MadGraph5 for Global QCD Analysis

Derek Sederman

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Global QCD Analysis and Parton Distribution Functions

- A fundamental goal in Global QCD analysis is to gain a better understanding of Parton Distribution Functions (PDFs).
- Each parton in the nucleon carries some fraction (x) of the nucleon's longitudinal momentum.
- PDFs describe the probability density of finding a specific parton with a given x at resolution scale Q^2.
- PDFs are inherently non-perturbative, thus can not be calculated with perturbative QCD.

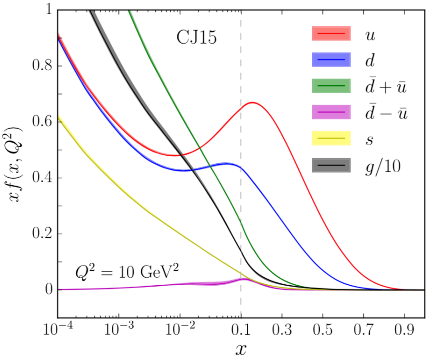

Figure: CJ15 PDFs from Accardi et al [2]

Determination of PDFs

$$\frac{df(x,Q)}{lnQ} = \int_x^1 \frac{dz}{z} P(z) f(\frac{x}{z}, Q)$$

- PDFs are characterized by a set of integral differential equations of the form shown above. To fin particular solution, the boundary conditions at some Q: f(x, Q, $\vec{a}$), must be known.

$$f^{(0)}(x, Q = 1GeV, \vec{a}) = Nx^a(1-x)^b(1 + c\sqrt{x} + ...)$$
$$\vec{a} = (N, a, b, c, ...)$$

- A PDF can be extracted by varying $\vec{a}$ in a fit to experimental data using QCD factorization theorems. For example, for a process of form,
  A + B $\rightarrow C + X$:

$$\frac{d\sigma^{expt}}{dO} = \int dx_a dx_b f_a(x_a) f_b(x_b) \frac{d\hat{\sigma}(x_a, x_b)}{dO}$$

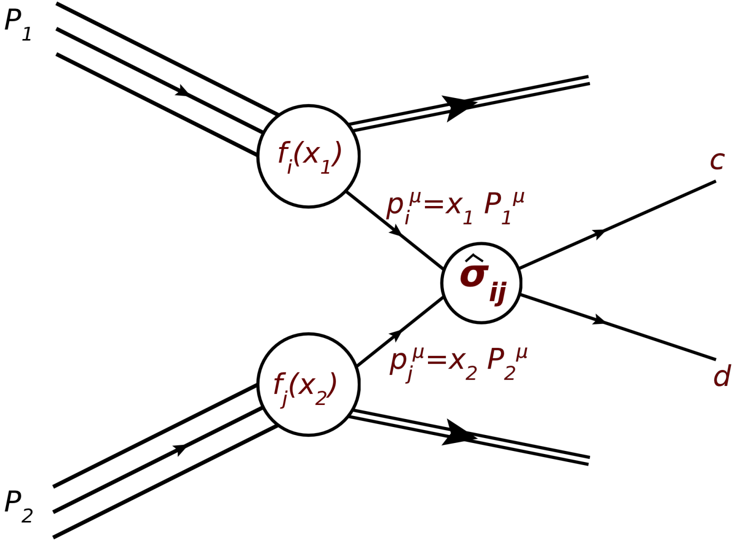

Figure: Diagram illustrating partonic and hadronic cross sections [4].

Mellin Transform

- The Mellin transform, a mathematical technique similar to the Fourier transform is defined by:

$$f_N(Q) = \int_0^1 dx\, x^{N-1} f(x, Q)$$

  - with inverse transform:

$$f(x, Q) = \frac{1}{2\pi i} \int_{C_n} dN\, x^{-N} f_N(Q)$$

  - where N is an arbitary complex number
- By applying the Mellin transform, the DGLAP equation simplifies to:

$$\frac{df(x, Q)}{lnQ} = P(z) f_N(Q)$$

- And for A + B $\rightarrow C + X$ we now have:

$$\frac{d\sigma^{expt}}{dO} = \frac{1}{(2\pi i)^2} \int dN \int dM f_{aN}(Q) f_{bM}(Q) \int dx_a dx_b x_a^{-N} x_b^{-M} \frac{d\hat{\sigma}(x_a, x_b)}{dO}$$

Partonic Cross Section Moments

- Now, part of the expression, $\int dx_a dx_b x_a^{-N} x_b^{-M} \frac{d\hat{\sigma}(x_a,x_b)}{dO} = \frac{d\hat{\sigma}^{NM}}{dO}$, is independent of the PDFs in the fit.
- These are the partonic cross section Mellin moments for a given N and M. Since they are PDF independent, they may be calculated separately.
- Double Mellin tables tabulate these moments for later use in PDF fitting.
- Performing the fits in Mellin space, where the relevant equations are simpler, allows for more efficient fitting of PDFs.

Partonic Cross Section

- The integral, $\int dx_a dx_b x_a^{-N} x_b^{-M} \frac{d\hat{\sigma}(x_a,x_b)}{dO} = \frac{d\hat{\sigma}^{NM}}{dO}$, is easy to perform once $\frac{d\hat{\sigma}(x_a,x_b)}{dO}$, the parton level differential cross section, is computed.
- We can obtain analytic expressions for these cross sections, however, they involve integrals over phase space that become difficult or impossible when cuts are implemented to match with experiment.
- Instead, $\frac{d\hat{\sigma}(x_a,x_b)}{dO}$ can be computed numerically with Monte Carlo event generation programs such as MCFM and MadGraph5.

MadGraph5

- MadGraph5 (MG5) contains multi-purpose collision event generation software, and is capable of computing tree level and one-loop amplitudes for a given process.
- The user specifies a given process and order, LO or next-to-leading order (NLO). MG5 generates the relevant diagrams and corresponding weights, then generates a specified number of events accordingly.
- MG5 can be interfaced with other programs such as MadSpin, Pythia and Delphes to handle decay chains, parton showering, and detector simulation respectively.

Example: W + c Production

- One process of interest is the production of a W boson and charm quark jet from proton-proton collisions.
- This process is of particular interest because it is sensitive to the strange and anti-strange quark distributions, which are not particularly well known.
- MG5, will generate almost 200 diagrams for this process at NLO, then run events accordingly.

Figure: W+c production LO diagrams [3]

Example: W + c Production

- After generating events with MG5, $\frac{d\hat{\sigma}}{dO}(x_a^i, x_b^i)$ can be extracted for each event i.
- Then, $\frac{d\hat{\sigma}^{NM}}{dO}$ is computed with the sum:

$$\sum_i (x_a^i)^{-N} (x_b^i)^{-M} \frac{d\hat{\sigma}}{dO}(x_a^i, x_b^i)$$

- This calculation is performed for pairs of complex numbers N and M ($\sim 10^5$) and tabulated in the double Mellin table.

Another Application of MG5: K Factor

- In some cases, higher order calculations are approximated using lower order calculations and a K factor.

$$K = \frac{\sigma^{NLO}}{\sigma^{LO}}$$

- The production of a W+ or W- boson in proton-antiproton collisions is of interest because at high η^{W}, this process can be used to constrain the high x range of the up and down quark PDFs.
- The Asymmetry of $W^{\pm}$ production is defined as:

$$A_w = \frac{\frac{d\sigma^{W+}}{d\eta^{W}} - \frac{d\sigma^{W-}}{d\eta^{W}}}{\frac{d\sigma^{W+}}{d\eta^{W}} + \frac{d\sigma^{W-}}{d\eta^{W}}}$$

- If $K^{W+} \approx K^{W-}$, then $A_W^{(LO)}$ can be used to approximate $A_W^{(NLO)}$

K Factor and $W^{\pm}$ Asymmetry

- At LO the asymmetry reduces to a fairly simple expression involving $\frac{d}{u}$:

$$A_w = \frac{\frac{d}{u}(x_2) - \frac{d}{u}(x_1)}{\frac{d}{u}(x_2) + \frac{d}{u}(x_1)}$$

- The relationship between $x_{1,2}$ and η_W is given by:

$$x_{1,2} = \frac{M_W}{\sqrt{s}} e^{\pm\eta^W}$$

- Thus, high η_W corresponds to high x_1

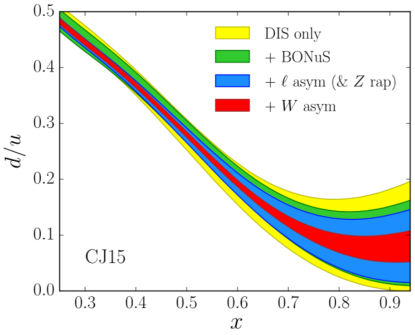

Figure: Illustration of the influence of various data sets on the uncertainty of $\frac{d}{u}$ from Accardi et al [2]

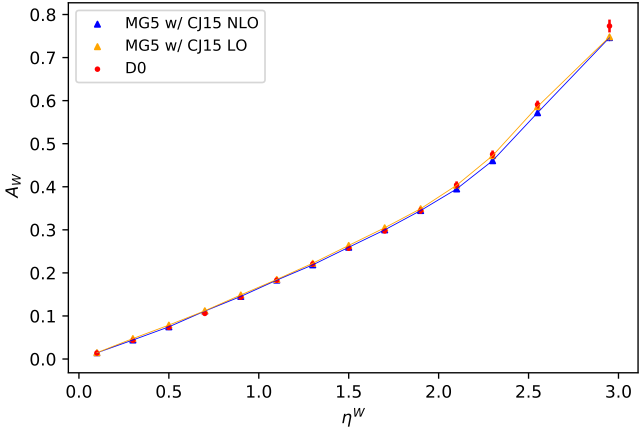

Figure: W boson asymmetry as a function of η^W with experimental data points from the D0 [1] collaboration.

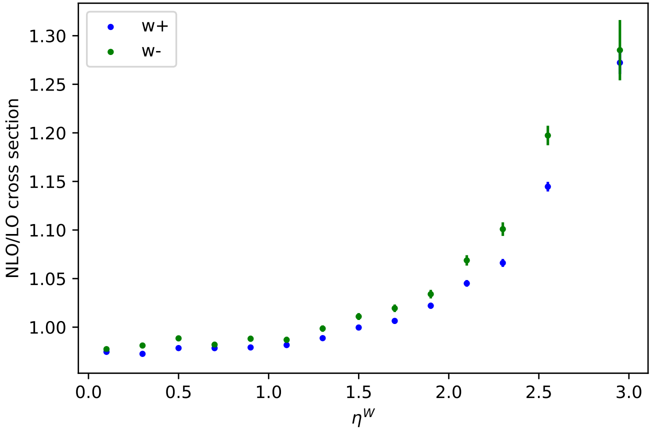

Figure: K factor for W+ and W- production as a function of W pseudorapidity

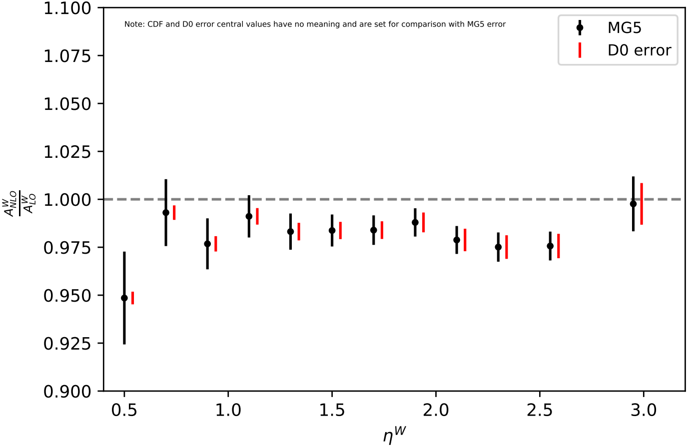

Figure: NLO/LO asymmetry ratio as a function of W pseuodrapidity.

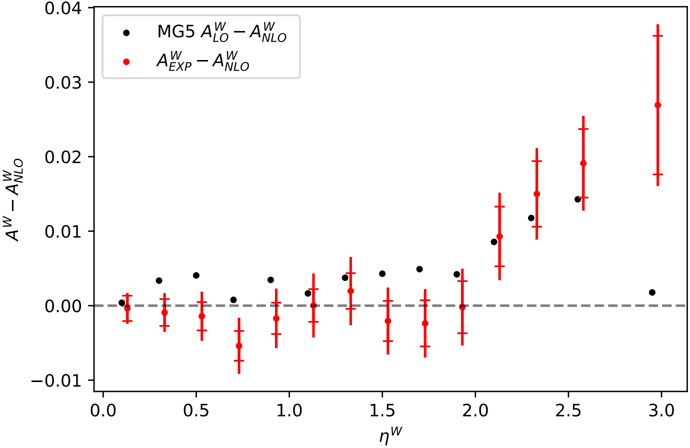

Figure: NLO, LO, experimental asymmetry comparison as a function of W pseuodrapidity.

Summary of MG5 use

- We would like to have efficient and accurate extraction of PDFs from experimental data.
- Fitting PDFs to data in Mellin space is much more efficient.
- This is because, in Mellin space, the parton level cross section moments factorize from the PDFs, allowing them to be computed separately.
- Applying phase space cuts complicates the analytic calculation of these cross sections.
- MG5 is used to compute various partonic cross section Mellin moments which are stored in double Mellin tables.
- MG5 can also be used to examine the accuracy of LO, NLO calculations for processes of interest
- This technology will be used in upcoming Global QCD analyses involving processes (e.g. W+c) that may provide new insight into the internal structure of the proton.

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