-1} j_1(\xi)\, j_2(0)$$

with

d_j : Dimension of the current

Z_j : Renormalization constant of the current

❑ Lattice cross sections – requirements:

✧ is calculable in lattice QCD with an Euclidean time

✧ has a well-defined continuum limit as the lattice spacing, $a \to 0$ and

✧ has the same and factorizable logarithmic CO divergences as PDFs

❑ Lattice cross sections – two-current correlations:

$$j_S(\xi) = \xi^2 Z_S^{-1}[\overline{\psi}_q \psi_q](\xi), \qquad j_V(\xi) = \xi Z_V^{-1}[\overline{\psi}_q \gamma\cdot\xi\psi_q](\xi),$$

$$j_{V'}(\xi) = \xi Z_{V'}^{-1}[\overline{\psi}_q \gamma\cdot\xi\psi_{q'}](\xi), \qquad j_G(\xi) = \xi^3 Z_G^{-1}[-\frac{1}{4}F_{\mu\nu}^c F_{\mu\nu}^c](\xi)\ , \ldots$$

❑ Lattice cross sections – quasi- and pseudo-PDFs:

$$\mathcal{O}_q(\xi) = Z_q^{-1}(\xi^2)\overline{\psi}_q(\xi)\,\gamma\cdot\xi\Phi(\xi,0)\,\psi_q(0) \qquad \Phi(\xi,0) = \mathcal{P}e^{-ig\int_0^1 \xi\cdot A(\lambda\xi)\,d\lambda}$$

A little bit more details, …

Ma and Qiu, arXiv:1404.6860
arXiv:1709.03018

❑ Lattice cross sections – definition:

$$\sigma_n(\xi^2, \omega, P^2) = \langle P|T\{\mathcal{O}_n(\xi)\}|P\rangle \qquad \omega = P\cdot\xi$$

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❑ Lattice cross sections – requirements:

✧ is calculable in lattice QCD with an Euclidean time

✧ has a well-defined continuum limit as the lattice spacing, $a \to 0$ and

✧ has the same and factorizable logarithmic CO divergences as PDFs

❑ Identify good lattice cross sections:

$$\overline{\sigma}_{\rm E}^{\rm Lat}(\xi_z, 1/a, P_z) \stackrel{\mathcal{Z}}{\longleftrightarrow} \sigma_{\rm E}(\xi_z, \tilde{\mu}^2, P_z)$$ – Renormalization

$$\Updownarrow$$

$$\sigma_{\rm M}(\xi_z, \tilde{\mu}^2, P_z) \stackrel{\mathcal{C}}{\longleftrightarrow} f_i(x, \mu^2)\,,$$ – Factorization

A "new" collaboration between lattice QCD and perturbative QCD!

Renormalization – summary

- **Take care by construction:**

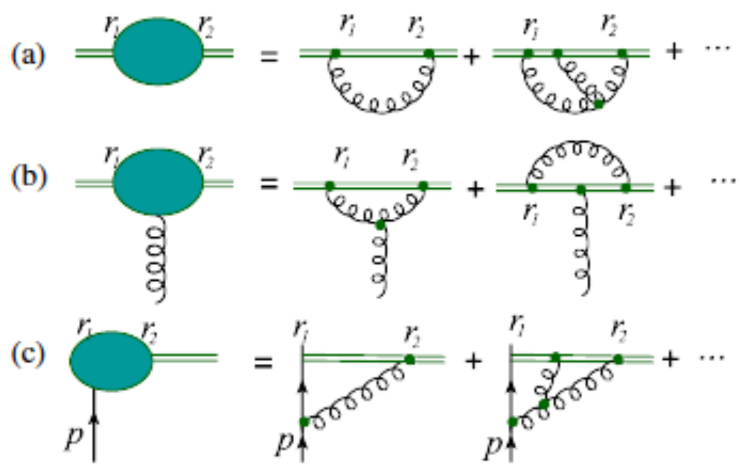
  Construct operators by using renormalizable, or conserved currents

- **Renormalization of quasi- and pseudo-PDFs:**

  Quasi-quark distributions is multiplicatively renormalizable

$$\tilde{q}_i^R(\xi_z, \mu^2, p_z) = e^{-C_i|\xi_z|} Z_{wi}^{-1} Z_{vi}^{-1} \tilde{q}_i(\xi_z, \mu^2, p_z)$$

Three classes of elementary divergent diagrams:



Ishikawa, Ma, Qiu and Yoshida
arXiv: 1701.03108

Pseudo-quark distributions takes care of the UV renormalization by

$$\mathcal{P}(x, \xi^2) \equiv \int \frac{d\nu}{2\pi} e^{ix\,\nu} \mathcal{M}_{p=p^0}(\nu, \xi^2)/\mathcal{M}_{p=p^0}(0, \xi^2)$$

Different matching

Renormalization – quasi-quark

❑ Coordinate-space definition:

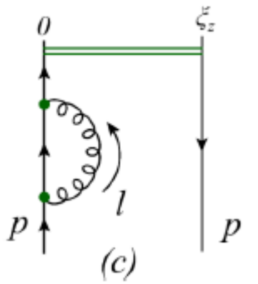
$$\tilde{F}_{q/p}(\xi_z, \tilde{\mu}^2, p_z) = \frac{e^{ip_z\xi_z}}{p_z}\langle h(p)|\overline{\psi}_q(\xi_z)\,\frac{\gamma_z}{2}\Phi^{(f)}_{n_z}(\{\xi_z,0\})\,\psi_q(0)|h(p)\rangle$$

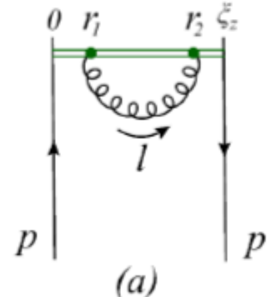
❑ Why the proof is hard:

- Because of z-direction dependence, Lorentz symmetry is broken, hard to exhaust all possible UV divergences
- Renormalization of composite operator is needed

❑ Broken Lorentz symmetry:

Both 3D and 4D loop-integration can generate UV divergences





$$l^\mu = \bar{l}^\mu + l_z n_z^\mu$$

UV: 4-D integration

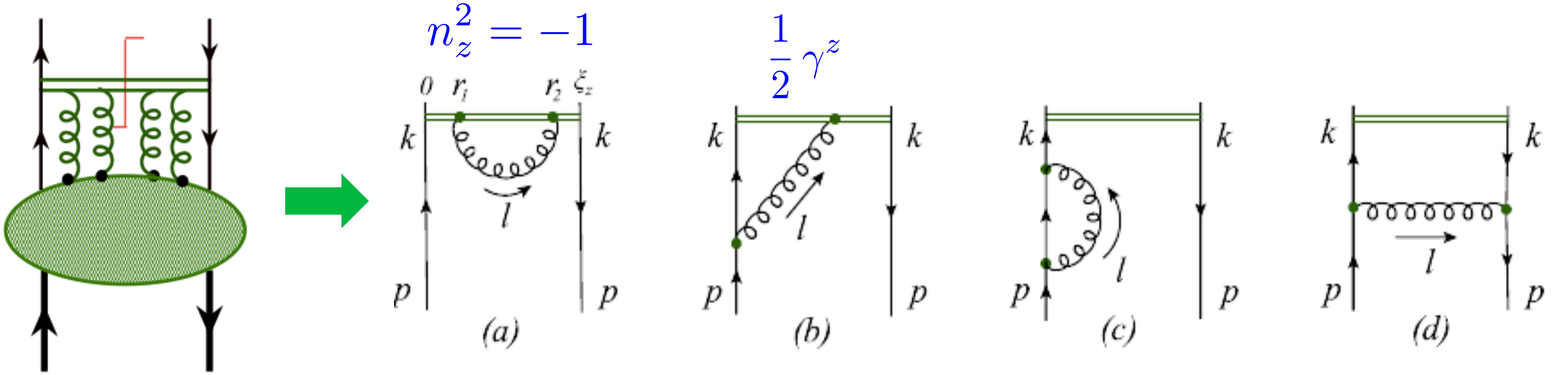
$$\int \frac{d^d l}{(2\pi)^d}\frac{1}{l^2(p-l)^2}$$

UV: 3-D integration

$$\int \frac{d^3\bar{l}}{l^2} = \int \frac{d^3\bar{l}}{\bar{l}^2 - l_z^2}$$

Renormalization – quasi-quark

- **Quasi-quark at one-loop:**



- **Fig. 1(a):**

$$M_{1a} = \frac{e^{ip_z\xi_z}}{p_z}\frac{1}{N_c}\mathrm{Tr}_c[T^aT^a]\int_a^{\xi_z-2a}dr_1\int_{r_1+a}^{\xi_z-a}dr_2$$
$$\times\int\frac{d^4l}{(2\pi)^4}e^{-ip_z\xi_z}\,e^{il_z(r_2-r_1)}\left(\frac{-ig_{\mu\nu}}{l^2}\right)$$
$$\times(-ig_s n_z^\mu)(-ig_s n_z^\nu)\mathrm{Tr}\left[\frac{1}{2}\not{p}\,\frac{1}{2}\gamma_z\right]$$
$$=\frac{\alpha_s C_F}{4i\pi^3}\int_a^{\xi_z-2a}dr_1\int_{r_1+a}^{\xi_z-a}dr_2\int d^4l\frac{e^{il_z(r_2-r_1)}}{l^2}$$

$$M_{1a}\stackrel{\mathrm{div}}{=}-\frac{\alpha_s C_F}{\pi}\frac{|\xi_z|}{a}+\frac{\alpha_s C_F}{\pi}\ln\frac{|\xi_z|}{a}$$

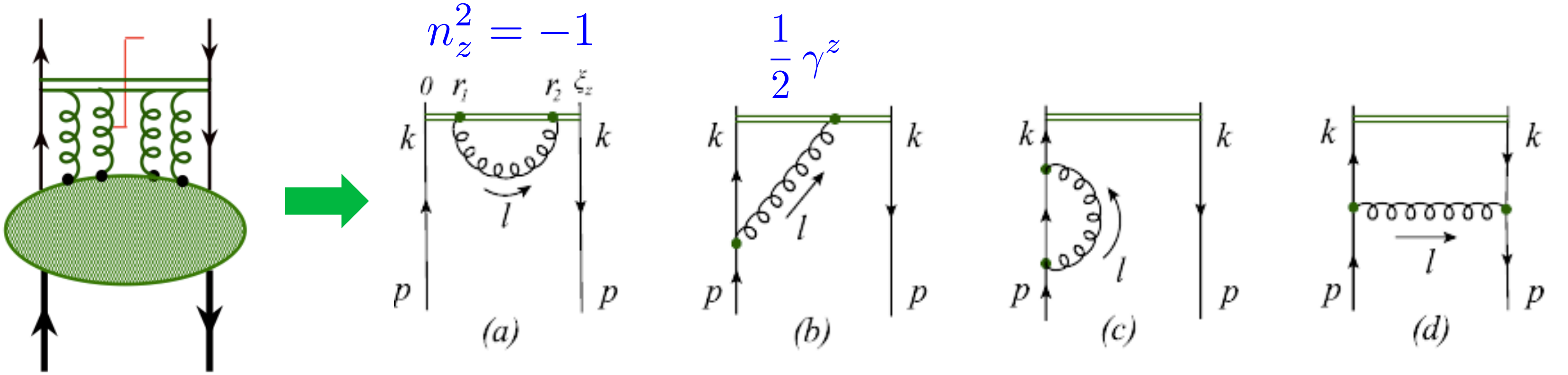
- Cutoff "a" between fields
- Conclusion independent of regulator
- 3D-integration: $d^4l = d^3\bar{l}\,dl_z$

$$\int\frac{d^3\bar{l}}{l^2} = \int\frac{d^3\bar{l}}{\bar{l}^2-l_z^2}$$
$$= \int d^3\bar{l}\left(\frac{1}{\bar{l}^2}+\frac{l_z^2}{(\bar{l}^2-l_z^2)\bar{l}^2}\right)$$
$$\int dl_z e^{il_z(r_2-r_1)} = 2\pi\delta(r_2-r_1)$$

1st term vanishes for $r_1 \neq r_2$

Renormalization – quasi-quark

❑ Quasi-quark at one-loop:



❑ Complete one-loop contribution:

$$M^{(1)} \stackrel{\text{div}}{=} M_{1a} + 2\times M_{1b} + 2\times\frac{1}{2}M_{1c} + M_{1d}$$
$$= \frac{\alpha_s C_F}{\pi}\left(-\frac{|\xi_z|}{a} + 2\ln\frac{|\xi_z|}{a} - \frac{1}{4\epsilon}\right).$$

✧ At one-loop, all 3D integrations are finite

✧ Divergence only come from the region when all momentum components go to infinity

➡ **Localized UV divergence in all directions!**

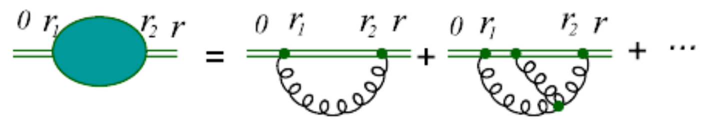
Very different from the UV behavior of normal PDFs: (1, λ^2, λ), $\lambda \to \infty$

Renormalization – quasi-quark

Ishikawa, Ma, Qiu, Yoshida (2017)

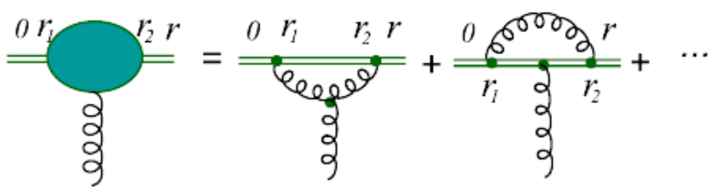
❑ Power counting and divergent sub-diagrams:

(a) - 1/a, ln(1/a):



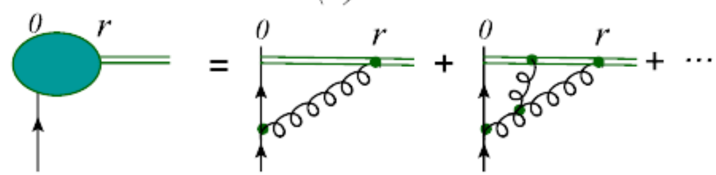
(a)

(b) - ln(1/a):



(b)

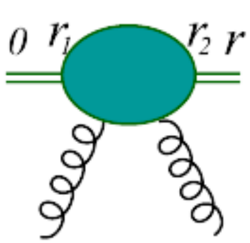
(c) - ln(1/a):

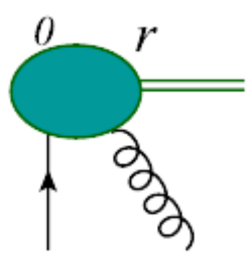


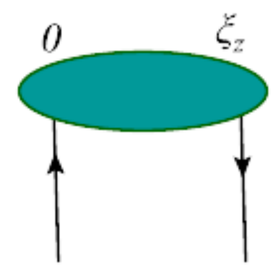
(c)

Happen only when all loop momenta go to infinity – localized!

❑ Example of convergent sub-diagrams:



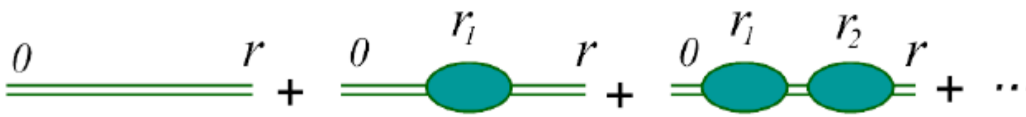




Renormalization – quasi-quark

Ishikawa, Ma, Qiu, Yoshida (2017)

❑ Power divergence:



$$1 + c\int_0^r dr_1 + c^2 \int_0^r dr_1 \int_{r_1}^r dr_2 + \cdots$$
$$= \mathcal{P}e^{\,c\int_0^r dr'} = e^{\,c\,r}\,,$$

- It is allowed to introduce an overall factor $e^{-c|\xi_z|}$ to remove all power UV divergences

❑ Interpretation:

- Mass renormalization of test particle

Dotsenko, Vergeles, NPB (1980)

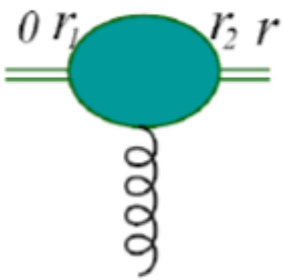
❑ Log divergence in from gauge link:

- Besides power divergence, there are also logarithmic UV divergences
- It is known that these divergences can be removed by a "wave function" renormalization of the test particle, Z_{wq}^{-1}.

Renormalization – quasi-quark

Ishikawa, Ma, Qiu, Yoshida (2017)

❑ Log divergence from gluon-gauge link vertex:



- Logarithmic UV: can be absorbed by the coupling constant renormalization of QCD.

❑ UV from vertex correction:

- The most dangerous UV diagram, may mix with other operators
- Locality of UV divergence: no dependence on $r_2 - r_1$ or p
- UV divergence is proportional to quark-gaugelink vertex at lowest order, with a constant coefficient
- A constant counter term is able to remove this UV divergence.

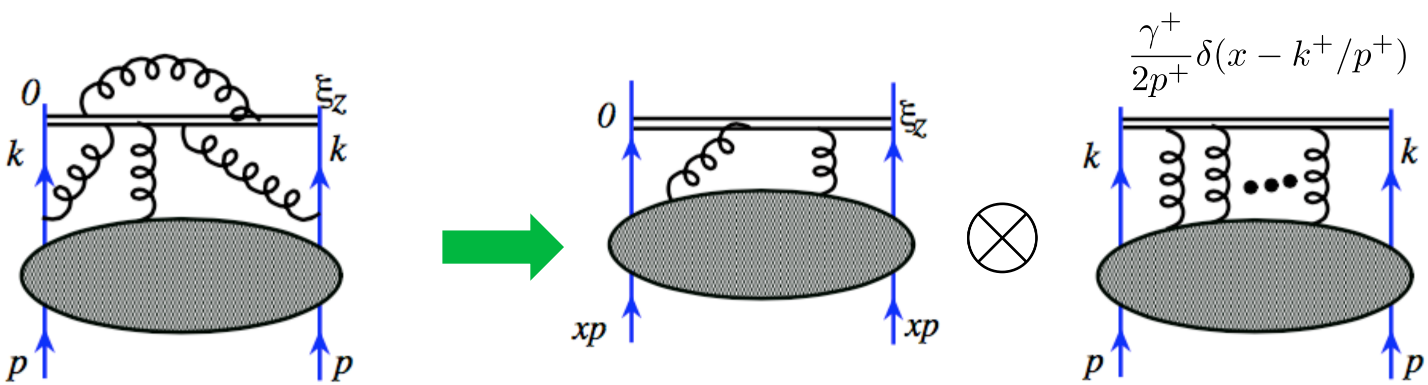
❑ Renormalization to all orders:

- Using bookkeeping forests subtraction method, the net effect is to introduce a constant multiplicative renormalizaton factor $Z_{vq}^{-1}$for the quark-gaugelink vertex.

Factorization

- ❑ Does the renormalized lattice cross section and quasi-PDFs share the same CO properties with PDFs?

- ❑ Can we extract PDFs from lattice cross section and/or renormalized quasi-PDFs reliably?



$+$ Corrections

Factorization

Ma and Qiu, arXiv:1404.6860
arXiv:1709.03018

- **Factorized formula for lattice cross section:**

$$\sigma_n(\omega,\xi^2,P^2) = \sum_a \int_{-1}^{1} \frac{dx}{x} f_a(x,\mu^2) \times K_n^a(x\omega,\xi^2,x^2P^2,\mu^2) + O(\xi^2\Lambda_{\rm QCD}^2)$$

with $f_a(x,\mu^2) = -f_a(-x,\mu^2)$

- **Steps needed to prove:**

Let ξ^2 be small but not vanishing, apply OPE to the operator,

$$\sigma_n(\omega,\xi^2,P^2) = \sum_{J=0}\sum_a W_n^{(J,a)}(\xi^2,\mu^2)\,\xi^{\nu_1}\cdots\xi^{\nu_J} \times \langle P|\mathcal{O}_{\nu_1\cdots\nu_J}^{(J,a)}(\mu^2)|P\rangle$$

Local, symmetric and traceless with spin *J*

with

$$\langle P|\mathcal{O}_{\nu_1\cdots\nu_J}^{(J,a)}(\mu^2)|P\rangle = 2A^{(J,a)}(\mu^2) \times (P_{\nu_1}\cdots P_{\nu_J} - \text{traces})$$

With reduced matrix element: $A^{(J,a)}(\mu^2) = \langle P|\mathcal{O}^{(J,a)}(\mu^2)|P\rangle$

→

$$\sigma_n(\omega,\xi^2,P^2) = \sum_{J=0}\sum_a W_n^{(J,a)}(\xi^2,\mu^2)\,2A^{(J,a)}(\mu^2) \times \Sigma_J(\omega,P^2\xi^2)$$

with $\Sigma_J(\omega,P^2\xi^2) \equiv \xi^{\nu_1}\cdots\xi^{\nu_J}(P_{\nu_1}\cdots P_{\nu_J} - \text{traces})$

$$= \sum_{i=0}^{[J/2]} C_{J-i}^i(\omega)^{J-2i}\left(-P^2\xi^2/4\right)^i$$

No approximation yet!

Factorization

Ma and Qiu, arXiv:1404.6860
arXiv:1709.03018

❑ Approximation – leading power/twist:

$$A^{(J,a)}(\mu^2) = \frac{1}{S_a}\int_{-1}^{1} dx\, x^{J-1} f_a(x,\mu^2)$$ With symmetry factor: $S_a = 1, 2$ for $a = q, g$

➡

$$\sigma_n(\omega,\xi^2,P^2) = \sum_a \int_{-1}^{1} \frac{dx}{x} f_a(x,\mu^2) \times K_n^a(x\omega,\xi^2,x^2P^2,\mu^2) + O(\xi^2\Lambda_{\rm QCD}^2)$$

with $$K_n^a = \sum_{J=1} \frac{2}{S_a} W_n^{(J,a)}(\xi^2,\mu^2)\,\Sigma_J(x\omega, x^2P^2\xi^2)$$

Note: our proof of factorization is valid only when $|\omega| \ll 1$ and $|p^2\xi^2| \ll 1$

❑ Extrapolate into large ω region:

- ✧ **Validity of OPE guarantees that σ_n is an analytic function of ω, so as its Taylor series of ω around ω=0, defined above**
- ✧ **If we fix ξ to be short-distance, while we increase ω by adjusting p, we can't introduce any new perturbative divergence**
- ✧ **That is, σ_n remains to be an analytic function of ω unless $\omega = \infty$**

Factorization holds for any finite value of ω and $p^2\xi^2$, if ξ is short-distance

Coefficient/matching functions

- **Matching coefficients for current-current correlators:**

$$K_n^a = \sum_{J=1} \frac{2}{S_a} W_n^{(J,a)}(\xi^2, \mu^2)\, \Sigma_J(x\omega, x^2 P^2 \xi^2)$$ ➡ **Need** $W_n^{(J,a)}(\xi^2,\mu^2)$

a) Calculate $K_n^a(x\omega, \xi^2, 0, \mu)$ - coefficient in CO factorization with $p^2=0$

b) Expand $K_n^a(x\omega, \xi^2, 0, \mu)$ in power series of $x\omega$

c) Extract $W_n^{(J,a)}(\xi^2,\mu^2)$ with $\Sigma_J(x\omega, 0) = (x\omega)^J$

- **LO matching:**

➡ $k^\mu = xp^\mu$, $p^2 = 0$

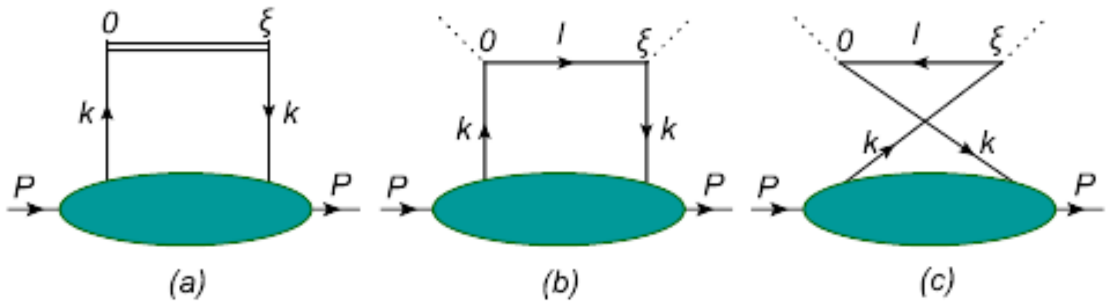


Fig. (a) $K_q^{q(0)}(x\omega, \xi^2, 0, \mu) = \frac{1}{2}\mathrm{Tr}[\not{k}\not{\xi}]e^{i\xi\cdot k} = 2x\omega e^{ix\omega}$ ➡ $W_q^{(J,q)} = i^{J-1}/(J-1)!$

Fig. (b,c) $M_b = \frac{i\xi^4}{2}\int \frac{d^4 l}{(2\pi)^4} \frac{\mathrm{Tr}[\not{k}\not{l}]e^{i\xi\cdot(k-l)}}{l^2+i\varepsilon} = \frac{i}{\pi^2} x\omega\, e^{ix\omega}$ $\qquad M_c = M_b^*$

➡ $\widetilde{K}^{q(0)}_{S/V/\widetilde{V}}(x\widetilde{\omega}, q^2, 0, \mu) = -2i\, \frac{x^2\widetilde{\omega}^2}{1 - x^2\widetilde{\omega}^2}$

Flavor change current
No crossing diagram

Connection to quasi- and pseudo-PDFs

- **Momentum-space version – Fourier transform:**

$$\widetilde{\sigma}_n(\widetilde{\omega}, q^2, P^2) \equiv \int \frac{d^4\xi}{\xi^4} e^{iq\cdot\xi} \sigma_n(P\cdot\xi, \xi^2, P^2)$$

With $\widetilde{\omega} \equiv \frac{2P\cdot q}{-q^2} = \frac{1}{x_B}$, and valid for $\widetilde{\omega}^2 < 1$

$$\widetilde{K}_n^a = \int \frac{d^4\xi}{\xi^4} e^{iq\cdot\xi} K_n^a(xP\cdot\xi, \xi^2, x^2P^2, \mu)$$

Care is needed for the physical region when $\widetilde{\omega}^2 > 1$

Contribution from large $\widetilde{\omega}^2 > 1$ region – poles and cuts

- **Comparison with other approaches:**

With $K_q^{q(0)}$ ➡ $\int \frac{d\omega}{\omega} \frac{e^{-ix\omega}}{4\pi} \sigma_q(\omega, \xi^2, P^2) \approx f_q(x, \mu)$

modulo $O(\alpha_s)$ corrections and higher twist corrections.

With $\xi_0 = 0$, the integral over $\omega = -\vec{\xi}\cdot\vec{P} = -|\vec{\xi}||\vec{P}|\cos\theta$

Quasi-PDFs: $\xi_0 = 0,\ \vec{p} = p_z,\ \vec{\xi} = \xi_z$ **with fixed** p_z

Pseudo-PDFs: $\xi_0 = 0,\ \vec{p} = p_z,\ \vec{\xi} = \xi_z$ **with fixed** ξ_z

One-loop example: quark →quark

Ma and Qiu, arXiv:1404.6860

- **Expand the factorization formula:**

$$\tilde{f}_{i/h}(\tilde{x},\tilde{\mu}^2,P_z) \approx \sum_j \int_0^1 \frac{dx}{x}\, \mathcal{C}_{ij}(\frac{\tilde{x}}{x},\tilde{\mu}^2,P_z)\, f_{j/h}(x,\mu^2)$$

To order α_s :

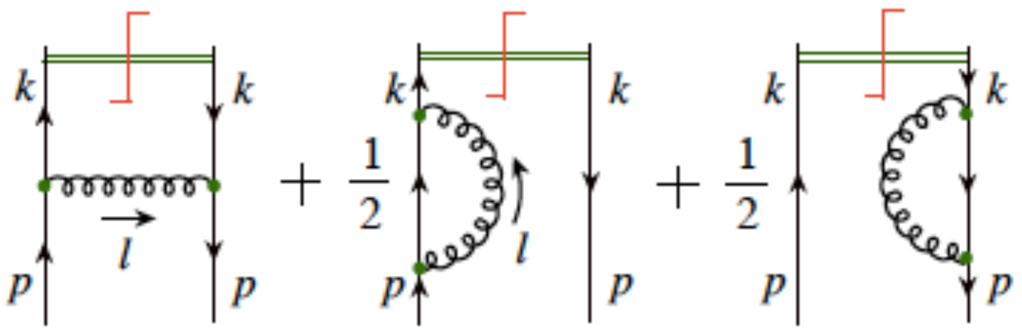
$$\tilde{f}^{(1)}_{q/q}(\tilde{x}) = f^{(0)}_{q/q}(x) \otimes \mathcal{C}^{(1)}_{q/q}(\tilde{x}/x) + f^{(1)}_{q/q}(x) \otimes \mathcal{C}^{(0)}_{q/q}(\tilde{x}/x)$$

$$\Rightarrow \quad \mathcal{C}^{(1)}_{q/q}(t,\tilde{\mu}^2,\mu^2,P_z) = \tilde{f}^{(1)}_{q/q}(t,\tilde{\mu}^2,P_z) - f^{(1)}_{q/q}(t,\mu^2)$$

- **Feynman diagrams:**

Same diagrams for both

$\tilde{f}_{q/q}$ and $f_{q/q}$



But, in different gauge: $n_z \cdot A = 0$ for $\tilde{f}_{q/q}$ $\quad n \cdot A = 0$ for $f_{q/q}$

- **Gluon propagator in n_z . A = 0:**

$$\tilde{d}^{\alpha\beta}(l) = -g^{\alpha\beta} + \frac{l^\alpha n_z^\beta + n_z^\alpha l^\beta}{l_z} - \frac{n_z^2\, l^\alpha l^\beta}{l_z^2}$$

with $n_z^2 = -1$

One-loop "quasi-quark" distribution in a quark

Ma and Qiu, arXiv:1404.6860

❑ Real + virtual contribution:

$$\tilde{f}^{(1)}_{q/q}(\tilde{x},\tilde{\mu}^2,P_z) = C_F\frac{\alpha_s}{2\pi}\frac{(4\pi)^\epsilon}{\Gamma(1-\epsilon)}\int_0^{\tilde{\mu}^2}\frac{dl_\perp^2}{l_\perp^{2+2\epsilon}}\int_{-\infty}^{+\infty}\frac{dl_z}{P_z}\left[\delta(1-\tilde{x}-y)-\delta(1-\tilde{x})\right]\left\{\frac{1}{y}\left(1-y+\frac{1-\epsilon}{2}y^2\right)\right.$$
$$\left.\times\left[\frac{y}{\sqrt{\lambda^2+y^2}}+\frac{1-y}{\sqrt{\lambda^2+(1-y)^2}}\right]+\frac{(1-y)\lambda^2}{2y^2\sqrt{\lambda^2+y^2}}+\frac{\lambda^2}{2y\sqrt{\lambda^2+(1-y)^2}}+\frac{1-\epsilon}{2}\frac{(1-y)\lambda^2}{[\lambda^2+(1-y)^2]^{3/2}}\right\}$$

where $y = l_z/P_z,\ \lambda^2 = l_\perp^2/P_z^2,\ C_F = (N_c^2-1)/(2N_c)$

❑ Cancelation of CO divergence:

$$\frac{y}{\sqrt{\lambda^2+y^2}}+\frac{1-y}{\sqrt{\lambda^2+(1-y)^2}} = 2\theta(0<y<1)-\left[\mathrm{Sgn}(y)\frac{\sqrt{\lambda^2+y^2}-|y|}{\sqrt{\lambda^2+y^2}}+\mathrm{Sgn}(1-y)\frac{\sqrt{\lambda^2+(1-y)^2}-|1-y|}{\sqrt{\lambda^2+(1-y)^2}}\right]$$

Only the first term is CO divergent for $0 < y < 1$, which is the same as the divergence of the normal quark distribution – necessary!

❑ UV renormalization:

Different treatment for the upper limit of $l_\perp^2$ integration - "scheme"

Here, a UV cutoff is used – other scheme is discussed in the paper

One-loop coefficient functions

Ma and Qiu, arXiv:1404.6860

- **MS scheme for $f_{q/q}(x,\mu^2)$:**

CO, UV IR finite!

$$\mathcal{C}^{(1)}_{q/q}(t,\tilde{\mu}^2,\mu^2,P_z) = \tilde{f}^{(1)}_{q/q}(t,\tilde{\mu}^2,P_z) - f^{(1)}_{q/q}(t,\mu^2)$$

$$\frac{\mathcal{C}^{(1)}_{q/q}(t)}{C_F\frac{\alpha_s}{2\pi}} = \left[\frac{1+t^2}{1-t}\ln\frac{\tilde{\mu}^2}{\mu^2}+1-t\right]_+ + \left[\frac{t\Lambda_{1-t}}{(1-t)^2}+\frac{\Lambda_t}{1-t}+\frac{\text{Sgn}(t)\Lambda_t}{\Lambda_t+|t|} - \frac{1+t^2}{1-t}\left[\text{Sgn}(t)\ln\left(1+\frac{\Lambda_t}{2|t|}\right)+\text{Sgn}(1-t)\ln\left(1+\frac{\Lambda_{1-t}}{2|1-t|}\right)\right]\right]_N$$

where $\Lambda_t = \sqrt{\tilde{\mu}^2/P_z^2+t^2}-|t|$, $\text{Sgn}(t)=1$ if $t\geq 0$, and -1 otherwise.

- **Generalized "+" description:** $t=\tilde{x}/x$

$$\int_{-\infty}^{+\infty} dt \left[g(t)\right]_N h(t) = \int_{-\infty}^{+\infty} dt\, g(t)\,[h(t)-h(1)]$$

For a testing function $h(t)$

Explicit verification of the CO factorization at one-loop

Note: $\Lambda_t \to \mathcal{O}\left(\frac{\widetilde{\mu}}{P_Z}\right)$ as $P_Z\to\infty$ the linear power UV divergence!

Summary and outlook

- "lattice cross sections" = single hadron matrix elements
  calculable in Lattice QCD, renormalizable + factorizable in QCD

  Going beyond the quasi-PDFs

- Extract PDFs by global analysis of data on "Lattice x-sections".
  Same should work for other distributions (TMDs, GPDs)

$$\widetilde{\sigma}_{\mathrm{E}}^{\mathrm{Lat}}(\tilde{x}, \frac{1}{a}, P_z) \approx \sum_i \int_0^1 \frac{dx}{x} f_{i/h}(x, \mu^2)\, \widetilde{C}_i(\frac{\tilde{x}}{x}, \frac{1}{a}, \mu^2, P_z)$$

- Conservation of difficulties – complementarity:
  High energy scattering experiments
  – less sensitive to large x parton distribution/correlation
  "Lattice factorizable cross sections"
  – more suited for large x PDFs, but limited to large x for now

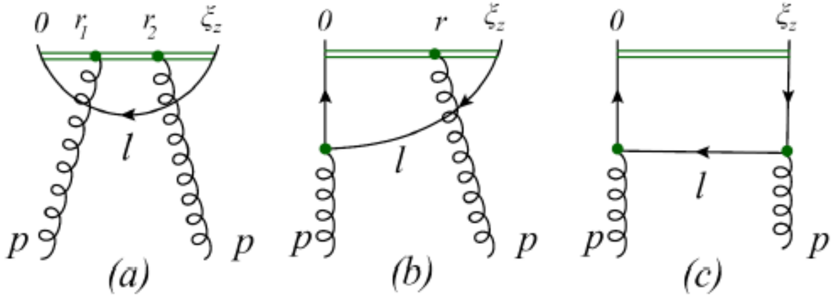
- Lattice QCD can be used to study hadron structure, but,
  more works are needed!

Thank you!

BACKUP SLIDES

Renormalization

- **Gluon-to-quark at one-loop:**



$$M_{2a} \propto \int_0^{\xi_z} dr_1 \int_{r_1}^{\xi_z} dr_2 \int d^4 l\, e^{-il_z\xi_z} \frac{l_z}{l^2}$$

$$= \frac{\xi_z^2}{2} \int dl_z\, e^{-il_z\xi_z}\, l_z \int d^3\bar{l} \left(\frac{1}{\bar{l}^2} + \frac{l_z^2}{(\bar{l}^2 - l_z^2)\bar{l}^2} \right)$$

- UV divergence from 3-D $\propto \delta'(\xi_z)$, vanishes for finite ξ_z

- **Caution for momentum-space version:**

Finite-term:

$$\frac{\xi_z^2}{2} \int dl_z\, e^{-il_z\xi_z}\, l_z \int d^3\bar{l} \frac{l_z^2}{(\bar{l}^2 - l_z^2)\bar{l}^2}$$

$$\propto \frac{\xi_z^2}{2} \int dl_z\, e^{-il_z\xi_z} \frac{l_z^3}{|l_z|} = \frac{2i}{\xi_z}$$

- Divergent as $\xi_z \to 0$
- Result in bad large $\tilde{x}$ behavior in momentum space