HADRONIC STRUCTURE OF THE NUCLEON

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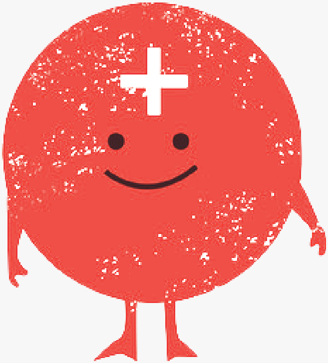
(QCDSF Collaboration)

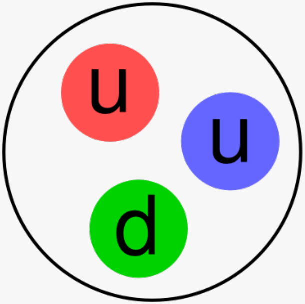


→ spin sum rule

$$\frac{1}{2}$$

→ momentum sum rule

1

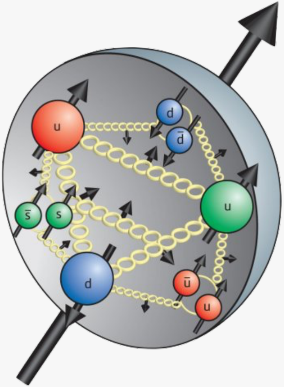


→ spin sum rule

$$\frac{1}{2} = \frac{1}{2}\sum_q \Delta q$$

→ momentum sum rule

$$1 = \sum_q \langle x \rangle_q$$



→ spin sum rule (Ji Decomposition)

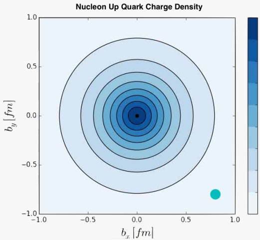
$$\frac{1}{2} = \frac{1}{2}\sum_q \Delta q + L_q + J_g$$

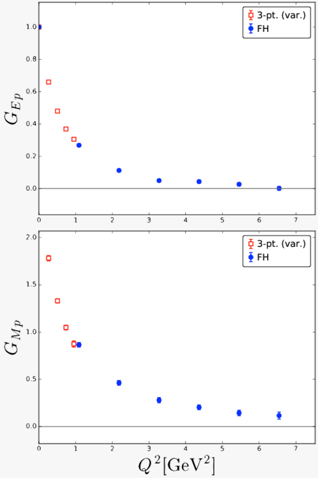
→ momentum sum rule

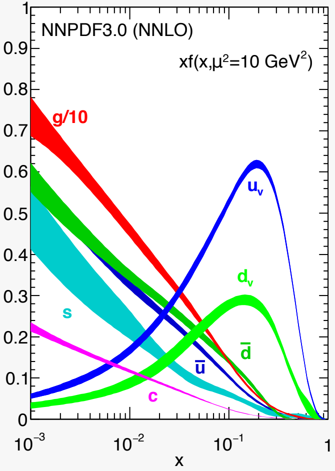
$$1 = \sum_q \langle x \rangle_q + \langle x \rangle_g$$

FORM FACTORS

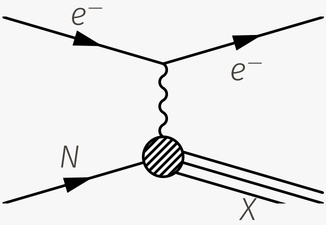
→ Contain information about charge radius (low Q^2) and distribution (high Q^2)



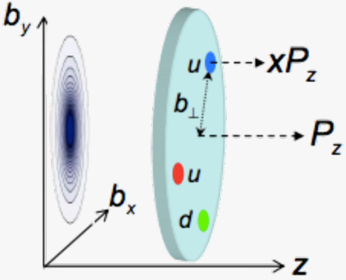




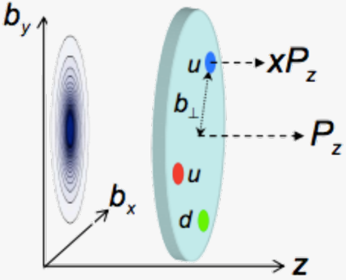
→ Contain information about longitudinal momentum of partons



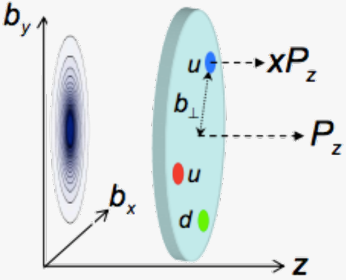
→ GPDs unify form factors and parton momentum fraction



- → GPDs unify form factors and parton momentum fraction
- → Want theoretical input for experiments to more completely understand strongly bound systems



- → GPDs unify form factors and parton momentum fraction
- → Want theoretical input for experiments to more completely understand strongly bound systems
- → Full image of GPDs on lattice still a challenge



LATTICE

FEYNMAN DIAGRAM CALCULATION

$$\begin{array}{ccc} \text{Hard problem} & \rightarrow & \text{Infinite series of problems} \\ \langle \text{Value} \rangle & = & \mathcal{O}(1) + \mathcal{O}(\alpha) + \mathcal{O}(\alpha^2) + \dots \end{array}$$

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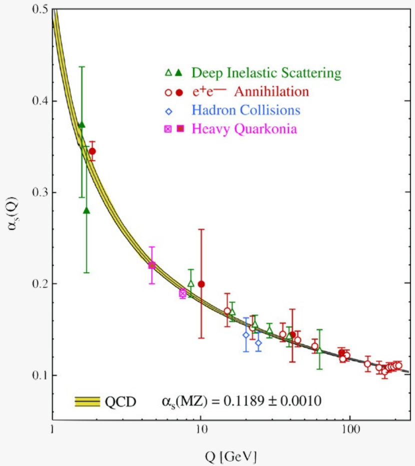

PATH INTEGRALS

$$\langle \mathcal{O} \rangle = \frac{1}{Z} \int \prod_f D\overline{\psi}_f D\psi_f DA_\mu \; \mathcal{O}\left[A_\mu, \overline{\psi}_f, \psi_f\right] \; e^{-S\left[A_\mu, \overline{\psi}_f, \psi_f\right]}$$

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All possible field configurations

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All possible field configurations

Field Configuration Importance

THE LATTICE

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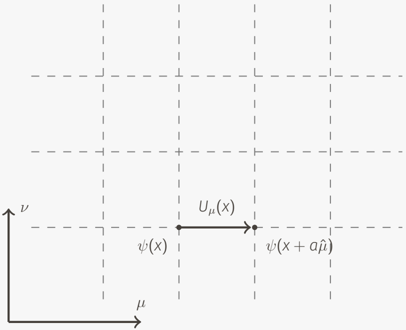

THE LATTICE

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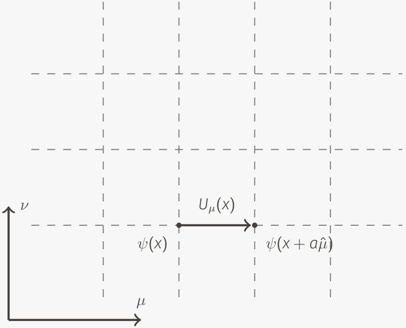


After discretisation and weighted Monte Carlo:

$$\langle \mathcal{O} \rangle = \frac{1}{N} \sum_i^N \mathcal{O}\left[U_\mu^{(i)}\right]$$

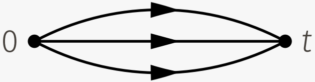
Weighted by

$$\prod_f \det\left[D_f\left(U_\mu\right)\right] e^{-S_g\left[U_\mu\right]}$$

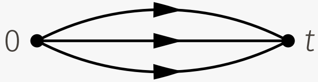
LATTICE CALCULATIONS

$\overline{\chi}(0)|\Omega\rangle$

0 •



$$G^{(2)}(t) = \langle \Omega | \chi(t) \overline{\chi}(0) | \Omega \rangle$$



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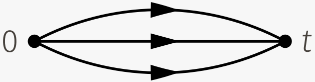
$$= \sum_X A_X e^{-E_X t}$$



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$$\to A_N e^{-E_N t}$$

GLUONS

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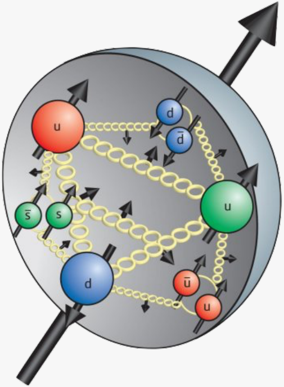

FORM FACTORS OF ENERGY MOMENTUM TENSOR

$$T^g_{\mu\nu} = \mathrm{Tr}_c\, G_{\mu\alpha} G_\nu{}^\alpha$$

$$\langle p' \mid T_{\mu\nu} \mid p\rangle = \mathcal{S}\overline{u}(p')\Big[\gamma_\mu P_\nu A_{20}\big(Q^2\big) + \frac{i\sigma_{\mu\alpha} q^\alpha}{2m_N} P_\nu B_{20}\big(Q^2\big) + \frac{q_\mu q_\nu}{m_N} C_{20}\big(Q^2\big)\Big] u(p)$$

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$$= \mathcal{S}\overline{u}(p')\left[P_\mu P_\nu M_2\left(Q^2\right) + \frac{i\sigma_{\mu\alpha} q^\alpha}{2m_N} P_\nu J\left(Q^2\right) + \frac{q_\mu q_\nu}{m_N} d_1\left(Q^2\right)\right] u(p)$$

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$$(A_{20} + B_{20})(0) = J$$

$$A_{20} = M_2$$

$$A_{20} + B_{20} = J$$

$$C_{20} = d_1$$

GENERATION

$$G^{(3)}(J, t, \tau, \mathsf{p}', \mathsf{q}) = \int d^3y e^{i\mathsf{q}\cdot\mathsf{y}} G^{(2)}(t, \mathsf{p}') \otimes \mathcal{O}(\mathsf{y}, \tau)$$

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→ No extra (expensive) quark propagator calculations

→ Take ratio R to remove time dependence

OPERATORS

$$T_{\mu\nu} = \mathrm{Tr}_c\, G_{\mu\alpha} G_{\nu}{}^{\alpha}$$

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T_{4i}	$(ExB)_i$

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Operator	Interpretation
T_{4i}	$(ExB)_i$
$T_{44} - \frac{1}{3}(T_{33} + T_{22} + T_{11})$	$\left(B^2 - E^2\right)$

FORM FACTOR REWRITTEN

→ Interested to extract $M_2\left(Q^2=0\right)$ and $J\left(Q^2=0\right)=\left(A_{20}+B_{20}\right)\left(Q^2=0\right)$

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$$\langle p' \mid T_{\mu\nu} \mid p\rangle = \mathcal{S}\overline{u}(p')\left[P_\mu P_\nu M_2\left(Q^2\right)+\frac{i\sigma_{\mu\alpha}q^\alpha}{2m_N}P_\nu J\left(Q^2\right)+\frac{q_\mu q_\nu}{m_N}d_1\left(Q^2\right)\right]u(p)$$

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→ Focus on operator T_{4i} and, $q_i=0$

Results

LATTICE DETAILS

$L^3 \times T$	β	κ	$m_\pi(MeV)$	N_{cfg}	$N_{src/cfg}$
$24^3 \times 48$	6.0	0.132	754.8(3)	2000	10

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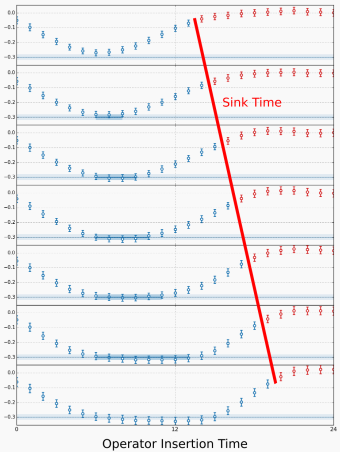
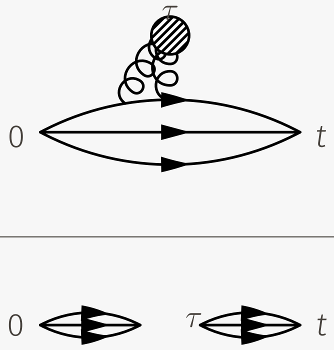
$\rightarrow$ Current from Clover Plaquette on Wilson flowed Gauge Fields (Lüscher 10.1007/JHEP08(2010)071)

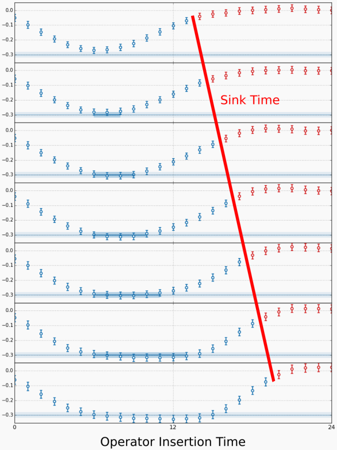
THREE POINT FUNCTION FIT

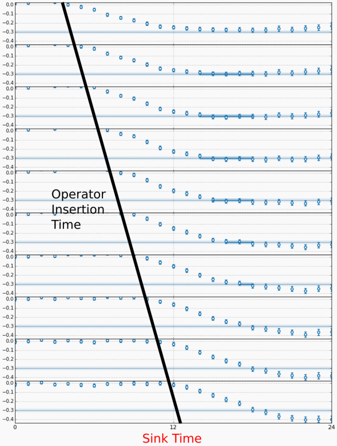




$$0 << \tau << t$$

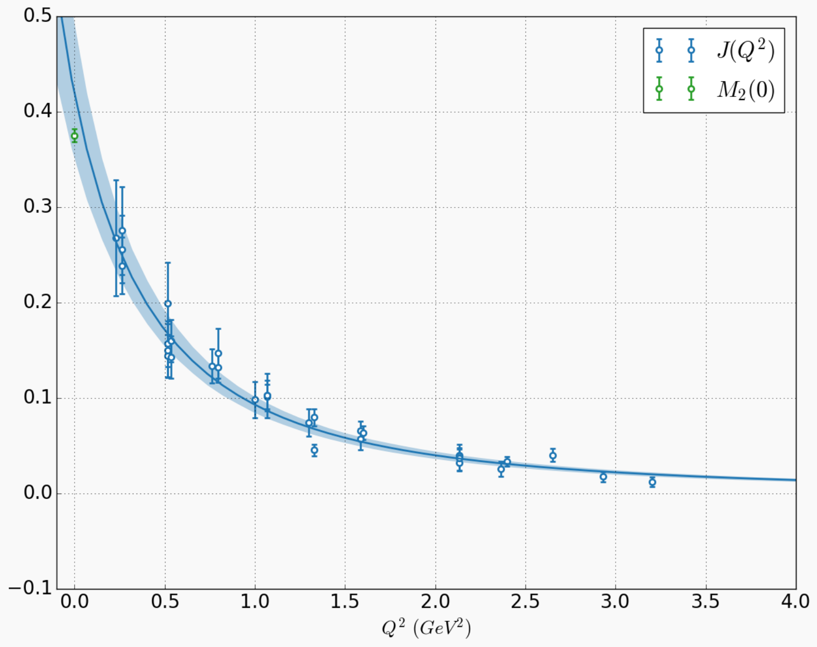
THREE POINT FUNCTION FIT



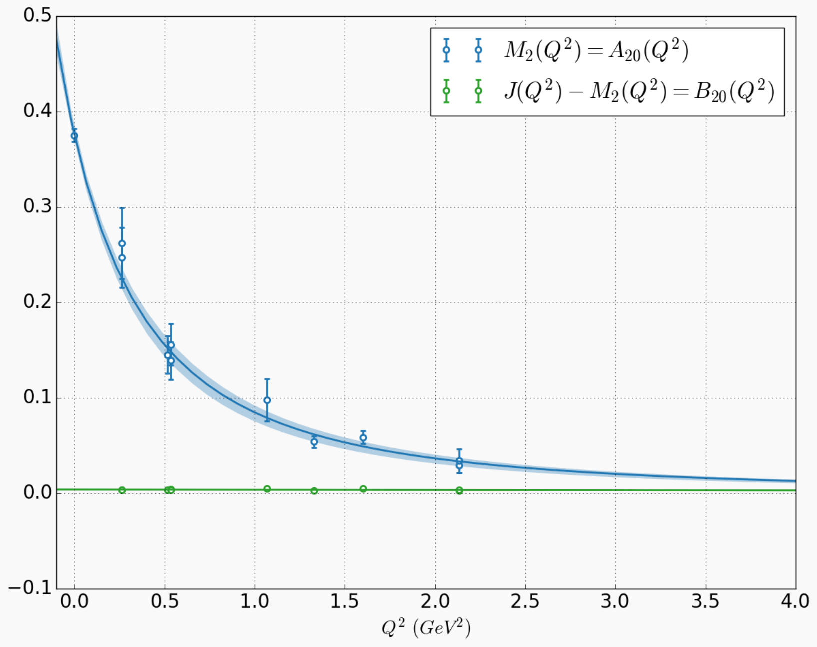


3888 SETS OF THREE POINT FUNCTIONS

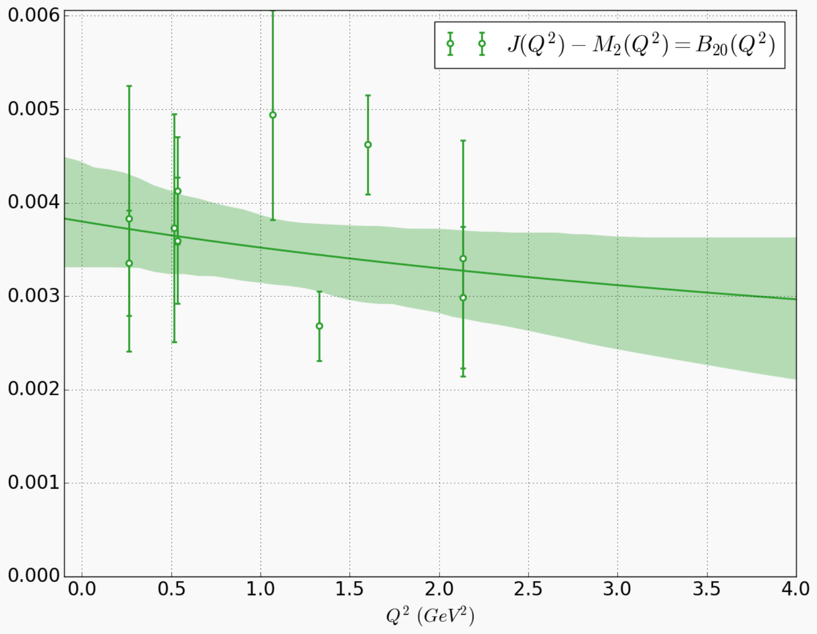
FOR 49 DATA POINTS



0.5
0.4
0.3
0.2
0.1
0.0
−0.1
0.0
0.5
1.0
1.5
2.0
2.5
3.0
3.5
4.0
Q^2 (GeV^2)
$J(Q^2)$
$M_2(0)$



$M_2(Q^2) = A_{20}(Q^2)$
$J(Q^2) - M_2(Q^2) = B_{20}(Q^2)$
0.5
0.4
0.3
0.2
0.1
0.0
−0.1
0.0
0.5
1.0
1.5
2.0
2.5
3.0
3.5
4.0
$Q^2\ (GeV^2)$


$J(Q^2) - M_2(Q^2) = B_{20}(Q^2)$
0.006
0.005
0.004
0.003
0.002
0.001
0.000
0.0
0.5
1.0
1.5
2.0
2.5
3.0
3.5
4.0
$Q^2\ (GeV^2)$

$$\frac{1}{2} = \frac{1}{2}\left(\sum_q A_{20}^q(0) + \sum_q B_{20}^q(0) + A_{20}^g(0) + B_{20}^g(0)\right)$$

$$1 = \sum_q \langle x \rangle_q + \langle x \rangle_g$$

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$\mu = 2\ GeV$	$B_{20}(0)$	J

SUM RULES

$$\frac{1}{2} = \frac{1}{2}\left(\sum_q A_{20}^q(0) + \sum_q B_{20}^q(0) + A_{20}^g(0) + B_{20}^g(0)\right)$$

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$\mu = 2\ GeV$	$B_{20}(0)$	J
$(u+d)_{con}$	≈ 0	0.3125(32)

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$\mu = 2\ GeV$	$B_{20}(0)$	J
$(u+d)_{con}$	≈ 0	0.3125(32)
gluon	0.00327(68)	0.209(29)

SUM RULES

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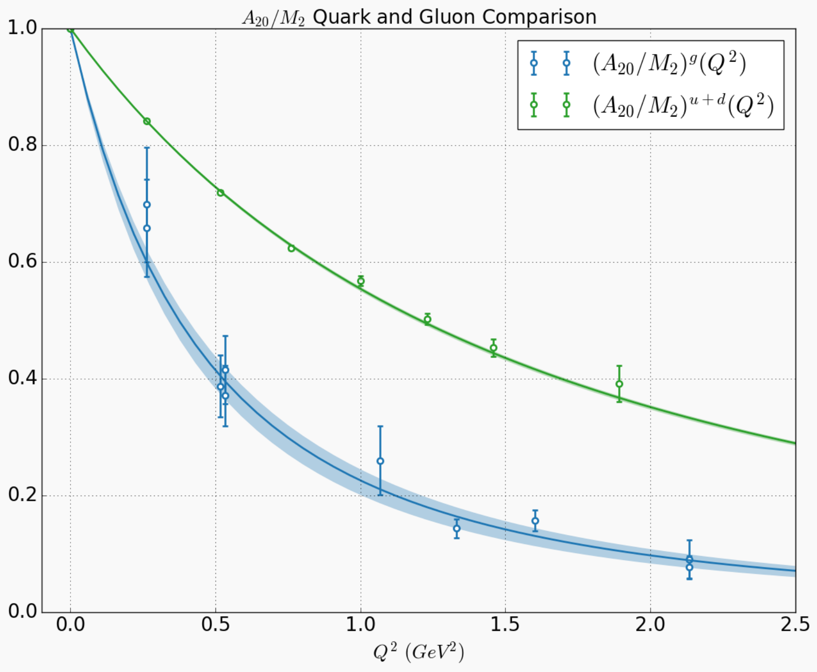
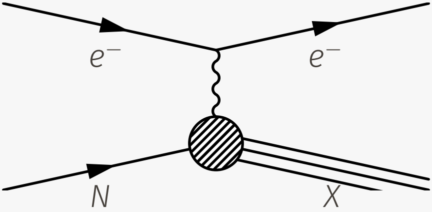
$$1 = \sum_q \langle x \rangle_q + \langle x \rangle_g$$

$\mu = 2\ GeV$	$B_{20}\,(0)$	J
$(u+d)_{con}$	≈ 0	0.3125(32)
gluon	0.00327(68)	0.209(29)
Total	-	0.522(29)


A_{20}/M_2 Quark and Gluon Comparison
$(A_{20}/M_2)^g(Q^2)$
$(A_{20}/M_2)^{u+d}(Q^2)$
1.0
0.8
0.6
0.4
0.2
0.0
0.0
0.5
1.0
1.5
2.0
2.5
Q^2 (GeV^2)

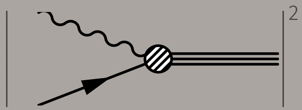
QUARKS

DEEP INELASTIC SCATTERING



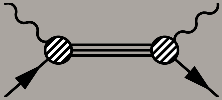
$$\omega = \frac{2p \cdot Q}{Q^2} = \frac{m_X^2 - m_N^2}{Q^2} + 1$$

Hadron Tensor



→ Hadron Tensor has structure functions F_1, F_2

Compton Amplitude

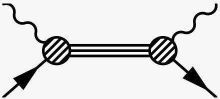


→ Compton Amplitude has Lorenzt-scalar functions T_1, T_2

$$F_i = \frac{1}{2\pi} Im T_i$$

COMPTON AMPLITUDE IN EXPERIMENT

$$T_{\mu\nu} = \rho_{ss'} \int d^4\xi e^{iq\cdot\xi} \langle p,s' \,|\, TJ_\mu(\xi)J_\nu(0) \,|p,s\rangle$$



COMPTON AMPLITUDE IN EXPERIMENT

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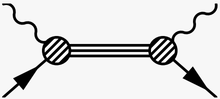

→ Two photon exchange part of Hydrogen spectroscopic transition's contribution to proton charge radius uncertainty

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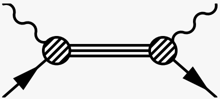

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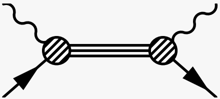

- → Two photon exchange part of Hydrogen spectroscopic transition's contribution to proton charge radius uncertainty
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- → Subtraction term $T(\omega = 0, Q^2)$

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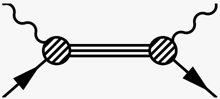

→ Two photon exchange part of Hydrogen spectroscopic transition's contribution to proton charge radius uncertainty

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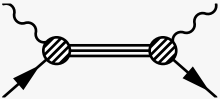

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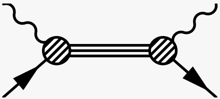

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- → Unpolarised Compton amplitude proton neutron difference contribution to P-N mass splitting
- → Subtraction term $T(\omega = 0, Q^2)$
    - Muonic hydrogen lamb shift uncertainty
    - P-N self energy uncertainty
    - Reggeon dominance hypothesis

COMPTON AMPLITUDE

$$
\begin{aligned}
T_{\mu\nu} &= \rho_{ss'} \int d^4\xi e^{iq\cdot\xi} \langle p, s' \,|\, TJ_\mu(\xi) J_\nu(0) \,| p, s\rangle \\
&= \left(-g_{\mu\nu} + \frac{q_\mu q_\nu}{q^2}\right) \boldsymbol{T_1} + \left(p_\mu - \frac{1}{2}\omega q_\mu\right)\left(p_\nu - \frac{1}{2}\omega q_\nu\right) \frac{\boldsymbol{T_2}}{\nu} \\
&+ \epsilon_{\mu\nu\alpha\beta} q^\alpha \left[\frac{s^\beta}{\nu} \boldsymbol{G_1} + \frac{\nu M s^\beta - s\cdot q p^\beta}{\nu^2} \boldsymbol{G_2}\right]
\end{aligned}
$$

where

$$
\nu = p \cdot Q
$$

$$
\omega = \frac{2p \cdot Q}{Q^2}
$$

COMPTON AMPLITUDE

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where

$$\nu = p \cdot Q$$
$$\omega = \frac{2p\cdot Q}{Q^2}$$

restrict to a subset

$$p_3 = q_3 = 0$$
$$\mu = \nu = 3$$
$$\rho = \frac{1}{2}\mathbb{I}$$

COMPTON AMPLITUDE

$$T_{33} = \int d^4\xi e^{iq\cdot\xi} \langle p, s' \,|\, TJ_3(\xi) J_3(0) \,| p, s\rangle$$
$$= T_1\left(\omega, Q^2\right)$$

where

$$\nu = p \cdot Q$$
$$\omega = \frac{2p \cdot Q}{Q^2}$$

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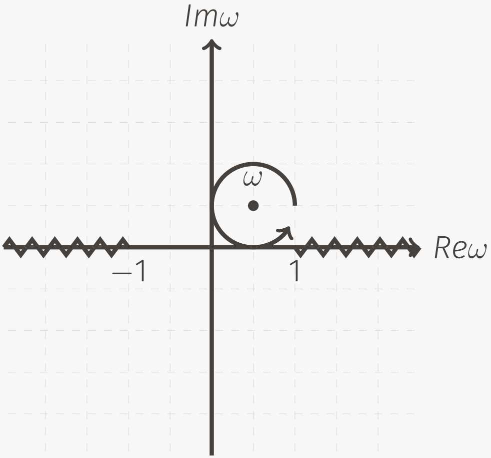
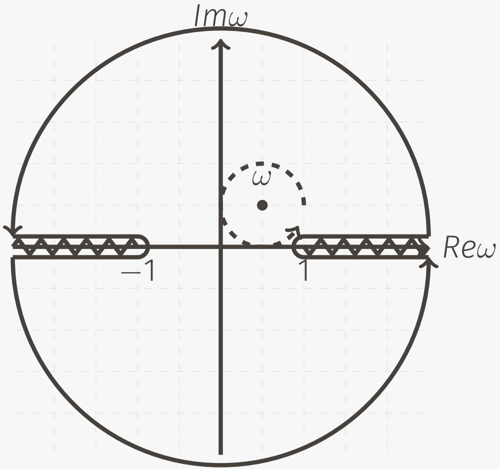
ANALYTIC STRUCTURE OF COMPTON AMPLITUDE



$$|\omega|^2 < 1$$

$$T\left(\omega, Q^2\right)$$



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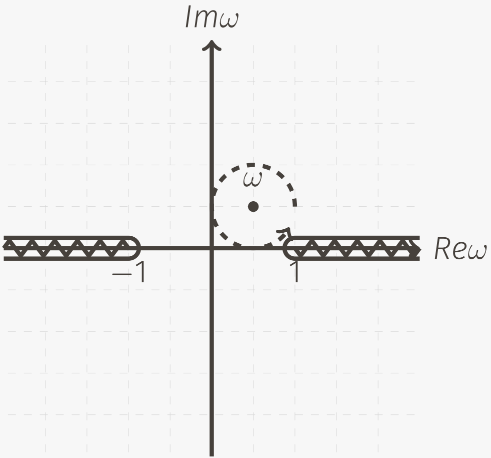
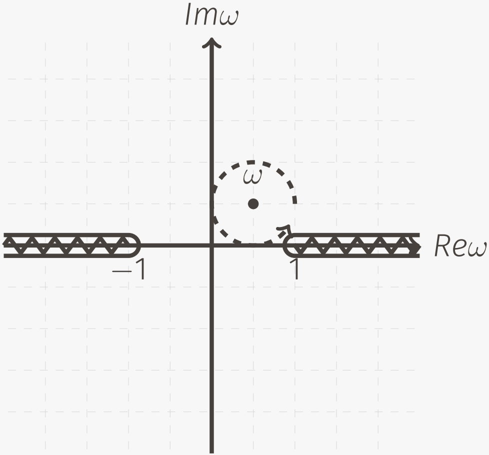


$$|\omega|^2 < 1$$

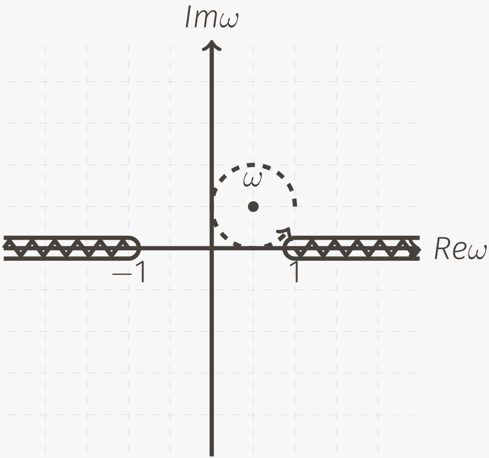
$$T\left(\omega, Q^2\right) - T\left(0, Q^2\right)$$

ANALYTIC STRUCTURE OF COMPTON AMPLITUDE



$$|\omega|^2 < 1$$

$$T\left(\omega, Q^2\right) - T\left(0, Q^2\right)$$
$$= \frac{4\omega^2}{2\pi} \int_1^\infty d\omega' \frac{ImT_1\left(\omega', Q^2\right)}{\omega'\left(\omega^2 - \omega'^2\right)}$$



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$$= 4\omega^2 \int_0^1 dxx \frac{F_1\left(x, Q^2\right)}{1 - (\omega x)^2}$$

$$\omega' = \frac{1}{x}$$

MOMENTS

Have

$$T_1\left(\omega, Q^2\right) - T_1\left(0, Q^2\right) = 4\omega^2 \int_0^1 dx \frac{xF_1\left(x, Q^2\right)}{1 - (\omega x)^2}$$

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use geometric sum

$$= \sum_n 4\omega^{2n} \int_0^1 dx x^{2n-1} F_1\left(x, Q^2\right)$$

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$$F_2\left(x, Q^2\right) = e_q^2 x\left(q\left(x\right) + \overline{q}\left(x\right)\right) \qquad \text{(Parton Model)}$$

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so

$$= \sum_n 2\omega^{2n} \left\langle x^{2n-1} \right\rangle$$

Let's take it to lattice QCD

LATTICE SITUATION

→ »Just« calculate the four point function
$\int d^3x d^4y d^4z e^{i\mathbf{p}\cdot\mathbf{x}} e^{iq\cdot y} e^{-iq\cdot z} \left\langle \chi(x)\, T\{J_\mu(y)\, J_\nu(z)\}\, \chi^\dagger(0) \right\rangle$

LATTICE SITUATION

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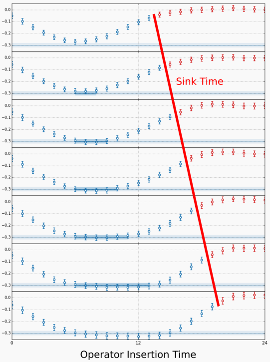
Just remember three point functions from our gluons and do it in an extra dimension



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→ Other techniques to get at PDFs
- Heavy quark currents
- Current-current correlators
- Quasi-PDFs
- Pseudo-PDFs
- “Good Lattice Cross Sections”

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- → Other techniques to get at PDFs
  - Heavy quark currents
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  - Quasi-PDFs
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  - “Good Lattice Cross Sections”
- → Complementary new method using Feynman-Hellmann theorem

FEYNMAN-HELLMANN THEOREM

→ Calculate matrix elements using two point methods, ie. energy eigenstates

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Modify Action

$$S \to S + \lambda \int d^4x \ \cos(\mathbf{q} \cdot \mathbf{x}) J(x)$$

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Modify Action

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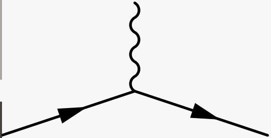
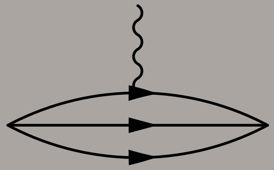
Feynman-Hellmann Theorem

$$\left.\frac{dE_{X,\mathbf{p}}}{d\lambda}\right|_{\lambda=0} = \frac{1}{2E_{X,\mathbf{p}}} \langle X, \mathbf{p} \,|\, J(0)\, | X, \mathbf{p} \pm \mathbf{q} \rangle$$



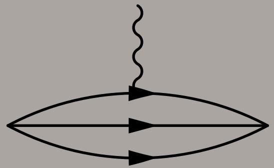
Connected



- → Modify quark propagator
- → High correlation for different λ

LATTICE CAVEAT

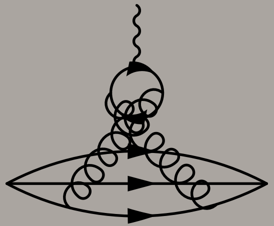
Connected



→ Modify quark propagator

→ High correlation for different λ

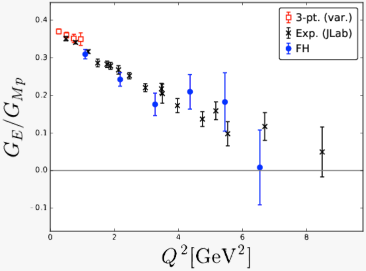
Disconnected

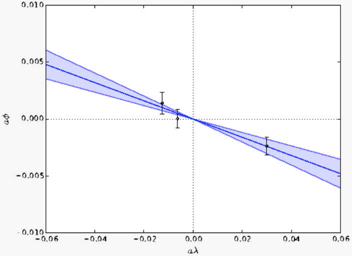


→ Modify weighting

→ No correlation for different λ

THINGS FEYNMAN-HELLMANN CAN DO





→ Large momentum EM form factors

→ Disconnected spin contribution to nucleon spin

FHT of 2nd Order

FHT SETUP

- $\rightarrow$ Two point function now dependent on λ.
- $\rightarrow$ Simplify calculation $\rightarrow$ no excited states,
  $\omega \neq 1 \implies \frac{\partial E_p}{\partial \lambda} = 0$

$$\int e^{-i\mathbf{p}\cdot\mathbf{x}} \left\langle \chi\left(x\right) \chi^{\dagger}\left(0\right) \right\rangle \approx A_p\left(\lambda\right) e^{-E_p(\lambda) x_4}$$

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→ Differentiate RHS

$$\left[\frac{\partial A_p}{\partial \lambda} - x_4 A_p \frac{\partial E_p}{\partial \lambda} \right] e^{-E_p(\lambda) x_4}$$

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- $\rightarrow$ Differentiate RHS

$$\left[\frac{\partial A_p}{\partial \lambda} - x_4 A_p \frac{\partial E_p}{\partial \lambda}\right] e^{-E_p(\lambda) x_4}$$

- $\rightarrow$ Twice

$$\left[\frac{\partial^2 A_p}{\partial \lambda^2} - \cancel{2x_4 \frac{\partial A_p}{\partial \lambda}\frac{\partial E_p}{\partial \lambda}} - x_4 A_p \frac{\partial^2 E_p}{\partial \lambda^2} + \cancel{x_4^2 A_p \left(\frac{\partial E_p}{\partial \lambda}\right)^2}\right] e^{-E_p(\lambda) x_4}$$

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→ Simplify calculation → no excited states,
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LHS

→ Differentiate LHS $\int e^{-i\mathbf{p}\cdot\mathbf{x}} \left\langle \chi(x)\, \chi^{\dagger}(0) \right\rangle$ twice, ignoring disconnected terms

$$\int e^{-i\mathbf{p}\cdot\mathbf{x}} \left\langle \chi(x)\, \chi^{\dagger}(0) \left(\frac{\partial S}{\partial \lambda}\right)^{2} \right\rangle$$

LHS

→ Differentiate LHS $\int e^{-i\mathbf{p}\cdot\mathbf{x}} \langle \chi(x)\,\chi^{\dagger}(0) \rangle$ twice, ignoring disconnected terms

$$\int e^{-i\mathbf{p}\cdot\mathbf{x}} \left\langle \chi(x)\,\chi^{\dagger}(0) \left(\frac{\partial S}{\partial \lambda} \right)^{2} \right\rangle$$

→ Pick $\Delta S(\lambda) = \int d^4y\, 2\cos(\mathbf{q}\cdot\mathbf{y})\, J(y)$

LHS

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$$\int e^{-i\mathbf{p}\cdot\mathbf{x}} \left\langle \chi(x)\, \chi^{\dagger}(0) \left(\frac{\partial S}{\partial \lambda} \right)^2 \right\rangle$$

→ Pick $\Delta S(\lambda) = \int d^4y\, 2\cos(\mathbf{q}\cdot\mathbf{y})\, J(y)$

→ Current outside operator has e^{-Ex_4} like time dependence $\implies$ corresponds to $\frac{\partial^2 A_p}{\partial \lambda^2}$

LHS

- $\rightarrow$ Differentiate LHS $\int e^{-i\mathbf{p}\cdot\mathbf{x}} \langle \chi(x) \chi^\dagger(0) \rangle$ twice, ignoring disconnected terms

$$\int e^{-i\mathbf{p}\cdot\mathbf{x}} \left\langle \chi(x) \chi^\dagger(0) \left(\frac{\partial S}{\partial \lambda}\right)^2 \right\rangle$$

- $\rightarrow$ Pick $\Delta S(\lambda) = \int d^4y \, 2\cos(\mathbf{q}\cdot\mathbf{y}) J(y)$
- $\rightarrow$ Current outside operator has e^{-Ex_4} like time dependence $\implies$ corresponds to $\frac{\partial^2 A_p}{\partial \lambda^2}$
- $\rightarrow$ Current inbetween operators has $x_4 e^{-Ex_4}$ like time dependence $\implies$ corresponds to $\frac{\partial^2 E_p}{\partial \lambda^2}$

LHS (CONT.)

→ LHS now

$$\int e^{-i\mathbf{p}\cdot\mathbf{x}} \int_0^{x_4} y_4 \int_0^{x_4} z_4 \int d^3y d^3z \, 4 \cos(\mathbf{q}\cdot\mathbf{y}) \cos(\mathbf{q}\cdot\mathbf{z}) \times \left\langle \chi(x) \, T\{J(y) J(z)\} \, \chi^\dagger(0) \right\rangle$$

LHS (CONT.)

→ LHS now

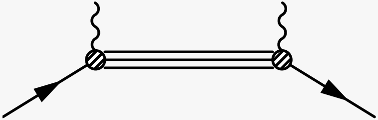
$$\int e^{-i\mathbf{p}\cdot\mathbf{x}} \int_0^{x_4} y_4 \int_0^{x_4} z_4 \int d^3y d^3z \, 4\cos\left(\mathbf{q}\cdot\mathbf{y}\right)\cos\left(\mathbf{q}\cdot\mathbf{z}\right)$$
$$\times \left\langle \chi\left(x\right) T\left\{J\left(y\right) J\left(z\right)\right\} \chi^\dagger\left(0\right)\right\rangle$$

→ Which becomes

$$\frac{A_p}{2E_p} x_4 e^{-E_p x_4} \int d^4\xi \, 2\cos\left(\mathbf{q}\cdot\xi\right) \left\langle p \,\middle|\, T\left\{J\left(\xi\right) J\left(0\right)\right\} \middle| p\right\rangle$$

Second Order FHT

$$\left.\frac{d^2E}{d\lambda^2}\right|_{\lambda=0} = -\frac{\left\langle p \,\middle|\, \int d^4\xi \; 2\cos\left(\mathbf{q}\cdot\xi\right) TJ\left(\xi\right) J\left(0\right) \middle|\, p\right\rangle}{2E}$$



$\rightarrow$ Need to renormalise T_{33}

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$$\boxed{T_{33}^{phys} = Z_V^2 T_{33}^{latt}}$$

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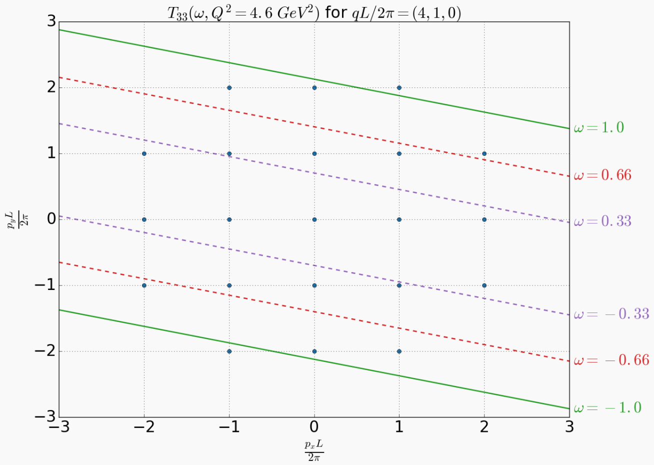
$$\boxed{T_{33}^{phys} = Z_V^2 T_{33}^{latt}}$$

Same renormalisation as for electro-magnetic form factor

DISCRETISED MOMENTUM



$$\rightarrow\ S \rightarrow S + \lambda \int d^4x\ \left(e^{iq\cdot x} + e^{-iq\cdot x}\right) \overline{q}\gamma_3 q$$

- $\rightarrow$ $S \rightarrow S + \lambda \int d^4x \; \left(e^{iq\cdot x} + e^{-iq\cdot x}\right) \overline{q}\gamma_3 q$
- $\rightarrow$ Do this for $\lambda = 0.0, \pm 0.0125, \pm 0.025, \pm 0.0375$ and calculate corresponding two point functions $C(\lambda, t)$ at $m_\pi \approx 470 MeV$

THE STRUCTURE FUNCTION RECIPE

- $S \to S + \lambda \int d^4x \, \left(e^{iq\cdot x} + e^{-iq\cdot x}\right) \overline{q}\gamma_3 q$
- Do this for $\lambda = 0.0, \pm 0.0125, \pm 0.025, \pm 0.0375$ and calculate corresponding two point functions $C(\lambda, t)$ at $m_\pi \approx 470 MeV$
- Take advantage of correlation $\frac{C(\lambda,t)C(-\lambda,t)}{C(0,t)^2} \propto e^{-2\Delta E_{even}(\lambda)t}$

THE STRUCTURE FUNCTION RECIPE

- $\rightarrow$ $S \rightarrow S + \lambda \int d^4x \; \left(e^{iq\cdot x} + e^{-iq\cdot x}\right) \overline{q}\gamma_3 q$
- $\rightarrow$ Do this for $\lambda = 0.0, \pm 0.0125, \pm 0.025, \pm 0.0375$ and calculate corresponding two point functions $C(\lambda, t)$ at $m_\pi \approx 470 MeV$
- $\rightarrow$ Take advantage of correlation $\frac{C(\lambda,t)C(-\lambda,t)}{C(0,t)^2} \propto e^{-2\Delta E_{even}(\lambda)t}$
- $\rightarrow$ $\Delta E_{even} = \frac{1}{2!}\lambda^2 \frac{d^2E}{d\lambda^2} + \frac{1}{4!}\lambda^4 \frac{d^4E}{d\lambda^4} + \cdots$

THE STRUCTURE FUNCTION RECIPE

- $S \to S + \lambda \int d^4x \, \left(e^{iq\cdot x} + e^{-iq\cdot x}\right) \overline{q}\gamma_3 q$
- Do this for $\lambda = 0.0, \pm 0.0125, \pm 0.025, \pm 0.0375$ and calculate corresponding two point functions $C\left(\lambda, t\right)$ at $m_\pi \approx 470 MeV$
- Take advantage of correlation $\frac{C(\lambda,t)C(-\lambda,t)}{C(0,t)^2} \propto e^{-2\Delta E_{even}(\lambda)t}$
- $\Delta E_{even} = \frac{1}{2!}\lambda^2 \frac{d^2E}{d\lambda^2} + \frac{1}{4!}\lambda^4 \frac{d^4E}{d\lambda^4} + \cdots$
- $\frac{d^2E}{d\lambda^2} \propto T_{33}\left(\omega = \frac{2\mathbf{p}\cdot\mathbf{q}}{Q^2}, Q^2\right)$

THE STRUCTURE FUNCTION RECIPE

- $\rightarrow$ $S \rightarrow S + \lambda \int d^4x \; \left(e^{iq\cdot x} + e^{-iq\cdot x}\right) \overline{q}\gamma_3 q$
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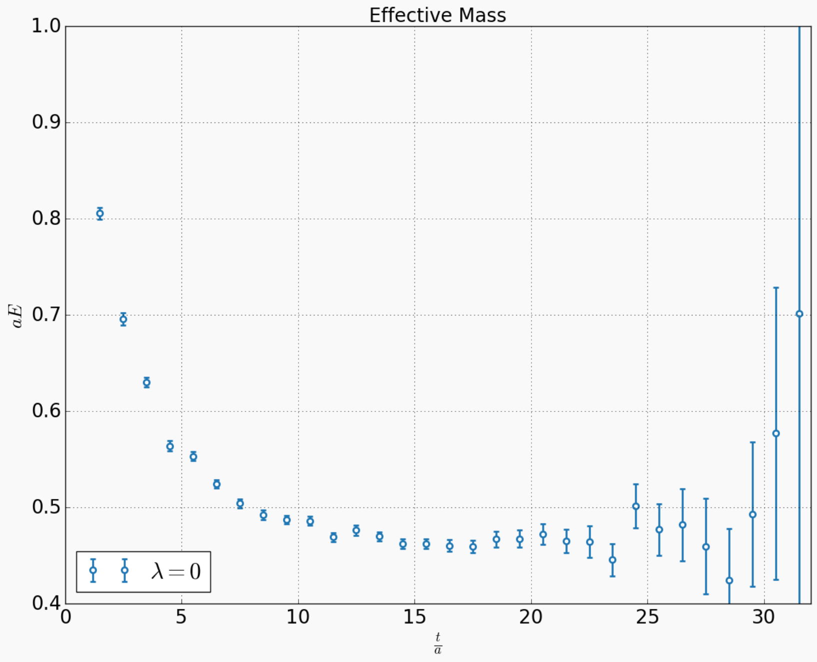
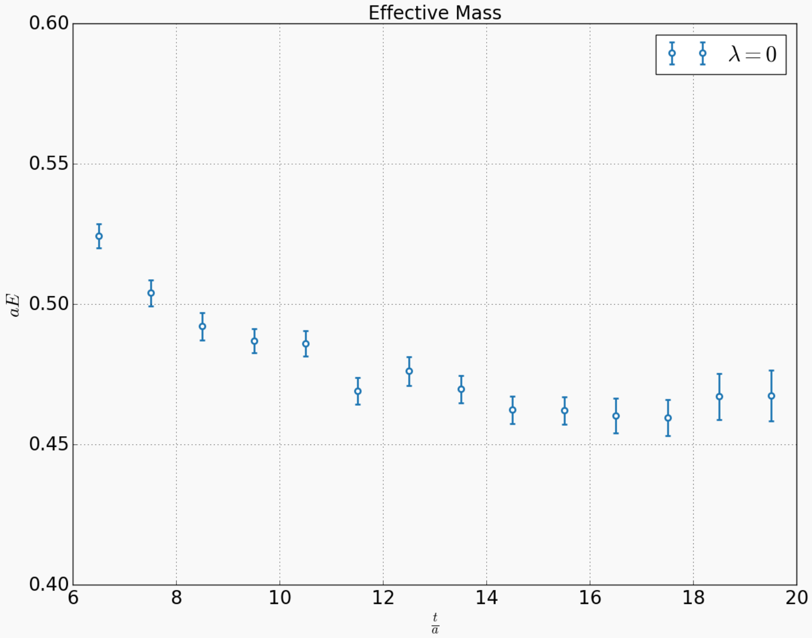
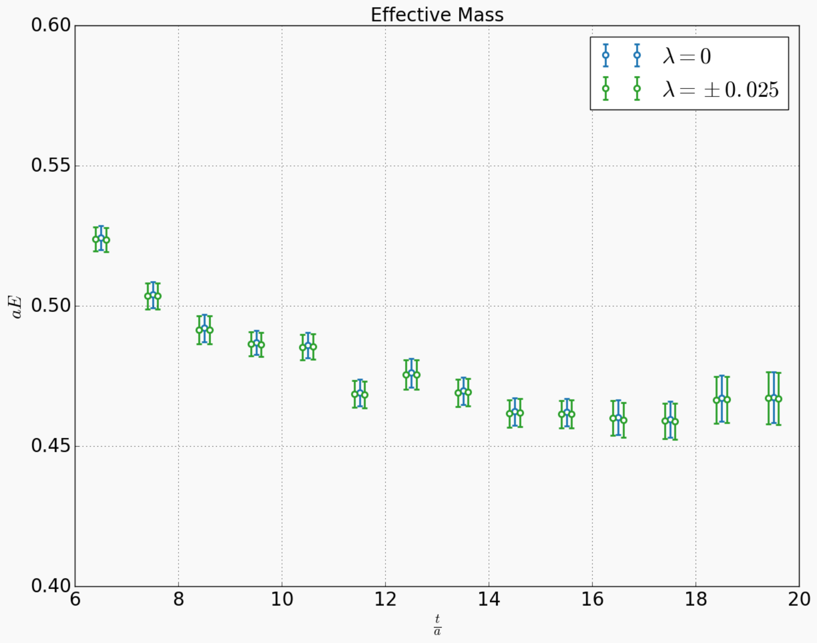
- $\rightarrow$ $\frac{d^2E}{d\lambda^2} \propto T_{33}\left(\omega = \frac{2\mathbf{p}\cdot\mathbf{q}}{Q^2}, Q^2\right)$
- $\rightarrow$ take $T_{33}^{uu} - T_{33}^{dd}$ and fit vs ω to get moments of $\langle x \rangle^{u-d}$

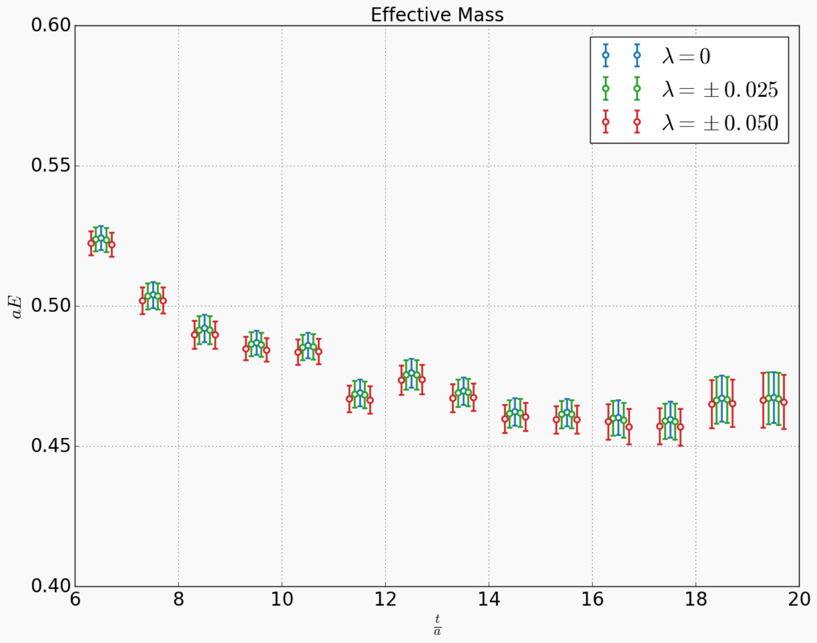


Effective Mass
1.0
0.9
0.8
0.7
0.6
0.5
0.4
aE
0
5
10
15
20
25
30
$\frac{t}{a}$
$\lambda = 0$

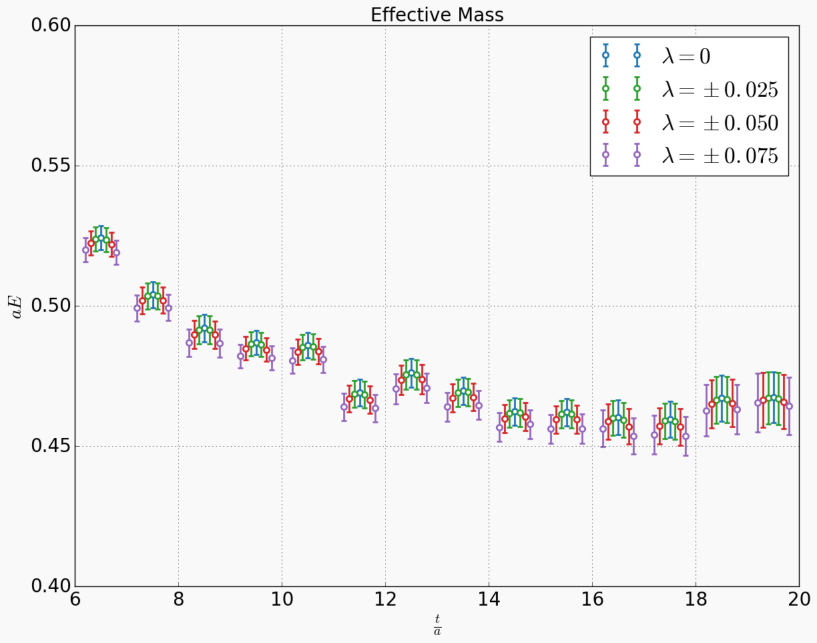


Effective Mass
0.60
0.55
0.50
0.45
0.40
aE
6
8
10
12
14
16
18
20
$\frac{t}{a}$
$\lambda = 0$

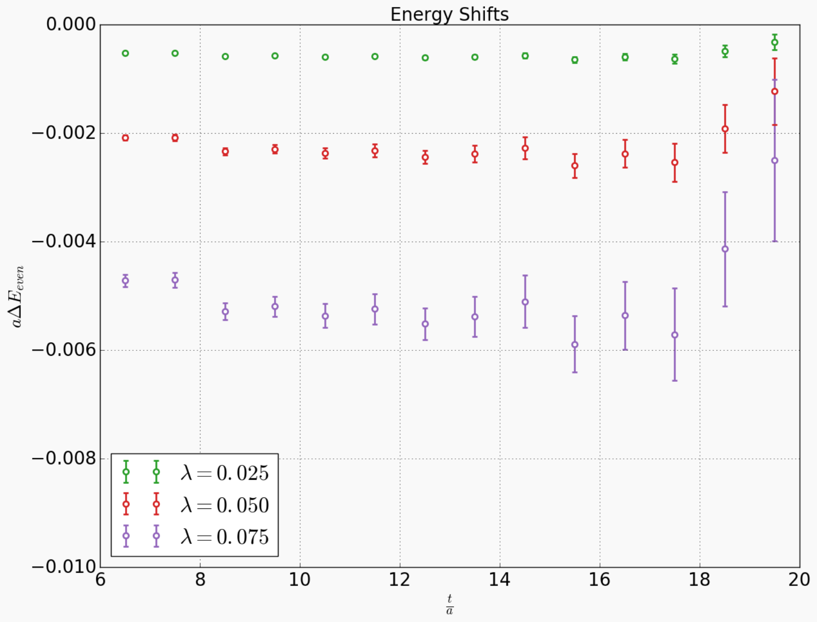

Effective Mass
aE
0.60
0.55
0.50
0.45
0.40
6
8
10
12
14
16
18
20
$\frac{t}{a}$
$\lambda = 0$
$\lambda = \pm 0.025$



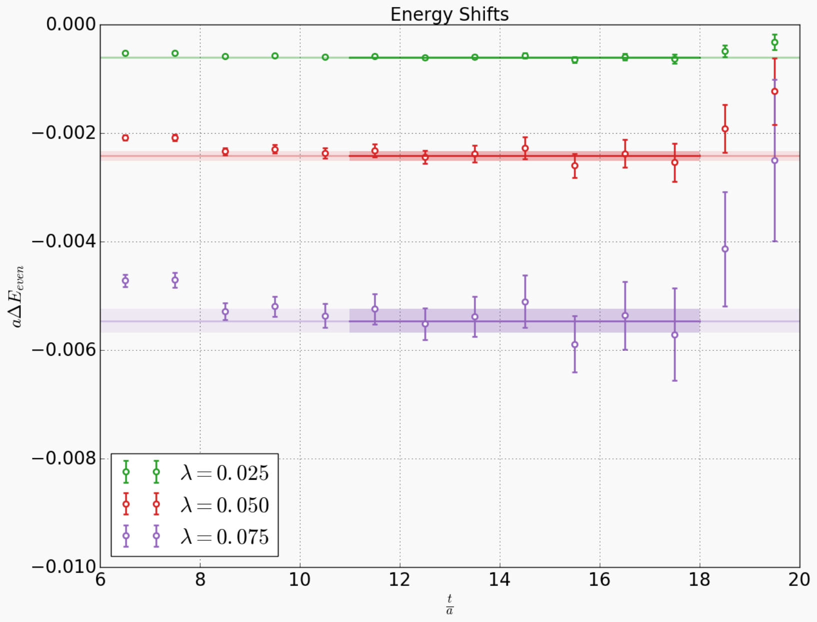
Effective Mass
0.60
0.55
0.50
0.45
0.40
aE
6
8
10
12
14
16
18
20
$\frac{t}{a}$
$\lambda = 0$
$\lambda = \pm 0.025$
$\lambda = \pm 0.050$

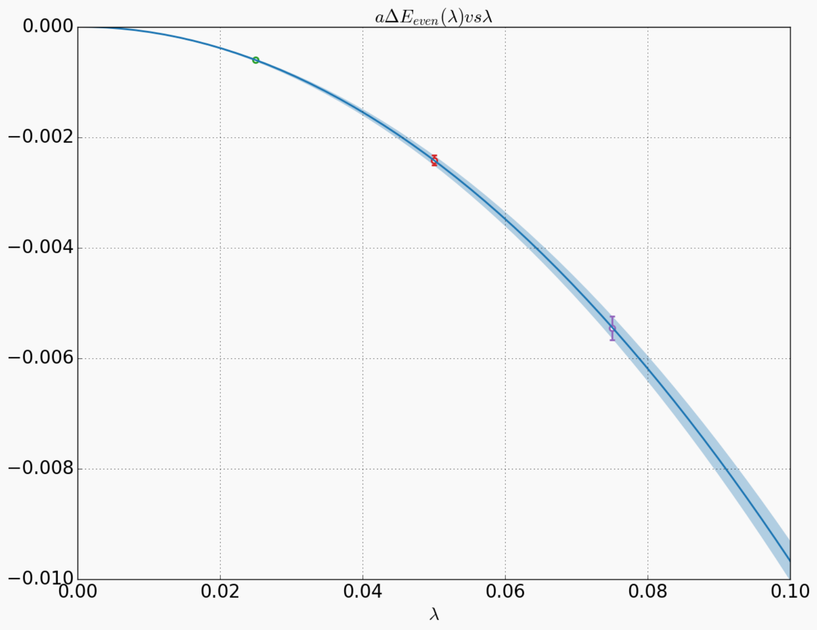


Effective Mass
$\lambda = 0$
$\lambda = \pm 0.025$
$\lambda = \pm 0.050$
$\lambda = \pm 0.075$
aE
$\frac{t}{a}$
0.60
0.55
0.50
0.45
0.40
6
8
10
12
14
16
18
20



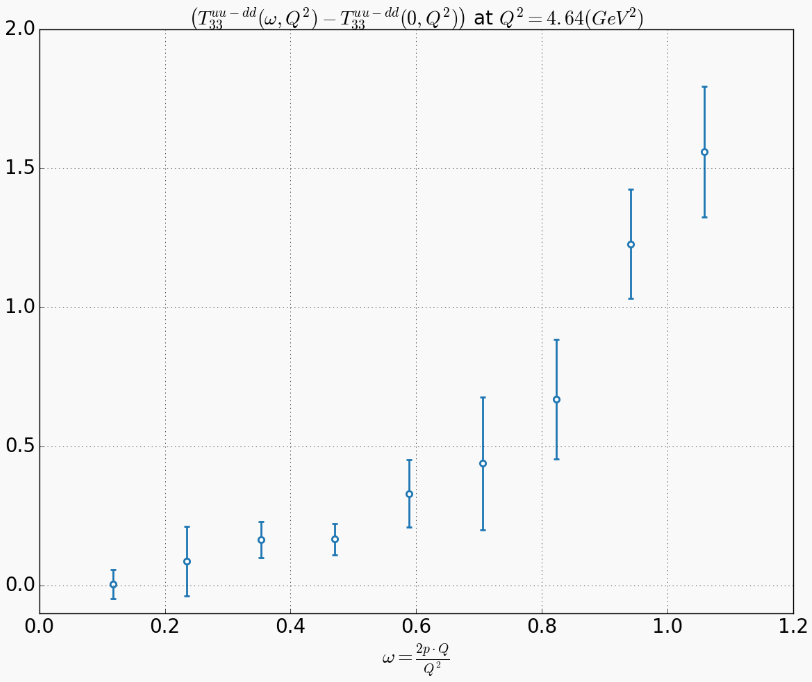
Energy Shifts
0.000
−0.002
−0.004
−0.006
−0.008
−0.010
$a\Delta E_{even}$
6
8
10
12
14
16
18
20
$\frac{t}{a}$
$\lambda = 0.025$
$\lambda = 0.050$
$\lambda = 0.075$


Energy Shifts
0.000
−0.002
−0.004
−0.006
−0.008
−0.010
6
8
10
12
14
16
18
20
$a\Delta E_{even}$
$\frac{t}{a}$
$\lambda = 0.025$
$\lambda = 0.050$
$\lambda = 0.075$

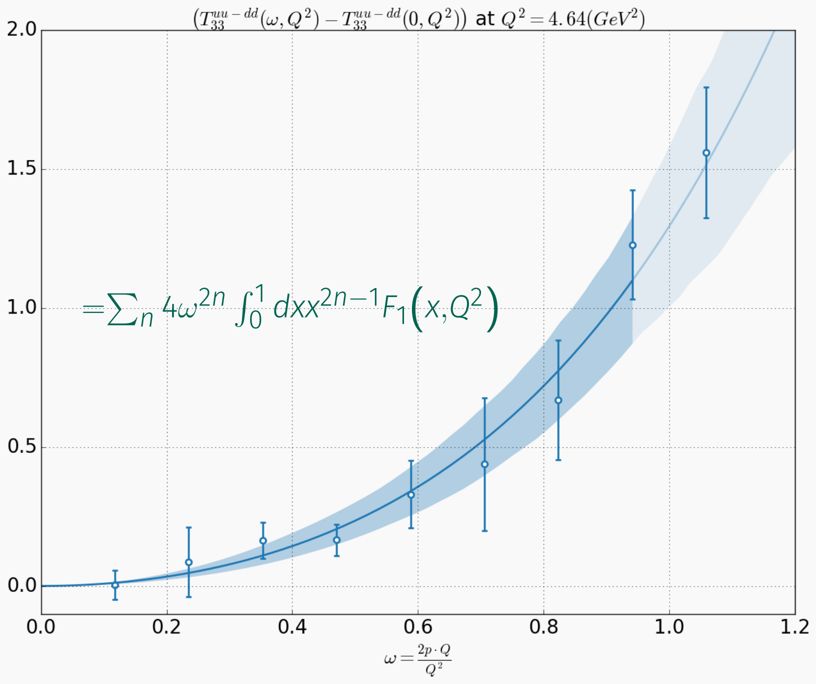


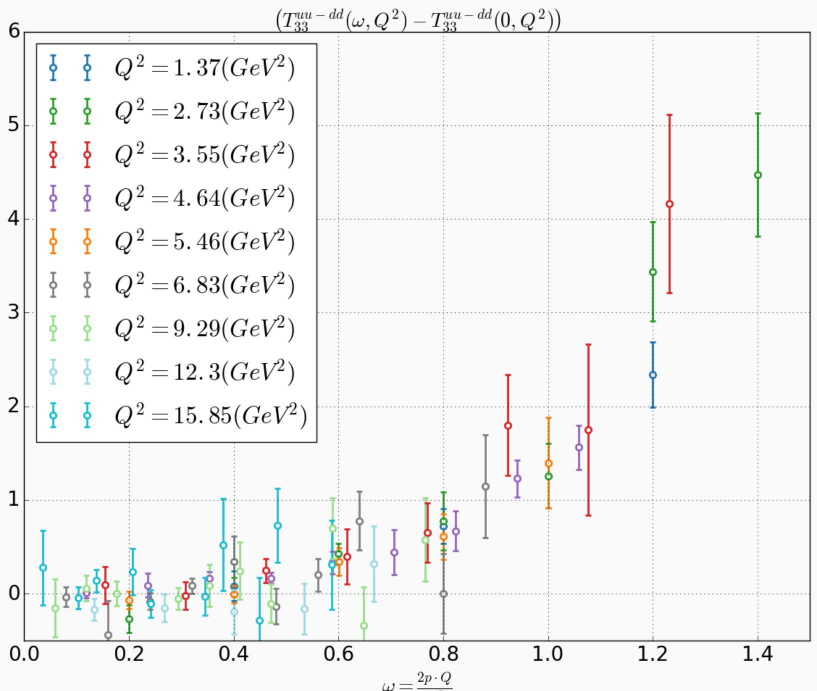
$a\Delta E_{even}(\lambda) vs \lambda$
0.000
−0.002
−0.004
−0.006
−0.008
−0.010
0.00
0.02
0.04
0.06
0.08
0.10
λ

79 **TOTAL SUCH FITS**

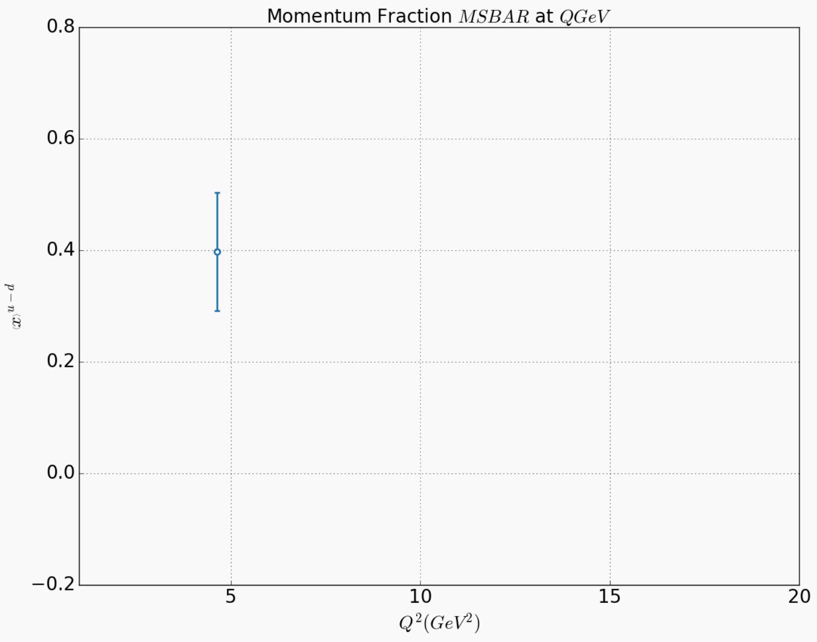


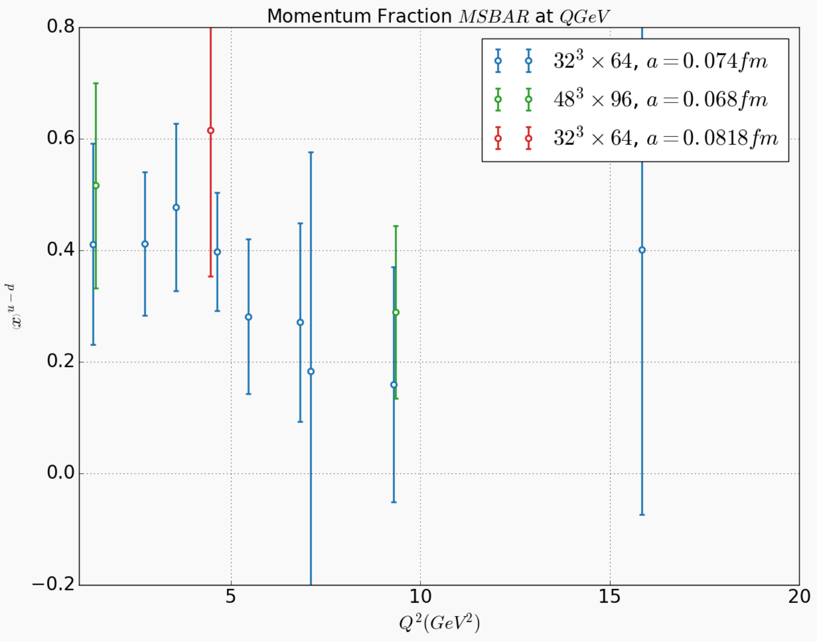
$\left(T_{33}^{uu-dd}(\omega,Q^2) - T_{33}^{uu-dd}(0,Q^2)\right)$ at $Q^2 = 4.64(GeV^2)$
2.0
1.5
1.0
0.5
0.0
0.0
0.2
0.4
0.6
0.8
1.0
1.2
$\omega = \frac{2p \cdot Q}{Q^2}$


$(T_{33}^{uu-dd}(\omega, Q^2) - T_{33}^{uu-dd}(0, Q^2))$ at $Q^2 = 4.64 (GeV^2)$
$= \sum_n 4\omega^{2n} \int_0^1 dx x^{2n-1} F_1(x, Q^2)$
2.0
1.5
1.0
0.5
0.0
0.0
0.2
0.4
0.6
0.8
1.0
1.2
$\omega = \frac{2p \cdot Q}{Q^2}$

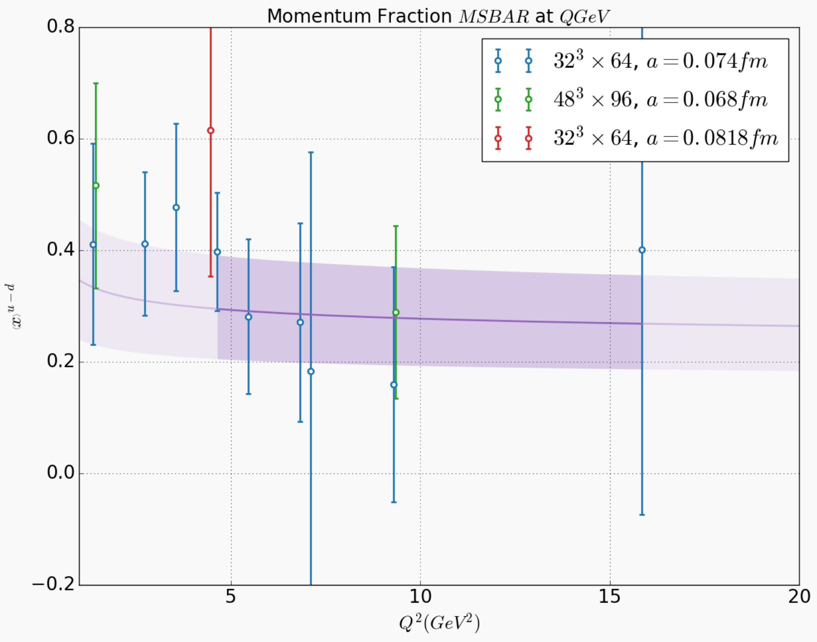


$(T_{33}^{uu-dd}(\omega, Q^2) - T_{33}^{uu-dd}(0, Q^2))$
$Q^2 = 1.37 (GeV^2)$
$Q^2 = 2.73 (GeV^2)$
$Q^2 = 3.55 (GeV^2)$
$Q^2 = 4.64 (GeV^2)$
$Q^2 = 5.46 (GeV^2)$
$Q^2 = 6.83 (GeV^2)$
$Q^2 = 9.29 (GeV^2)$
$Q^2 = 12.3 (GeV^2)$
$Q^2 = 15.85 (GeV^2)$
0
1
2
3
4
5
6
0.0
0.2
0.4
0.6
0.8
1.0
1.2
1.4

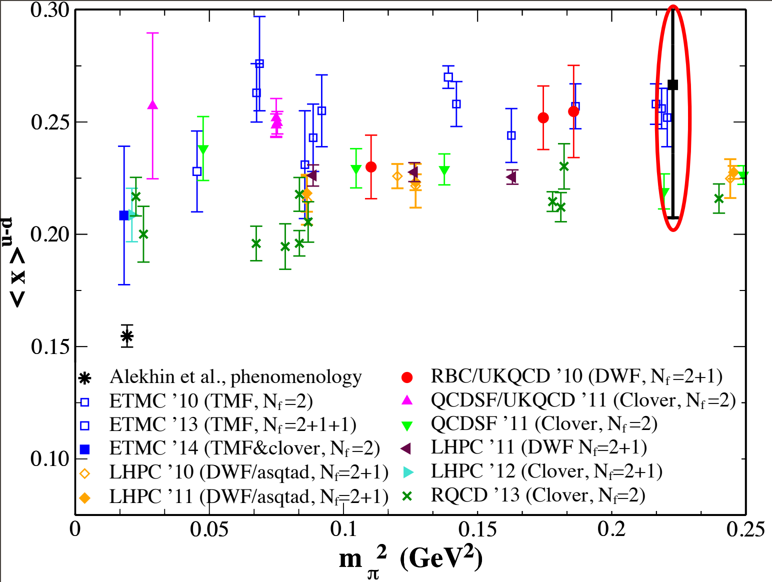

Momentum Fraction $MSBAR$ at $QGeV$
0.8
0.6
0.4
0.2
0.0
−0.2
$\langle x \rangle_{u-d}$
5
10
15
20
$Q^2(GeV^2)$



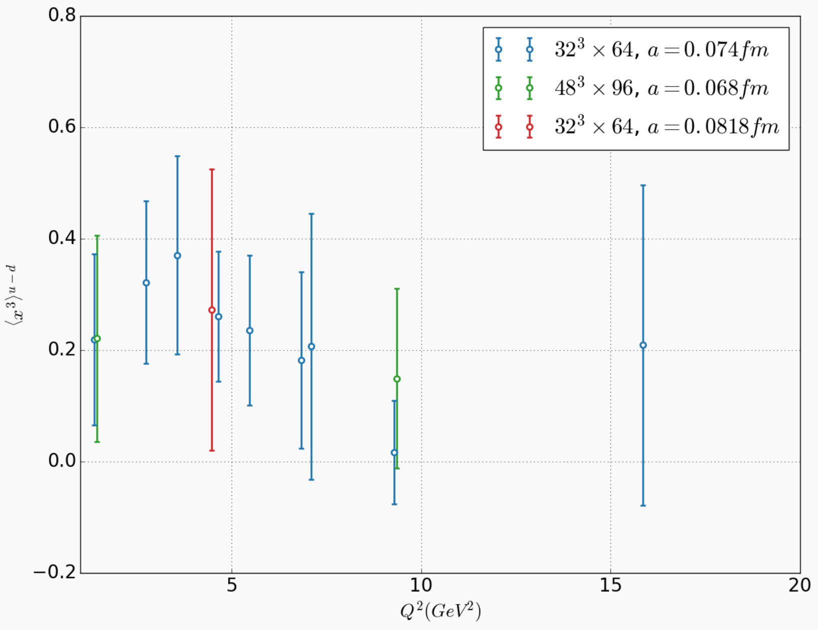
Momentum Fraction $MSBAR$ at $QGeV$
$32^3 \times 64$, $a = 0.074 fm$
$48^3 \times 96$, $a = 0.068 fm$
$32^3 \times 64$, $a = 0.0818 fm$
$\langle x \rangle^{u-d}$
$Q^2 (GeV^2)$
0.8
0.6
0.4
0.2
0.0
−0.2
5
10
15
20



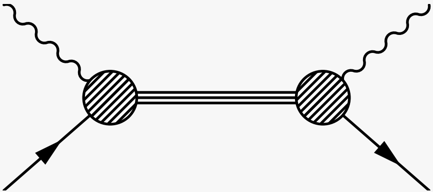
Momentum Fraction $MSBAR$ at $QGeV$
$32^3 \times 64$, $a = 0.074fm$
$48^3 \times 96$, $a = 0.068fm$
$32^3 \times 64$, $a = 0.0818fm$
$\langle x \rangle^{u-d}$
$Q^2(GeV^2)$
0.8
0.6
0.4
0.2
0.0
−0.2
5
10
15
20



Constantinou [arXiv: 1511.00214]



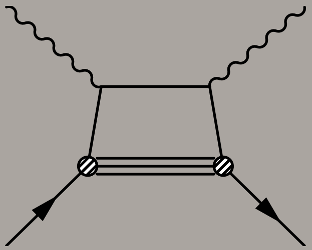
0.8
0.6
0.4
0.2
0.0
−0.2
5
10
15
20
$Q^2(GeV^2)$
$\langle x^3 \rangle_{u-d}$
$32^3 \times 64,\ a = 0.074 fm$
$48^3 \times 96,\ a = 0.068 fm$
$32^3 \times 64,\ a = 0.0818 fm$



→ Non local operator in terms of a series of local operators

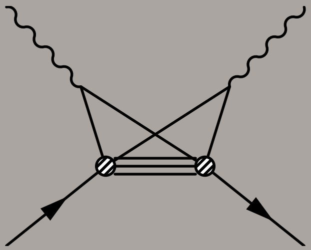
→ $\mathcal{N} = c_1\mathcal{O}_1 + \frac{c_2}{Q^2}\mathcal{O}_2 + \frac{c_3}{Q^4}\mathcal{O}_3 + \cdots$

"Handbag" Diagram



Twist 2 or Leading Twist

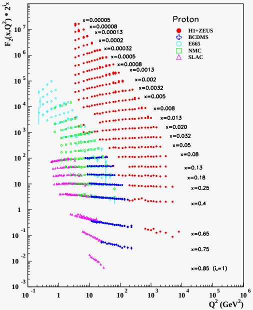
"Cat's Ears" Diagram



Higher Twist

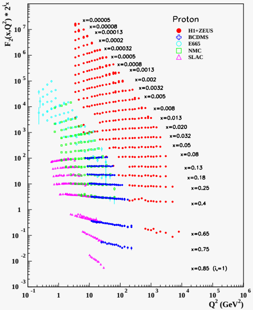
→ Experiment limited in quark content of targets (Ignoring heavier quark terms)

$$T^{proton} = \frac{4}{9}T^{uu} + \frac{1}{9}T^{dd} - \frac{2}{9}T^{ud+du}$$
$$T^{neutron} = \frac{1}{9}T^{uu} + \frac{4}{9}T^{dd} - \frac{2}{9}T^{ud+du}$$

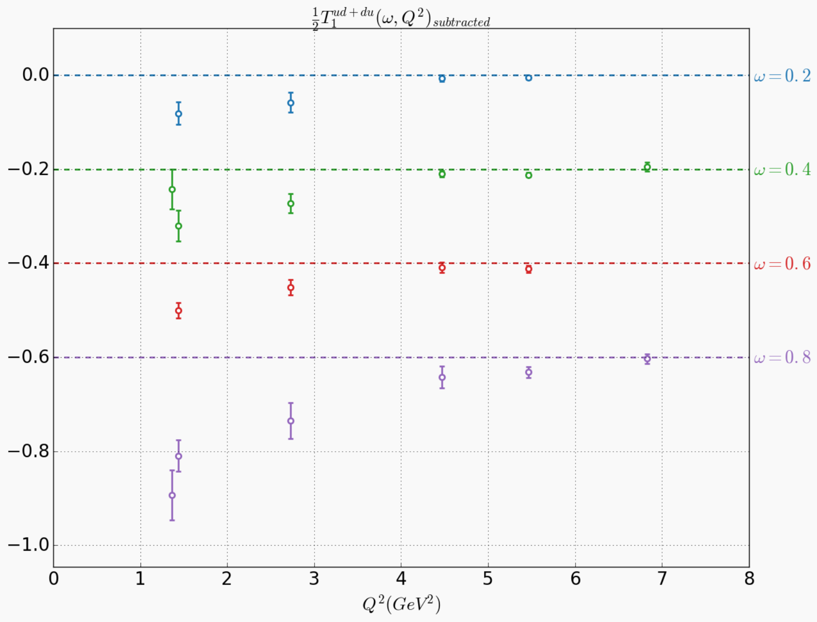


→ Experiment limited in quark content of targets (Ignoring heavier quark terms)

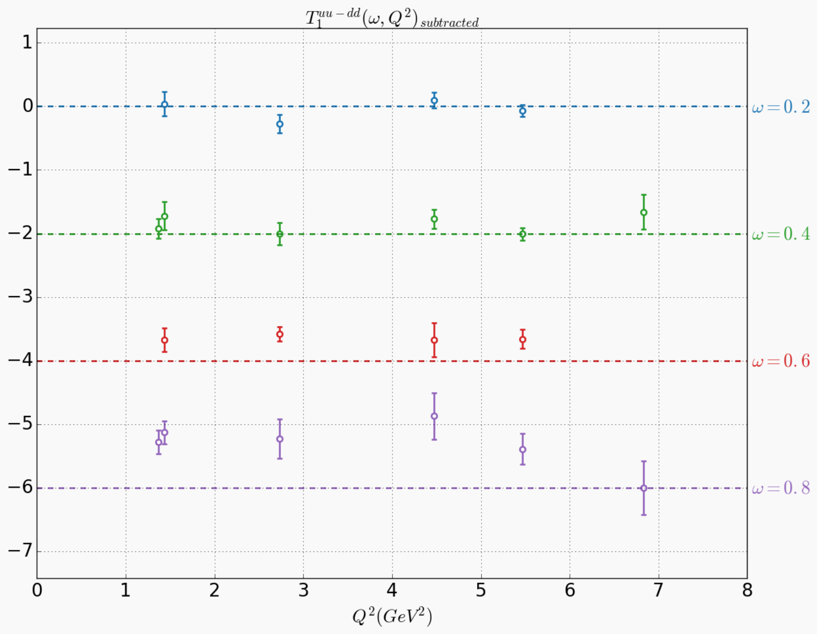
$$T^{proton} = \frac{4}{9}T^{uu} + \frac{1}{9}T^{dd} - \frac{2}{9}T^{ud+du}$$
$$T^{neutron} = \frac{1}{9}T^{uu} + \frac{4}{9}T^{dd} - \frac{2}{9}T^{ud+du}$$



→ Lattice extraction of more combinations T^{uu}, T^{dd}, $T^{uu} + T^{dd} + T^{ud+du}$, $T^{uu} + T^{dd} - T^{ud+du}$



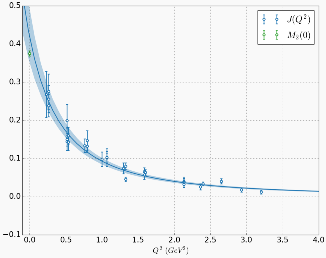
$\frac{1}{2}T_1^{ud+du}(\omega, Q^2)_{subtracted}$
0.0
−0.2
−0.4
−0.6
−0.8
−1.0
0
1
2
3
4
5
6
7
8
$Q^2(GeV^2)$
$\omega = 0.2$
$\omega = 0.4$
$\omega = 0.6$
$\omega = 0.8$

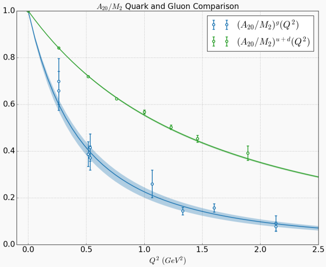


$T_1^{uu-dd}(\omega, Q^2)_{subtracted}$
1
0
−1
−2
−3
−4
−5
−6
−7
0
1
2
3
4
5
6
7
8
$Q^2(GeV^2)$
$\omega = 0.2$
$\omega = 0.4$
$\omega = 0.6$
$\omega = 0.8$

CONCLUSION

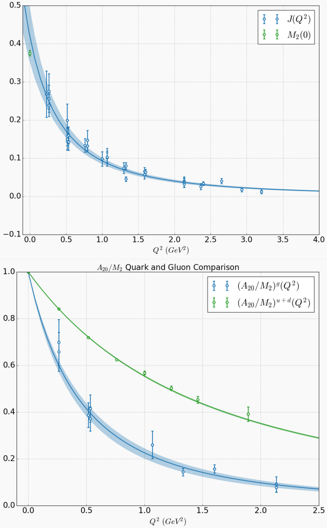
GLUON STRUCTURE





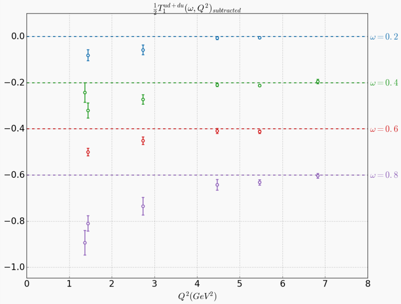
→ (A_{20}/M_2), J and B_{20} for a large range of Q^2 using Wilson flow

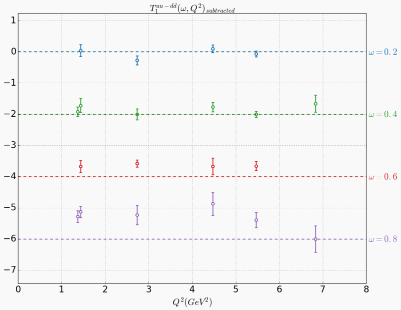
GLUON STRUCTURE



- → (A_{20}/M_2), J and B_{20} for a large range of Q^2 using Wilson flow
- → Want to extract (C_{20}/d_1) with the other operator

QUARK STRUCTURE





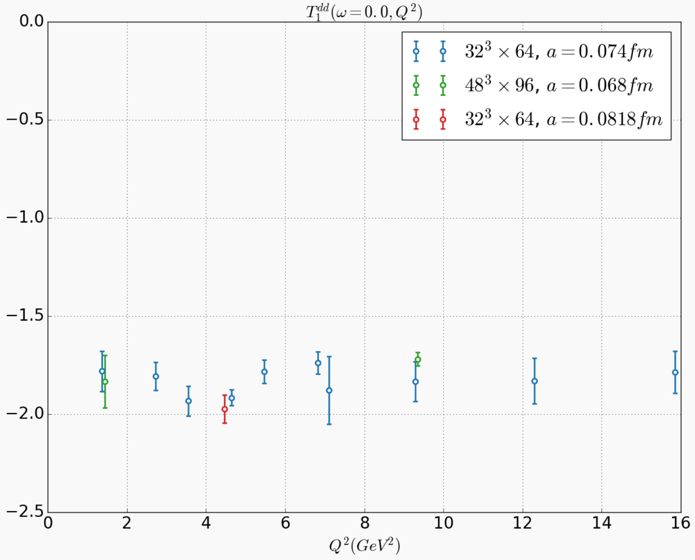
→ Looked at higher twist contributions, and quark flavour decomposition

→ Want to determine F_2, g_1 and g_2

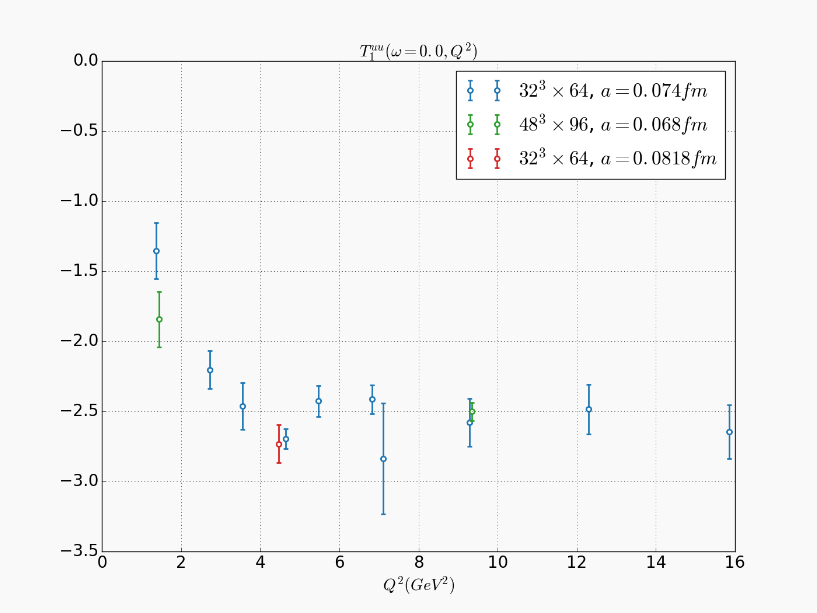
Thank You

BACKUP

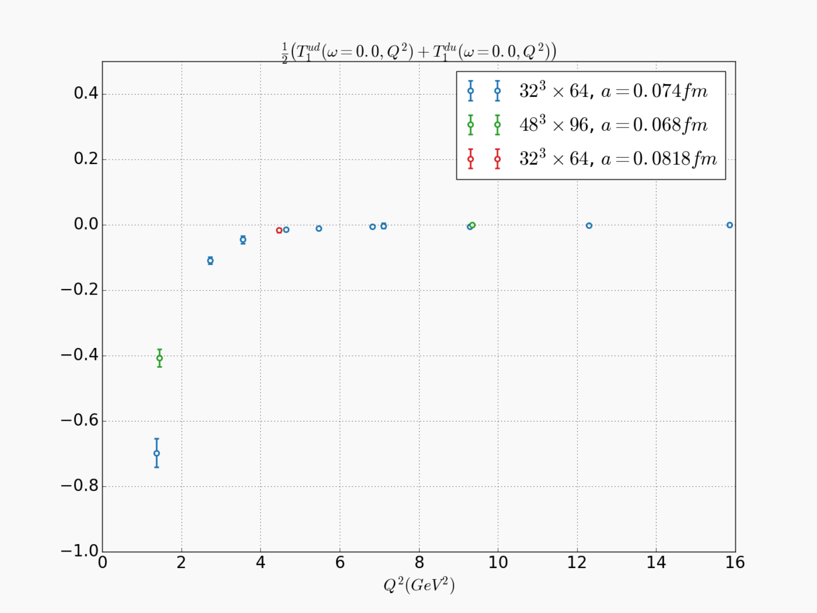
What about subtraction terms?



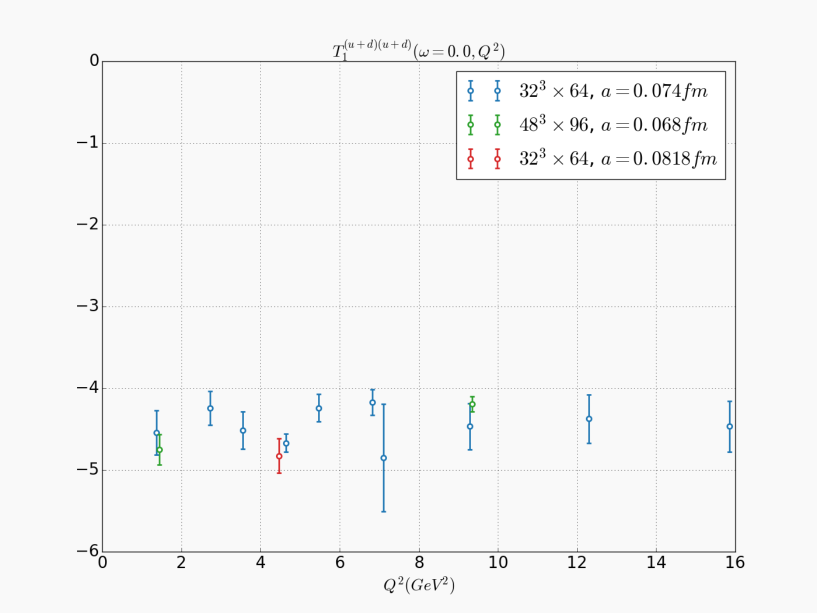
$T_1^{dd}(\omega = 0.0, Q^2)$
0.0
−0.5
−1.0
−1.5
−2.0
−2.5
0
2
4
6
8
10
12
14
16
$Q^2(GeV^2)$
$32^3 \times 64,\ a = 0.074fm$
$48^3 \times 96,\ a = 0.068fm$
$32^3 \times 64,\ a = 0.0818fm$



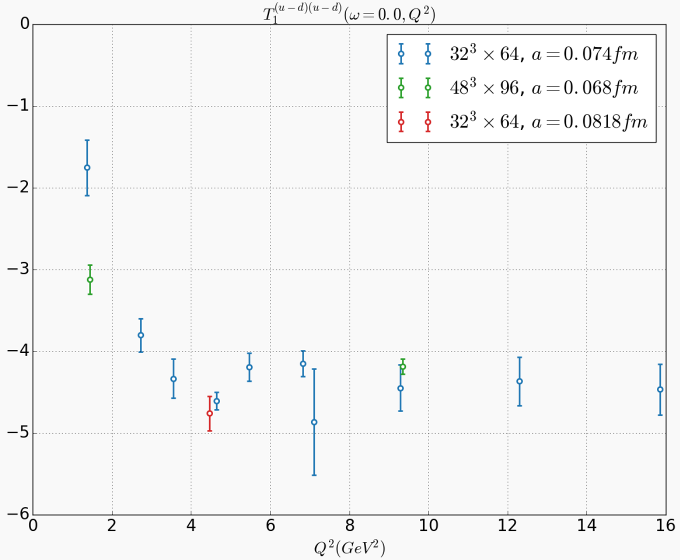
$T_1^{uu}(\omega=0.0, Q^2)$
0.0
−0.5
−1.0
−1.5
−2.0
−2.5
−3.0
−3.5
0
2
4
6
8
10
12
14
16
$Q^2(GeV^2)$
$32^3 \times 64,\ a=0.074fm$
$48^3 \times 96,\ a=0.068fm$
$32^3 \times 64,\ a=0.0818fm$



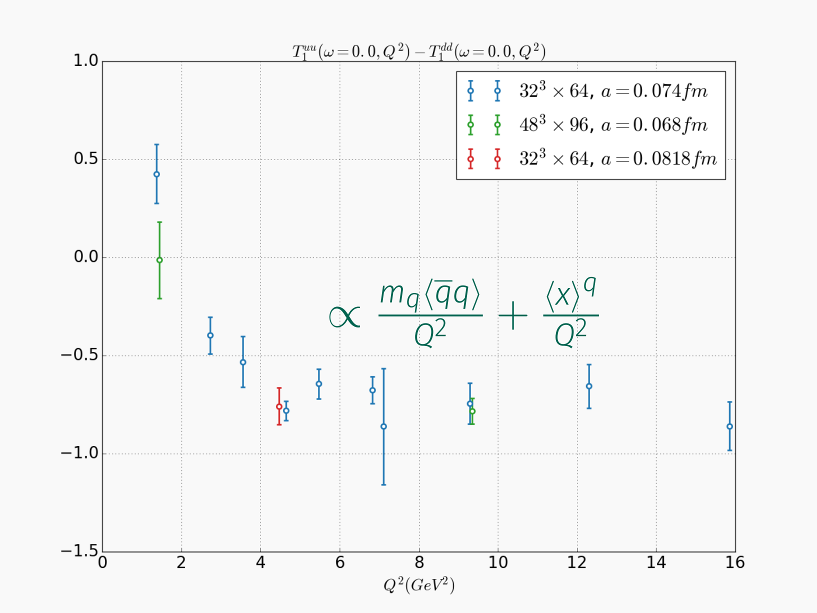
$\frac{1}{2}(T_1^{ud}(\omega=0.0,Q^2)+T_1^{du}(\omega=0.0,Q^2))$
$32^3\times64,\ a=0.074fm$
$48^3\times96,\ a=0.068fm$
$32^3\times64,\ a=0.0818fm$
0.4
0.2
0.0
−0.2
−0.4
−0.6
−0.8
−1.0
0
2
4
6
8
10
12
14
16
$Q^2(GeV^2)$



$T_1^{(u+d)(u+d)}(\omega = 0.0, Q^2)$
$32^3 \times 64,\ a = 0.074fm$
$48^3 \times 96,\ a = 0.068fm$
$32^3 \times 64,\ a = 0.0818fm$
0
−1
−2
−3
−4
−5
−6
0
2
4
6
8
10
12
14
16
$Q^2(GeV^2)$



$T_1^{(u-d)(u-d)}(\omega = 0.0, Q^2)$
0
−1
−2
−3
−4
−5
−6
0
2
4
6
8
10
12
14
16
$Q^2 (GeV^2)$
$32^3 \times 64,\ a = 0.074 fm$
$48^3 \times 96,\ a = 0.068 fm$
$32^3 \times 64,\ a = 0.0818 fm$



$T_1^{uu}(\omega=0.0, Q^2) - T_1^{dd}(\omega=0.0, Q^2)$
$32^3 \times 64,\ a=0.074fm$
$48^3 \times 96,\ a=0.068fm$
$32^3 \times 64,\ a=0.0818fm$
$\propto \frac{m_q \langle \bar{q}q \rangle}{Q^2} + \frac{\langle x \rangle^q}{Q^2}$
$Q^2(GeV^2)$
1.0
0.5
0.0
−0.5
−1.0
−1.5
0
2
4
6
8
10
12
14
16