Fate of the neutron-deuteron virtual state as an Efimov level

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Jefferson Laboratory Theory Seminar, Mar 19, 2018

Outline

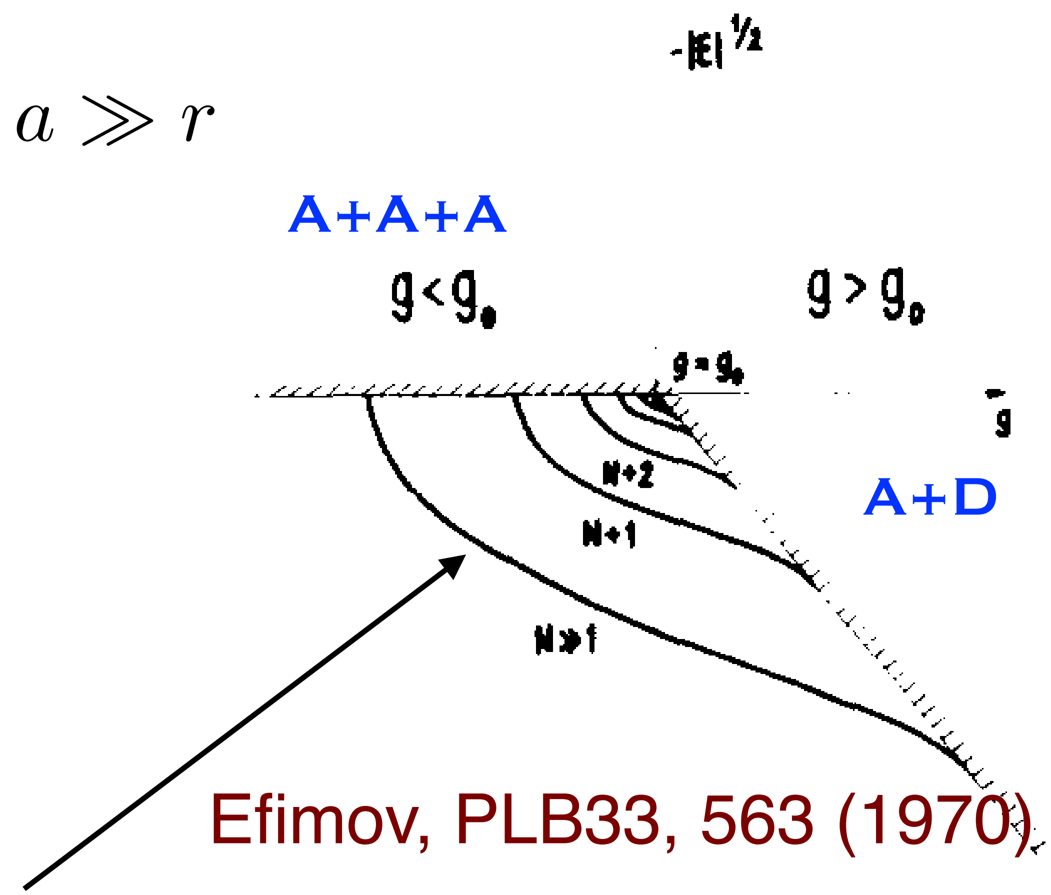
- Background
    - Efimov levels, experiments, 3-nucleon systems
- Effective Field Theory
    - Pionless EFT as the fundamental theory
    - Halo EFT of deuteron
- Results
- Conclusions

Efimov levels

- Two body interaction $gV(r)$
- At $g = g_0,$ scattering length $a \gg r$
- Three-body bound states

$$\# \sim \frac{1}{\pi} \ln\left(\frac{|a|}{r}\right)$$

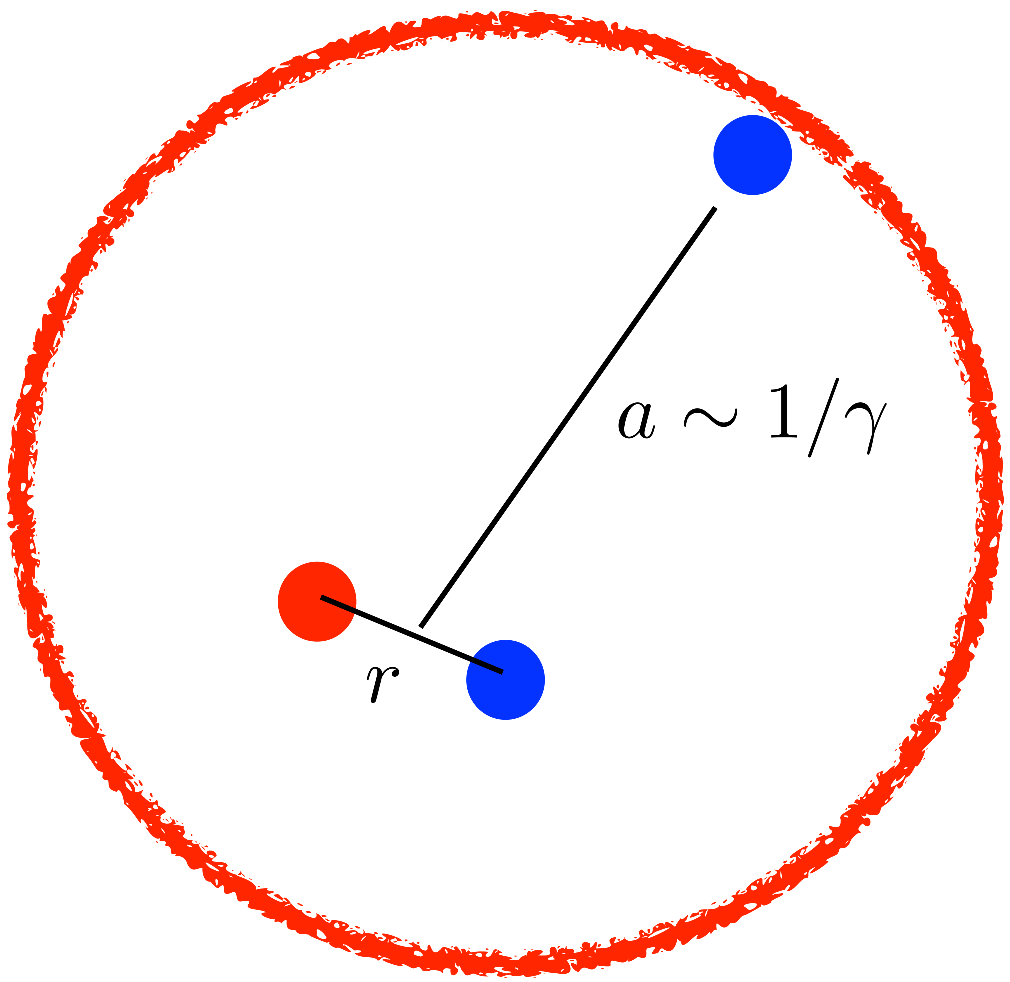
$$\text{between } \frac{1}{ma^2} \text{ and } \frac{1}{mr^2}$$

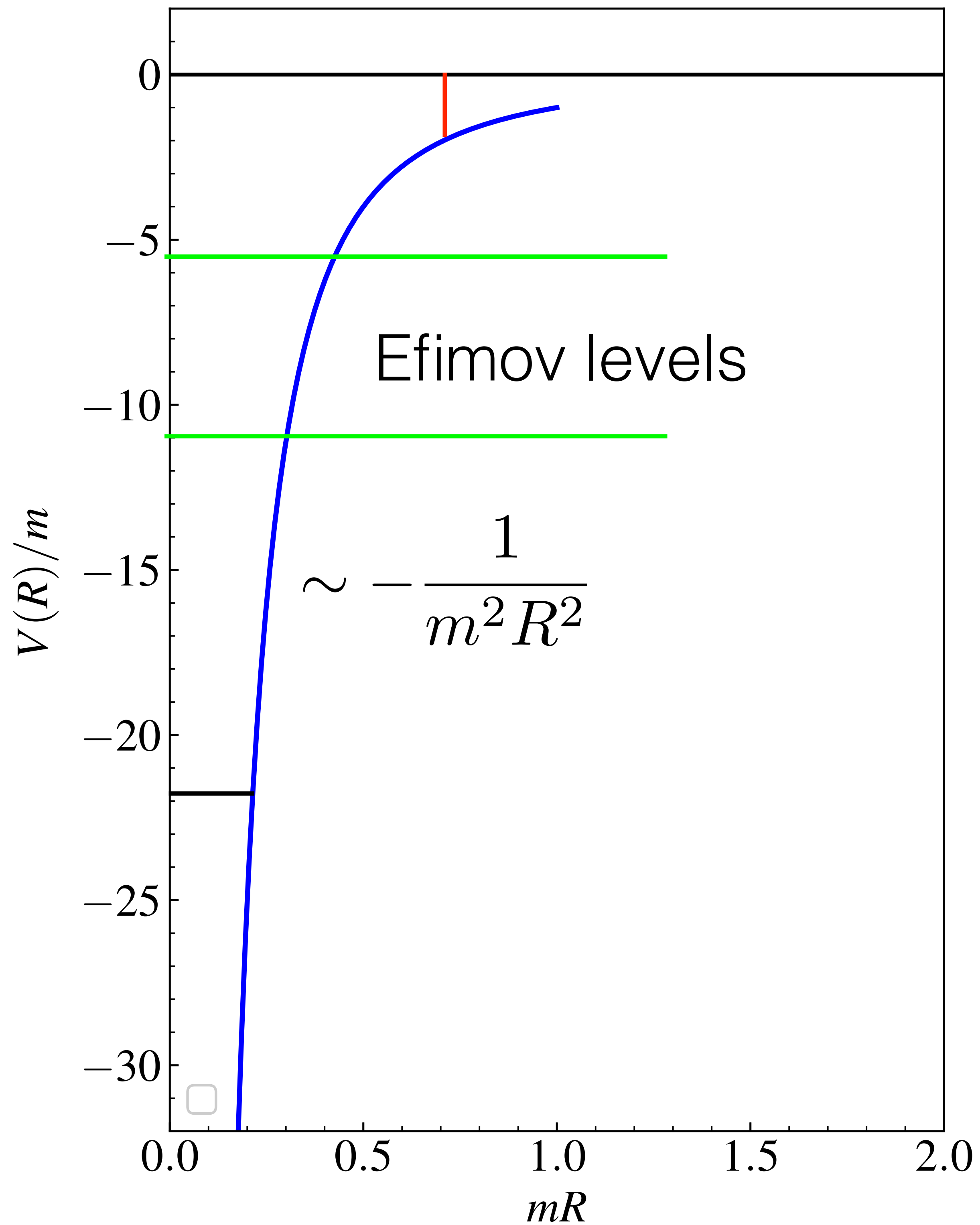


Efimov, PLB33, 563 (1970)

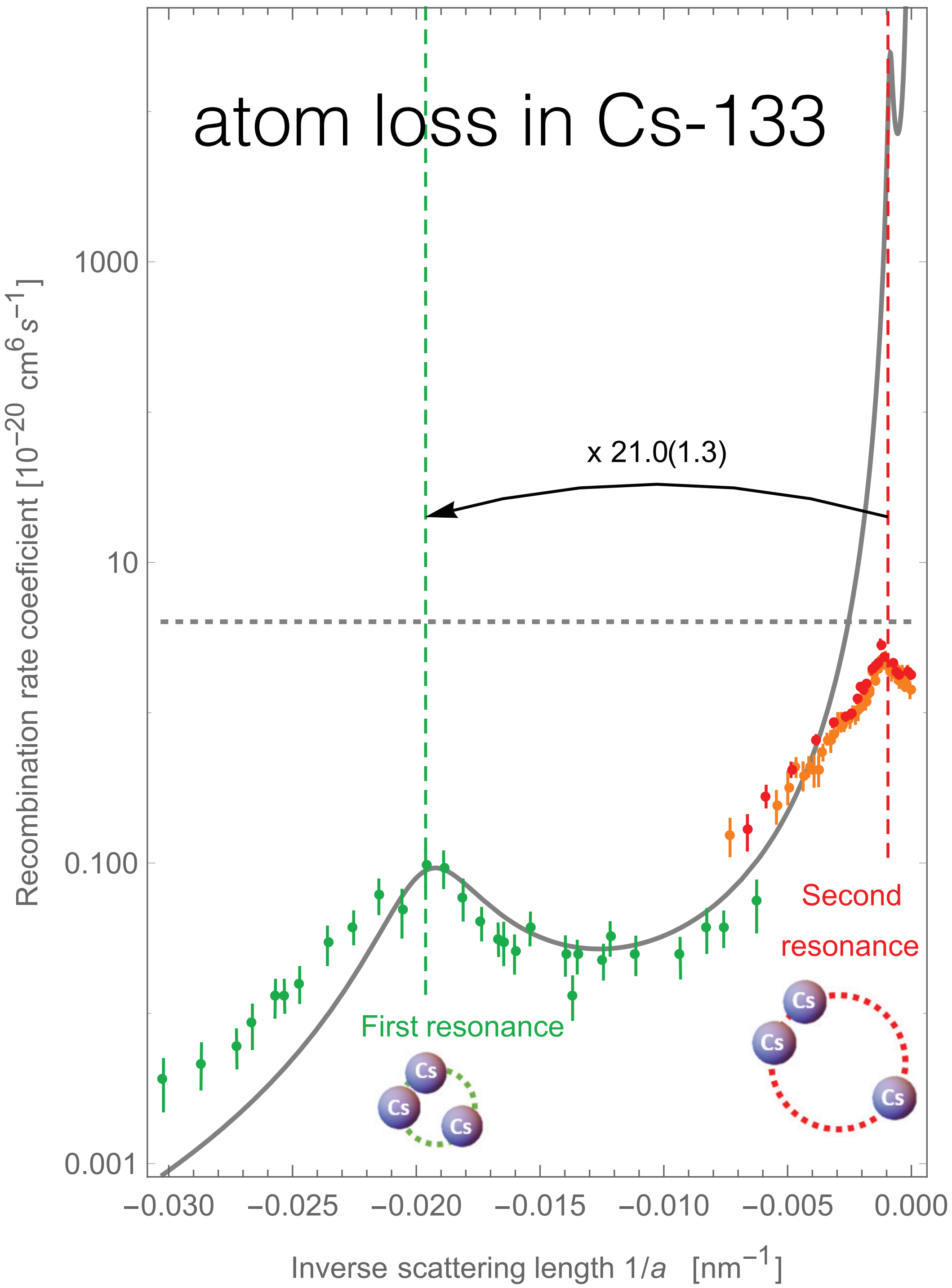
geometrical scaling

Why does it happen?



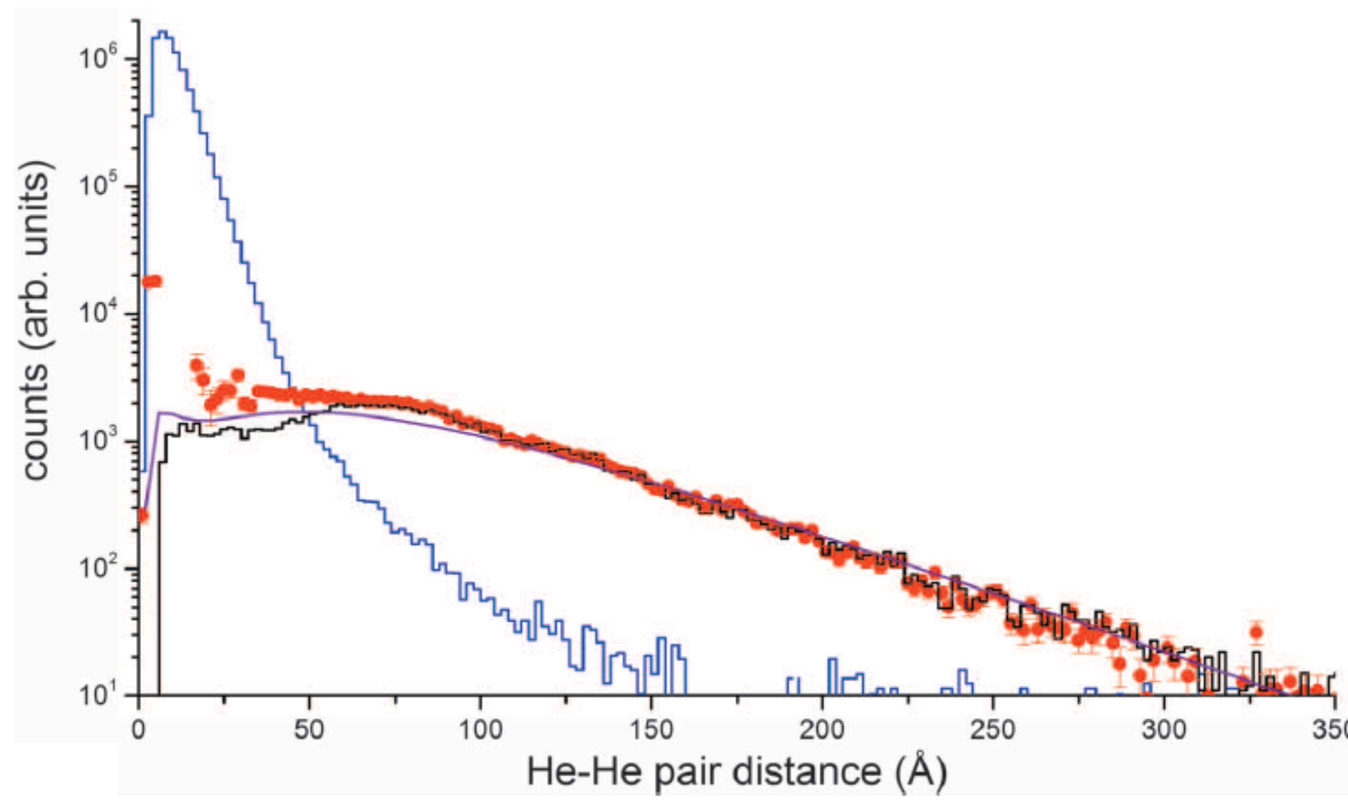


Cold Atom Experiments



Kraemer et al. Nature 440, 315 (2006)

He-4 trimers



Kunitski et al. Science 348, 551 (2015)

Naidon and Endo review paper Rep. Prog. Phys. 80,56001 (2017)

Nuclear Systems?

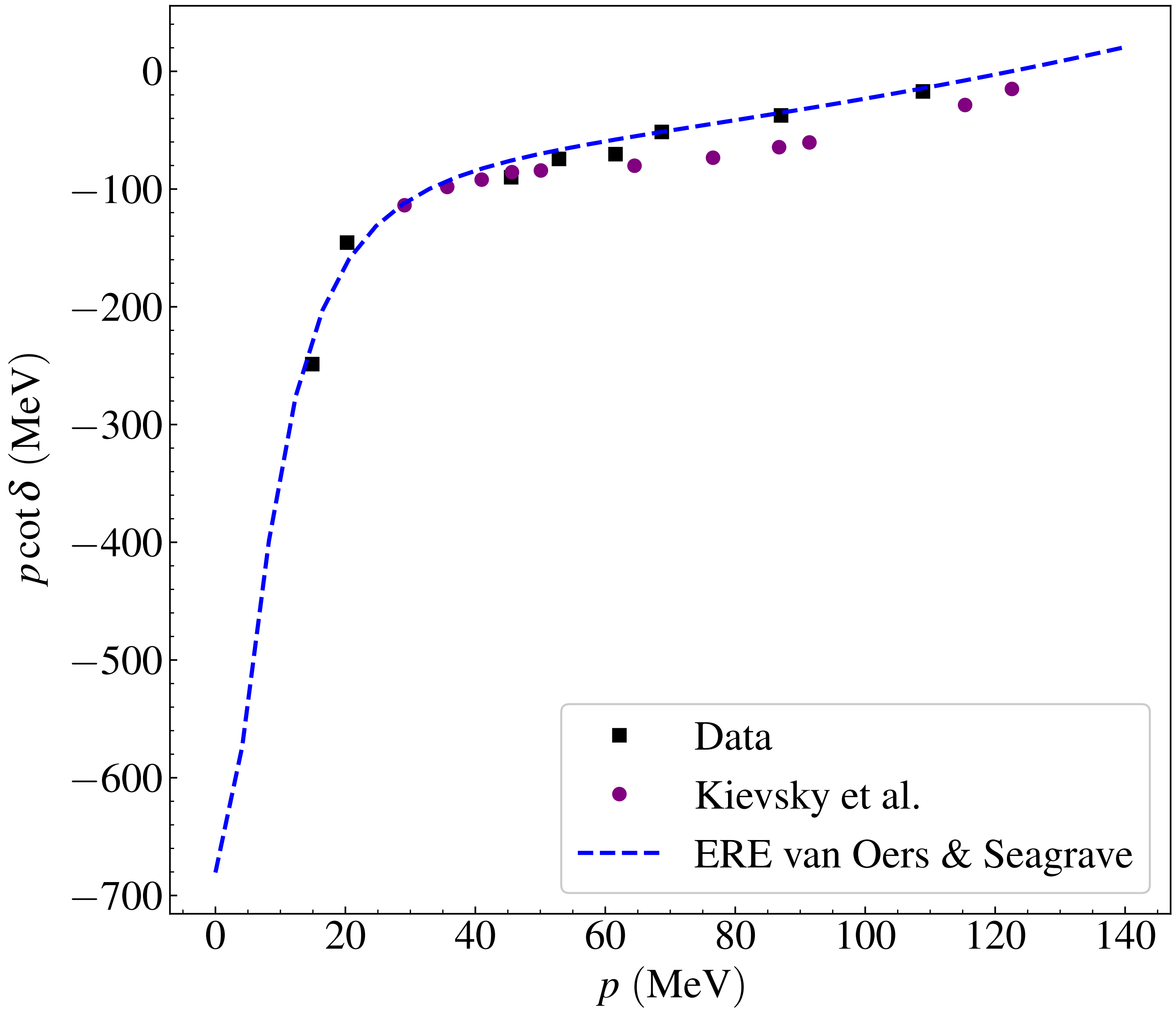
- Triton binding energy ~ 8.5 MeV, deeper state ~ 4.4 GeV.

  Forget that!

- Coulomb force — introduces new scale. Only very light systems

However, neutron-deuteron scattering does have a virtual state
Girard and Fuda (1979), Adhikari and Torreao (1983)

Neutron-Deuteron Virtual State



van Oers & Seagrave (1967):

$$p\cot\delta = \frac{-1/a + rp^2/2 + \cdots}{p^2 + p_0^2}$$

Pole at $p^2 = -p_0^2 \sim -100\ \text{MeV}^2$

Shallow virtual state B~ 0.5 MeV

Data: Ref. [1] and [2] in Phys. Lett. 562 (1967)

Virtual State as Efimov Level?

Accumulation of 3-body Efimov levels near unitarity: $|a| \to \infty, r \to 0$

1. Achieve unitarity theoretically (not feasible experimentally)
   - Want a model-independent method
   - Universally applicable
2. Model-independent description of shallow virtual state
   - Derive the modified ERE below deuteron breakup

For first task: use pionless EFT that produces triton and virtual state as *"the fundamental theory"* to generate **"data"**

For second task: formulate a low energy theory with fundamental deuteron fields (a halo EFT)

EFT: the long and short of it

- Identify degrees of freedom

$$\mathcal{L} = c_0\,\mathrm{O}^{(0)} + c_1\,\mathrm{O}^{(1)} + c_2\,\mathrm{O}^{(2)} + \cdots$$ expansion in

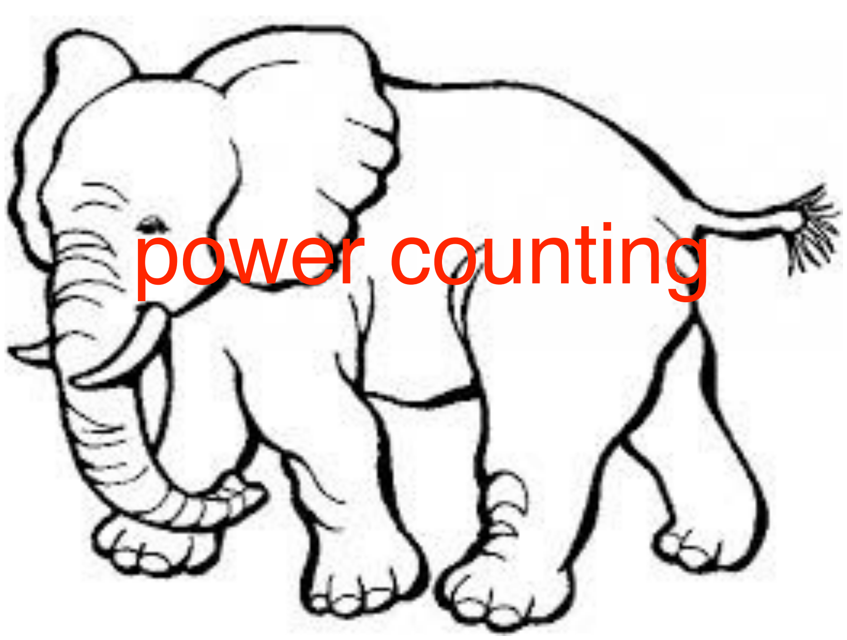
Hide UV ignorance- short distance

IR explicit- long distance

- Determine c_n from data (elastic, inelastic)

- EFT : ERE + currents + relativistic corrections

Not just Ward-Takahashi identity



Pionless EFT — $\not{\pi}$EFT

nucleon-nucleon scattering

$$i\mathcal{A}(p) = \frac{2\pi}{\mu}\frac{i}{p\cot\delta_0 - ip} = \frac{2\pi}{\mu}\frac{i}{-1/a + \frac{r}{2}p^2 + \cdots - ip}$$

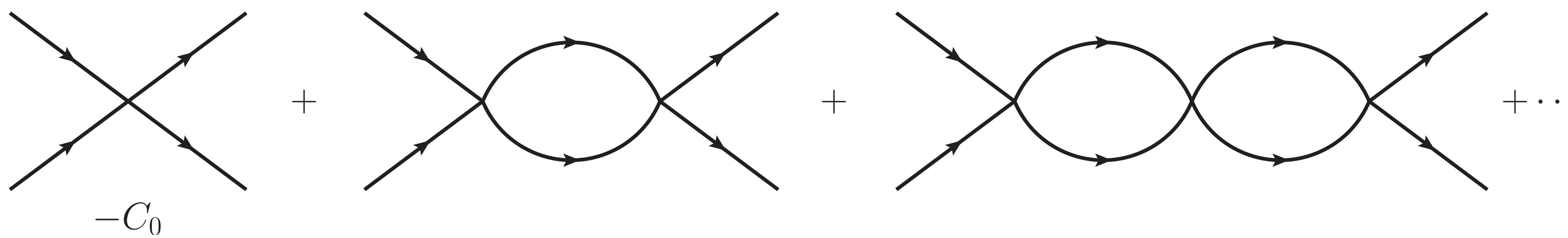
$$\approx -\frac{2\pi}{\mu}\frac{i}{1/a + ip}\left[1 + \frac{rp^2/2}{1/a + ip} + \cdots\right]$$, for a ~ 1/p >> r

Example: neutron-proton scattering

$$^1S_0 : a = -\,23.8 \text{ fm}, \quad r = 2.73 \text{ fm},$$
$$^3S_1 : a = +\,5.42 \text{ fm}, \quad r = 1.75 \text{ fm}.$$

Construct $\not{\pi}$ EFT

- Non-relativistic nucleons
- Short ranged interaction — point-like interaction



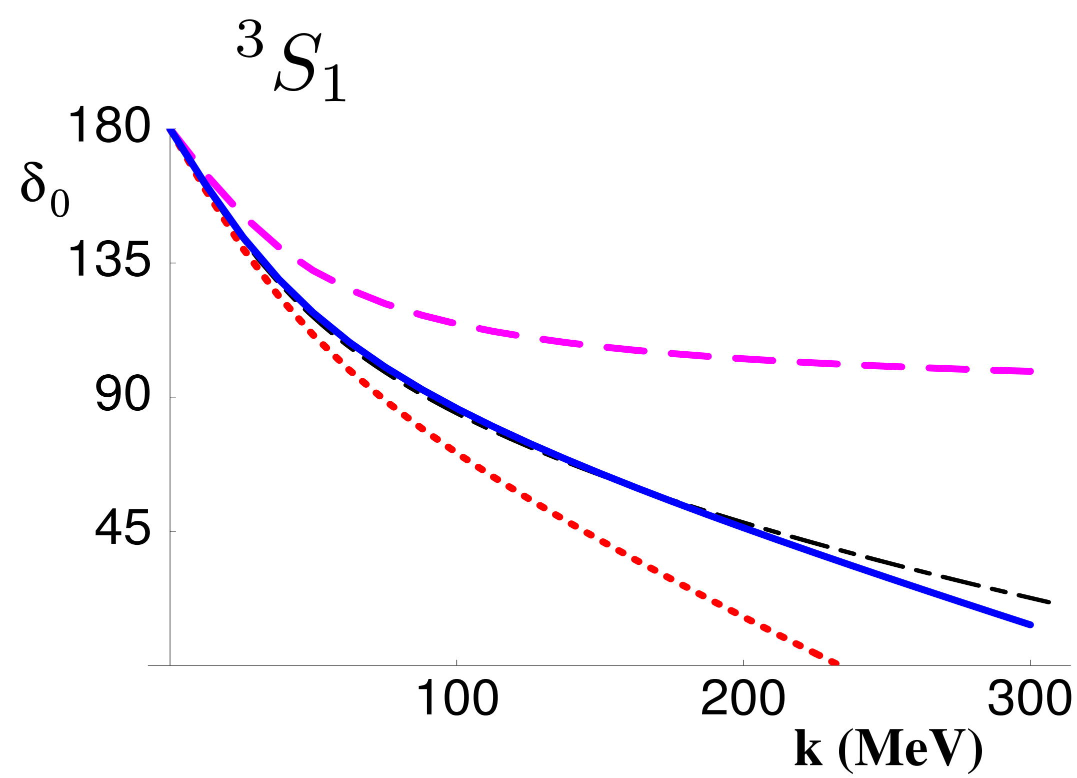
Weinberg ’90
Bedaque, van Kolck ’97
Kaplan, Savage, Wise ’98

$$i\mathcal{A}(p) = \frac{-i}{\frac{1}{C_0} + i\frac{\mu}{2\pi}p} \Rightarrow C_0 \sim \frac{2\pi a}{\mu}$$

$$1/a \sim p \sim Q \ll 1/r \sim \Lambda \sim m_\pi$$

power-counting $C_0 \sim 1/Q$

single fine-tuning (rho-pion physics)

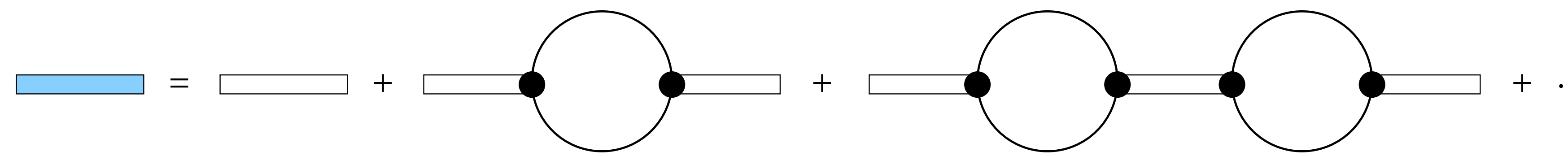


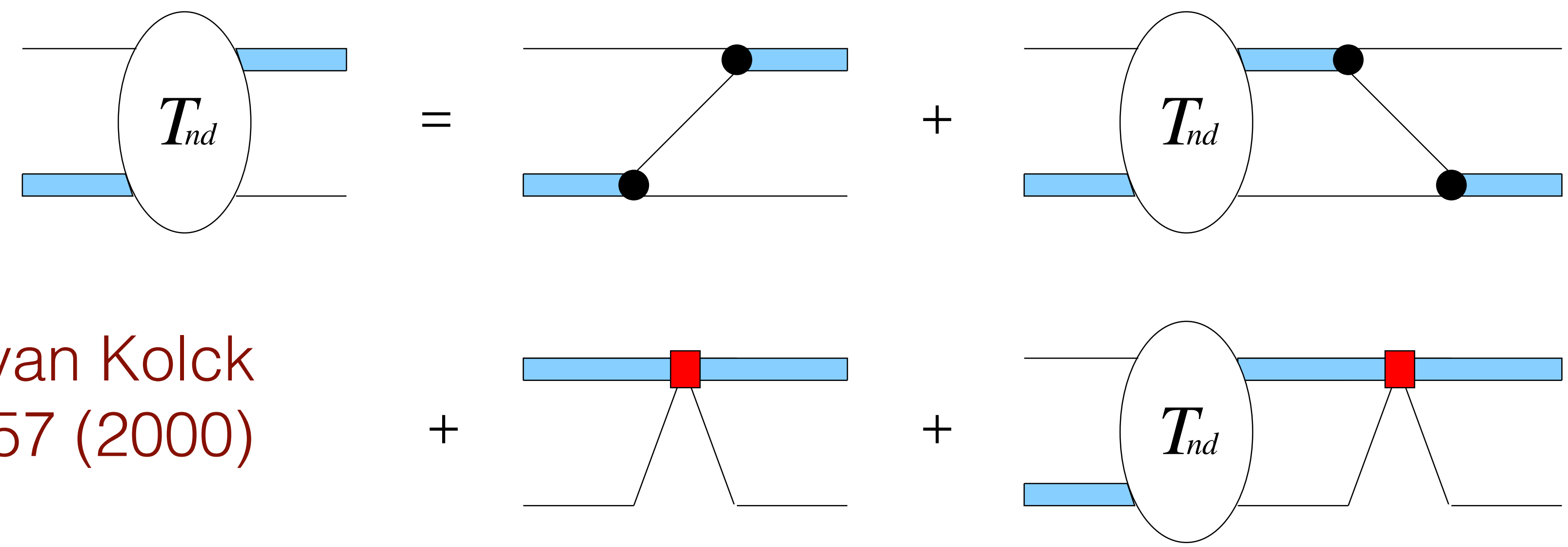
Chen, Rupak, Savage (1999)
Phillips, Rupak, Savage (1999)

Neutron-Deuteron Scattering

dimer-formulation (auxiliary field)

$$C_0 \leftrightarrow \frac{g^2}{\Delta}$$





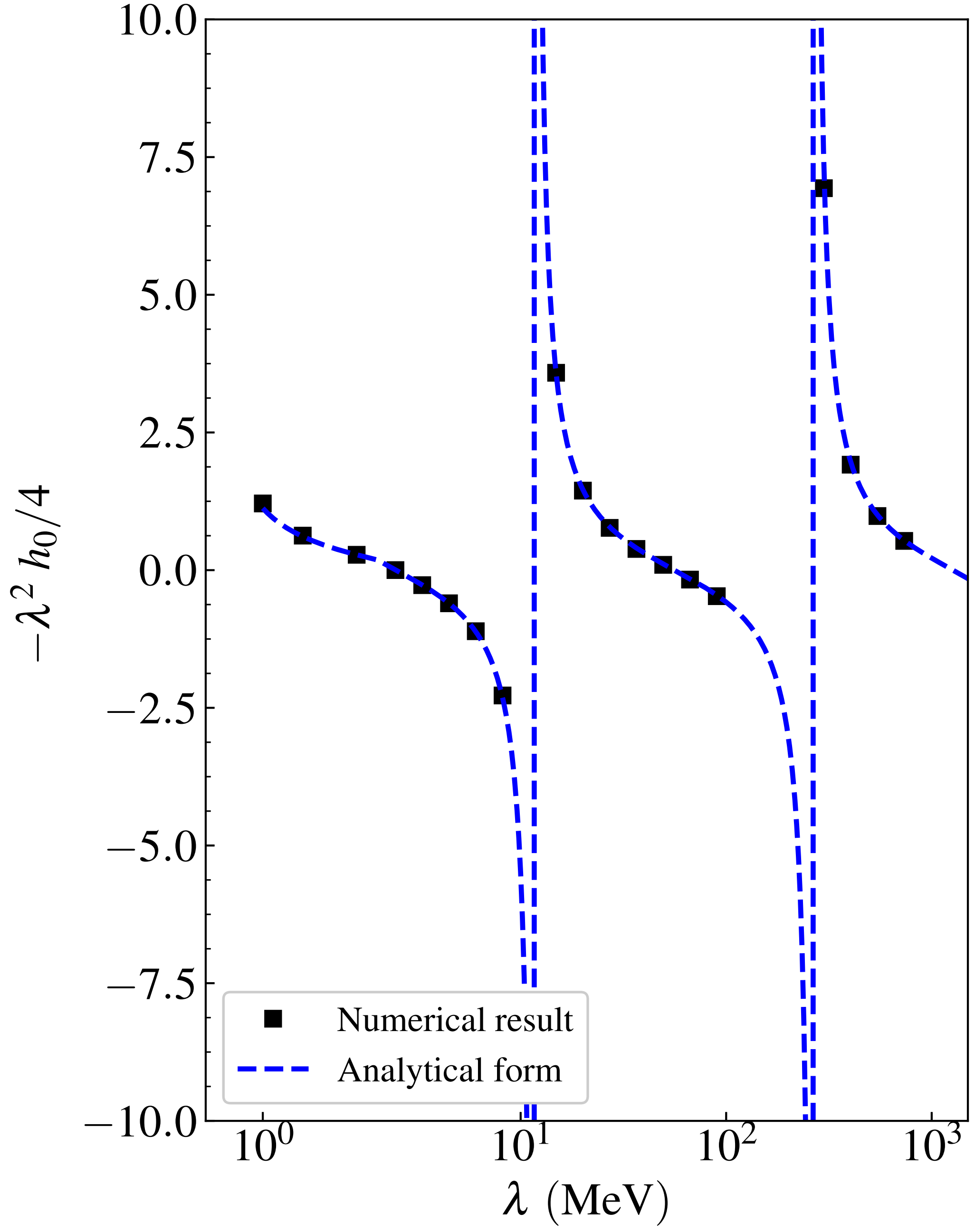
Bedaque, Hammer, van Kolck
Nucl. Phys. A 676, 357 (2000)

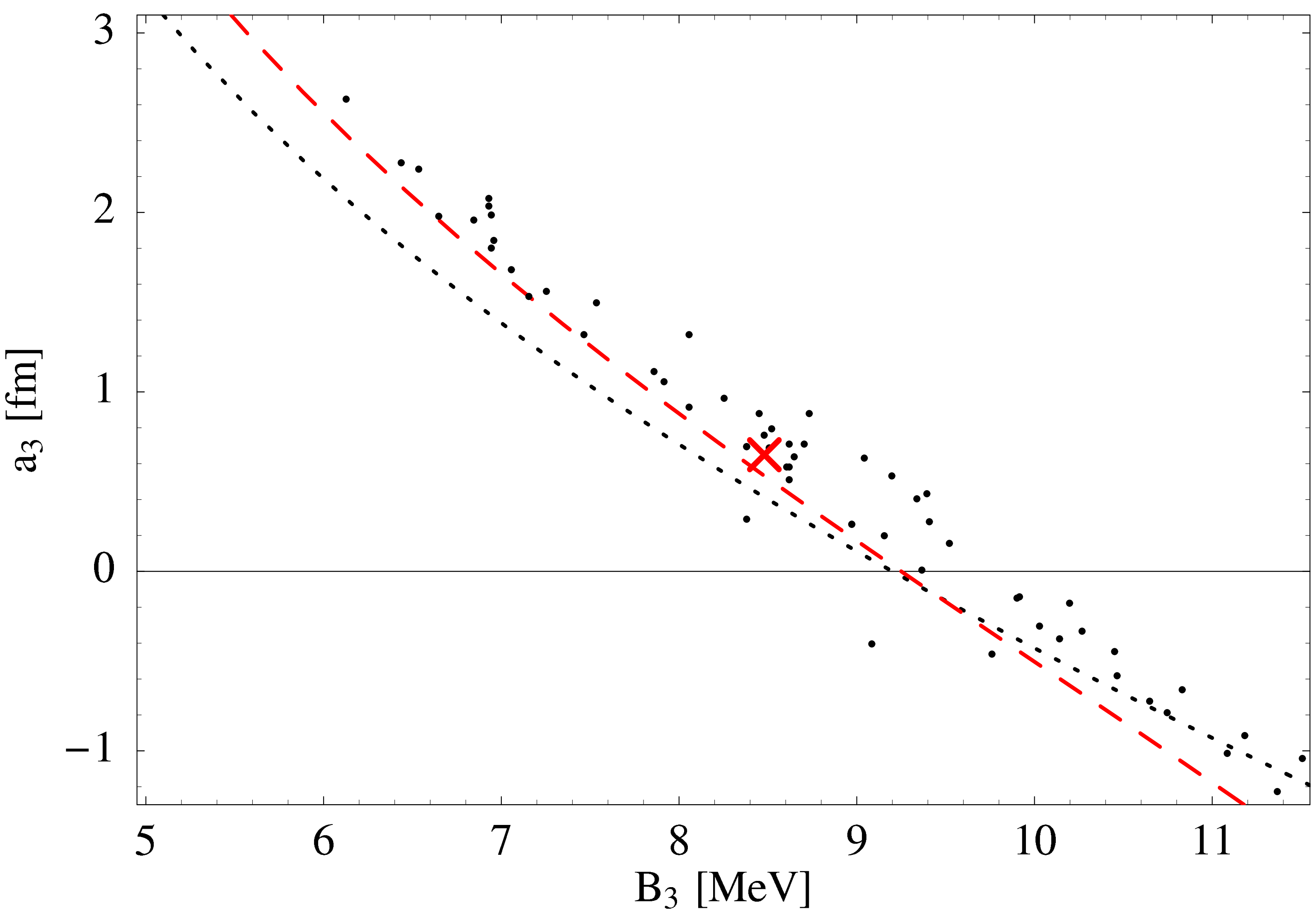
$$h_0(\lambda) \sim -\frac{4}{\lambda^2}\frac{\sin\left[s_0 \ln(\lambda/\lambda^*) - \tan^{-1}(s_0)\right]}{\sin\left[s_0 \ln(\lambda/\lambda^*) + \tan^{-1}(s_0)\right]},$$

$$s_0 \approx 1.0062$$

3-nucleon coupling
— limit cycle, Wilson (1971)
— Phillips line (1968)

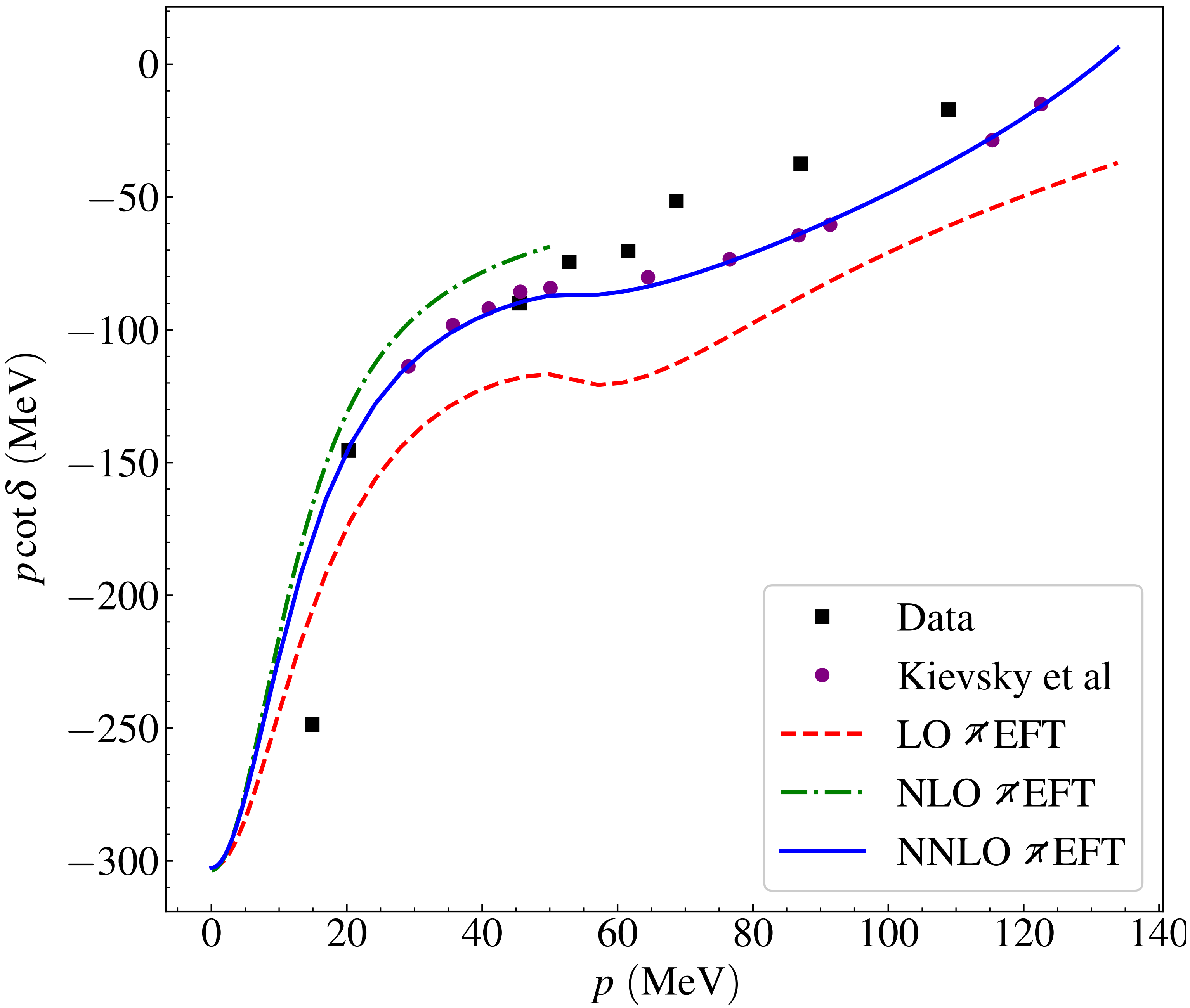
Limit Cycle, Phillips line





Bedaque, Rupak, Grießhammer, Hammer
Nucl. Phys. A 714, 589 (2003)

Neutron-Deuteron in pionless EFT



<u>pionless EFT Input</u>

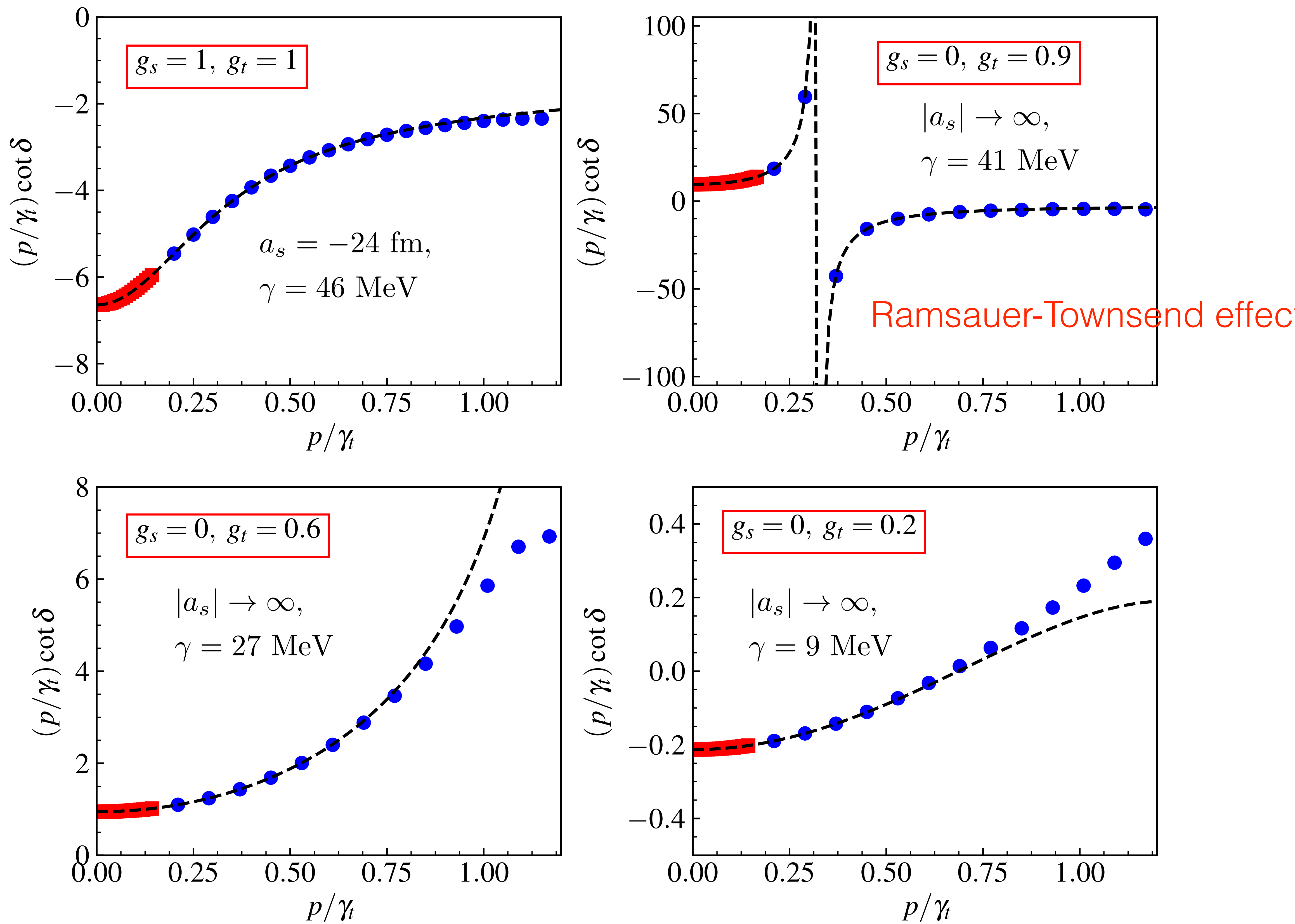
$$\mathrm{LO}:\ \gamma,\ a_s,\ a_3$$
$$\mathrm{NLO}:\ \mathrm{LO}+\ r_t,\ r_s$$
$$\mathrm{NNLO}:\ \mathrm{NLO}+\ B_3$$

Bedaque, Rupak, Grießhammer, Hammer (2003)

NLO: S. König, J. Vanesse

Next proceed to derive a theory with fundamental deuteron fields below breakup

Phase Shift


$g_s = 1,\ g_t = 1$
$a_s = -24$ fm,
$\gamma = 46$ MeV
$(p/\gamma_t)\cot\delta$
p/γ_t
$g_s = 0,\ g_t = 0.9$
$|a_s| \to \infty$,
$\gamma = 41$ MeV
Ramsauer-Townsend effect
$g_s = 0,\ g_t = 0.6$
$|a_s| \to \infty$,
$\gamma = 27$ MeV
$g_s = 0,\ g_t = 0.2$
$|a_s| \to \infty$,
$\gamma = 9$ MeV

Halo EFT and modified ERE

$$\mathcal{L} = n^\dagger [i\partial_0 + \frac{\boldsymbol{\nabla}^2}{2m_N}]n + d_a^\dagger [i\partial_0 + \frac{\boldsymbol{\nabla}^2}{2m_d}]d_a + \sum_{i=1}^{2} \psi^{(i)\dagger}[\Delta_i + c_i(i\partial_0 + \frac{\boldsymbol{\nabla}^2}{2M})]\psi^{(i)}$$

$$+ \sum_{i=1}^{2} \sqrt{\frac{2\pi}{\mu}} [\psi^{(i)\dagger} \frac{\sigma_a}{\sqrt{3}} n d_a + \mathrm{h.\,c.}],$$

Introduce two auxiliary fields!

<u>neutron-deuteron amplitude:</u>

$$iT_t(p) = \frac{2\pi}{\mu} \frac{i}{-\left[\frac{1}{\Delta_1 + c_1 p^2/(2\mu)} + \frac{1}{\Delta_2}\right]^{-1} - ip} = \frac{2\pi}{\mu} \frac{i}{p\cot\delta - i}$$

calculate as a 2-body amplitude

Generate modified ERE:

$$p_0^2 = 2\mu \frac{\Delta_1 + \Delta_2}{c_1}, \qquad \frac{1}{a} = \frac{\Delta_1 \Delta_2}{\Delta_1 + \Delta_2}, \qquad -\frac{2}{r} = 2\mu \frac{\Delta_1 + \Delta_2}{c_1 \Delta_2},$$

Halo EFT Power-Counting

- Breakdown scale Λ set by deuteron breakup momentum
- Zero of T-matrix at ***Q***
- Virtual state momentum **א**

Initially: $Q^2 \sim 100 \text{ MeV}^2 \ll \aleph^2 \sim 900 \text{ MeV}^2 \ll \Lambda^2 \sim 2500 \text{ MeV}^2$

We define: $\gamma_t \equiv g_t\gamma \approx 45.7 g_t \text{ MeV}, \quad \gamma_s \equiv g_s/a_s = -8.3 g_s \text{ MeV}$

Approach unitarity as: $(g_s = 0, g_t \to 0)$

As we tune the pionless EFT, $\boldsymbol{Q^2}$ gets smaller, changes sign and approaches Λ^2 and exceeds it.

Power-counting has to account for the varying relative size of $\boldsymbol{Q^2}$ (and other fine tunings)

Power-Counting Continued

Consider 3 intervals

$$0.7 \lesssim g_t \lesssim 1: \;\; \text{small } a \sim Q^2/(\aleph\Lambda^2), \text{large } r \sim \Lambda^2/(\aleph Q^2)$$

$\Delta_1 \sim \Delta_2$ and $c_2 \ll c_1$ small shape parameter

$Q \gtrsim \Lambda$

$$0.3 \lesssim g_t \lesssim 0.7: \;\; \text{large } a \sim r \sim 1/\aleph$$

$\Delta_2 \gg \Delta_1$ and still $c_2 \ll c_1$

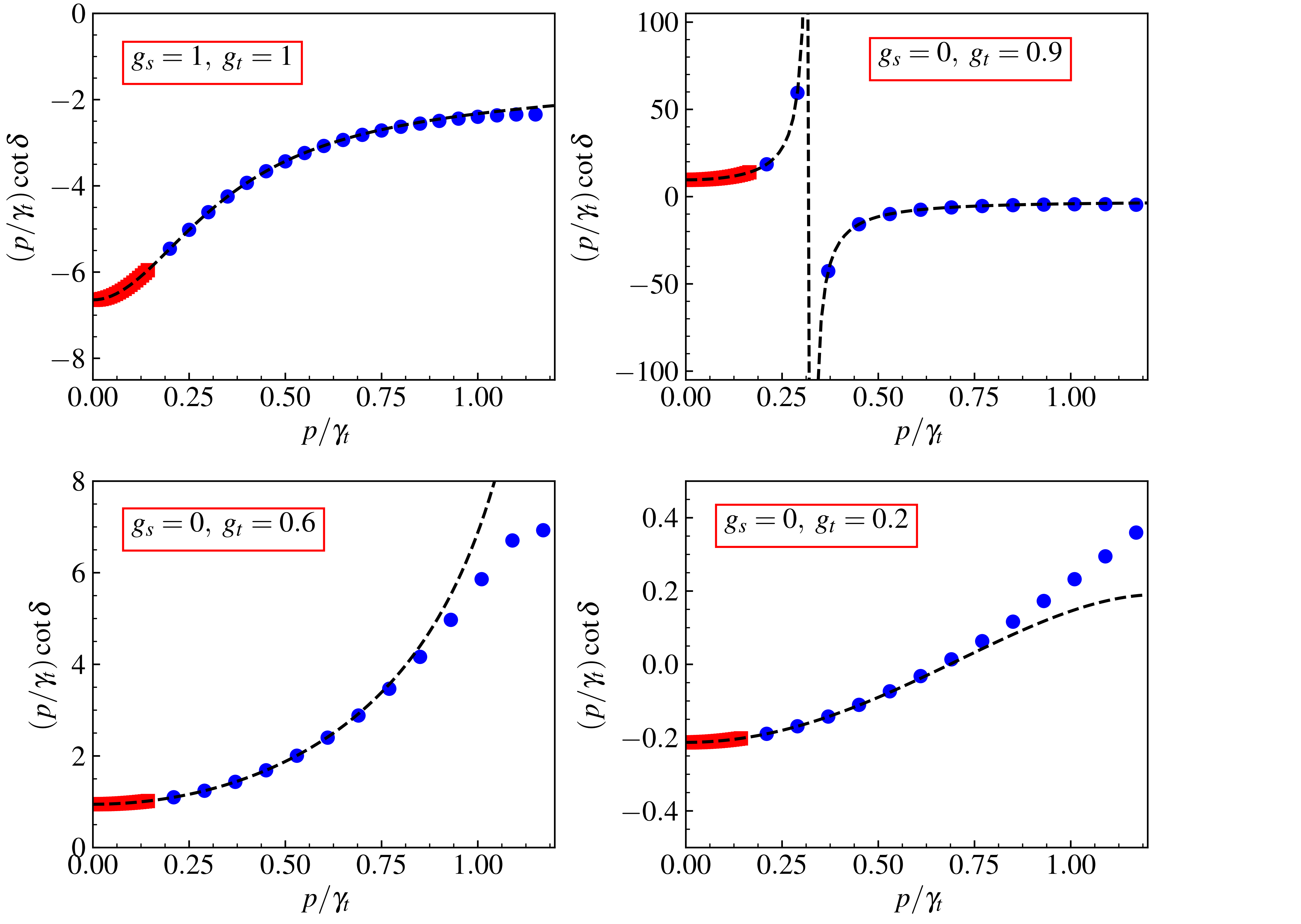
Second auxiliary field decouples: regular ERE

$$0.1 \lesssim g_t \lesssim 0.3: \;\; \text{large } a \sim 1/\aleph \text{ and } r \lesssim 1/\Lambda$$

Familiar unitary limit EFT with a single auxiliary field

Continue on?

Phase Shift Again



Virtual, Bound and Resonance States

Look at analytic structure of the S-matrix

$$S_t(p) = e^{2i\delta(p)} = 1 + \frac{i2p}{p\cot\delta - ip} = 1 + i\frac{\mu p}{\pi}T_t(p) = 1 + \frac{i2p}{\frac{-1/a + rp^2/2}{p^2+p_0^2} - ip}$$

$$= -\frac{(p+i\pi_1)(p+i\pi_2)(p+i\pi_3)}{(p-i\pi_1)(p-i\pi_2)(p-i\pi_3)}$$

Interpretation of the three poles in halo EFT:

$$\pi_1 + \pi_2 + \pi_3 = -\frac{r}{2}p_0^2, \qquad \pi_1\pi_2 + \pi_2\pi_3 + \pi_3\pi_1 = -p_0^2, \qquad \pi_1\pi_2\pi_3 = -\frac{p_0^2}{a}$$

3rd root not relevant as $\pi_3 \gg \Lambda$

1st root is the shallow virtual state

2nd root on positive imaginary axis … triton?

No, a redundant pole.

Redundant Pole

We look at the residue of the S-matrix near the poles

$$S_t(p) \sim \sum_i \frac{R_i}{p - i\pi_i} + \text{regular pieces},$$

Normalization of bound and virtual states

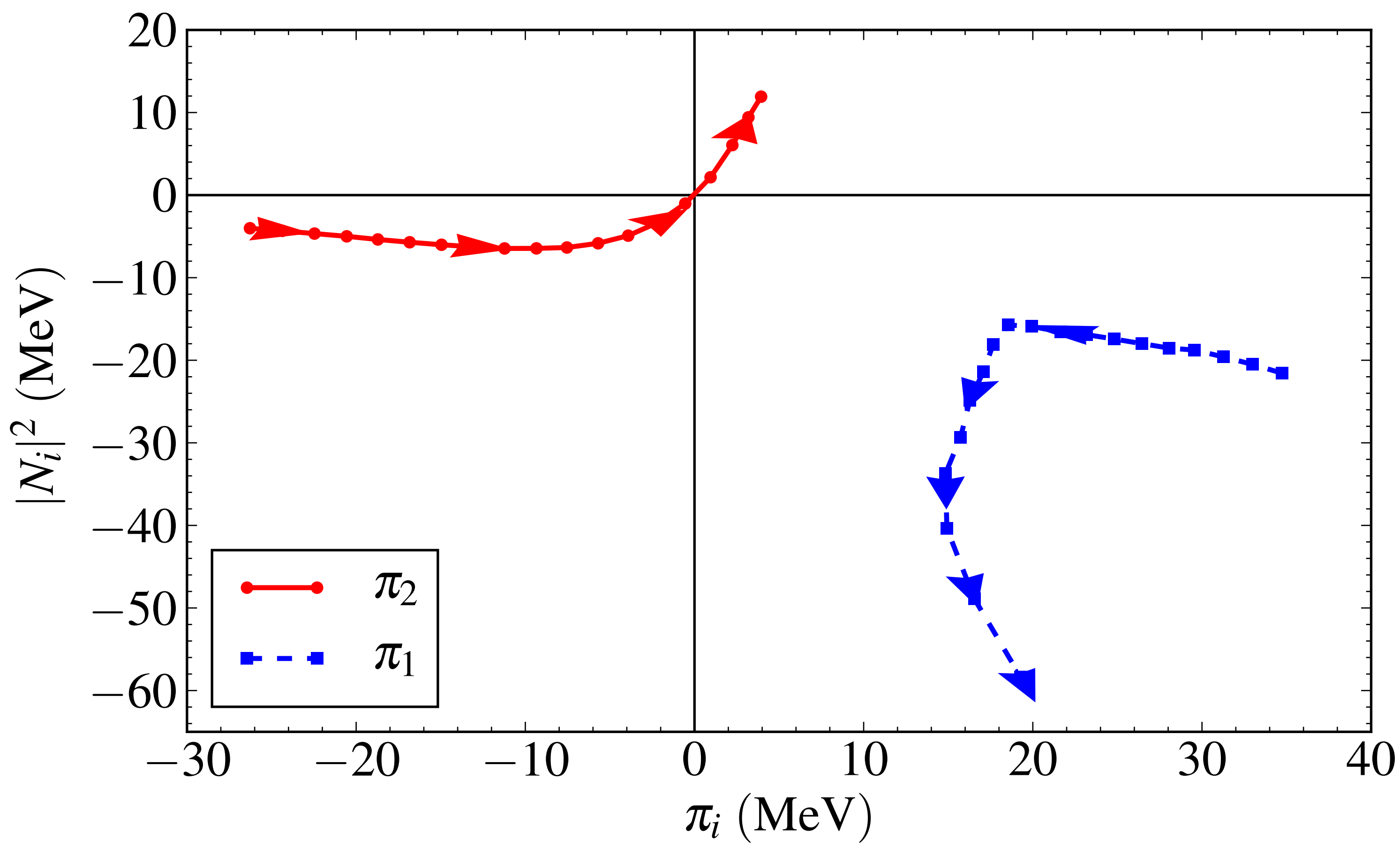
$$|N_1|^2 = iR_1 = \frac{2\pi_1(\pi_1^2 - p_0^2)}{(\pi_1 - \pi_2)(\pi_1 - \pi_3)},$$

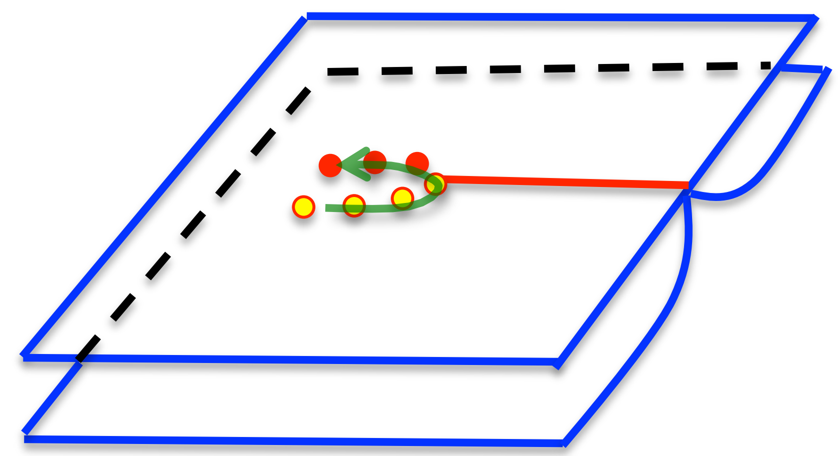
$$|N_2|^2 = iR_2 = \frac{2\pi_2(\pi_2^2 - p_0^2)}{(\pi_2 - \pi_1)(\pi_2 - \pi_3)} < 0.$$

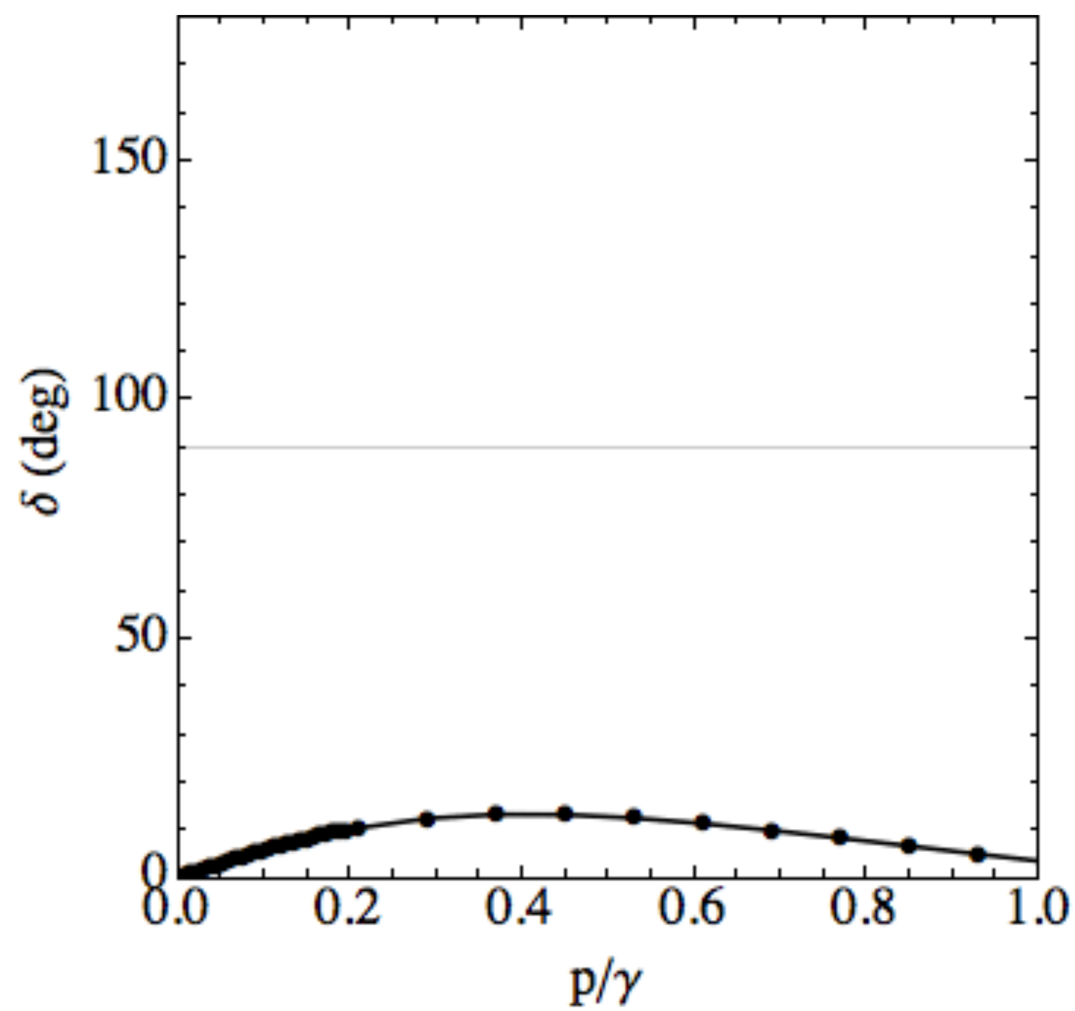
$\pi_2 > 0$ is called a redundant/shadow pole

Ma, Phys. Rev. 69, 668 (1946)

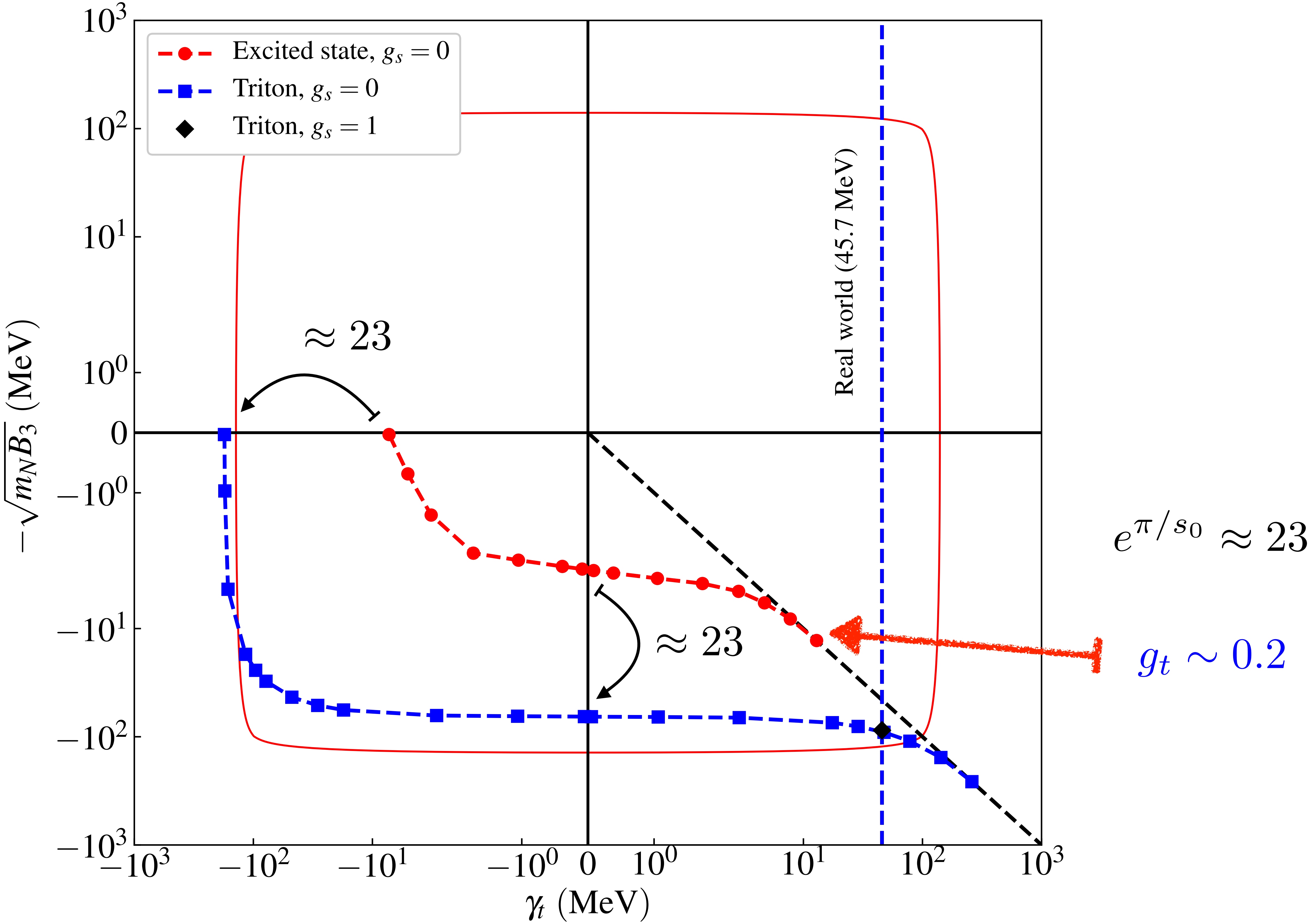
Virtual State to Efimov Level







Efimov Levels



Conclusions

- Efimov level emerged from the n-d virtual state near unitarity
- Model-independent analysis using a halo EFT
- Claim the mechanism for emergence of Efimov levels is universal
    - Atomic systems
    - lattice QCD at unphysical quark masses
- radiative capture in n-d, p-d system for Big Bang Nucleosynthesis