# Power corrections to TMD factorization

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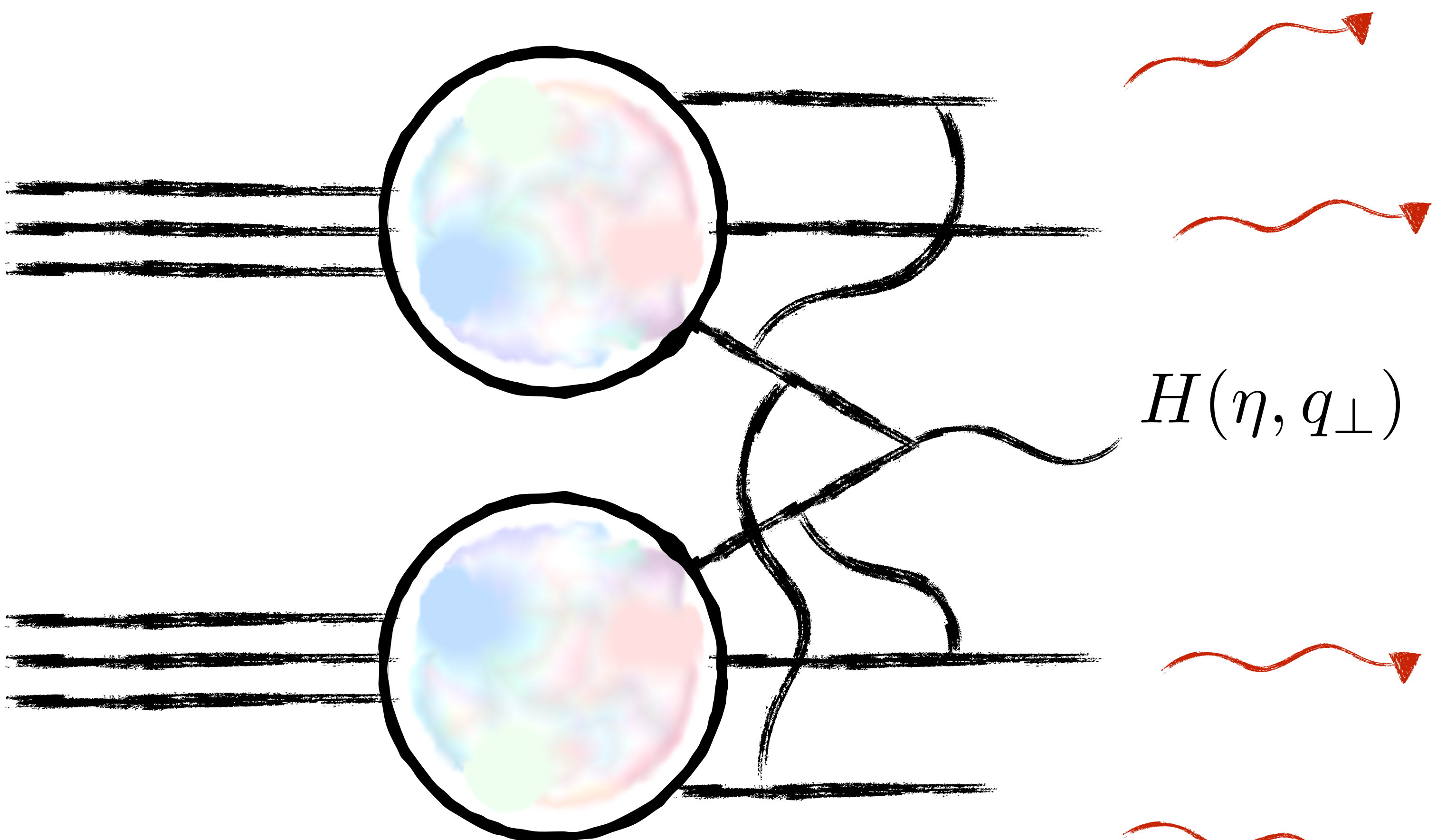
# TMD factorization

$$\frac{d\sigma}{d\eta d^2 q_\perp} = \sum_f \int d^2 b_\perp e^{i(q,b)_\perp} \mathcal{D}_{f/A}(x_A, b_\perp, \eta) \mathcal{D}_{f/B}(x_B, b_\perp, \eta) \sigma(ff \to H)$$

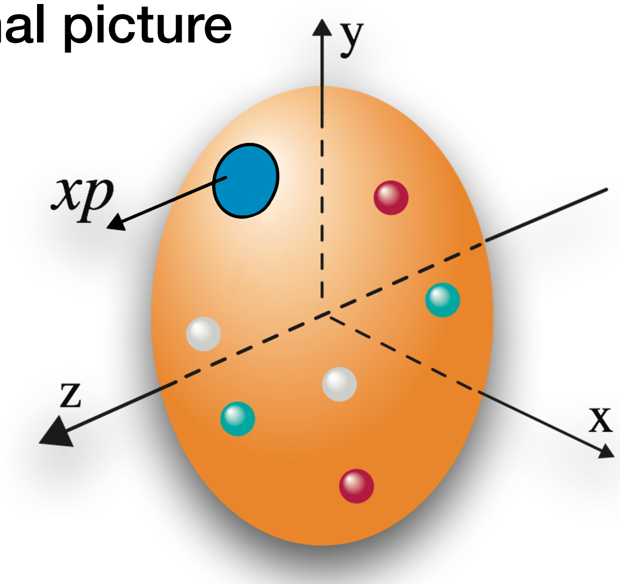


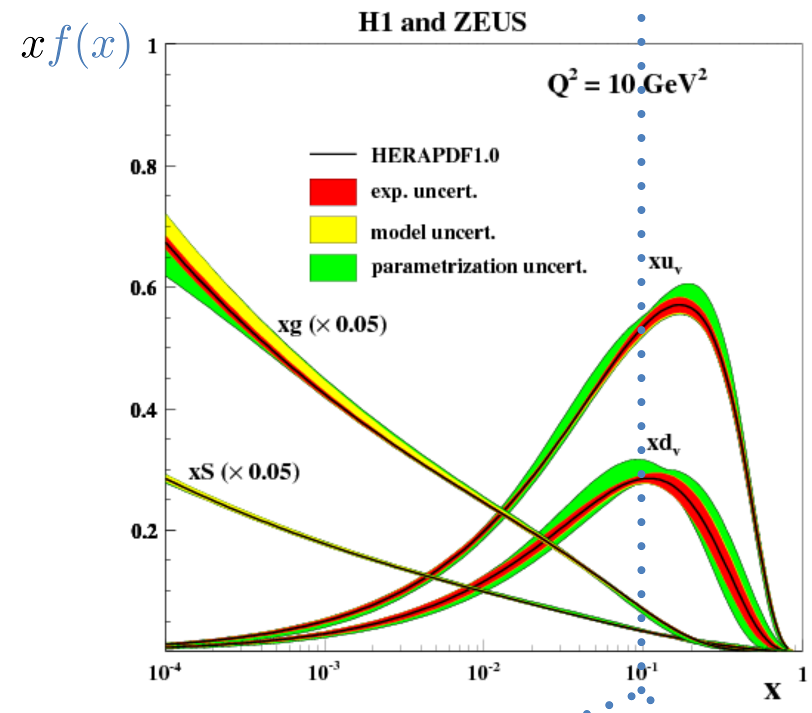


J.C. Collins, D.E. Soper and G. Sterman, Phys. Lett. B 109 (1982) 388;
J.C. Collins, D.E. Soper and G.F. Sterman, Nucl. Phys. B 250 (1985) 199;
G.T. Bodwin, Phys. Rev. D 31 (1985) 10;
X.-d. Ji, J.-p. Ma and F. Yuan, Phys. Rev. D 71 (2005) 034005;
M.G. Echevarria, A. Idilbi and I. Scimemi, JHEP 07 (2012) 002

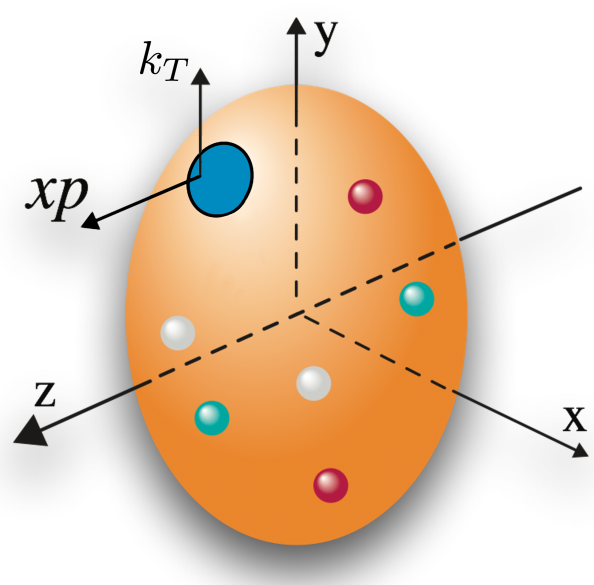
# Distribution functions

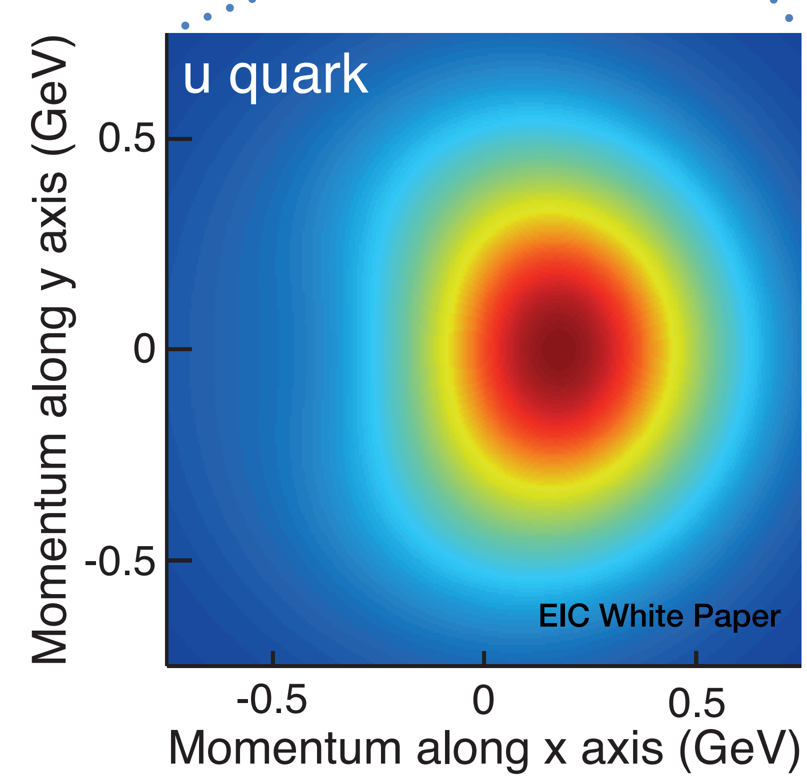
One-dimensional picture



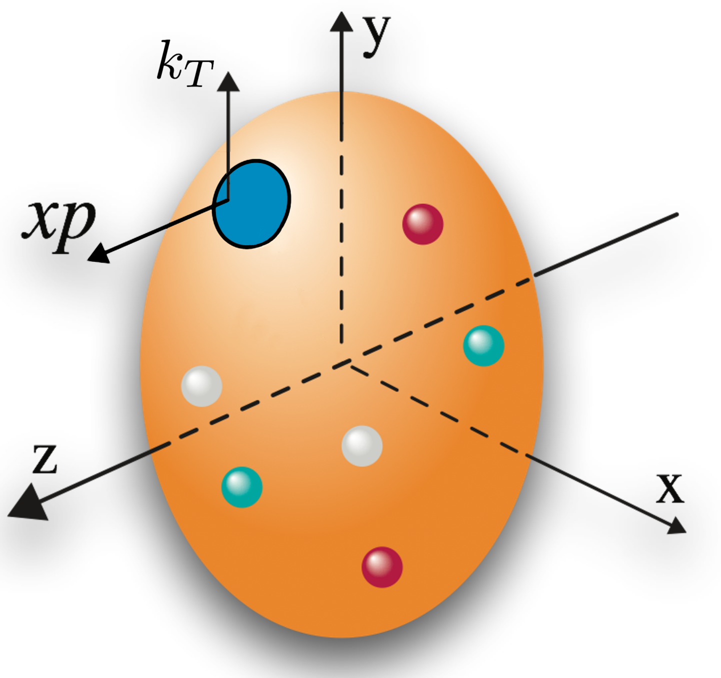

Three-dimensional picture
(Transverse momentum dependent distribution functions)

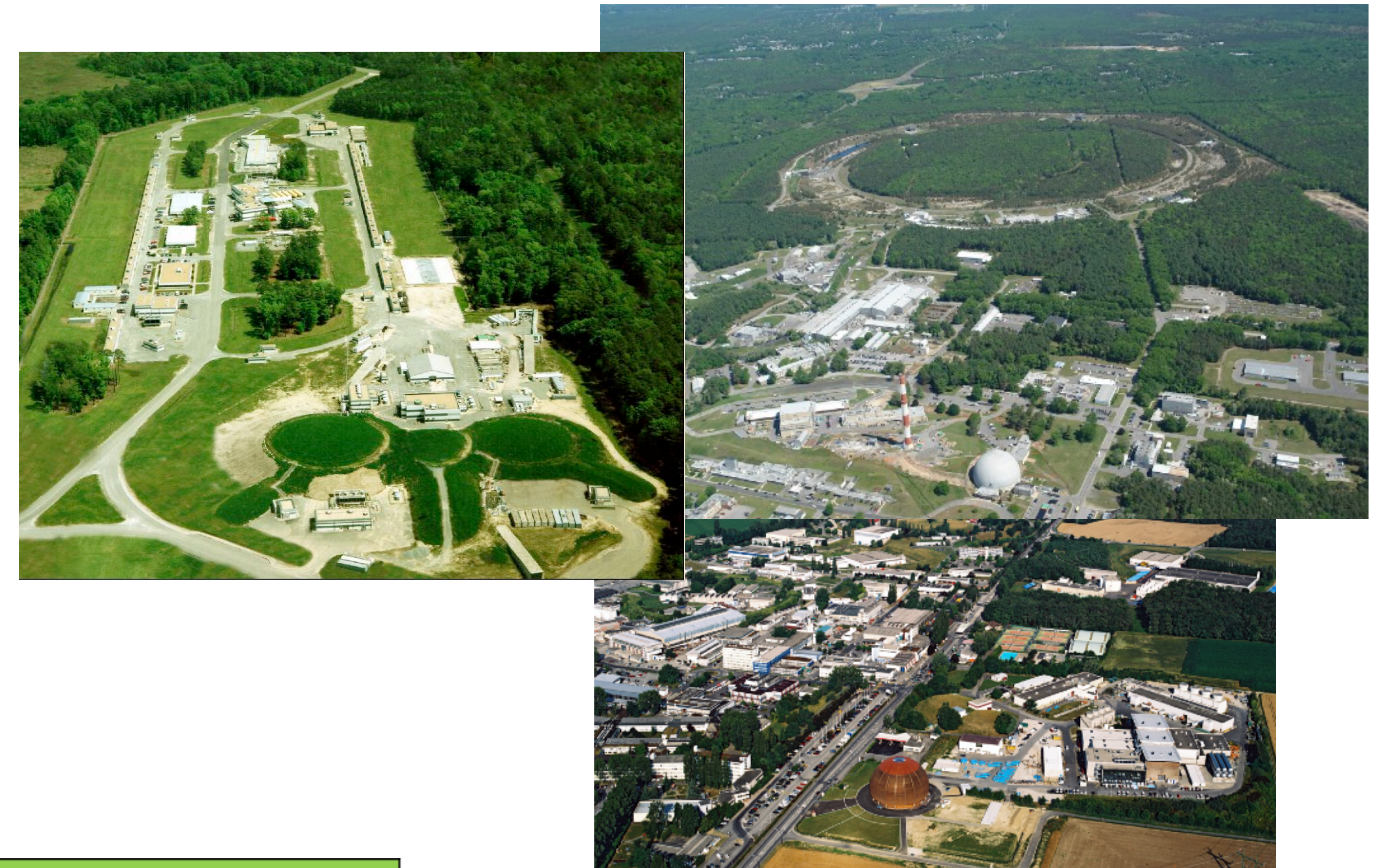


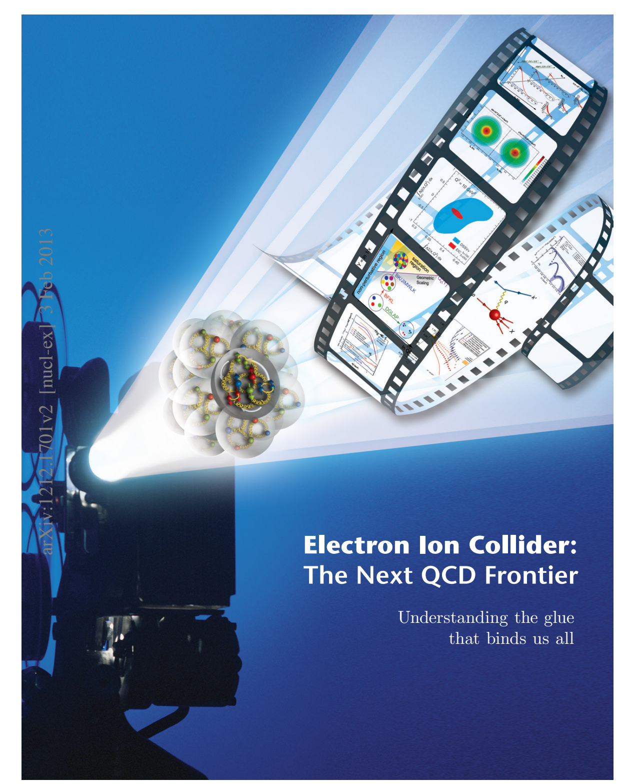

TMD factorization

# Quark TMDs



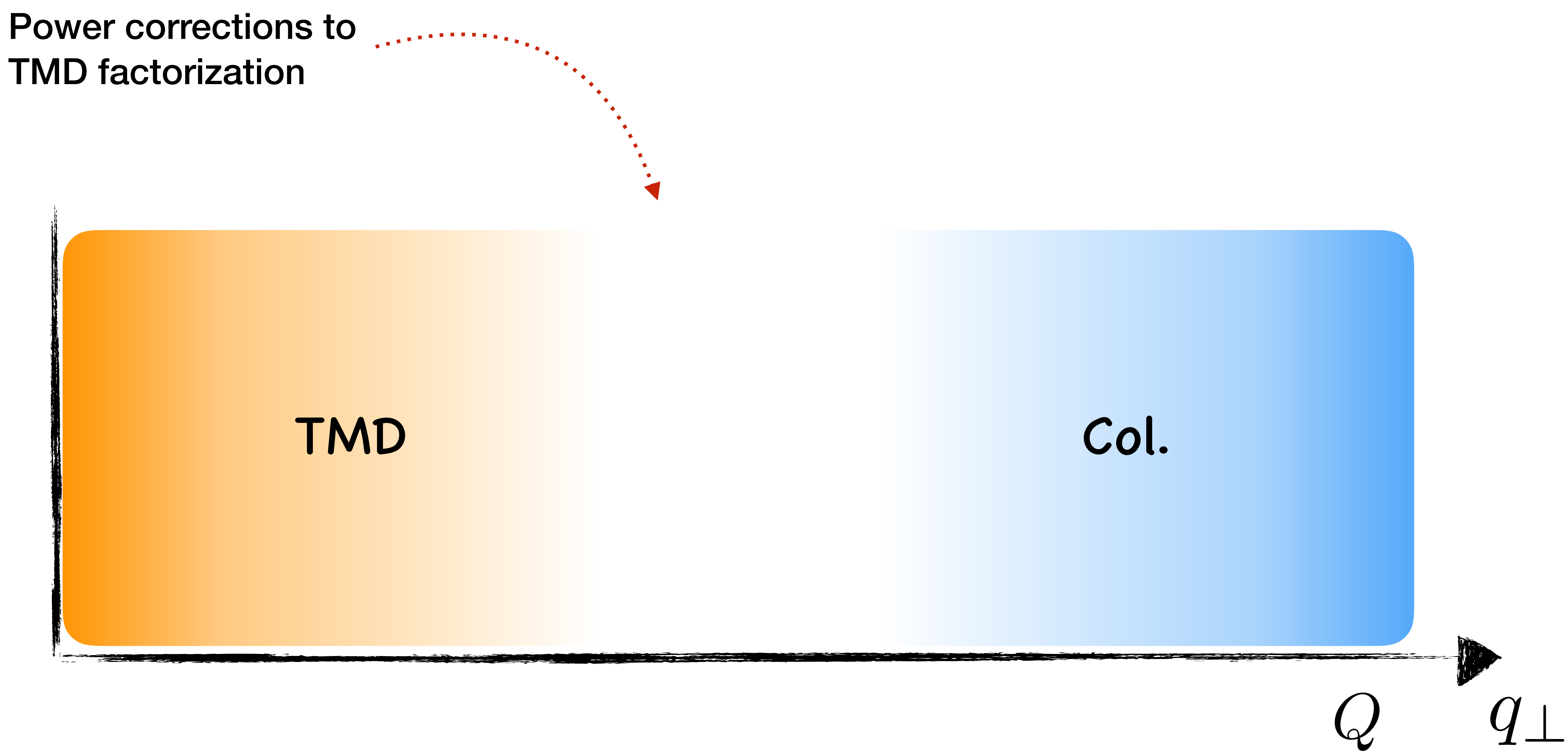



| Nucleon Polarization | Quark Polarization: Unpolarized (U) | Quark Polarization: Longitudinally Polarized (L) | Quark Polarization: Transversely Polarized (T) |
|---|---|---|---|
| U | $f_1(x,k_T^2)$ | | $h_1^{\perp}(x,k_T^2)$ Boer-Mulders |
| L | | $g_1(x,k_T^2)$ Helicity | $h_{1L}^{\perp}(x,k_T^2)$ Long-Transversity |
| T | $f_1^{\perp}(x,k_T^2)$ Sivers | $g_{1T}(x,k_T^2)$ Trans-Helicity | $h_1(x,k_T^2)$ Transversity; $h_{1T}^{\perp}(x,k_T^2)$ Pretzelosity |



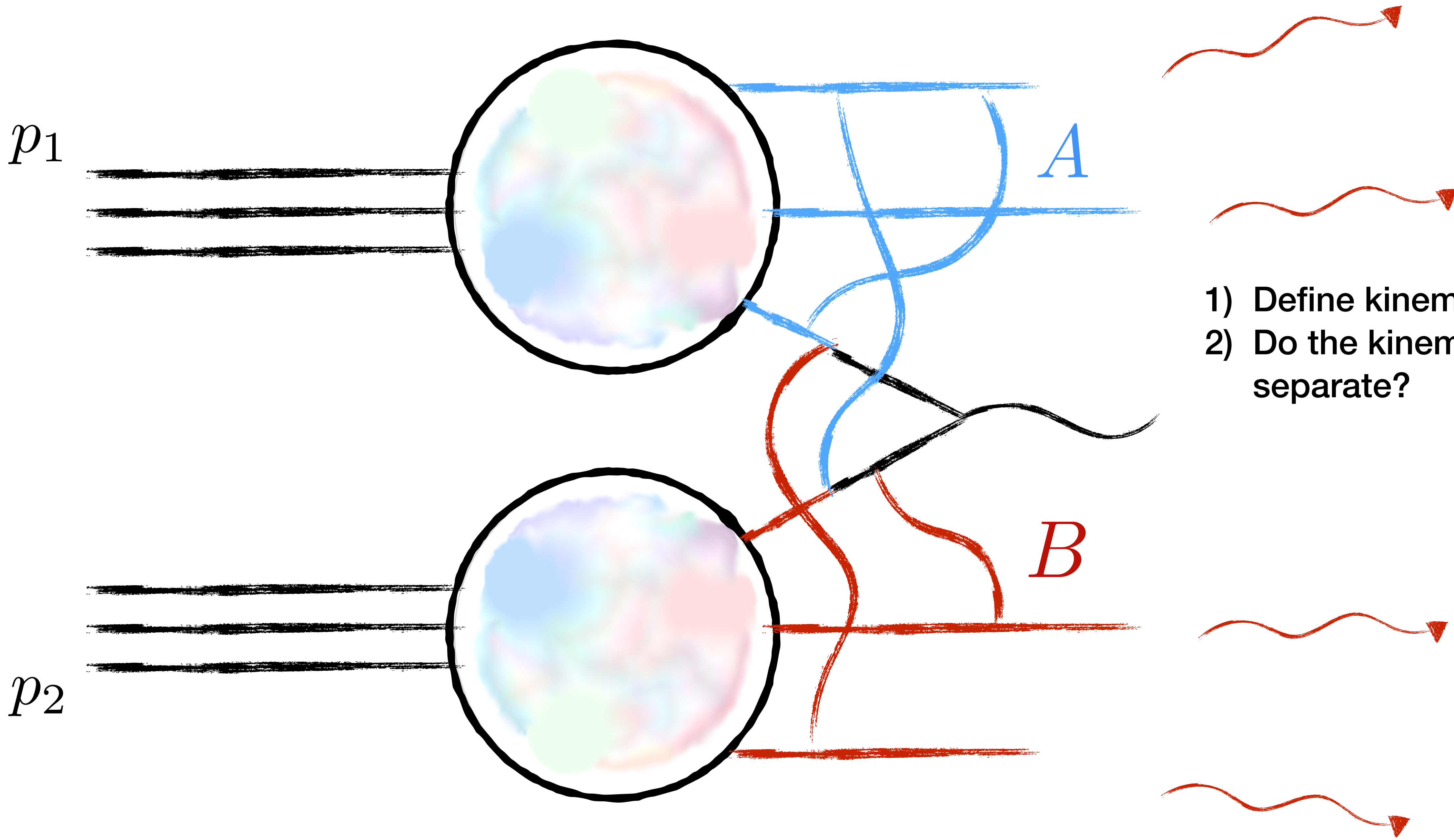
# TMD vs. collinear factorization

$$\frac{d\sigma}{d\eta d^2 q_\perp} = \sum_f \int d^2 b_\perp e^{i(q,b)_\perp} \mathcal{D}_{f/A}(x_A, b_\perp, \eta) \mathcal{D}_{f/B}(x_B, b_\perp, \eta) \sigma(ff \to H)$$



# Kinematic modes

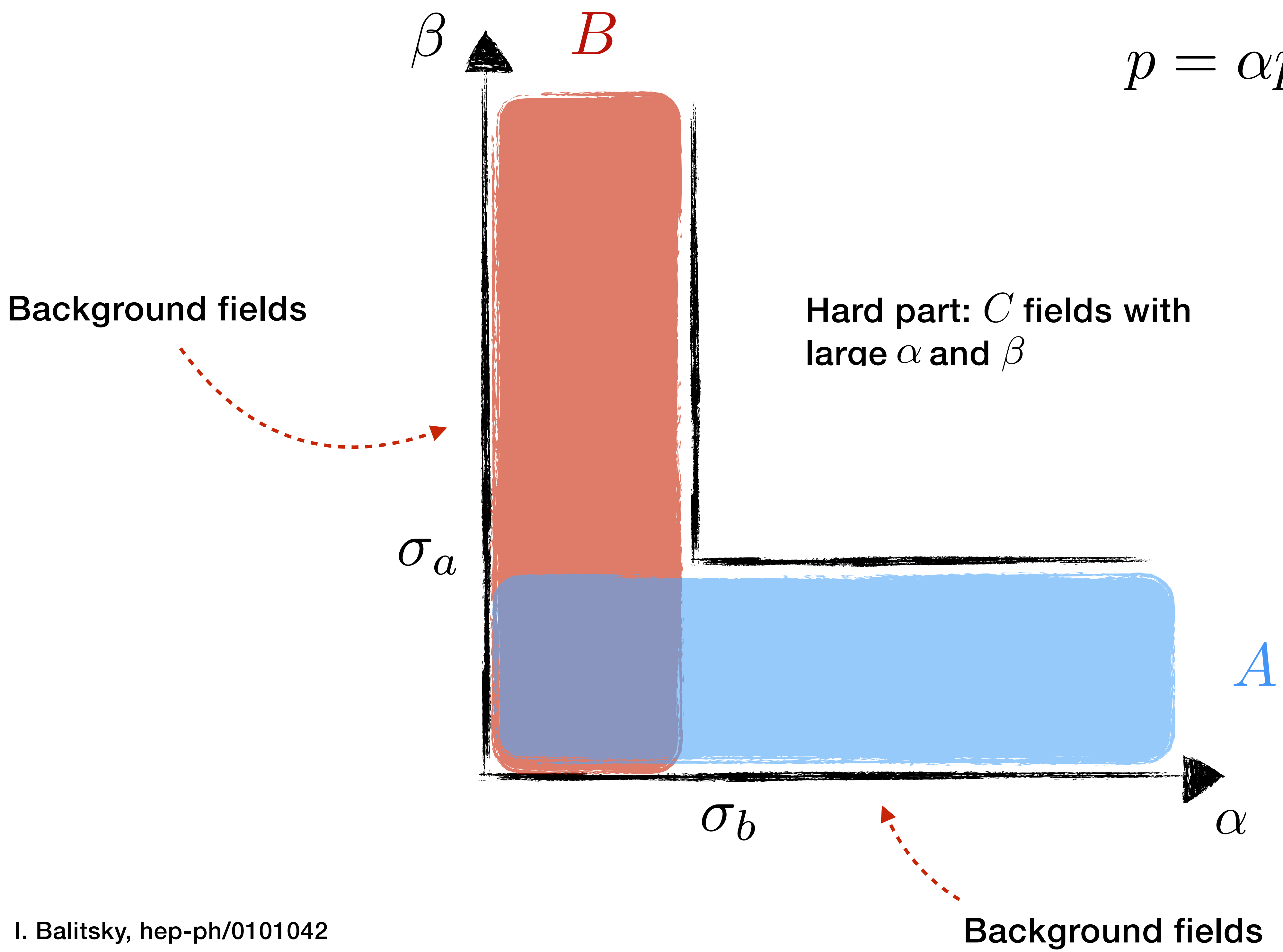
$$\frac{d\sigma}{d\eta d^2 q_\perp} = \sum_f \int d^2 b_\perp e^{i(q,b)_\perp} \mathcal{D}_{f/A}(x_A, b_\perp, \eta) \mathcal{D}_{f/B}(x_B, b_\perp, \eta) \sigma(ff \to H)$$




1) Define kinematical modes
2) Do the kinematics modes separate?

# Separation of kinematic modes

$$p = \alpha p_1 + \beta p_2 + p_\perp$$




I. Balitsky, hep-ph/0101042

# Cut-off parameters

We work at the tree level. There is no large logarithms

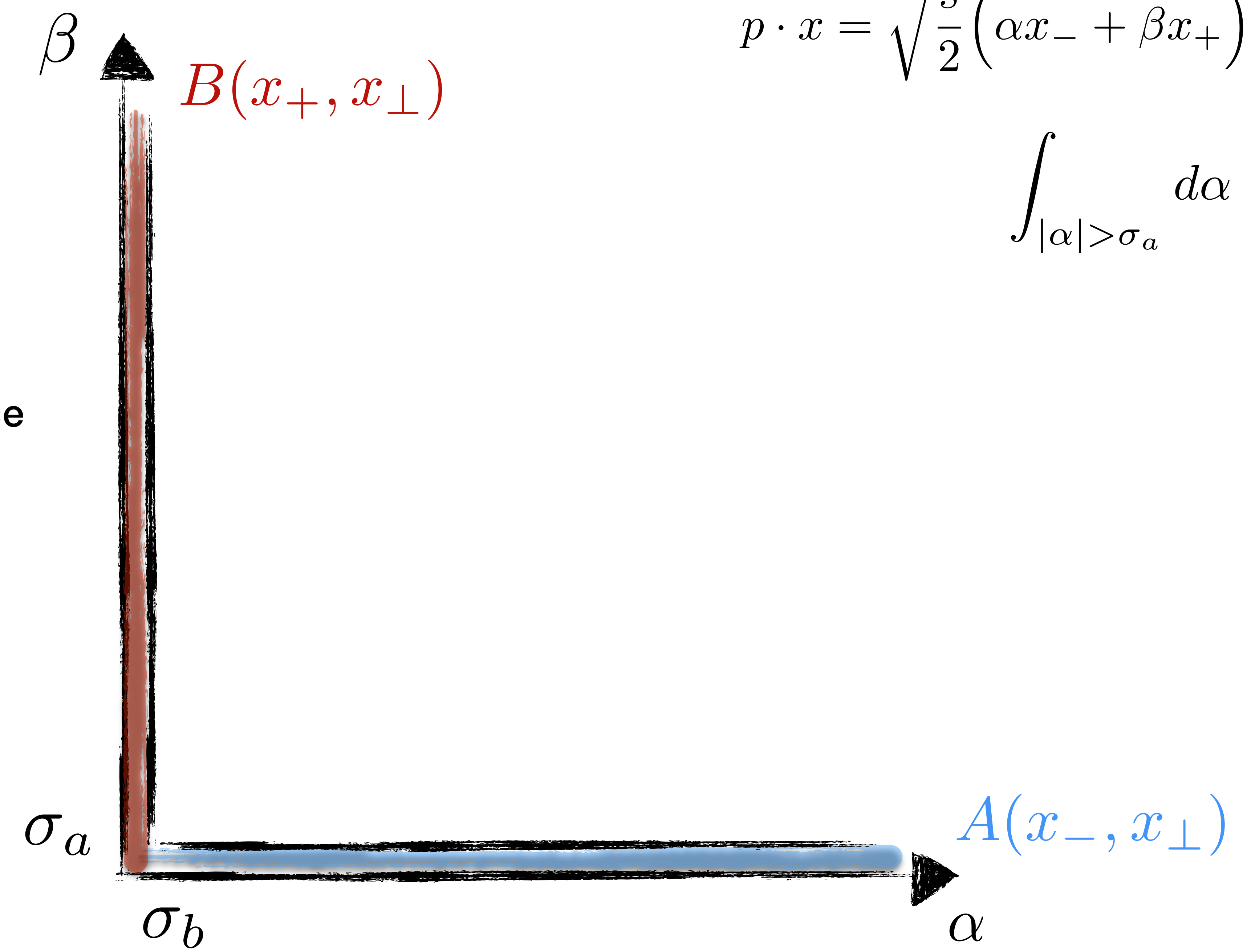
$$\ln \frac{\sigma_a \sigma_b s}{M_Z^2}$$

We neglect dependence on cut-off parameters

$$\sigma_a, \sigma_b \to 0$$

$$p \cdot x = \sqrt{\frac{s}{2}} \left( \alpha x_- + \beta x_+ \right) - (p, x)_\perp$$

$$\int_{|\alpha| > \sigma_a} d\alpha \to \int_{-\infty}^{\infty} d\alpha$$

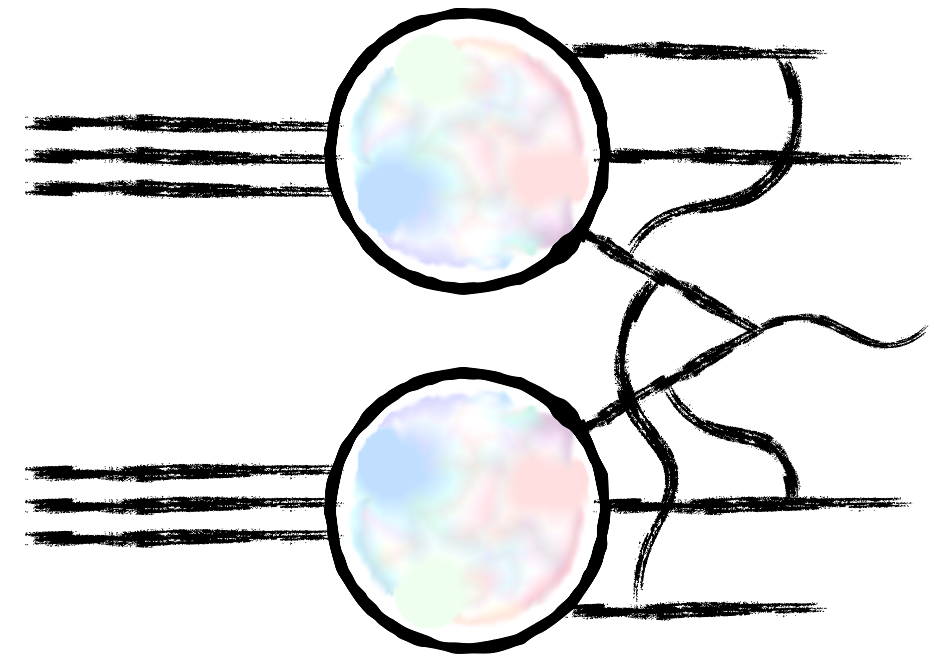


# Z boson production

$$\mathcal{L}_Z = \int dx \; J_\mu Z^\mu(x)$$

$$J_\mu = -\frac{e}{2s_W c_W} \sum_f \bar{\psi}_f \gamma_\mu (g_f^V - g_f^A \gamma_5) \psi_f$$

$$d\sigma = \frac{\pi}{2s} \frac{d^3q}{E_q} [-W(p_A, p_B, q)]$$



$$W(p_A, p_B, q) \stackrel{\text{def}}{=} \frac{1}{(2\pi)^4} \int d^4x \; e^{-iqx} \langle p_A, p_B | J_\mu(x) J^\mu(0) | p_A, p_B \rangle$$

**Calculate the hadronic tensor and present the result in a factored form**

# Hadronic tensor and functional integral

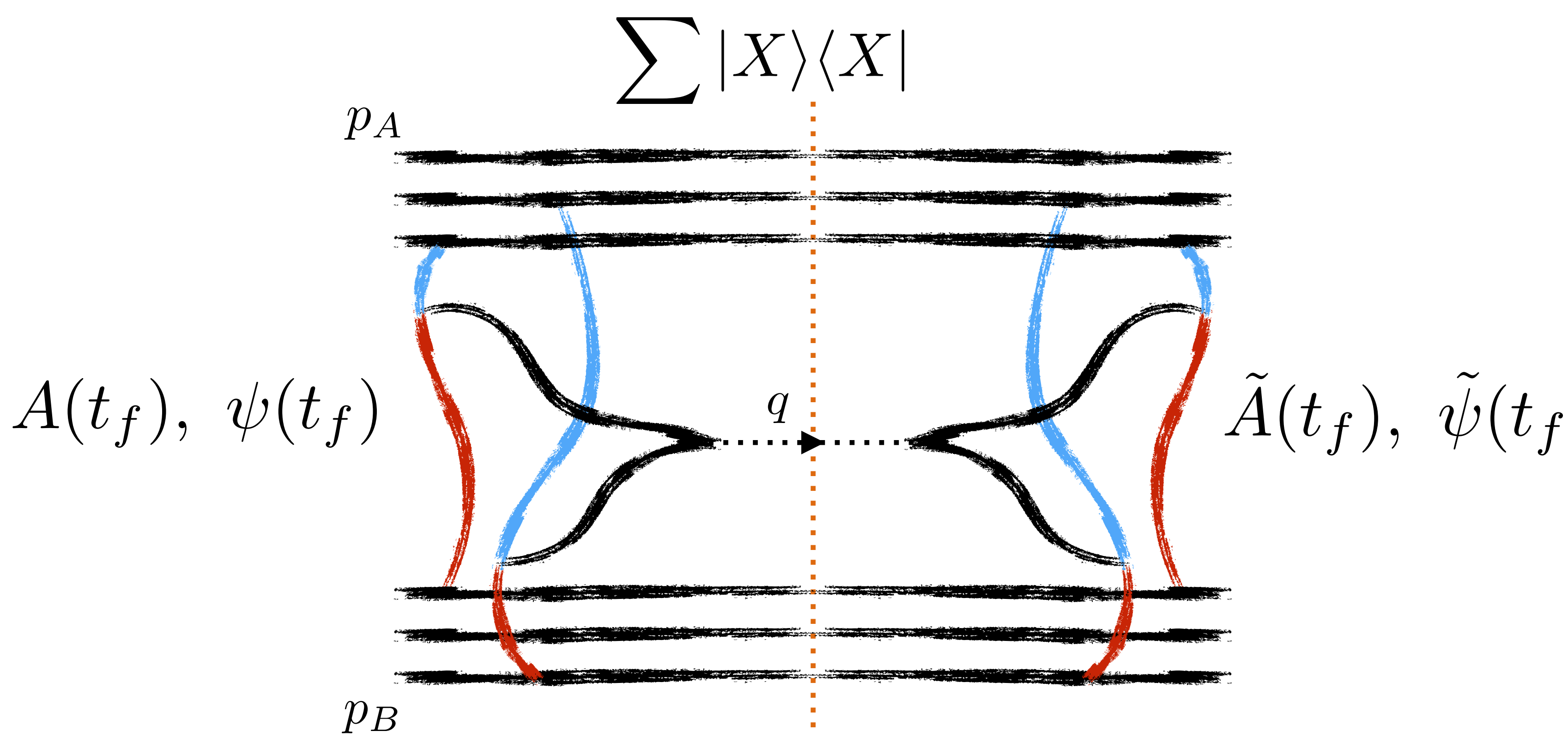
**Functional integral representation for hadronic tensor**

$$
\begin{aligned}
(2\pi)^4 W(p_A, p_B, q) \;&=\; \sum_X \int d^4x \; e^{-iqx} \langle p_A, p_B | J^\mu(x) | X \rangle \langle X | J_\mu(0) | p_A, p_B \rangle \\
&= \lim_{\substack{t_f \to \infty \\ t_i \to -\infty}} \int d^4x \; e^{-iqx} \int^{\tilde{A}(t_f) = A(t_f)} D\tilde{A}_\mu D A_\mu \int^{\tilde{\psi}(t_f) = \psi(t_f)} D\tilde{\bar{\psi}} D\tilde{\psi} D\bar{\psi} D\psi \; \Psi^*_{p_A}(\tilde{\vec{A}}(t_i), \tilde{\psi}(t_i)) \\
&\times \Psi^*_{p_B}(\tilde{\vec{A}}(t_i), \tilde{\psi}(t_i)) e^{-iS_{\mathrm{QCD}}(\tilde{A}, \tilde{\psi})} e^{iS_{\mathrm{QCD}}(A, \psi)} \tilde{J}_\mu(x) J^\mu(0) \Psi_{p_A}(\vec{A}(t_i), \psi(t_i)) \Psi_{p_B}(\vec{A}(t_i), \psi(t_i))
\end{aligned}
$$

**Boundary conditions**

$$\tilde{\psi}(t_f) = \psi(t_f)$$

$$\tilde{A}(t_f) = A(t_f)$$



# Separation of modes and functional integral

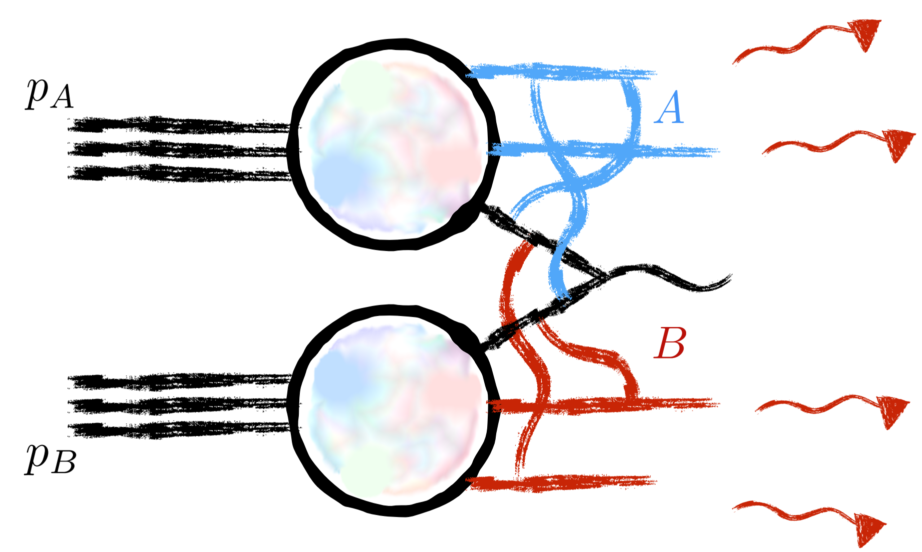
$$W(p_A,p_B,q) \;=\; \frac{1}{(2\pi)^4}\int d^4x e^{-iqx}\int^{\tilde{A}(t_f)=A(t_f)} D\tilde{A}_\mu DA_\mu \int^{\tilde{\psi}_a(t_f)=\psi_a(t_f)} D\bar{\psi}_a D\psi_a$$
$$\times\; D\bar{\tilde{\psi}}_a D\tilde{\psi}_a e^{-iS_{\rm QCD}(\tilde{A},\tilde{\psi}_a)} e^{iS_{\rm QCD}(A,\psi_a)} \Psi^*_{p_A}(\vec{\tilde{A}}(t_i),\tilde{\psi}_a(t_i))\Psi_{p_A}(\vec{A}(t_i),\psi(t_i))$$
$$\times \int^{\tilde{B}(t_f)=B(t_f)} D\tilde{B}_\mu DB_\mu \int^{\tilde{\psi}_b(t_f)=\psi_b(t_f)} D\bar{\psi}_b D\psi_b D\bar{\tilde{\psi}}_b D\tilde{\psi}_b$$
$$\times\; e^{-iS_{\rm QCD}(\tilde{B},\tilde{\psi}_b)} e^{iS_{\rm QCD}(B,\psi_b)} \Psi^*_{p_B}(\vec{\tilde{B}}(t_i),\tilde{\psi}_b(t_i))\Psi_{p_B}(\vec{B}(t_i),\psi_b(t_i))$$
$$\times \int DC_\mu \int^{\tilde{C}(t_f)=C(t_f)} D\tilde{C}_\mu \int D\bar{\psi}_C D\psi_C \int^{\tilde{\psi}_c(t_f)=\psi_c(t_f)} D\bar{\tilde{\psi}}_C D\tilde{\psi}_C \; \tilde{J}_{C\mu}(x) J^\mu_C(0)\, e^{-i\tilde{S}_C + iS_C}$$

**QCD action in two background fields**

$$S_C = S_{QCD}(A+B+C) - S_{QCD}(A) - S_{QCD}(B)$$

**Integrals over background fields depend on each other through "interaction" with C fields**

**Integrals are independent when integral over C fields has factorized form**



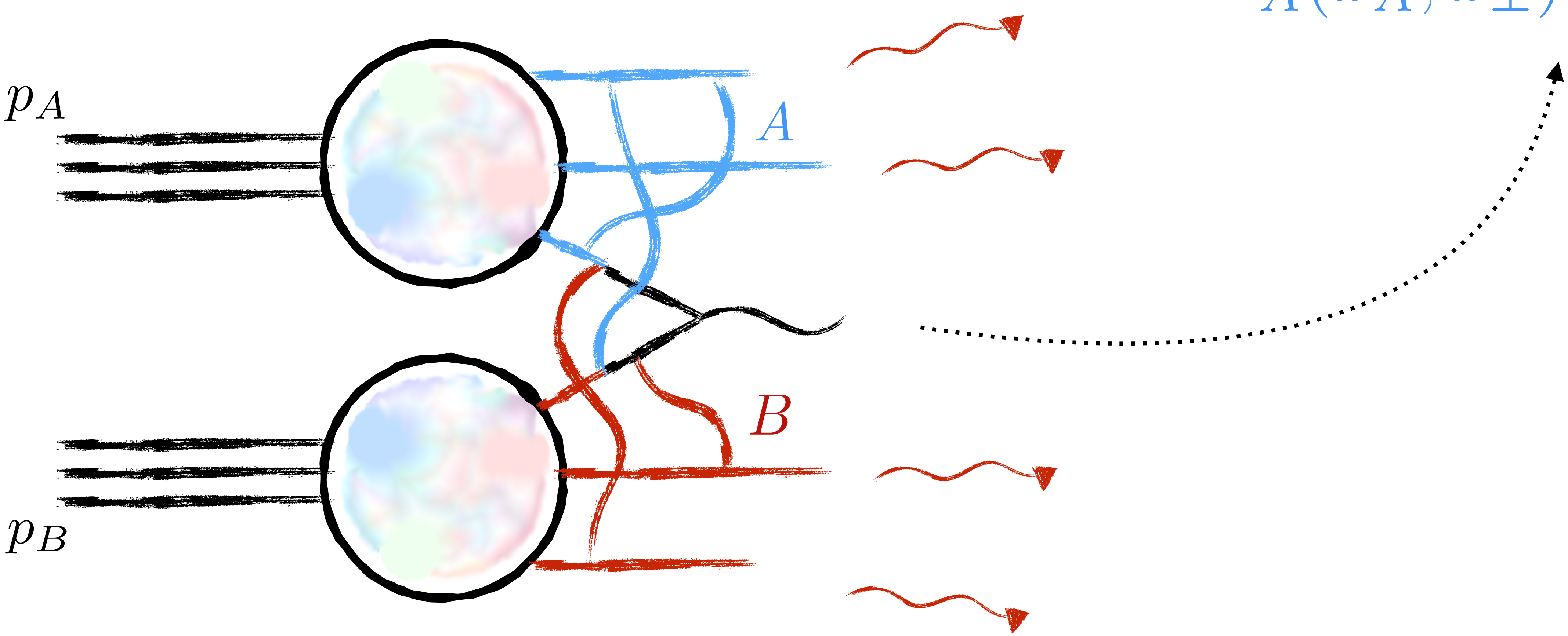
# Integration over "central" fields

**Calculate functional integral at tree level**

$$\int DC_\mu \int^{\tilde{C}(t_f)=C(t_f)} D\tilde{C}_\mu \int D\bar{\psi}_C D\psi_C \int^{\tilde{\psi}_c(t_f)=\psi_c(t_f)} D\tilde{\bar{\psi}}_C D\tilde{\psi}_C \; \tilde{J}_{C\mu}(x) J^\mu_C(0) \; e^{-i\tilde{S}_C+iS_C}$$

**Show that it has factorized form**

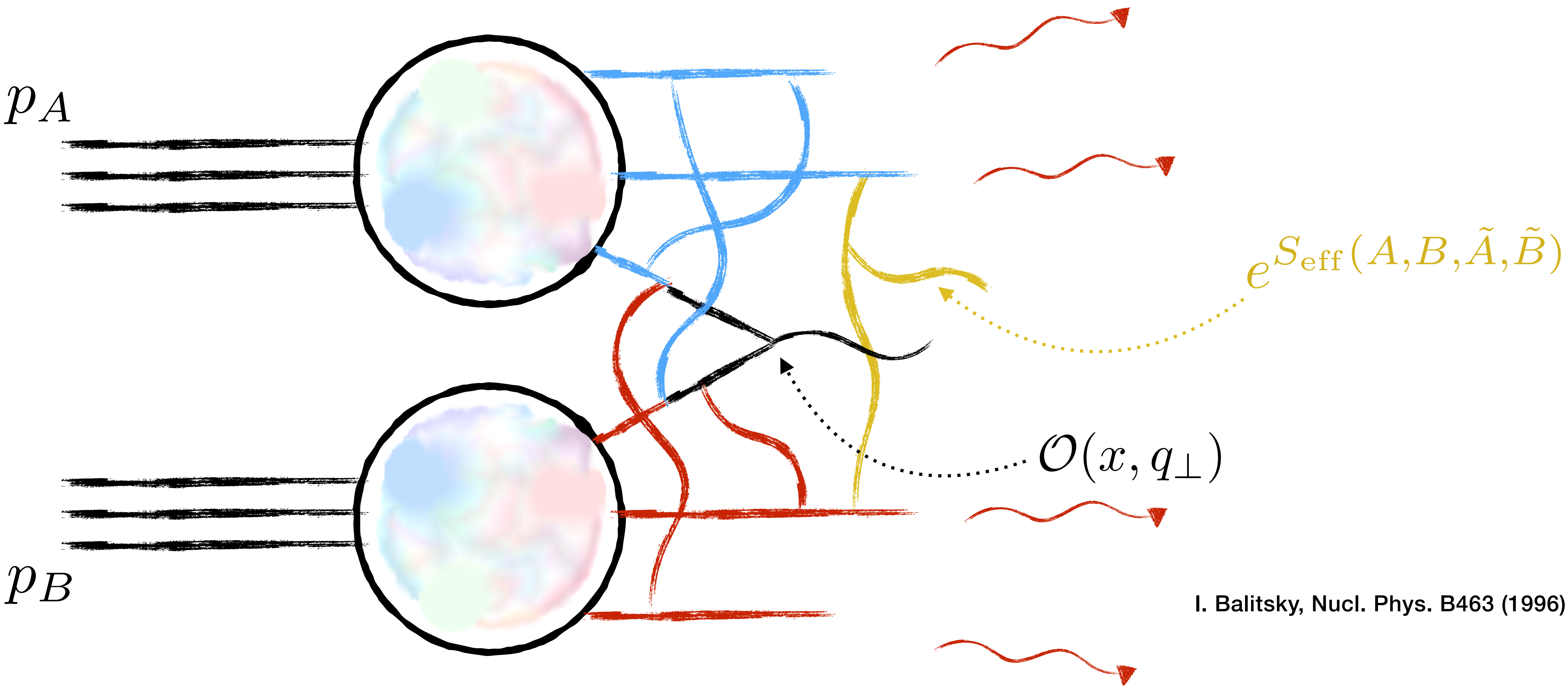
$$\Phi_A(x_A, x_\perp)\Phi_B(x_B, x_\perp)$$



# Glauber exchanges

$$\int DC_\mu \int^{\tilde{C}(t_f)=C(t_f)} D\tilde{C}_\mu \int D\bar{\psi}_C D\psi_C \int^{\tilde{\psi}_c(t_f)=\psi_c(t_f)} D\tilde{\bar{\psi}}_C D\tilde{\psi}_C \; \tilde{J}_{C\mu}(x) J^\mu_C(0)\; e^{-i\tilde{S}_C+iS_C}$$

$$= \; e^{S_{\rm eff}(\bar{A},\bar{B},\tilde{\bar{A}},\tilde{\bar{B}})}\mathcal{O}(q,x;A,\psi_a,\tilde{A},\tilde{\psi}_a;B,\psi_b,\tilde{B},\tilde{\psi}_b)$$




I. Balitsky, Nucl. Phys. B463 (1996)

# Glauber exchanges

The effective action is known only up to first few orders of perturbation theory:

$$e^{S_{\rm eff}(A,B,\tilde{A},\tilde{B})}\Big|_{A=\tilde{A},B=\tilde{B}} = 1$$

Boundary conditions for functional integrals:

$$\tilde{A}(t_f) = A(t_f)\Big|_{t_f\to\infty} \qquad \tilde{\psi}(t_f) = \psi(t_f)\Big|_{t_f\to\infty}$$

Kinematic approximation for background fields:

$$A(x_-, x_\perp)$$

$$\tilde{A}(x_-, x_\perp)$$

boundary conditions

$$A(x_-, x_\perp) = \tilde{A}(x_-, x_\perp)\Big|_{x_+\to\infty}$$

$$A(x_-, x_\perp) = \tilde{A}(x_-, x_\perp)$$

Functional integrals are not independent

# Connected diagrams

$$\int DC_\mu \int^{\tilde{C}(t_f)=C(t_f)} D\tilde{C}_\mu \int D\bar{\psi}_C D\psi_C \int^{\tilde{\psi}_c(t_f)=\psi_c(t_f)} D\tilde{\bar{\psi}}_C D\tilde{\psi}_C \; \tilde{J}^{C\mu}(x) J^C_\mu(0)\; e^{-i\tilde{S}_C + iS_C}$$
$$= \mathcal{O}(q, x; A, \psi_a; B, \psi_b)$$

**Calculate at the tree level and show factorized form**

$$\hat{\mathcal{O}}(q, x; \hat{A}, \hat{\psi}_a; \hat{B}, \hat{\psi}_b) \;=\; \sum_{m,n} \int dz_m dz'_n c_{m,n}(q, x) \hat{\Phi}_A(z_m) \hat{\Phi}_B(z'_n)$$

**Hadronic tensor:**

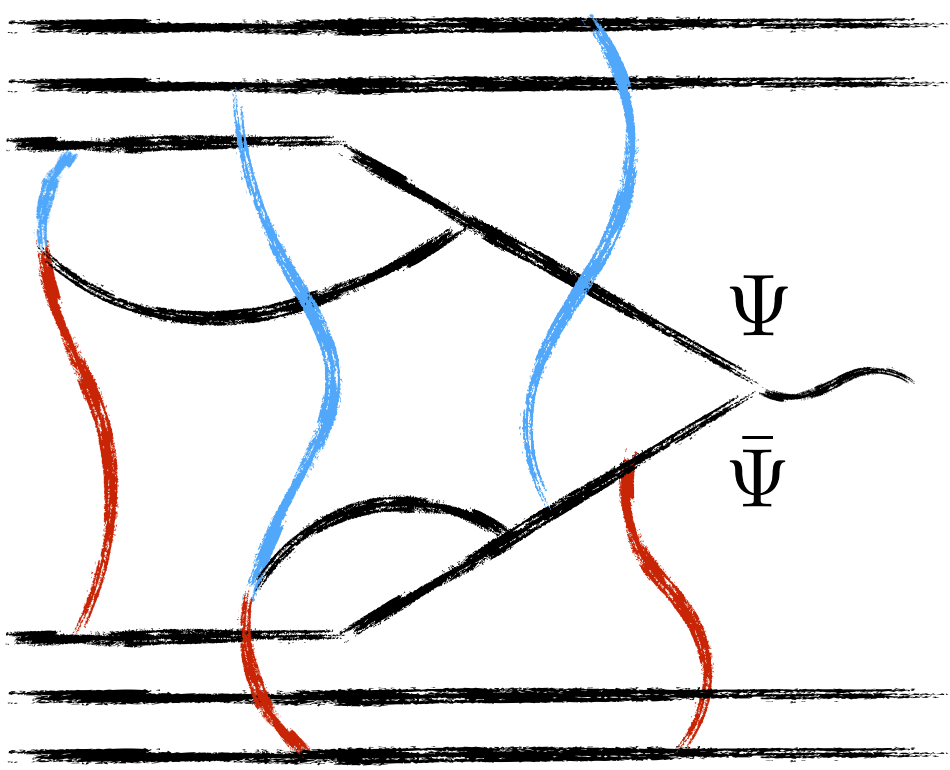
$$W \;=\; \frac{1}{(2\pi)^4} \int d^4x e^{-iqx} \sum_{m,n} \int dz_m \int dz'_n c_{m,n}(q, x) \langle p_A | \hat{\Phi}_A(z_m) | p_A \rangle \langle p_B | \hat{\Phi}_B(z'_n) | p_B \rangle$$

# Equations of motion

To calculate $\mathcal{O}(q,x;A,\psi_a;B,\psi_b)$ we find solution of equations of motion

$$(i\,\partial\!\!\!/ + g A\!\!\!/ + g B\!\!\!/ + g\, C\!\!\!/)(\psi_A^f + \psi_B^f + \psi_C^f) \;=\; 0$$

$$D^\nu F^a_{\mu\nu}(A+B+C) \;=\; g\sum_f (\bar\psi_A^f + \bar\psi_B^f + \bar\psi_C^f)\gamma_\mu t^a(\psi_A^f + \psi_B^f + \psi_C^f)$$




Background fields satisfy equations of motion:

$$i\,D\!\!\!/_A\psi_A \;=\; 0,\quad D_A^\nu A^a_{\mu\nu} \;=\; g\sum_f \bar\psi_A^f\gamma_\mu t^a\psi_A^f$$

$$i\,D\!\!\!/_B\psi_B \;=\; 0,\quad D_B^\nu B^a_{\mu\nu} \;=\; g\sum_f \bar\psi_B^f\gamma_\mu t^a\psi_B^f$$

We will use solution of the equations of motion

$$\mathcal{A} = A + B + C \qquad \Psi = \psi_A + \psi_B + \psi_C$$

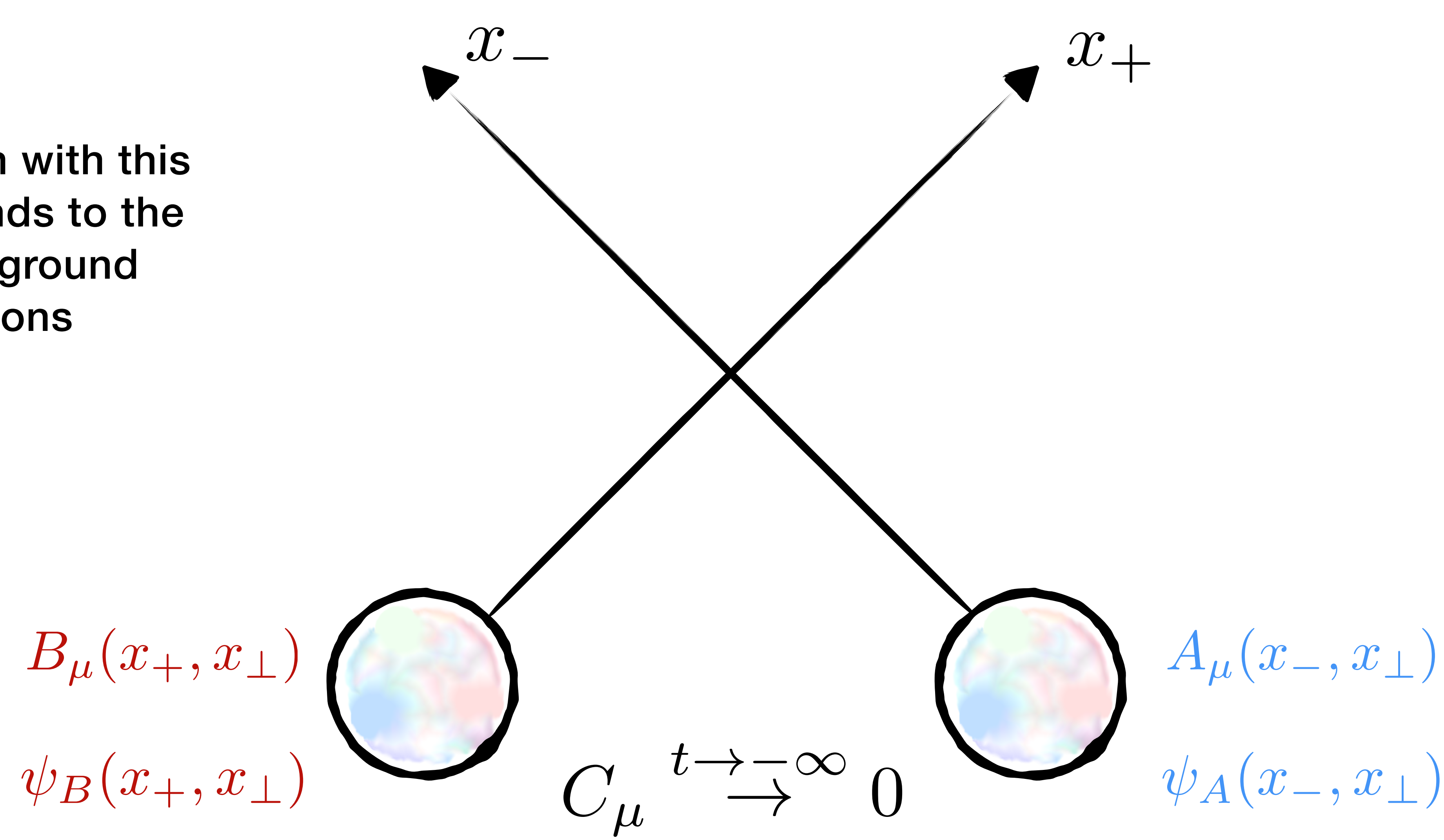
to calculate currents in the hadronic tensor

# Boundary conditions

$$\mathcal{A}_\mu(x) \overset{x_+\to-\infty}{=} A_\mu(x_-, x_\perp), \qquad \Psi(x) \overset{x_+\to-\infty}{=} \psi_A(x_-, x_\perp)$$

$$\mathcal{A}_\mu(x) \overset{x_-\to-\infty}{=} B_\mu(x_+, x_\perp), \qquad \Psi(x) \overset{x_-\to-\infty}{=} \psi_B(x_+, x_\perp)$$

**Solution of equations of motion with this boundary conditions corresponds to the sum of set of diagrams in background field with retarded Green functions**




J.P. Blaizot, F. Gelis, and R. Venugopalan, Nucl. Phys. A743, 13 (2004);
F. Gelis, T. Lappi, and R. Venugopalan, Phys. Rev. D 78, 054019 (2008)

# Perturbative solution. First order

We can construct perturbation solution of the equation of motion

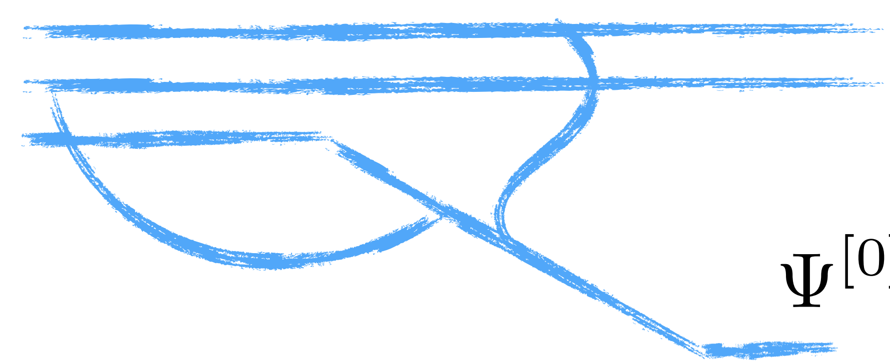
$$\mathcal{A}_\mu(x) = \mathcal{A}_\mu^{[0]}(x) + \mathcal{A}_\mu^{[1]}(x) + \mathcal{A}_\mu^{[2]}(x) + \dots$$

$$\Psi(x) = \Psi^{[0]}(x) + \Psi^{[1]}(x) + \Psi^{[2]}(x) + \dots$$

The leading order is trivial:

$$\mathcal{A}_\mu^{[0]}(x) \;=\; A_\mu(x_-, x_\perp) + B_\mu(x_+, x_\perp)$$

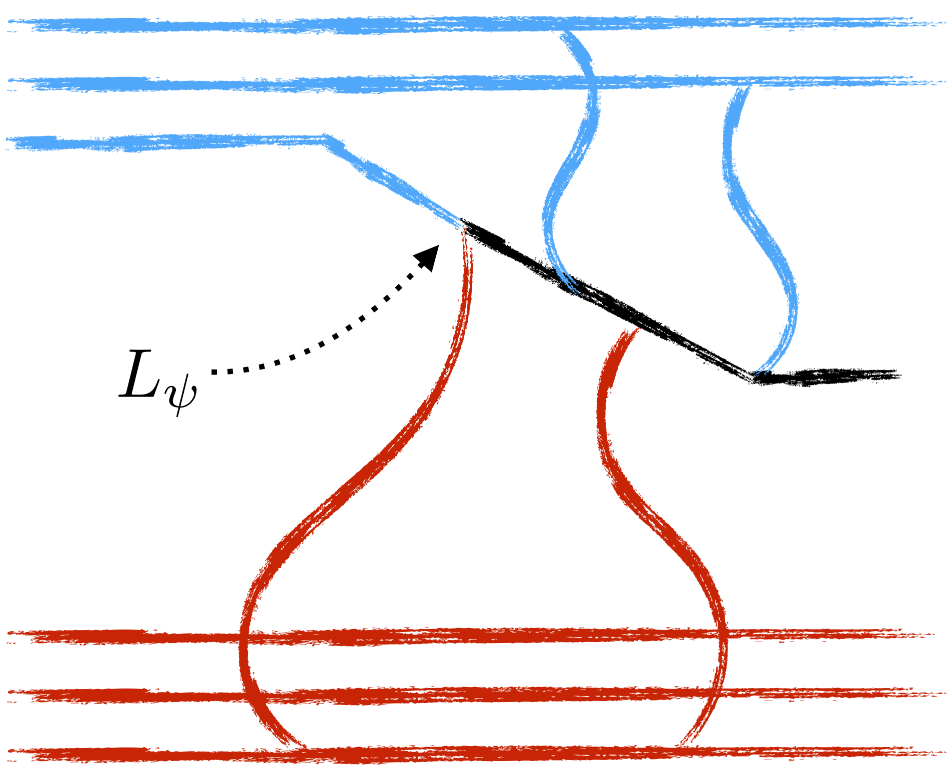
$$\Psi^{[0]}(x) \;=\; \psi_A(x_-, x_\perp) + \psi_B(x_+, x_\perp)$$



$\Psi^{[0]}(x)$

Background fields satisfy classical equations of motions

# Perturbative solution. Second order




$$\Psi^{[1]} = -\frac{1}{\mathcal{P}} L_\psi$$

quark propagates in two background fields

Covariant derivative for two background fields:

$$\mathcal{P}_\mu \equiv i\partial_\mu + gA_\mu + gB_\mu$$

Creation of the central sector fields from the background is described by the linear term

$$L_\psi \equiv \mathcal{P}\Psi^{[0]}$$

The gluon field in the second order of the perturbative expansion has a similar structure

$$\mathcal{A}_\mu^{[1]} = \frac{1}{\mathcal{P}^2 g^{\mu\nu} + 2ig\mathcal{F}^{[0]\mu\nu}} L^\nu$$

$$L_\mu^a \equiv \mathcal{D}^\xi \mathcal{F}_{\xi\mu}^{[0]a} + g\bar{\Psi}^{[0]}\gamma_\mu t^a \Psi^{[0]}$$

We should prove TMD factorization in all orders of perturbation theory. Need a new expansion parameter

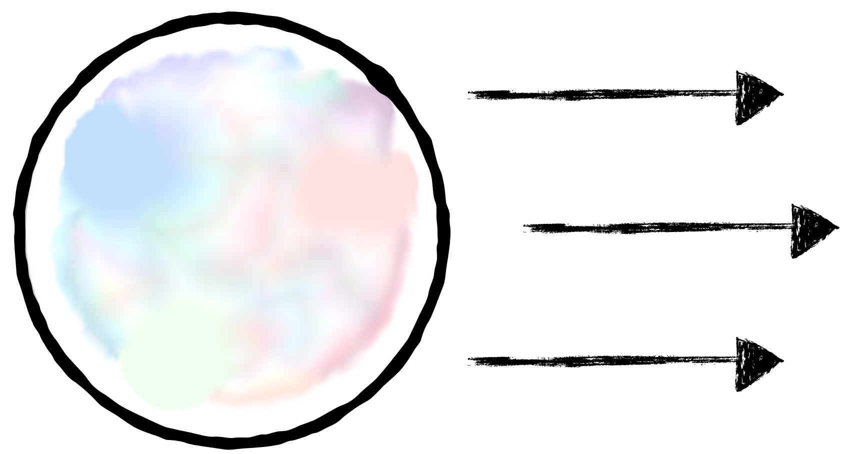
# Parametrization of external fields. Lorentz boost

Hadron in the rest frame:

$$A_+ \sim m, \quad A_- \sim m, \quad A_i \sim m$$

Boost the system

Look at the limit: $s \to \infty$



Expansion parameter:

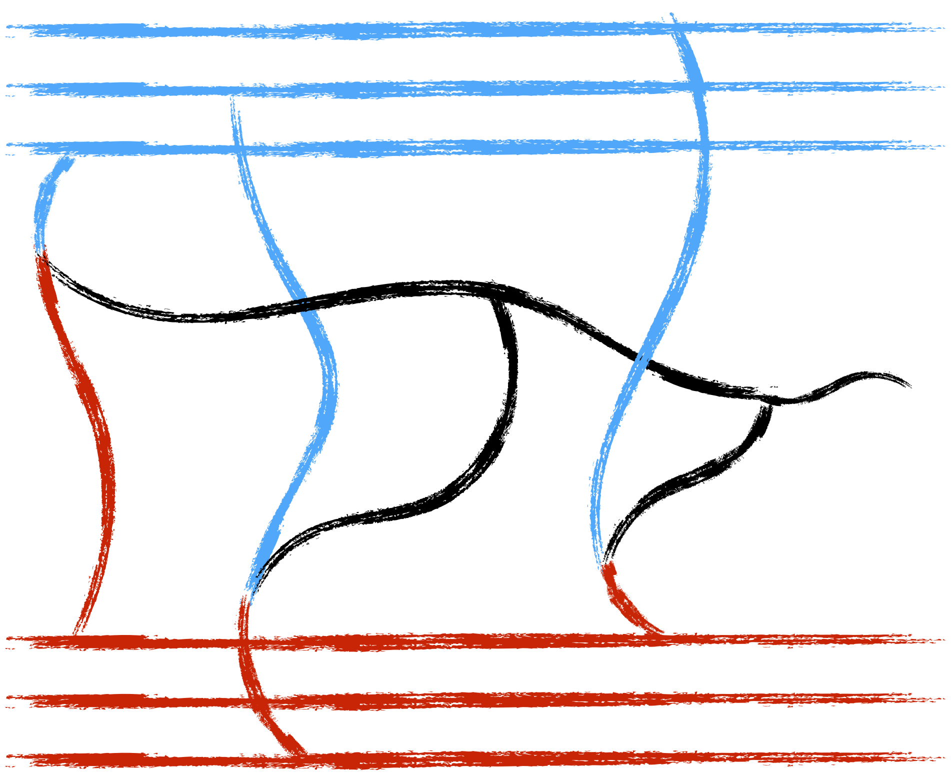
$$m^2/s \sim p_\perp^2/s$$

$$A_+ \sim m^2/\sqrt{s}, \quad A_- \sim \sqrt{s}, \quad A_i \sim m$$

Use this parametrization to separate different contributions

# Large component of the background field

$A_-(x_+, x_\perp) \sim \sqrt{s}$



$B_+(x_-, x_\perp) \sim \sqrt{s}$

It is convenient to eliminate large component of the background field by gauge rotation

The gauge matrix satisfies boundary conditions:

$$\Omega(x) \stackrel{x_+ \to -\infty}{=} [x_-, -\infty]^{A_+}$$

$$\Omega(x) \stackrel{x_- \to -\infty}{=} [x_+, -\infty]^{A_-}$$

It is possible to reconstruct first few terms of the matrix at arbitrary point:

$$\Omega = \frac{1}{2}[x_+, -\infty][x_-, -\infty] + \frac{1}{2}[x_-, -\infty][x_+, -\infty] + \dots$$

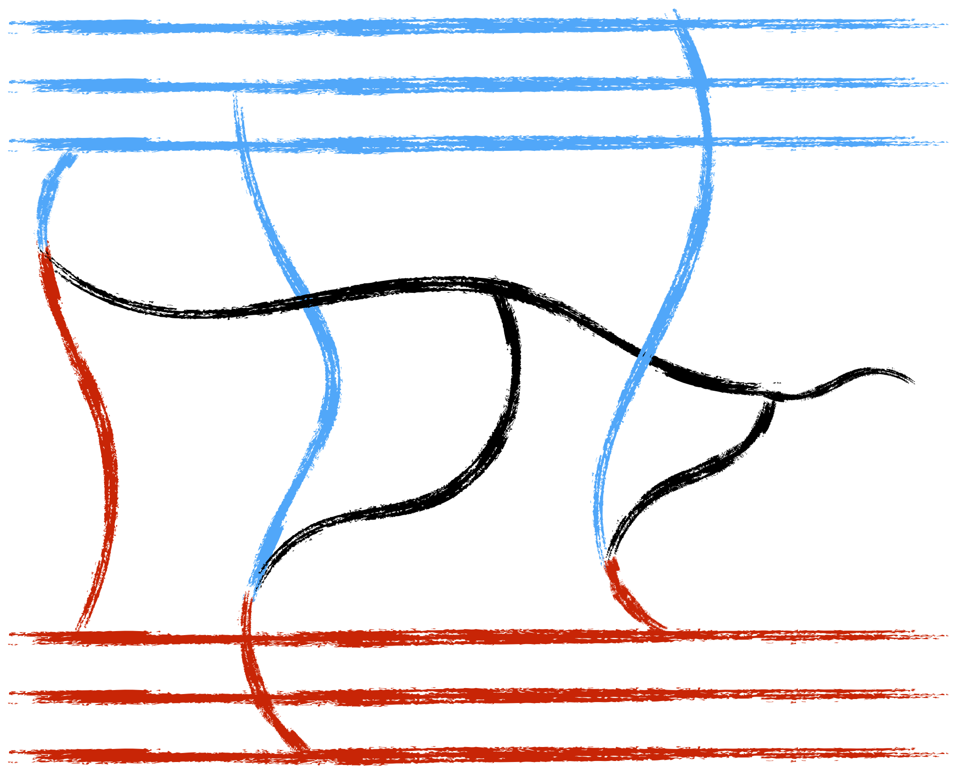
# Gauge rotation of the background fields

**New background fields**

$$\bar{A}_\mu = \Omega^\dagger(x)\Big(\frac{i}{g}\partial_\mu + A_\mu(x)\Big)\Omega(x)$$

$$\psi_{\bar{A}}(x_-, x_\perp) = \Omega^\dagger \psi_A(x_-, x_\perp)$$

$\bar{A}_\mu(x_-, x_\perp)$ $\psi_{\bar{A}}(x_-, x_\perp)$



$\bar{B}_\mu(x_+, x_\perp)$ $\psi_{\bar{B}}(x_+, x_\perp)$

**New initial conditions:**

$$\mathcal{A}_\mu(x) \overset{x_+ \to -\infty}{\to} \bar{A}_\mu(x_-, x_\perp), \quad \psi(x) \overset{x_+ \to -\infty}{\to} \psi_{\bar{A}}(x_-, x_\perp)$$

$$\mathcal{A}_\mu(x) \overset{x_- \to -\infty}{\to} \bar{B}_\mu(x_+, x_\perp), \quad \psi(x) \overset{x_- \to -\infty}{\to} \psi_{\bar{B}}(x_+, x_\perp)$$

**New background fields**

$$\bar{B}_\mu = \Omega^\dagger(x)\Big(\frac{i}{g}\partial_\mu + B_\mu(x)\Big)\Omega(x)$$

$$\psi_{\bar{B}}(x_-, x_\perp) = \Omega^\dagger \psi_B(x_+, x_\perp)$$

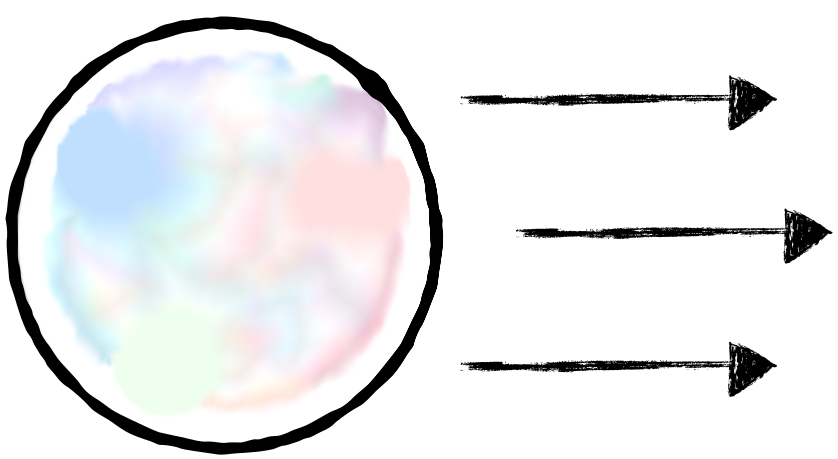
# Parametrization of fields after gauge rotation

$\bar{A}_-(x_-, x_\perp) \sim m^2/\sqrt{s}, \quad \bar{A}_+(x_-, x_\perp) = 0, \quad \bar{A}_i(x_-, x_\perp) \sim m$

$\not{p}_1 \psi_{\bar{A}}(x_-, x_\perp) \ \sim \ m^{5/2} \qquad \gamma_i \psi_{\bar{A}}(x_-, x_\perp) \ \sim \ m^{3/2}$

$\not{p}_2 \psi_{\bar{A}}(x_-, x_\perp) \ \sim \ s\sqrt{m}$

**Consider YM equation in these background fields**



$\bar{B}_-(x_+, x_\perp) = 0; \quad \bar{B}_+(x_+, x_\perp) \sim m^2/\sqrt{s}; \quad \bar{B}_i(x_*, x_\perp) \sim m$

$\not{p}_1 \psi_{\bar{B}}(x_+, x_\perp) \ \sim \ s\sqrt{m} \qquad \gamma_i \psi_{\bar{B}}(x_+, x_\perp) \ \sim \ m^{3/2}$

$\not{p}_2 \psi_{\bar{B}}(x_+, x_\perp) \ \sim \ m^{5/2}$

# Parametrization of perturbation solution

$$\mathcal{A}^{[0]}_\mu(x) \;=\; \bar{A}_\mu(x_-, x_\perp) + \bar{B}_\mu(x_+, x_\perp)$$

$$\mathcal{A}^{[1]}_\mu \;=\; \frac{1}{\mathcal{P}^2 g^{\mu\nu} + 2ig\mathcal{F}^{[0]\mu\nu}} L^\nu$$

We regroup terms of the perturbative solution using new expansion parameter: $m^2/s$

gluon propagator in two background fields

operator constructed from background fields

**Perturbative expansion of the gluon propagator in two background fields**

$$(x|\frac{1}{\mathcal{P}^2 g^{\mu\nu} + 2ig\mathcal{F}^{[0]\mu\nu} + i\epsilon p_0}|y) \;\equiv\; (x|\frac{1}{p^2 + i\epsilon p_0}|y) - g(x|\frac{1}{p^2 + i\epsilon p_0}\mathcal{O}_{\mu\nu}\frac{1}{p^2 + i\epsilon p_0}|y)$$
$$+\; g^2(x|\frac{1}{p^2 + i\epsilon p_0}\mathcal{O}_{\mu\xi}\frac{1}{p^2 + i\epsilon p_0}\mathcal{O}^{\xi}{}_{\nu}\frac{1}{p^2 + i\epsilon p_0}|y) + \ldots$$

Separation of terms with the new parameter

$$\sim 1 + \frac{m^2}{s} + \left(\frac{m^2}{s}\right)^2 + \left(\frac{m^2}{s}\right)^3 + \ldots$$

$$(x|\frac{1}{p_{||}^2 + i\epsilon p_0}|y)$$

Do we need these terms?

# Parametrization of the linear term

**Use explicit form of the linear term**

$$L_\psi \;\equiv\; \not{\mathcal{P}}\Psi^{[0]} \;=\; L_\psi^{(0)} + L_\psi^{(1)}$$

**Expansion in terms of the new parameter**

$$L_\psi^{(0)} \;=\; g\gamma^i \bar{A}_i \psi_{\bar{B}} + g\gamma^i \bar{B}_i \psi_{\bar{A}}, \qquad L_\psi^{(1)} \;=\; g\sqrt{\frac{2}{s}}\; \not{p}_2 \bar{A}_- \psi_{\bar{B}} + g\sqrt{\frac{2}{s}}\; \not{p}_1 \bar{B}_+ \psi_{\bar{A}}$$

**Power counting:** $L_\psi^{(0)} \;\sim\; m^{5/2}, \qquad L_\psi^{(1)} \;\sim\; \dfrac{m^{9/2}}{s}$

**Linear term for gluons:**

$$L_\mu^a \;\equiv\; \mathcal{D}^\xi \mathcal{F}_{\xi\mu}^{[0]a} + g\bar{\Psi}^{[0]}\gamma_\mu t^a \Psi^{[0]} \;=\; L_\mu^{(-1)a} + L_\mu^{(0)a} + L_\mu^{(1)a}$$

$$\sim s^{1/2} m^2 + \frac{m^4}{s^{1/2}} + \frac{m^6}{s^{3/2}}$$

# Solution of the equations of motion

**Equation of motion:**

$$(i\,\not\partial + g\not\bar{A} + g\not\bar{B} + g\not{C})\Psi \;=\; 0$$

parametrization of the perturbative solution in terms of the new parameter

**Perturbative solution:**

$$\Psi(x) \;=\; \Psi^{[0]}(x) + \Psi^{[1]}(x) + \Psi^{[2]}(x) + \ldots \;=\; \Psi_A^{(0)} + \Psi_B^{(0)} + \Psi_A^{(1)} + \Psi_B^{(1)} + \ldots$$

**Leading order solution:**

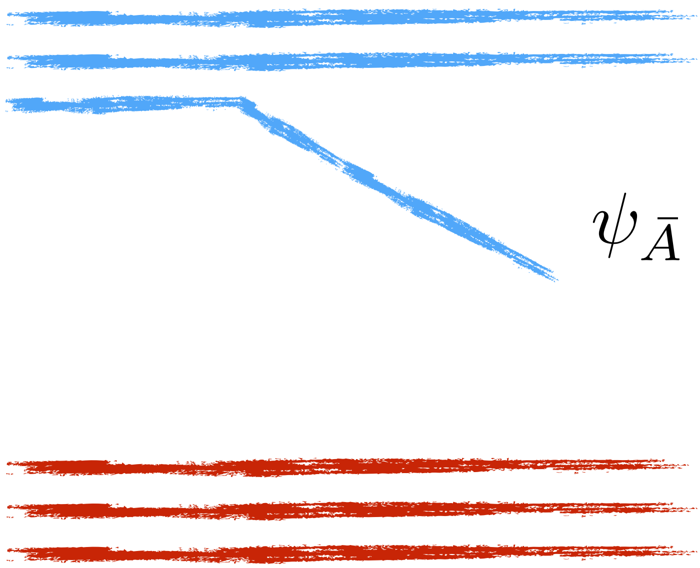
$$\Psi_A^{(0)} \;=\; \psi_{\bar{A}} + \Xi_{2A} \sim m^{3/2}, \qquad \Xi_{2A} \;=\; -\frac{g\,\not p_2}{s}\gamma^i \bar{B}_i \frac{1}{\alpha + i\epsilon}\psi_{\bar{A}}$$

$$\Psi_B^{(0)} \;=\; \psi_{\bar{B}} + \Xi_{1B} \sim m^{3/2}, \qquad \Xi_{1B} \;=\; -\frac{g\,\not p_1}{s}\gamma^i \bar{A}_i \frac{1}{\beta + i\epsilon}\psi_{\bar{B}}$$

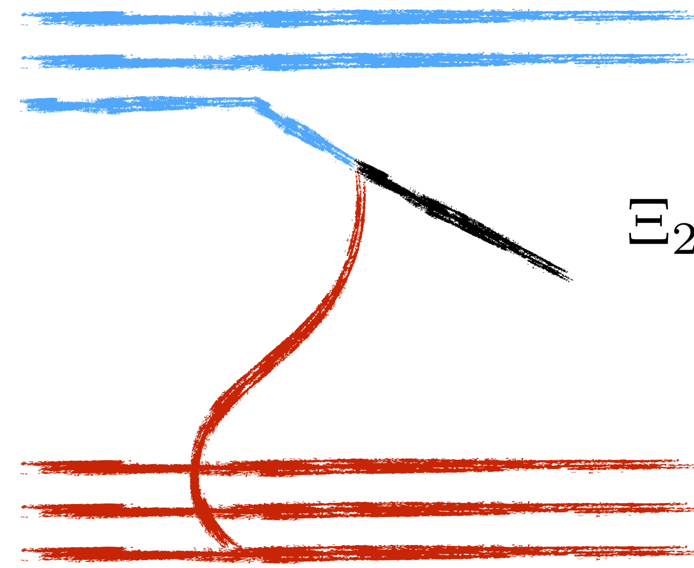
We use this solution to calculate hadronic tensor

# The structure of the leading order solution

$$\Psi_A^{(0)} = \psi_{\bar{A}} + \Xi_{2A}$$

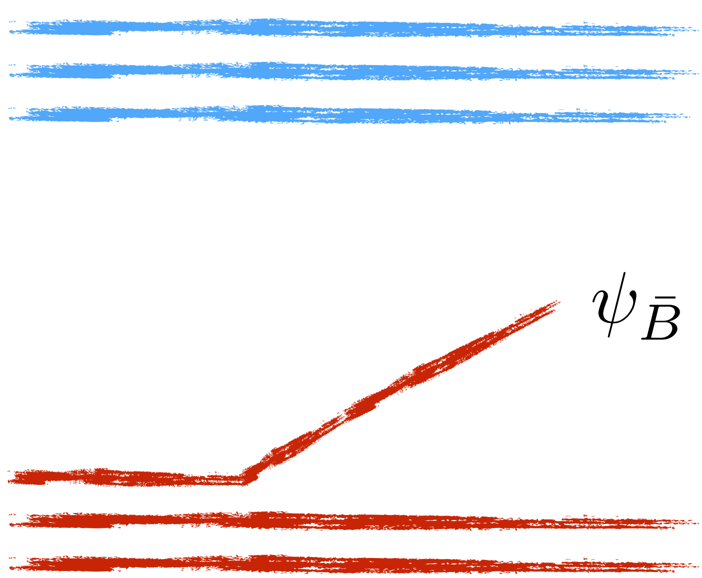


$\psi_{\bar{A}} \sim m^{3/2}$

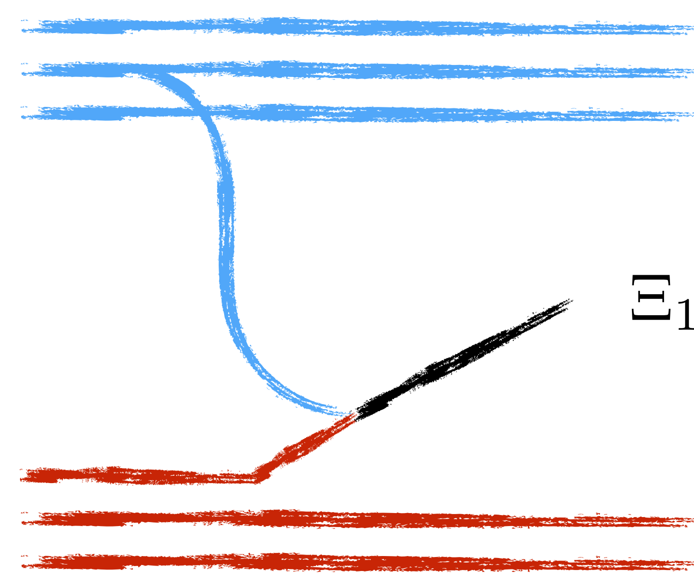


$\Xi_{2A} \sim m^{3/2}$

$$\Psi_B^{(0)} = \psi_{\bar{B}} + \Xi_{1B}$$



$\psi_{\bar{B}} \sim m^{3/2}$



$\Xi_{1B} \sim m^{3/2}$

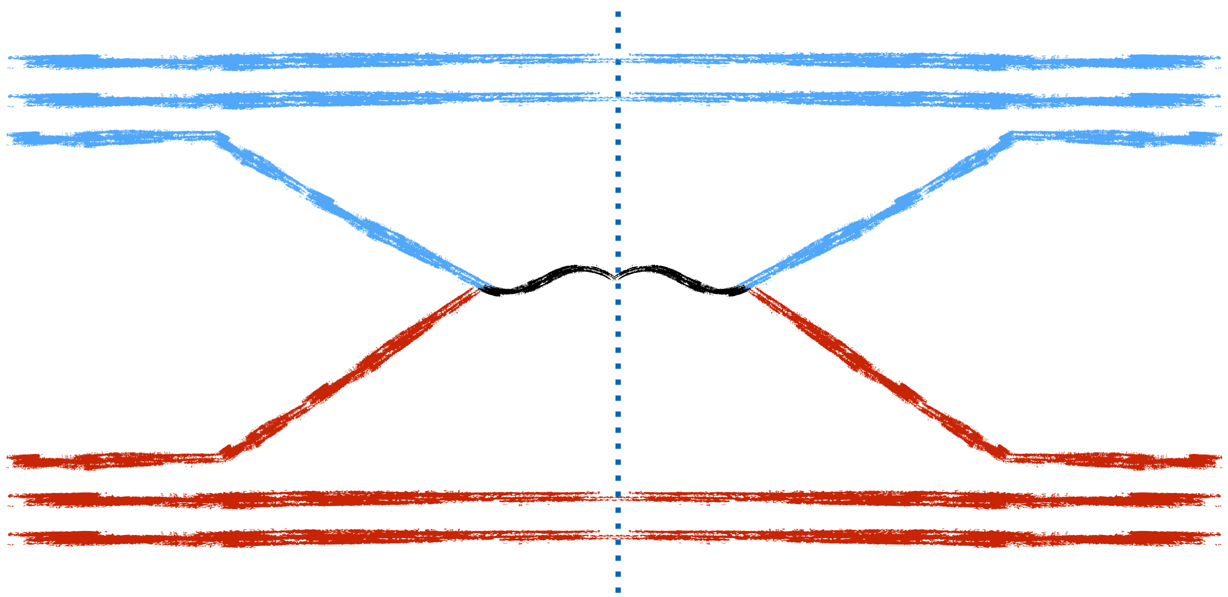
# Hadronic tensor in the leading order

$$W(\alpha_z,\beta_z,x_\perp) \equiv \frac{1}{(2\pi)^4}\int dx_+dx_-\ e^{-i\sqrt{\frac{s}{2}}\alpha_z x_- - i\sqrt{\frac{s}{2}}\beta_z x_+}\langle p_A,p_B|J_\mu(x_+,x_-,x_\perp)J^\mu(0)|p_A,p_B\rangle$$

**In the leading order we use**

$$\Psi_A^{(0)} = \psi_{\bar{A}}$$

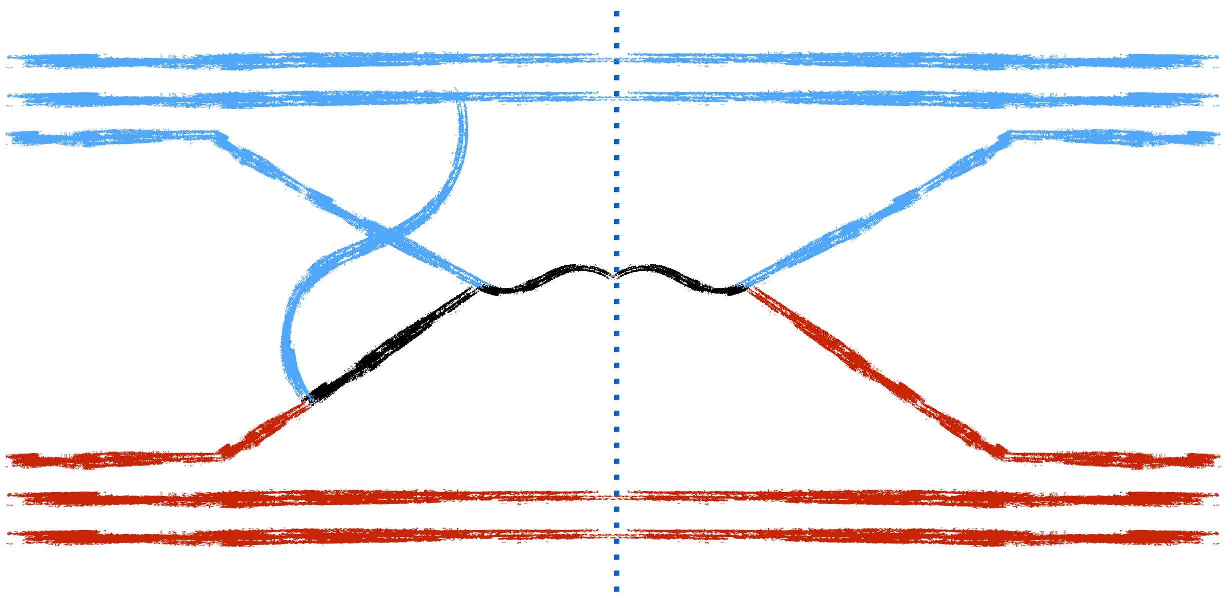
$$\Psi_B^{(0)} = \psi_{\bar{B}}$$



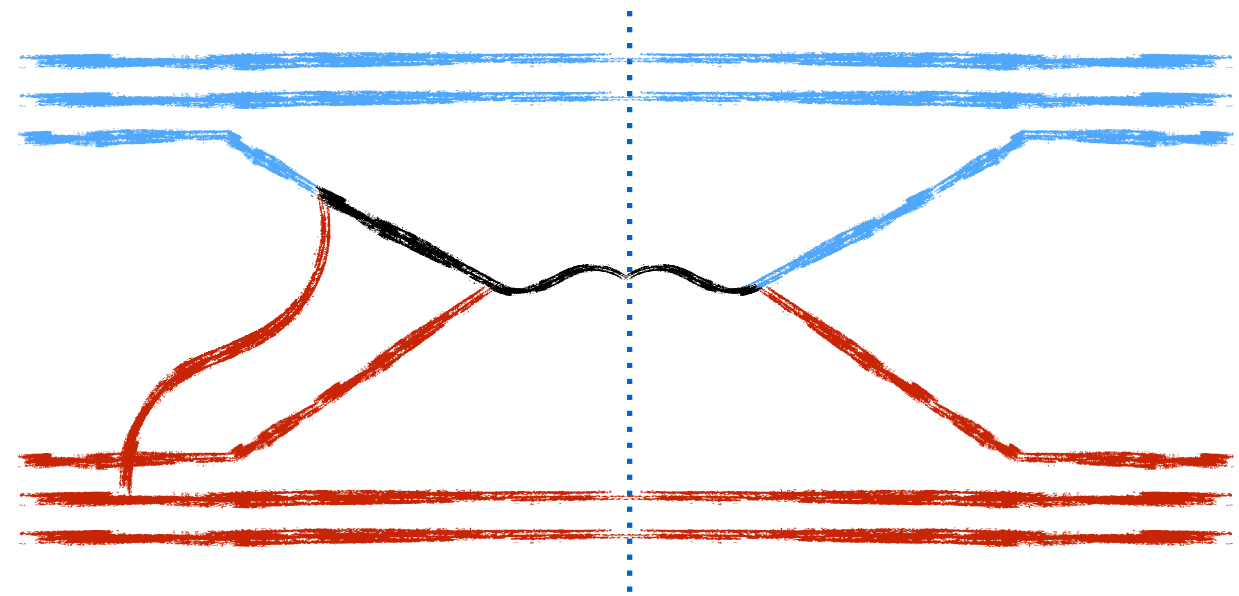
$$W^{\rm lt}(\alpha_z,\beta_z,q_\perp) = -\frac{e^2}{8s_W^2c_W^2N_c}\int d^2k_\perp\Big(\big\{(1+a_u^2)\big[f_1^u(\alpha_z,k_\perp)\bar{f}_1^u(\beta_z,q_\perp-k_\perp) + \bar{f}_1^u(\alpha_z,k_\perp)f_1^u(\beta_z,q_\perp-k_\perp)\big]\big\} + \{u\leftrightarrow c\} + \{u\leftrightarrow d\} + \{u\leftrightarrow s\}\Big)$$

$$a_{u,c} = (1-\frac{8}{3}s_W^2) \qquad a_{d,s} = (1-\frac{4}{3}s_W^2)$$

# Suppressed contributions I



$\Psi_A^{(0)} = \psi_{\bar{A}}$ $\quad$ $\Psi_A^{(0)} = \Xi_{2A}$



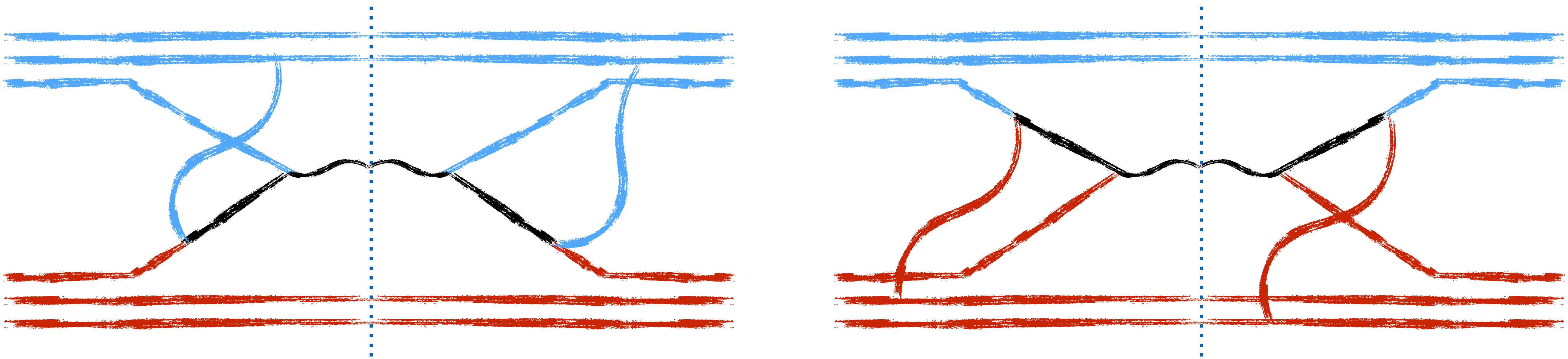
$\Psi_B^{(0)} = \Xi_{1B}$ $\quad$ $\Psi_B^{(0)} = \psi_{\bar{B}}$

**This contribution** $\sim \dfrac{q_\perp^2}{\alpha_z s} W^{\rm lt} \ll \dfrac{q_\perp^2}{Q^2} W^{\rm lt}$

**Looks like a leading power correction, but suppressed in the kinematic limit**

$$s \gg Q^2 \gg Q_\perp^2$$

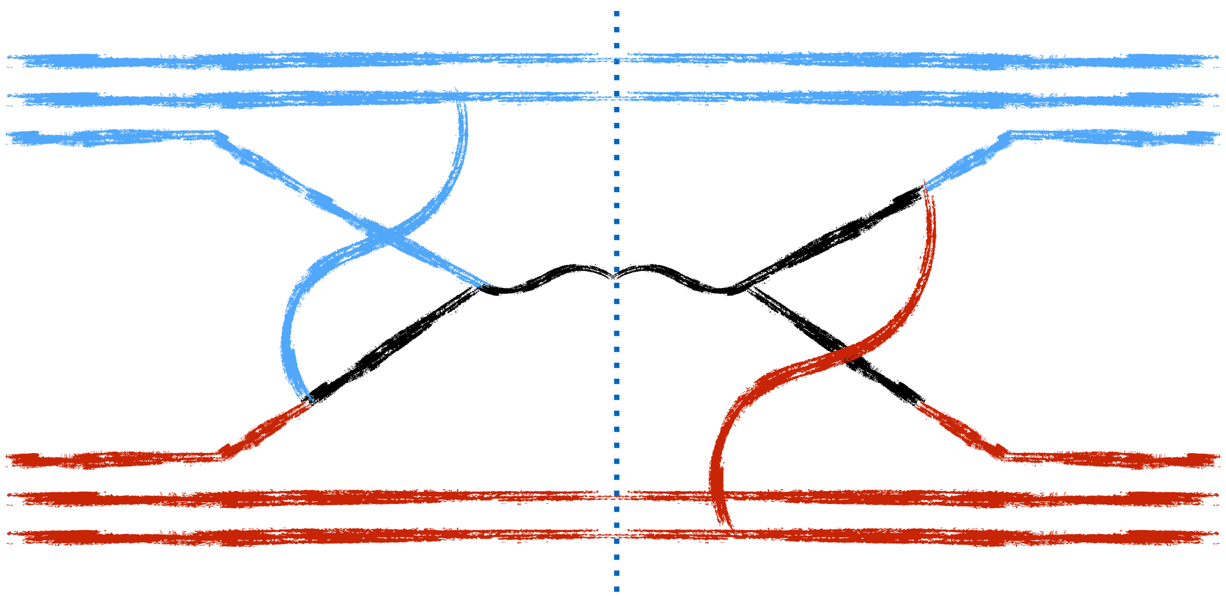
# Suppressed contributions II

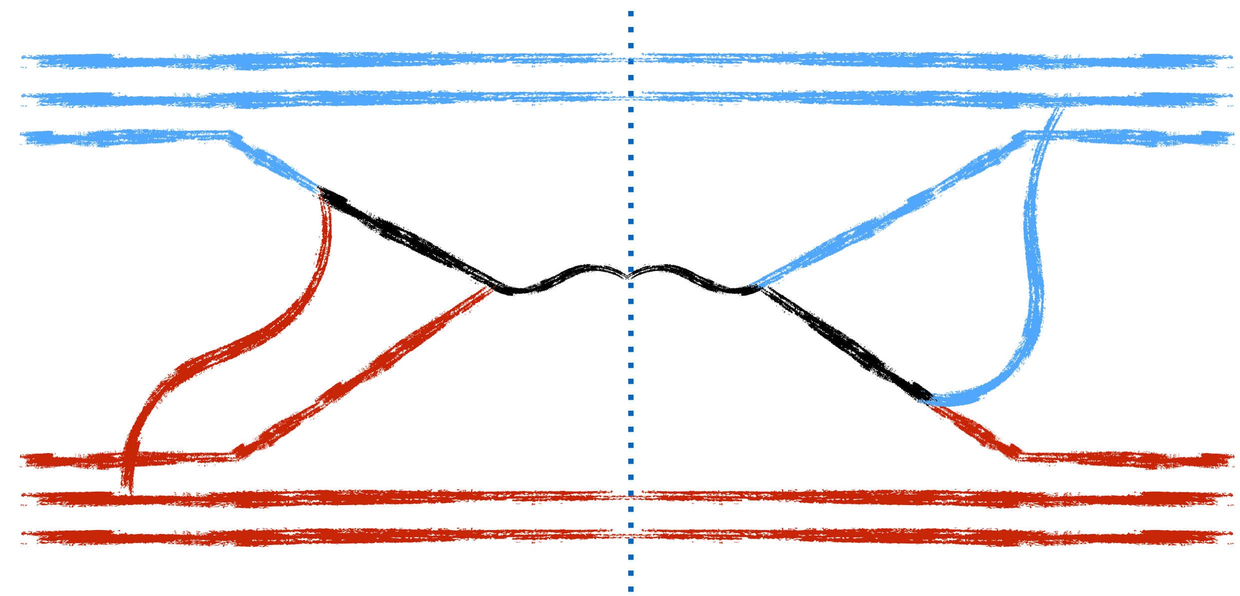


$\Psi_A^{(0)} = \psi_{\bar{A}}$ $\Psi_A^{(0)} = \Xi_{2A}$ $\Psi_B^{(0)} = \Xi_{1B}$ $\Psi_B^{(0)} = \psi_{\bar{B}}$

**This contribution is suppressed as** $\sim \dfrac{m^4}{s^2} W^{\mathrm{lt}}$

# Leading in 1/N$_c$ contribution





$\Psi_A^{(0)} \;=\; \Xi_{2A}$ $\qquad\qquad$ $\Psi_B^{(0)} \;=\; \Xi_{1B}$

$$W(\alpha_z,\beta_z,q_\perp) = \frac{e^2}{4s_W^2c_W^2N_cQ^2}\int d^2k_\perp\Big[\Big\{(1+a_u^2)(k,q-k)_\perp f_1^u(\alpha_z,k_\perp)\bar{f}_1^u(\beta_z,q_\perp-k_\perp)$$
$$+\;\frac{1}{m^2}(1-a_u^2)k_\perp^2(q-k)_\perp^2 h_{1u}^\perp(\alpha_z,k_\perp)\bar{h}_{1u}^\perp(\beta_z,q_\perp-k_\perp)+\;(\alpha_z\leftrightarrow\beta_z)\Big\}$$
$$+\;\Big\{u\leftrightarrow c\Big\}+\Big\{u\leftrightarrow d\Big\}+\Big\{u\leftrightarrow s\Big\}\Big]$$

**Leading power correction to TMD factorization**

$$\sim\frac{q_\perp^2}{Q^2}W^{\rm lt}$$

# Leading in 1/$N_c$ contribution

$$W(\alpha_z,\beta_z,q_\perp) = \frac{e^2}{4s_W^2 c_W^2 N_c Q^2}\int d^2k_\perp \Big[\Big\{(1+a_u^2)(k,q-k)_\perp f_1^u(\alpha_z,k_\perp)\bar f_1^u(\beta_z,q_\perp-k_\perp)$$
$$+ \frac{1}{m^2}(1-a_u^2)k_\perp^2(q-k)_\perp^2 h_{1u}^\perp(\alpha_z,k_\perp)\bar h_{1u}^\perp(\beta_z,q_\perp-k_\perp) + (\alpha_z\leftrightarrow\beta_z)\Big\}$$
$$+ \Big\{u\leftrightarrow c\Big\} + \Big\{u\leftrightarrow d\Big\} + \Big\{u\leftrightarrow s\Big\}\Big]$$

1) It is a leading power correction from the point of view of parametrization

2) Leading contribution in kinematic limit

$$s \gg Q^2 \gg Q_\perp^2$$

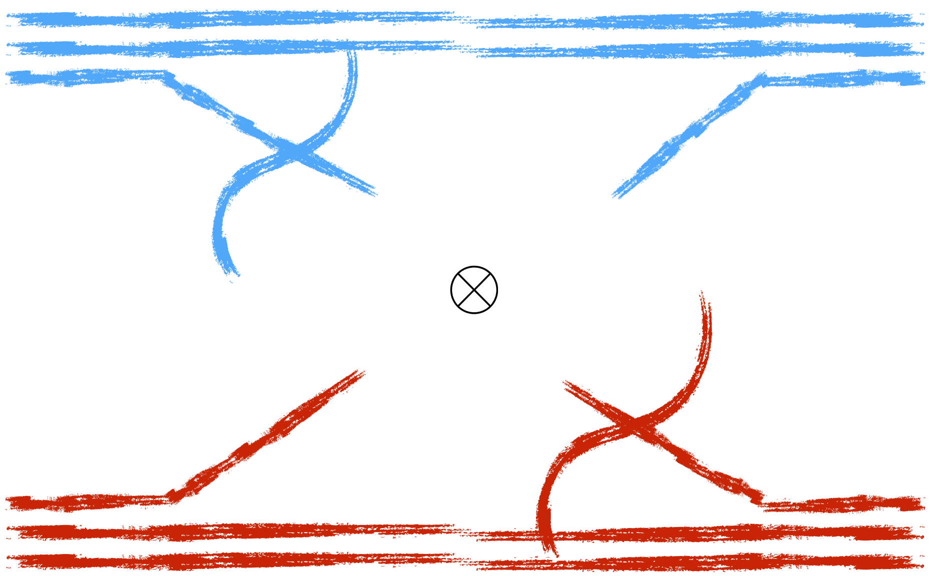
(the contribution is $\sim \frac{q_\perp^2}{Q^2}W^{\rm lt}$ )

3) The contribution is proportional to $\frac{1}{N_c}$

(all other important terms $\sim \frac{1}{N_c^2},\ \frac{1}{N_c^3}$)

4) The leading power correction to TMD factorization can be expressed in terms of leading twist distribution functions. This is the most important observation.

# Leading twist distribution functions



$f_1(\alpha_z, k_\perp) \qquad h_1^\perp(\alpha_z, k_\perp)$

$f_1(\beta_z, k_\perp) \qquad h_1^\perp(\beta_z, k_\perp)$

**The structure of the TMD operator**

$$\frac{g}{8\pi^3 s}\sqrt{\frac{s}{2}}\int dx_- dx_\perp e^{-i\alpha\sqrt{\frac{s}{2}}x_- + ik_\perp x_\perp}\langle A|\bar\psi_{\bar A}(x_-, x_\perp)\ \not p_2\left[\bar A_i(0) + i\gamma_5\tilde{\bar A}_i(0)\right]\psi_{\bar A}(0)|A\rangle = -k_i f_1(\alpha, k_T^2)$$

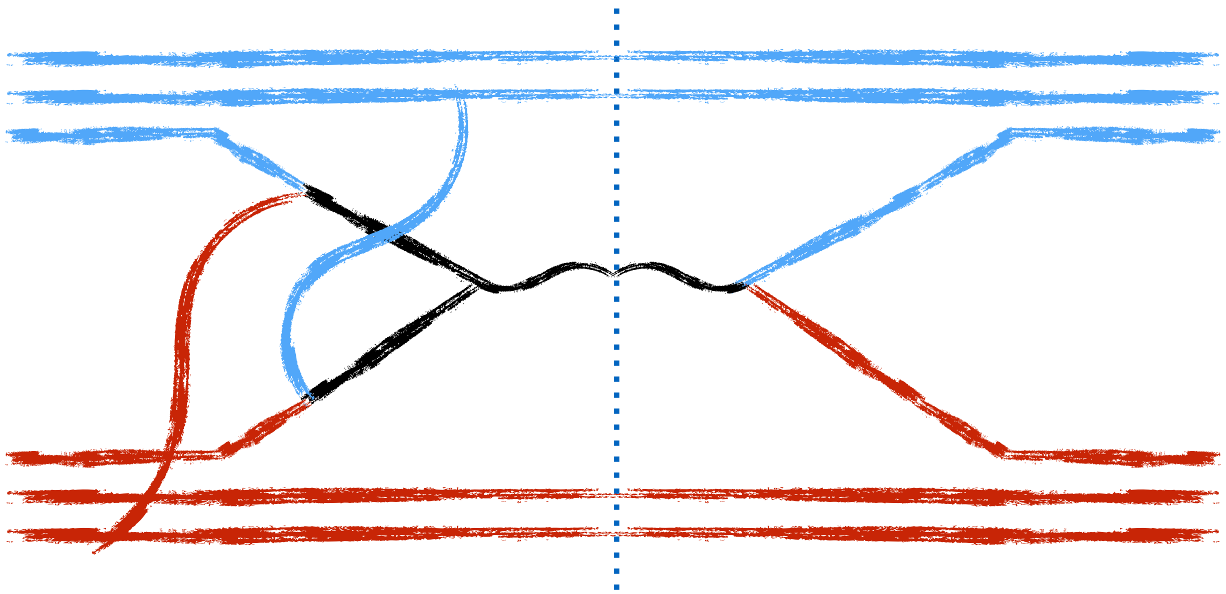
**Leading twist TMD operator is constructed from quark fields only**

$$f_1(\alpha, k_T^2) = \frac{1}{2}\int\frac{dx_- d^2x_\perp}{(2\pi)^3}e^{-i\alpha\sqrt{\frac{s}{2}}x_- + ik_\perp x_\perp}\langle A|\bar\psi_{\bar A}(x)\gamma_+\psi_{\bar A}(0)|A\rangle$$

R. D. Tangerman and P. J. Mulders,
Phys. Rev. D51 (1995)

**The structure of Wilson lines can be restored by gauge rotation**

# Contribution suppressed by color



1) Introduce higher twist distribution functions

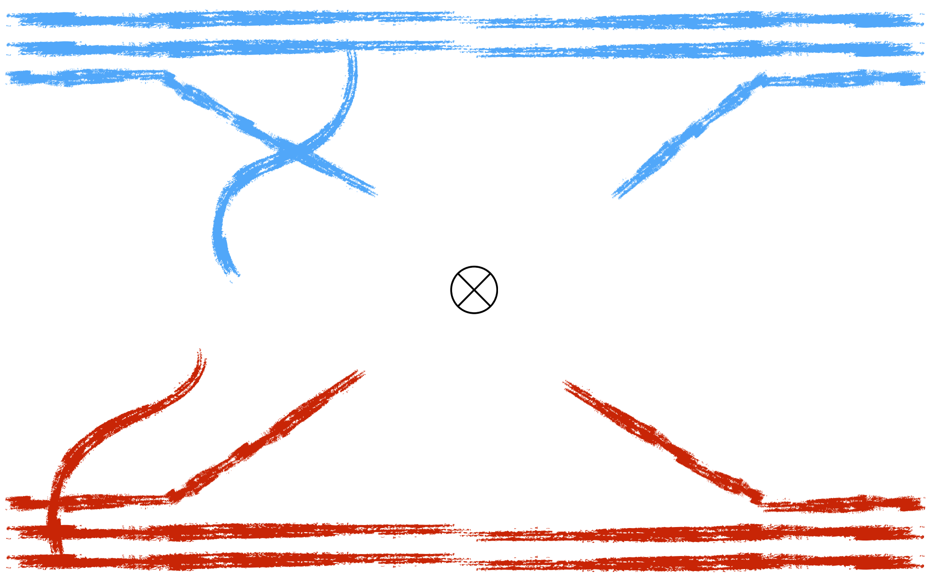
2) The contribution is suppressed as $\frac{1}{N_c^3}$

$$W(\alpha_z, \beta_z, q_\perp) \; = \; -\frac{e^2}{4s_W^2 c_W^2 N_c (N_c^2-1)Q^2} \int d^2k_\perp \; k_\perp^2 (q-k)_\perp^2$$

$$\times \; \Big\{ \frac{1}{m^2}(a_u^2-1)[h_u^{\rm tw3}(\alpha_z, k_\perp)\bar{h}_u^{\rm tw3}(\beta_z, q_\perp - k_\perp) + \tilde{h}_u^{\rm tw3}(\alpha_z, k_\perp)\bar{\tilde{h}}_u^{\rm tw3}(\beta_z, q_\perp - k_\perp)]$$

$$+ \; (\alpha_z \leftrightarrow \beta_z)\Big\} \; + \; \Big\{u \leftrightarrow c\Big\} + \Big\{u \leftrightarrow d\Big\} + \Big\{u \leftrightarrow s\Big\}\Big]$$

# New class of TMD distribution functions



$$h_f^{tw3}(\alpha, k_\perp^2) \qquad \tilde{h}_f^{tw3}(\alpha, k_\perp^2)$$

$$h_f^{tw3}(\beta, k_\perp^2) \qquad \tilde{h}_f^{tw3}(\beta, k_\perp^2)$$

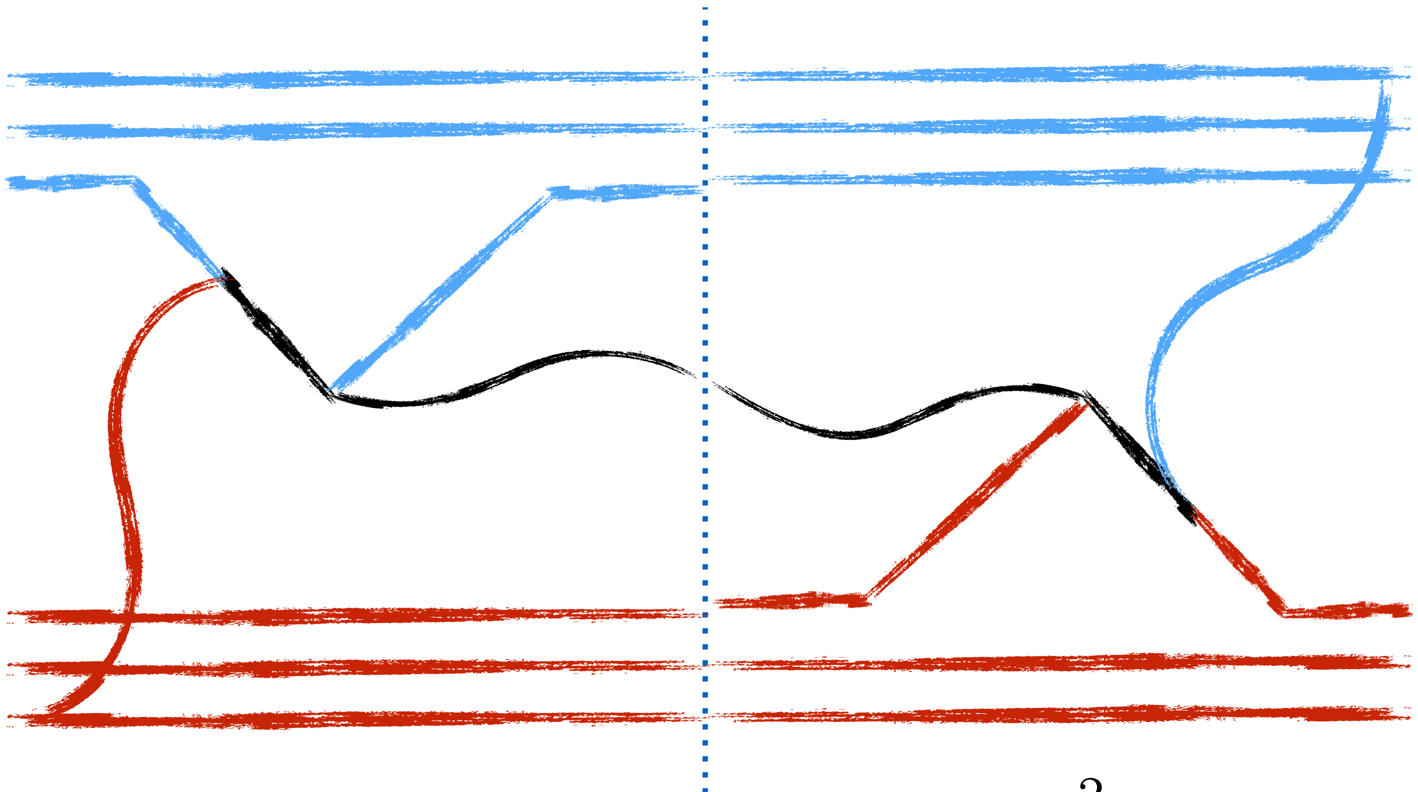
**Higher twist distribution functions**

$$\frac{g}{8\pi^3 s}\sqrt{\frac{s}{2}}\int dx_\perp dx_-\ e^{-i\alpha\sqrt{\frac{s}{2}}x_- + i(k,x)_\perp}\langle A|\bar{\psi}_{\bar{A}}(x_-, x_\perp)\ \not{p}_2\gamma^i\Big\{\bar{A}_i(0)\psi_{\bar{A}}(0) +\ \bar{F}_{+i}(0)\int_{-\infty}^0 dx'_-\ \psi_{\bar{A}}(x'_-, 0_\perp)\Big\}|A\rangle$$

$$=\ i\frac{k_\perp^2}{m}\big[h_f^{\rm tw3}(\alpha, k_\perp^2) + i\tilde{h}_f^{\rm tw3}(\alpha, k_\perp^2)\big]$$

**The structure of Wilson lines can be restored by gauge rotation**

# Contribution with gluon exchanges



The contribution is suppressed as $\dfrac{1}{N_c^2}$

$$
\begin{aligned}
W(\alpha_z,\beta_z,q_\perp) \;=\; & \frac{e^2}{8s_W^2c_W^2(N_c^2-1)Q^2}\int d^2k_\perp\;(k,q-k)_\perp\Big[\Big\{2(1+a_u^2)\\
\times\;& \big[j_{1u}^{\rm tw3}(\alpha_z,k_\perp)j_{2u}^{\rm tw3}(\beta_z,q_\perp-k_\perp)-\;\tilde{j}_{1u}^{\rm tw3}(\alpha_z,k_\perp)\tilde{j}_{2u}^{\rm tw3}(\beta_z,q_\perp-k_\perp)\big]\\
+\;& (1-a_u^2)\big[j_{1u}^{\rm tw3}(\alpha_z,k_\perp)j_{1u}^{\rm tw3}(\beta_z,q_\perp-k_\perp)+\tilde{j}_{1u}^{\rm tw3}(\alpha_z,k_\perp)\tilde{j}_{1u}^{\rm tw3}(\beta_z,q_\perp-k_\perp)\\
+\;& j_{2u}^{\rm tw3}(\alpha_z,k_\perp)j_{2u}^{\rm tw3}(\beta_z,q_\perp-k_\perp)+\tilde{j}_{2u}^{\rm tw3}(\alpha_z,k_\perp)\tilde{j}_{2u}^{\rm tw3}(\beta_z,q_\perp-k_\perp)\big]\;+\;\alpha_z\leftrightarrow\beta_z\Big\}\\
+\;& \Big\{u\leftrightarrow c\Big\}+\Big\{u\leftrightarrow d\Big\}+\Big\{u\leftrightarrow s\Big\}\Big]
\end{aligned}
$$

The TMD operator is constructed from quark and gluon fields

# Power correction to TMD factorization

leading twist contribution

leading power correction

$$W(\alpha_z,\beta_z,q_\perp) = -\frac{e^2}{8s_W^2c_W^2N_c}\int d^2k_\perp\Big[\Big\{(1+a_u^2)\Big[1-2\frac{(k,q-k)_\perp}{Q^2}\Big]$$
$$\times f_{1u}(\alpha_z,k_\perp)\bar f_{1u}(\beta_z,q_\perp-k_\perp) + 2(a_u^2-1)\frac{k_\perp^2(q-k)_\perp^2}{m^2Q^2}h_{1u}^\perp(\alpha_z,k_\perp)\bar h_{1u}^\perp(\beta_z,q_\perp-k_\perp)$$
$$+\frac{2k_\perp^2(q-k)_\perp^2}{(N_c^2-1)Q^2m^2}(a_u^2-1)[h_u^{\rm tw3}(\alpha_z,k_\perp)\bar h_u^{\rm tw3}(\beta_z,q_\perp-k_\perp)+\tilde h_u^{\rm tw3}(\alpha_z,k_\perp)\bar{\tilde h}_u^{\rm tw3}(\beta_z,q_\perp-k_\perp)]$$
$$-\frac{N_c}{N_c^2-1}\frac{(k,q-k)_\perp}{Q^2}\Big(2(1+a_u^2)[j_{1u}^{\rm tw3}(\alpha_z,k_\perp)j_{2u}^{\rm tw3}(\beta_z,q_\perp-k_\perp)-\tilde j_{1u}^{\rm tw3}(\alpha_z,k_\perp)\tilde j_{2u}^{\rm tw3}(\beta_z,q_\perp-k_\perp)]$$
$$+(1-a_u^2)[j_{1u}^{\rm tw3}(\alpha_z,k_\perp)j_{1u}^{\rm tw3}(\beta_z,q_\perp-k_\perp)+j_{2u}^{\rm tw3}(\alpha_z,k_\perp)j_{2u}^{\rm tw3}(\beta_z,q_\perp-k_\perp)$$
$$+\tilde j_{1u}^{\rm tw3}(\alpha_z,k_\perp)\tilde j_{1u}^{\rm tw3}(\beta_z,q_\perp-k_\perp)+\tilde j_{2u}^{\rm tw3}(\alpha_z,k_\perp)\tilde j_{2u}^{\rm tw3}(\beta_z,q_\perp-k_\perp)]\Big)$$
$$+(\alpha_z\leftrightarrow\beta_z)\Big\} + \Big\{u\leftrightarrow c\Big\}+\Big\{u\leftrightarrow d\Big\}+\Big\{u\leftrightarrow s\Big\}\Big] + O\Big(\frac{m^8}{s}\Big)$$

power correction suppressed by $1/N_c^2$

power correction (gluon contribution) suppressed by $1/N_c$

I. Balitsky, A.T. (2018)

# Estimations

**perturbative tails of TMDs:**

$$f_1(\alpha_z, k_\perp^2) \simeq \frac{f(\alpha_z)}{k_\perp^2}, \quad h_1^\perp(\alpha_z, k_\perp^2) \simeq \frac{m^2 h(\alpha_z)}{k_\perp^4}, \qquad \bar{f}_1 \simeq \frac{\bar{f}(\alpha_z)}{k_\perp^2}, \quad \bar{h}_1^\perp \simeq \frac{m^2 \bar{h}(\alpha_z)}{k_\perp^4}$$

$$f_1(\beta_z, k_\perp^2) \simeq \frac{f(\beta_z)}{k_\perp^2}, \quad h_1^\perp(\beta_z, k_\perp^2) \simeq \frac{m^2 h(\beta_z)}{k_\perp^4}, \qquad \bar{f}_1 \simeq \frac{\bar{f}(\beta_z)}{k_\perp^2}, \quad \bar{h}_1^\perp \simeq \frac{m^2 \bar{h}(\beta_z)}{k_\perp^4}$$

J. Zhou, F. Yuan and Z.-T. Liang, Phys. Rev. D78 (2008)

**Leading order power correction:**

relative weight of the power correction

$$W(\alpha_z, \beta_z, q_\perp) \simeq -\frac{e^2}{8 s_W^2 c_W^2 N_c} \int d^2k_\perp \frac{1}{k_\perp^2 (q-k)_\perp^2} \left[1 - 2\frac{(k, q-k)_\perp}{Q^2}\right]$$
$$\times \left[\left\{(1+a_u^2)[f_u(\alpha_z)\bar{f}_u(\beta_z) + \bar{f}_u(\alpha_z) f_u(\beta_z)]\right\} + \left\{u \leftrightarrow c\right\} + \left\{u \leftrightarrow d\right\} + \left\{u \leftrightarrow s\right\}\right]$$

Power corrections are important in the region where transverse momentum is not too small

# Conclusion

$$W(\alpha_z,\beta_z,q_\perp) \;=\; -\frac{e^2}{8s_W^2c_W^2N_c}\int d^2k_\perp\Big[\Big\{(1+a_u^2)\Big[1-2\frac{(k,q-k)_\perp}{Q^2}\Big]$$
$$\times\ [f_{1u}(\alpha_z,k_\perp)\bar{f}_{1u}(\beta_z,q_\perp-k_\perp)+\bar{f}_{1u}(\alpha_z,k_\perp)f_{1u}(\beta_z,q_\perp-k_\perp)]$$
$$+2(a_u^2-1)\frac{k_\perp^2(q-k)_\perp^2}{m^2Q^2}[h_{1u}^\perp(\alpha_z,k_\perp)\bar{h}_{1u}^\perp(\beta_z,q_\perp-k_\perp)+\bar{h}_{1u}^\perp(\alpha_z,k_\perp)h_{1u}^\perp(\beta_z,q_\perp-k_\perp)]\Big\}$$
$$+\Big\{u\leftrightarrow c\Big\}+\Big\{u\leftrightarrow d\Big\}+\Big\{u\leftrightarrow s\Big\}\Big]\ +\ O(\frac{m^8}{s})$$

I. Balitsky and A.T., JHEP 07 (2017); hep-ph/1712.09389

1) We calculated leading power correction to TMD factorization in Z boson production

2) We calculated hadronic tensor at the tree level using solution of the equations of motion

3) We constructed solution of the equations of motion by expansion in parameter $m^2/s$

4) We found leading contribution in kinematic limit $s \gg Q^2 \gg Q_\perp^2$

5) We found that the leading term can be expressed in terms of leading twist distribution functions