Fate of the neutron-deuteron virtual state as an Efimov level

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Outline

• Background
  • Efimov levels, experiments, 3-nucleon systems
• Effective Field Theory
  • Pionless EFT as the fundamental theory
  • Halo EFT of deuteron
• Results
• Conclusions
Efimov levels

- Two body interaction $gV(r)$
- At $g = g_0$, scattering length $a \gg r$
- Three-body bound states
  \[
  \# \sim \frac{1}{\pi} \ln \left( \frac{|a|}{r} \right)
  \]
  between $\frac{1}{ma^2}$ and $\frac{1}{mr^2}$

Efimov, PLB33, 563 (1970)

geometrical scaling
Why does it happen?

\[ a \sim \frac{1}{\gamma} \]

\[ V(R)/m \sim -\frac{1}{m^2R^2} \]

Efimov levels
Cold Atom Experiments

atom loss in Cs-133

He-4 trimers


Nuclear Systems?

• Triton binding energy \( \sim 8.5 \text{ MeV} \), deeper state \( \sim 4.4 \text{ GeV} \).
  
  Forget that!

• Coulomb force — introduces new scale. Only very light systems

However, neutron-deuteron scattering does have a virtual state
Girard and Fuda (1979), Adhikari and Torreao (1983)
Neutron-Deuteron Virtual State

van Oers & Seagrave (1967):

\[ p \cot \delta = \frac{-1/a + rp^2/2 + \cdots}{p^2 + p_0^2} \]

Pole at \( p^2 = -p_0^2 \sim -100 \text{ MeV}^2 \)

Shallow virtual state \( B \sim 0.5 \text{ MeV} \)

Virtual State as Efimov Level?

Accumulation of 3-body Efimov levels near unitarity: \( |a| \to \infty, r \to 0 \)

1. Achieve unitarity theoretically (not feasible experimentally)
   - Want a \textbf{model-independent} method
   - \textbf{Universally} applicable

2. Model-independent description of shallow virtual state
   - \textbf{Derive} the modified ERE below deuteron breakup

For first task: use pionless EFT that produces triton and virtual state as \textbf{“the fundamental theory”} to generate \textbf{“data”}

For second task: formulate a low energy theory with fundamental deuteron fields (a halo EFT)
EFT: the long and short of it

- Identify degrees of freedom

\[ \mathcal{L} = c_0 \, O^{(0)} + c_1 \, O^{(1)} + c_2 \, O^{(2)} + \cdots \]  

- Determine \( c_n \) from data (elastic, inelastic)

- EFT: ERE + currents + relativistic corrections

Hide UV ignorance - short distance  
IR explicit - long distance

Not just Ward-Takahashi identity
nucleon-nucleon scattering

\[ iA(p) = \frac{2\pi}{\mu} \frac{i}{p \cot \delta_0 - ip} = \frac{2\pi}{\mu} \frac{i}{-1/a + \frac{r}{2}p^2 + \cdots - ip} \]

\[ \approx -\frac{2\pi}{\mu} \frac{i}{1/a + ip} \left[ 1 + \frac{rp^2/2}{1/a + ip} + \cdots \right] , \text{ for } a \sim 1/p >> r \]

Example: neutron-proton scattering

\[ ^1S_0 : a = -23.8 \text{ fm}, \quad r = 2.73 \text{ fm}, \]

\[ ^3S_1 : a = +5.42 \text{ fm}, \quad r = 1.75 \text{ fm}. \]
Construct π EFT

- Non-relativistic nucleons
- Short ranged interaction — point-like interaction

\[ iA(p) = \frac{-i}{C_0} + i \frac{\mu}{2\pi p} \Rightarrow C_0 \sim \frac{2\pi a}{\mu} \]

\[ 1/a \sim p \sim Q \ll 1/r \sim \Lambda \sim m_\pi \]

power-counting \( C_0 \sim 1/Q \)

single fine-tuning (rho-pion physics)
Neutron-Deuteron Scattering

dimer-formulation (auxiliary field)

$C_0 \leftrightarrow \frac{g^2}{\Delta}$

$T_{nd} = + + + + \ldots$

Bedaque, Hammer, van Kolck

$h_0(\lambda) \sim - \frac{4 \sin \left[ s_0 \ln(\lambda/\lambda^*) - \tan^{-1}(s_0) \right]}{\lambda^2 \sin \left[ s_0 \ln(\lambda/\lambda^*) + \tan^{-1}(s_0) \right]}$,

$s_0 \approx 1.0062$

3-nucleon coupling
— limit cycle, Wilson (1971)
— Phillips line (1968)
Limit Cycle, Phillips line

\[-\lambda^2 h_0/4\]

\[\lambda (\text{MeV})\]

\[B_{3} [\text{MeV}]\]

\[a_3 [\text{fm}]\]

Numerical result
Analytical form

Bedaque, Rupak, Grießhammer, Hammer
Neutron-Deuteron in pionless EFT

Next proceed to derive a theory with fundamental deuteron fields below breakup

pionless EFT Input

LO : \( \gamma, a_s, a_3 \)

NLO : LO + \( r_t, r_s \)

NNLO : NLO + \( B_3 \)


NLO: S. König, J. Vanesse
Phase Shift

\[ g_s = 1, g_t = 1 \]

\[ a_s = -24 \text{ fm}, \quad \gamma = 46 \text{ MeV} \]

\[ (p/\gamma) \cot \delta \]

\[ p/\gamma \]

\[ g_s = 0, g_t = 0.9 \]

\[ |a_s| \rightarrow \infty, \quad \gamma = 41 \text{ MeV} \]

\[ (p/\gamma) \cot \delta \]

\[ p/\gamma \]

\[ g_s = 0, g_t = 0.6 \]

\[ |a_s| \rightarrow \infty, \quad \gamma = 27 \text{ MeV} \]

\[ (p/\gamma) \cot \delta \]

\[ p/\gamma \]

\[ g_s = 0, g_t = 0.2 \]

\[ |a_s| \rightarrow \infty, \quad \gamma = 9 \text{ MeV} \]

\[ (p/\gamma) \cot \delta \]

\[ p/\gamma \]

Ramsauer-Townsend effect
Halo EFT and modified ERE

\[ \mathcal{L} = n^\dagger [i \partial_0 + \frac{\nabla^2}{2m_N}] n + d_a^\dagger [i \partial_0 + \frac{\nabla^2}{2m_d}] d_a + \sum_{i=1}^{2} \psi^{(i)}^\dagger [\Delta_i + c_i (i \partial_0 + \frac{\nabla^2}{2M})] \psi^{(i)} \]

\[ + \sum_{i=1}^{2} \sqrt{\frac{2\pi}{\mu}} \left[ \frac{\sigma}{\sqrt{3}} nd_a + \text{h. c.} \right], \]

Introduce two auxiliary fields!

neutron-deuteron amplitude:

\[ i T_t(p) = \frac{2\pi \mu}{\mu} \frac{i}{\left[ \frac{1}{\Delta_1 + c_1 p^2/(2\mu)} + \frac{1}{\Delta_2} \right]^{-1} - ip} = \frac{2\pi}{\mu} \frac{i}{p \cot \delta - i} \]

Generate modified ERE: calculate as a 2-body amplitude

\[ p_0^2 = 2\mu \frac{\Delta_1 + \Delta_2}{c_1}, \quad \frac{1}{a} = \frac{\Delta_1 \Delta_2}{\Delta_1 + \Delta_2}, \quad -\frac{2}{r} = 2\mu \frac{\Delta_1 + \Delta_2}{c_1 \Delta_2}, \]
Halo EFT Power-Counting

- Breakdown scale $\Lambda$ set by deuteron breakup momentum
- Zero of T-matrix at $Q$
- Virtual state momentum $\mathcal{N}$

Initially:  $Q^2 \sim 100 \text{ MeV}^2 \ll \mathcal{N}^2 \sim 900 \text{ MeV}^2 \ll \Lambda^2 \sim 2500 \text{ MeV}^2$

We define:  $\gamma_t \equiv g_t \gamma \approx 45.7 g_t \text{ MeV}, \quad \gamma_s \equiv g_s / a_s = -8.3 g_s \text{ MeV}$

Approach unitarity as:  $(g_s = 0, g_t \to 0)$

As we tune the pionless EFT, $Q^2$ gets smaller, changes sign and approaches $\Lambda^2$ and exceeds it.

Power-counting has to account for the varying relative size of $Q^2$ (and other fine tunings)
Power-Counting Continued

Consider 3 intervals

$0.7 \lesssim g_t \lesssim 1$ : small $a \sim Q^2/(\Lambda^2)$, large $r \sim \Lambda^2/(\Lambda Q^2)$

$\Delta_1 \sim \Delta_2$ and $c_2 \ll c_1$ small shape parameter

$0.3 \lesssim g_t \lesssim 0.7$ : large $a \sim r \sim 1/\Lambda$

$\Delta_2 \gg \Delta_1$ and still $c_2 \ll c_1$

Second auxiliary field decouples: regular ERE

$0.1 \lesssim g_t \lesssim 0.3$ : large $a \sim 1/\Lambda$ and $r \lesssim 1/\Lambda$

Familiar unitary limit EFT with a single auxiliary field

Continue on?
Phase Shift Again

\[
\frac{p}{\gamma t} \cot \delta = 1, \quad g_t = 1
\]

\[
\frac{p}{\gamma t} \cot \delta = 0, \quad g_t = 0.9
\]

\[
\frac{p}{\gamma t} \cot \delta = 0, \quad g_t = 0.6
\]

\[
\frac{p}{\gamma t} \cot \delta = 0, \quad g_t = 0.2
\]
Virtual, Bound and Resonance States

Look at analytic structure of the S-matrix

\[ S_t(p) = e^{2i\delta(p)} = 1 + \frac{i2p}{p \cot \delta - ip} = 1 + \frac{i\mu p}{\pi} T_t(p) = 1 + \frac{i2p}{\frac{-1/a + rp^2/2}{p^2 + p_0^2} - ip} \]

\[ = - \frac{(p + i\pi_1)(p + i\pi_2)(p + i\pi_3)}{(p - i\pi_1)(p - i\pi_2)(p - i\pi_3)} \]

Interpretation of the three poles in halo EFT:

\[ \pi_1 + \pi_2 + \pi_3 = -\frac{r}{2}p_0^2, \quad \pi_1\pi_2 + \pi_2\pi_3 + \pi_3\pi_1 = -p_0^2, \quad \pi_1\pi_2\pi_3 = -\frac{p_0^2}{a} \]

3rd root not relevant as \( \pi_3 \gg \Lambda \)

1st root is the shallow virtual state

2nd root on positive imaginary axis ... triton?

No, a redundant pole.
Redundant Pole

We look at the residue of the S-matrix near the poles

\[ S_t(p) \sim \sum_i \frac{R_i}{p - i\pi_i} + \text{regular pieces}, \]

Normalization of bound and virtual states

\[ |N_1|^2 = iR_1 = \frac{2\pi_1(\pi_1^2 - p_0^2)}{(\pi_1 - \pi_2)(\pi_1 - \pi_3)}, \]
\[ |N_2|^2 = iR_2 = \frac{2\pi_2(\pi_2^2 - p_0^2)}{(\pi_2 - \pi_1)(\pi_2 - \pi_3)} < 0. \]

\( \pi_2 > 0 \) is called a redundant/shadow pole

Ma, Phys. Rev. 69, 668 (1946)
Virtual State to Efimov Level

\[ |N_i|^2 \text{ (MeV)} \]

\[ \pi_i \text{ (MeV)} \]

\[ \delta \text{ (deg)} \]

\[ \frac{p}{\gamma} \]
Efimov Levels

\[ e^{\pi/s_0} \approx 23 \]

\[ g_t \sim 0.2 \]

Excited state, \( g_s = 0 \)
Triton, \( g_s = 0 \)
Triton, \( g_s = 1 \)

\[ -\sqrt{m_N B_3} \text{ (MeV)} \]

\[ \gamma_t \text{ (MeV)} \]
Conclusions

• Efimov level emerged from the n-d virtual state near unitarity
• Model-independent analysis using a halo EFT
• Claim the mechanism for emergence of Efimov levels is universal
  • Atomic systems
  • lattice QCD at unphysical quark masses
• radiative capture in n-d, p-d system for Big Bang Nucleosynthesis