$\eta' \to \eta\pi\pi$ in unitarized Resonance Chiral Theory

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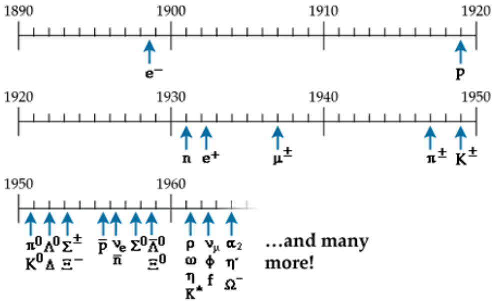
Outline

1. **Introduction: framework and motivation**
2. **Structure of the decay amplitude**
3. **Fits to experimental data**
   - ChPT at one loop with resonances
   - Unitarization of the ChPT amplitude
4. **Summary**

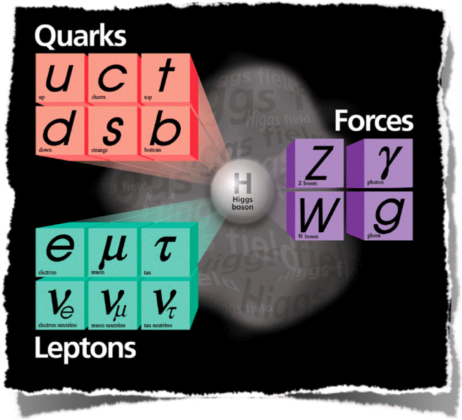
The Dawn of the Standard Model



- By early 30's, after the discovery of **electron, proton, neutron and positron**, we had a reasonable **description of particle physics**
- The discovery of **the muon** was unexpected: this new particle, **did not fit in!**
- To make things worse, **a plethora of new** strongly interacting **particles** (pion, kaons, etc), was soon **discovered**
- How to **make sense** of this "chaos"?

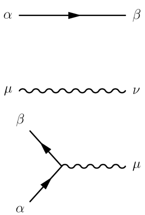
The Standard Model: a history of success

- The **Standard Model** of particle physics explains a wide variety of phenomena in a unified framework: **Quantum Field Theory**
- **Matter content** composed by six **quarks** and six **leptons** organised in three families
- **Interactions** among matter particles are **driven by gauge bosons**: γ (electromagnetism), W and Z (weak force) and gluons (strong interactions)
- The last ingredient is the **Higgs** boson, provides a **mechanism** by which particles acquire **mass**



Quantum Electrodynamics

- Describes how **charged particles** and **photons interact**
- Based on simple rules (Feynman diagrams), **compute terms** at any order as a **perturbative expansion** in the small QED

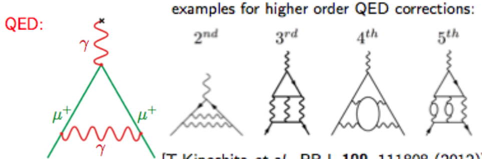


$$\rightarrow \left(\frac{i}{\not p - m + i\varepsilon}\right)_{\beta\alpha}$$

$$\rightarrow \frac{-i\eta_{\mu\nu}}{p^2 + i\varepsilon}$$

$$\rightarrow -ie\gamma^{\mu}_{\beta\alpha}(2\pi)^4\delta^{(4)}(p_1+p_2+p_3).$$

- Some of the most **precise calculations** ever done have been **obtained in QED**



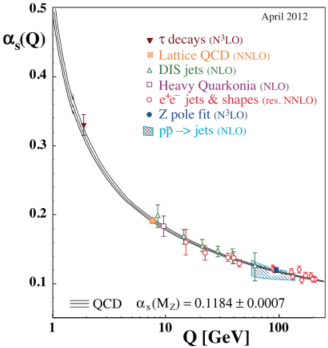
[T.Kinoshita *et al.*, PR L **109**, 111808 (2012)]

$a_e^{QED}(theory) = 1159652181.78(77)\cdot 10^{-12}$

$a_e(exp) = 1159652180.73(28)\cdot 10^{-12}$

Quantum Chromodynamics

- Hadrons interact strongly: could perturbation theory be applied to describe strong interactions?
- **Quantum Chromodynamics** is a renormalizable QFT but
  1. with **asymptotic freedom**: it looks like QED, but only at very high energies
  2. with **confinement**: at low energies the gluons bind the quarks together



Chiral Perturbation Theory

Gasser and Leutwyler, Nucl. Phys. B 250, 465 (1985)

- Low-energy EFT of QCD for light mesons i.e. $\pi^{\pm,0}$, $K^{\pm}$, K^0, $\bar{K}^0, \eta_8$ associated to $SU(3)_L \otimes SU(3)_R \xrightarrow{\text{SCSB}} SU(3)_V$ exhibited by QCD
- Perturbative expansion in terms of p^2 and m_q: $\mathcal{L} = \mathcal{L}_2 + \mathcal{L}_4 + \ldots$

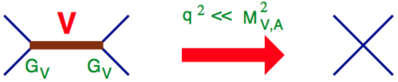
$$\mathcal{L}_2 = \frac{F^2}{4}\langle u_\mu u^\mu \rangle + \frac{F^2}{4}\langle \chi_+ \rangle \,, \quad \phi = \begin{pmatrix} \frac{1}{\sqrt{2}}\pi_3 + \frac{1}{\sqrt{6}}\eta_8 & \pi^+ & K^+ \\ \pi^- & -\frac{1}{\sqrt{2}}\pi_3 + \frac{1}{\sqrt{6}}\eta_8 & K^0 \\ K^- & \bar{K}^0 & -\frac{2}{\sqrt{6}}\eta_8 \end{pmatrix},$$

$$u^2 = e^{i\frac{\sqrt{2}\phi}{F}} \,, \quad \chi = 2B\mathcal{M} \,, \quad \chi_\pm = u^\dagger \chi u^\dagger \pm u \chi^\dagger u \,, \quad u_\mu = i u^\dagger D_\mu U u^\dagger \,,$$

$$\mathcal{L}_4 = L_1 \langle u_\mu u^\mu \rangle^2 + L_2 \langle u_\mu u^\nu \rangle \langle u^\mu u_\nu \rangle + L_3 \langle u_\mu u^\mu u_\nu u^\nu \rangle + \ldots$$

⚠ η_1 not included due to the axial anomaly

⚠ Valid $\frac{p^2}{M_R^2} < 1$: polynomial cannot reproduce resonance poles



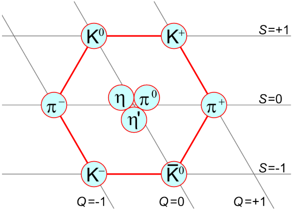
$$\frac{G_V^2}{M_V^2} \propto L_{1,2,3}$$

Large-N_C $U(3)$ ChPT

Kaiser and Leutwyler, EPJC 17, 623 (2000)

- Axial Anomaly is absent; η_1 as the ninth Goldstone boson
- Degrees of freedom: $\pi^{\pm,0}$, $K^{\pm}$, K^0, $\bar{K}^0$ and the η and η'

$$\begin{pmatrix}\eta\\ \eta'\end{pmatrix} = \begin{pmatrix}\cos\theta & -\sin\theta\\ \sin\theta & \cos\theta\end{pmatrix}\begin{pmatrix}\eta_8\\ \eta_1\end{pmatrix}$$



- Simultaneous triple expansion in terms of $p^2 \sim m_q \sim 1/N_C$

$$\mathcal{L}_\chi = \frac{F^2}{4}\langle u_\mu u^\mu\rangle + \frac{F^2}{4}\langle \chi_+\rangle + \frac{F^2}{3} m_1^2 \ln^2 \det u\,,$$

$$\Phi = \begin{pmatrix} \frac{1}{\sqrt{2}}\pi_3 + \frac{1}{\sqrt{6}}\eta_8 + \frac{1}{\sqrt{3}}\eta_1 & \pi^+ & K^+ \\ \pi^- & -\frac{1}{\sqrt{2}}\pi_3 + \frac{1}{\sqrt{6}}\eta_8 + \frac{1}{\sqrt{3}}\eta_1 & K^0 \\ K^- & \bar{K}^0 & -\frac{2}{\sqrt{6}}\eta_8 + \frac{1}{\sqrt{3}}\eta_1 \end{pmatrix}.$$

⚠ η' heavier than some resonances $\frac{M^2_{\eta'}}{M^2_R} > 1$

Resonance Chiral Theory

Ecker, Gasser, Pich and de Rafael, Nucl. Phys. B 321, 311 (1989)

- Resonance as explicit degrees of freedom

$$\boxed{\mathcal{L}_{\mathrm{R}\chi\mathrm{T}} = \mathcal{L}^{p^2} + \mathcal{L}_S + \mathcal{L}_{\mathrm{kin}}^S}$$

$$\mathcal{L}_S = c_d\langle S_8 u_\mu u^\mu\rangle + c_m\langle S_8\chi_+\rangle + \tilde{c}_d S_1\langle u_\mu u^\mu\rangle + \tilde{c}_m S_1\langle\chi_+\rangle\,,$$

$$S_8 = \begin{pmatrix} \frac{1}{\sqrt{2}}a_0^0 + \frac{1}{\sqrt{6}}\sigma_8 & a_0^+ & \kappa^+ \\ a_0^- & -\frac{1}{\sqrt{2}}a_0^0 + \frac{1}{\sqrt{6}}\sigma_8 & \kappa^0 \\ \kappa^- & \bar{\kappa}^0 & -\frac{2}{\sqrt{6}}\sigma_8 \end{pmatrix}, \qquad S_1 = \sigma_1$$

$$\mathcal{L}_S = \frac{2c_d}{f^2}\langle S_8(\partial_\mu\Phi)(\partial^\mu\Phi)\rangle + 4B_0 c_m[\langle S_8\mathcal{M}\rangle - \frac{1}{4f^2}\langle S_8(\Phi^2\mathcal{M} + \mathcal{M}\Phi^2 + 2\Phi\mathcal{M}\Phi)\rangle]$$
$$+\frac{2\tilde{c}_d}{f^2}S_1\langle(\partial_\mu\Phi)(\partial^\mu\Phi)\rangle + 4B_0\tilde{c}_m S_1[\langle\mathcal{M}\rangle - \frac{1}{4f^2}\langle(\phi^2\mathcal{M} + \mathcal{M}\Phi^2 + 2\Phi\mathcal{M}\Phi)\rangle]$$

⚠ Resonances spoils power counting, no systematic EFT but a model based on the Large-N_C limit as a guideline

Recent experimental activity on η and η' physics

- Phenomenology of η and η' among their main objectives

A2	CLAS	BESIII	LHCb
$\eta \to e^+e^-\gamma$ [1]	$\eta \to e^+e^-\gamma$ [4]	$\eta' \to e^+e^-\gamma$ [5]	$\eta^{(\prime)} \to \pi^+\pi^-$ [12]
$\eta \to \pi^0\gamma\gamma$ [2]	$\eta \to \pi^+\pi^-\gamma$ [4]	$\eta' \to \pi^0\gamma\gamma$ [6]	
$\eta' \to \eta\pi^0\pi^0$ [3]	$\eta \to e^+e^-e^+e^-$ [4]	$\eta' \to \omega e^+e^-$ [7]	
	$\eta \to \pi^+\pi^-e^+e^-$ [4]	$\eta' \to 4\pi$ [8]	
		$\eta' \to 3\pi$ [9]	
		$\eta' \to \pi^+\pi^-e^+e^-$ [10]	
		$\eta' \to \eta\pi\pi$ [11]	

- Experimental precision for η-η' observables is increasing
- Better theoretical predictions are demanded
- To have a better and more complete knowledge of QCD at low-energies

[1] Adlarson et.al. Phys.Rev. C 94 6 065206 (2016) ; [2] Nefknes et.al. Phys.Rev. C 90 2 025206 (2014)
[3] Adlarson et.al. ArXiv: 1709.04230
[4] Adlarson et.al. Phys.Rev. C 94 6 065206 (2016) ; [5] Ablikim et.al. Phys.Rev. D 92 1 012001 (2015)
[6] Ablikim et.al. Phys.Rev. D96 (2017) ; [7] Ablikim et.al. Phys.Rev. D 92 5 051101 (2015)
[8] Ablikim et.al. Phys.Rev.Lett. 112 251801 (2014) ; [9] Ablikim et.al. Phys.Rev.Lett. 118 1 012001 (2017)
[10] Ablikim et.al. Phys.Rev. D 87 9 092011 (2013) ; [11] Ablikim et.al. ArXiv: 1709.04627
[12] Aaji et.al. Phys.Lett. B 764 233 (2017)

Motivation for $\eta' \to \eta\pi\pi$

- Main decay channel of the η': $\mathrm{BR}(\eta' \to \eta\pi^0\pi^0) = 22.3(8)\%$, $\mathrm{BR}(\eta' \to \eta\pi^+\pi^-) = 42.9(7)\%$ PDG [2017]
- Cannot be described within $SU(3)$ ChPT
- Advantageous laboratory to test any of its extensions Large-N_C $U(3)$ ChPT and Resonance Chiral Theory
- G-parity conservation prevents vectors to contribute: analysis of the properties of scalar resonances i.e. $\sigma, f_0(980), a_0(980)$
- Study of the η-η' mixing
- Access $\pi\eta$ scattering and phase-shift
- New data very recently released the A2 and BESIII collaborations

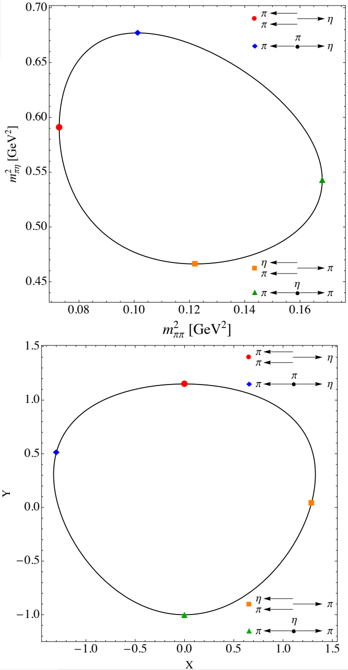
Kinematics and Dalitz plot variables

- $s = (p_{\eta'} - p_\eta)^2$
- $t = (p_{\eta'} - p_{\pi^+})^2$
- $u = (p_{\eta'} - p_{\pi^-})^2$
- $s + t + u = m_{\eta'}^2 + m_\eta^2 + 2m_\pi^2$
  $\Rightarrow$ only two independent variables,
  e.g. s and $t - u \propto \cos\theta_s$

- $X = \frac{\sqrt{3}}{Q}(T_{\pi_1} - T_{\pi_2}) = \frac{\sqrt{3}}{Q}(u - t)$
- $Y = \frac{m_\eta + 2m_\pi}{m_\pi}\frac{T_\eta}{Q} - 1$

$$= \frac{m_\eta + 2m_\pi}{m_\pi}\frac{(m_{\eta'} - m_\eta)^2 - s}{2m_{\eta'}Q} - 1$$

- $Q = m_{\eta'} - m_\eta - 2m_\pi$



Dalitz plot parameters: current state-of-the-art

- Dalitz plot to compare experiment and theory

$$|M(X,Y)|^2 = |N|^2\left(1 + aY + bY^2 + cX + dX^2 + \dots\right)$$

- a, b, c, d are the Dalitz plot parameters

$\eta' \to \eta\pi^0\pi^0$	$a[Y]$	$b[Y^2]$	$c[X]$	$d[X^2]$
GAMS-4 (2009)	$-\mathbf{0.067(16)(4)}$	$-0.064(29)(5)$	$=0$	$-0.067(20)(3)$
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Borasoy $et.al.$'05	$-0.127(9)$	$-0.049(36)$	0	$0.011(21)$
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$\eta' \to \eta\pi\pi$: Leading order

- ChPT Lagrangian at $\mathcal{O}(p^2)$

$$\mathcal{L}^{p^2} = \frac{F_\pi^2}{4}\langle u_\mu u^\mu \rangle + \frac{F_\pi^2}{4}\langle \chi_+ \rangle + \frac{F_\pi^2}{3} m_1^2 \ln^2 \det u$$

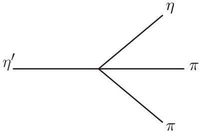
- Expanding in powers of Φ

$$\mathcal{L}^{p^2} = \frac{1}{2}\langle \partial_\mu \Phi \partial^\mu \Phi \rangle + \frac{1}{12f^2}\underline{\langle (\Phi(\partial_\mu \Phi) - (\partial_\mu \Phi)\Phi)\,(\Phi(\partial^\mu \Phi) - (\partial^\mu \Phi)\Phi)\rangle}$$

$$+B_0\left\{-\langle \mathcal{M}\Phi^2 \rangle + \underline{(1/6f^2)\langle \mathcal{M}\Phi^4 \rangle}\right\} + \mathcal{O}\left(\frac{\Phi^6}{f^4}\right)$$

no contribution

$$\Downarrow$$

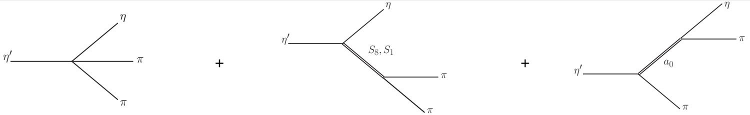
- 

$$= \boxed{\mathcal{M}^{\mathrm{LO}}_{\eta' \to \eta\pi\pi} = \frac{M_\pi^2}{6F_\pi^2}\left(2\sqrt{2}\cos 2\theta - \sin 2\theta\right)}$$

	$BR(\eta' \to \eta\pi^+\pi^-)$	$BR(\eta' \to \eta\pi^0\pi^0)$
Leading Order	1.1%	0.6%
PDG 2018	$42.6(7)$	$22.8(8)\%$

- Reason for this difference: amplitude is chirally suppressed (vanishes when $M_\pi^2 \to 0$)
- Higher order effects? • Resonances exchanges (a_0, f_0, σ) • $\pi\pi$, $\pi\eta$ final state interactions

$\eta' \to \eta\pi\pi$: Scalar Resonance contributions



c_d: dominant

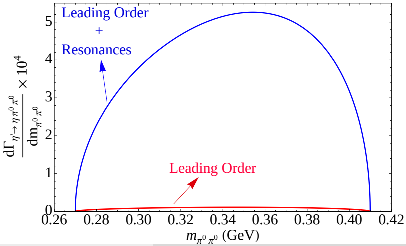
c_m: suppressed ($\propto m_\pi^2$)

$$\mathcal{M}^{\text{Res}}(s,t,u) = \frac{\left(2\sqrt{2}\cos 2\theta - \sin 2\theta\right)}{9F_\pi^4} \times \Bigg\{ -\frac{24\tilde{c}_m\left(m_K^2 - m_\pi^2\right)\left(2\tilde{c}_m m_\pi^2 + \tilde{c}_d\left(s - 2m_\pi^2\right)\right)}{M_{S_1}^2 - s}$$

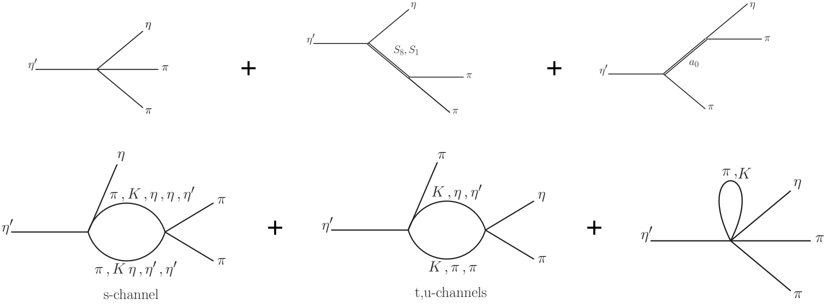
$$-\frac{\left(2c_m\left(m_\pi^2 - 4m_K^2\right) + 3c_d\left(m_\eta^2 + m_{\eta'}^2 - s\right)\right)\left(2c_m m_\pi^2 + c_d\left(s - 2m_\pi^2\right)\right)}{M_{S_8}^2 - s} + \frac{12c_d c_m m_\pi^2}{M_{S_8}^2}\left(m_\pi^2 - m_K^2\right)$$

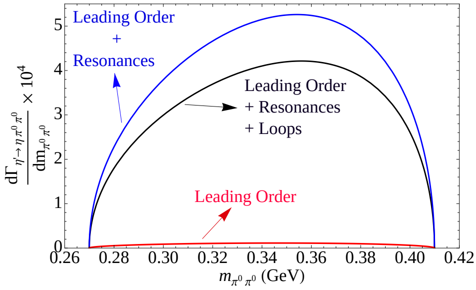
$$+\frac{3\left(4{c_m}^2 m_\pi^4 - 2c_d c_m m_\pi^2\left(m_\eta^2 + m_{\eta'}^2 + 2m_\pi^2 - 2t\right) + {c_d}^2\left(m_\eta^2 + m_\pi^2 - t\right)\left(m_{\eta'}^2 + m_\pi^2 - t\right)\right)}{M_{a_0}^2 - t} + (t \leftrightarrow u)\Bigg\}.$$

Source	c_m	c_d	BR (%)
$a_0 \to \eta\pi$ (Guo'09)	80	26	~ 16
Res. saturation (Ecker'88)	42	32	~ 35
$K\pi$ scattering	43	30	~ 27
Jamin et.al. '00	76.7	24.8	~ 13
	85	13	~ 1
$PP \to PP$			
Guo et.al '11	31.5	15.6	~ 1
Guo et.al '12	41.9	19.8	~ 4
Pich et.al. '14	41.1	39.8	~ 85
PDG			22.8(8)



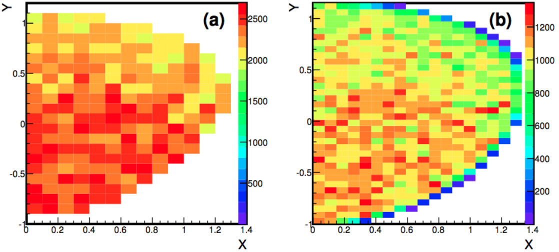
$\eta' \to \eta\pi\pi$: Scalar Resonance and loop contributions





Fits to experimental data

A2 Coll. 1709.04230



- We relate the experimental Dalitz plot data with the differential decay distribution from theory through

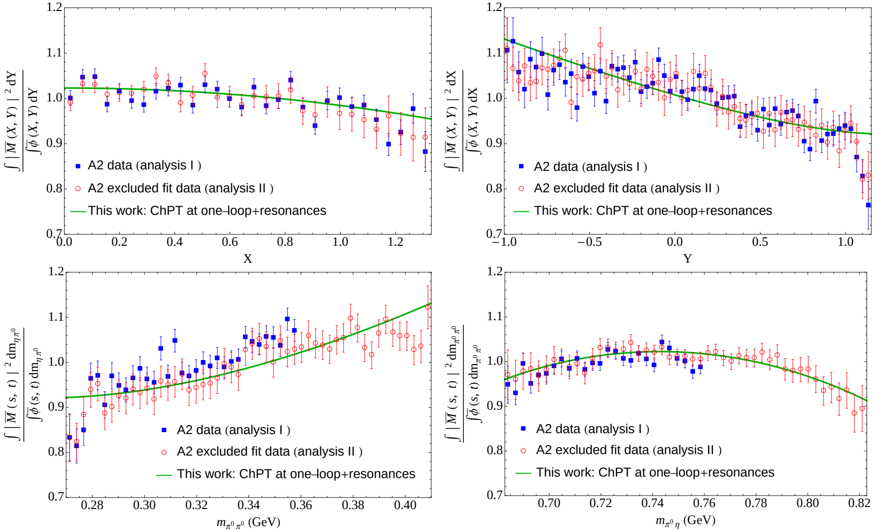
$$\frac{d^2 N_{\rm events}}{dX dY} = \frac{d\Gamma(\eta' \to \eta\pi^0\pi^0)}{dX dY} \frac{N_{\rm events}}{\Gamma_{\eta'}\bar{B}(\eta' \to \eta\pi^0\pi^0)} \Delta X \Delta Y \,,$$

- $N_{\rm events} = 463066$ (analysis I) and 473044 (analysis II)
- $\Delta X = \Delta Y = 0.10$

Fits to experimental data: ChPT at one loop with resonances

- Fit 1: $M_S = M_{S_8} = M_{S_1} = M_{a_0}$ and $\tilde{c}_{d,m} = c_{d,m}/\sqrt{3}$ with $c_d = c_m$

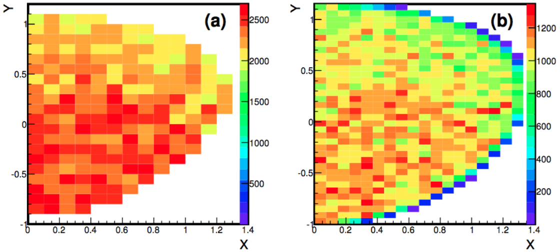
$$\text{Fit parameters}: \quad M_S = 973(5)\,\text{MeV}\,, \quad c_d = c_m = 30.1(4)\,\text{MeV}\,, \quad \chi^2_{\text{dof}} = 1.22$$



Fits to experimental data: ChPT at one loop with resonances

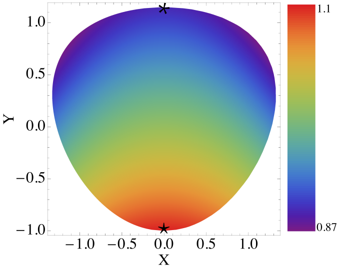
- Dalitz plot slope parameters

A2 Coll. 1709.04230



Dalitz Parameters (A2):

$$a[Y] = -0.074(8)(6)$$
$$b[Y^2] = -0.063(14)(5)$$
$$d[X^2] = -0.050(9)(5)$$



$$\begin{array}{l} \pi \longleftarrow \\ \pi \longleftarrow \end{array} \; * \longrightarrow \eta$$

$$\pi \longleftarrow \overset{\star}{\eta} \longrightarrow \pi$$

Dalitz Parameters (this work):

$$a[Y] = -0.095(6)$$
$$b[Y^2] = 0.005(1)$$
$$d[X^2] = -0.037(5)$$

Fits to experimental data: ChPT at one loop with resonances

- Fit 2: $M_S = M_{S_8} = M_{S_1} = M_{a_0}$ and $\tilde{c}_{d,m} = c_{d,m}/\sqrt{3}$ with $c_d \neq c_m$

$$\text{Fit parameters}: \quad M_S = 954(47)\,\text{MeV}\,, \quad c_d = 28.0(4.6)\,\text{MeV}\,, c_m = 53.4(52.0)\,\text{MeV}\,, \chi^2_{\text{dof}} = 1.23$$

$$\textbf{Dalitz parameters}: \quad a = -0.093(45)\,, \quad b = 0.004(3)\,, \quad d = -0.039(18)$$

- Fit 3: $M_S = M_{S_8} = M_{S_1} = M_{a_0}$ and $c_m = 41.1(1)$ $\tilde{c}_m = 18.9(9)$ (Pich'14)

$$\text{Fit parameters}: \quad M_S = 968(11)\,\text{MeV}\,, \quad c_d = 29.8(9)\,\text{MeV}\,, \quad \tilde{c}_d = 21.2(1.2)\,\text{MeV}\,, \chi^2_{\text{dof}} = 1.24$$

$$\textbf{Dalitz parameters}: \quad a = -0.092(5)\,, \quad b = 0.004(2)\,, \quad d = -0.041(11)$$

- Fit 4: $M_S = M_{S_8} = M_{S_1} = M_{a_0}$ with $c_m = 80(20)$ and $\tilde{c}_{d,m} = c_{d,m}/\sqrt{3}$ (Guo'09)

$$\text{Fit parameters}: \quad M_S = 926(5)(25)\,\text{MeV}\,, \quad c_d = 25.7(4)(1.9)\,\text{MeV}\,, \chi^2_{\text{dof}} = 1.22$$

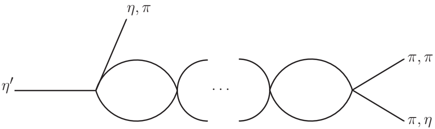
$$\textbf{Dalitz parameters}: \quad a = -0.090(7)(3)\,, \quad b = 0.004(1)(0)\,, \quad d = -0.041(7)(1)$$

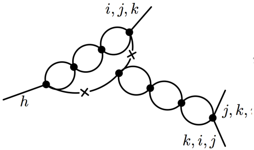
- The **Dalitz plot parameters remain stable** independently of the allowed parameters to fit
- We also have tried a fit letting **all couplings to float** i.e. $c_m, \tilde{c}_m, c_d$ and $\tilde{c}_d$, but in this case the **fit becomes unstable** since there are too many parameters to fit

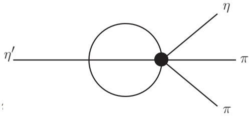
Unitarity

- Unitarity relation

$$\mathrm{Im}\mathcal{M}_{\eta'\to\eta\pi\pi} = \frac{1}{2}\sum_n (2\pi)^4\,\delta^4\left(p_\eta + p_1 + p_2 - p_n\right)\mathcal{T}^*_{n\to\eta\pi\pi}\mathcal{M}_{\eta'\to n}$$







~~3-body FSI~~

- Restrict to 2-particle rescattering

$$\mathrm{Im}\mathcal{M}^I_{\eta'\to\eta\pi\pi}(s,t,u) = \frac{1}{2!2(2\pi)^2}\int \frac{dq^3_\pi}{2q^0_\pi}\frac{dq^3_\pi}{2q^0_\pi}\delta^4(q_\pi + q_\pi - p_1 - p_2)\mathcal{T}^I_{\pi\pi\to\pi\pi}(s,\theta'_s)^*\mathcal{M}^I_{\eta'\to\eta\pi\pi}(s,\theta''_s,\phi''_s$$

$$+\frac{1}{2(2\pi)^2}\int \frac{dq^3_\pi}{2q^0_\pi}\frac{dq^3_\eta}{2q^0_\eta}\delta^4(q_\pi + q_\eta - p_1 - p_\eta)\mathcal{T}^I_{\pi\eta\to\pi\eta}(t,\theta'_t)^*\mathcal{M}^I_{\eta'\to\eta\pi\pi}(s,\theta''_t,\phi''_t)$$

$$+\frac{1}{2(2\pi)^2}\int \frac{dq^3_\eta}{2q^0_\eta}\frac{dq^3_\pi}{2q^0_\pi}\delta^4(q_\eta + q_\pi - p_2 - p_\eta)\mathcal{T}^I_{\eta\pi\to\eta\pi}(u,\theta'_u)^*\mathcal{M}^I_{\eta'\to\eta\pi\pi}(u,\theta''_u,\phi''_u)$$

Unitarity

- Partial waves decompostion ($\mathcal{A} = \mathcal{T}, \mathcal{M}$)

$$\mathcal{A}^I(s, \cos\theta) = \sum_J 32\pi(2J+1)P_J(\cos\theta)a^{IJ}(s)\,,$$

- Integrating over the momentum and then using the relation

$$\int d\Omega' P_J(\cos\theta')P_{J'}(\cos\theta'') = \frac{4\pi}{2J+1}\delta_{JJ'}P_J(\cos\theta)$$

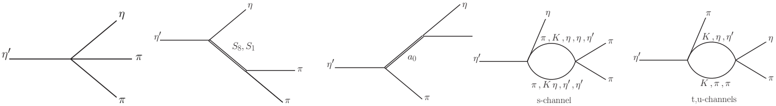
- Two-particle unitarity relation for the partial-wave decay amplitude

$$\begin{aligned}
\mathrm{Im}\left(m^{IJ}_{\eta'\to\eta\pi\pi}(s,t,u)\right) \quad &= \quad \sigma_\pi(s)t^{IJ}_{\pi\pi\to\pi\pi}(s)^* m^{IJ}_{\eta'\to\eta\pi\pi}(s)\theta(s-4m_\pi^2) \\
&+ \frac{\lambda^{1/2}(t, m_\pi^2, m_\eta^2)}{t} t^{IJ}_{\pi\eta\to\pi\eta}(t)^* m^{IJ}_{\eta'\to\eta\pi\pi}(t)\theta(t-(m_\pi+m_\eta)^2) \\
&+ \frac{\lambda^{1/2}(u, m_\pi^2, m_\eta^2)}{u} t^{IJ}_{\pi\eta\to\pi\eta}(u)^* m^{IJ}_{\eta'\to\eta\pi\pi}(u)\theta(u-(m_\pi+m_\eta)^2)
\end{aligned}$$

N/D unitarisation method applied to $\eta' \to \eta\pi\pi$

- Amplitude at one-loop in Large-N_C $U(3)$ ChPT with resonances

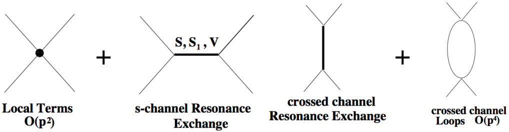
$\mathcal{M}^{\eta'\to\eta\pi\pi}(s) =$



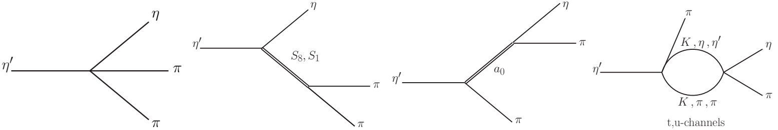
- N/D representation of $\mathcal{M}^{\eta'\to\eta\pi\pi}(s)$

$$m^{IJ}_{\eta'\to\eta\pi\pi}(s) = [1 + N^{IJ}_{\pi\pi}(s)g_{\pi\pi}(s)]^{-1} R^{IJ}_{\eta'\to\eta\pi\pi}(s)$$

$N^{IJ}_{\pi\pi\to\pi\pi}(s) =$



$R^{IJ}_{\eta'\to\eta\pi\pi}(s) =$



$g_{\pi\pi}(s) = a_{\pi\pi}(\mu) -$  $= \frac{1}{16\pi^2}\left(a_{\pi\pi}(\mu) + \log\frac{m_\pi^2}{\mu^2} - \sigma(s)\log\frac{\sigma(s)-1}{\sigma(s)+1}\right)$

N/D applied to $\eta' \to \eta\pi\pi$

- Amplitude at one-loop in Large-N_C $U(3)$ ChPT with resonances

$$\mathcal{M}^{\eta'\to\eta\pi\pi}(s) = \mathcal{M}(s)^{(2)} + \mathcal{M}(s)^{\mathrm{Res(s,t,u)}} + \mathcal{M}(s)^{\mathrm{Loop(s,t,u)}} \quad (2)$$

- N/D representation of Eq. (2)

$$m^{IJ}_{\eta'\to\eta\pi\pi}(s) = \left[1 + N^{IJ}_{\pi\pi}(s) g_{\pi\pi}(s)\right]^{-1} R^{IJ}_{\eta'\to\eta\pi\pi}(s) \quad (3)$$

where

$$\begin{aligned} N^{IJ}_{\pi\pi}(s) &= t^{IJ}_{\pi\pi}(s)^{(2)+\mathrm{Res+Loop}}\,, \\ R^{IJ}_{\eta'\to\eta\pi\pi}(s) &= m^{IJ}_{\eta'\to\eta\pi\pi}(s)^{(2)+\mathrm{Res+Loop}}\,, \end{aligned}$$

- Chiral expansion of Eq. (3) leads

$$m^{IJ}_{\eta'\to\eta\pi\pi}(s) = m^{IJ}_{\eta'\to\eta\pi\pi}(s)^{(2)+\mathrm{Res+Loop}} - t^{IJ}_{\pi\pi}(s)^{(2)} g_{\pi\pi}(s) m^{IJ}_{\eta'\to\eta\pi\pi}(s)^{(2)} + \cdots$$

$$\mathrm{Im}\left(m^{IJ}_{\eta'\to\eta\pi\pi}(s)\right)\Big|^{p^4} = t^{IJ}_{\pi\pi}(s)^{(2)} \sigma_\pi(s) m^{IJ}_{\eta'\to\eta\pi\pi}(s)^{(2)}$$

N/D applied to $\eta' \to \eta\pi\pi$

- Amplitude at one-loop in Large-N_C $U(3)$ ChPT with resonances

$$\mathcal{M}^{\eta'\to\eta\pi\pi}(s) = \mathcal{M}(s)^{(2)} + \mathcal{M}(s)^{\mathrm{Res(s,t,u)}} + \mathcal{M}(s)^{\mathrm{Loop(s,t,u)}} \tag{4}$$

- N/D representation of Eq. (2)

$$m^{IJ}_{\eta'\to\eta\pi\pi}(s) = \left[1 + N^{IJ}_{\pi\pi}(s) g_{\pi\pi}(s)\right]^{-1} R^{IJ}_{\eta'\to\eta\pi\pi}(s) \tag{5}$$

where

$$\begin{aligned} N^{IJ}_{\pi\pi}(s) &= t^{IJ}_{\pi\pi}(s)^{(2)+\mathrm{Res+Loop}} \,, \\ R^{IJ}_{\eta'\to\eta\pi\pi}(s) &= m^{IJ}_{\eta'\to\eta\pi\pi}(s)^{(2)+\mathrm{Res+Loop}} \,, \end{aligned}$$

- Chiral expansion of Eq. (3) leads

$$m^{IJ}_{\eta'\to\eta\pi\pi}(s) = m^{IJ}_{\eta'\to\eta\pi\pi}(s)^{(2)+\mathrm{Res+Loop}} - t^{IJ}_{\pi\pi}(s)^{(2)} g_{\pi\pi}(s) m^{IJ}_{\eta'\to\eta\pi\pi}(s)^{(2)} + \cdots$$

$$\boxed{\mathrm{Im}\left(m^{IJ}_{\eta'\to\eta\pi\pi}(s)\right) = t^{IJ}_{\pi\pi}(s)\sigma_\pi(s) m^{IJ}_{\eta'\to\eta\pi\pi}(s)}$$

Partial waves

- Unitarized amplitude in terms of the S-and-D-waves

$$\mathcal{M}^{I=0}_{\eta'\to\eta\pi\pi}(s,\cos\theta_s) = \sum_J 32\pi(2J+1)P_J(\cos\theta_s)m^{IJ}(s)$$

$$= 32\pi P_0(\cos\theta_s)\frac{m^{00}(s)}{1+g_{\pi\pi}(s)t^{00}_{\pi\pi}(s)} + 160\pi P_2(\cos\theta_s)\frac{m^{02}(s)}{1+g_{\pi\pi}(s)t^{02}_{\pi\pi}(s)}$$

$$m^{IJ}(s) = \frac{1}{32\pi}\frac{s}{\lambda(s,m^2_{\eta'},m^2_\eta)^{1/2}\lambda(s,m^2_\pi,m^2_\pi)^{1/2}}\int_{t_{\min}}^{t_{\max}} dt P_J(\cos\theta_s)\mathcal{M}^I(s,t,u)$$

$$\cos\theta_s = -\frac{s\left(m^2_{\eta'}+m^2_\eta+2m^2_\pi-s-2t\right)}{\lambda(s,m^2_{\eta'},m^2_\eta)^{1/2}\lambda(s,m^2_\pi,m^2_\pi)^{1/2}}\,,$$

$$P_0(\cos\theta_s) = 1\,,\quad P_2(\cos\theta_s) = \frac{1}{2}\Big[-1+3(\cos\theta_s)^2\Big] \propto X^2$$

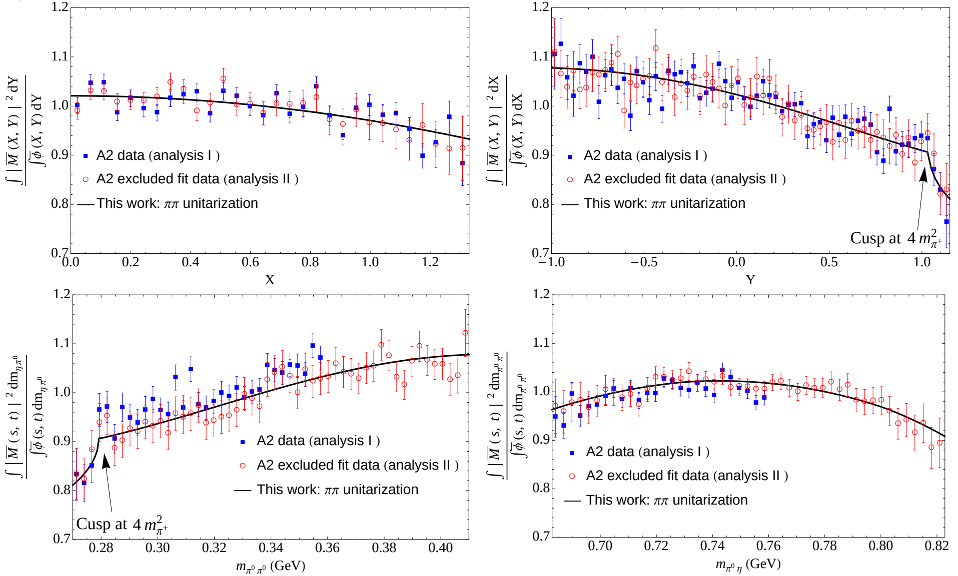
Fits to experimental data: $\pi\pi$ final-state interactions effects

Parameter	Fit A	Fit B	Fit C	A2 Collaboration
M_S	$1001(24)$	$988(10)$	$930(7)(39)$	
c_d	$29.5(1.8)$	$28.6(7)$	$23.5(6)(2.4)$	
c_m	$= c_d$	$= \mathbf{41.1(1)}$	$= \mathbf{80(20)}$	
$\tilde{c}_d$	$17.0(1.0)$	$16.5(4)$	$13.6(4)(1.4)$	
$\tilde{c}_m$	$= \tilde{c}_d$	$= \mathbf{18.9(9)}$	$46.2(11.5)$	
$a_{\pi\pi}(\mu)$	$0.73(25)$	$0.24(12)$	$0.41(11)(19)$	
$\chi^2_{\rm dof}$	$220.4/197 \sim 1.12$	$220.1/197 \sim 1.12$	$220.1/197 \sim 1.12$	
$a[Y]$	$\mathbf{-0.075(9)}$	$-0.075(7)$	$-0.074(7)(1)$	$\mathbf{-0.074(8)(6)}$
$b[Y^2]$	$\mathbf{-0.051(1)}$	$-0.056(1)$	$-0.053(1)(1)$	$\mathbf{-0.063(14)(5)}$
$c[X]$	$\mathbf{0}$	0	0	$\mathbf{0}$
$d[X^2]$	$\mathbf{-0.049(14)}$	$-0.050(4)$	$-0.049(4)(1)$	$\mathbf{-0.050(9)(5)}$

Parameter	Fit A (ChPT)	Fit B (ChPT)	Fit C (ChPT)	A2 Collaboration
M_S	$973(5)$	$992(7)$	$926(5)(25)$	
c_d	$30.1(4)$	$32.9(5)$	$25.7(4)(1.9)$	
c_m	$= c_d$	$= 41.1(1)$	$= 80(20)$	
$\tilde{c}_d$	$17.4(2)$	$18.4(3)$	$14.8(2)(1.1)$	
$\tilde{c}_m$	$= \tilde{c}_d$	$= 18.9(9)$	$46.2(11.5)$	
$\chi^2_{\rm dof}$	$242.2/198 \sim 1.22$	$246.4/198 \sim 1.24$	$242.3/198 \sim 1.22$	
$a[Y]$	$\mathbf{-0.095(6)}$	$-0.083(6)$	$-0.090(7)(2)$	$\mathbf{-0.074(8)(6)}$
$b[Y^2]$	$\mathbf{0.005(1)}$	$-0.001(1)$	$0.004(1)(0)$	$\mathbf{-0.063(14)(5)}$
$c[X]$	$\mathbf{0}$	0	0	$\mathbf{0}$
$d[X^2]$	$\mathbf{-0.037(5)}$	$-0.057(5)$	$-0.041(7)(1)$	$\mathbf{-0.050(9)(5)}$

Fits to experimental data: $\pi\pi$ final-state interactions effects

·) Cusp seen for the first time in $\eta' \to \eta\pi^0\pi^0$



$\pi\eta$ final state interactions effects

- N/D representation accounting for $\pi\eta$ FSI

$$m^{IJ}_{\eta'\to\eta\pi\pi}(t,u) = [1 + N^{IJ}_{\pi\eta}(t)g_{\pi\eta}(t)]^{-1}R^{IJ}_{\eta'\to\eta\pi\pi}(t) + [1 + N^{IJ}_{\pi\eta}(u)g_{\pi\eta}(u)]^{-1}R^{IJ}_{\eta'\to\eta\pi\pi}(u)\,,$$

$$\begin{aligned} N^{IJ}_{\pi\eta}(t) &= t^{IJ}_{\pi\eta}(t)^{(2)+\text{Res+Loop}}\,, \\ R^{IJ}_{\eta'\to\eta\pi\pi}(t) &= m^{IJ}_{\eta'\to\eta\pi\pi}(t)^{(2)+\text{Res+Loop}}\,, \end{aligned}$$

- Perturbative expansion supplemented by S-wave $\pi\eta$ FSI

$$\begin{aligned} &\mathcal{M}^{I=1}_{\eta'\to\eta\pi\pi}(s,t,u,\cos\theta_t,\cos\theta_u) = \mathcal{M}(s,t,u)^{(2)+\text{Res+Loop}} \\ &+32\pi P_0(\cos\theta_t)\frac{m^{10}_{\eta'\to\eta\pi\pi}(t)^{(2)+\text{Res+Loop}}}{1+g_{\pi\eta}(t)t^{10}_{\pi\eta}(t)^{(2)+\text{Res+Loop}}} \\ &+32\pi P_0(\cos\theta_u)\frac{m^{10}_{\eta'\to\eta\pi\pi}(u)^{(2)+\text{Res+Loop}}}{1+g_{\pi\eta}(u)t^{10}_{\pi\eta}(u)^{(2)+\text{Res+Loop}}} \\ &-32\pi P_0(\cos\theta_t)m^{10}_{\eta'\to\eta\pi\pi}(t)^{(2)+\text{Res+Loop}} - 32\pi P_0(\cos\theta_u)m^{10}_{\eta'\to\eta\pi\pi}(u)^{(2)+\text{Res+Loop}} \end{aligned}$$

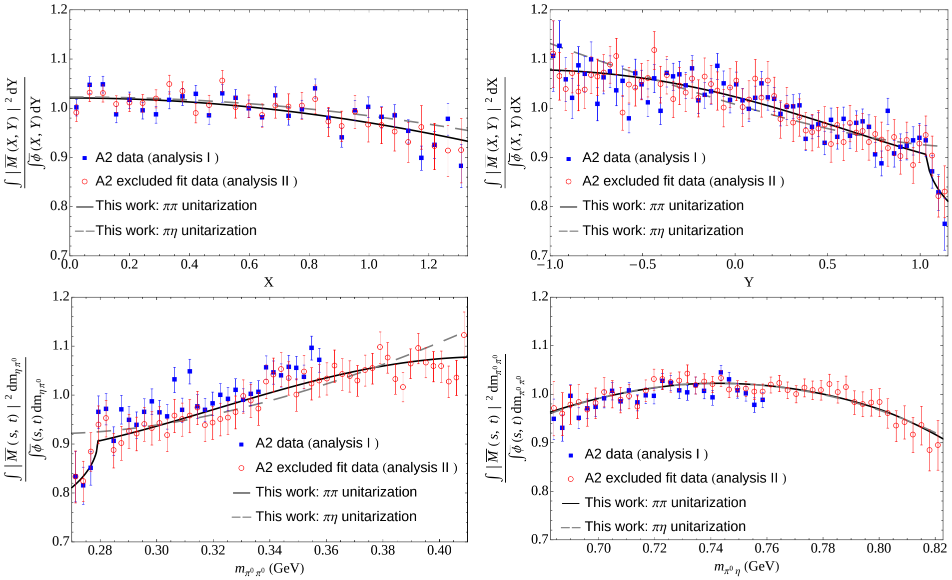
Fits to experimental data: $\pi\eta$ final-state interactions effects

- Fit restrictions: $M_S = M_{S_8} = M_{S_1} = M_{a_0}$ and $a_{\pi\eta} = 2.0^{+3.1}_{-3.4}$ (Guo'11)

Parameter	Fit A	Fit B	Fit C	Fit D
M_S	$979(7)(11)$	$950(18)(22)$	$996(8)(8)$	$933(7)(39)$
c_d	$30.3(3)(3)$	$27.3(1.5)(4.1)$	$32.1(6)(4)$	$25.9(5)(2.3)$
c_m	$= c_d$	$65.3(11.4)(28.5)$	$= 41.1(1)$	$= 80(20)$
$\tilde{c}_d$	$17.5(2)(2)$	$15.8(9)(2.4)$	$18.5(4)(2)$	$15.0(3)(1.3)$
$\tilde{c}_m$	$= \tilde{c}_d$	$37.7(6.6)(16.5)$	$= 18.9(9)$	$46.2(11.5)$
$\chi^2_{\rm dof}$	$242.9/198 \sim 1.23$	$242.6/197 \sim 1.23$	$245.9/198 \sim 1.24$	$242.7/197 \sim 1.23$
$a[Y]$	$\mathbf{-0.095(6)(5)}$	$-0.091(8)(6)$	$-0.083(7)(3)$	$-0.089(7)(8)$
$b[Y^2]$	$\mathbf{0.005(1)(1)}$	$0.004(1)(0)$	$-0.001(1)(1)$	$0.004(1)(0)$
$c[X]$	$\mathbf{0}$	0	0	0
$d[X^2]$	$\mathbf{-0.034(5)(3)}$	$-0.037(6)(5)$	$-0.053(5)(7)$	$-0.038(6)(6)$

Parameter	Fit A (ChPT)	Fit B (ChPT)	Fit C (ChPT)	Fit D (ChPT)
M_S	$973(5)$	$954(47)$	$992(7)$	$926(5)(25)$
c_d	$30.1(4)$	$28.0(4.6)$	$32.9(5)$	$25.7(4)(1.9)$
c_m	$= c_d$	$53.4(52.0)$	$= 41.1(1)$	$= 80(20)$
$\tilde{c}_d$	$17.4(2)$	16.1	$18.4(3)$	$14.8(2)(1.1)$
$\tilde{c}_m$	$= \tilde{c}_d$	30.8	$= 18.9(9)$	$46.2(11.5)$
$\chi^2_{\rm dof}$	$242.2/198 \sim 1.22$	$242.0/197 \sim 1.23$	$246.4/198 \sim 1.24$	$242.3/198 \sim 1.22$
$a[Y]$	$\mathbf{-0.095(6)}$	$-0.093(45)$	$-0.083(6)$	$-0.090(7)(2)$
$b[Y^2]$	$\mathbf{0.005(1)}$	$0.004(3)$	$-0.001(1)$	$0.004(1)(0)$
$c[X]$	$\mathbf{0}$	0	0	0
$d[X^2]$	$\mathbf{-0.037(5)}$	$-0.039(18)$	$-0.057(5)$	$-0.041(7)(1)$

Fits to experimental data: $\pi\eta$ final-state interactions



Inclusion of $\pi\pi$ and $\pi\eta$ final-state interactions

- Perturbative expansion + S-and D-wave $\pi\pi$ and S-wave $\pi\eta$ FSI

$$\mathcal{M}(s,t,u,\cos\theta_{s,t,u}) = \mathcal{M}(s,t,u)^{(2)+\mathrm{Res}+\mathrm{Loop}}$$

$$+32\pi P_0(\cos\theta_s)\frac{m^{00}_{\eta'\to\eta\pi\pi}(s)^{(2)+\mathrm{Res}+\mathrm{Loop}}}{1+g_{\pi\pi}(s)t^{00}_{\pi\pi}(s)^{(2)+\mathrm{Res}+\mathrm{Loop}}}$$

$$+160\pi P_2(\cos\theta_s)\frac{m^{02}_{\eta'\to\eta\pi\pi}(s)^{(2)+\mathrm{Res}+\mathrm{Loop}}}{1+g_{\pi\pi}(s)t^{02}_{\pi\pi}(s)^{(2)+\mathrm{Res}+\mathrm{Loop}}}$$

$$-32\pi P_0(\cos\theta_s)m^{00}_{\eta'\to\eta\pi\pi}(t)^{(2)+\mathrm{Res}+\mathrm{Loop}} - 160\pi P_2(\cos\theta_s)m^{02}_{\eta'\to\eta\pi\pi}(u)^{(2)+\mathrm{Res}+\mathrm{Loop}}$$

$$+32\pi P_0(\cos\theta_t)\frac{m^{10}_{\eta'\to\eta\pi\pi}(t)^{(2)+\mathrm{Res}+\mathrm{Loop}}}{1+g_{\pi\eta}(t)t^{10}_{\pi\eta}(t)^{(2)+\mathrm{Res}+\mathrm{Loop}}}$$

$$+32\pi P_0(\cos\theta_u)\frac{m^{10}_{\eta'\to\eta\pi\pi}(u)^{(2)+\mathrm{Res}+\mathrm{Loop}}}{1+g_{\pi\eta}(u)t^{10}_{\pi\eta}(u)^{(2)+\mathrm{Res}+\mathrm{Loop}}}$$

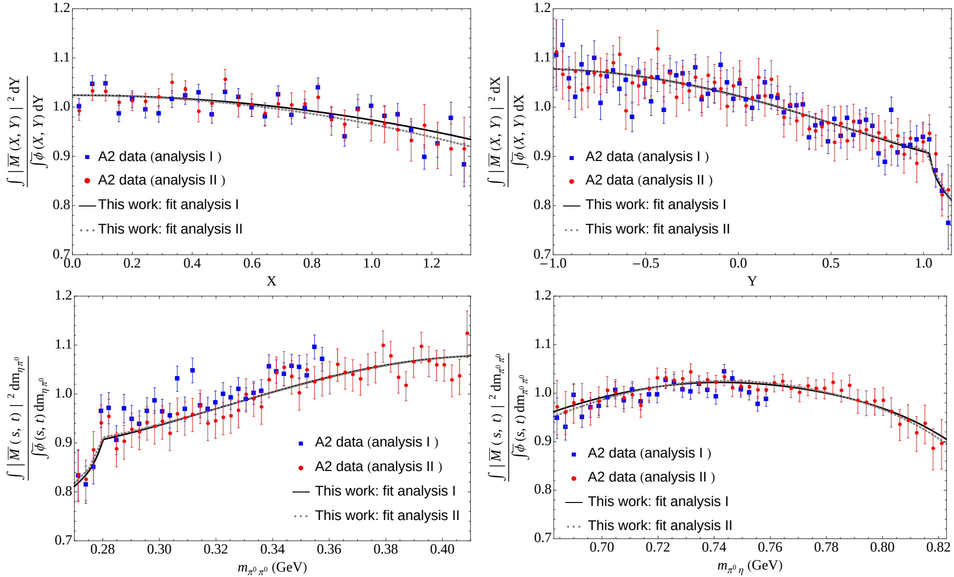
$$-32\pi P_0(\cos\theta_t)m^{10}_{\eta'\to\eta\pi\pi}(t)^{(2)+\mathrm{Res}+\mathrm{Loop}} - 32\pi P_0(\cos\theta_u)m^{10}_{\eta'\to\eta\pi\pi}(u)^{(2)+\mathrm{Res}+\mathrm{Loop}}\,.$$

Fits to experimental data: $\pi\pi$ and $\pi\eta$ final-state interactions

- Fit restrictions: $M_S = M_{S_8} = M_{S_1} = M_{a_0}$ and $a_{\pi\eta} = 2.0^{+3.1}_{-3.4}$ (Guo'11)

Parameter	Analysis I Fit 1	Analysis I Fit 2	Analysis II Fit 1	Analysis II Fit 2
M_S	$1009(54)(14)$	$996(6)(12)$	$1030(71)(17)$	$1014(15)(16)$
c_d	$29.9(3.8)(6)$	$29.1(4)(9)$	$31.5(4.8)(7)$	$30.5(1.0)(1.0)$
c_m	$= c_d$	$= 41.1(1)$	$= c_d$	$= 41.1(1)$
$\tilde{c}_d$	$17.3(2.2)(4)$	$16.8(2)(2)$	$18.2(2.8)(4)$	$17.6(6)(6)$
$\tilde{c}_m$	$= \tilde{c}_d$	$= 18.9(9)$	$= \tilde{c}_d$	$= 18.9(9)$
$a_{\pi\pi}$	$0.73(49)(4)$	$0.32(8)(9)$	$0.97(56)(5)$	$0.52(14)(12)$
χ^2_{dof}	1.12	1.12	1.23	1.23
$a[Y]$	$\mathbf{-0.075(7)(4)}$	$\mathbf{-0.074(6)(4)}$	$\mathbf{-0.071(6)(5)}$	$\mathbf{-0.071(6)(5)}$
$b[Y^2]$	$\mathbf{-0.050(1)(1)}$	$\mathbf{-0.055(1)(1)}$	$\mathbf{-0.050(2)(1)}$	$\mathbf{-0.054(1)(1)}$
$c[X]$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$
$d[X^2]$	$\mathbf{-0.047(8)(2)}$	$\mathbf{-0.049(2)(2)}$	$\mathbf{-0.056(6)(1)}$	$\mathbf{-0.056(6)(2)}$
$\kappa_{03}[Y^3]$	0.001	0.003	0.001	0.002
$\kappa_{21}[YX^2]$	-0.004	-0.005	-0.005	-0.005
$\kappa_{22}[Y^2X^2]$	0.001	0.002	0.002	0.002

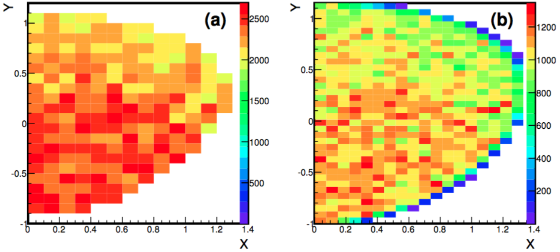
Fits to experimental data: $\pi\pi$ and $\pi\eta$ final-state interactions



Fits to experimental data

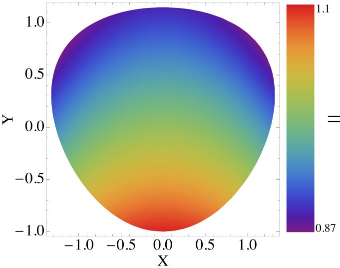
- Dalitz plot slope parameters

A2 Coll. 1709.04230

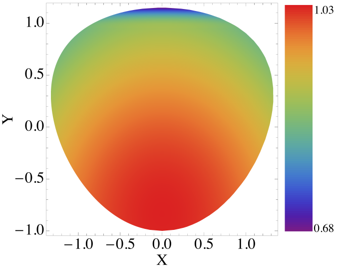


Dalitz Parameters (A2)

$$a[Y] = -0.074(8)(6)$$
$$b[Y^2] = -0.063(14)(5)$$
$$d[X^2] = -0.050(9)(5)$$



$\Rightarrow$

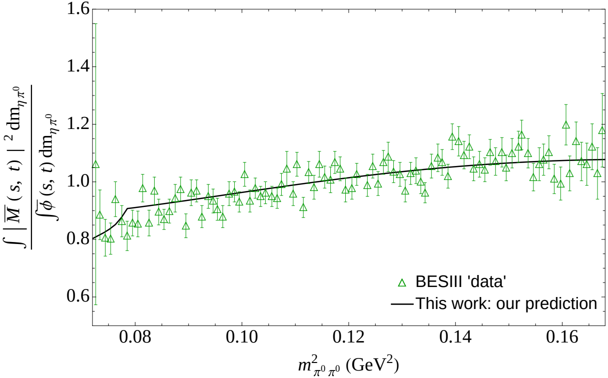


Dalitz Parameters

$$a[Y] = -0.095(6)$$
$$b[Y^2] = 0.005(1)$$
$$d[X^2] = -0.037(5)$$
$$\Downarrow$$
$$a[Y] = -0.073(7)(5)$$
$$b[Y^2] = -0.052(1)(2)$$
$$d[X^2] = -0.052(8)(5)$$

Comparison with BESIII 2017 experimental data

- Data is not publicly available
- Less events than A2 $(351016$ vs $1.2 \cdot 10^5)$
- No cusp structure seen
- Contrarty to A2 both $\eta' \to \eta\pi^0\pi^0$ and $\eta' \to \eta\pi^+\pi^-$ are measured



BESIII Coll. 1709.04627

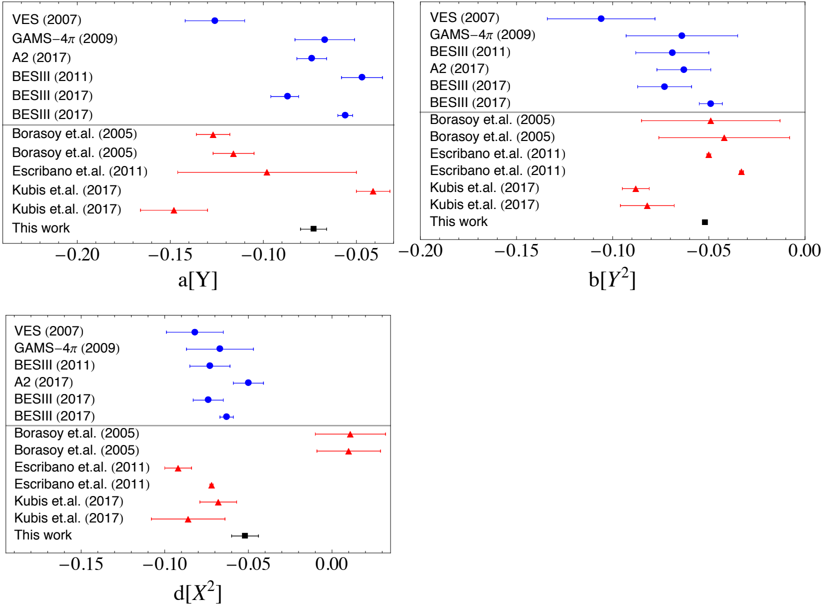
$\chi^2_{\mathrm{dof}} \sim 1.1$

Summary

- $\eta' \to \eta\pi\pi$ analyzed within $U(3)$ ChPT at one-loop with resonances
- We have illustrated a method (N/D) to resumme two-particle FSI
- Dalitz plot parameters:
  - Y-variable is linear in s: Importance of $\pi\pi$ FSI
  - X-variable appear in the form $\cos\theta_s = X f(Y)$: Importance of the D-wave
  - $\pi\eta$ FSI effects are small

Experiment	a$[Y]$	b$[Y^2]$	c$[X]$	d$[X^2]$
GAMS4π (c=0) '09	$-0.067(16)(4)$	$-0.064(29)(5)$	0	$-0.067(20)(3)$
VES '07	$-0.127(16)(8)$	$-0.106(28)(14)$	$0.015(11)(14)$	$-0.082(17)(8)$
BESIII '11	$-0.047(11)(3)$	$-0.069(19)(9)$	$0.019(11)(3)$	$-0.073(12)(3)$
A2'17	$-0.074(8)(6)$	$-0.063(14)(5)$	—	$-0.050(9)(5)$
BESIII'17	$-0.087(9)(6)$	$-0.073(14)(5)$	0	$-0.074(9)(4)$
BESIII'17	$-0.056(4)(3)$	$-0.049(6)(6)$	$2.7(2.4)(1.8)\cdot 10^{-3}$	$-0.063(4)(4)$
Previous Estimates				
Borasoy et.al.'05	$-0.127(9)$	$-0.049(36)$	0	$0.011(21)$
Borasoy et.al.'05	$-0.116(11)$	$-0.042(34)$	0	$0.010(19)$
Escribano et.al.'10	$-0.098(48)$	$-0.050(1)$	0	$-0.092(8)$
Escribano et.al.'10	$-0.098(48)$	$-0.033(1)$	0	$-0.072(1)$
Fariborz et.al.'14	-0.024	0.0001	0	-0.029
Kubis $et.al.$'17	$-0.148(18)$	$-0.082(7)$	0	$-0.068(11)$
Kubis $et.al.$'17	$-0.041(9)$	$-0.088(7)$	0	$-0.086(22)$
This talk				
Resonances	$-0.096(9)$	$0.002(1)$	0	$-0.036(6)$
Resonances+loops	$-0.095(6)$	$0.005(1)$	0	$-0.037(5)$
$\pi\pi$ FSI	$-0.075(9)$	$-0.051(1)$	0	$-0.049(14)$
$\pi\eta$ FSI	$-0.095(6)(5)$	$0.005(1)(1)$	0	$-0.034(5)(3)$
$\pi\pi + \pi\eta$	$\mathbf{-0.073(7)(5)}$	$\mathbf{-0.052(1)(2)}$	$\mathbf{0}$	$\mathbf{-0.052(8)(5)}$

Summary


VES (2007)
GAMS−4π (2009)
A2 (2017)
BESIII (2011)
BESIII (2017)
BESIII (2017)
Borasoy et.al. (2005)
Borasoy et.al. (2005)
Escribano et.al. (2011)
Kubis et.al. (2017)
Kubis et.al. (2017)
This work
−0.20
−0.15
−0.10
−0.05
a[Y]
VES (2007)
GAMS−4π (2009)
BESIII (2011)
A2 (2017)
BESIII (2017)
BESIII (2017)
Borasoy et.al. (2005)
Borasoy et.al. (2005)
Escribano et.al. (2011)
Escribano et.al. (2011)
Kubis et.al. (2017)
Kubis et.al. (2017)
This work
−0.20
−0.15
−0.10
−0.05
0.00
b[Y^2]
VES (2007)
GAMS−4π (2009)
BESIII (2011)
A2 (2017)
BESIII (2017)
BESIII (2017)
Borasoy et.al. (2005)
Borasoy et.al. (2005)
Escribano et.al. (2011)
Escribano et.al. (2011)
Kubis et.al. (2017)
Kubis et.al. (2017)
This work
−0.15
−0.10
−0.05
0.00
d[X^2]

Summary

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- We have illustrated a method (N/D) to resumme two-particle FSI
- Dalitz plot parameters:
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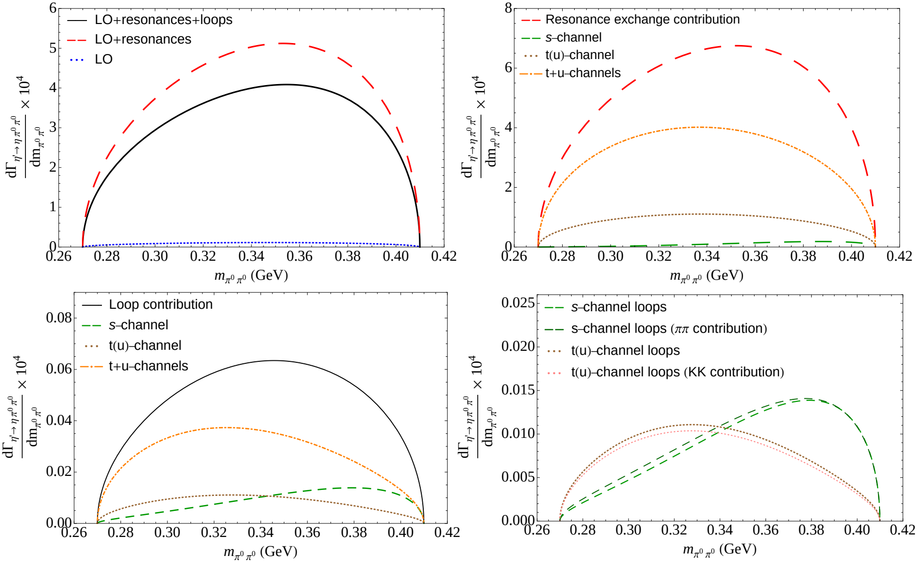
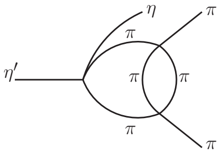
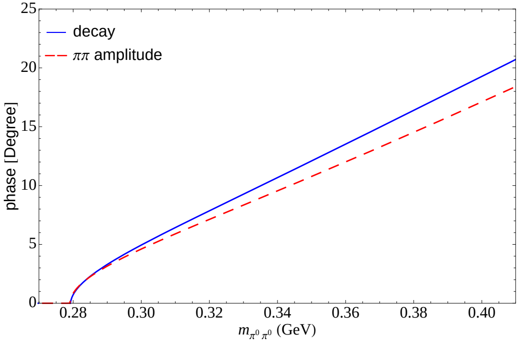
  - X-variable appear in the form $\cos\theta_s = Xf(Y)$: Importance of the D-wave
  - $\pi\eta$ FSI effects are small
- We are in position to provide our parameterization to experimental groups
- Same method can be applied for $\eta^{(\prime)} \to 3\pi$ decays

Hierarchy of the contributions



Unitarity Violations



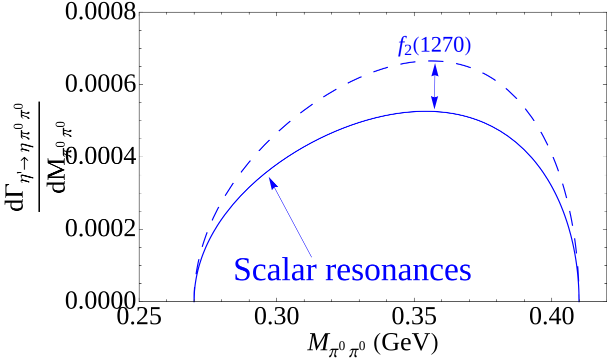


Tensor Resonance contributions

Ecker and Zauner, EPJC 52, 315 (2007)

$$\mathcal{L}_T = -\frac{1}{2}\langle T_{\mu\nu} D_T^{\mu\nu,\rho\sigma} T_{\rho\sigma}\rangle + g_T\langle T_{\mu\nu}\{u^\mu, u^\nu\}\rangle + \beta\langle T_\mu^\mu u_\nu u^\nu\rangle$$

$$g_T = 28\,\mathrm{MeV}\,, \quad \beta = -g_T\,, \quad M_T = 1300\,\mathrm{MeV}$$



Masses and couplings

Source	M_{S_8}	M_{S_1}	M_{a_0}	c_d	c_m
$a_0 \to \eta\pi$ (Guo et.al. '09)	980	$= M_{S_8}$	$= M_{S_8}$	26	80
res. saturation (Ecker et.al. '88)	983	$= M_{S_8}$	$= M_{S_8}$	32	42
$K\pi$ scattering (Jamin et.al. '00)					
	1400	$= M_{S_8}$	$= M_{S_8}$	30	43
	1190	$= M_{S_8}$	$= M_{S_8}$	45.4	$= c_d$
	1260	$= M_{S_8}$	$= M_{S_8}$	24.8	76.7
	1360	$= M_{S_8}$	$= M_{S_8}$	13	85
$PP \to PP$ ($P = \pi, K, \eta$)					
Guo et.al '11	1370^{+132}_{-57}	1063^{+53}_{-31}	$= M_{S_8}$	$15.6^{+4.2}_{-3.4}$	$31.5^{+19.5}_{-22.5}$
Guo et.al '12	1397^{+73}_{-61}	1100^{+30}_{-63}	$= M_{S_8}$	$19.8^{+2.0}_{-5.2}$	$41.9^{+3.9}_{-9.2}$
Ledwig et.al. '14	$1279(9)$	$808.9(4)$	$= M_{S_8}$	$39.8(1)$	$41.1(1)$
This work					
Resonances+loops	$972(6)$	$= M_{S_8}$	$= M_{S_8}$	$29.9(4)$	$= c_d$
Resonances+loops	953	$= M_{S_8}$	$= M_{S_8}$	27.8	53.2
$\pi\pi$ final state interactions	$998(19)$	$= M_{S_8}$	$= M_{S_8}$	$29.3(1.2)$	$= c_d$
	1021	$= M_{S_8}$	$= M_{S_8}$	32.5	$4c_m c_d = f^2$
$\pi\eta$ final state interactions	$978(7)$	$= M_{S_8}$	$= M_{S_8}$	$29.8(7)$	$= c_d$
$\pi\pi$+$\pi\eta$ final state interactions	$990(95)$	$= M_{S_8}$	$= M_{S_8}$	$32.0(6.8)$	$= c_d$