$1/N_c$ expansion for baryons in the N=2 band

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Jlab, June 20, 2018

Willemyns Scoccola, arXiv:1804.07840

Willemyns Schat, PRD **95**.094007

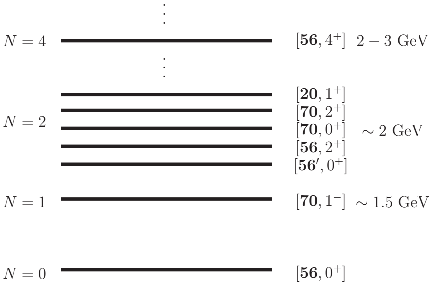
Outline

Baryon classification scheme

Baryon classification scheme $\longrightarrow$ Quark model

In the QM

Baryons belong to the $SU(6) \times O(3)$ irreducible representations.
They organize in bands of the harmonic oscillator.



Experimental data on band $\mathcal{N} = 2$ states

$O(3) \times SU(2N_f)$	$SU(3) \times SU(2)$	J^P	$s = 0$	$s = -1$, $I = 0$	$s = -1$, $I = 1$	$s = -2$	$s = -3$
$[\mathbf{56}', 0^+]$	$^2 8$	$1/2^+$	$N(1440)$	$\Lambda(1600)$	$\Sigma(1660)$	$\Xi(1690)$	
	$^4 10$	$3/2^+$	$\Delta(1600)$**		$\Sigma(1690)$**		
$[\mathbf{56}, 2^+]$	$^2 8$	$3/2^+$	$N(1720)$	$\Lambda(1890)$			
		$5/2^+$	$N(1680)$	$\Lambda(1820)$	$\Sigma(1915)$		
	$^4 10$	$1/2^+$	$\Delta(1910)$				
		$3/2^+$	$\Delta(1920)$		$\Sigma(2080)$**		
		$5/2^+$	$\Delta(1905)$				
		$7/2^+$	$\Delta(1950)$		$\Sigma(2030)$		
$[\mathbf{70}, 0^+]$	$^2 8$	$1/2^+$	$N(1710)$	$\Lambda(1810)$	$\Sigma(1880)$**		
	$^4 8$	$3/2^+$	$N(1540)$*				
	$^2 10$	$1/2^+$	$\Delta(1550)$				
	$^2 1$	$1/2^+$	$N(2100)$**				
$[\mathbf{70}, 2^+]$	$^2 8$	$5/2^+$		$\Lambda(2100)$			
	$^4 8$	$7/2^+$	$N(1990)$**				
$[\mathbf{20}, 1^+]$	$^2 8$	$1/2^+$	$N(2010)$				
	$^4 8$	$3/2^+$	$N(2040)$*				

Table : Experimental data on baryonic resonances associated to irreducible representations of $O(3) \times SU(2N_f)$ and their $SU(3) \times SU(2)$ composition.

[1] M. Tanabashi et al. (Particle Data Group), Phys. Rev. D 98, 030001 (2018)

Experimental data on band $\mathcal{N} = 2$ states

$O(3) \times SU(2N_f)$	$SU(3) \times SU(2)$	J^P	$s = 0$	$s = -1$, $I = 0$	$s = -1$, $I = 1$	$s = -2$	$s = -3$
$[\mathbf{56}', 0^+]$	$^2 8$	$1/2^+$	$N(1440)$	$\Lambda(1600)$	$\Sigma(1660)$	$\Xi(1690)$	
	$^4 10$	$3/2^+$	$\Delta(1600)$**		$\Sigma(1690)$**		
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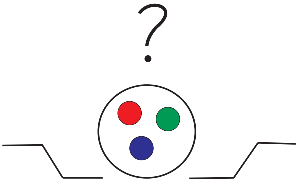
$[\mathbf{20}, 1^+]$	$^2 8$	$1/2^+$	$N(2010)$				
	$^4 8$	$3/2^+$	$N(2040)$*				

Lattice calculations seem to confirm this classification scheme

[1] Edwards Dudek Richards Wallace, PRD **84**.074508 ('11)

[1] Edwards *et al.*, PRD **87**.054506 ('13)

Why is the constituent QM so successful?



- Lattice QCD $\longrightarrow$ missing states + $O(3) \times SU(2N_f)$ assignment
- Large N_c QCD $\longrightarrow$ Analytic approach

Spin-flavor symmetry for baryons in large N_c QCD

Gervais, Sakita and Dashen, Manohar found that in the large N_c limit a spin-flavor "contracted" $SU(2N_f)_c$ arises for ground state baryons.

$SU(2N_f)$	$SU(2N_f)_c$
$[S^i, T^a] = 0,$ $[S^i, S^j] = i\epsilon^{ijk} S^k, \quad [T^a, T^b] = if^{abc} T^c,$ $[S^i, G^{ja}] = i\epsilon^{ijk} G^{ka}, \quad [T^a, G^{ib}] = if^{abc} G^{ic},$ $[G^{ia}, G^{jb}] = \frac{i}{4}\delta^{ij} f^{abc} T^c + \frac{i}{2N_f}\delta^{ab}\epsilon^{ijk} S^k + \frac{i}{2}\epsilon^{ijk} d^{abc} G^{kc}$	$[S^i, T^a] = 0,$ $[S^i, S^j] = i\epsilon^{ijk} S^k, \quad [T^a, T^b] = if^{abc} T^c,$ $[S^i, X_0^{ja}] = i\epsilon^{ijk} X_0^{ka}, \quad [T^a, X_0^{ib}] = if^{abc} X_0^{ic}$ $[X_0^{ia}, X_0^{jb}] = 0$

$$X_0^{ia} = \lim_{N_c \to \infty} \frac{G^{ia}}{N_c} .$$

$$SU(2N_f)\ (\text{QM}) \xrightarrow[N_c \to \infty]{} SU(2N_f)_c$$

[2] J. L. Gervais and B. Sakita, Phys. Rev. Lett. **52**, 87 (1984), Phys. Rev. D **30**, 1795 (1984).

[3] R. F. Dashen and A. V. Manohar, Phys. Lett. B **315**, 425 (1993)

Towers of large N_c states

States in the large N_c limit organize into towers:
$SU(2N_f)_c$ symmetry → towers **K** ("grand spin")

For $N_f = 2$

$\mathbf{K} = \mathbf{L}$ for S
$\mathbf{K} = \mathbf{L} + \mathbf{1}$ for MS,A

For $[\text{“}\mathbf{70}'', 2^+]$ → three towers $K = 1, 2, 3$

For $N_f = 3$ this relation is not trivial

$1/N_c$ expansion for excited baryons

$$\text{Excited baryon}: \underbrace{\text{excited quark}}_{\ell} + \underbrace{N_c - 1 \text{ quarks core}}_{\text{Symmetric in spin-flavor}}.$$

Quark and core couple to form $O(3) \times SU(2N_f)$ multiplets

Building blocks for operators:

$$s,\ t,\ g, \quad S_c,\ T_c,\ G_c$$

These spin-flavor generators are coupled to the angular momentum operator ℓ.

Large N_c operators

Using reduction rules one can find an operator basis for excited states. The mass operator can be expressed at leading order in N_c as

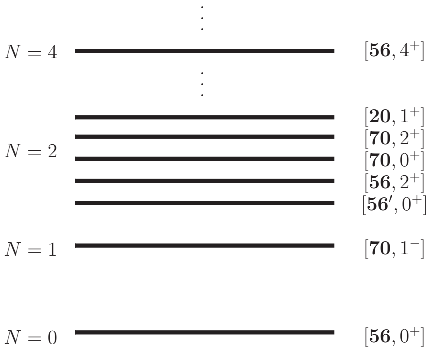
$$H = \sum_{i=1}^{5} c_i^{\mathbf{T},\mathbf{T}'} O_i + \mathcal{O}(1/N_c)$$

$$\begin{aligned}
O_1 &= (N_c 1)^{[0,\mathbf{1}]} \\
O_2 &= \left(\ell^{(1)} s\right)^{[0,\mathbf{1}]} \\
O_3 &= \left(\ell^{(2)} \left(g G_c\right)^{[2,\mathbf{1}]}\right)^{[0,\mathbf{1}]} \\
O_4 &= \frac{1}{N_c} \left(\ell^{(1)} \left(t G_c\right)^{[1,\mathbf{1}]}\right)^{[0,\mathbf{1}]} \\
O_5 &= \frac{1}{N_c} \left(t T_c\right)^{[0,\mathbf{1}]}
\end{aligned}$$

(No $SU(3)$ breaking)

[4] Carlson Carone Goity Lebed, PRD **59**.114008 ('99)

States studied in the $1/N_c$ expansion



$\mathcal{N} = 2$

Carlson Carone PLB **484**.260

Goity Schat Scoccola PLB **564**.83

Matagne Stancu PLB **631**.7

Matagne Stancu PRD **74**.034014

$\mathcal{N} = 1$

Pirjol Schat PRD **67**.096009

Goity Schat Scoccola PRD **66**.114014

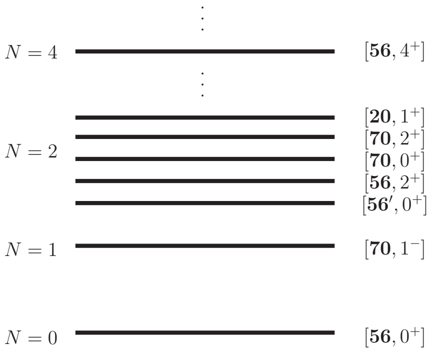
Schat Goity Scoccola PRL **88**.102002

Carlson Carone Goity Lebed PRD **59**.114008

$\mathcal{N} = 0$

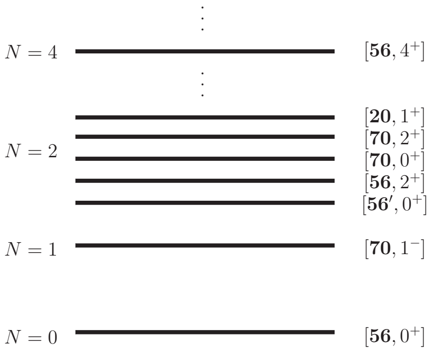
Witten NPB **160**.57 ('79)

States studied in the $1/N_c$ expansion



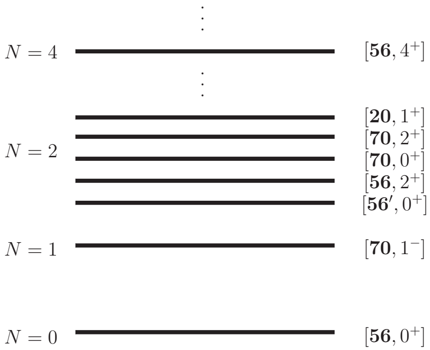
- In nature these states appear mixed

States studied in the $1/N_c$ expansion



- In nature these states appear mixed
- In large N_c QCD too

States studied in the $1/N_c$ expansion



- In nature these states appear mixed
- In large N_c QCD too

Configuration mixing effects are not N_c suppressed

[5] Goity, Yad. Fiz. 68.655 (2005).

Large N_c operators

The mass operator can be expressed as

$$H = \sum_{i=1}^{5} c_i^{\mathbf{T},\mathbf{T}'} O_i + \mathcal{O}(1/N_c)$$

At leading order in N_c taking $l \to \xi$

$$\begin{aligned}
O_1 &= (N_c 1)^{[0,\mathbf{1}]} \\
O_2 &= \left(\xi^{(1)} s\right)^{[0,\mathbf{1}]} \\
O_3 &= \left(\xi^{(2)} \left(gG_c\right)^{[2,\mathbf{1}]}\right)^{[0,\mathbf{1}]} \\
O_4 &= \frac{1}{N_c}\left(\xi^{(1)} \left(tG_c\right)^{[1,\mathbf{1}]}\right)^{[0,\mathbf{1}]} \\
O_5 &= \frac{1}{N_c}\left(tT_c\right)^{[0,\mathbf{1}]}
\end{aligned}$$

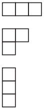
Large N_c baryons

For $N_c = 3$:

56 $\rightarrow$ Symmetric

70 $\rightarrow$ Mixed-symmetric

20 $\rightarrow$ Antisymmetric

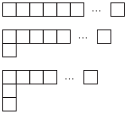


For large N_c:

"**56**" $\rightarrow$ Symmetric

"**70**" $\rightarrow$ Mixed-symmetric

"**20**" $\rightarrow$ Antisymmetric



$\mathbf{8} = (\mathbf{1}, \mathbf{1})$

1 1

1 2 1

1 1

$"\mathbf{8}" = \left(1, \frac{N_c - 1}{2}\right)$

1 1

1 2 1

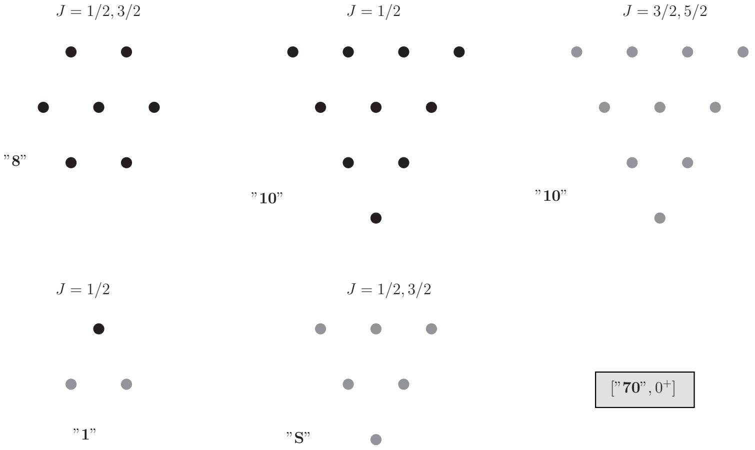
1 2 2 1

1 2 2 2 1

1 2 2 2 2 1

⋰ ⋱

Large N_c baryons



Building the states

- Symmetric and Mixed-symmetric states

$$\mathbf{S} = \mathbf{S}_c + \mathbf{1/2}$$

$$|S;\mathbf{R}\rangle_{S,MS} \;=\; \sum_{\eta=\pm 1/2} c_{sym}(p,S,\eta)|S;\mathbf{R};S_c = S+\eta\rangle$$

- Antisymmetric states

$$\left|S,\mathbf{R}\right\rangle_A = \sum_i c_i \left|([S_{c_i},\mathbf{R}_{c_i}]_{MS}\;\;\mathbf{q})^{[S,\mathbf{R}]}\right\rangle,$$

where $\mathbf{q}\equiv[1/2,\mathbf{3}]$.

Towers with configuration mixing

By calculating the eigenvalues of the 24 mass matrices we found that all the S and MS states of the $\mathcal{N} = 2$ band have only nine masses which can be expressed as

$$
\begin{aligned}
m_0 &= \bar{c}_1^{S_0} N_c, \\
m_{1\pm} &= \bar{m}_1 \pm \delta_1, \\
m_{2\pm} &= \bar{m}_2 \pm \delta_2, \\
m_3 &= \bar{c}_1^{MS_2} N_c + c_2^{MS_2} - \frac{2}{7} c_3^{MS_2} \\
m_{\frac{1}{2}} &= \bar{c}_1^{MS_0} N_c - 3 c_5^{MS_0} \\
m_{\frac{3}{2}} &= \bar{c}_1^{MS_2} N_c - \frac{3}{2} \bar{c}_2^{MS_2} + 3 c_4^{MS_2} - 3 c_5^{MS_2} \\
m_{\frac{5}{2}} &= \bar{c}_1^{MS_2} N_c + \bar{c}_2^{MS_2} - 2 c_4^{MS_2} - 3 c_5^{MS_2}
\end{aligned}
$$

Towers with configuration mixing

$$
\begin{aligned}
m_{1\pm} &= \bar{m}_1 \pm \delta_1, \\
m_{2\pm} &= \bar{m}_2 \pm \delta_2,
\end{aligned}
$$

where

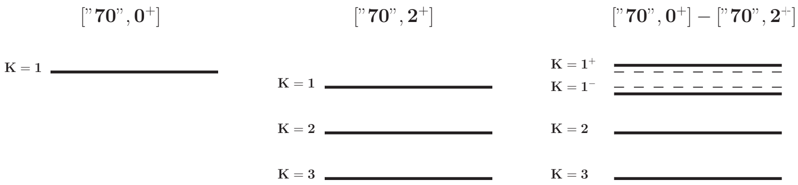
$$
\begin{aligned}
\delta_1 &= \sqrt{\left(\frac{1}{2}\left(\bar{c}_1^{MS_0} - \bar{c}_1^{MS_2}\right) N_c + \frac{3}{4}\bar{c}_2^{MS_2} + \frac{1}{2} c_3^{MS_2}\right)^2 + 2\left(c_3^{MS_0,MS_2}\right)^2} \\
\delta_2 &= \sqrt{\left(\frac{1}{2}\left(\bar{c}_1^{S_2} - \bar{c}_1^{MS_2}\right) N_c + \frac{1}{4}\bar{c}_2^{MS_2} - \frac{1}{2} c_3^{MS_2}\right)^2 + 2\left(\bar{c}_2^{S_2,MS_2}\right)^2}
\end{aligned}
$$

$$O_2 = \left(\xi^{(1)} s\right)^{[0,\mathbf{1}]} \quad \text{mixes} \quad [\text{“}\mathbf{70}\text{”}, 0^+] - [\text{“}\mathbf{70}\text{”}, 2^+]$$

$$O_3 = \left(\xi^{(2)} (g G_c)^{[2,\mathbf{1}]}\right)^{[0,\mathbf{1}]} \quad \text{mixes} \quad [\text{“}\mathbf{56}\text{”}, 2^+] - [\text{“}\mathbf{70}\text{”}, 2^+]$$

Towers with configuration mixing

Configuration mixing effects



$$\delta_1 = \sqrt{\left(\frac{1}{2}\left(\bar{c}_1^{MS_0} - \bar{c}_1^{MS_2}\right) N_c + \frac{3}{4}\bar{c}_2^{MS_2} + \frac{1}{2}c_3^{MS_2}\right)^2 + 2\left(c_3^{MS_0,MS_2}\right)^2}$$

K number assignment

We know how to get the K number for the nonstrange baryons:
$\mathbf{K} = \mathbf{L}$ for S and $\mathbf{K} = \mathbf{L} + \mathbf{1}$ for MS

$$
\begin{aligned}
m_0 &= \bar{c}_1^{S_0} N_c, && K = 0 \\
m_{1\pm} &= \bar{m}_1 \pm \delta_1, && K = 1 \\
m_{2\pm} &= \bar{m}_2 \pm \delta_2, && K = 2 \\
m_3 &= \bar{c}_1^{MS_2} N_c + c_2^{MS_2} - \frac{2}{7} c_3^{MS_2} && K = 3 \\
m_{\frac{1}{2}} &= \bar{c}_1^{MS_0} N_c - 3 c_5^{MS_0} && K = ? \\
m_{\frac{3}{2}} &= \bar{c}_1^{MS_2} N_c - \frac{3}{2} \bar{c}_2^{MS_2} + 3 c_4^{MS_2} - 3 c_5^{MS_2} && K = ? \\
m_{\frac{5}{2}} &= \bar{c}_1^{MS_2} N_c + \bar{c}_2^{MS_2} - 2 c_4^{MS_2} - 3 c_5^{MS_2} && K = ?
\end{aligned}
$$

Nucleon-meson scattering picture

For the present purpose it is sufficient to consider the case of π or η mesons scattering off a ground-state band baryon.

$$S = (-1)^{\ell-\ell'} \frac{\sqrt{D(\mathbf{R}_B)D(\mathbf{R}_{B'})}}{D(\mathbf{R}_s)} \sum_{\substack{I,I',Y\in\mathbf{8},\\ I''\in\mathbf{R}_s}} (-1)^{I+I'+Y}\hat{I}''$$

$$\times \left(\begin{array}{cc||c} \mathbf{R}_B & \mathbf{8} & \mathbf{R}_s\,\gamma_s \\ S_B\,\frac{N_c}{3} & IY & I''\;Y+\frac{N_c}{3} \end{array} \right) \left(\begin{array}{cc||c} \mathbf{R}_B & \mathbf{8} & \mathbf{R}_s\,\gamma_s \\ I_B\,Y_B & I_\phi\,Y_\phi & I_s\,Y_s \end{array} \right)$$

$$\times \left(\begin{array}{cc||c} \mathbf{R}_{B'} & \mathbf{8} & \mathbf{R}_s\,\gamma'_s \\ S_{B'}\,\frac{N_c}{3} & I'Y & I''\;Y+\frac{N_c}{3} \end{array} \right) \left(\begin{array}{cc||c} \mathbf{R}_{B'} & \mathbf{8} & \mathbf{R}_s\,\gamma'_s \\ I_{B'}\,Y_{B'} & I_{\phi'}\,Y_{\phi'} & I_s\,Y_s \end{array} \right)$$

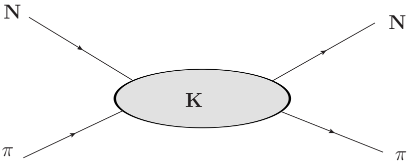
$$\times \sum_K \hat{K} \left\{ \begin{array}{ccc} K & I'' & J_s \\ S_B & \ell & I \end{array} \right\} \left\{ \begin{array}{ccc} K & I'' & J_s \\ S_{B'} & \ell' & I' \end{array} \right\} \tau^{II'Y}_{K\ell\ell'}$$

[6] Cohen Lebed, PLB 619.115 ('05)

Nucleon-meson scattering picture

For the present purpose it is sufficient to consider the case of π or η mesons scattering off a ground-state band baryon.

We consider the scatterings that can produce a given baryon and calculate the possible K numbers of the resonances:



We can test the compatibility between the two pictures.

[7] nonstrange $[\mathbf{70}, L]$ → Cohen Lebed, PRD **68**.056003 ('03)

[8] complete $[\mathbf{70}, 1]$ → Cohen Lebed, PRD **72**.056001 ('05)

Operators for antisymmetric states

What about the antisymmetric states?
Recall the operator basis

$$
\begin{aligned}
O_1 &= (N_c 1)^{[0,\mathbf{1}]} \\
O_2 &= \left(\xi^{(1)} s\right)^{[0,\mathbf{1}]} \\
O_3 &= \left(\xi^{(2)} \, (g G_c)^{[2,\mathbf{1}]}\right)^{[0,\mathbf{1}]} \\
O_4 &= \frac{1}{N_c} \left(\xi^{(1)} \, (t G_c)^{[1,\mathbf{1}]}\right)^{[0,\mathbf{1}]} \\
O_5 &= \frac{1}{N_c} \, (t T_c)^{[0,\mathbf{1}]}
\end{aligned}
$$

States with containing cores with different symmetry cannot mix with this basis!

Operators for antisymmetric states

For the nonstrange antisymmetric states $\mathbf{K} = \mathbf{L} + \mathbf{1}$

Operator basis

$$\begin{aligned}
O_1 &= (N_c 1)^{[0,\mathbf{1}]} \\
O_2 &= \left(\xi^{(1)} s\right)^{[0,\mathbf{1}]} \\
O_3 &= \left(\xi^{(2)} (g G_c)^{[2,\mathbf{1}]}\right)^{[0,\mathbf{1}]}
\end{aligned}$$

Towers for antisymmetric states

The tower structure found for the nonstrange antisymmetric states and their strange partners in the flavor multiplet is given by

$$
\begin{aligned}
m_0 &= c_1 N_c - c_2 - \frac{5}{24} c_3 \\
m_1 &= c_1 N_c - \frac{1}{2} c_2 + \frac{5}{48} c_3 \\
m_2 &= c_1 N_c + \frac{1}{2} c_2 - \frac{1}{48} c_3
\end{aligned}
$$

Note: This structure is the same as the one found for the $[\text{``}\mathbf{70}\text{''}, 1^-]$ multiplet.
Furthermore, not only the tower structure coincides but also the m_i and the matrices expressions are identical.

[9] Cohen Lebed, PRD 68.056003 ('03)

Towers for antisymmetric states

From the resonance picture we get 3 towers for the antisymmetric strange states:

$$K = \frac{1}{2} \quad : \quad \left(\Lambda^{\mathbf{1}}_{1/2}, \Xi^{\mathbf{1}}_{1/2}\right)$$
$$K = \frac{3}{2} \quad : \quad \left(\Lambda^{\mathbf{1}}_{3/2}, \Xi^{\mathbf{1}}_{3/2}\right)$$
$$K = \frac{5}{2} \quad : \quad \left(\Lambda^{\mathbf{1}}_{5/2}, \Xi^{\mathbf{1}}_{5/2}\right)$$

Towers for antisymmetric states

From the resonance picture we get 3 towers for the antisymmetric strange states:

$$K = \frac{1}{2} \quad : \quad \left(\Lambda^{\mathbf{1}}_{1/2}, \Xi^{\mathbf{1}}_{1/2}\right) \qquad \qquad \left(\Lambda^{\mathbf{1}}_{1/2}, \Xi^{\mathbf{1}}_{1/2}\right)$$
$$K = \frac{3}{2} \quad : \quad \left(\Lambda^{\mathbf{1}}_{3/2}, \Xi^{\mathbf{1}}_{3/2}\right) \qquad \qquad \left(\Lambda^{\mathbf{1}}_{3/2}, \Xi^{\mathbf{1}}_{3/2}\right)$$
$$K = \frac{5}{2} \quad : \quad \left(\Lambda^{\mathbf{1}}_{5/2}, \Xi^{\mathbf{1}}_{5/2}\right)$$

Towers for antisymmetric states

From the resonance picture we get 3 towers for the antisymmetric strange states:

$$K = \frac{1}{2} \quad : \quad \left(\Lambda^{\mathbf{1}}_{1/2}, \Xi^{\mathbf{1}}_{1/2}\right) \qquad\qquad \left(\Lambda^{\mathbf{1}}_{1/2}, \Xi^{\mathbf{1}}_{1/2}\right)$$

$$K = \frac{3}{2} \quad : \quad \left(\Lambda^{\mathbf{1}}_{3/2}, \Xi^{\mathbf{1}}_{3/2}\right) \qquad\qquad \left(\Lambda^{\mathbf{1}}_{3/2}, \Xi^{\mathbf{1}}_{3/2}\right)$$

$$K = \frac{5}{2} \quad : \quad \left(\Lambda^{\mathbf{1}}_{5/2}, \Xi^{\mathbf{1}}_{5/2}\right)$$

A completely spurious tower arises

Summary and conclusions I

- We found that symmetric and mixed-symmetric states in the $N = 2$ band organize into 9 towers
- We built the antisymmetric states and found that they organize into 3 (+3) towers
- Confirmed that the core+quark approach is suitable for S and MS states.
- We found that the core+quark approach is suitable for antisymmetric states. Effects from more complex constructions are N_c suppressed.

Summary and conclusions II

- We found configuration mixing effects only on flavor multiplets containing nonstrange states: "**8**" and "**10**" (but not on "**1**" or "**S**")
- Configuration mixing happens between ["**70**", 0^+] – ["**70**", 2^+] and ["**56**", 2^+] – ["**70**", 2^+].
- Configuration mixing of the ["**20**", 1^+] is a $\mathcal{O}(1/N_c)$ effect.
- Quark+core picture is consistent with the resonance picture for the $\mathcal{N} = 2$ band.

Next steps

- Break $SU(3)$, consider NLO corrections, fit coefficients to data