Renormalisation of Operators.

Renormalised OPI Green's $f^{=}$ with n external (fermion) legs are related to the Bare Green's function through
massless

$$\tilde{\Gamma}^{(n)}(p_1,\ldots,p_n;g,\mu) = Z_\psi^{n/2}\,\Gamma_B^{(n)}(p_1,\ldots,p_n;g_B)$$

This satisfies the renormalisation gp. equation

$$\left(\mu\frac{\partial}{\partial\mu} + \beta\frac{\partial}{\partial g} - n\gamma_\psi\right)\tilde{\Gamma}^{(n)}(p_1,\ldots,p_n;g,\mu) = 0 \qquad (1)$$

where
$$\gamma_\psi = Z_\psi^{-\frac{1}{2}}\,\mu\frac{\partial}{\partial\mu}Z_\psi^{\frac{1}{2}}$$

$$\beta = \mu\frac{\partial g}{\partial\mu}$$

(*) just arises from the independence of the bare Green's $f^{=}$ on the renormalisation scale.

We can extend this idea to allow for composite operators O_i by adding source term $O_i(x)\,J_i(x)$ to the Lagrangian in the functional integral.

The resulting Green's $f^{=}$ then has the following rel$^{=}$ between Ren. & bare Greens $f^{=}$

one of our operators in the above.

$$\tilde{\Gamma}^{(n)}_{O_i}(p_1,\ldots,p_n;g,\mu) = Z_\psi^{n/2}\,Z_{ij}\,\Gamma^{(n)}_{O_j,B}(p_1,\ldots,p_n;g_B) \qquad (2)$$

where the momentum associated with the operator O_i is zero.

$O_{Bi} = Z_{ij}\,O_j$ ← operators can mix

The important point is that operators can mix under renormalisation. However, only operators of the same dimension can mix.

Renorm. of operators

This is clear when you consider that renormalization is logarithmic.

Then we have a corresponding renormalisation gp. eq: for the Green's f² $\tilde{\Gamma}^{(n)}_{O_i}(p_1,\dots,p_n;g,\mu)$

$$\left[\delta_{ij}\left(\mu\frac{\partial}{\partial\mu}+\beta\frac{\partial}{\partial g}-n\gamma_\psi\right)-\gamma_{ij}\right]\tilde{\Gamma}^{(n)}_{O_j}(p_1,\dots,p_n;g,\mu)=0 \qquad (3)$$

where $\quad \gamma_{ij}=\left(\mu\frac{\partial}{\partial\mu}Z_{ik}\right)Z^{-1}_{kj}$

Let us now apply this to the ~~current~~ OPE for the product of vector currents.

We have written

$$i\,T_{\mu\nu}=\sum_i C_i(q,g,\mu)\,\langle N(p)|O_i|N(p)\rangle\big|_\mu .$$

In ~~terms of~~ the above language in this case

$$i\,\tilde{\Gamma}^{(n)}_{J_\mu J_\nu}(q,-q,p_1,\dots,p_n;g,\mu)=\sum_i C_i(q,g,\mu)\,\tilde{\Gamma}^{(n)}_{O_i}(p_1,\dots,p_n;g,\mu) \quad (4)$$

Now $\gamma_J=0$; anomalous dimension of conserved quantities is zero, as we shall see explicitly for J_μ later.

$$\text{So}\quad\left(\mu\frac{\partial}{\partial\mu}+\beta\frac{\partial}{\partial g}-n\gamma_\psi\right)\tilde{\Gamma}^{(n)}_{J_\mu J_\nu}(q,-q,p_1,\dots,p_n)=0$$

Apply to r.h.s. of (4)

$$\xcancel{\sum_i\left\{\left(\mu\frac{\partial}{\partial\mu}+\beta\frac{\partial}{\partial g}-n\gamma_\psi\right)C_i(q)\right\}\tilde{\Gamma}^{(n)}_{O_i}(p_1,\dots,p_n;g,\mu)+C_i(q)\left[\mu\frac{\partial}{\partial\mu}+\beta\frac{\partial}{\partial g}-n\gamma_\psi\right]\tilde{\Gamma}^{(n)}_{O_i}(p_1,\dots,p_n;g,\mu)\}=0}$$

Renorm. of operators

Apply to R.H.S of (4)

$$\sum_i \Big\{ \Big[\Big(\mu \frac{\partial}{\partial\mu} + \beta \frac{\partial}{\partial g} \Big) C_i(q, \mu, g) \Big] \tilde{\Gamma}^{(n)}_{O_i}(p_1, \ldots, p_n; g, \mu)$$

$$+ \; C_i(q, \mu, g) \Big[\Big(\mu \frac{\partial}{\partial\mu} + \beta \frac{\partial}{\partial g} \Big) \tilde{\Gamma}^{(n)}_{O_i}(p_1, \ldots, p_n; g, \mu) \Big] - n\gamma_\psi C_i(q, \mu, g)$$

$$\tilde{\Gamma}^{(n)}_{O_i}(p_1, \ldots, p_n; g, \mu) \Big\} = 0$$

Now $\Big\{ \mu \frac{\partial}{\partial\mu} + \beta \frac{\partial}{\partial g} - n\gamma_\psi \Big\} \tilde{\Gamma}^{(n)}_{O_i}(p_1, \ldots, p_n; g, \mu) = \gamma_{ij} \tilde{\Gamma}^{(n)}_{O_j}(p_1, \ldots, p_n; g, \mu)$

So the ren. gp. eqn for the C's is

$$\Big\{ \Big(\mu \frac{\partial}{\partial\mu} + \beta \frac{\partial}{\partial g} \Big) \delta_{ij} + \gamma_{ji} \Big\} C_j(q, \mu, g) = 0 \qquad (5)$$

N.B.

1) Does not depend on anom. dim. of external legs → no Z_ψ here
2) Anom. dim. opposite to that of the operator.

Evolution

We have

$$\mu_2(n, Q^2) \equiv \int_0^1 dx\, x^{n-2}\, F_2(x) = \underset{\substack{\uparrow \\ \text{dimensionless}}}{C^{n,2}}\ \bar{A}^n$$

Suppose anomalous dimension matrix diagonal

Callan-Symanzik eq:

dimensionless

$$\left\{ \mu \frac{\partial}{\partial \mu} + \beta \frac{\partial}{\partial g} + \gamma(n,g) \right\} C^{n,2}\left[\frac{Q}{\mu}, g\right]$$

Has solution (bjp) ← cf Collins, ...

$$C^{n,2}\left[\frac{Q}{\mu}, \frac{g^2(\mu)}{4\pi}\right] = e^{\int_0^t d \ln \frac{Q'}{\mu}\, \gamma[n, g(Q'^2)]}\ C^{n,2}[1, \alpha_s(Q^2)]$$

where $t = \frac{1}{2} \ln \frac{Q^2}{\mu^2}$.

Assume we are in perturbative regime

$$\gamma[n,g] = \gamma_0(n)\, g^2(Q'^2)$$

where $g^2(Q'^2) = \dfrac{1}{b_0 \ln Q'^2/\Lambda^2}$

~~That~~ Then $g^2(Q'^2) \sim \dfrac{1}{b_0 \ln Q'^2/\Lambda^2} = \dfrac{1}{b_0 \left\{ \ln \mu^2/\Lambda^2 + \ln \frac{Q'^2}{\mu^2} \right\}}$

So

$$C^{n,2}\left[\frac{Q}{\mu}, \frac{g^2(\mu)}{4\pi}\right] \sim e^{\frac{\gamma_0(n)}{2b_0} \ln \frac{\ln Q^2/\Lambda^2}{\ln \mu^2/\Lambda^2}}\ C^{n,2}[1, \alpha_s(Q^2)]$$

$$\sim \left(1 + \ln Q^2/\mu^2 \cdot \frac{1}{\ln \mu^2/\Lambda^2}\right)^{\frac{\gamma_0(n)}{2b_0}} C^{n,2}[1, \alpha_s(Q^2)]$$

$$\cancel{\sim \left[1 + a \ln Q^2/\mu^2\ \alpha_s(\mu^2)\right]^{\frac{\gamma_0(n)}{2b_0}} C^{n,2}[1, 0].}$$

i.e. $$C^{n,2}\left[\frac{Q}{\mu}, \frac{g^2(\mu)}{4\pi}\right] \sim \left(\ln \frac{Q^2}{\mu^2}\right)^{\frac{\gamma_0(n)}{2b_0}} C^{n,2}[1,0].$$

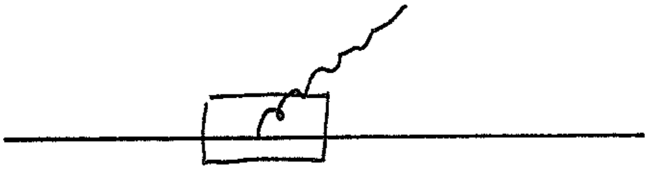
$$\Theta \quad \frac{C^{n,2}\left(\frac{Q}{\mu}, \frac{g^2}{4\pi}\right)}{C^{n,2}\left(\frac{Q_0}{\mu}, \frac{g^2}{4\pi}\right)} \sim \left(\frac{\ln \frac{Q^2}{\mu^2}}{\ln \frac{Q_0^2}{\mu^2}}\right)^{\frac{\gamma_0(n)}{2b_0}}$$

Operator renormalization

Which operators are diagonal?

The non-singlet structure function

Consider quark in proton; can emit a gluon



So in principle gluon and quark dist$^{=}$ can mix in a nucleon.

However splitting function is independent of quark flavour. (and $q \to \bar{q}$).

Thus if we choose non-singlet operators

$$O_{NS}^{\mu_1 \ldots \mu_n} = \frac{i^{n-1}}{2^{n-1}} \bar{\Psi} \gamma_{\mu_1} \overleftrightarrow{D}_{\mu_2} \ldots \overleftrightarrow{D}_{\mu_n} \cancel{\gamma_5} \lambda^{\text{flavour}} \Psi$$

where for 2 flavours λ^{flavour} might be, say, τ_3, the mixing with gluon dist$^{=}$ cancels

Therefore the diagonal Operators are the NS (nonsinglet) operators.

Feynman rules for operators.

Because we wish to symmetrise over the indices and remove traces, consider the operator in the form.

$$\tilde{O}^{(n)}_{NS} = \Delta_{\mu_1}\cdots\Delta_{\mu_n}\, O^{\mu_1\cdots\mu_n}_{NS} \quad (*)$$ where Δ_μ satisfies $\Delta^2 = 0$.

Trivial exercise to see that this projects out the traceless, symmetric part of the operator

To lowest order (tree level) $\tilde{O}^{(n)}_{NS} \to$

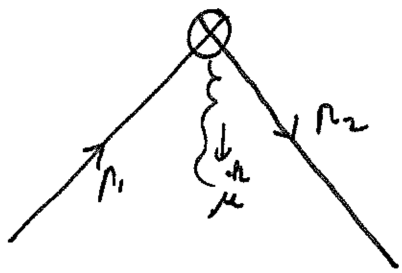
$$\to \not{\Delta}\,(p\cdot\Delta)^{n-1}$$

i.e. we just replace the $iD_\mu \to i\partial_\mu \to p_\mu$

now $D_\mu = \partial_\mu - igT^aA^a_\mu$

$$\to \quad iD_\mu \to i\partial_\mu + gT^aA^a_\mu$$

Thus we have a first order operator



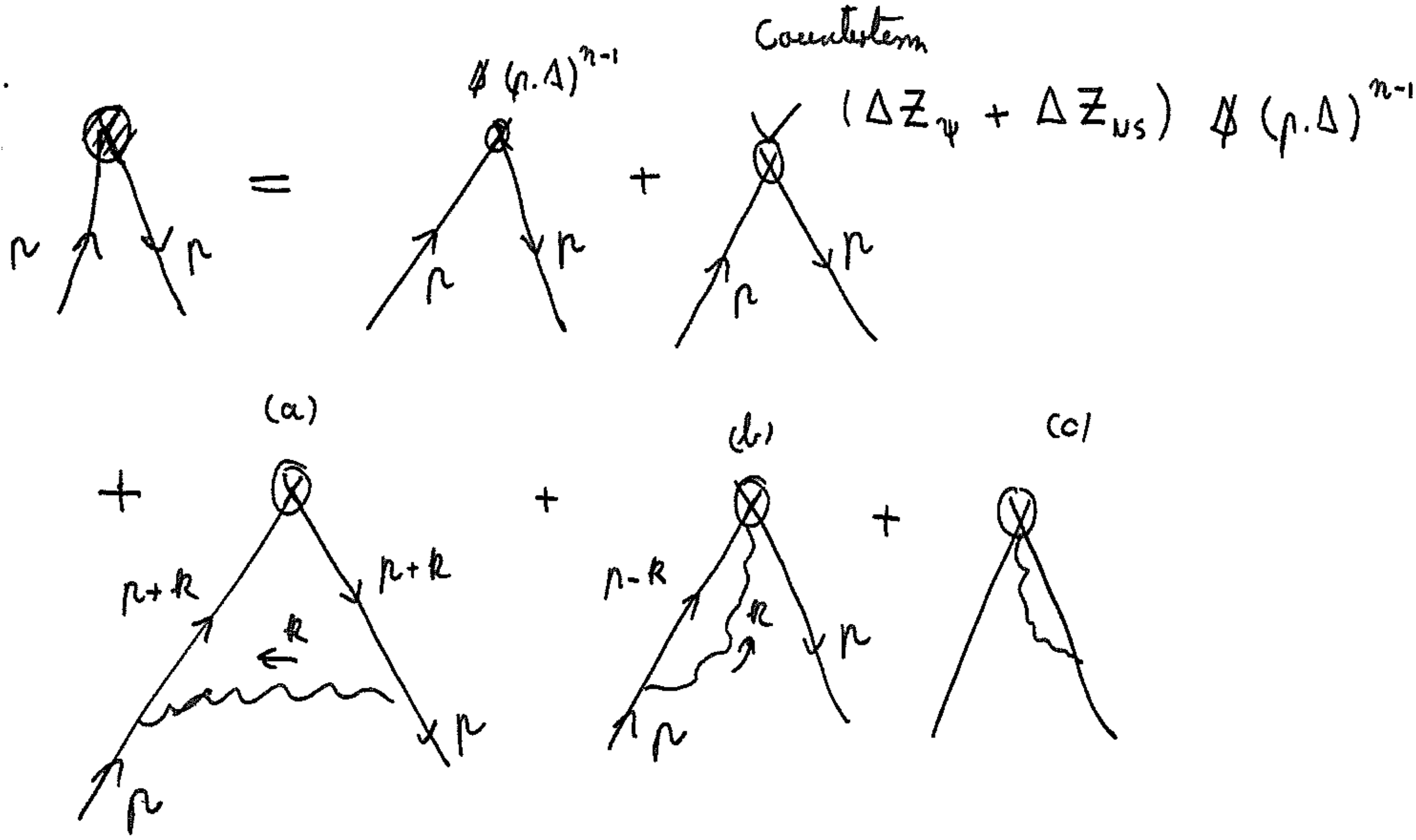
$$\to +gT^a\Delta_\mu\,\not{\Delta}\sum_{j=0}^{n-2}\underbrace{(\Delta\cdot p_1)^j\,(\Delta\cdot p_2)^{n-j-2}}$$

~~we can avoid~~
we can obtain the order g term from any of the D_μ's

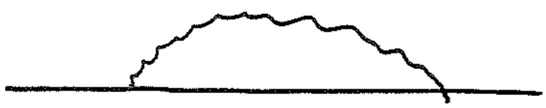
Consider operator renormalisation

$$\Gamma^{(2)}(p,-p;g,\mu) = Z_{NS}\, Z_\psi\, \Gamma_B^{(n)}(p,-p;g)$$

i.e.



Z_ψ is obtained from



Begin by looking at diag (a).

$$\rightarrow (ig)^2 \int \frac{d^{2\omega}k}{(2\pi)^{2\omega}} \gamma_\mu \frac{i(\not p+\not k)}{(p+k)^2} \not\Delta [\Delta.(p+k)]^{n-1} \frac{i(\not p+\not k)}{(p+k)^2}$$

$$\times \gamma^\mu \frac{-i}{k^2} (T^a T^a)_{ij}$$

$$\rightarrow -i^5 g^2 C_F \delta_{ij} \int \frac{d^{2\omega}k}{(2\pi)^{2\omega}} \left[\frac{1}{(p+k)^2}\right]^2 \frac{1}{k^2} \gamma_\mu (\not p+\not k) \not\Delta [\Delta.(p+k)]^{n-1} (\not p+\not k)\gamma^\mu$$

Now $(T^a T^a)_{ij} = C_F \delta_{ij}$

Feynman params.

Now $\frac{1}{A^\alpha} \cdot \frac{1}{B^\beta} = \frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)} \int_0^1 dx \frac{x^{\alpha-1}(1-x)^{\beta-1}}{\{xA+(1-x)B\}^{\alpha+\beta}}$

$$\rightarrow -i g^2 C_F \delta_{ij} \frac{\Gamma(3)}{\Gamma(2)\Gamma(1)} \int_0^1 dx \, (1-x) \int \frac{d^{2\omega}k}{(2\pi)^{2\omega}}$$

$$\times \underbrace{\{xk^2 + (1-x)(p+k)^2\}}^{-3} \gamma_\mu (\not p+\not k) \not\Delta [\Delta.(p+k)]^{n-1} (\not p+\not k)\gamma^\mu$$

$$\{xk^2 + (1-x)[2p.k + k^2 + p^2]\}$$

$$k^2 + (1-x).2p.k + (1-x)p^2$$

$$[k + (1-x)p]^2 + \cancel{(1-x)}^2 (1-x)[1-1+x]p^2$$

write as immediately

$$\rightarrow [k+(1-x)p]^2 + x(1-x)p^2$$

Let $k + (1-x)p = l \qquad k = l - (1-x)p$

$$\to -ig^2 C_F \, \delta_{ij} \quad 2\int_0^1 dx\,(1-x) \int \frac{d^{2\omega}l}{(2\pi)^{2\omega}}$$

$$\times \left\{ l^2 + x(1-x)p^2 \right\}^{-3} \gamma_\mu (\not{l} + x\not{p}) \not{\Delta} \left[\Delta.(l+xp)\right]^{n-1} (\not{l}+x\not{p}) \gamma^\mu \quad -*$$

Now we have removed the linear term in p from the denominator of the integrand

See Ramond, reg.

0 in degener
$\partial_\mu v_\mu = 0$ degenerate quark.

Consider ~~to~~ an integral of the form

$$\int \frac{d^{2\omega}l}{(2\pi)^{2\omega}} \quad \frac{l_{\mu_1} \ldots l_{\mu_n}}{\{l^2 + x(1-x)p^2\}^A}$$

purely f^2 of p^2

purely isotropic in p

If n is odd, the integral vanishes, since must be proportional to p_μ, and integrand is symmetric under $p \to -p$.

If n even, then integral must be some product of δ-f=

Thus ~~the~~ we can replace $[\Delta.(l+xp)]^{n-1}$ by $[\Delta.xp]^{n-1}$ in $(*)$ — since $\Delta^2 = 0$

So $(*) \to -2ig^2 C_F \, \delta_{ij} \int_0^1 dx\,(1-x) \int \frac{d^{2\omega}l}{(2\pi)^{2\omega}}$

$$\cdot\, x^{n-1} (\Delta.p)^{n-1} \quad \{l^2 + x(1-x)p^2\}^{-3} \underbrace{\gamma_\mu \not{l} \not{\Delta} \not{l} \gamma^\mu}$$

require co-eff of $(\Delta.p)^{n-1}$

$$\to -2ig^2 C_F \, \delta_{ij} \int_0^1 dx\,(1-x)\, x^{n-1} (\Delta.p)^{n-1} \gamma_\mu \gamma_\nu \not{\Delta} \gamma_\sigma \gamma^\mu$$

$$\times \int \frac{d^{2\omega}l}{(2\pi)^{2\omega}} \quad \frac{l_\nu \, l_\sigma}{[l^2 + x(1-x)p^2]^{-3}}$$

Look up this integral over l in your favourite book & identify the leading co-efficient of $\frac{1}{\varepsilon}$

$$\boxed{\begin{array}{l} \rightarrow \quad \frac{g^2}{16\pi^2} C_F \frac{2}{n(n+1)} (\Delta \cdot p)^{n-1} \not{\Delta}\, \delta_{ij} \, N_\varepsilon \\ \text{where } \underbrace{N_\varepsilon = \frac{2}{\varepsilon} - \gamma_E + \ln 4\pi}_{} \end{array}}$$

Euler const.

$$\int \frac{d^{2\omega} l}{(2\pi)^{2\omega}} \cdot \frac{l_\sigma l_\sigma}{[l^2 + x(1-x)p^2]^{-3}}$$

$$\rightarrow \frac{1}{(4\pi)^\omega} \cdot \frac{1}{\Gamma(3)} \cdot \frac{1}{2} \delta_{\sigma\sigma} \frac{\Gamma(2-\omega)}{\{x(1-x)p^2\}^{2-\omega}}$$

Now look at (b)

$$\cancel{\rightarrow - i^3 g^2 C_F \delta}$$

$$\rightarrow \int \frac{d^{2\omega} k}{(2\pi)^{2\omega}} \cdot g \, T^a \Delta_\mu \not{\Delta} \sum_{j=0}^{n-2} (\Delta \cdot p)^j [\Delta \cdot (p-k)]^{n-j-2}$$

$$\cdot \frac{i(\not{p} + \not{k})}{(p+k)^2} \, i g \gamma^\mu T^a \frac{-i}{p^2}$$

leading behaviour

$$\rightarrow \quad \frac{g^2}{16\pi^2} N_\varepsilon C_F \delta_{ij} \left(-2 \sum_{l=2}^{n} \frac{1}{l} \right) (\Delta \cdot p)^{n-1} \not{\Delta}$$

(c) gives exactly the same cont.

(d)+(e) $\rightarrow$ Brian's lecture.

$$Z_\psi = 1 - C_F \frac{g^2}{16\pi^2} \delta_{ij} N_\varepsilon$$

Combining the above

$$(a)+(b)+(c) \rightarrow +\frac{g^{*2}}{16\pi^2} N_\epsilon C_F \delta_{ij} \left\{ \frac{2}{n(n+1)} - 4\sum_{l=2}^{n} \frac{1}{l} \right\}$$

i.e. "$\overline{MS}$"

$$\Delta Z_\psi + \Delta Z_{O_N} = -\frac{g^{*}}{16\pi^2} N_\epsilon C_F \delta_{ij} \left\{ \frac{2}{n(n+1)} - 4\sum_{l=2}^{n} \frac{1}{l} + 4 \right\}$$

i.e.

$$\Delta Z_{O_N} = -\frac{g^{*}}{16\pi^2} N_\epsilon C_F \delta_{ij} \left\{ \frac{2}{n(n+1)} - 4\sum_{l=1}^{n} \frac{1}{l} + 3 \right\}$$

Recall $g^{*} = \mu^{-\frac{\epsilon}{2}} \hat{g}$ where $\hat{g}$ dimensionless

$$\gamma_{O_N} = \mu \frac{\partial}{\partial\mu} \ln Z$$

to

$$\gamma(n) = -\frac{2g^{*2} C_F}{16\pi^2} \left\{ -3 - \frac{2}{n(n+1)} + 4\sum_{l=1}^{n} \frac{1}{l} \right\} = g^{*2}\gamma_0(n$$

~~Letting $d(n) = \frac{\gamma(n)}{2b_0}$~~

~~$$d(n) = \frac{16}{33-2n_f} \left\{ \frac{1}{2n(n+1)} + \frac{3}{4} - \sum_{l=1}^{n} \frac{1}{l} \right\}$$~~

~~where $$\frac{C_n^2\left(\frac{Q}{\mu}, \frac{g^2}{4\pi}\right)}{C_n^2\left(\frac{Q_0}{\mu}, \frac{g^2}{4\pi}\right)} \sim \left\{ \frac{\alpha_s(Q_0^2)}{\alpha_s(Q^2)} \right\}^{d(n)}$$~~

~~So the theory higher moments~~

~~As $Q^2 \to \infty$, $\alpha_s(Q^2) \to 0$; $d(n)$ is -ve for higher moment~~

~~Thus higher moments get smaller~~

~~note $d(1) = 0 \Rightarrow$ vector current conserved.~~

Recall from last time

At this order, independent of scheme.

renormalized coupling

$$\gamma(n,g) = -\frac{2g^2 C_F}{16\pi^2}\left\{-3 - \frac{2}{n(n+1)} + 4\sum_1^n \frac{1}{l}\right\} = \gamma_0(n)\, g^2$$

Recall the Callan-Symanzik eq:

$$\left\{\mu\frac{\partial}{\partial\mu} + \beta\frac{\partial}{\partial g} + \gamma(n,g)\right\} C^{n,2}\left[\frac{Q}{\mu}, g\right] = 0$$

with solution

$$C^{n,2}\left[\frac{Q}{\mu}, \frac{g^2(\mu)}{4\pi}\right] = e^{\int_0^t d\ln\frac{Q'}{\mu}\,\gamma[n, g(Q'^2)]}\; C^{n,2}\left[1, \frac{g^2(Q^2)}{4\pi}\right]$$

with $t = \frac{1}{2}\ln\frac{Q^2}{\mu^2}$

Now write $\gamma[n,g] = \gamma_0[n]\, g^2$

$$g^2(Q'^2) = \frac{1}{b_0 \ln\left(\frac{Q'^2}{\Lambda^2}\right)} = \frac{1}{2b_0\left[\ln\frac{Q'}{\mu} + \ln\frac{\mu}{\Lambda}\right]} \;;\quad b_0 = \frac{33 - 2n_f}{16\pi^2}$$

$11 N_c$

Λ_{QCD}.

So $$C^{n,2}\left[\frac{Q}{\mu}, \frac{g^2(\mu)}{4\pi}\right] = e^{\int_0^{\frac{1}{2}\ln\frac{Q^2}{\mu^2}} dt\, \frac{\gamma_0[n]}{2b_0}\frac{1}{t + \ln\frac{\mu}{\Lambda}}}\; C^{n,2}\left[1, \frac{g^2(Q^2)}{4\pi}\right]$$

$$= e^{\frac{\gamma_0[n]}{2b_0}\ln\left\{\frac{\ln\frac{Q}{\mu} + \ln\frac{\mu}{\Lambda}}{\ln\frac{\mu}{\Lambda}}\right\}}.\; C^{n,2}\left[1, \frac{g^2(Q^2)}{4\pi}\right].$$

i.e. $C^{n,2}\left[\frac{Q}{\mu}, \frac{g^2(\mu)}{4\pi}\right] = \left\{1 + \underbrace{\frac{1}{\ln\frac{\mu}{\Lambda}}}_{g} \ln\frac{Q}{\mu}\right\}^{\frac{\gamma_0[n]}{2b_0}} C^{n,2}\left[1, \frac{g^2(Q)}{4\pi}\right]$

Let $Q^2 \to \infty$

$$C^{n,2}\left[\frac{Q}{\mu}, \frac{g^2(\mu)}{4\pi}\right] = \left\{g^2(\mu) \ln\frac{Q}{\mu}\right\}^{\frac{\gamma_0[n]}{2b_0}} \underbrace{C^{n,2}[1,0]}$$

asymptotic freedom
tree-level
co-efficient

More importantly

$$\frac{C^{n,2}\left[\frac{Q}{\mu}, \frac{g^2(\mu)}{4\pi}\right]}{C^{n,2}\left[\frac{Q_0}{\mu}, \frac{g^2(\mu)}{4\pi}\right]} \sim \left\{\frac{\ln\frac{Q}{\mu}}{\ln\frac{Q_0}{\mu}}\right\}^{\frac{\gamma_0[n]}{2b_0}}$$

$\Rightarrow$ anomalous dimension describes <u>evolution</u> of co-efficient f^n.

$$d[n] \equiv \frac{\gamma_0[n]}{2b_0} = \frac{16}{33-2n_f}\left\{\frac{1}{2n(n+1)} + \frac{3}{4} - \sum_{l=1}^{n}\frac{1}{l}\right\}.$$

$n=1$; $d[1] = 0$

$n > 1$ $\qquad d[n] < 0$

Higher moments of non-singlet str. f^2 <u>decrease</u> as $Q \to \infty$.

From def$^{=}$ of str. f$^{=}$

$$\int x^{n-2} F_2^{NS}(x) = C_{NS}^{n,2} A_{NS}^{n}$$

Let $\quad F_2^{NS}(x) = x f(x)$

Then $\quad \int dx\, x^{n-1} f_{NS}(x) = C_{NS}^{n,2} \bar{A}_{NS}^{n}$

So $\quad \int dx\, f_{NS}(x) =$ const., independent of q^2 — (*)

Let $\quad q_v(x) = q(x) - \bar{q}(x)$; number of quarks in proton less number of anti-quarks

$\downarrow$

NS dist$^{=}$

$q_s(x) = q(x) + \bar{q}(x)$. $\qquad$ sea quark dist$^{=}$

$g(x)$ $\qquad$ gluon dist$^{=}$

We've only studied non-singlet ("valence" quark) dist$^{=}$

$$(*) \Rightarrow \left.\begin{aligned} \int u_v\, dx &= 2 \\ \int d_v\, dx &= 1 \end{aligned}\right\}$$ number of up/down quarks in proton.

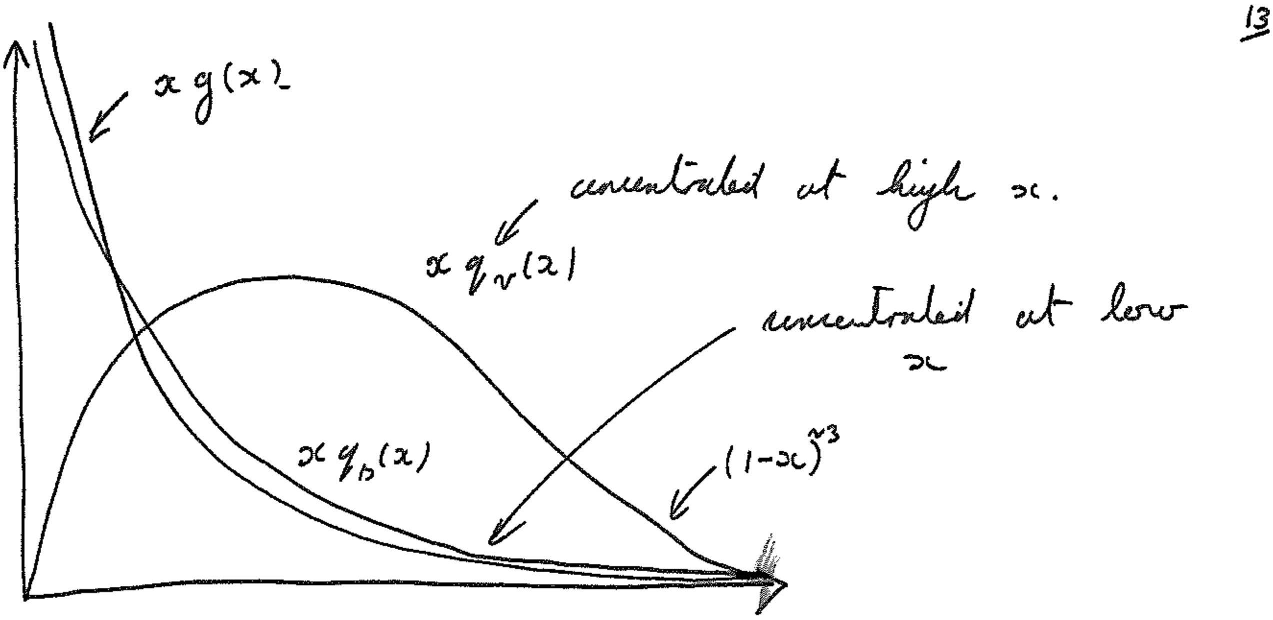
For singlet dist$^{=}$, we have anomalous dimension matrix

$D[n]$

mixing gluon and (sea) quark dist$^{=}$

For $n = 2$; zero eigenvalue

$\Rightarrow \sum_i \int dx\, x\, f_{i/p}(x) = 1$; total momentum carried by all constituents is const $\equiv 1$



Measure str. f= at typical $Q^2 \sim 5\ \mathrm{GeV}^2$, for $x > 0.1, 0.2$ say.
Interested in studying processes at, say, SSC.

Energy $\sim$ 10 TeV
Produced particles have, say, $Q^2 \sim 100\ \mathrm{GeV}$.

$\Big\}$ $\nu \to \infty$, Q^2 fixed

Therefore very small x.

Primary interest is in structure f= at small x. Not in perturbative regime in current experiments.
Can invert eq= for moments; to yield Altarelli – Parisi eq= for str. f=

$$\frac{d\,q_v(x,t)}{dt} = \frac{\alpha(t)}{4\pi} \int_x^1 \frac{dy}{y}\, q_v(y,t)\, P^{(0)}_{NS}\left(\frac{x}{y}\right)$$

where $t = \frac{1}{2} \ln \frac{Q^2}{\mu^2}$

Crucial point is that evolution of q_v at x depends only on q_v at $y \geqslant x. \to$ str. f= evolve down in q^2. ballon etc.

~~Rela~~ Have not studied polarized structure functi

$$q(x) = q_{\uparrow}(x) - q_{\downarrow}(x)$$, according to whether the spin of the parton is ~~alle~~ aligned or anti-aligned with that of proton.

EMC effect

$$\sum_{i=q,\bar{q}} \int \cancel{q} \{ f_i^+(x) - f_i^-(x) \} \simeq \cancel{\&} \; 0$$

N.B.

$$\cancel{\sum} \int \{ f_v^+(x) - f_v^-(x) \} \simeq 0.5$$

"All spin ~~inf~~ of proton is carried by quarks, gluons."

Graham Ross etc.

First moment of polarized quark dist = mixes with glue dist = through anomaly.