Energy-momentum tensor for unpolarized proton target

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The goal

The quantity of our interest is the energy momentum tensor (EMT) on unpolarized proton state,

$$\langle T^{\mu\nu} \rangle = \frac{1}{2} \sum_{s=\uparrow,\downarrow} \frac{\langle p', s | \hat{T}^{\mu\nu}(0) | p, s \rangle}{\sqrt{2p'_0 \, 2p_0}} ,$$

which Fourier transformation leads to the EMT in the position space,

$$\widetilde{\langle T^{\mu\nu} \rangle}(x) = \int \frac{d^3\vec{\Delta}}{(2\pi)^3} \, e^{i\Delta \cdot x} \, \langle T^{\mu\nu} \rangle ,$$

where $\Delta = p' - p$, $P = \frac{1}{2}(p' + p)$ and $t = \Delta^2$.

M. V. Polyakov, Phys. Lett. B555, 57 (2003)
C. Lorcé, L. Mantovani, B. Pasquini, Phys. Lett. B776, 38 (2018)

The matrix element of the general local asymmetric energy–momentum tensor for a spin-1/2 target reads

$$\left\langle p', s' \right| \hat{T}^{\mu\nu}(0) |p, s\rangle =$$

$$= \bar{u}(p', s') \left\{ \frac{P^\mu P^\nu}{M} A(t) + \frac{\Delta^\mu \Delta^\nu - \eta^{\mu\nu}\Delta^2}{M} C(t) + M\eta^{\mu\nu} \bar{C}(t) \right.$$

$$+ \frac{P^\mu i\sigma^{\nu\lambda}\Delta_\lambda}{4M} \Big[A(t) + B(t) + D(t) \Big]$$

$$\left. + \frac{P^\nu i\sigma^{\mu\lambda}\Delta_\lambda}{4M} \Big[A(t) + B(t) - D(t) \Big] \right\} u(p, s) \, .$$

X.-D. Ji, Phys. Rev. Lett., 78, 610 (1997)
C. Lorcé, L. Mantovani, B. Pasquini, Phys. Lett. B776, 38 (2018)

The study of the EMT became especially important after obtaining by Ji a relation between the EMT and GPDs

$$\int_{-1}^{1} dx\, H(x,\xi,t) = A(t) + 4\xi^2 C(t)\,,$$
$$\int_{-1}^{1} dx\, E(x,\xi,t) = B(t) - 4\xi^2 C(t)\,.$$

Besides this, it is know that $D(t) = -g_A(t)$, the axial form factor, and $\bar{C}(t)$ can be related to the scalar form factor.

X.-D. Ji, Phys. Rev. Lett., 78, 610 (1997)
X.-D. Ji, Phys. Rev. D55, 7114 (1997)
B.L.G. Bakker, E. Leader, T.L. Trueman, Phys. Rev. D 70, 114001 (2004)

Motivation

The study of the EMT is important because:

- $T^{\mu\nu}$ is a fundamental quantity, which allows to access for example a spin decomposition.
- DVCS gives a way to experimentally measure $T^{\mu\nu}$, *e.g.* JLab.
- Its form factors have a clear interpretation as spatial densities ($\vec{\Delta}$ is related to $\vec{r}$).
- EMT form factors and GPDs constrains each other.
- Studding EMT form factors one has an access to the limit $t = \Delta^2 \to 0$, which is excluded experimentally.

E. Leader, C. Lorcé, Phys.Rept. 541 (2014) 163

Outline

To better understand these formal definitions we write the EMT in a position space $x = (x_0, \vec{x})$ as

$$\widetilde{\langle T^{\mu\nu} \rangle}(x) = \int \frac{d^3\vec{\Delta}}{(2\pi)^3} e^{+i\Delta_0 x_0 - i\vec{\Delta}\cdot\vec{x}} \langle T^{\mu\nu} \rangle \ ,$$

where $\Delta_0 = \dfrac{\vec{P} \cdot \vec{\Delta}}{P_0}$ and we consider some special cases:

(a) forward limit (FL), $\vec{\Delta} = \vec{0}$;

(b) Breit frame (BF), $\vec{P} = \vec{0}$;

(c) elastic frame (EF), $\vec{P} \cdot \vec{\Delta} = 0$;

(d) infinite momentum frame (IMF), $\vec{P} \cdot \vec{\Delta} = 0$ and $|\vec{P}| \to \infty$.

Since in all these frames energy transfer $\Delta^0 = 0$, they all lead to computing the static (average of time) EMT.

$$\widetilde{\langle T^{\mu\nu}\rangle}(\vec{x}) = \int dx_0 \widetilde{\langle T^{\mu\nu}\rangle}(x)$$

$$= \int \frac{d^3\vec{\Delta}}{(2\pi)^3} \left(\int dx_0\, e^{i\Delta_0 x_0} \right) e^{-i\vec{\Delta}\cdot\vec{x}} \,\langle T^{\mu\nu}\rangle$$

$$= \int \frac{d^3\vec{\Delta}}{(2\pi)^3} e^{-i\vec{\Delta}\cdot\vec{x}} \,\langle T^{\mu\nu}\rangle \Big|_{\Delta_0=0}$$

The definition of $\langle T^{\mu\nu}\rangle$ originate from derivation based on the Wigner distribution. We can define the Wigner distribution of the proton state of momentum $\vec{P}$ at space point $\vec{X}$ following,

$$\rho(\vec{X}, \vec{P}) = \int \frac{d^3\vec{\Delta}}{(2\pi)^3} \frac{e^{-i\vec{\Delta}\cdot\vec{X}}}{\sqrt{2p_0'\,2p_0}} \left| P - \frac{\Delta}{2} \right\rangle \left\langle P + \frac{\Delta}{2} \right| .$$

Then,

$$\widetilde{\langle T^{\mu\nu}\rangle}(x) = \mathrm{Tr}[\hat{T}^{\mu\nu}(x)\,\rho(\vec{0}, \vec{P})] = \int \frac{d^3\vec{\Delta}}{(2\pi)^3} e^{i\Delta\cdot x} \langle T^{\mu\nu}\rangle .$$

E. P. Wigner, Phys. Rev. 40, 749 (1932)

(a) Forward limit

The EMT in the FL, $\vec{\Delta} = \vec{0}$, reads

$$\langle T^{\mu\nu} \rangle \Big|_{\mathrm{FL}} = \int d^3\vec{r}\, \widetilde{\langle T^{\mu\nu} \rangle}(\vec{r}) = A(0) \frac{P^\mu P^\nu}{P_0} + \bar{C}(0) \frac{M^2}{P_0} \eta^{\mu\nu} \,,$$

where $P_0 \big|_{FL} = \sqrt{M^2 + \vec{P}^{\,2}}$.

C. Lorcé, Eur. Phys. J. C78, 120 (2018)
C. Eckart, Phys. Rev. 58, 919 (1940)

We can divide $\hat{T}^{\mu\nu}(x)$ into quark and gluon contributions, where

$$\hat{T}_q^{\mu\nu}(x) = \frac{1}{2}\hat{\bar{\psi}}(x)\gamma^{\mu}\, i\overleftrightarrow{D}^{\nu}\hat{\psi}(x)\,,$$
$$\hat{T}_g^{\mu\nu}(x) = -2\mathrm{Tr}[G^{\mu\lambda}(x)G^{\nu}{}_{\lambda}(x)] + \frac{1}{2}g^{\mu\nu}\mathrm{Tr}[G^{\rho\sigma}(x)G_{\rho\sigma}(x)]\,.$$

Then each EMT form-factor is a sum of quark and gluon contribution, e.g $A(t) = A_q(t) + A_g(t)$ and $\bar{C}(t) = \bar{C}_q(t) + \bar{C}_g(t)$.

The conservation of the EMT implies,

$$\partial_{\nu}T^{\mu\nu} = 0 \implies \bar{C}(t) = 0.$$

Proper velocity and volume

The EMT in the FL has the same structure as the perfect fluid density

$$\theta^{\mu\nu} = (\epsilon + p) u^{\mu} u^{\nu} - p\,\eta^{\mu\nu}\,,$$

where ϵ is the energy density, p is the isotropic pressure, and $u^{\mu} = P^{\mu}/M$ is the four-velocity.
The physical velocity $\vec{v}$ is related to the proper velocity, $\vec{u}$, by $\frac{M}{P_0}$,

$$\vec{v} = \frac{M}{P_0}\vec{u} = \frac{M}{\sqrt{M^2 + M^2\vec{u}^{\,2}}}\,\vec{u} = \frac{\vec{u}}{\sqrt{1+\vec{u}^{\,2}}}\,,$$

similar, the volume $d^3\vec{r}$ is related to the proper volume $d^3\mathcal{V}$,

$$d^3\vec{r} = \frac{M}{P_0} d^3\mathcal{V} = \frac{d^3\mathcal{V}}{\sqrt{1+\vec{u}^{\,2}}}\,.$$

C. Lorcé, Eur. Phys. J. C78, 120 (2018)

Interpretation of $A_{q/g}(0)$ and $\bar{C}_{q/g}(0)$

Observing that

$$\int d^3\mathcal{V}\, p_{q/g} = -\bar{C}_{q/g}(0) M\,,$$

$$\int d^3\mathcal{V}\, \epsilon_{q/g} = \Big[A_{q/g}(0) + \bar{C}_{q/g}(0)\Big] M\,.$$

And because

$$\bar{C}(t) = \bar{C}_q(t) + \bar{C}_g(t) = 0 \quad \text{and} \quad A(0) = A_g(0) + A_q(0) = 1\,,$$

we have

$$\int d^3\mathcal{V}\,(p_q + p_g) = 0\,, \qquad \text{and} \qquad \int d^3\mathcal{V}\,(\epsilon_q + \epsilon_g) = M\,.$$

C. Lorcé, Eur. Phys. J. C78, 120 (2018)

The EMT in the BF, $\vec{P} = \vec{0}$, reads

$$\langle T^{\mu\nu}\rangle\Big|_{\mathrm{BF}} = \left[MA(t) - \frac{\vec{\Delta}^{\,2}}{4M}B(t)\right]\eta^{\mu 0}\eta^{\nu 0} + \left[M\bar{C}(t) + \frac{\vec{\Delta}^2}{M}C(t)\right]\eta^{\mu\nu} + \frac{\Delta^\mu\Delta^\nu}{M}C(t)\,,$$

where $P_0\big|_{BF} = \sqrt{M^2 + \frac{\vec{\Delta}^2}{4}}$. This expression has the same structure as the anisotropic fluid density,

$$\Theta^{\mu\nu} = (\epsilon + p_t)u^\mu u^\nu - p_t\eta^{\mu\nu} + (p_r - p_t)\chi^\mu\chi^\nu\,,$$

in the rest frame: $u^\mu = \eta^{\mu 0}$ and $\chi^\mu = (0, \vec{r}/|\vec{r}|)$, where $(p_r - p_t)$ is a pressure anisotropy.

S.S. Bayin, Astrophys. J. 303, 101 (1986)

The Fourier transforms of the EMT leads to

$$p_r(r) = -M\tilde{\bar{C}}(r) + \frac{1}{M}\frac{2}{r}\frac{d\tilde{C}(r)}{dr},$$

$$p_t(r) = -M\tilde{\bar{C}}(r) + \frac{1}{M}\Big(\frac{1}{r}\frac{d\tilde{C}(r)}{dr} + \frac{d^2\tilde{C}(r)}{dr^2}\Big),$$

$$\begin{aligned}\epsilon(r) = M\tilde{A}(r) &+ \frac{1}{4M}\Big(\frac{2}{r}\frac{d\tilde{B}(r)}{dr} + \frac{d^2\tilde{B}(r)}{dr^2}\Big) \\ &+ M\tilde{\bar{C}}(r) - \frac{1}{M}\Big(\frac{2}{r}\frac{d\tilde{C}(r)}{dr} + \frac{d^2\tilde{C}(r)}{dr^2}\Big).\end{aligned}$$

The EMT in the EF, $\vec{P}\cdot\vec{\Delta} = \vec{0}$, reads

$$\langle T^{\mu\nu}\rangle\Big|_{\text{EF}} = \left\{\left[1-\frac{\vec{P}^{\,2}}{N^2}\right]A(t) - \frac{\vec{\Delta}_\perp^{\,2}}{4N^2}\left[A(t)+B(t)\right]\right\}\frac{P^\mu P^\nu}{M}$$
$$+\left[1-\frac{\vec{P}^{\,2}}{N^2}\right]\left\{\left[M\bar{C}(t)+\frac{\vec{\Delta}_\perp^{\,2}}{M}C(t)\right]\eta^{\mu\nu}+\frac{\Delta^\mu\Delta^\nu}{M}C(t)\right\}$$
$$-\frac{\vec{\Delta}_\perp^{\,2}}{8N^2}\left[A(t)+B(t)\right]\left[\eta^{0\mu}P^\nu+\eta^{0\nu}P^\mu\right]$$
$$+\frac{\vec{\Delta}_\perp^{\,2}}{8N^2}D(t)\left[\eta^{0\mu}P^\nu-\eta^{0\nu}P^\mu\right],$$

where $N^2 = P_0(M+P_0)$ and $P_0\big|_{\text{EF}} = \sqrt{M^2+\vec{P}^{\,2}+\frac{\vec{\Delta}_\perp^{\,2}}{4}}$.

C. Lorcé, L. Mantovani, B. Pasquini, Phys. Lett. B776, 38 (2018)

Interpretation of $D(t)$

The EMT in the EF is not symmetric,

$$\left\langle T^{[\mu\nu]} \right\rangle \Big|_{\text{EF}} = \frac{\Delta_\perp^2}{4N^2} D(t)\, \eta^{0\mu} P^\nu = -i\Delta_\lambda \left\langle S^{\lambda\mu\nu} \right\rangle ,$$

where spin tensor reads

$$\left\langle S^{\lambda\mu\nu} \right\rangle = -i\, \frac{g_A(t)}{4N^2} \epsilon^{\lambda\mu\nu\sigma} \epsilon_{\sigma 0\alpha\beta} P^\alpha \Delta^\beta .$$

Thus $D(t) = -g_A(t)$ and

$$\vec{S}\,(\vec{r}_\perp) = -i \int \frac{d^2\Delta_\perp}{(2\pi)^2} e^{-i\vec{\Delta}_\perp \cdot \vec{r}_\perp} \frac{D(t)}{4N^2} \left(\vec{P} \times \vec{\Delta}_\perp \right) \Big|_{\text{EF}} .$$

$$= \left(\vec{P} \times \vec{\partial}_\perp \right) \int \frac{d^2\Delta_\perp}{(2\pi)^2} e^{-i\vec{\Delta}_\perp \cdot \vec{r}_\perp} \frac{D(t)}{4N^2}$$

C. Lorcé, L. Mantovani, B. Pasquini, Phys. Lett. B776, 38 (2018)

In the IMF, where $\vec{P} \cdot \vec{\Delta} = \vec{0}$ and $|\vec{P}| \to \infty$, thus

$$P^\mu \simeq |\vec{P}|(\eta^{\mu 0} + \eta^{\mu 3}) + \frac{M^2 + \frac{\vec{\Delta}_\perp^{\,2}}{4}}{2|\vec{P}|}\eta^{\mu 0}$$

$$N^2 = P_0^2 + P_0\, M \simeq |\vec{P}|^2 + M\,|\vec{P}|\,,$$

where $\simeq$ means that terms $\mathcal{O}(\vec{P}^{\,-2})$ have been suppressed.

We also introduce a notation, that for each vector/tensor v^μ,

$$v^\pm = \frac{1}{\sqrt{2}}(v^0 \pm v^3)\,.$$

S. Weinberg, Phys. Rev. 150, 1313 (1966)

The EMT in the IMF reads,

$$\langle T^{++} \rangle \approx 2|\vec{P}\,|A\,,$$

$$\langle T^{+-} \rangle \simeq \frac{1}{2|\vec{P}\,|}\left[\left(M^2 + \frac{\vec{\Delta}_\perp^{\,2}}{4}\right)A + 2\left(M^2\bar{C} + \vec{\Delta}_\perp^{\,2}C\right) - \frac{\vec{\Delta}_\perp^{\,2}}{4}(A+B+D)\right],$$

$$\langle T^{ij} \rangle \simeq \frac{1}{|\vec{P}\,|}\left[\Delta_\perp^i\Delta_\perp^j C + \left(M^2\bar{C} + \vec{\Delta}_\perp^{\,2}C\right)\eta^{ij}\right] \quad \text{for } i,j \in \{1,2\}\,.$$

It is easy to make Fourier transform.

The EMT in the DYF, $\Delta^+ = 0$, reads,

$$\langle T^{++} \rangle = P^+ A\,,$$

$$\langle T^{+-} \rangle = \frac{1}{2P^+}\left[\left(M^2 + \frac{\vec{\Delta}_\perp^{\,2}}{4}\right)A + 2\left(M^2\bar{C} + \vec{\Delta}_\perp^{\,2} C\right) - \frac{\vec{\Delta}_\perp^{\,2}}{4}(A + B + D)\right],$$

$$\langle T^{ij} \rangle = \frac{1}{P^+}\left[\Delta_\perp^i \Delta_\perp^j C + \left(M^2\bar{C} + \vec{\Delta}_\perp^{\,2} C\right)\eta^{ij}\right] \quad \text{for } i,j \in \{1,2\}\,.$$

P.A.M. Dirac, Rev. Mod. Phys. 21, 392 (1949) http://partons.cea.fr

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PARTONS provides a necessary bridge between models of GPDs and experimental data measured in various exclusive channels, like Deeply Virtual Compton Scattering (DVCS) and Hard Exclusive Meson Production (HEMP). The experimental programme devoted to study GPDs has been carrying out by several experiments, like HERMES at DESY (closed), COMPASS at CERN, Hall-A and CLAS at JLab. GPD subject will be also a key component of the physics case for the expected Electron Ion Collider (EIC).

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Scalar diquark model

The SDQM is defined by

$$\psi_{\uparrow}^{\Uparrow}(X,k_\perp) = \left(M + \frac{m}{X}\right)\varphi(X,k_\perp) = \psi_{\downarrow}^{\Downarrow}(X,k_\perp)\,,$$
$$\psi_{\uparrow}^{\Downarrow}(X,k_\perp) = \frac{k_1 - i\,k_2}{X}\varphi(X,k_\perp) = -\psi_{\downarrow}^{\Uparrow}(X,k_\perp)^{\dagger}\,,$$

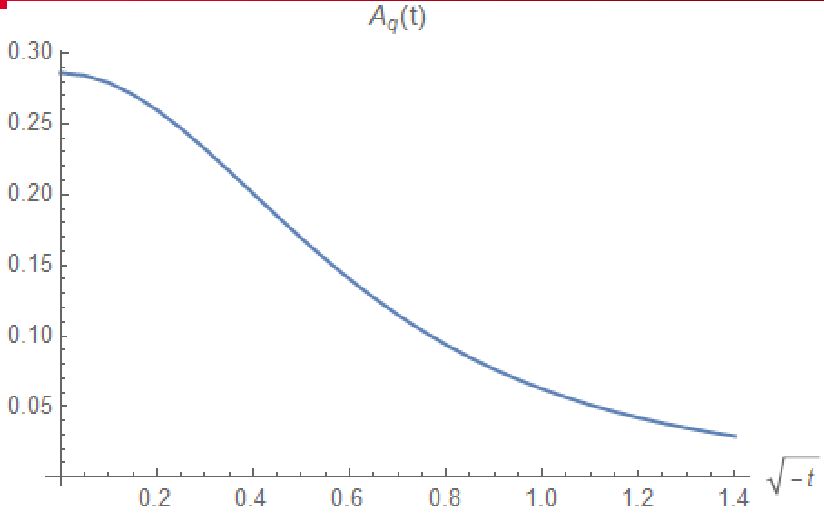
where

$$\varphi(X,k_\perp) = g\frac{M^{2p}}{\sqrt{1-X}}X^{-p}\left(M^2 - \frac{k_\perp^2 + m^2}{X} - \frac{k_\perp^2 + \lambda^2}{1-X}\right)^{-p-1}\,.$$

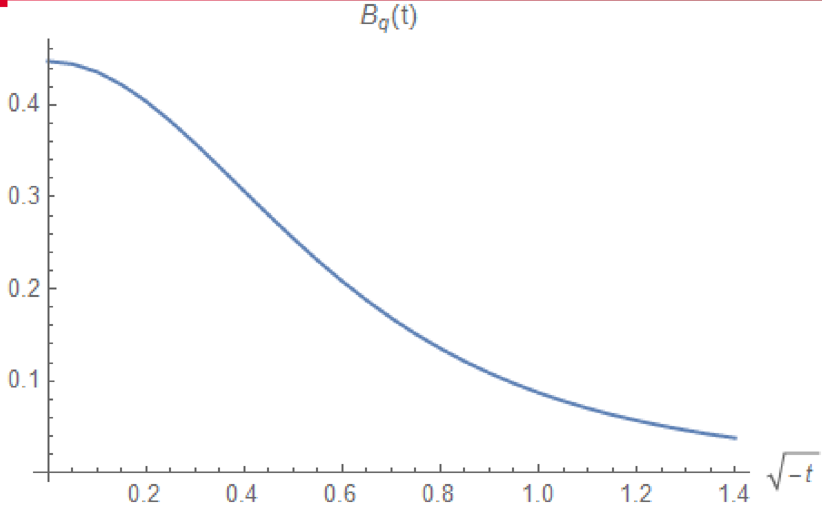
Reasonable results for:

$$p = 1 \qquad \lambda = 0.75\text{GeV} \qquad m = 0.45\text{GeV} \qquad M = 1GeV$$

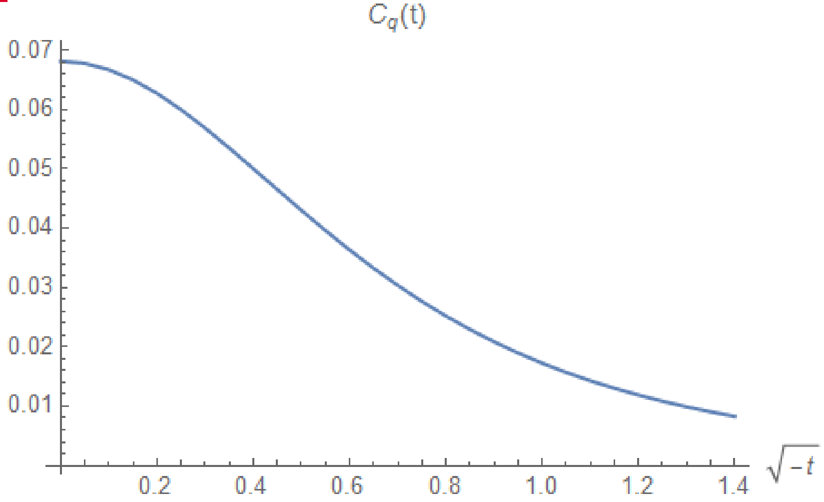
D.S. Hwang, D. Mueller, Phys.Lett. B660 (2008) 350

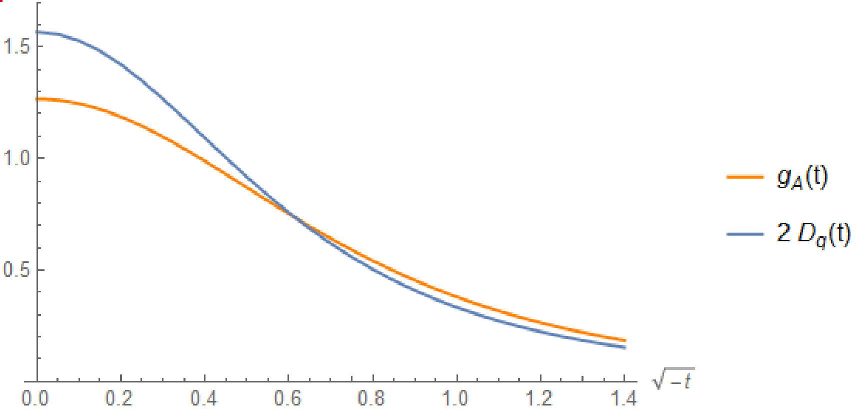


$A_q(t)$
0.30
0.25
0.20
0.15
0.10
0.05
0.2
0.4
0.6
0.8
1.0
1.2
1.4
$\sqrt{-t}$



$B_q(t)$
0.4
0.3
0.2
0.1
0.2
0.4
0.6
0.8
1.0
1.2
1.4
$\sqrt{-t}$


$C_q(t)$
0.07
0.06
0.05
0.04
0.03
0.02
0.01
0.2
0.4
0.6
0.8
1.0
1.2
1.4
$\sqrt{-t}$



$g_A(t) = \frac{g_A}{1-t/M_A^2}$, where $g_A = 1.3$ and $M_A = 1.1$ GeV.

A.V. Belitsky, A.V. Radyushkin, Phys.Rept. 418 (2005) 1

Conclusion

Study of the $\langle T^{\mu\nu} \rangle$ in different frames of reference gives us clear interpretation of the EMT form factors

- $A(t)$, $B(t)$, $C(t)$, $\bar{C}(t)$ are related to the energy and pressure density,
- $C(t)$ is related to the anisotropy pressure,
- $D(t)$ is related to the spin density.

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