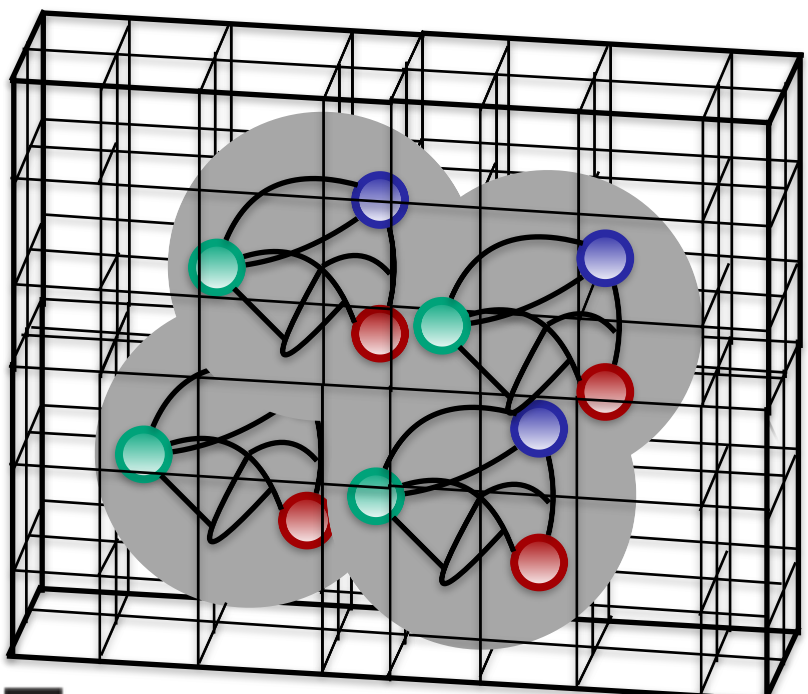
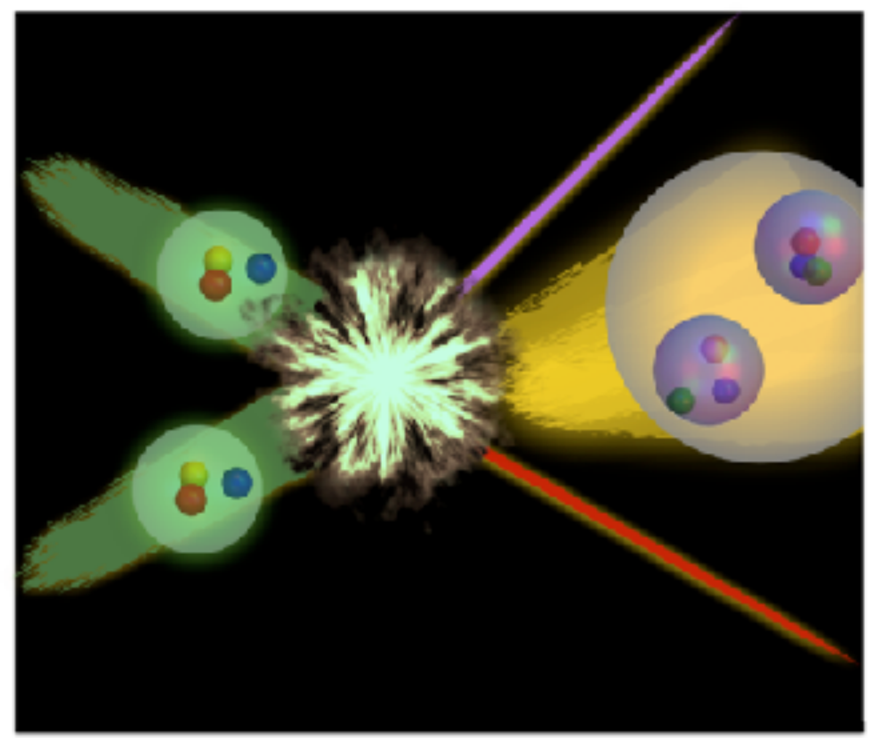
Interacting with Nuclei in the Standard Model and Beyond

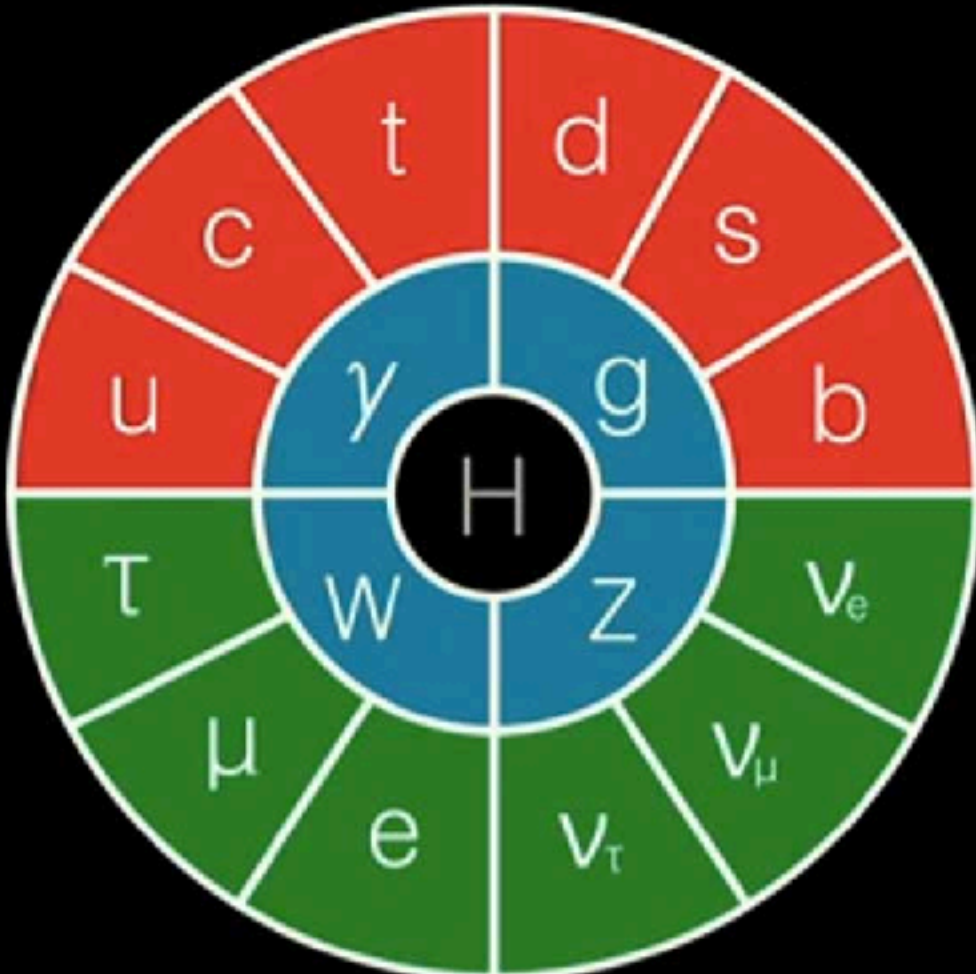
JLab Theory Seminar, Feb 26, 2018

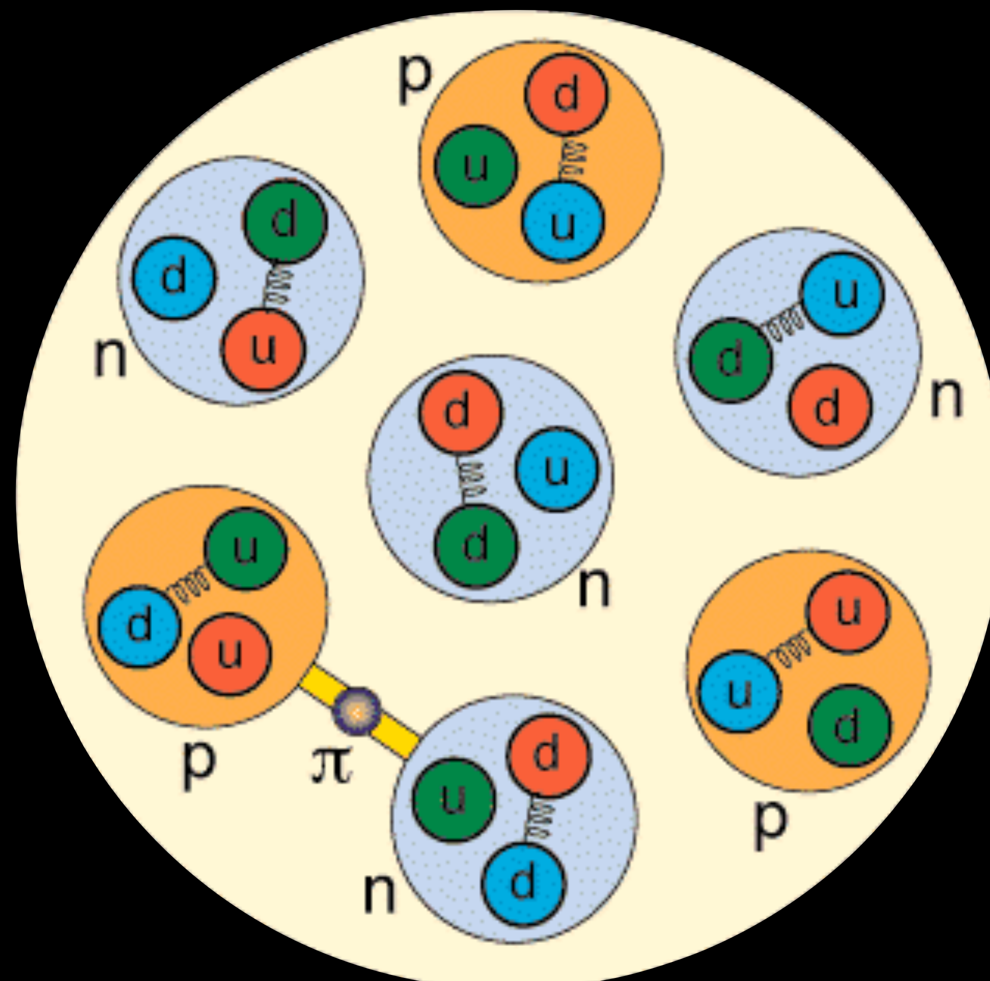
Michael Wagman





Big Picture — The Standard Model



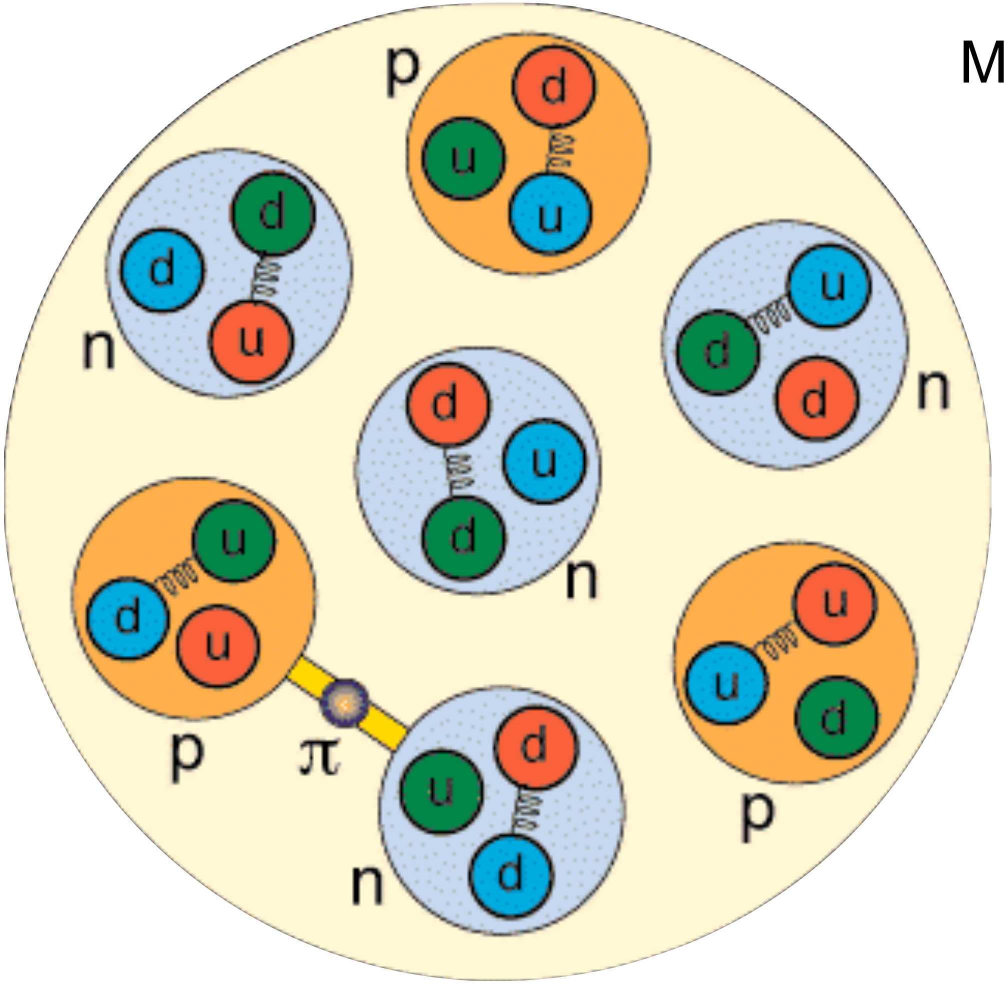


What’s the ultraviolet completion of the Standard Model?

Can the Standard Model explain the emergent complexity of matter?

Nuclei

In nature, nuclei behave (to a crude approximation) like collections of non-interacting nucleons



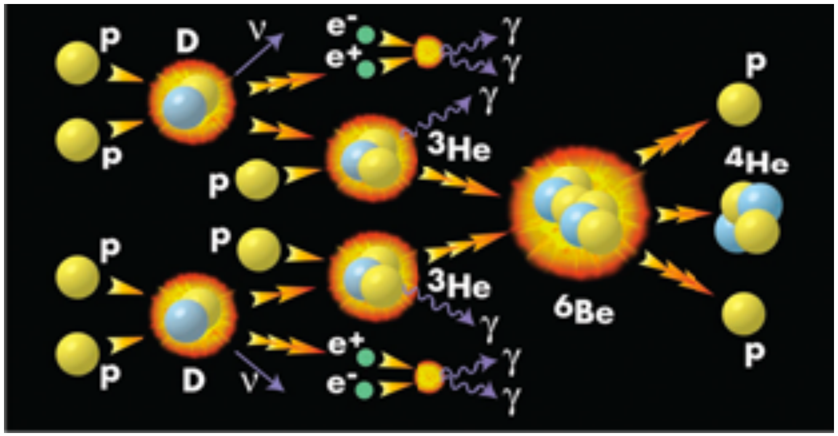
Masses and charges of nuclei are close to sums of mass and charges of free nucleons

…why?

- Generic emergent property of non-Abelian gauge theory?

- Quirk due to the values of quark masses in nature?

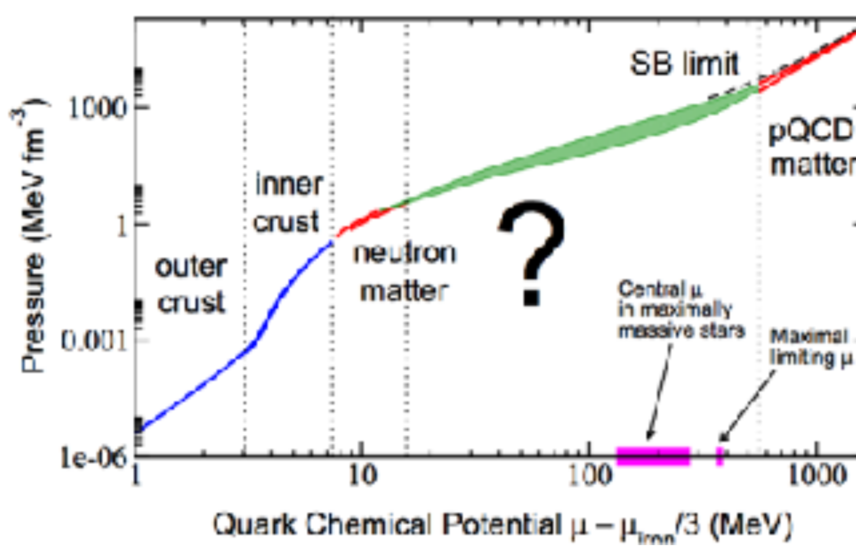
Nuclear Physics from QCD



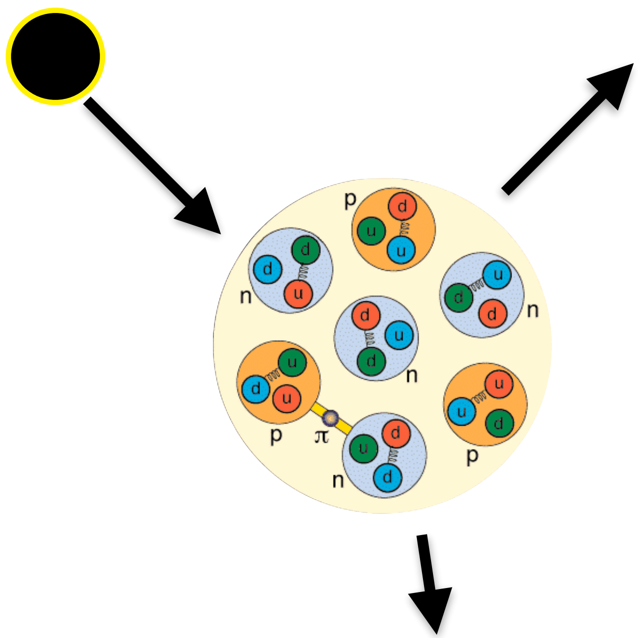
QCD matrix elements determine cross-sections for fusion reactions powering stellar burning

QCD equation of state determines properties of neutron star matter

QCD structure functions determine the distributions of quarks and gluons in nuclei

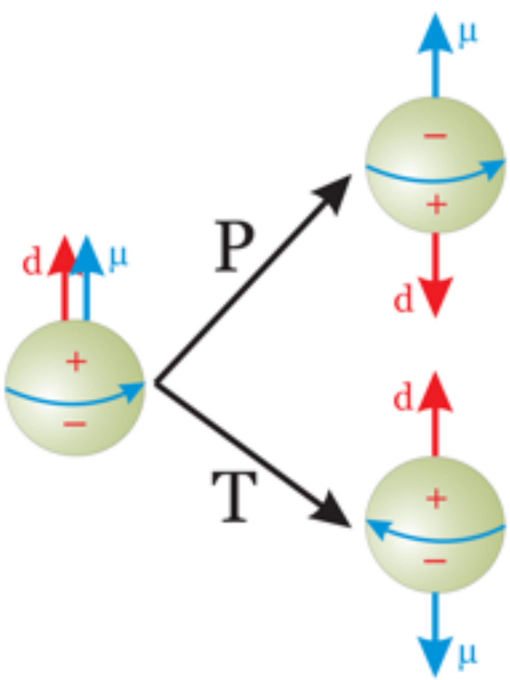


Nuclei as New Physics Probes



Dark matter direct detection experiments constrain cross-sections of nuclear recoils from dark matter collisions

Electric dipole moments nearly zero in Standard Model, larger in theories with extra CP violation that can explain the matter-antimatter asymmetry



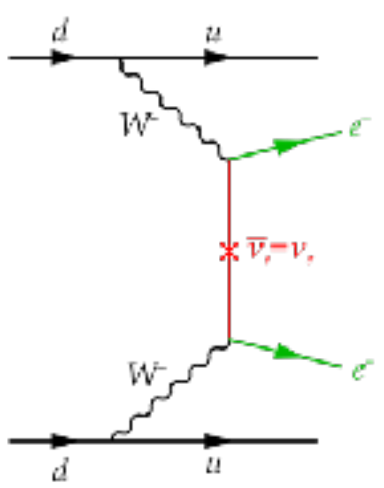
Neutrinoless Double-Beta Decay

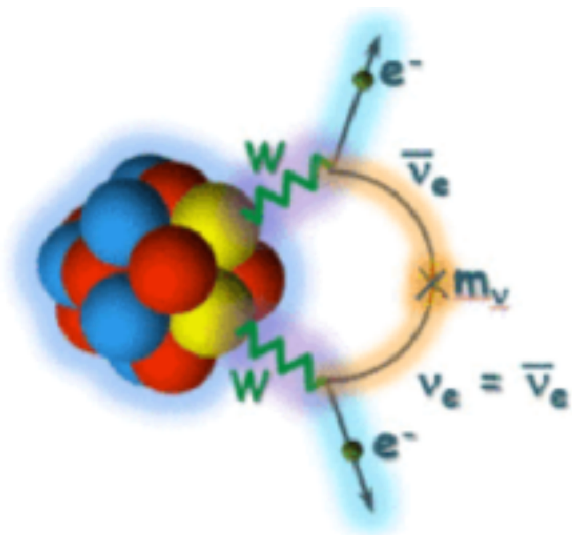
Neutrino masses provide evidence for physics beyond the Standard Model

Magnitude of neutrino masses and Majorana vs Dirac nature unknown

$$\mathcal{L}_{SMEFT} = \mathcal{L}_{SM} + \frac{1}{\Lambda}(\ell H)^2 + \ldots$$

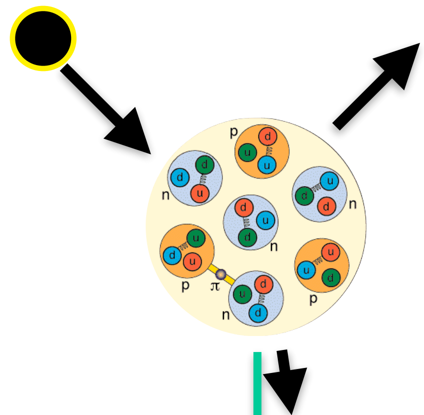
A Majorana neutrino mass would imply that lepton number is not conserved and that neutrinoless double-beta decay is possible

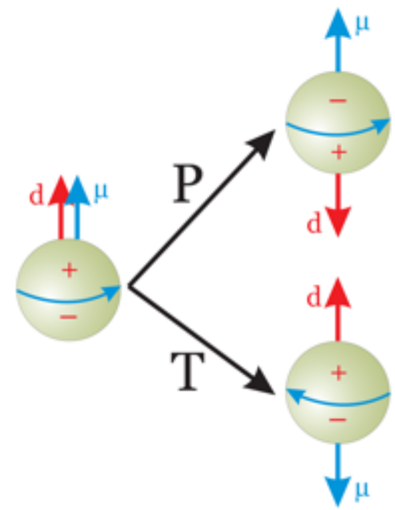




Nuclear Matrix Elements

Matrix elements of currents in nuclei are needed to predict structure and reactions and to connect experiment to BSM theory



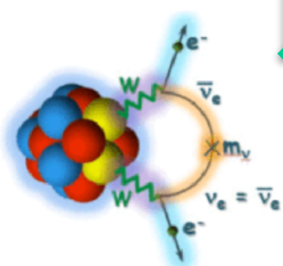


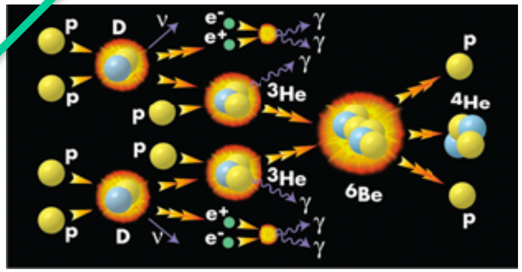
$$< A|J_X|A >$$

$$X = S, A, T, V, P$$

$$< A|J_X J_{X'}|A >$$

..





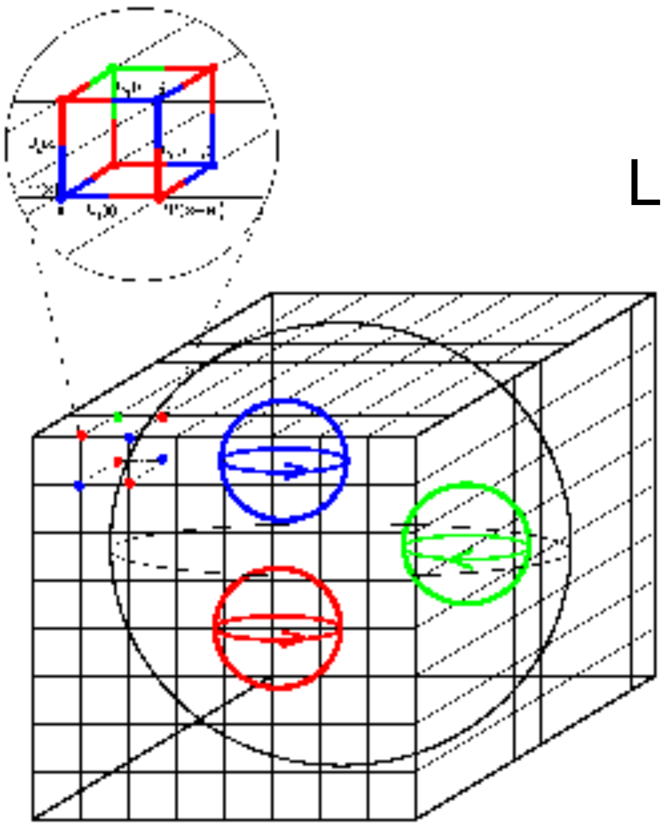
$$V \sim 1$$

$$P \sim 0$$

Lattice QCD

For a finite volume with a discrete lattice of spacetime points, QFT path integrals are large but finite dimensional integrals

$$< A|J_X|A >= \int \mathcal{D}U\mathcal{D}\overline{q}\mathcal{D}q\; e^{-S(q,\bar{q},U)}\; A\; J_X\; A^\dagger$$

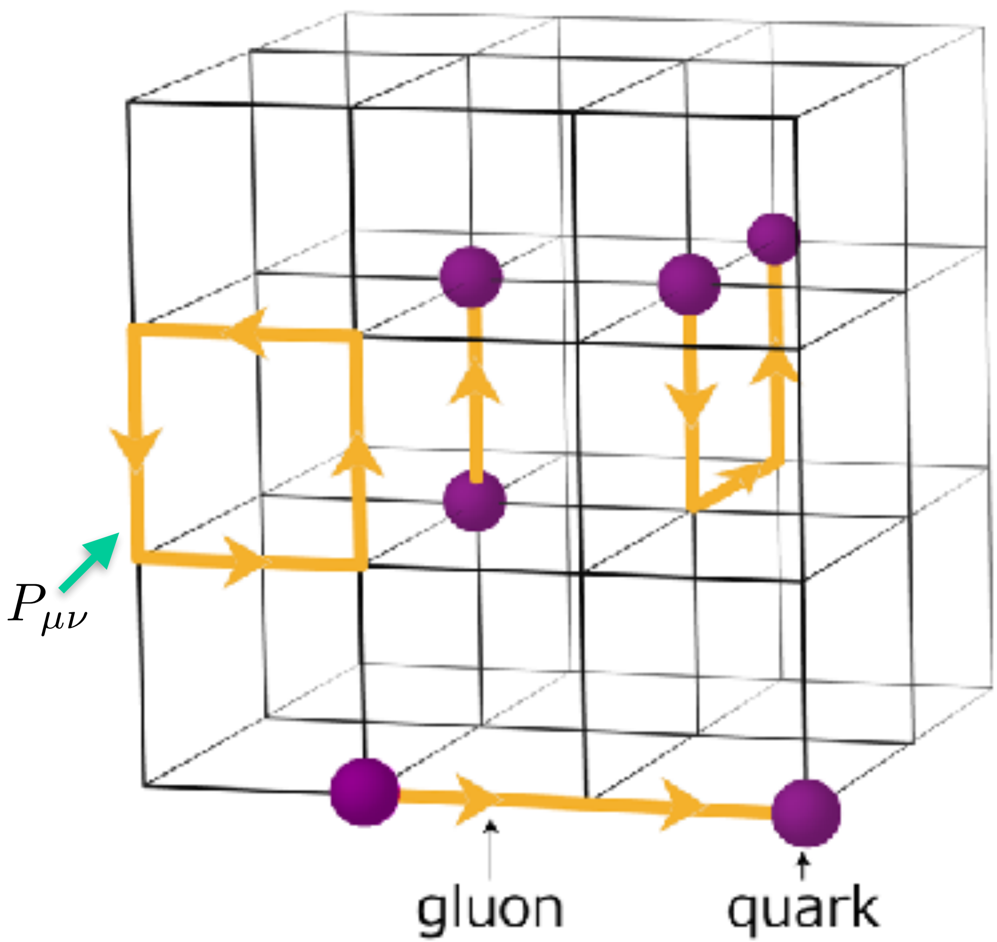


Lattice QCD — stochastically sampling QCD field configurations with probability $\mathcal{P}(U) = e^{-S(U)}$ on very large but finite computers

Gluon Field Configurations

QCD action can be discretized with quark fields on lattice sites and gluon fields associated with links between sites

$$U_\mu = e^{i \int A_\mu dx_\mu}$$



Action defined in terms of gauge invariant loops such as "plaquette"

$$P_{\mu\nu}(x) = U_\mu(x) U_\nu(x+\hat{\mu}) \times U^\dagger_\mu(x+\hat{\mu}+\hat{\nu}) U^\dagger_\nu(x+\hat{\nu})$$

$$S(U) = \frac{1}{g^2} \sum_{x,\mu,\nu} \mathrm{Tr}[1 - P_{\mu\nu}(x)]$$

Pure gauge theory Monte Carlo

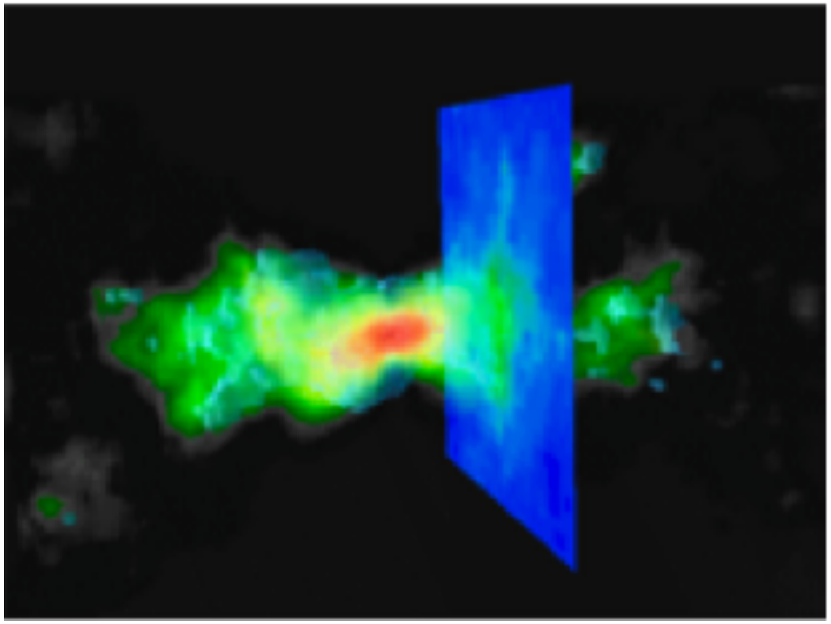
$$\mathcal{P}(U) = e^{-S(U)}$$

Additional determinant for quarks

Quark Propagators

Quark action — free fermions interacting
with a background gluon field

$$S_q = \sum_x \bar{q}(x)[\gamma_\mu D_\mu(U) + m]q(x)$$



R. Gupta

Quark propagator

$$C_q(t,U) = \frac{1}{Z_q}\int \mathcal{D}\bar{q}\mathcal{D}q\; e^{-S_q(q,\bar{q},U)}q(t)\bar{q}(0) = (\gamma_\mu D_\mu(U) + m)^{-1}$$

Numerically calculate inverse for Monte Carlo ensemble of gluon fields

Construct hadron correlation functions from products of quark propagators

Hadron Correlation Functions

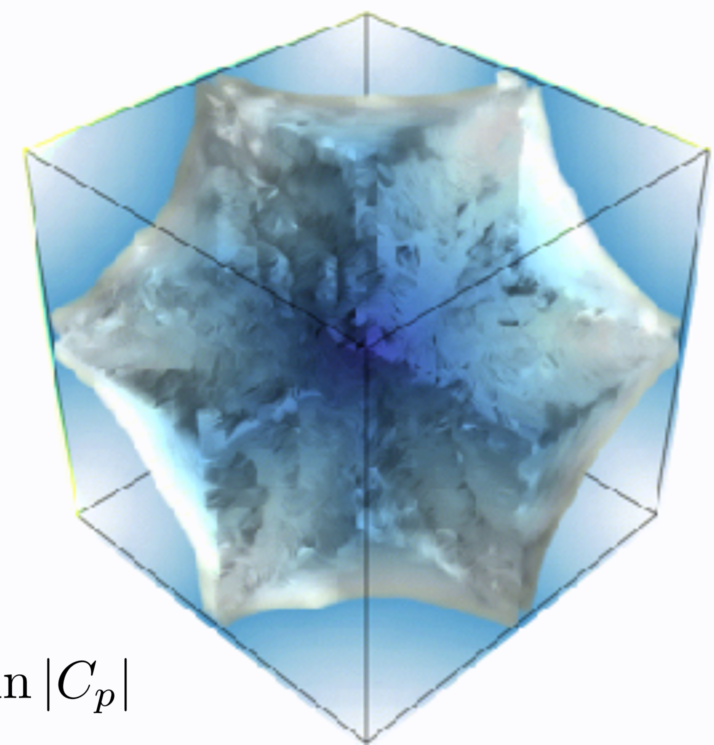
Creation/annihilation operators for hadrons built from products of quark fields with appropriate quantum numbers

$$\pi \sim \bar{u}\gamma_5 d \qquad\qquad p \sim (uC\gamma_5 d)u$$

Hadronic correlation functions (e.g. proton propagator) in each gluon field configuration built from products of quark propagators.

$$C_\pi \sim C_u C_d^\dagger \qquad\qquad C_p \sim C_u^2 C_d$$

QCD proton propagator obtained by averaging this over gluon field configurations



$\ln |C_p|$

$\ln |C_p|$

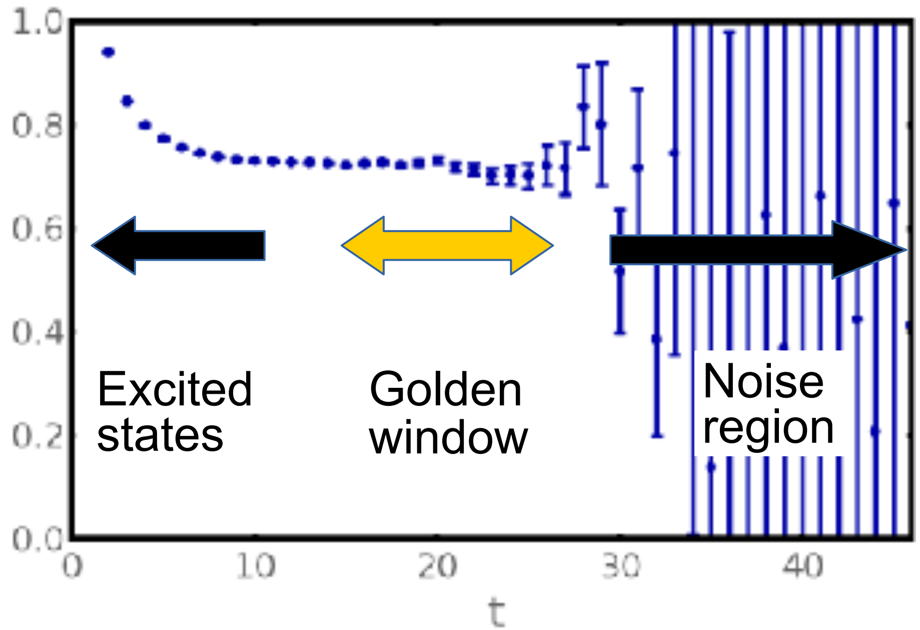
The Proton Mass

Inserting complete set of states relates proton propagator to QCD spectrum

$$\langle\Omega|p(t)p^\dagger(0)|\Omega\rangle = \sum_n \left|\langle n|p^\dagger(0)|\Omega\rangle\right|^2 e^{-E_n t} \sim e^{-M_p t}$$

Proton mass obtained from large Euclidean time behavior of proton propagator

$$M_p(t) = -\frac{\partial}{\partial t} C_p$$



Different choices of proton creation operator have different overlap with QCD proton ground state. Excited state contamination reduced by improving proton operator

Nuclear Physics from LQCD

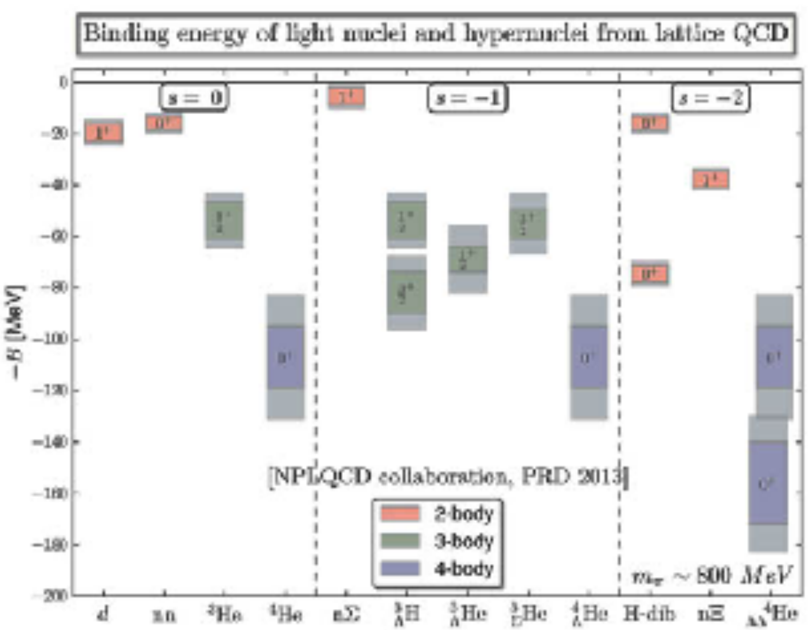
Quark field contractions factorially bad in baryon number; symmetries help

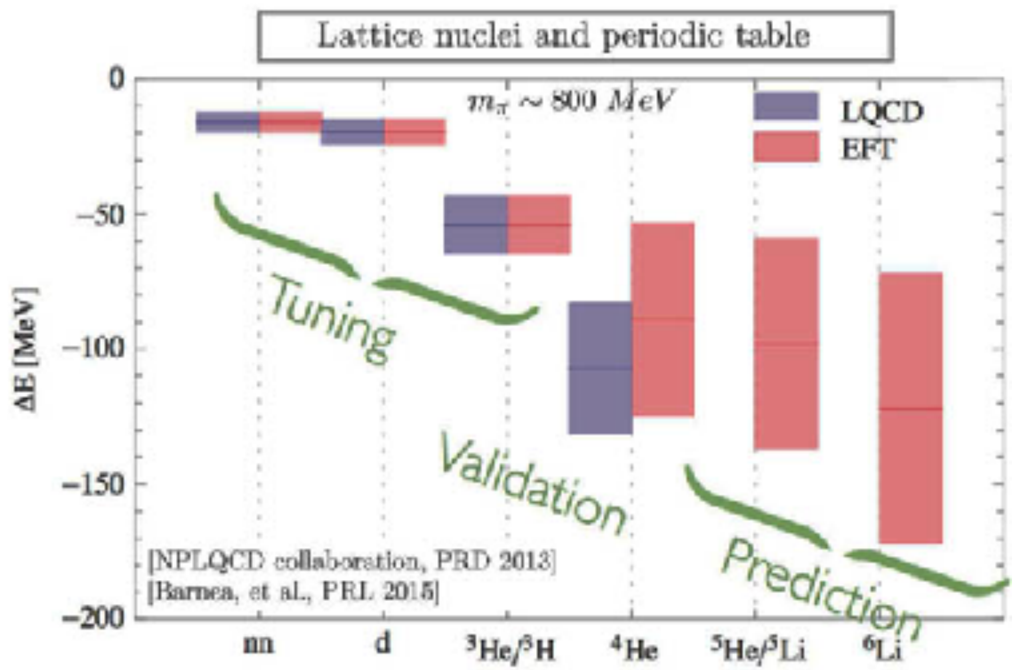
Doi, Endres (2012) Detmold, Orginos (2012)

Exponentially worse signal-to-noise ratios for more nucleons and lighter quarks

Parisi (1984) Lepage (1989)

Binding energies of small nuclei computed from LQCD with heavy quarks, extended to larger nuclei using effective field theory and many-body methods

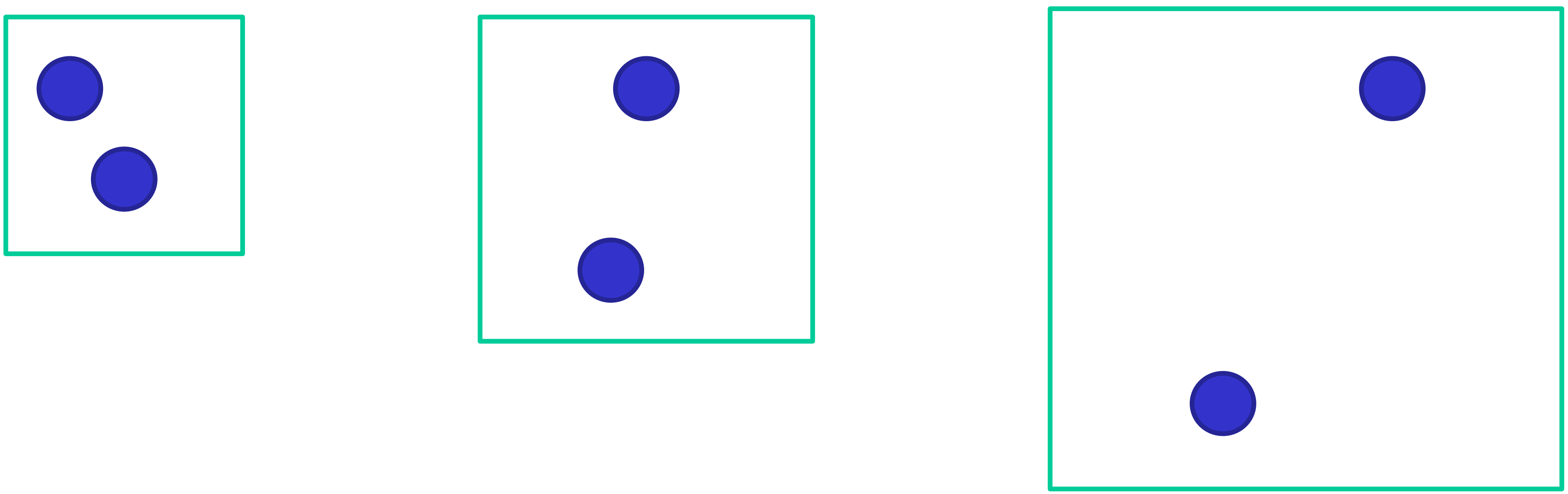




Two Nucleons in a Box

Real-time scattering processes can’t be directly calculated with Monte Carlo importance sampling because of sign problem

Instead exploit finite volume — short-range interactions more probable in smaller boxes Lüscher (1986), extended by many others

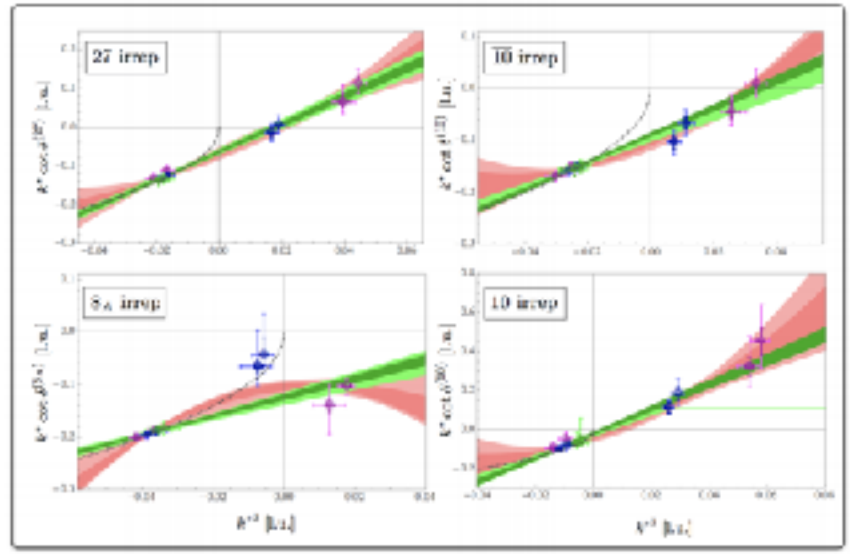


Interaction strength related to volume-dependence of total system energy

Finite-volume quantization conditions allow calculations of phase shifts

Baryon-Baryon Scattering

Baryon-baryon low-energy scattering determined from finite-volume spectra at unphysical heavy quark masses



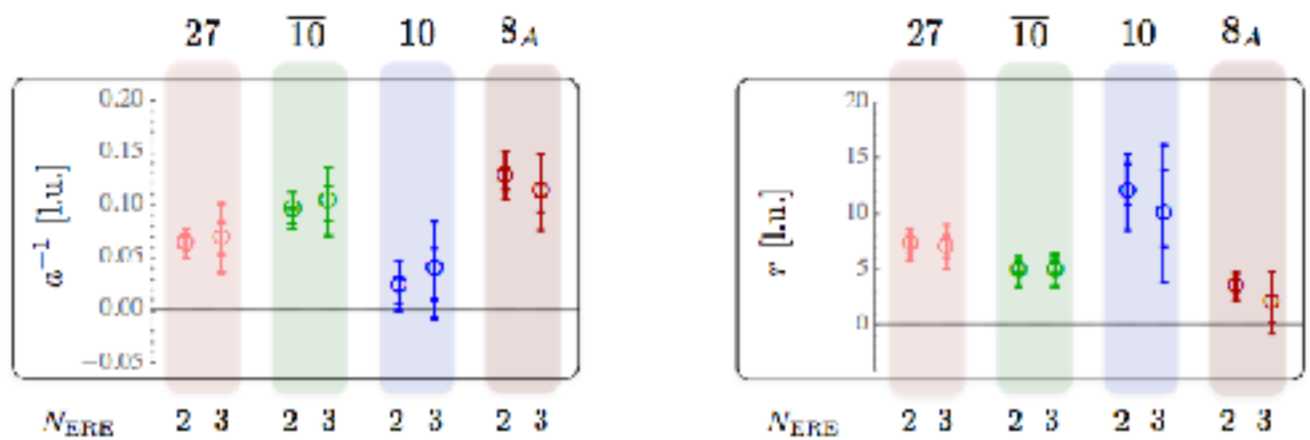
Quenched:
Gupta, Patel, Sharpe (1993)
Fukugita, Kuramashi, Mino, Okawa, Ukawa (1994)

Dynamical:
Beane, Bedaque, Orginos, Savage (2006)
Inoue, Ishii, Aoki et al (2010)
…

$SU(6)$ and $SU(16)$ approximate symmetries of nuclear forces observed at heavy quark mass

All spin-flavor channels appear close to unitary fixed point (technically unnatural)

$N_f = 3,\ m_N = 0.806$ GeV, $a = 0.145(2)$ fm

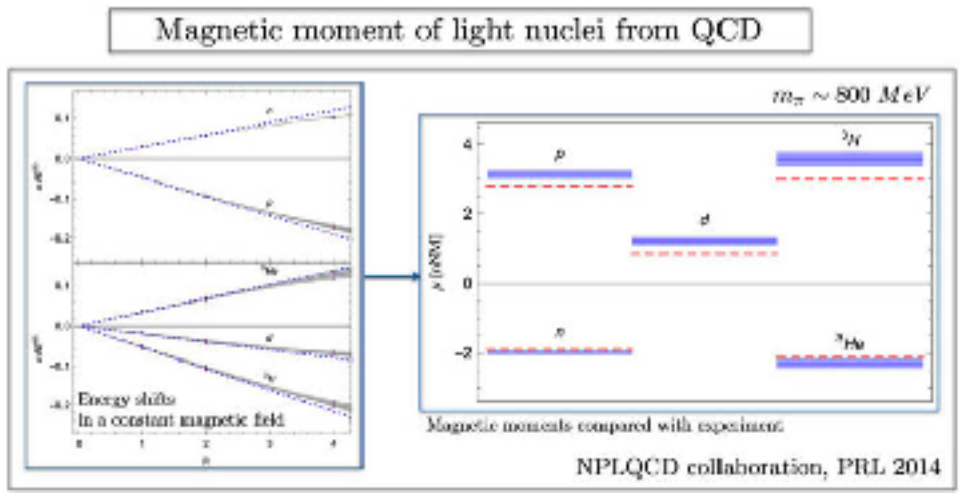


MW, Winter, Chang, Davoudi, Detmold, Orginos, Savage, Shanahan (2017)

Two Nucleons in a Box with a Magnetic Field

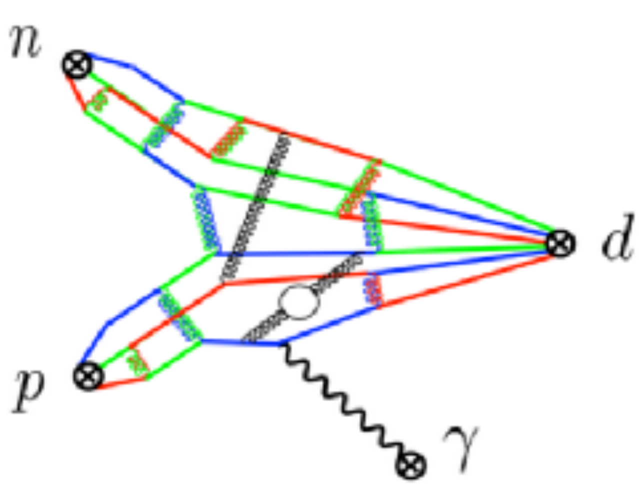
Classical background fields can be included in LQCD calculations

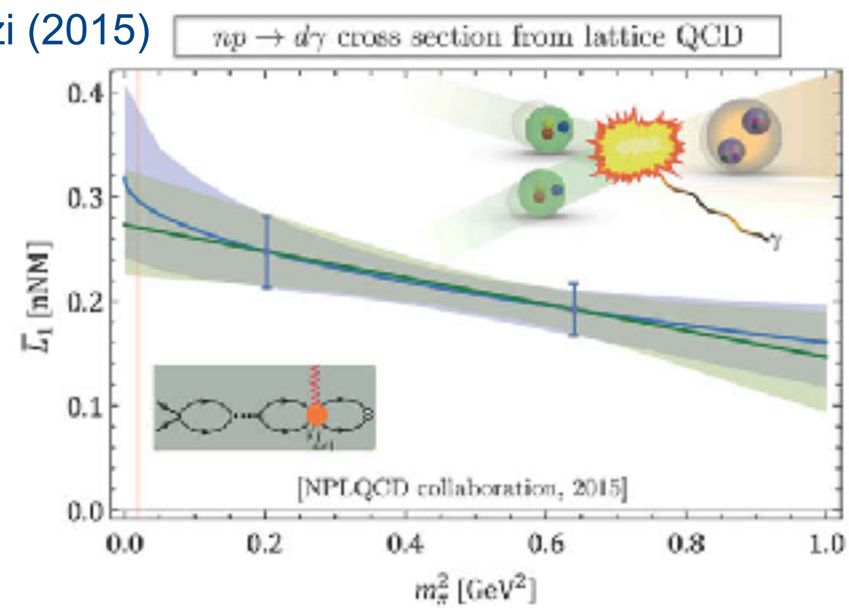
Linear response to magnetic field allows calculation of magnetic moments



Electromagnetic fusion reaction rates also calculable from linear response

Beane, Chang, Detmold, Orginos, Parreño, Savage, Tiburzi (2015)



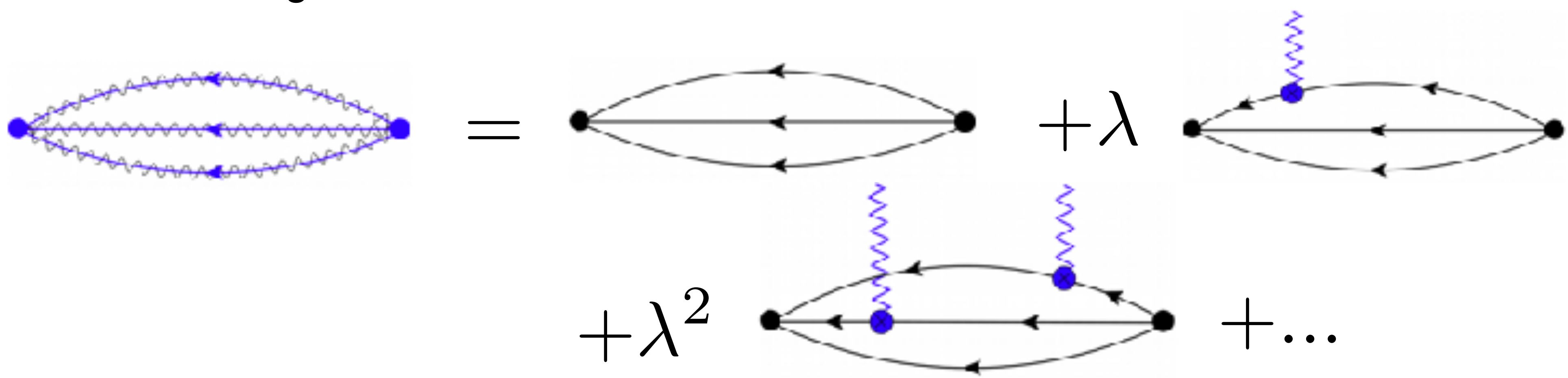


Two Nucleons in a Box with an Axial Field

Instead of putting classical background field in action and fitting linear response,

linearized background fields obtained by adding currents to quark propagators

Linear response of composite particle given exactly from linear combinations of different background fields where all nonlinear terms cancel

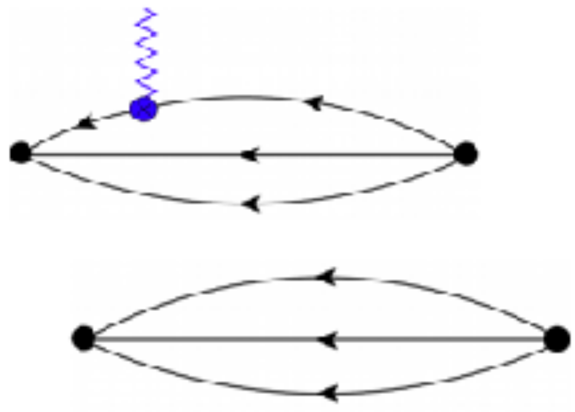


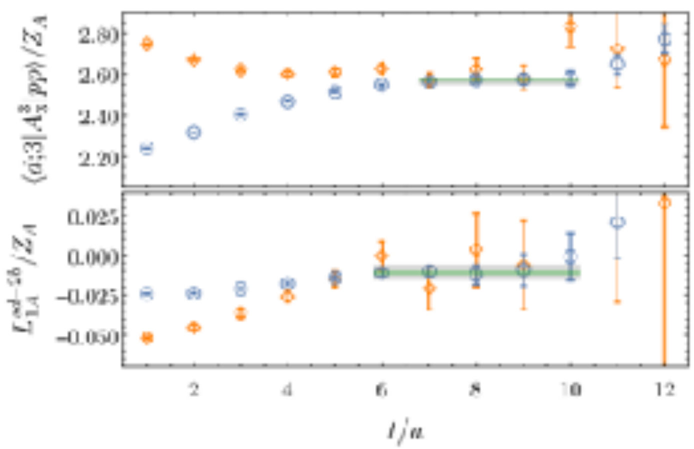
Savage, Shanahan, Tiburzi, MW, Winter, Beane, Chang, Davoudi, Detmold, Orginos (2016)

Bouchard, Chang, Kurth, Orginos, Walker-Loud (2016)

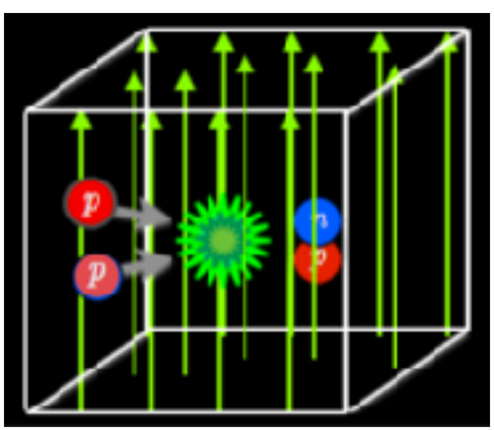
Proton-Proton Fusion

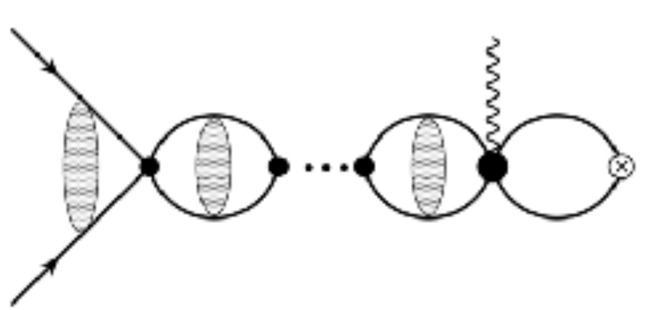
Bound dineuteron state mixes with bound deuteron state in the pretense of background axial field (or with the insertion of a single axial current)

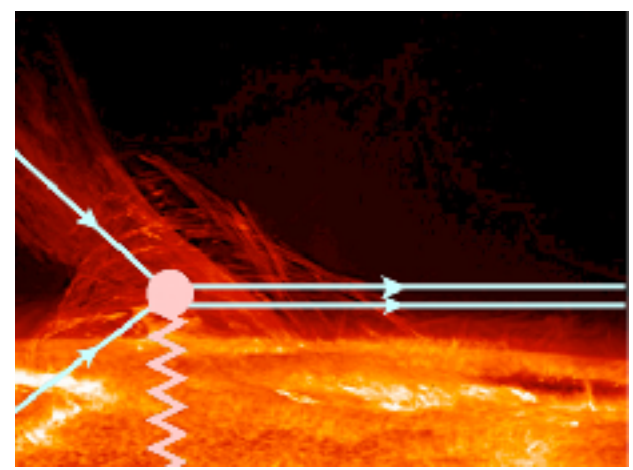




Static axial responses of nuclei determine “quenching” of axial charge in nuclei, transition matrix element for proton-proton fusion





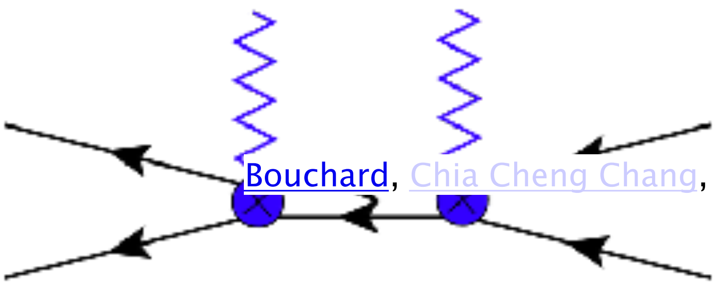


Savage, Shanahan, Tiburzi, MW, Winter, Beane, Chang, Davoudi, Detmold, Orginos (2016)

Double-Beta Decay from LQCD

Background axial fields can induce quadratic as well as linear response

LQCD calculations of (kinematically forbidden) $nn \to pp$ double-beta decay can determine short-distance multi-nucleon effects needed in calculations of larger nuclei with effective field theory or many-body methods



Bouchard, Chia Cheng Chang, Thorsten Kurth, Kostas Orginos, Andre Walker-Loud

Isotensor axial polarizability identified as a potentially significant contribution to double-beta decay neglected in previous many-body calculations

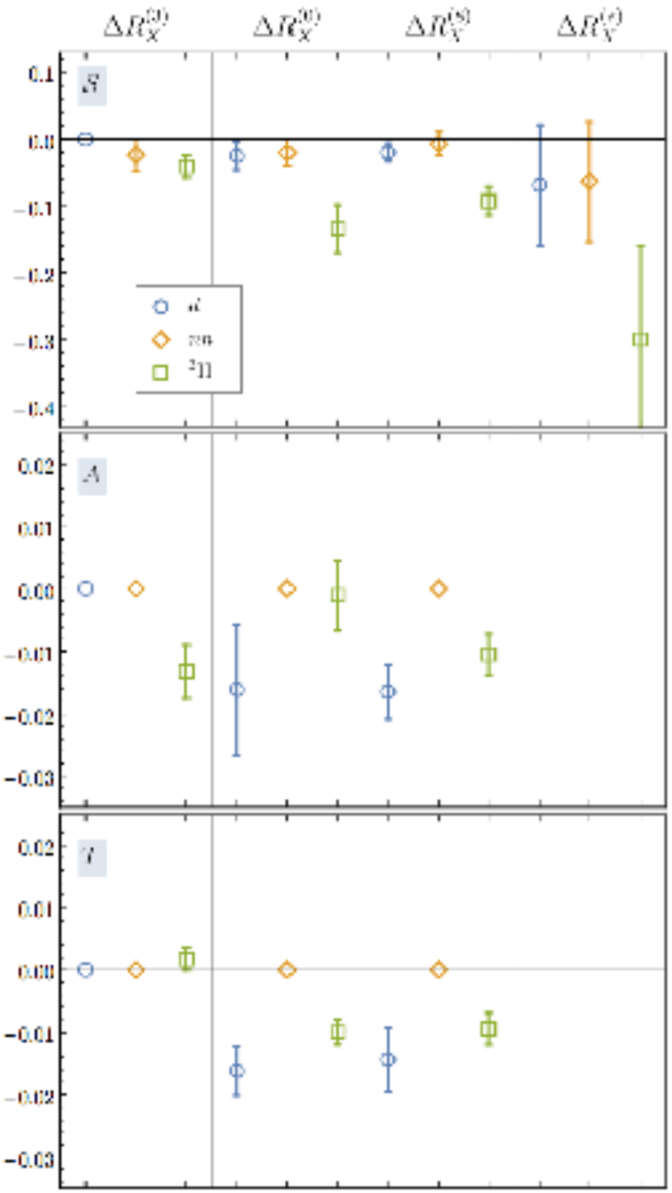
Shanahan, Tiburzi, MW, Winter, Chang, Davoudi, Detmold, Orginos, Savage (2017)

Also LQCD work on (Weinberg power counting) double-beta decay potentials

Berkowitz, Kurth, Nicholson, Joo, Rinaldi, Strother, Vranas, Walker-Loud (2015)

Further calculations needed to assess convergence of power-counting possibilities

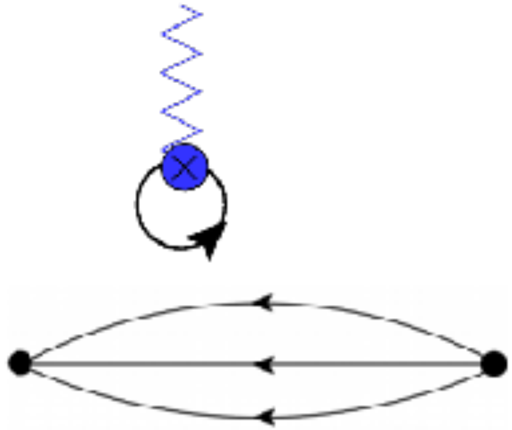
Nuclei in Background Fields

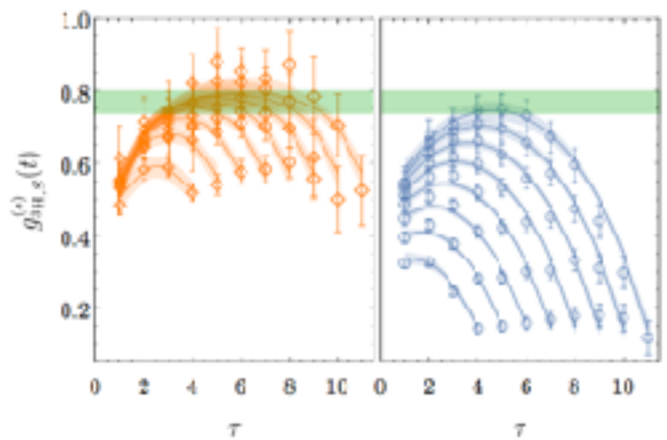


Full flavor decomposition of A=1-3 static response to scalar, tensor, and axial fields calculated

Technically challenging disconnected diagrams computed with state-of-the-art methods

Gambhir, Stathopoulos, and Orginos (2016)





Small nuclei at heavy quark mass “look like” non-interacting nucleons plus few-percent-level corrections

Chang, Davoudi, Detmold, Gambhir, Orginos, Savage, Shanahan, Tiburzi, MW, Winter (2017)

Axial and Tensor Quenching

Isovector axial charge shows 1-2% nuclear modification to 3H beta-decay rate with heavier quark masses (5% in nature)

Tensor charge shows nuclear EDMs include similar 1-2% nuclear modifications, important for disentangling different possible BSM contributions to EDMs

	d	pp	^{3}H	Expect.
$R_A^{(0)}$	1.98(1)	–	0.999(6)	$2S_3$
$R_A^{(3)}$	–	–	0.987(4)	$4T_3S_3$
$R_A^{(8)}$	1.983(4)	–	0.990(3)	$2S_3$
$R_A^{(s)}$	–	–	–	BS_3
$R_S^{(0)}$	1.97(2)	1.98(2)	2.87(4)	B
$R_S^{(3)}$	–	1.98(2)	0.96(2)	$2T_3$
$R_S^{(8)}$	1.98(1)	1.99(2)	2.90(2)	$2T_8$
$R_S^{(s)}$	1.93(9)	1.94(9)	2.70(14)	B
$R_T^{(0)}$	1.984(4)	–	0.990(2)	$2S_3$
$R_T^{(3)}$	–	–	1.002(2)	$4T_3S_3$
$R_T^{(8)}$	1.986(5)	–	0.991(3)	$2S_3$
$R_T^{(s)}$	–	–	–	BS_3

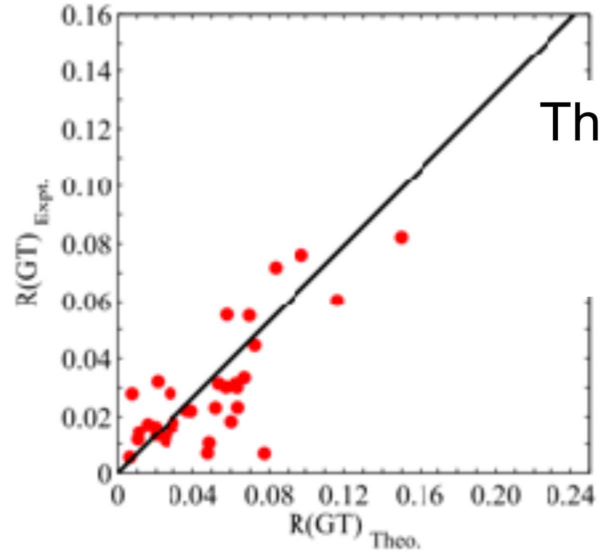
Nuclear quenching (rather than enhancement) appears generic

Scalar quenching significantly larger than axial and tensor

Chang, Davoudi, Detmold, Gambhir, Orginos, Savage, Shanahan, Tiburzi, MW, Winter (2017)

Scalar Quenching

Kumar, Srivastava, and Li (2016)



In nature, 5% quenching of axial charge in ^3He grows to ~30% quenching for A=30-60

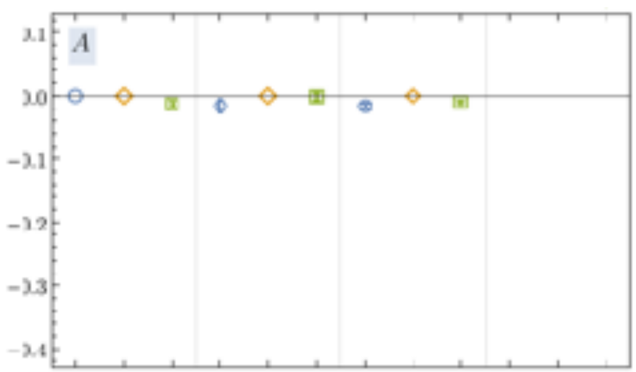
Theory vs experiment for A = 30 - 60 nuclei

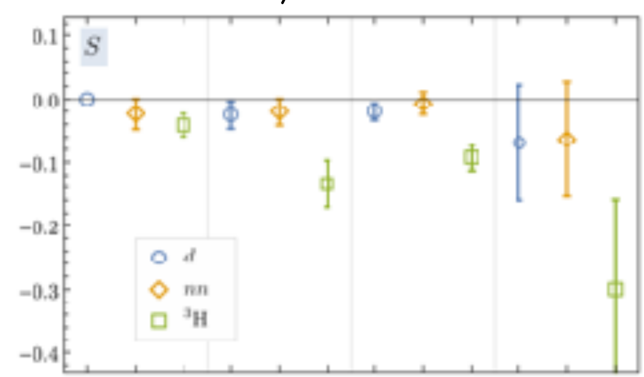
At $m_\pi \approx 806 \text{ MeV}$:

Scalar quenching larger than axial for light nuclei, ^3He has ~10% scalar vs ~1% axial

Similar hierarchy at physical quark mass would imply that scalar quenching should not be neglected in direct detection, isotope shift searches, etc

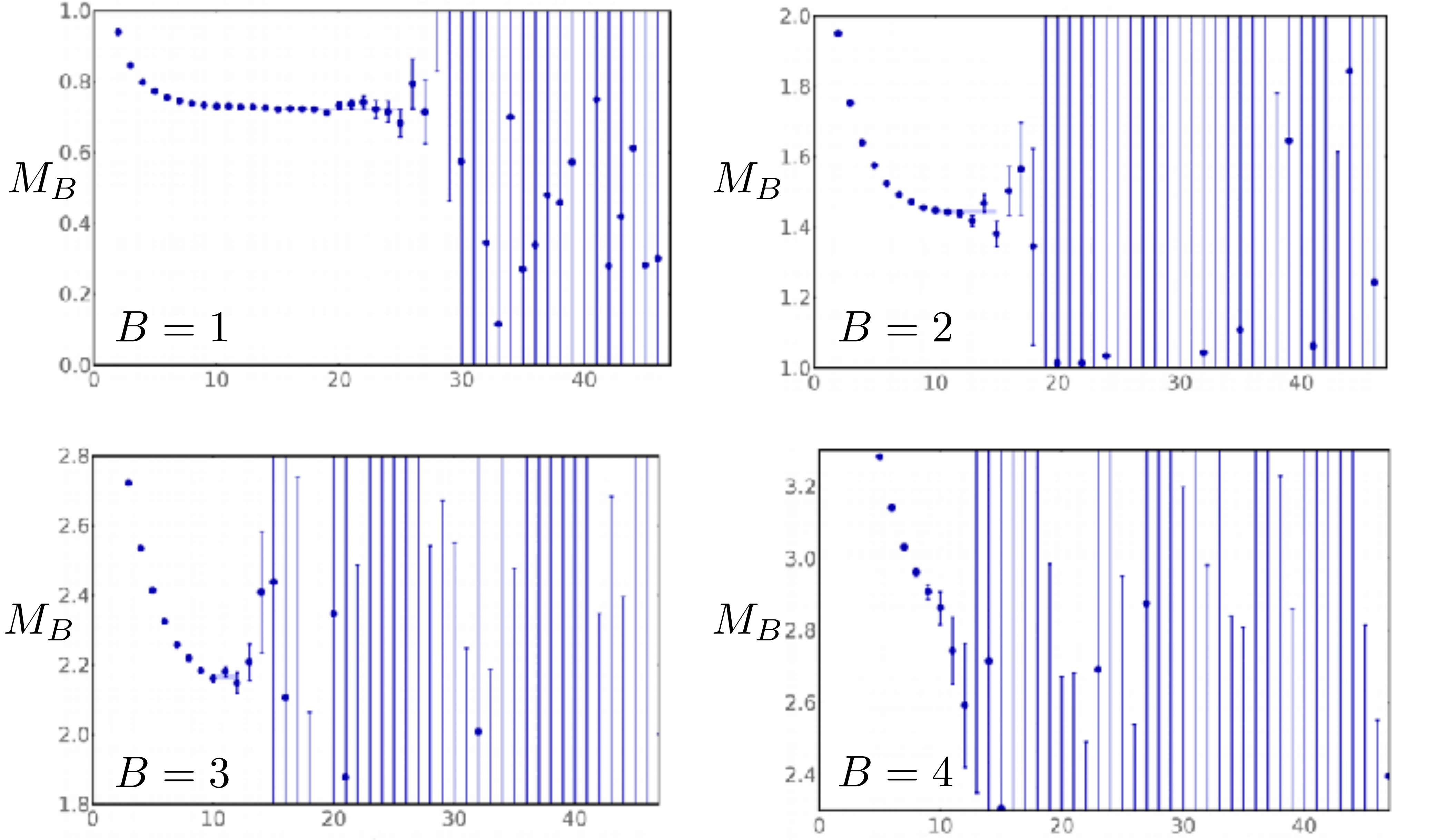
$$\sigma_A \neq A^2 \sigma_N \ ?$$





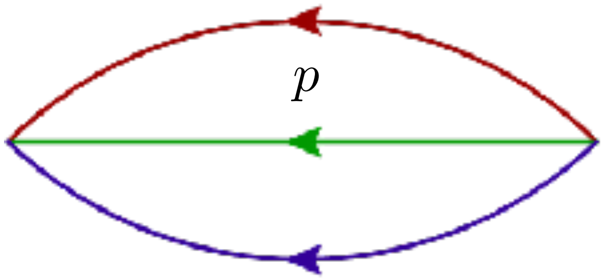
Nuclei are Noisy

$m_\pi \sim 450$ MeV $\qquad N \sim 100,000$



The Signal-to-Noise Problem

"Noise" in Monte Carlo measurements represents quantum fluctuations in observables

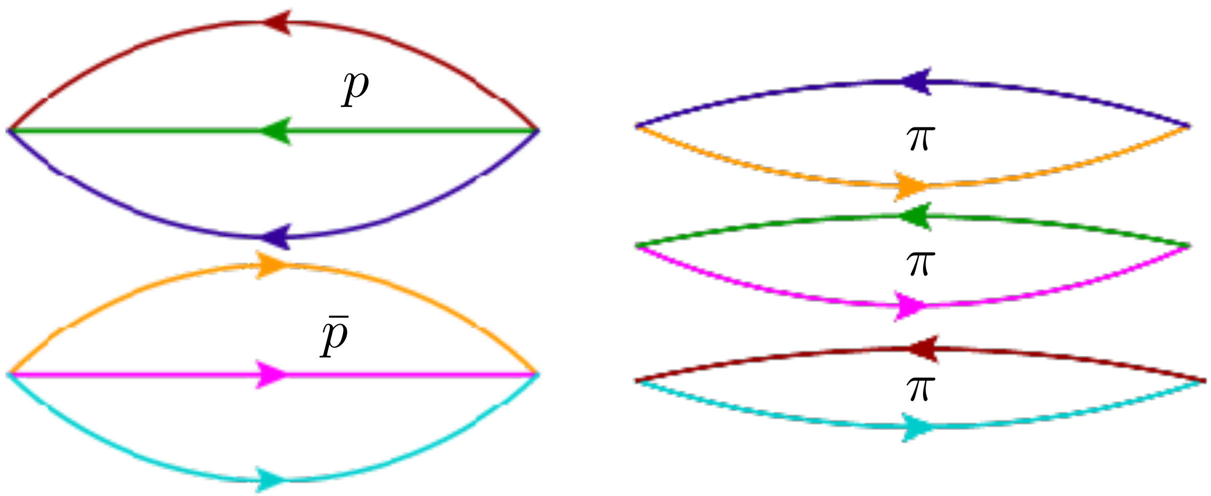


$$C_p \sim e^{-M_p t}$$

Late-time behavior of nucleon variance determined by lowest energy state with the right quantum numbers Parisi (1984), Lepage (1989)

$$\mathrm{Var}(C_p) \sim \left\langle |p(t)p^\dagger(0)|^2 \right\rangle$$

$$\sim e^{-3m_\pi t}$$



Signal-to-noise problem

$$\frac{C_p}{\sqrt{\mathrm{Var}(C_p)}} \sim e^{-(M_p - \frac{3}{2}m_\pi)t}$$

Proton signal-to-noise problem related to a sign problem MW, Savage (2016)

The Sign(al-to-Noise) Problem

Statistical estimation of an exponentially decaying average phase always has exponential StN degradation

$$\mathrm{StN}(e^{i\theta_i(t)}) \sim \frac{\left\langle e^{i\theta_i(t)} \right\rangle}{\sqrt{\left\langle \left| e^{i\theta_i(t)} \right|^2 \right\rangle}} \sim \left\langle e^{i\theta_i(t)} \right\rangle \sim e^{-m_\theta t}$$

Average correlation functions are real. Individual correlation functions in generic gauge fields are complex

$$C_i(t) = e^{R_i(t) + i\theta_i(t)}$$

Standard LQCD methods equivalent to reweighting a complex action

$$G(t) = \langle C_i(t) \rangle = \int \mathcal{D}U \; e^{-S(U) + R(t,U_i) + i\theta(t,U_i)} \; = \frac{1}{N} \sum_{i=1}^{N} e^{R_i(t) + i\theta_i(t)}$$

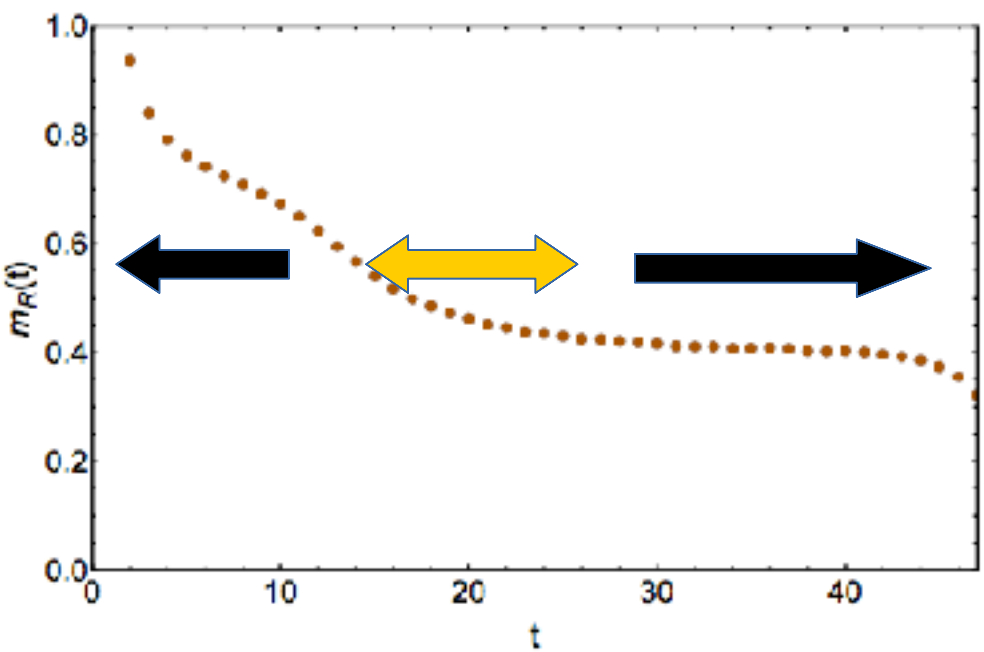
Is the LQCD signal-to-noise problem in all or part a sign problem?

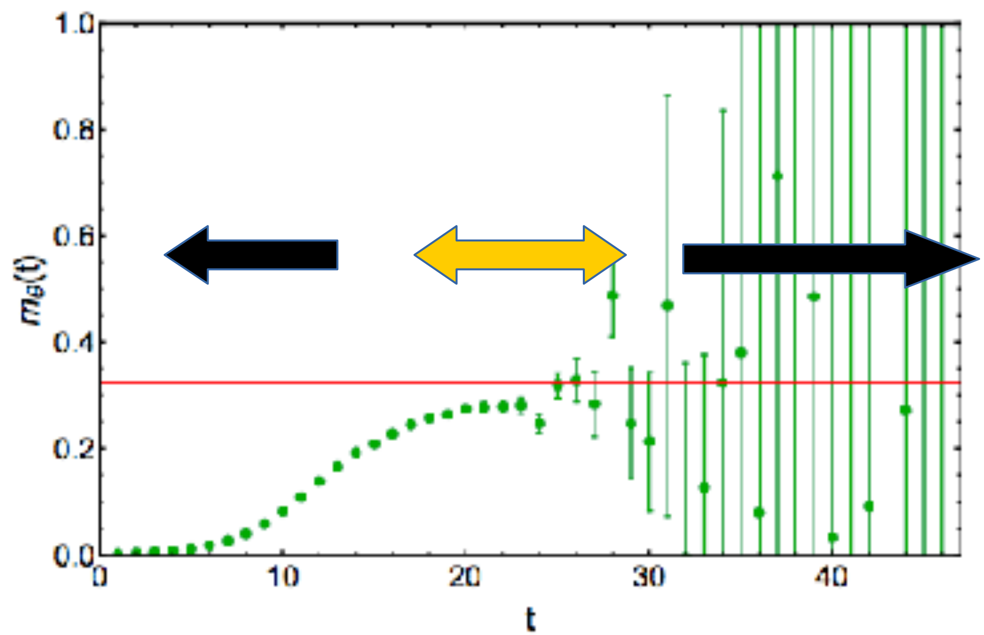
Magnitude - Phase Decomposition

Magnitude and phase of generic hadron correlation functions are (empirically) approximately decorrelated

$$m_R(t) = \ln\left(\frac{\left\langle e^{R_i(t)}\right\rangle}{\left\langle e^{R_i(t+1)}\right\rangle}\right) \qquad m_\theta(t) = \ln\left(\frac{\left\langle e^{i\theta_i(t)}\right\rangle}{\left\langle e^{i\theta_i(t+1)}\right\rangle}\right)$$

$$\sim \frac{3}{2}m_\pi \qquad \sim m_N - \frac{3}{2}m_\pi$$

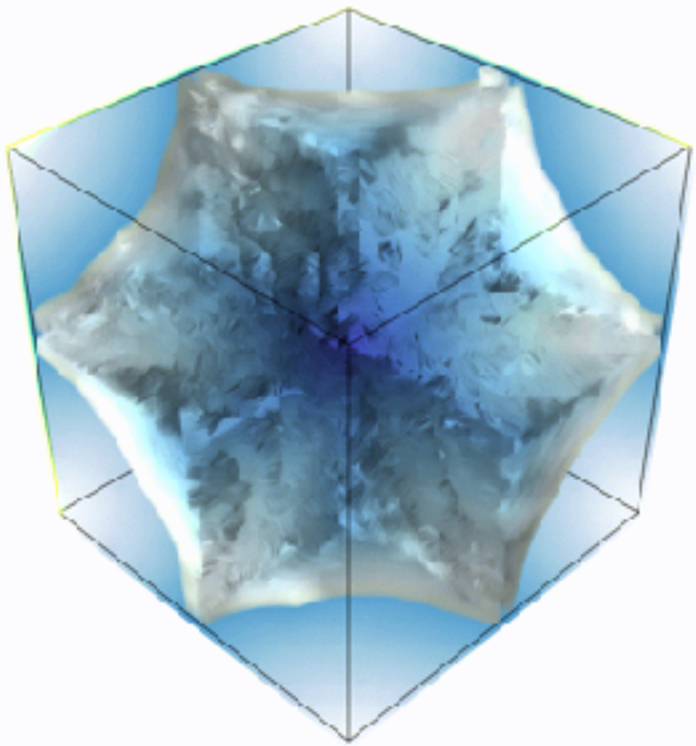




MW and Savage (2016)

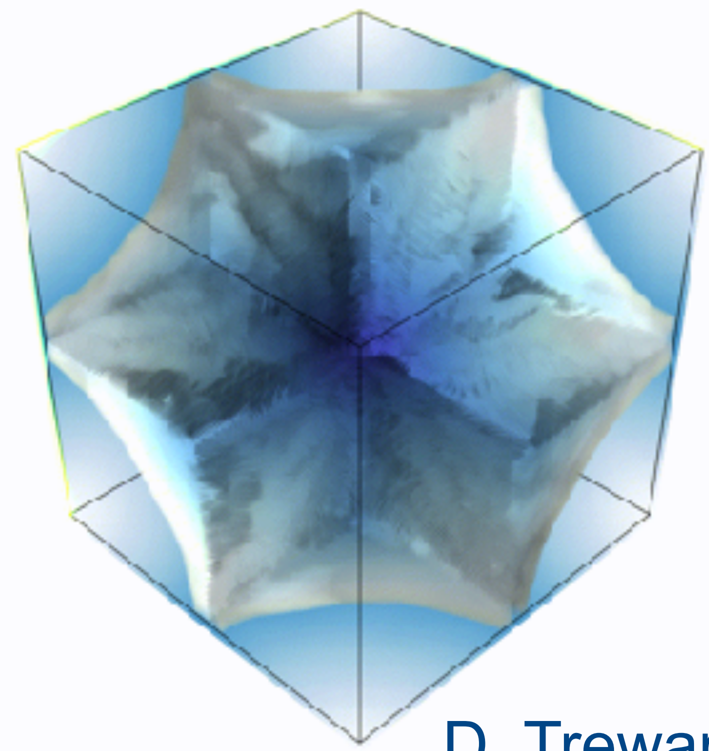
Nucleon Log-Magnitude

Similar long-range structures in log-magntiude of nucleon and pion correlation functions



$$\ln |C_i(t)|$$

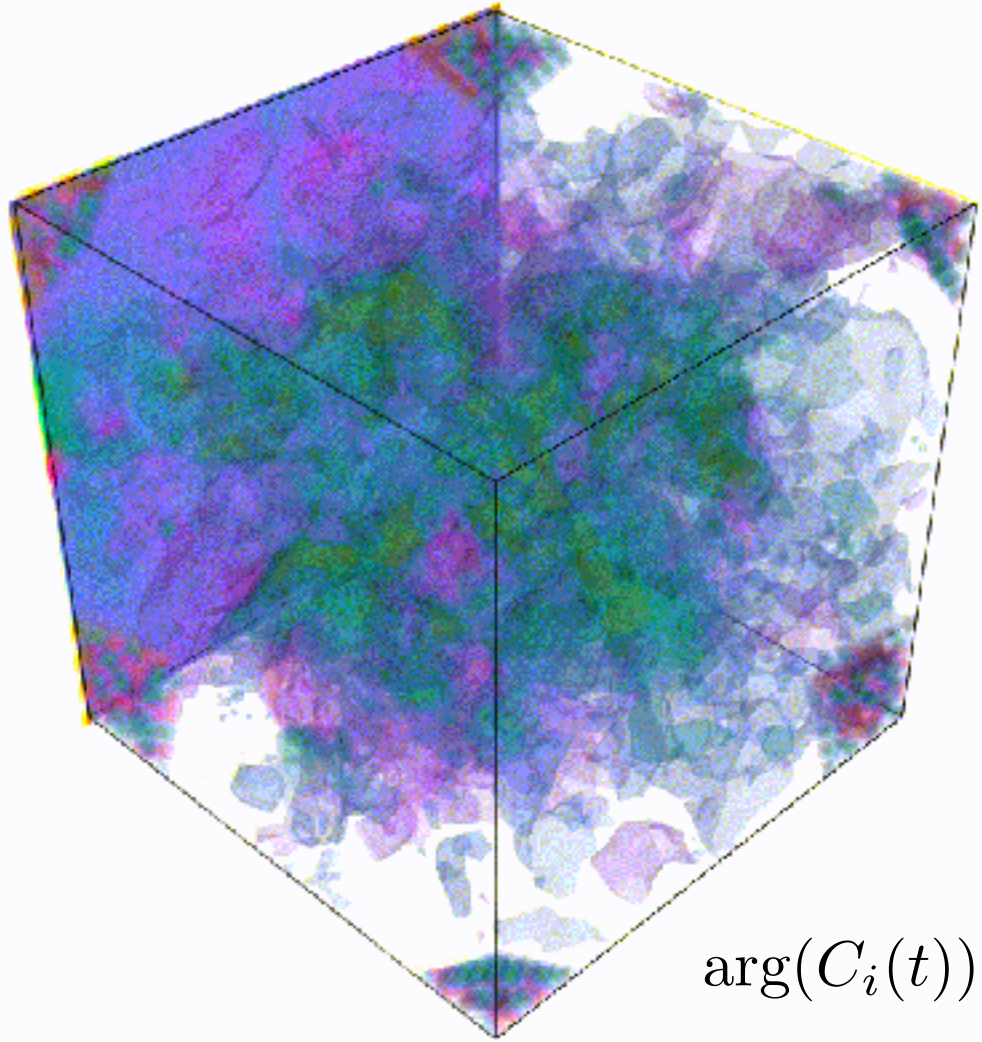
$$\ln C_i^{\pi}(t)$$



D. Trewartha

Nucleon Phase

The difference between the nucleon mass and 3/2 the pion mass comes from destructively interfering phase fluctuations



D. Trewartha

LQCD Correlation Function Statistics

Generic real, positive correlation functions, as well as early-time nucleons in LQCD, are log-normally distributed

Hamber, Marinari, Parisi and Rebbi (1983)

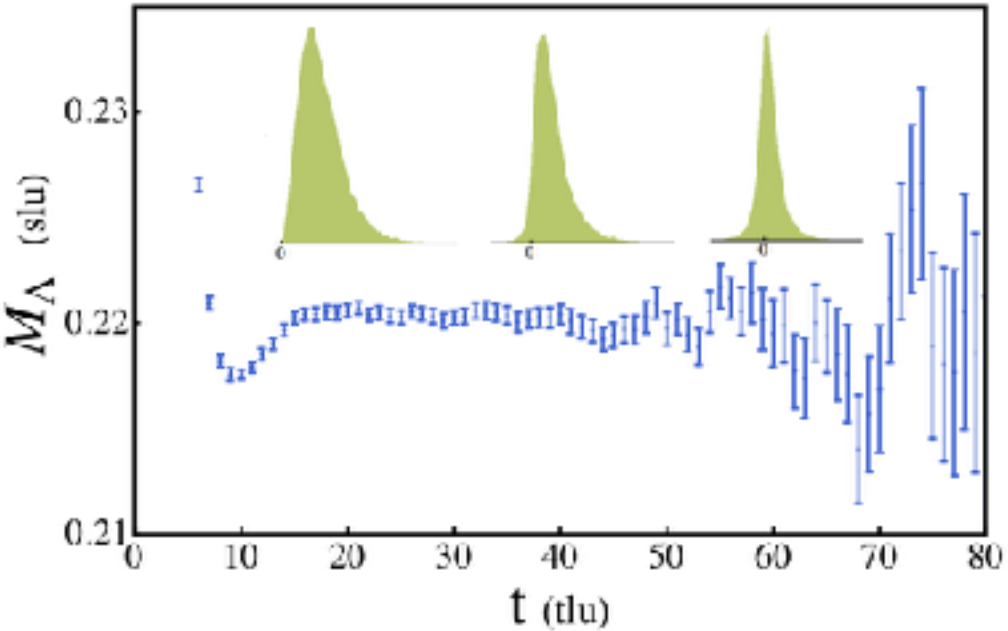
Guagnelli, Marinari, and Parisi (1990)

Endres, Kaplan, Lee and Nicholson (2011)

DeGrand (2012)

Log-normal distributions arise in models with only two-body interactions and products of generic random numbers

Late-time nucleon real part is not log-normal. Moment analysis by Savage predicts broad, symmetric distribution



Beane, Detmold, Orginos, Savage (2015)

Complex Log-Normal Distributions

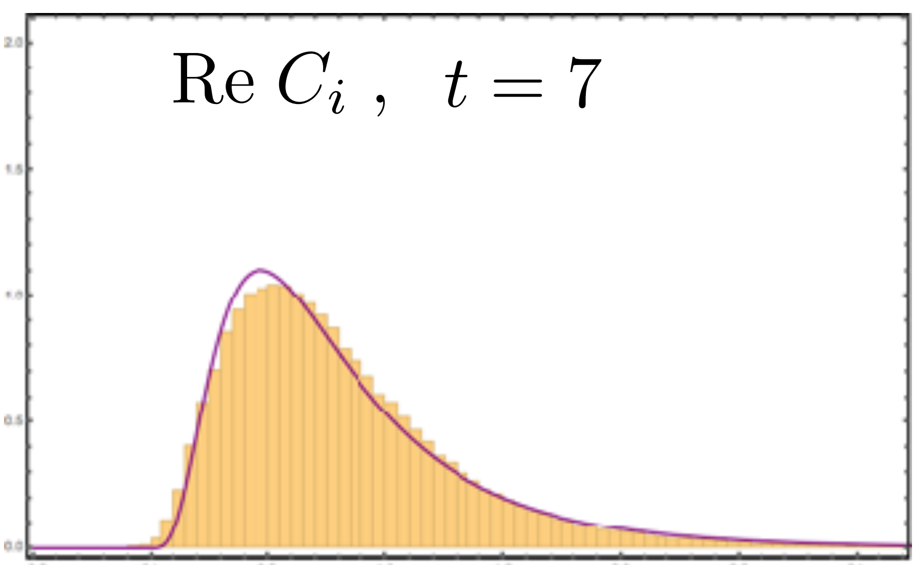
Phase is a circular random variable only defined modulo 2π

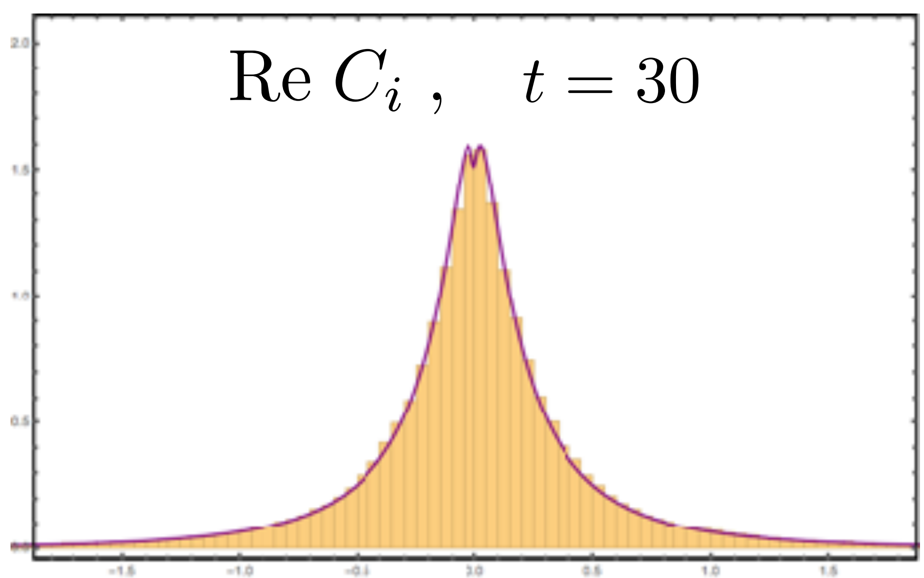
Summing over periodic images of Gaussian provides a wrapped normal distribution suitable for a circular random variable

See e.g. N. I. Fisher, "Statistical Analysis of Circular Data" (1995)

Real part of nucleon correlation functions well-described by marginalization of "complex log-normal distribution"

$$C_i(t) = e^{R_i(t) + i\theta_i(t)} \qquad \mathcal{P}(R_i, \theta_i) = e^{-(R_i - \mu_R)^2/(2\sigma_R^2)} \sum_{n=-\infty}^{\infty} e^{-n^2\theta_i^2/(2\sigma_\theta^2)}$$





MW and Savage (2016)

Circular Statistics and the Noise Region

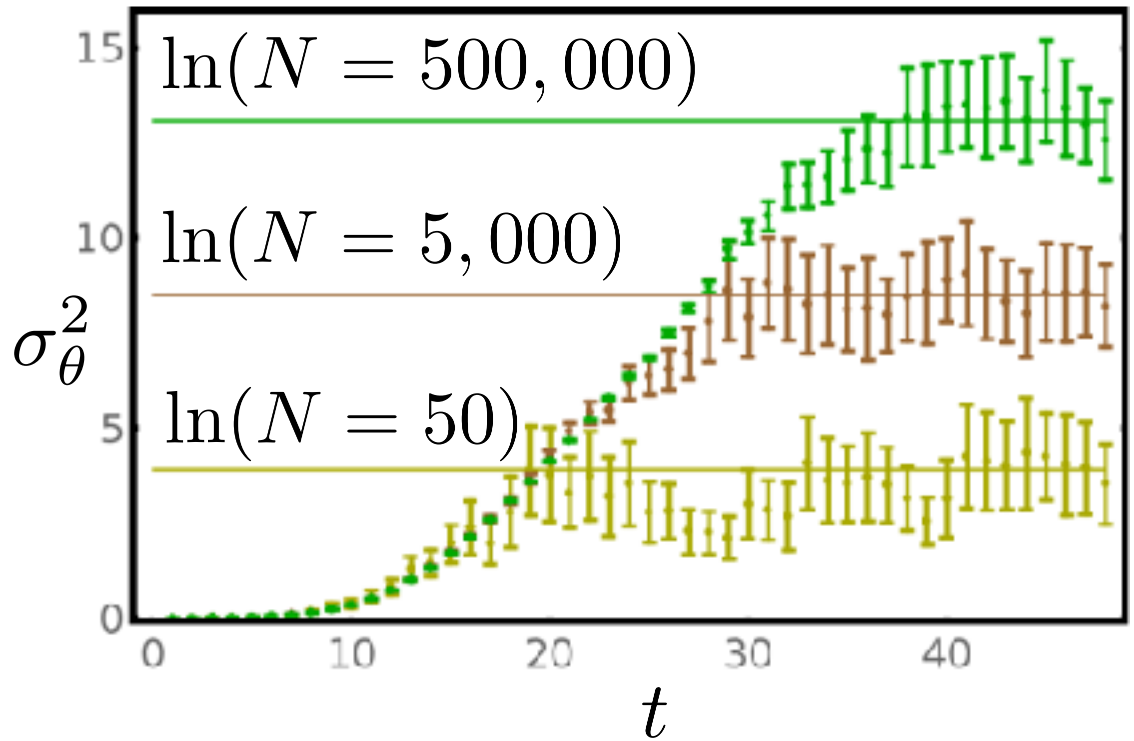
Circular random variables have different properties than random real numbers. Finite sample effects obstruct parameter inference unless

$$\frac{1}{N}\sum_{i=1}^{N}\cos\theta_i > \frac{1}{\sqrt{N}}$$

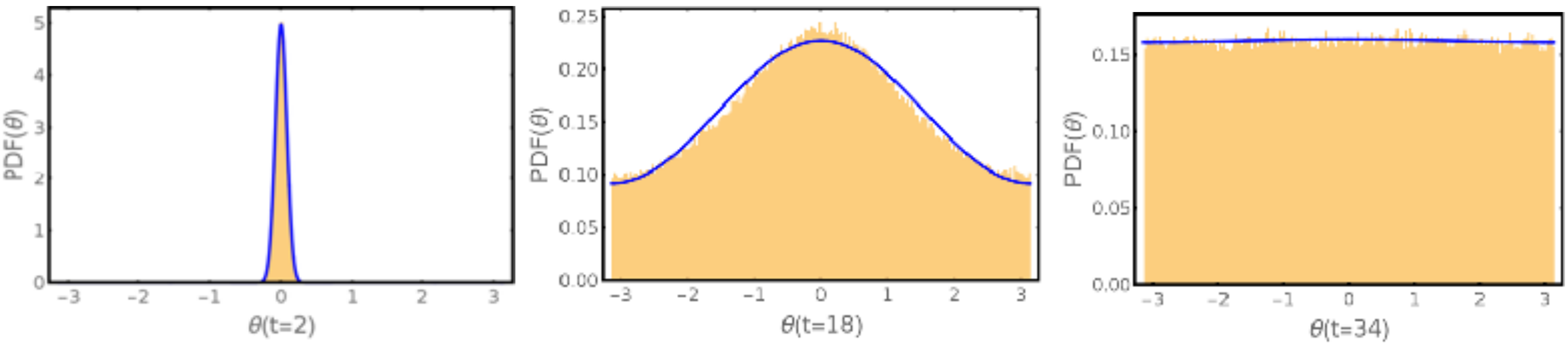
Avoiding finite sample effects requires

$$N > e^{\sigma_\theta^2} \sim e^{2(m_N - \frac{3}{2}m_\pi)t}$$

This will violated in a late-time "noise region" where standard estimators become unreliable



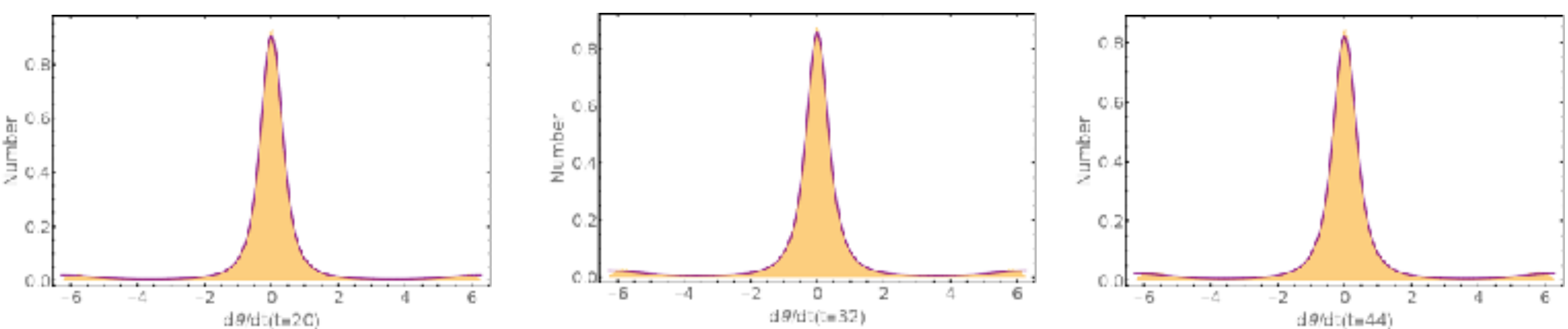
Lévy Flights on the Unit Circle



Phase wrapped normally distributed at all times

Phase and log-magnitude time derivatives approach time independent, heavy-tailed wrapped stable distributions at late times

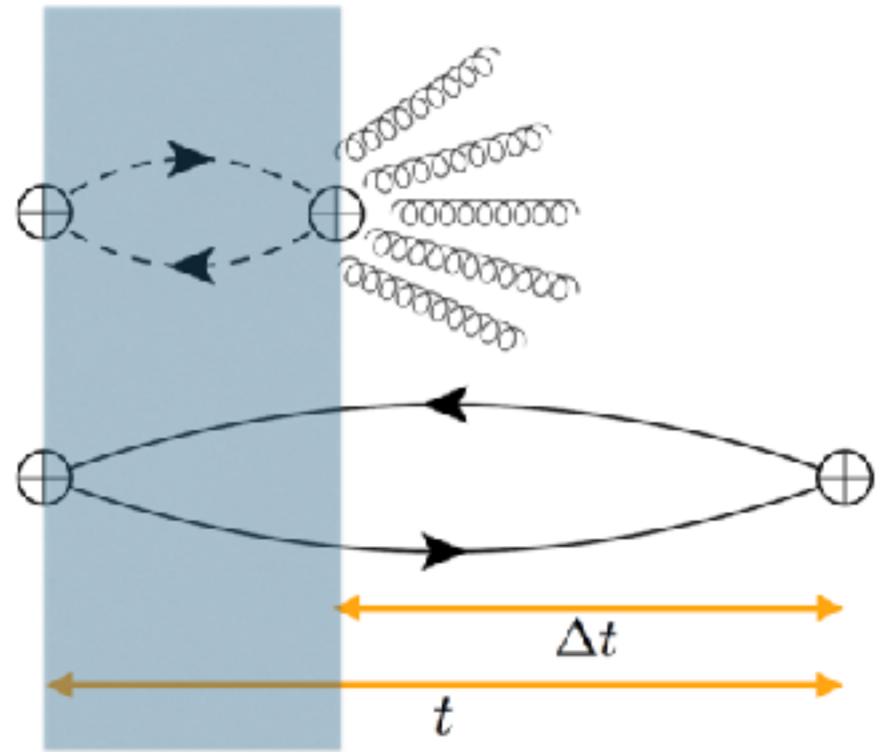
Independent samples from these distributions describe Lévy flights



Phase Reweighting

Calculate phase differences over fixed-length intervals Δt, not entire path t

$$G^{\theta}(t,\Delta t) = \left\langle C_i(t) e^{-i\theta_i(t-\Delta t)} \right\rangle$$



StN set by phase difference interval, not full evolution time

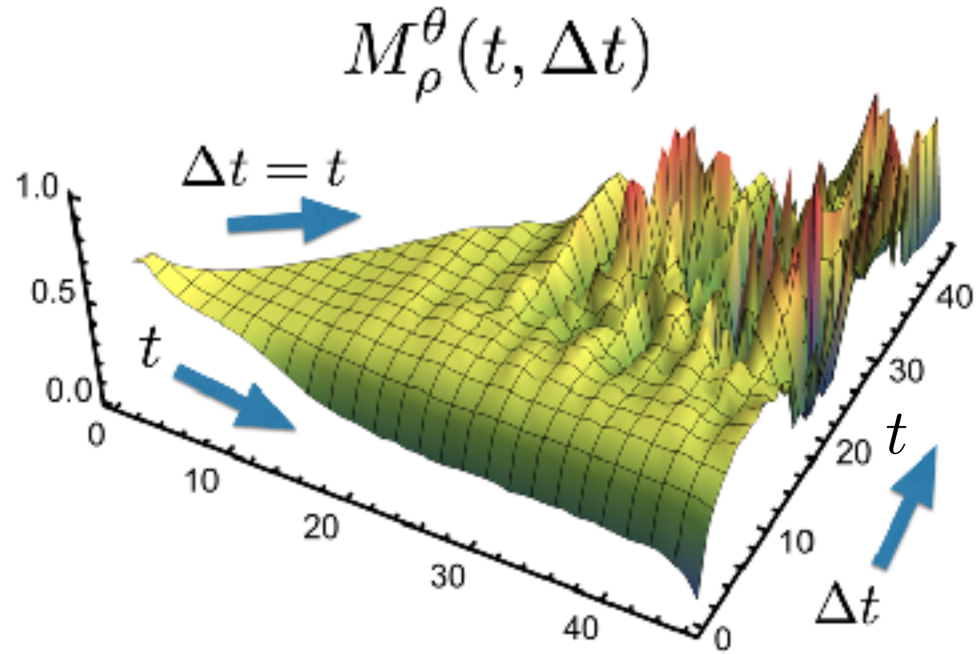
$$\mathrm{StN} \sim e^{-(m_N - \frac{3}{2} m_\pi)\Delta t}$$

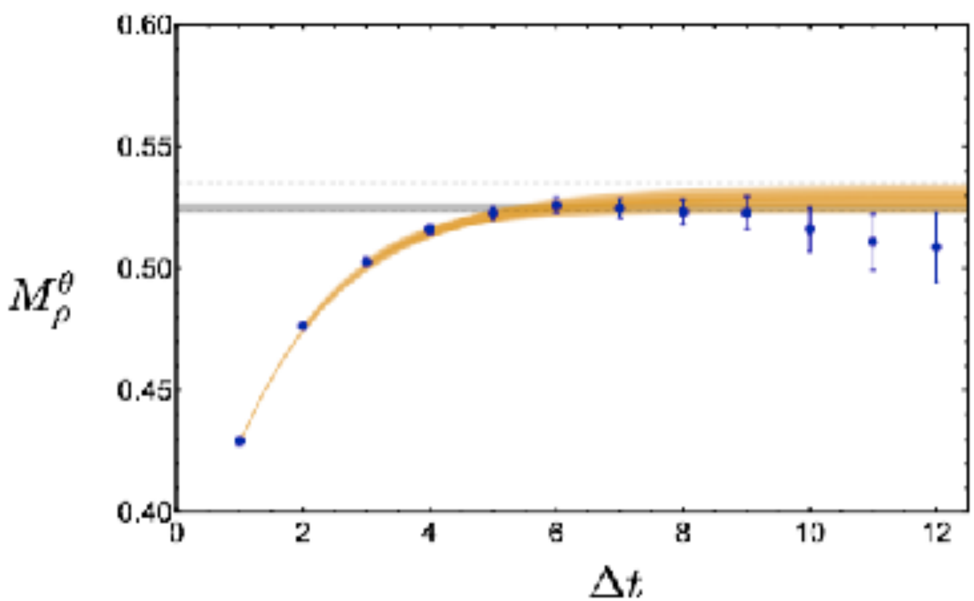
Results exact in limit $\Delta t \to t$, can be recovered by extrapolating results for a range of Δt

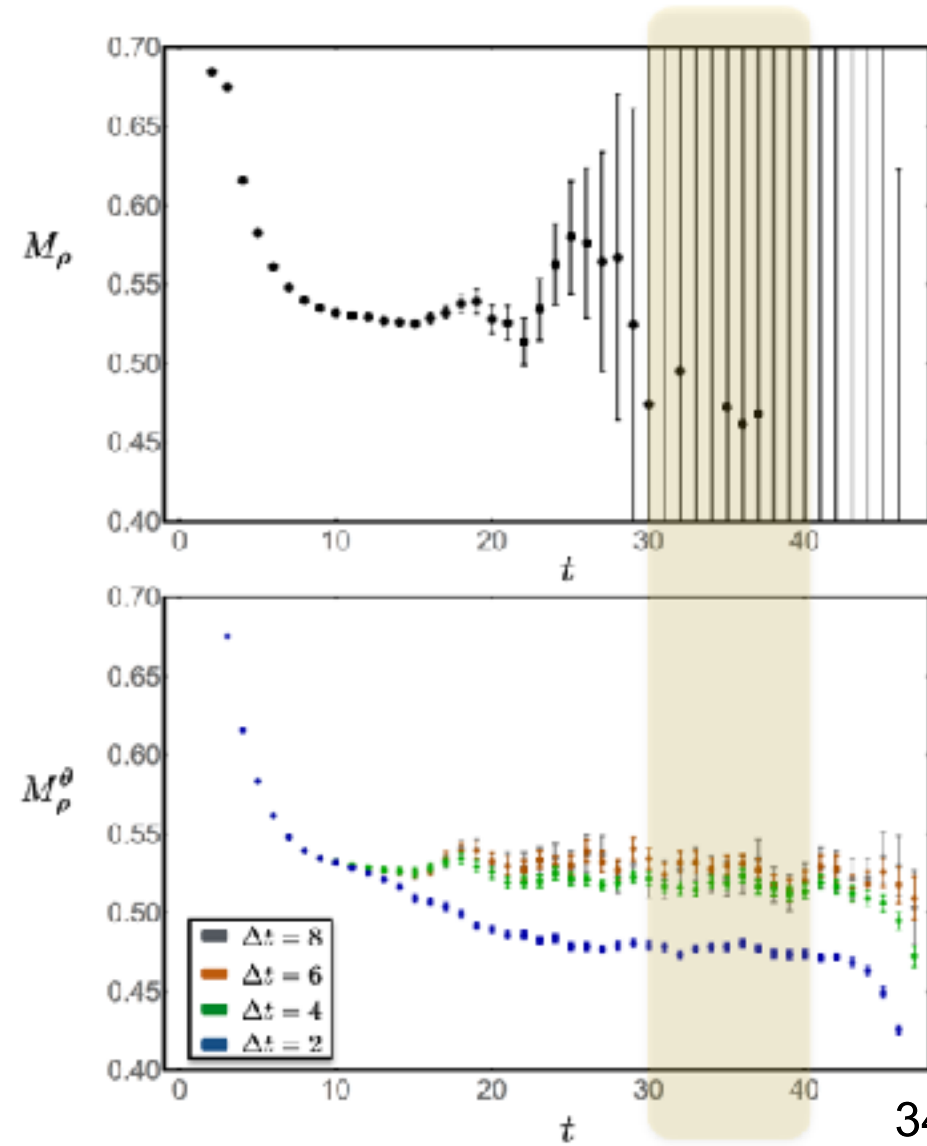
$$G^{\theta}(t,t) = \langle C_i(t) \rangle = G(t)$$

MW and Savage (2017)

Meson Phase Reweighting



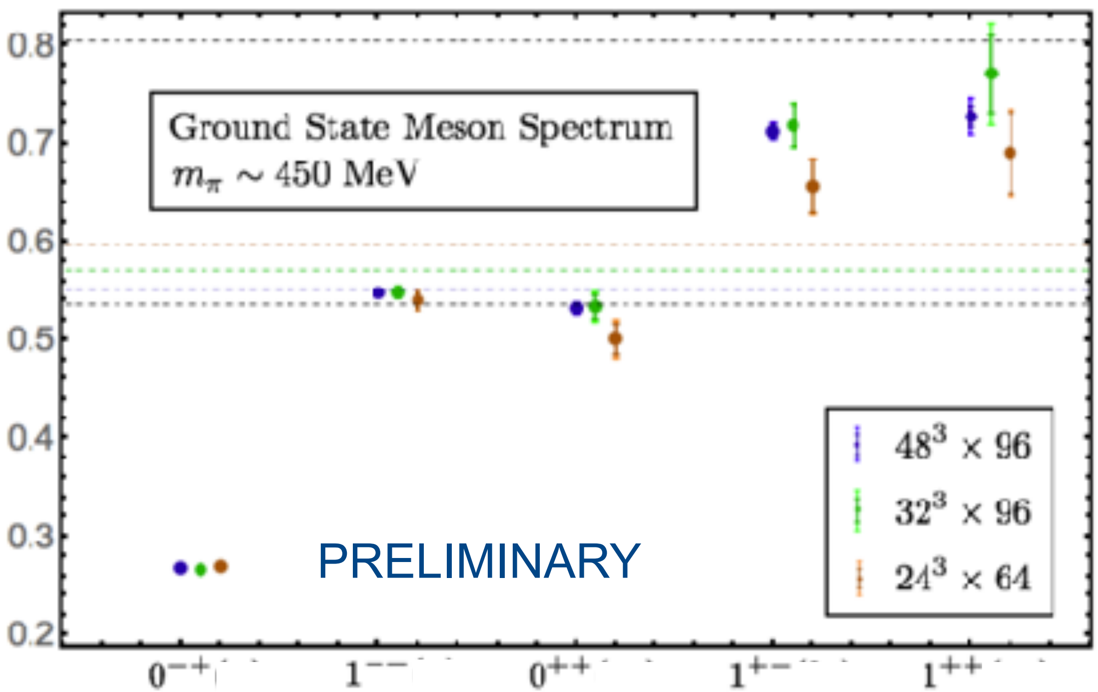




Phase-Reweighting as a Background Field

Phase-reweighting alternatively described as including a background gauge field with zero field strength

$$U_\mu(x) \to e^{i\theta(x)-i\theta(x-\hat{\mu})}U_\mu(x) \quad \Rightarrow \quad C_i^B(x,y) \to e^{N_c i(\theta(x)-\theta(y))}C_i^B(x,y)$$



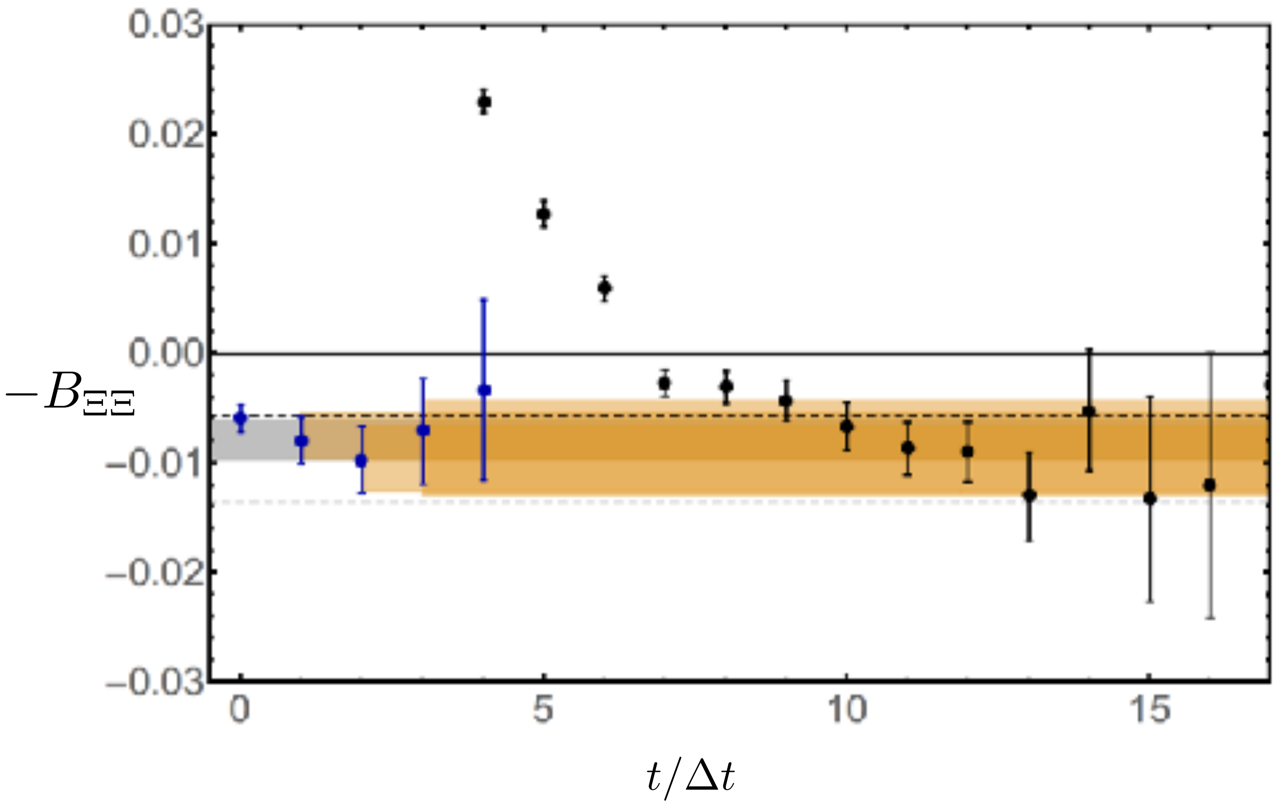
Background coupling to $U(1)_B$ for baryons, $U(1)_{u-d} \subset SU(2)_V$ for isovector mesons

Total isospin no longer good quantum number for meson case, isovector and isoscalar mesons mix

Phase-Reweighting Sources

No symmetry-breaking issues for baryons

In appropriate coordinates, phase-reweighted correlation functions are standard correlation functions involving a non-standard source



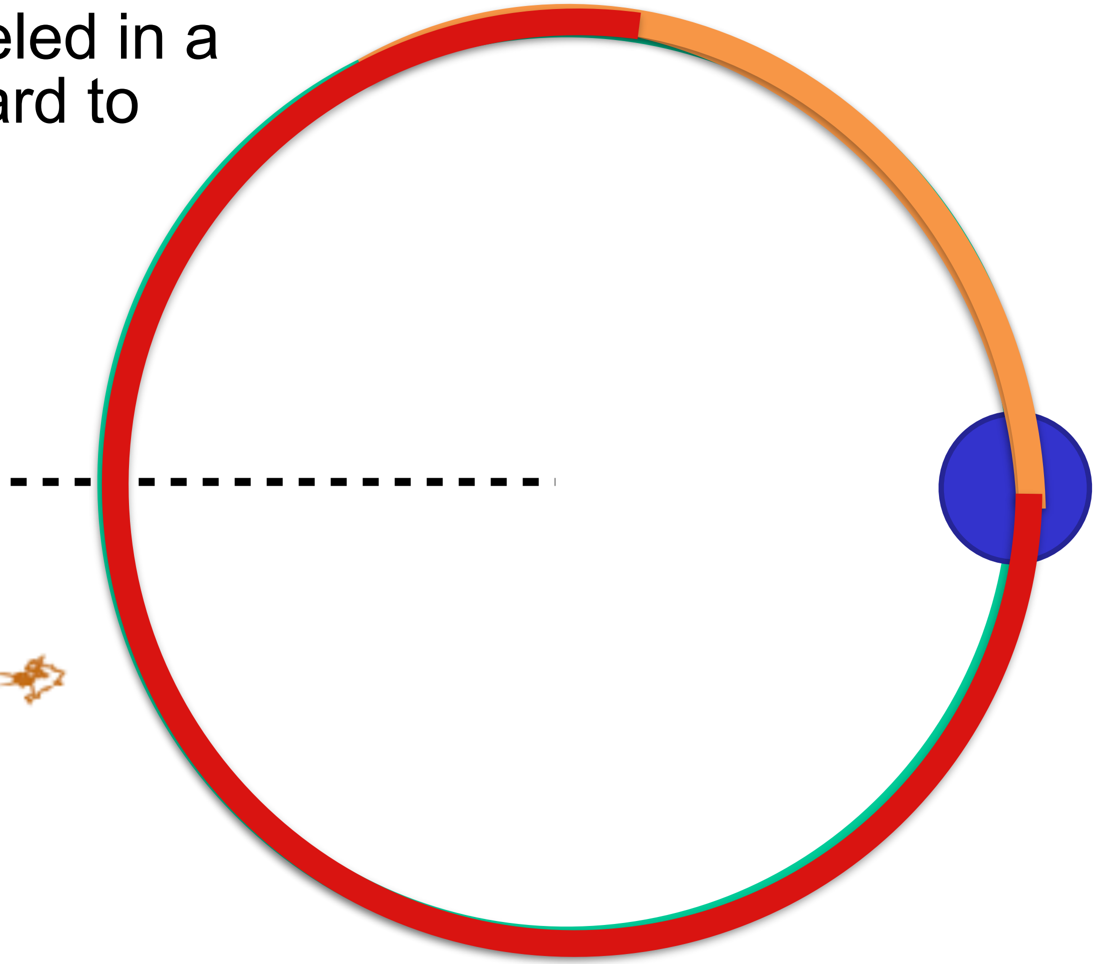
Analogous to generalized pencil-of-functions (GPoF) methods using time-evolved sources

"Freezing" phase during source creation leads to modest precision increases over GPoF

Winding Walks

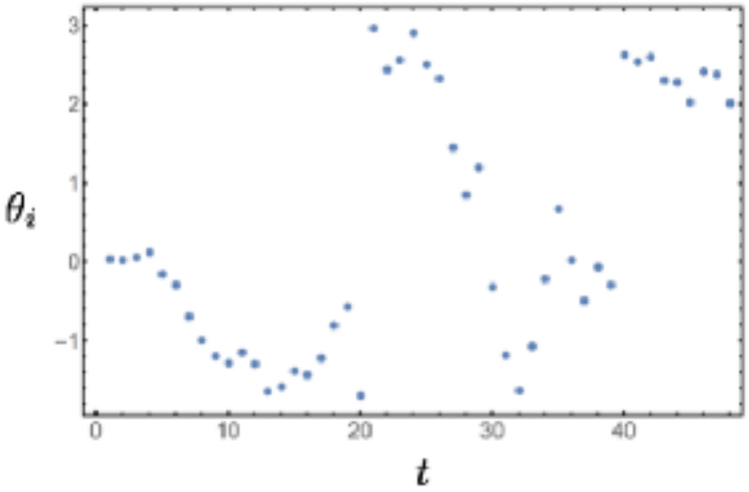
Phase identified as source of StN problem, but phase reweighting still has exponentially hard extrapolation, doesn't "solve" StN problem

Generic issue: The distance traveled in a long circular random walk is hard to estimate from time series data



Branch cut crossings cannot be reconstructed with coarse resolution

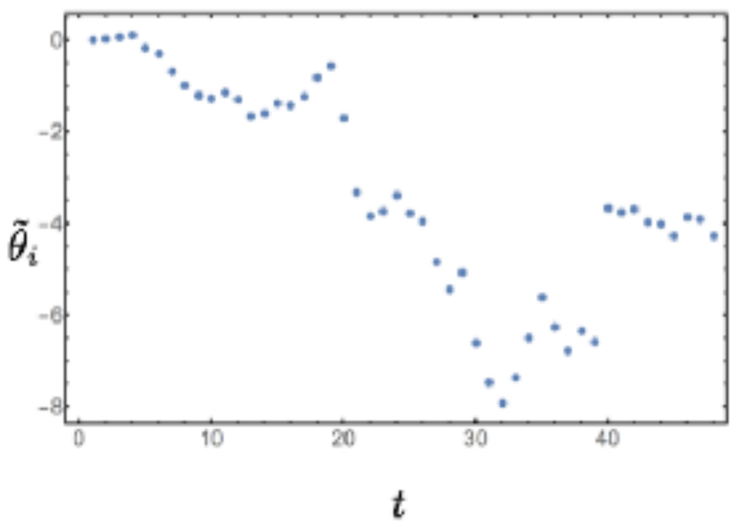
Phase Unwrapping



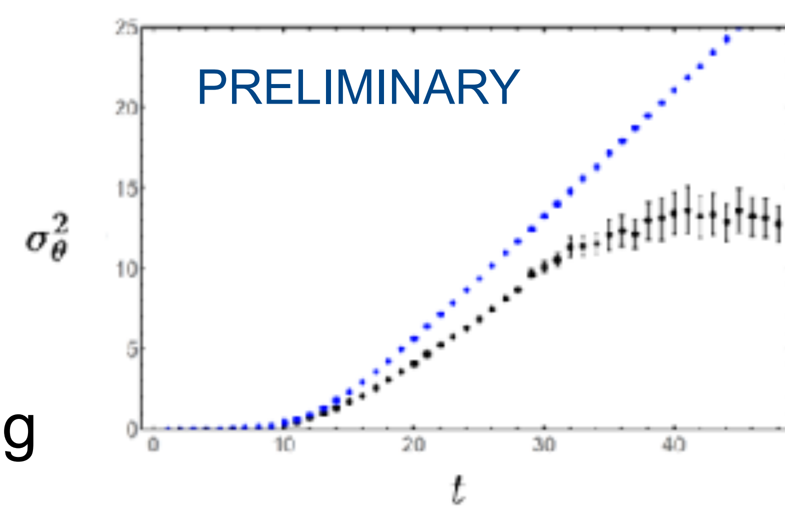
If phase change at each (discrete) time $< \pi$ then phase can be “unwrapped”

$$\tilde{\theta}_i(t) = \sum_{t'=0}^{t} \theta_i(t') - \theta_i(t'-1) + 2\pi\nu_i(t'),$$

Unwrapped phase is a real (not circular) random variable



No exponential StN degradation when estimating unwrapped moments



Accumulation of numerical errors leads to incorrect results at coarse lattice spacing

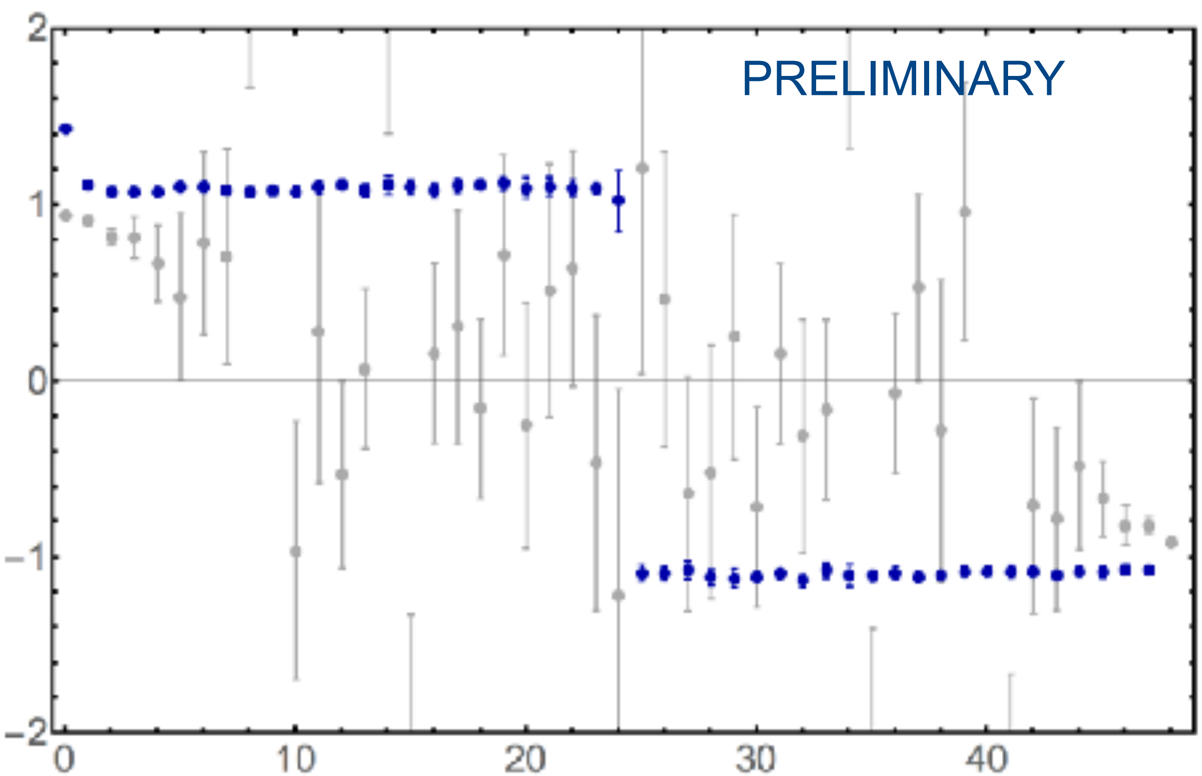
Scalar Field Phase Unwrapping

Toy model: one dimensional complex harmonic oscillator

Simple phase unwrapping algorithms reproduce analytical results with no exponential StN degradation

Successful unwrapping at sufficiently fine lattice spacing

Periodic boundary conditions subtle

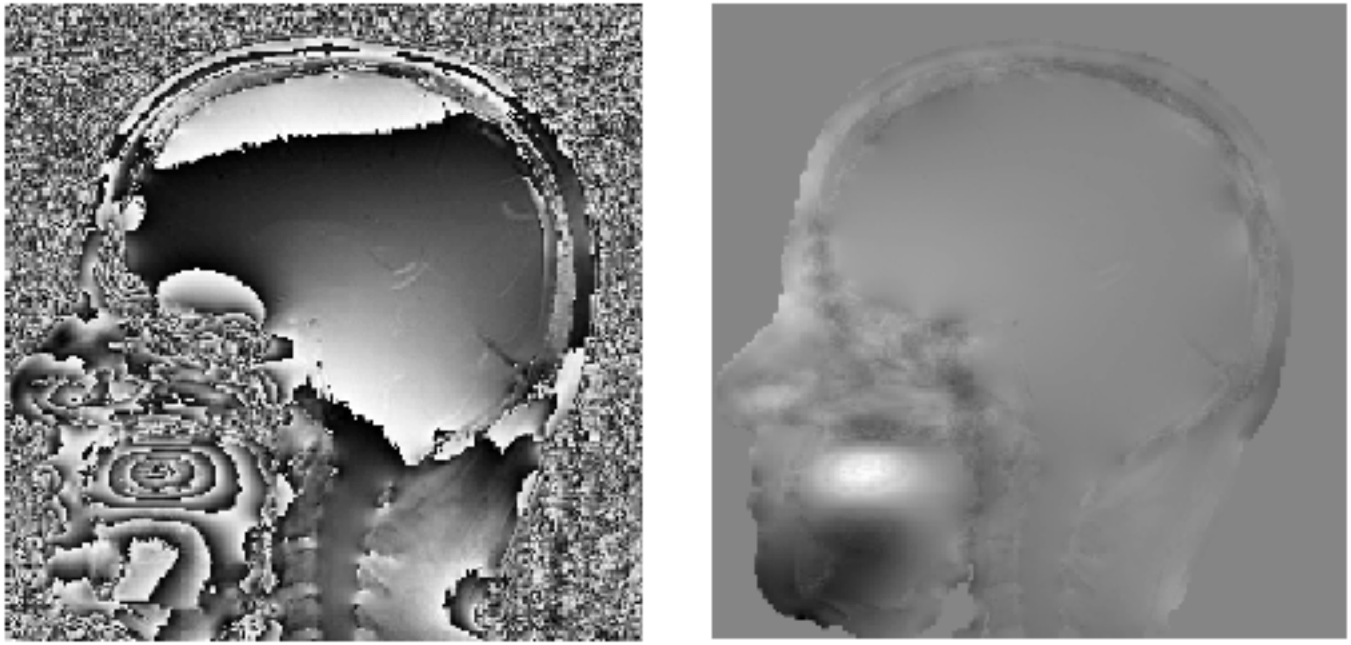


2D Phase Unwrapping

Similar problem arises in other fields where time series of circular random variables are measured

"Phase unwrapping" used to reconstruct magnetic field phase information in MRI

2D avoids 1D accumulation of errors problem



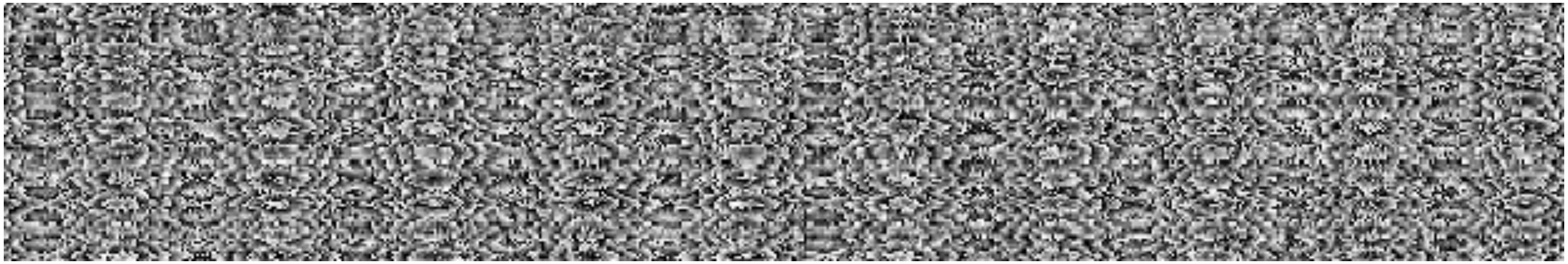
(a) Ying (2003) (b)

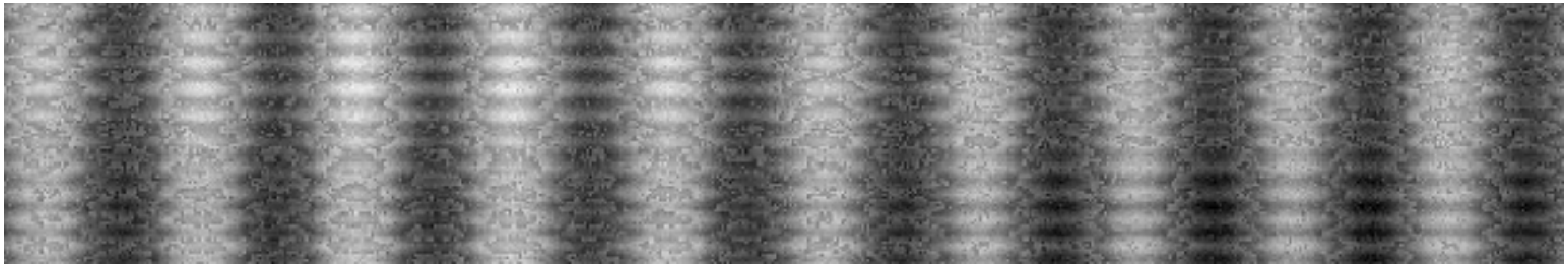
4D Phase Unwrapping

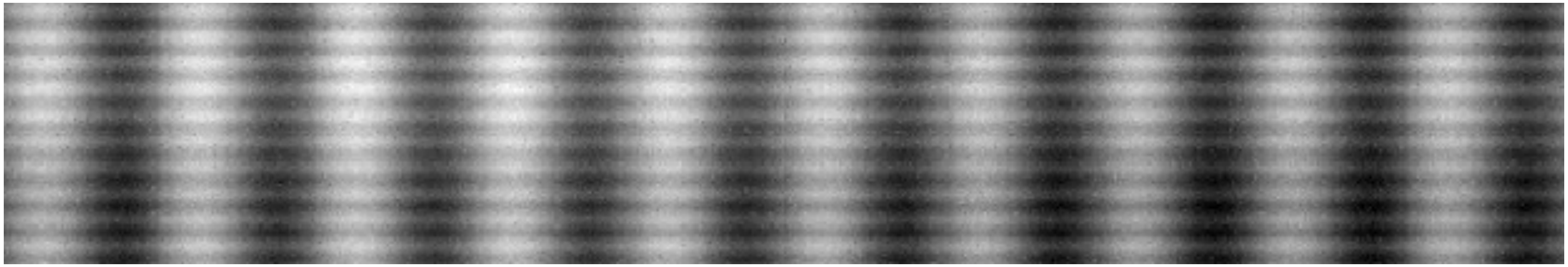
Unwrapping coordinate-space phase instead of momentum-space leads to a 4D unwrapping problem in LQCD

2D algorithms in literature can be generalized to 4D

MIT Grad Student Gurtej Kanwar:







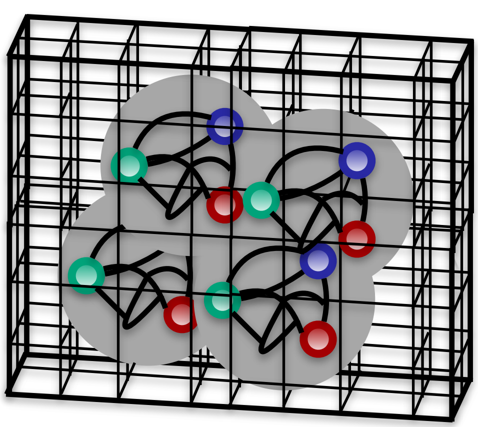
Conclusion

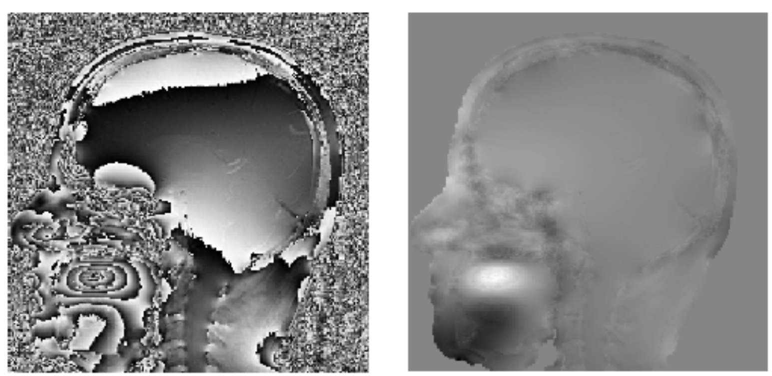
Lattice QCD is determining nuclear structure and reactions from first principles in universes with heavier quark mass

In a universe with $m_\pi \approx 806\ \mathrm{MeV}$:

- Nuclear forces in all flavor-channels close to a unitary fixed point (technically unnatural), light nuclei slightly deeper bound than in nature
- Light nuclei look like collections of nucleons plus few-percent corrections
- Scalar quenching may be important for dark matter direct detection

New ideas needed to mitigate the sign problem and permit efficient calculations at lighter quark masses. Can phase unwrapping help?





(a) (b)