Heavy and heavy-light mesons with the Covariant Spectator Theory

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Motivation

- Intense experimental activity to explore meson structure at LHC, *BABAR*, Belle, CLEO, and soon at GlueX (JLab) and PANDA (GSI)
- Search for exotic mesons (hybrids, glueballs, … maybe $q\bar{q}$?)
- Need to understand also "conventional" $q\bar{q}$-mesons in more detail
- Study production mechanisms, transition form factors
(also important for hadronic contributions to light-by-light scattering)

Theory: a huge amount of work has already been done on meson structure (LQCD, BS/DSE, constrained dynamics two-body Dirac equation, BLFQ, relativized Schrödinger equation, …)

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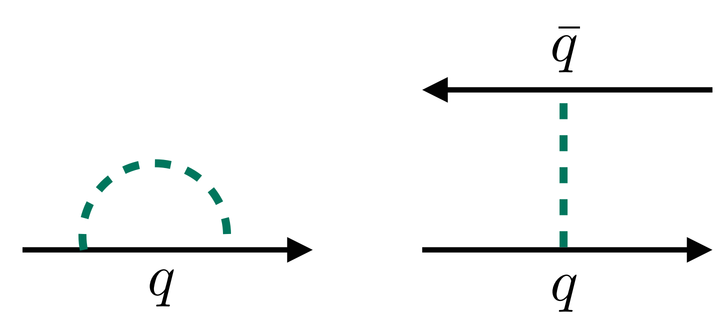
Guiding principles of our approach (CST - Covariant Spectator Theory):

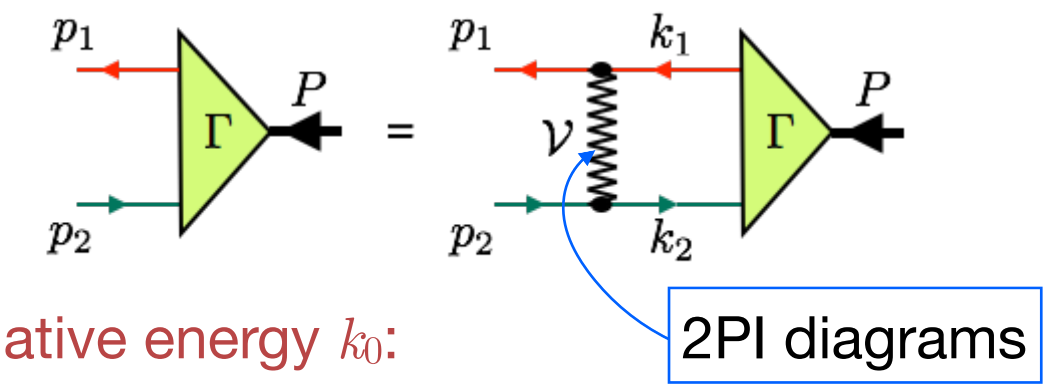
- Find $q\bar{q}$ interaction that can be used in all mesons (unified model)
- Relativistic covariance (work in Minkowski space)
- Confinement through a confining interaction kernel, which should reduce to linear+Coulomb in the nonrelativistic limit
- Learn about the Lorentz structure of the confining interaction
- Quark masses are dynamic: self-interaction should be consistent with $q\bar{q}$ interaction
- Chiral symmetry: massless pion in chiral limit, satisfy the axialvector Ward-Takahashi identity

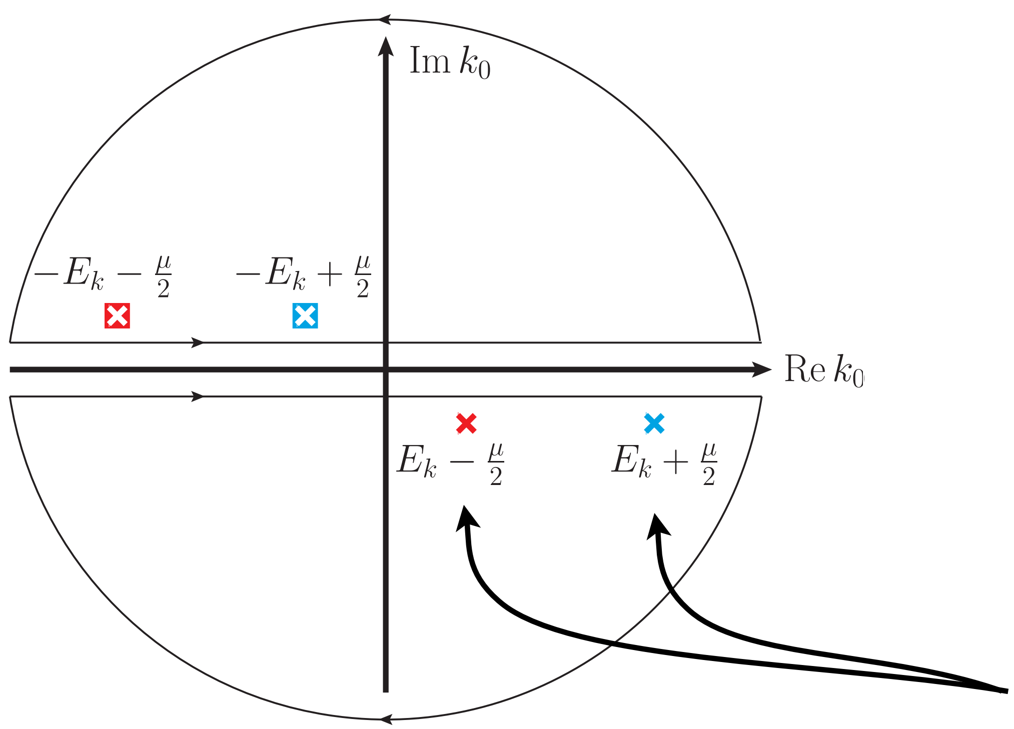
Huge mass variation:
from pions (~0.14 GeV)
to bottomonium (> 10 GeV)



CST equation for two-body bound states

Bethe-Salpeter equation for $q\bar{q}$ bound-state with mass μ





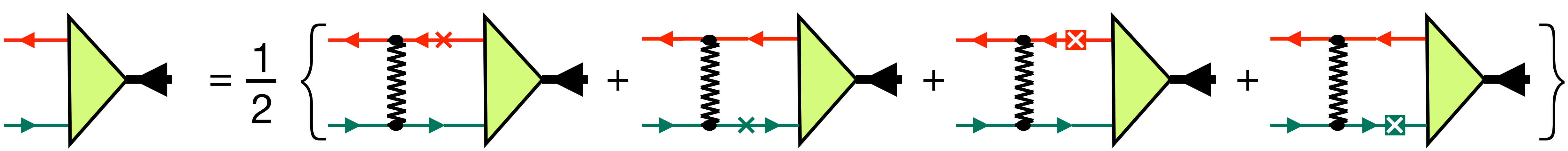
Integration over relative energy k_0:

- Keep only pole contributions from constituent particle propagators
- Poles from particle exchanges appear in higher-order kernels (usually neglected — tend to cancel)
- Reduction to 3D loop integrations, but covariant
- Correct one-body limit

If bound-state mass μ is small:
both poles are close together (both important)

Symmetrize pole contributions from both half planes: charge conjugation symmetry

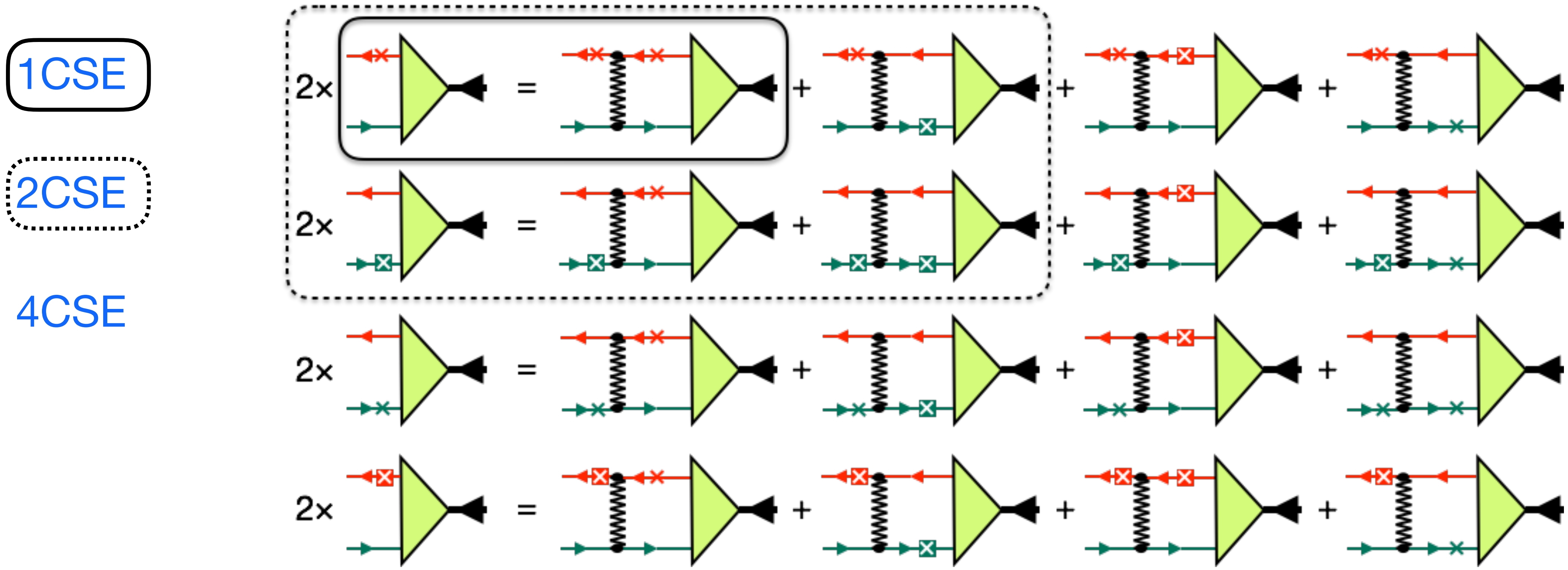
BS vertex (approx.) CST vertices



Once the four CST vertices (with one quark on-shell) are all known, one can use this equation to get the vertex function for other momenta (also Euclidean).

CST equations

Closed set of equations when external legs are systematically placed on-shell



Solutions: bound state masses μ and corresponding vertex functions Γ

One-channel spectator equation (1CSE):

- Particularly appropriate for unequal masses
- Numerical solutions easier (fewer singularities)
- But not charge-conjugation symmetric

Two-channel spectator equation (2CSE):

- Restores charge-conjugation symmetry
- Additional singularities in the kernel

Four-channel spectator equation (4CSE):

- Necessary for light bound states (pion!)

All have smooth one-body limit (Dirac equation) and nonrelativistic limit (Schrödinger equation).

The covariant kernel

Our kernel: $\mathcal{V}(p,k;P) = \frac{3}{4}\mathbf{F}_1\cdot\mathbf{F}_2 \sum_K V_K(p,k;P)\,\Theta_1^{K(\mu)} \otimes \Theta^K_{2(\mu)}$

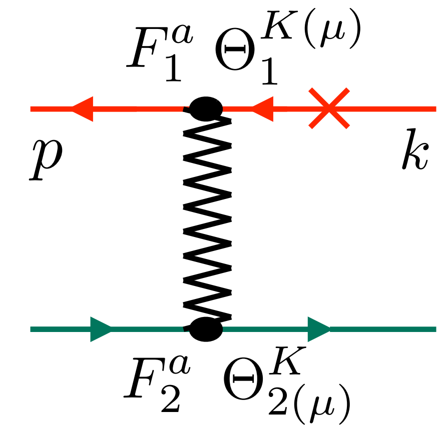
$F_a = \frac{1}{2}\lambda_a$ color SU(3) generators

1 for $q\bar{q}$ color singlets

momentum dependence

Lorentz structure

$\Theta_i^{K(\mu)} = \mathbf{1}_i, \gamma_i^5, \gamma_i^\mu$



- Confining interaction: Lorentz (scalar + pseudoscalar) mixed with vector
  Coupling strength σ, mixing parameter y

$y=0$	pure S+PS
$y=1$	pure V

for correct nonrelativistic limit

$$\mathcal{V}_\mathrm{L}(p,k;P) = \left[(1-y)\left(\mathbf{1}_1\otimes\mathbf{1}_2 + \gamma_1^5\otimes\gamma_2^5\right) - y\,\gamma_1^\mu\otimes\gamma_{\mu 2}\right] V_\mathrm{L}(p,k;P)$$

equal weight (constraint from chiral symmetry)

→ E.P. Biernat et al., PRD **90**, 096008 (2014)

- One-gluon exchange with constant coupling strength α_s
  + Constant interaction (in r-space) with strength C } Lorentz vector

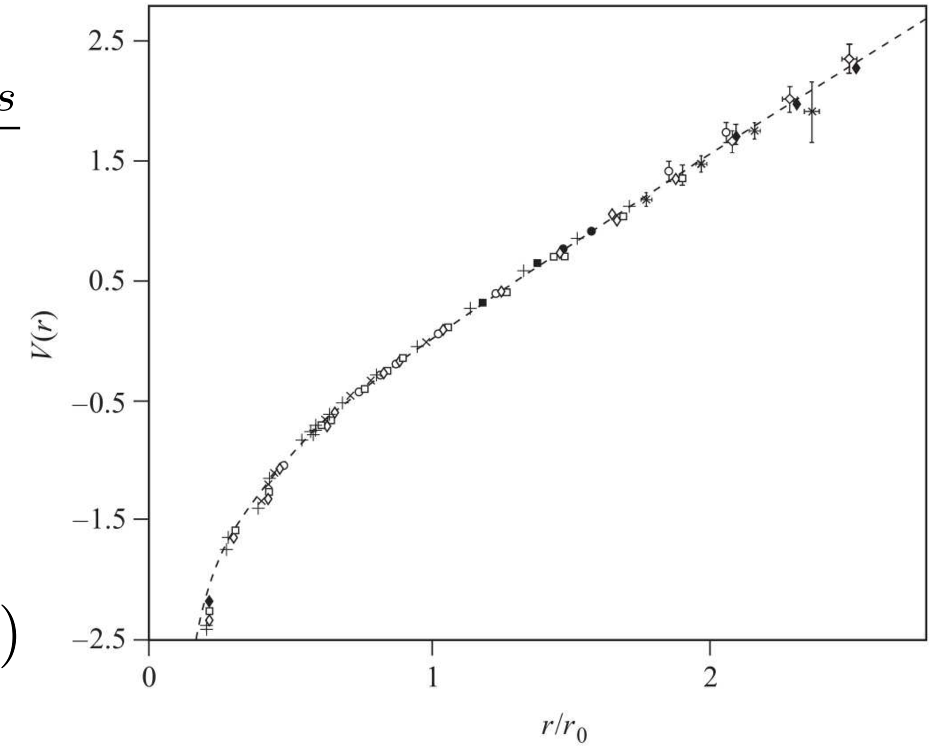
$$\mathcal{V}_\mathrm{OGE}(p,k;P) + \mathcal{V}_\mathrm{C}(p,k;P) = -\gamma_1^\mu\otimes\gamma_{2\mu}\left[V_\mathrm{OGE}(p,k;P) + V_\mathrm{C}(p,k;P)\right]$$

Confining potential in momentum space

Phenomenological $q\bar{q}$ kernel

Inspired by Cornell potential: $V(r) = \sigma r - C - \dfrac{\alpha_s}{r}$

Static QCD potential from the lattice



Allton et al, UKQCD Collab., PRD **65**, 054502 (2002)

NR linear potential in momentum space:

Fourier transform of screened potential

Usually: $$\sigma r = \lim_{\epsilon\to 0} \sigma \frac{\partial^2}{\partial \epsilon^2} \frac{e^{-\epsilon r}}{r}$$

But simpler: $$\sigma r = \lim_{\epsilon\to 0} -\frac{\sigma}{\epsilon}\left(e^{-\epsilon r} - 1\right) \equiv \tilde{V}_A(r) - \tilde{V}_A(0)$$

FT: $$V_L(\mathbf{q}) = V_A(\mathbf{q}) - (2\pi)^3 \delta(\mathbf{q}) \int \frac{d^3 q'}{(2\pi)^3} V_A(\mathbf{q}')$$

with $$V_A(\mathbf{q}) = -\frac{8\pi\sigma}{\mathbf{q}^4}$$

Leitão, Stadler, Peña, Biernat, PRD **90**, 096003 (2014)
Gross, Milana, PRD **43**, 2401 (1991)
Savkli, Gross, PRC **63**, 035208 (2001)

$$\langle V_L \phi \rangle(\mathbf{p}) = \int \frac{d^3k}{(2\pi)^3} V_L(\mathbf{p}-\mathbf{k})\phi(\mathbf{k}) = -8\pi\sigma \int \frac{d^3k}{(2\pi)^3} \frac{\phi(\mathbf{k}) - \phi(\mathbf{p})}{(\mathbf{p}-\mathbf{k})^4}$$

any regular function

highly singular

automatic subtraction

only a Cauchy principal value singularity remains

Schrödinger equation with linear potential in momentum space

Wave functions expanded in basis of B-splines

Great test case: exact solutions are known in r-space for S-waves (Airy functions)

Binding energies in units of $(\sigma^2/2m_R)^{1/3}$ m_R … reduced mass

radial excitations ↓

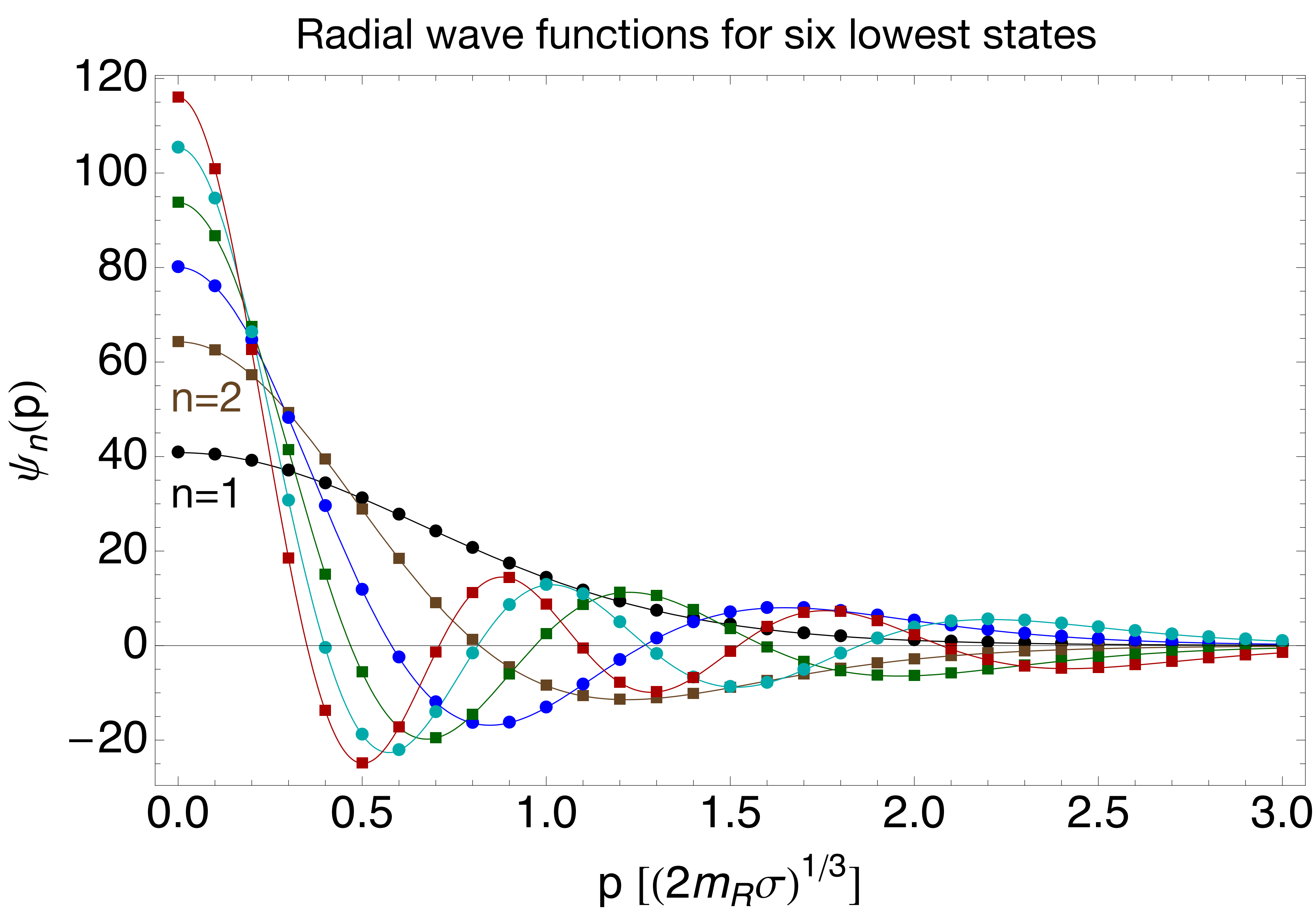
Number of splines in basis →

n	$N=12$	$N=16$	$N=20$	$N=24$	$N=36$	$N=48$	$N=64$	Exact
1	2.338121	2.338108	2.338108	2.338107	2.338107	2.338107	2.338108	2.338107
2	4.088498	4.087976	4.087953	4.087950	4.087947	4.087949	4.087949	4.087949
3	5.527017	5.520928	5.520601	5.520568	5.520559	5.520559	5.520560	5.520560
4	6.794183	6.788208	6.787047	6.786787	6.786710	6.786707	6.786708	6.786708
5	8.002342	7.956598	7.947220	7.944767	7.944146	7.944135	7.944134	7.944134
6	9.626868	9.156258	9.046241	9.026388	9.022727	9.022657	9.022651	9.022651
7	11.435079	10.273394	10.083415	10.048670	10.040511	10.040201	10.040177	10.040174
8	12.099834	11.147565	11.027556	11.028855	11.009868	11.008626	11.008534	11.008524
9	14.993451	12.941736	12.318324	12.105283	11.940068	11.936344	11.936044	11.936016
10	19.122419	15.309248	13.997541	13.138047	12.839002	12.829770	12.828860	12.828777

Schrödinger equation with linear potential in momentum space

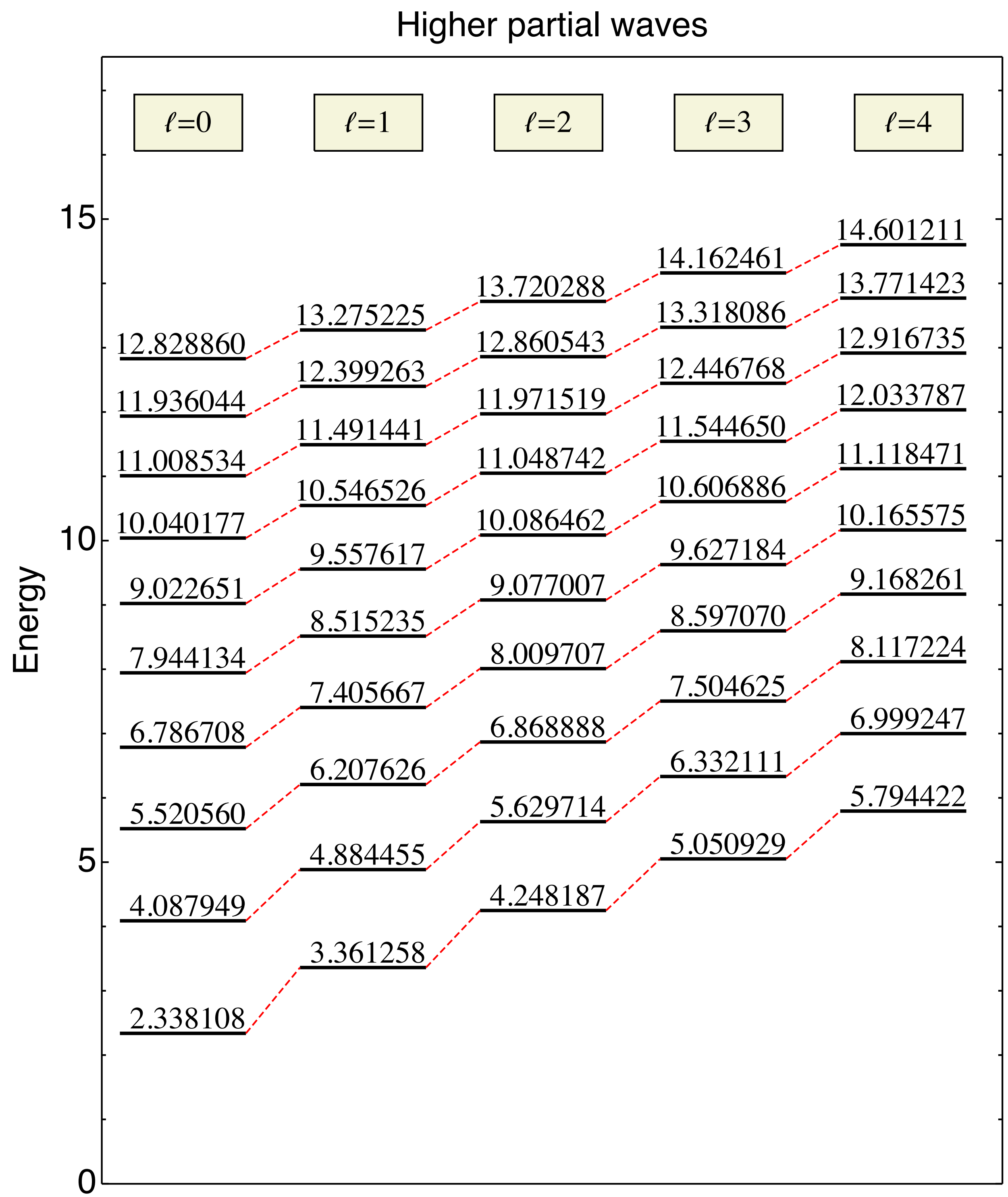
Radial wave functions in momentum space (with N=64)

Lines are our numerical solutions
Symbols are Fourier transforms of exact r-space solutions



Schrödinger equation with linear potential in momentum space

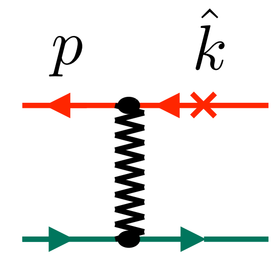
Works well also for higher partial waves



Covariant confining kernel in CST

- Covariant generalization: $\mathbf{q}^2 \to -q^2$

This leads to a kernel that acts like



initial state: either quark or antiquark onshell

$$\langle V_L \phi \rangle(p) = \int \frac{d^3k}{(2\pi)^3} \frac{m}{E_k} V_L(p,\hat{k})\phi(\hat{k}) = -8\pi\sigma \int \frac{d^3k}{(2\pi)^3} \frac{m}{E_k} \frac{\phi(\hat{k}) - \phi(\hat{k}_R)}{(p-\hat{k})^4}$$

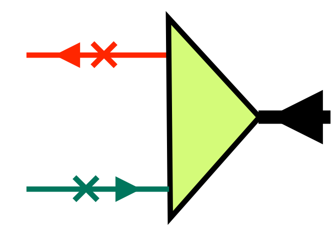
$\hat{k} = (E_k, \mathbf{k})$ on mass shell

Complication: Singularity not only when $\mathbf{k} = \mathbf{p}$

$\hat{k}_R = (E_{k_R}, \mathbf{k}_R)$ $\qquad$ $\mathbf{k}_R = \mathbf{k}_R(p_0, \mathbf{p})$ ← value of $\mathbf{k}$ at which kernel becomes singular

- Does it still confine?

Yes: the vertex function vanishes if both quarks are on-shell!



= 0

More details: Savkli, Gross, PRC **63**, 035208 (2001)

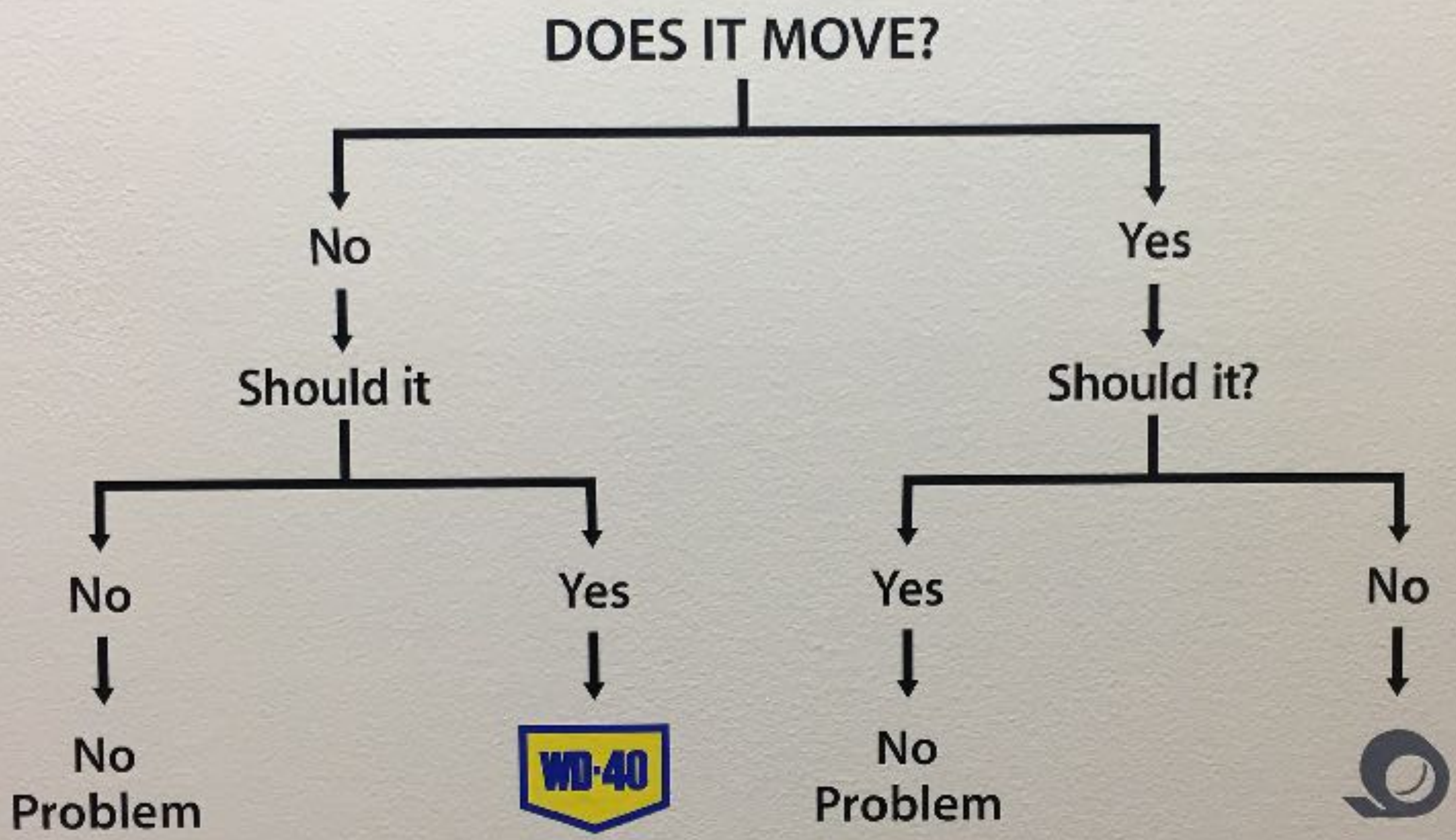
$$\langle V_L \rangle = \int \frac{d^3k}{(2\pi)^3} \frac{m}{E_k} V_L(p,\hat{k}) = 0$$

← important property

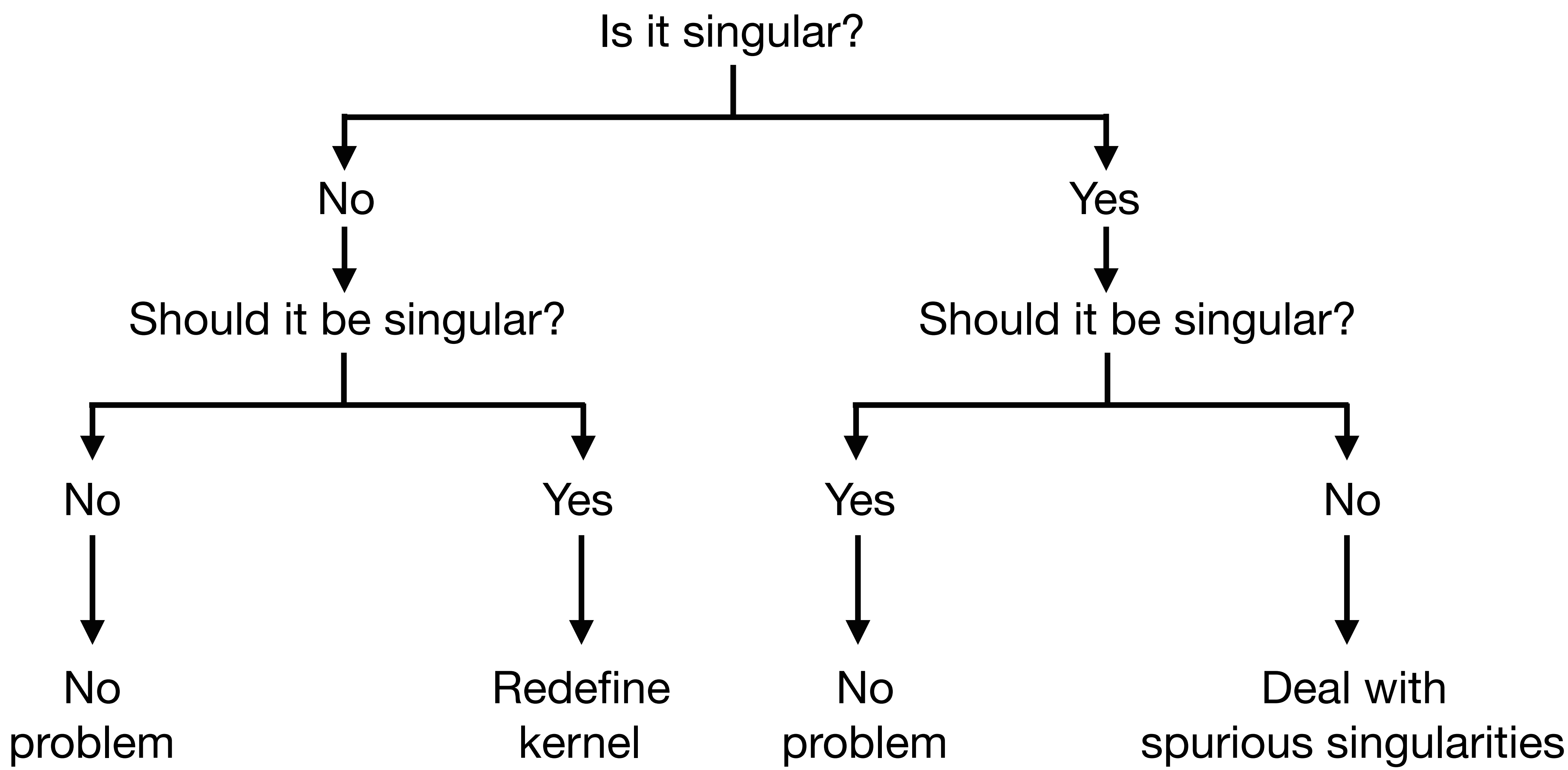
corresponds to $\tilde{V}_L^{\mathrm{nr}}(r=0) = 0$

- But is there always a singularity?

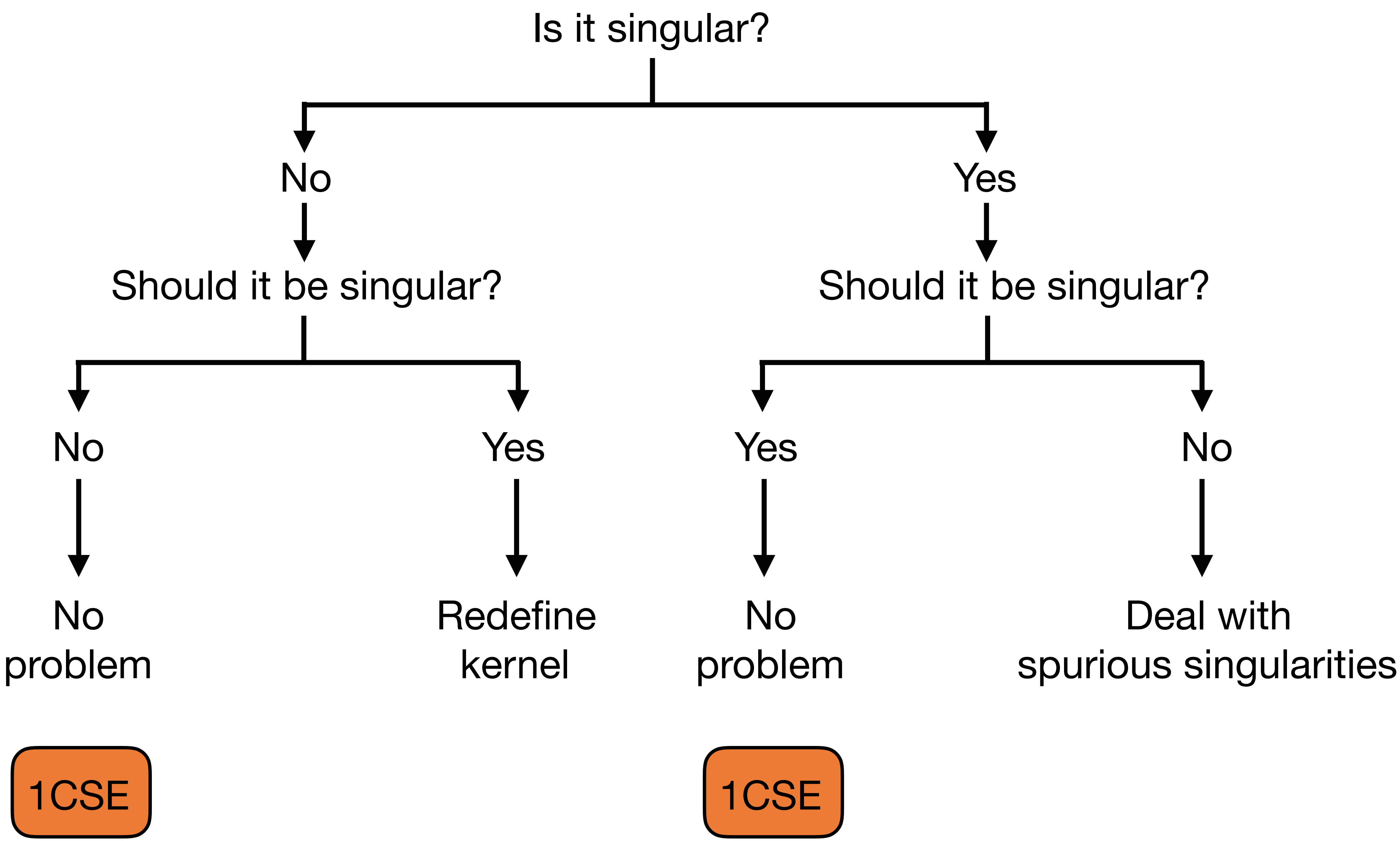
Engineering Flowchart



Relativistic kernel flowchart

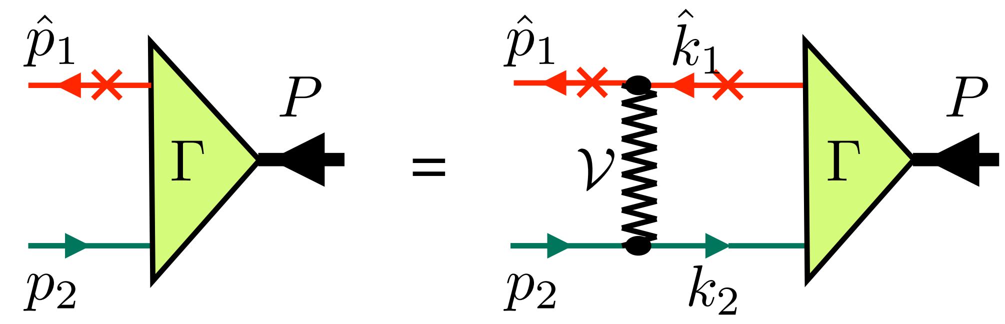


Relativistic kernel flowchart



The One-Channel Spectator Equation (1CSE)

We solve the 1CSE for heavy and heavy-light systems



- Should work well for bound states with at least one heavy quark
- Much easier to solve numerically than 2CSE or 4CSE
- C-parity splitting small in heavy quarkonia
- For now with constant constituent quark masses (quark self-energies will be included later)

$$\Gamma(\hat{p}_1,p_2) = -\int \frac{d^3k}{(2\pi)^3}\frac{m_1}{E_{1k}}\sum_K V_K(\hat{p}_1,\hat{k}_1)\Theta_1^{K(\mu)}\frac{m_1+\hat{\not{k}}_1}{2m_1}\Gamma(\hat{k}_1,k_2)\frac{m_2+\not{k}_2}{m_2^2-k_2^2-i\epsilon}\Theta^K_{2(\mu)}$$

$$E_{ik}=\sqrt{m_i^2+\mathbf{k}^2}$$

- Momentum-dependence of kernels is also simpler

$$V_{\rm L}(\hat{p}_1,\hat{k}_1) = -8\sigma\pi\left[\frac{1}{(\hat{p}_1-\hat{k}_1)^4} - \frac{E_{p_1}}{m_1}(2\pi)^3\delta^3(\mathbf{p}_1-\mathbf{k}_1)\int\frac{d^3k_1'}{(2\pi)^3}\frac{m_1}{E_{k_1'}}\frac{1}{(\hat{p}_1-\hat{k}_1')^4}\right]$$

$$V_{\rm OGE}(\hat{p}_1,\hat{k}_1) = -\frac{4\pi\alpha_s}{(\hat{p}_1-\hat{k}_1)^2}$$

$$V_{\rm C}(\hat{p}_1,\hat{k}_1) = (2\pi)^3\frac{E_{k_1}}{m_1}C\delta^3(\mathbf{p}_1-\mathbf{k}_1)$$

- Linear and OGE kernels need to be regularized
  We chose Pauli-Villars regularizations with parameter $\Lambda = 2m_1$

CST vertex functions

$$P^\mu = p_1 - p_2 \qquad \rho^\mu = \frac{p_1 + p_2}{2} \qquad \Lambda(p_i) = \frac{m_i + \not{p}_i}{2m_i}$$

Pseudoscalar mesons

$$\Gamma^P(p_1,p_2) = \Gamma_1^P(p_1,p_2)\gamma^5 + \Gamma_2^P(p_1,p_2)\Lambda(-p_1)\gamma^5 + \Gamma_3^P(p_1,p_2)\gamma^5\Lambda(-p_2) + \Gamma_4^P(p_1,p_2)\Lambda(-p_1)\gamma^5\Lambda(-p_2)$$

Scalar mesons

$$\Gamma^S(p_1,p_2) = \Gamma_1^S(p_1,p_2) + \Gamma_2^S(p_1,p_2)\Lambda(-p_1) + \Gamma_3^S(p_1,p_2)\Lambda(-p_2) + \Gamma_4^S(p_1,p_2)\Lambda(-p_1)\Lambda(-p_2)$$

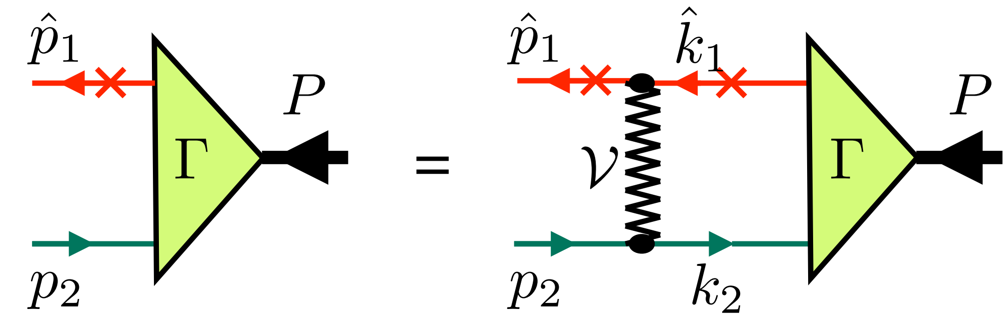
Vector mesons

$$\begin{aligned}\Gamma^{VT\mu}(p_1,p_2) =&\Gamma_1^V(p_1,p_2)\gamma^{T\mu} + \Gamma_2^V(p_1,p_2)\Lambda(-p_1)\gamma^{T\mu} + \Gamma_3^V(p_1,p_2)\gamma^{T\mu}\Lambda(-p_2) \\ &+ \Gamma_4^V(p_1,p_2)\Lambda(-p_1)\gamma^{T\mu}\Lambda(-p_2) + \Gamma_5^V(p_1,p_2)\rho^{T\mu} + \Gamma_6^V(p_1,p_2)\Lambda(-p_1)\rho^{T\mu} \\ &+ \Gamma_7^V(p_1,p_2)\rho^{T\mu}\Lambda(-p_2) + \Gamma_8^V(p_1,p_2)\Lambda(-p_1)\rho^{T\mu}\Lambda(-p_2)\end{aligned}$$

Axialvector mesons

$$\begin{aligned}\Gamma^{AT\mu}(p_1,p_2) =&\Gamma_1^A(p_1,p_2)\gamma^{T\mu}\gamma^5 + \Gamma_2^A(p_1,p_2)\Lambda(-p_1)\gamma^{T\mu}\gamma^5 + \Gamma_3^A(p_1,p_2)\gamma^{T\mu}\gamma^5\Lambda(-p_2) \\ &+ \Gamma_4^A(p_1,p_2)\Lambda(-p_1)\gamma^{T\mu}\gamma^5\Lambda(-p_2) + \Gamma_5^A(p_1,p_2)\rho^{T\mu}\gamma^5 + \Gamma_6^A(p_1,p_2)\Lambda(-p_1)\rho^{T\mu}\gamma^5 \\ &+ \Gamma_7^A(p_1,p_2)\rho^{T\mu}\gamma^5\Lambda(-p_2) + \Gamma_8^A(p_1,p_2)\Lambda(-p_1)\rho^{T\mu}\gamma^5\Lambda(-p_2)\end{aligned}$$

Solution of the 1CSE



- Work in rest frame of the bound state $P = (\mu, \mathbf{0})$
- Use ρ-spin decomposition of the propagator

$$\frac{m_2 + \not{k}_2}{m_2^2 - k_2^2 - i\epsilon} = \frac{m_2}{E_{2k}} \sum_{\rho,\lambda_2} \rho \frac{u_2^\rho(\mathbf{k},\lambda_2)\bar{u}_2^\rho(\mathbf{k},\lambda_2)}{E_{2k} - \rho k_{20} - i\epsilon}$$

$u^+(\mathbf{k},\lambda) \equiv u(\mathbf{k},\lambda)$
$u^-(\mathbf{k},\lambda) \equiv v(-\mathbf{k},\lambda)$
ρ-spinors with helicity λ

$$E_{ik} = \sqrt{m_i^2 + \mathbf{k}^2}$$

- Project 1CSE onto ρ-spin helicity channels

$$\Gamma^{+\rho'}_{\lambda\lambda'}(p) \equiv \bar{u}_1^+(\mathbf{p},\lambda)\Gamma(p)u_2^{\rho'}(\mathbf{p},\lambda')$$

$$\Theta^{K,\rho\rho'}_{i,\lambda\lambda'}(\mathbf{p},\mathbf{k}) \equiv \bar{u}_i^\rho(\mathbf{p},\lambda)\Theta_i^K u_i^{\rho'}(\mathbf{k},\lambda')$$

spinor matrix elements of vertices

- Define relativistic wave functions

$$\Psi^{+\rho}_{\lambda\lambda'}(p) \equiv \sqrt{\frac{m_1 m_2}{E_{1p}E_{2p}}} \frac{\rho}{E_{2p} - \rho(E_{1p} - \mu)} \Gamma^{+\rho}_{\lambda\lambda'}(p)$$

The 1CSE becomes a generalized **linear** EV problem for the mass eigenvalues μ

$$(E_{1p} - \rho_2 E_{2p})\Psi^{+\rho_2}_{\lambda_1\lambda_2}(\mathbf{p}) - \sum_{K\lambda_1'\lambda_2'\rho_2'} \int \frac{d^3k}{(2\pi)^3} N_{12}(p,k) V_K(\mathbf{p},\mathbf{k}) \Theta^{K,++}_{1,\lambda_1\lambda_1'}(\mathbf{p},\mathbf{k}) \Psi^{+\rho_2'}_{\lambda_1'\lambda_2'}(\mathbf{k}) \Theta^{K,\rho_2'\rho_2}_{2,\lambda_2'\lambda_2}(\mathbf{k},\mathbf{p})$$
$$= \mu \Psi^{+\rho_2}_{\lambda_1\lambda_2}(\mathbf{p})$$

Solution of the 1CSE

- Normalization

$$2\mu = N_c \sum_{\lambda_1\lambda_2\rho_2} \int \frac{d^3p}{(2\pi)^3} \left[\Psi^{+\rho_2}_{\lambda_1\lambda_2}(\mathbf{p})\right]^\dagger \Psi^{+\rho_2}_{\lambda_1\lambda_2}(\mathbf{p})$$

(kernel independent of *P*)

- Switch to basis of eigenstates of total orbital angular momentum *L* and of total spin *S* (not necessary, but useful for spectroscopic identification of solutions)

$$\Psi^{+\rho_2}_{\lambda_1\lambda_2}(\mathbf{p}) = \sum_j \psi^{\rho_2}_j(p)\chi^\dagger_{\lambda_1}(\hat{\mathbf{p}})\, K^{\rho_2}_j(\hat{\mathbf{p}})\, \chi_{\lambda_2}(\hat{\mathbf{p}})$$

J^P	$K_1^-(\hat{\mathbf{p}})$	Wave	$K_2^-(\hat{\mathbf{p}})$	Wave	$K_1^+(\hat{\mathbf{p}})$	Wave	$K_2^+(\hat{\mathbf{p}})$	Wave
0^-	$\mathbf{1}$	S	-	-	$\boldsymbol{\sigma}\cdot\hat{\mathbf{p}}$	P	-	-
0^+	$\boldsymbol{\sigma}\cdot\hat{\mathbf{p}}$	P	-	-	$\mathbf{1}$	S	-	-
1^-	$\boldsymbol{\sigma}\cdot\hat{\boldsymbol{\xi}}$	S	$\frac{1}{\sqrt{2}}\left(3\boldsymbol{\xi}\cdot\hat{\mathbf{p}}\,\boldsymbol{\sigma}\cdot\hat{\mathbf{p}} - \boldsymbol{\sigma}\cdot\hat{\boldsymbol{\xi}}\right)$	D	$\sqrt{3}\boldsymbol{\xi}\cdot\hat{\mathbf{p}}$	P_s	$\sqrt{\frac{3}{2}}\left(\boldsymbol{\sigma}\cdot\hat{\boldsymbol{\xi}}\,\boldsymbol{\sigma}\cdot\hat{\mathbf{p}} - \boldsymbol{\xi}\cdot\hat{\mathbf{p}}\right)$	P_t
1^+	$\sqrt{3}\boldsymbol{\xi}\cdot\hat{\mathbf{p}}$	P_s	$\sqrt{\frac{3}{2}}\left(\boldsymbol{\sigma}\cdot\hat{\boldsymbol{\xi}}\,\boldsymbol{\sigma}\cdot\hat{\mathbf{p}} - \boldsymbol{\xi}\cdot\hat{\mathbf{p}}\right)$	P_t	$\boldsymbol{\sigma}\cdot\hat{\boldsymbol{\xi}}$	S	$\frac{1}{\sqrt{2}}\left(3\boldsymbol{\xi}\cdot\hat{\mathbf{p}}\,\boldsymbol{\sigma}\cdot\hat{\mathbf{p}} - \boldsymbol{\sigma}\cdot\hat{\boldsymbol{\xi}}\right)$	D

$$J^P = 0^\pm \qquad \int_0^\infty dp\, p^2 \left[\psi_S^2(p) + \psi_D^2(p)\right] = 1$$

$$J^P = 1^\pm \qquad \int_0^\infty dp\, p^2 \left[\psi_S^2(p) + \psi_D^2(p) + \psi_{P_s}^2(p) + \psi_{P_t}^2(p)\right] = 1$$

Normalization of radial wave functions → probabilities of partial waves

- Expand radial wave functions in a basis of B-splines (modified for correct asymptotic behavior) and solve eigenvalue problem → expansion coefficients and mass eigenvalues

Solution of the 1CSE

- Normalization $$2\mu = N_c \sum_{\lambda_1\lambda_2\rho_2} \int \frac{d^3p}{(2\pi)^3} \left[\Psi^{+\rho_2}_{\lambda_1\lambda_2}(\mathbf{p})\right]^\dagger \Psi^{+\rho_2}_{\lambda_1\lambda_2}(\mathbf{p})$$

  (kernel independent of *P*)

- Switch to basis of eigenstates of total orbital angular momentum *L* and of total spin *S* (not necessary, but useful for spectroscopic identification of solutions)

$$\Psi^{+\rho_2}_{\lambda_1\lambda_2}(\mathbf{p}) = \sum_j \psi^{\rho_2}_j(p)\chi^\dagger_{\lambda_1}(\hat{\mathbf{p}})\, K^{\rho_2}_j(\hat{\mathbf{p}})\,\chi_{\lambda_2}(\hat{\mathbf{p}})$$

J^P	$K_1^-(\hat{\mathbf{p}})$	Wave	$K_2^-(\hat{\mathbf{p}})$	Wave	$K_1^+(\hat{\mathbf{p}})$	Wave	$K_2^+(\hat{\mathbf{p}})$	Wave
0^-	$\mathbf{1}$	S	-	-	$\boldsymbol{\sigma}\cdot\hat{\mathbf{p}}$	P	-	-
0^+	$\boldsymbol{\sigma}\cdot\hat{\mathbf{p}}$	P	-	-	$\mathbf{1}$	S	-	-
1^-	$\boldsymbol{\sigma}\cdot\hat{\boldsymbol{\xi}}$	S	$\frac{1}{\sqrt{2}}\left(3\boldsymbol{\xi}\cdot\hat{\mathbf{p}}\,\boldsymbol{\sigma}\cdot\hat{\mathbf{p}}-\boldsymbol{\sigma}\cdot\hat{\boldsymbol{\xi}}\right)$	D	$\sqrt{3}\boldsymbol{\xi}\cdot\hat{\mathbf{p}}$	P_s	$\sqrt{\frac{3}{2}}\left(\boldsymbol{\sigma}\cdot\hat{\boldsymbol{\xi}}\,\boldsymbol{\sigma}\cdot\hat{\mathbf{p}}-\boldsymbol{\xi}\cdot\hat{\mathbf{p}}\right)$	P_t
1^+	$\sqrt{3}\boldsymbol{\xi}\cdot\hat{\mathbf{p}}$	P_s	$\sqrt{\frac{3}{2}}\left(\boldsymbol{\sigma}\cdot\hat{\boldsymbol{\xi}}\,\boldsymbol{\sigma}\cdot\hat{\mathbf{p}}-\boldsymbol{\xi}\cdot\hat{\mathbf{p}}\right)$	P_t	$\boldsymbol{\sigma}\cdot\hat{\boldsymbol{\xi}}$	S	$\frac{1}{\sqrt{2}}\left(3\boldsymbol{\xi}\cdot\hat{\mathbf{p}}\,\boldsymbol{\sigma}\cdot\hat{\mathbf{p}}-\boldsymbol{\sigma}\cdot\hat{\boldsymbol{\xi}}\right)$	D

relativistic components

$$J^P = 0^\pm \qquad \int_0^\infty dp\, p^2 \left[\psi_S^2(p) + \psi_D^2(p)\right] = 1$$

$$J^P = 1^\pm \qquad \int_0^\infty dp\, p^2 \left[\psi_S^2(p) + \psi_D^2(p) + \psi_{P_s}^2(p) + \psi_{P_t}^2(p)\right] = 1$$

Normalization of radial wave functions
→ probabilities of partial waves

- Expand radial wave functions in a basis of B-splines (modified for correct asymptotic behavior) and solve eigenvalue problem → expansion coefficients and mass eigenvalues

Data sets used in least-square fits of meson masses

	State	$J^{P(C)}$	Mass (MeV)	S1	S2	S3
$b\bar{b}$	$\Upsilon(4S)$	1^{--}	10579.4 ± 1.2		•	•
	$\chi_{b1}(3P)$	1^{++}	10512.1 ± 2.3			•
	$\Upsilon(3S)$	1^{--}	10355.2 ± 0.5		•	•
	$\eta_b(3S)$	0^{-+}	10337			
	$h_b(2P)$	1^{+-}	10259.8 ± 1.2			•
	$\chi_{b1}(2P)$	1^{++}	$10255.46\pm0.22\pm0.50$			•
	$\chi_{b0}(2P)$	0^{++}	$10232.5\pm0.4\pm0.5$		•	•
	$\Upsilon(1D)$	1^{--}	10155			
	$\Upsilon(2S)$	1^{--}	10023.26 ± 0.31		•	•
	$\eta_b(2S)$	0^{-+}	9999 ± 4	•	•	•
	$h_b(1P)$	1^{+-}	9899.3 ± 0.8			•
	$\chi_{b1}(1P)$	1^{++}	$9892.78\pm0.26\pm0.31$			•
	$\chi_{b0}(1P)$	0^{++}	$9859.44\pm0.42\pm0.31$		•	•
	$\Upsilon(1S)$	1^{--}	9460.30 ± 0.26		•	•
	$\eta_b(1S)$	0^{-+}	9399.0 ± 2.3	•	•	•
$b\bar{c}$	$B_c(2S)^{\pm}$	0^-	6842 ± 6			•
	B_c^+	0^-	6275.1 ± 1.0	•	•	•
$b\bar{s}$	$B_{s1}(5830)$	1^+	5828.63 ± 0.27			•
$b\bar{q}$	$B_1(5721)^{+,0}$	1^+	5725.85 ± 1.3			•
$b\bar{s}$	B_s^*	1^-	5415.8 ± 1.5		•	•
	B_s^0	0^-	5366.82 ± 0.22	•	•	•
$b\bar{q}$	B^*	1^-	5324.65 ± 0.25		•	•
	$B^{\pm,0}$	0^-	5279.45	•	•	•

	State	$J^{P(C)}$	Mass (MeV)	S1	S2	S3
$c\bar{c}$	$X(3915)$	0^{++}	3918.4 ± 1.9		•	•
	$\psi(3770)$	1^{--}	3773.13 ± 0.35		•	•
	$\psi(2S)$	1^{--}	3686.097 ± 0.010		•	•
	$\eta_c(2S)$	0^{-+}	3639.2 ± 1.2	•	•	•
	$h_c(1P)$	1^{+-}	3525.38 ± 0.11			•
	$\chi_{c1}(1P)$	1^{++}	3510.66 ± 0.07			•
	$\chi_{c0}(1P)$	0^{++}	3414.75 ± 0.31		•	•
	$J/\Psi(1S)$	1^{--}	3096.900 ± 0.006		•	•
	$\eta_c(1S)$	0^{-+}	2983.4 ± 0.5	•	•	•
$c\bar{s}$	$D_{s1}(2536)^{\pm}$	1^+	2535.10 ± 0.06			•
	$D_{s1}(2460)^{\pm}$	1^+	2459.5 ± 0.6			•
$c\bar{q}$	$D_1(2420)^{\pm,0}$	1^+	2421.4			•
	$D_0^*(2400)^0$	0^+	2318 ± 29		•	•
$c\bar{s}$	$D_{s0}^*(2317)^{\pm}$	0^+	2317.7 ± 0.6		•	•
	$D_s^{*\pm}$	1^-	2112.1 ± 0.4		•	•
$c\bar{q}$	$D^*(2007)^0$	1^-	2008.62			•
$c\bar{s}$	$D_s^{\pm}$	0^-	1968.27 ± 0.10	•	•	•
$c\bar{q}$	$D^{\pm,0}$	0^-	1867.23	•	•	•

S1: 9 PS mesons

S2: 25 PS+V+S mesons

S3: 39 PS+V+S+AV mesons

q represents a light quark (u or d)

We use $m_u = m_d \equiv m_q$

Global fits with fixed quark masses and y=0

S. Leitão, A. S., M. T. Peña, E. Biernat, Phys. Lett. B **764** (2017) 38

First step: we perform global fits to the heavy + heavy-light meson spectrum

Adjustable model parameters: σ α_s C

Model parameters not adjusted in the fits:

Constituent quark masses (in GeV) m_b=4.892, m_c=1.600, m_s=0.448, m_q=0.346

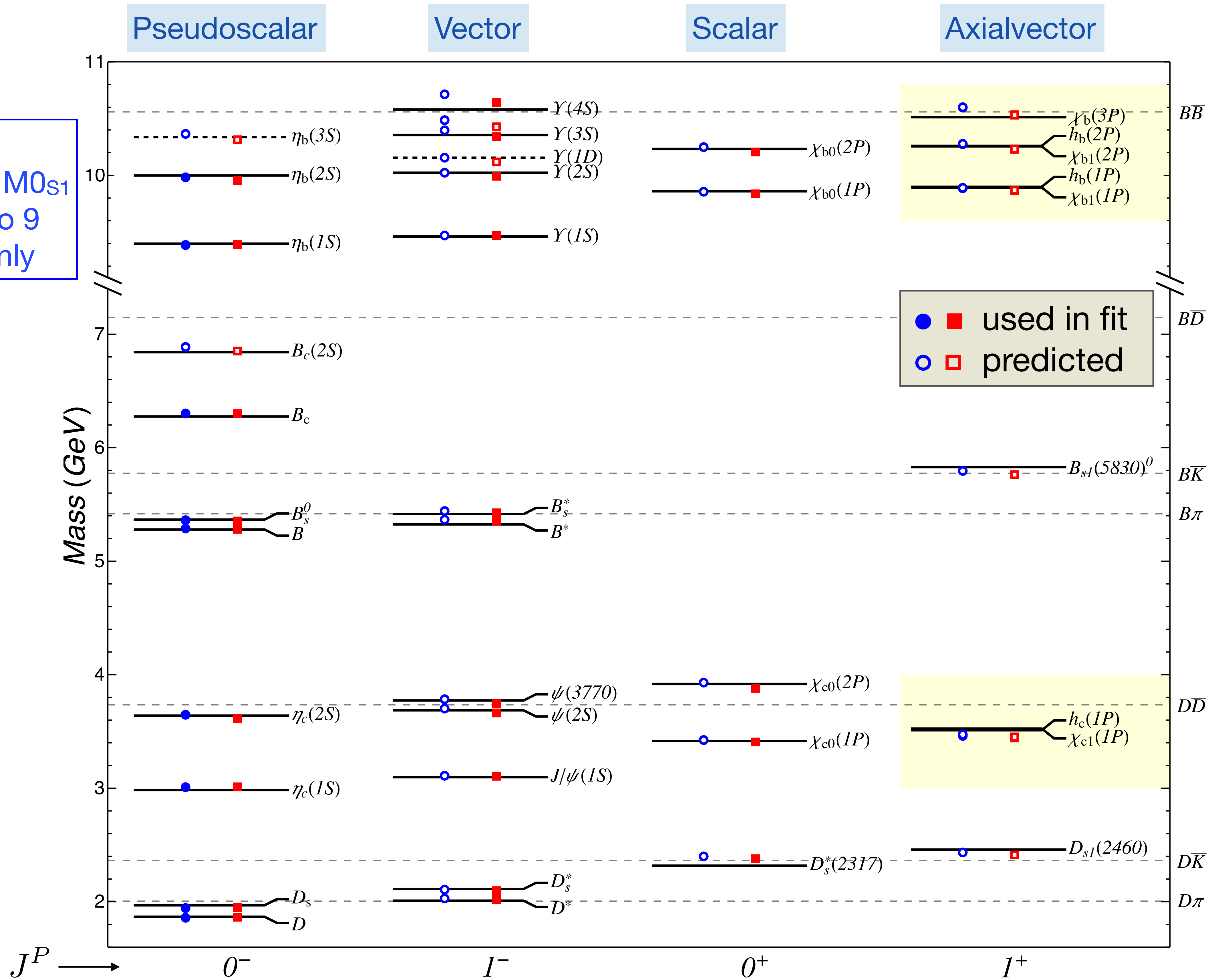
Scalar + pseudoscalar confinement $y = 0$

- Model $M0_{S1}$: fitted to 9 pseudoscalar meson masses only
- Model $M0_{S2}$: fitted to 25 pseudoscalar, vector, and scalar meson masses

(Previously called models P1 and PSV1)

Global fits with fixed quark masses and scalar confinement (y=0)

Blue:
model M0$_{S1}$
fitted to 9
0^-(P) only



Red:
model M0$_{S2}$
fitted to 25
P+S+V

Global fits with fixed quark masses and y=0

The results of the two fits are remarkably similar!

Model	σ [GeV^2]	α_s	C [GeV]
M0$_{S1}$	0.2493	0.3643	0.3491
M0$_{S2}$	0.2247	0.3614	0.3377

→

rms differences to experimental masses (set S3):

Model	$\Delta_{\rm rms}$ [GeV]
M0$_{S1}$	0.037
M0$_{S2}$	0.036

- Kernel parameters are already well determined through pseudoscalar states ($J^P = 0^-$)

Almost 100% L=0, S=0
(S-wave, spin singlet)

$$\langle 0^- | \mathbf{L}\cdot\mathbf{S} | 0^- \rangle = 0$$ Spin-orbit force vanishes

$$\langle 0^- | S_{12} | 0^- \rangle = 0$$ Tensor force vanishes

$$\langle 0^- | \mathbf{S}_1\cdot\mathbf{S}_2 | 0^- \rangle = -3/4$$ Spin-spin force acts in singlet only

- Good test for a covariant kernel:

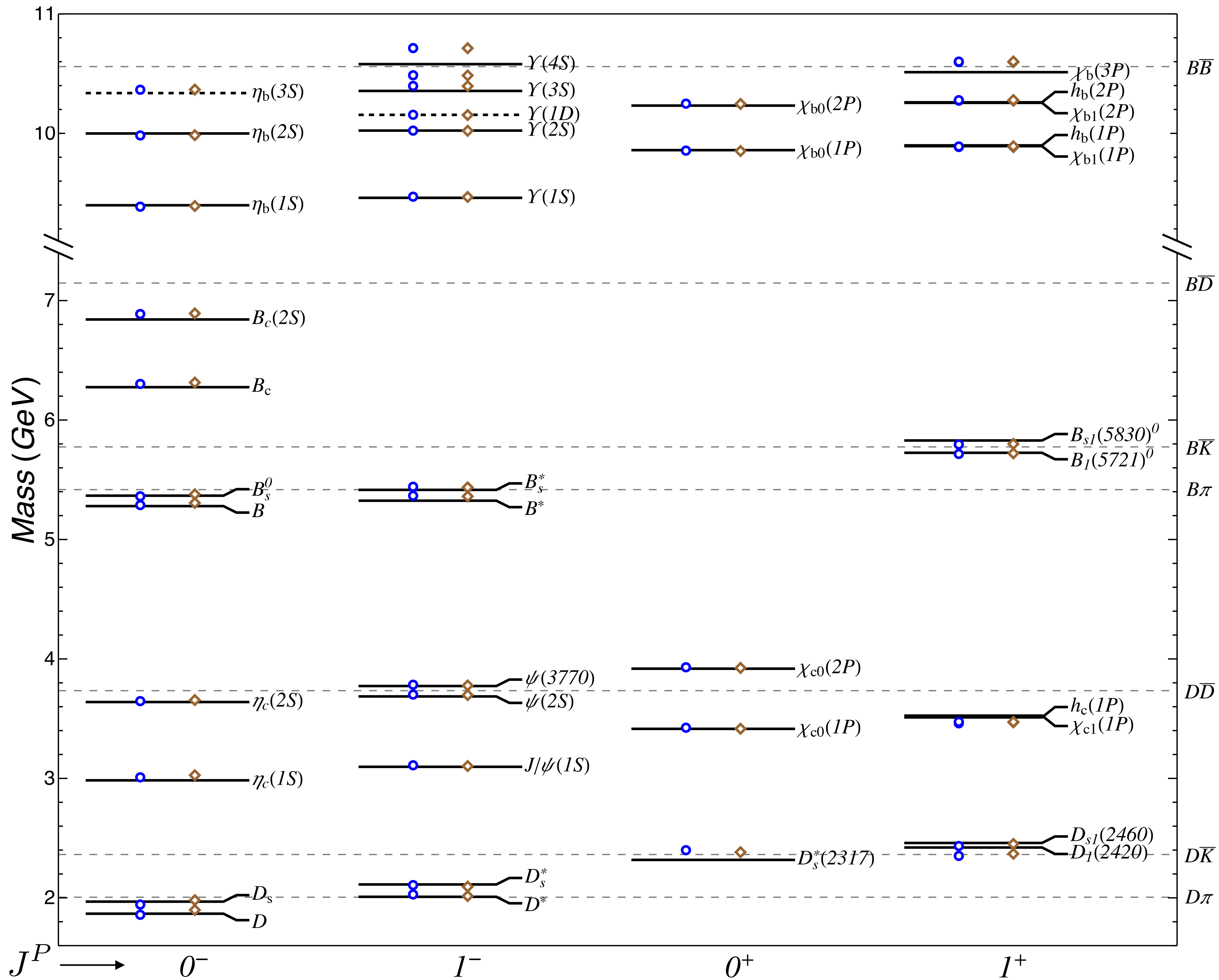
Pseudoscalar states do not constrain spin-orbit and tensor forces, and cannot separate spin-spin from central force.

But they should be determined through covariance.

Model M0$_{S1}$ indeed predicts spin-dependent forces correctly!

Leitão, AS, Peña, Biernat, Phys. Lett. B **764** (2017) 38

Importance of PS coupling in the confining kernel



Confining interaction (with y=0)

$$\left(\mathbf{1}_1 \otimes \mathbf{1}_2 + \gamma_1^5 \otimes \gamma_2^5\right) V_L$$

S PS

Model $M0_{S1}$

- S+PS
- S only (no refitting)

PS effect very small:

- a few MeV in bottomonium
- max: ~40 MeV in D mesons

Fits with variable quark masses and confinement (S+PS)-V mixing *y*

In a new series of fits we treat quark masses and mixing parameter *y* as adjustable parameters.

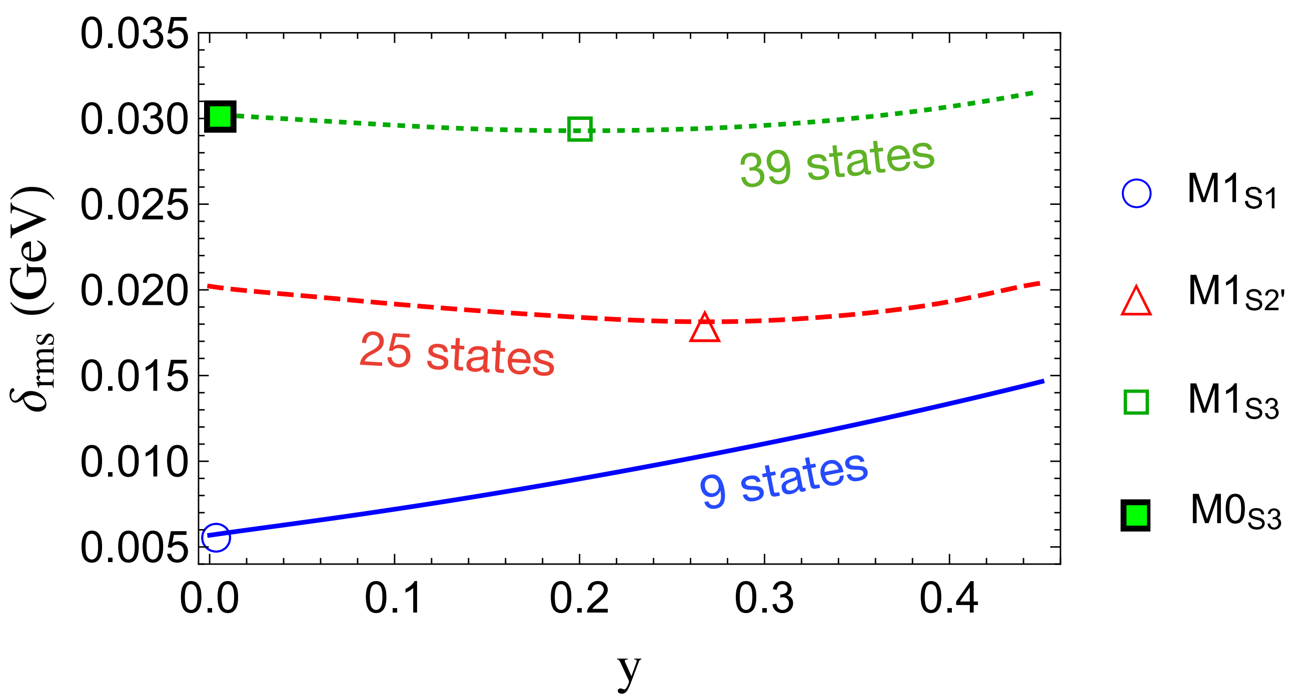
Model	Symbol	σ [GeV^2]	α_s	C [GeV]	y	m_b [GeV]	m_c [GeV]	m_s [GeV]	m_q [GeV]	N	$\delta_{\rm rms}$ [GeV]	$\Delta_{\rm rms}$ [GeV]
$\rm M0_{S1}$		0.2493	0.3643	0.3491	**0.0000**	**4.892**	**1.600**	**0.4478**	**0.3455**	9	0.017	0.037
$\rm M1_{S1}$	○	0.2235	0.3941	0.0591	0.0000	4.768	1.398	0.2547	0.1230	9	0.006	0.041
$\rm M0_{S2}$		0.2247	0.3614	0.3377	**0.0000**	**4.892**	**1.600**	**0.4478**	**0.3455**	25	0.028	0.036
$\rm M1_{S2}$		0.1893	0.4126	0.1085	0.2537	4.825	1.470	0.2349	0.1000	25	0.022	0.033
$\rm M1_{S2'}$	△	0.2017	0.4013	0.1311	0.2677	4.822	1.464	0.2365	0.1000	24	0.018	0.033
$\rm M1_{S3}$	□	0.2022	0.4129	0.2145	0.2002	4.875	1.553	0.3679	0.2493	39	0.030	0.030
$\rm M0_{S3}$	■	0.2058	0.4172	0.2821	**0.0000**	4.917	1.624	0.4616	0.3514	39	0.031	0.031

include AV states in fit ($\rm M1_{S3}$, $\rm M0_{S3}$)

Parameters in **bold** were not varied during the fit

y held fixed, other parameters refitted

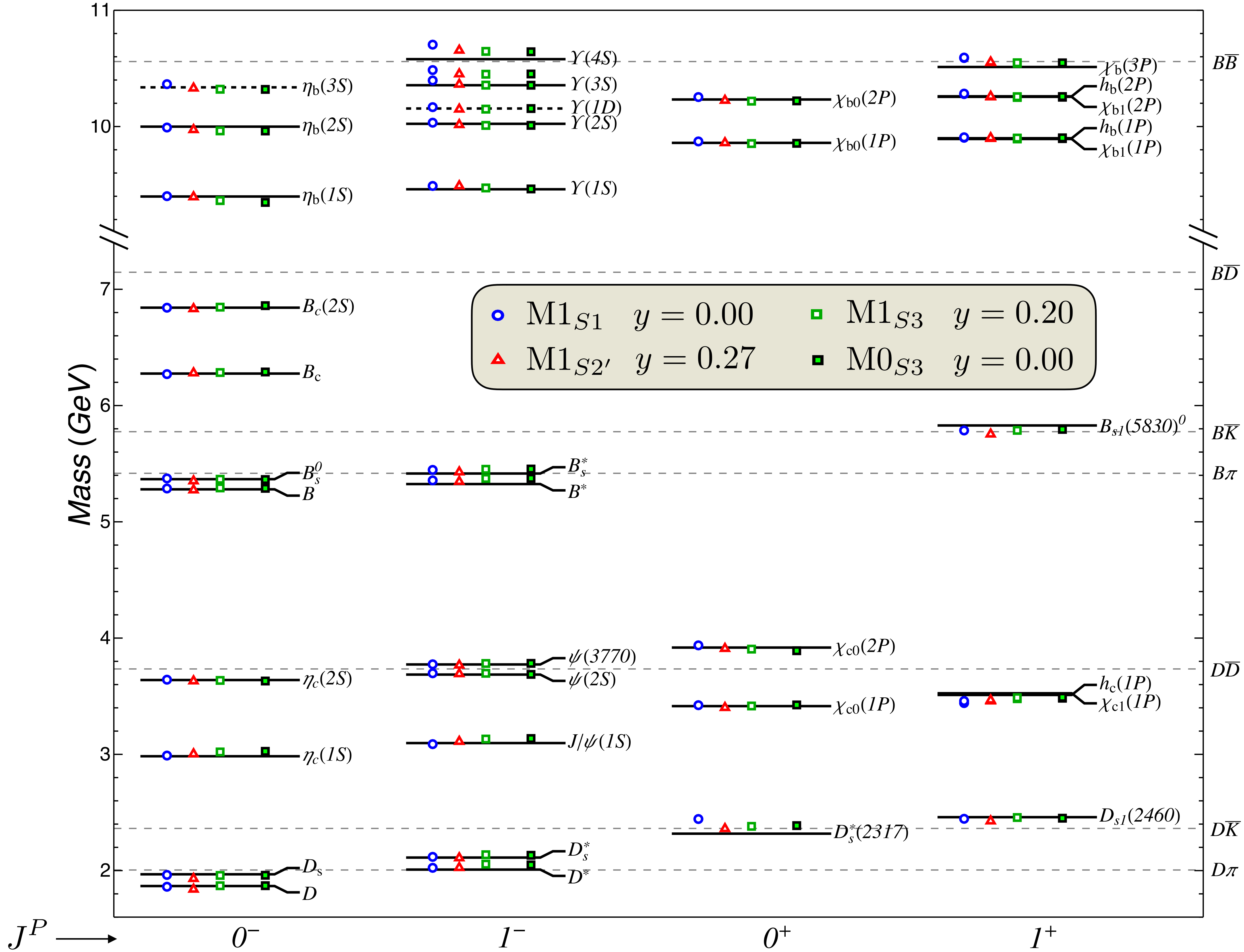
rms difference to fitted masses



- Quality of fits not much improved
- Best model $\rm M1_{S3}$ has y=0.20, but minimum is very shallow

y and quark masses are not much constrained by the mass spectrum.

Mass spectra of heavy and heavy-light mesons

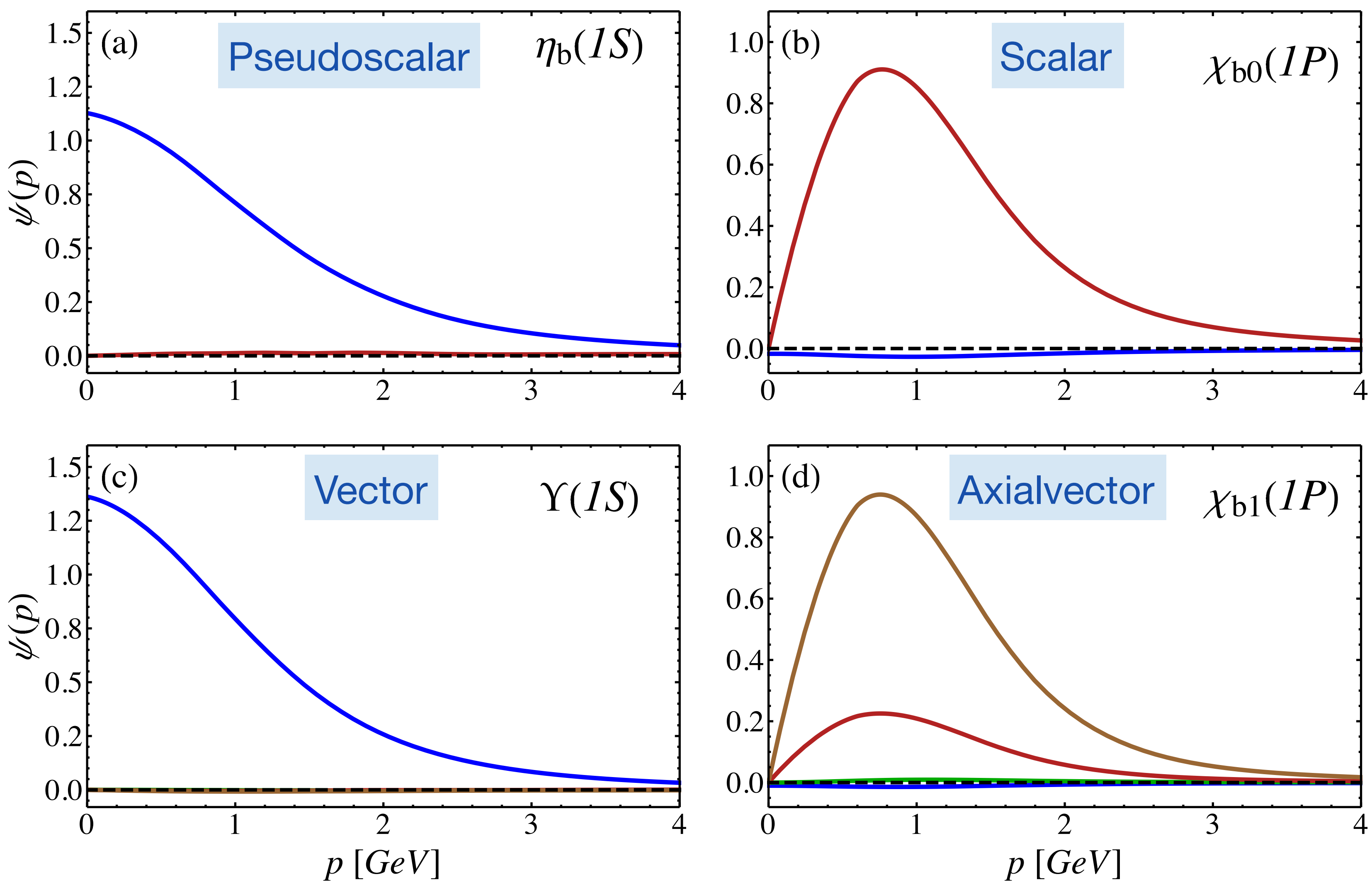


Numerical convergence ($M1_{S3}$)

Meson	J^P	n	Number of splines 12	24	36	48	64
$b\bar{b}$	0^-	1	9.37765	9.37886	9.37917	9.37931	9.37940
		2	9.96915	9.96932	9.96938	9.96939	9.96939
		3	10.33061	10.32623	10.32623	10.32622	10.32621
		4	10.61822	10.61660	10.61646	10.61643	10.61641
$b\bar{b}$	1^-	1	9.47414	9.47411	9.47409	9.47407	9.47406
		2	10.01186	10.01147	10.01141	10.01138	10.01135
		3	10.14699	10.14692	10.14702	10.14714	10.14731
		4	10.36325	10.35767	10.35758	10.35755	10.35751
$c\bar{c}$	0^-	1	3.02240	3.02341	3.02380	3.02400	3.02414
		2	3.63778	3.63814	3.63832	3.63843	3.63850
		3	4.09893	4.09910	4.09925	4.09933	4.09938
		4	4.49972	4.49926	4.49940	4.49947	4.49952
$c\bar{c}$	1^-	1	3.13139	3.13154	3.13163	3.13169	3.13174
		2	3.69834	3.69840	3.69847	3.69853	3.69857
		3	3.75095	3.75366	3.75659	3.75966	3.76395
		4	4.14245	4.14248	4.14257	4.14263	4.14267
$c\bar{q}$	0^-	1	1.86997	1.87122	1.87182	1.87217	1.87247
		2	2.51166	2.51196	2.51213	2.51227	2.51242
		3	2.99045	2.99065	2.99071	2.99079	2.99090
		4	3.40197	3.40221	3.40225	3.40232	3.40241
$c\bar{q}$	1^-	1	2.05555	2.05597	2.05612	2.05620	2.05626
		2	2.61323	2.61365	2.61383	2.61397	2.61411
		3	2.65564	2.65763	2.66005	2.66273	2.66654
		4	3.06017	3.06073	3.06096	3.06115	3.06135

Bottomonium ground-state wave functions

Calculated with model M1$_{S3}$



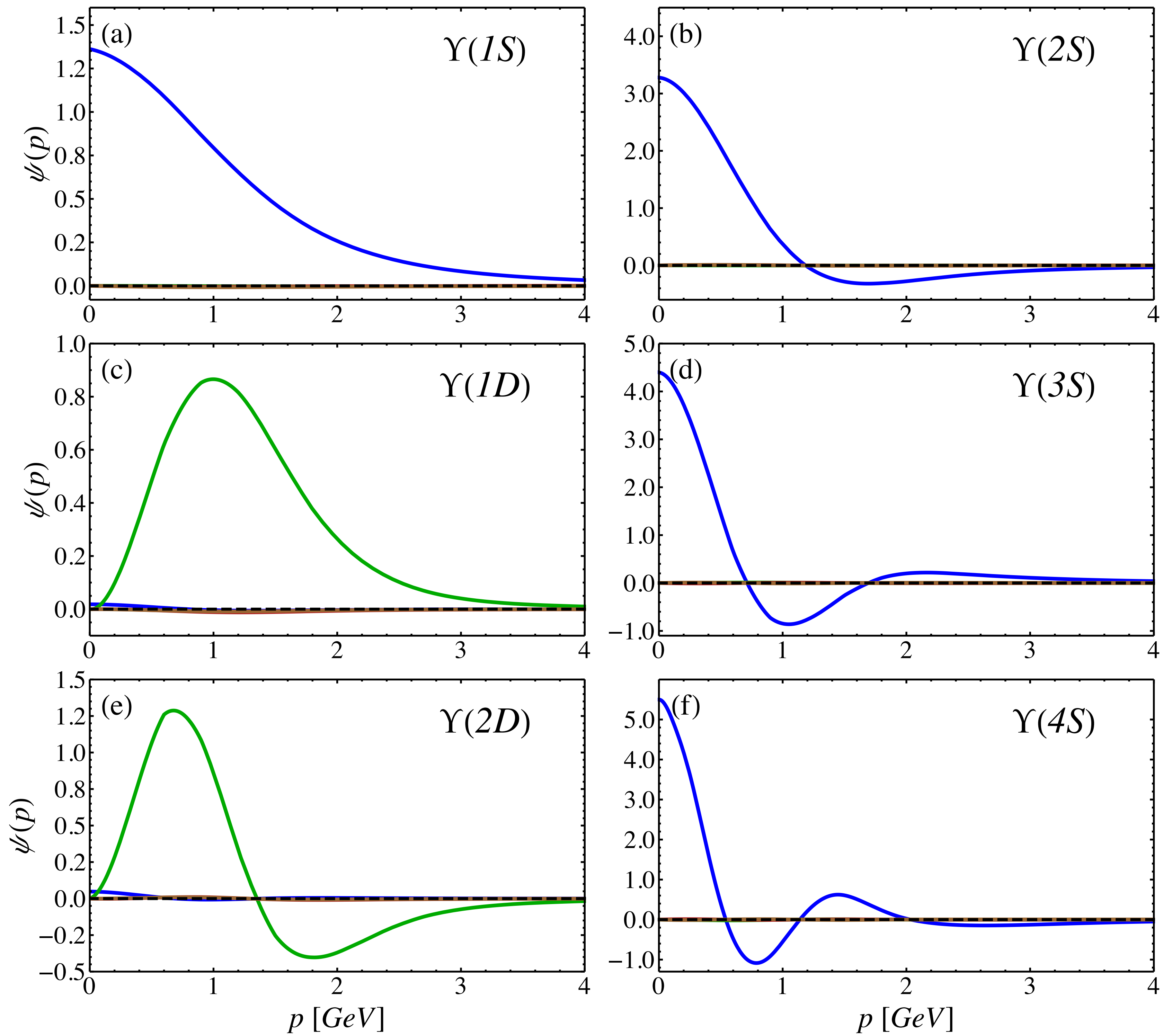
Partial waves
- S
- P

Partial waves
- S
- D
- P_t (spin triplet)
- P_s (spin singlet)

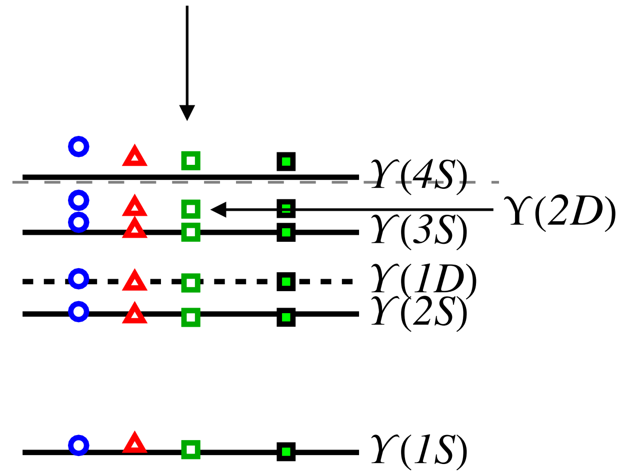
Relativistic wave function components are very small

Radial excitations in vector bottomonium

Wave functions of excited states look reasonable



Model M1S3

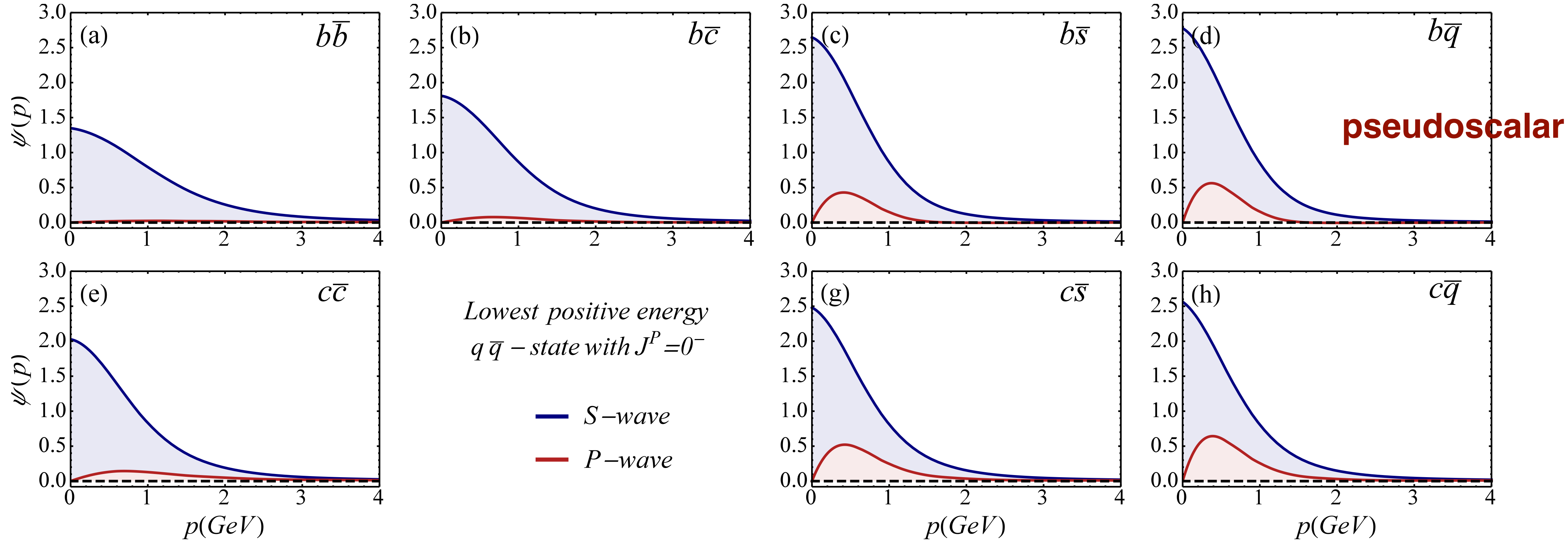


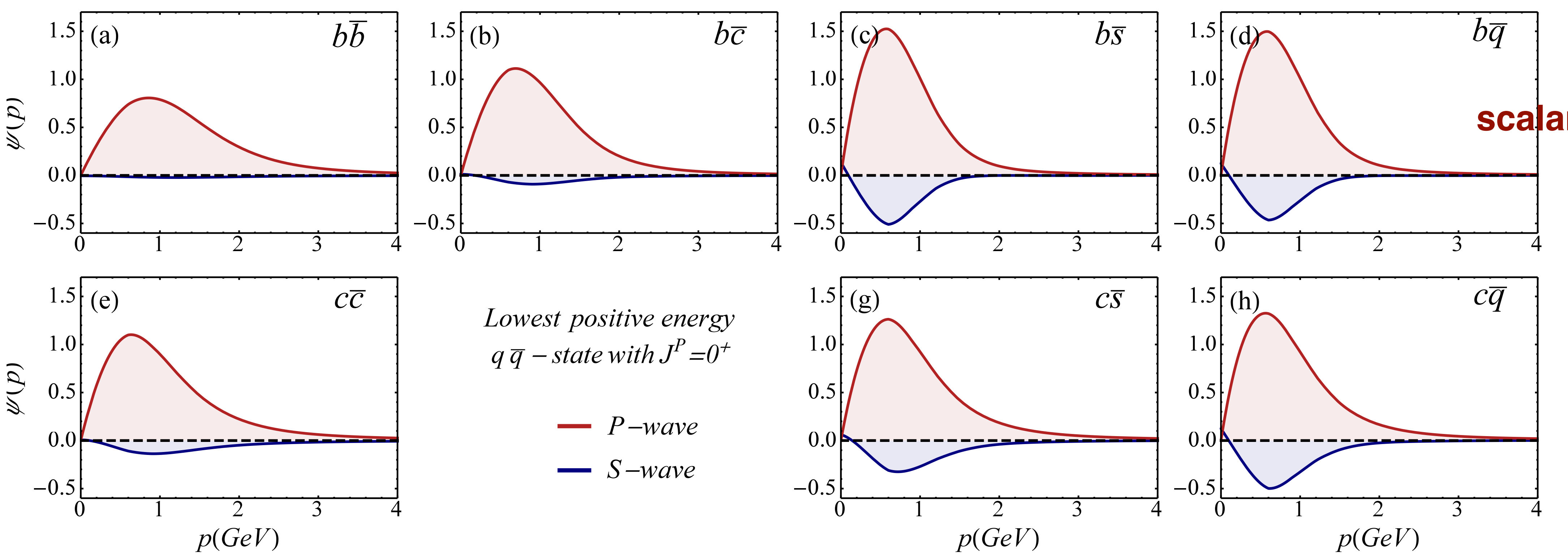
Partial waves

- S
- D
- P_t (spin triplet)
- P_s (spin singlet)

Importance of relativistic components

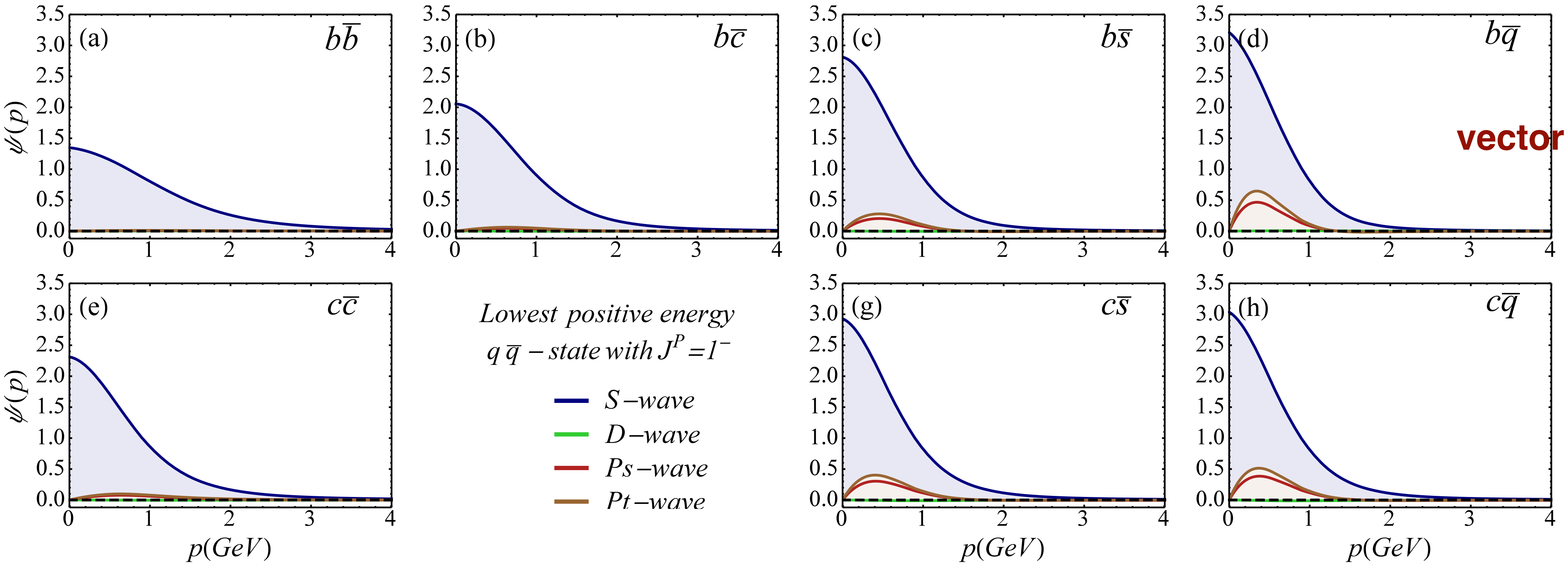
Ground-state wave functions of model $M1_{S3}$.

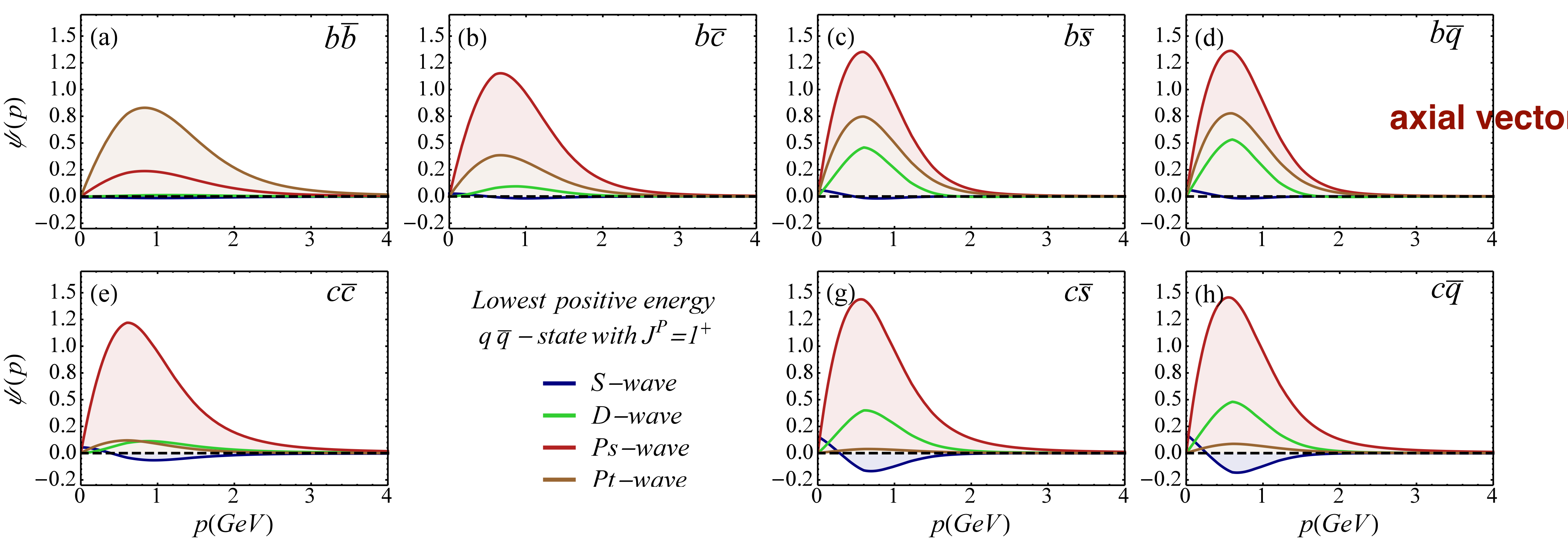




Importance of relativistic components

Ground-state wave functions of model M1$_{S3}$.





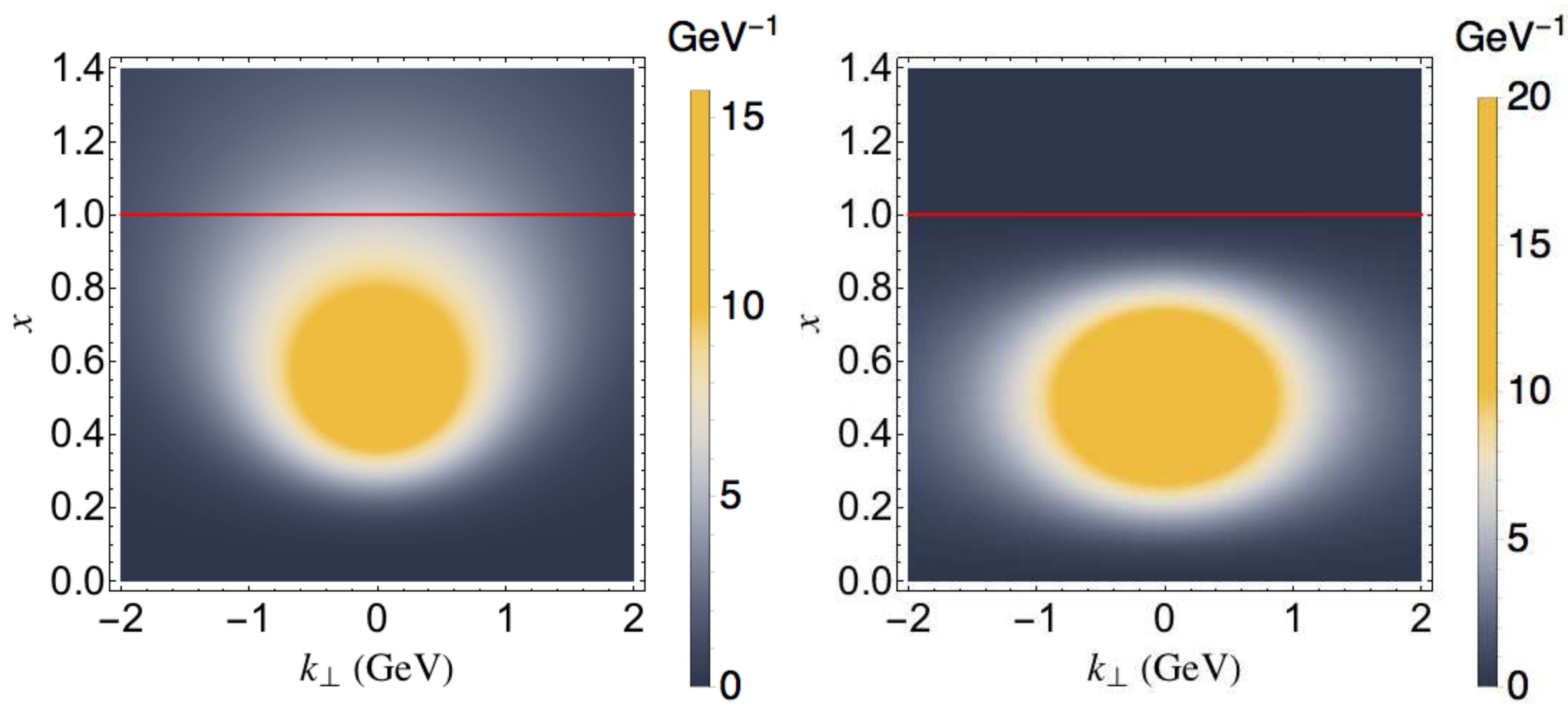
CST light-front wave functions

Leitão, Li, Maris, Peña, AS, Vary, Biernat, EPJC **77**, 696 (2017); arXiv:1705.06178

Comparison of CST and BLFQ wave functions

Calculated CST-LFWF, mapped with the Brodsky-Huang-Lepage prescription (map.)

Example: wave function of J/ψ (1S) with λ=0



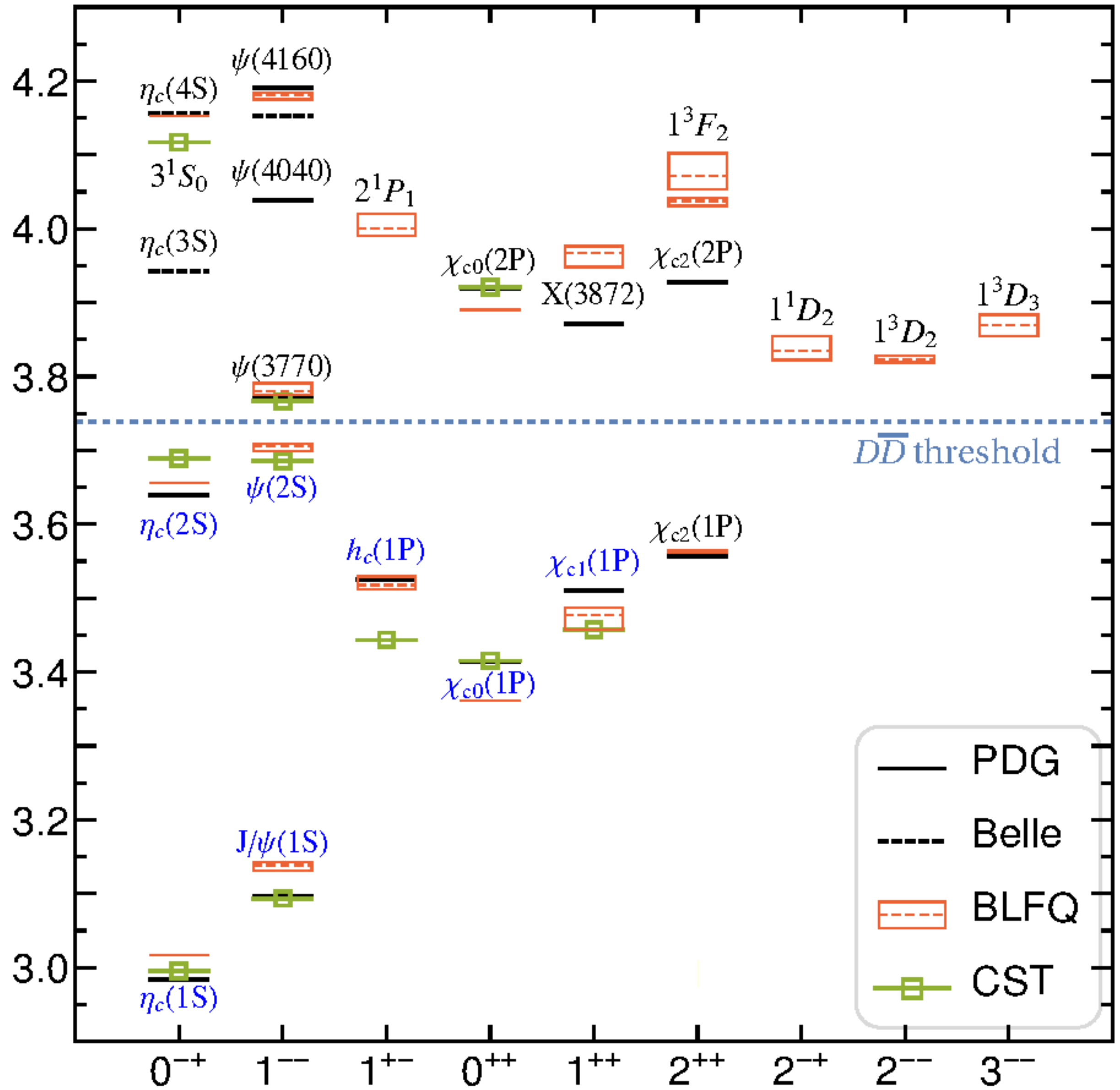
$$x = \frac{k_1^+}{P^+} = \frac{E_k + k^3}{M} = \frac{\sqrt{m^2 + \boldsymbol{k}_\perp^2 + (k^3)^2} + k^3}{M}$$

$$x = \frac{k^+}{P^+} \equiv \frac{E_k + k^3}{2E_k} = \frac{1}{2} + \frac{k^3}{2\sqrt{k_\perp^2 + (k^3)^2 + m^2}}$$

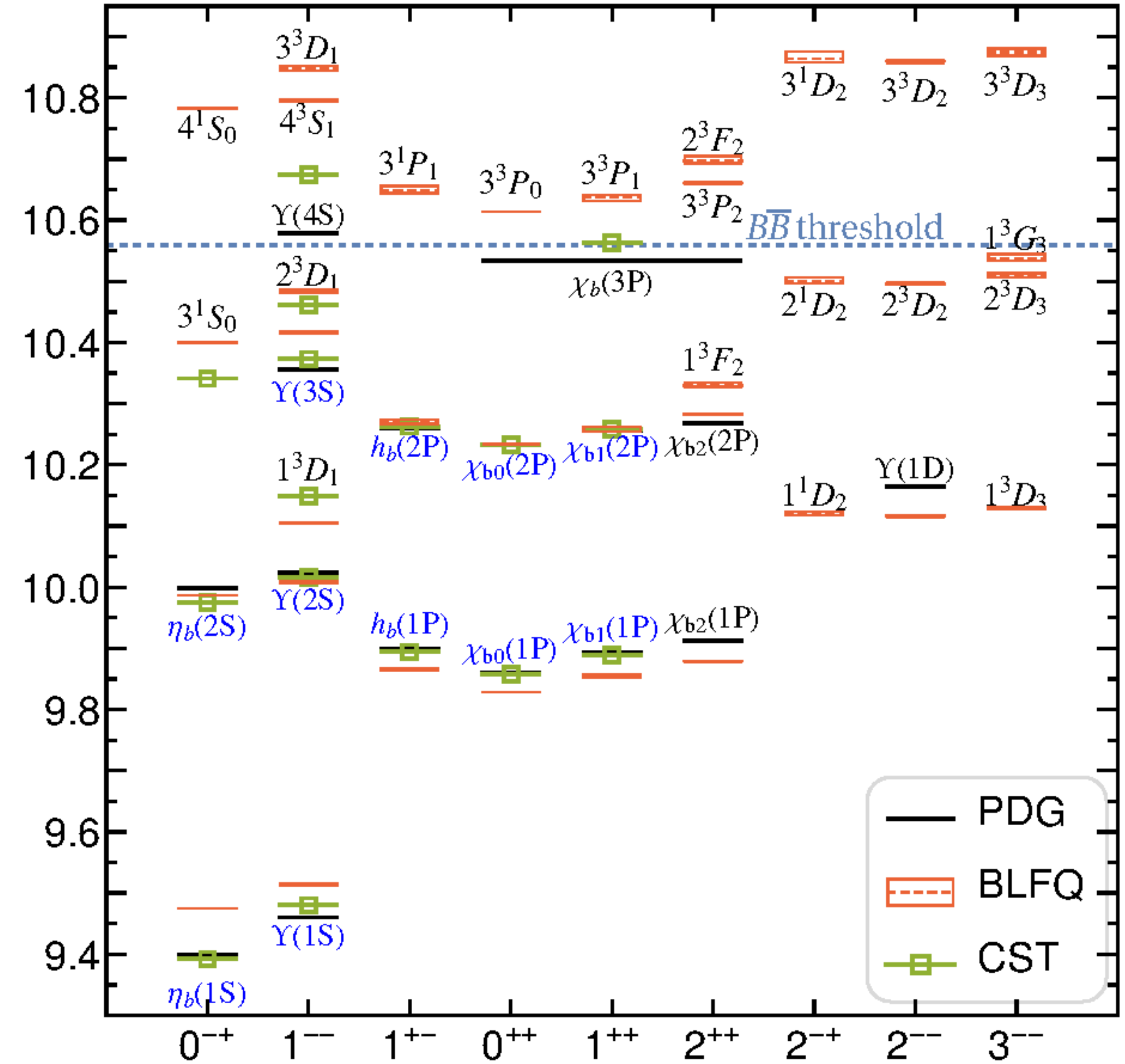
BHL prescription

Quarkonium spectrum with BLFQ and CST

Charmonium



Bottomonium



Rms differences (in MeV) between the calculated and experimental masses shown in blue

	Charmonium	Bottomonium
BLFQ	33	39
CST	42	11

Comparison between BLFQ and CST light front wave functions

BLFQ: Basis Light Front Quantization

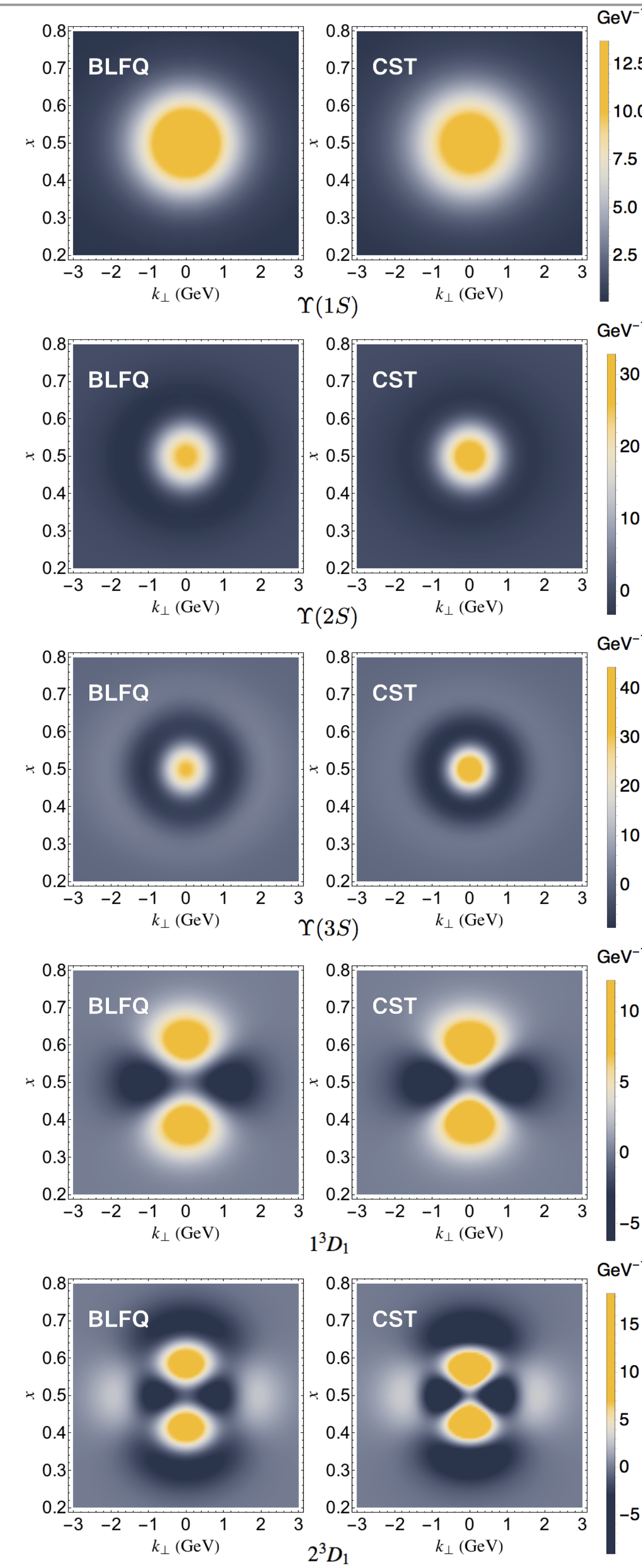
- Effective Hamiltonian from light-front holography
- Contains confining interaction
- Minkowski space

Y. Li, P. Maris, J. Vary, PRD **96**, 016022 (2017)

Leitão, Li, Maris, Peña, AS, Vary, Biernat, EPJC **77**, 696 (2017); arXiv:1705.06178

Vector bottomonium wave functions, dominant components (S=1)

Wave functions are remarkably similar



BLFQ and CST distribution amplitudes

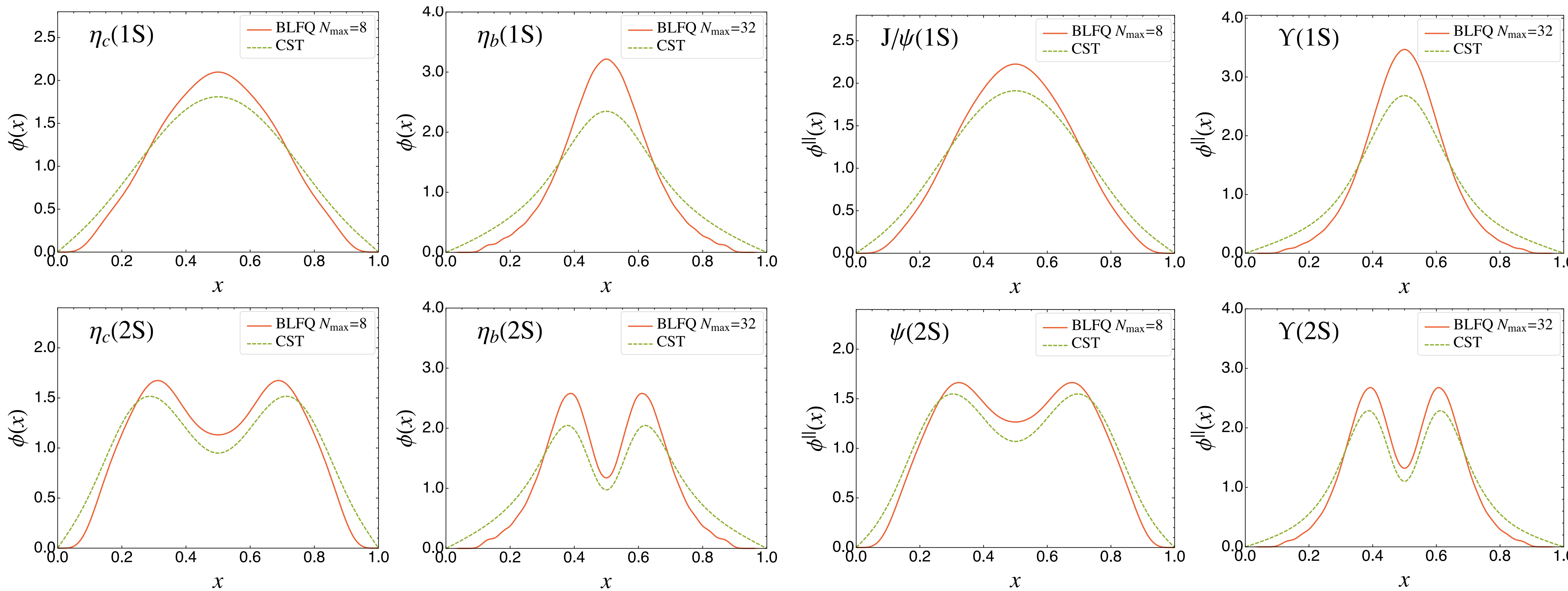
Leading twist distribution amplitudes from BLFQ and CST (map.) wave functions

$$\frac{f_{P,V}}{2\sqrt{2Nc}}\phi_{P,V^{||}}(x;\mu) = \frac{1}{\sqrt{x(1-x)}}\int_0^{k_\perp\le\mu}\frac{d^2\mathbf{k}_\perp}{2(2\pi)^3}\psi^{\lambda=0}_{\uparrow\downarrow\mp\downarrow\uparrow}(\mathbf{k}_\perp,x)$$

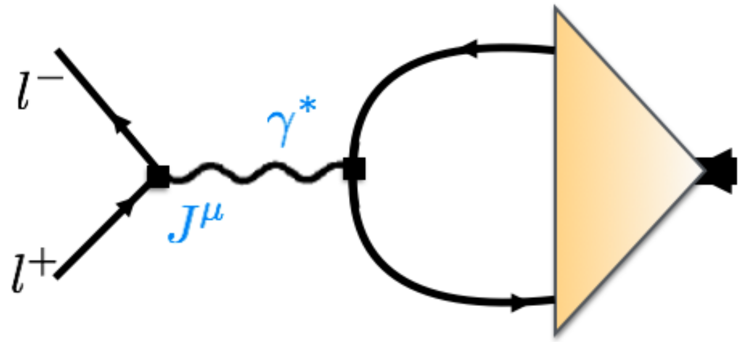
- PS
+ V

Pseudoscalar quarkonia | **Vector quarkonia**

Charmonium | Bottomonium | Charmonium | Bottomonium



Heavy quarkonium decay constants



Very precise measurements for some charmonium and bottomonium PS and V states (no data for S and AV)

Nonrelativistic: depend on $\Psi(r=0)$ (only S-waves contribute)

Relativistic: all partial waves can contribute

Pseudoscalar mesons

$$f_P = \frac{1}{\pi}\sqrt{\frac{N_c}{2\mu_P}}\int_0^\infty dk\, k^2 \sqrt{\left(1+\frac{m_1}{E_{1k}}\right)\left(1+\frac{m_2}{E_{2k}}\right)}\left[(1-\tilde{k}_1\tilde{k}_2)\psi_s(k) + (\tilde{k}_1+\tilde{k}_2)\psi_p(k)\right]$$

Vector mesons

$$f_V = \frac{1}{\pi}\sqrt{\frac{N_c}{2\mu_V}}\int_0^\infty dk\, k^2 \sqrt{\left(1+\frac{m_1}{E_{1k}}\right)\left(1+\frac{m_2}{E_{2k}}\right)}\Bigg[(1+\frac{1}{3}\tilde{k}_1\tilde{k}_2)\psi_s(k) - \frac{2\sqrt{2}}{3}\tilde{k}_1\tilde{k}_2\psi_d(k) + \frac{1}{\sqrt{3}}(\tilde{k}_1+\tilde{k}_2)\psi_{p_s}(k) + \sqrt{\frac{2}{3}}(\tilde{k}_2-\tilde{k}_1)\psi_{p_t}(k)\Bigg]$$

$$\tilde{k}_i \equiv \frac{|\mathbf{k}_i|}{E_{ik}+m_i}$$

Quarkonium decay constants (preliminary results)

Refit with stronger cut-off in OGE kernel (spectrum almost unchanged)

Quark content	n	Meson	$J^{P(C)}$	PDG	Lattice	DSE I	DSE II	BLFQ	$\mathrm{M}_{Q\bar{Q}}\Lambda_{\mathrm{OGE}}$ (this work)
$b\bar{b}$	1	$\eta_b(1S)$	0^{-+}	–	667^{+6}_{-6}	773	756	589	795
	2	$\eta_b(2S)$	0^{-+}	–	–	419(8)	285	427	596
	3	$\eta_b(3S)$	0^{-+}	–	–	534(57)	333	331	536
	4	$\eta_b(4S)$	0^{-+}	–	–	–	40(15)	–	503
	1	$\Upsilon(1S)$	1^{--}	689^{+5}_{-5}	649^{+31}_{-31}	768	707	689	703
	2	$\Upsilon(2S)$	1^{--}	479^{+4}_{-4}	481^{+39}_{-39}	467(17)	393	484	573
	3	1^3D_1	1^{--}	–	–	41(7)	371(2)	4.2	26
	4	$\Upsilon(3S)$	1^{--}	414^{+4}_{-4}	–	–	9(5)	366	536
	5	2^3D_1	1^{--}	–	–	–	165(50)	–	38
	6	$\Upsilon(4S)$	1^{--}	328^{+17}_{-18}	–	–	20(15)	–	518
$c\bar{c}$	1	$\eta_c(1S)$	0^{-+}	330^{+13}_{-13}	393^{+9}_{-9}	401	378	368	547
	2	$\eta_c(2S)$	0^{-+}	211^{+35}_{-42}	–	244(12)	82	280	461
	3	$\eta_c(3S)$	0^{-+}	–	–	145(145)	206	–	417
	4	$\eta_c(4S)$	0^{-+}	–	–	–	87	–	387
	1	J/ψ	1^{--}	407^{+5}_{-5}	405^{+6}_{-6}	450	411	404	525
	2	$\psi(2S)$	1^{--}	290^{+2}_{-2}	–	30(3)	155	290	531
	3	$\psi(3770)$	1^{--}	97.7^{+3}_{-3}	–	118(91)	45	0.9	98

Summary

- With the simplest, one-channel CST equation and a few global parameters, we get a very nice description of the heavy and heavy-light meson spectrum
- (S+PS) confining kernel with ~ 0%—30% admixture of V coupling is compatible with the data
- In heavy quarkonia, we find remarkable similarities between CST LFWF (with BHL prescription) and BLFQ LFWF by Li, Vary, Maris, even in excited states
- Decay constants are very sensitive to details — stronger constraints on kernel

Next steps:

- Include dynamical quark mass (mass function) from quark self-interaction
- Inclusion of running quark-gluon coupling
- Calculation of tensor mesons (spin ≥ 2)
- Extension of current model to the light-quark sector (requires 4-channel eq.)
- Calculation of parton distribution functions
- Calculate relativistic quark-antiquark states with exotic J^{PC}