Hadron Structure

David Richards

Jefferson Laboratory

Hadron Structure

How are

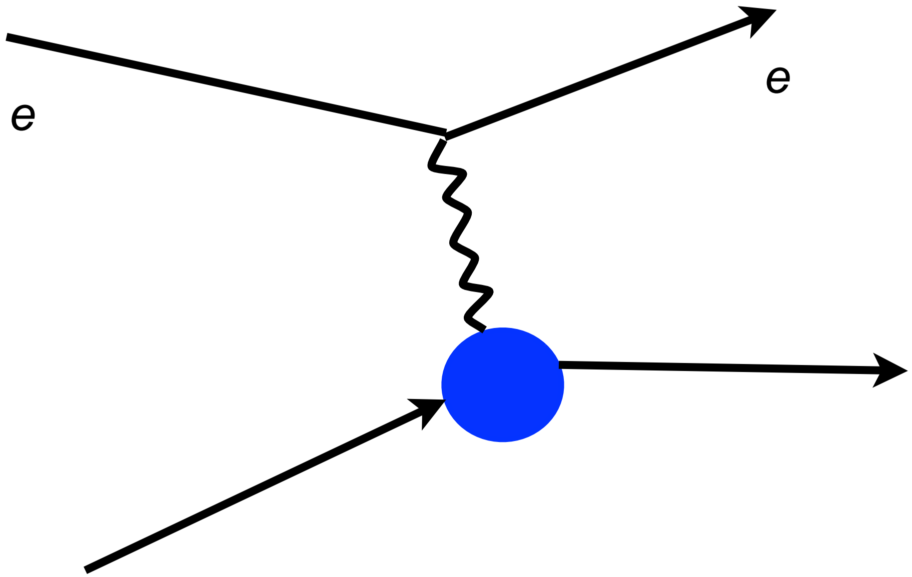
- charge and currents
- momentum
- spin and angular momentum

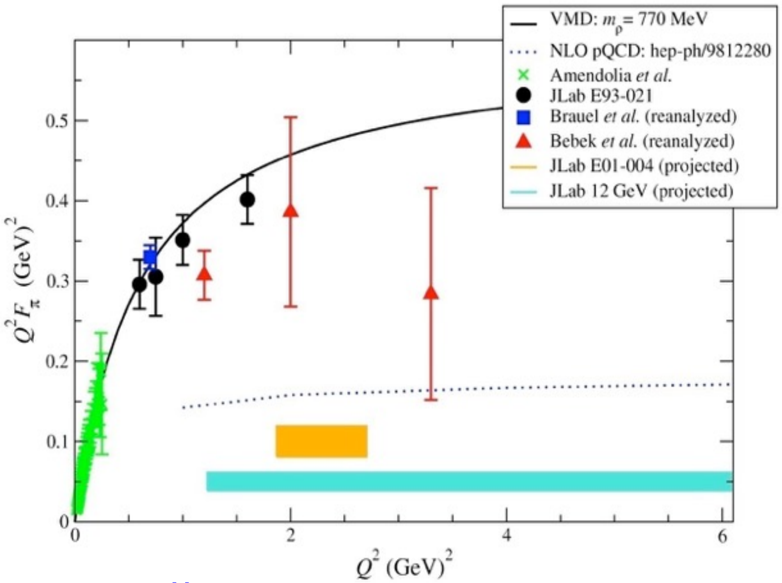
apportioned amongst the quarks and gluons that make up a hadron?

Encapsulated in

- electromagnetic form factors
- unpolarized structure functions and Transverse-momentum-dependent distributions (TMDs)
- polarized structure functions, Generalized Parton Distributions (GPDs), TMDs

Paradigm: Pion EM form factor





$$\langle \pi(\vec{p}_f) \mid V_\mu(0) \mid \pi(\vec{p}_i) \rangle = (p_i + p_f)_\mu F(Q^2)$$

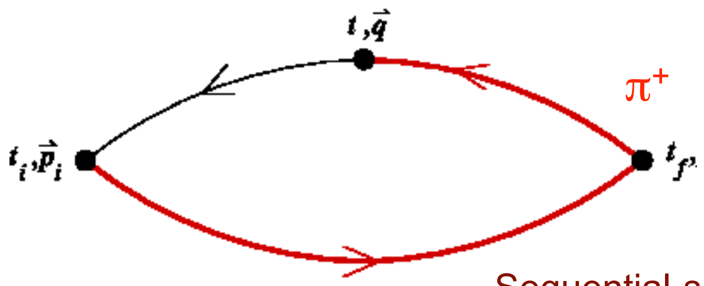
where

$$\begin{aligned} V_\mu &= \frac{2}{3}\bar{u}\gamma_\mu u - \frac{1}{3}\bar{d}\gamma_\mu d \\ -Q^2 &= [E_\pi(\vec{p}_f) - E_\pi(\vec{p}_i)]^2 - (\vec{p}_f - \vec{p}_i)^2 \end{aligned}$$

Anatomy of a Matrix Element Calculation - I

Pion Interpolating Operator

$$\begin{aligned} \phi(x) &= \bar{d}(x)\gamma_5 u(x) \\ \phi^\dagger(x) &= -\bar{u}(x)\gamma_5 d(x) \\ V_\mu(x) &= e_u \bar{u}(x)\gamma_\mu u(x) + e_d \bar{d}(x)\gamma_\mu d(x). \end{aligned}$$



$$\Gamma_{\pi^+\mu\pi^+}(t_f, t; \vec{p}, \vec{q}) = \sum_{\vec{x},\vec{y}} \langle 0|\phi(\vec{x}, t_f) V_\mu(\vec{y}, t)\phi^\dagger(\vec{0}, 0)|0\rangle e^{-i\vec{p}\cdot\vec{x}} e^{-i\vec{q}\cdot\vec{y}},$$

$$V_\mu^{\text{cont}} = Z_V V_\mu^{\text{lattice}}; Z_V = 1 \quad \text{for conserved current}$$

Anatomy of a Matrix Element Calculation - II

Construction of three-point function

Introduce quark propagators

$$U^{ij}_{\alpha\beta}(x,y) = \langle u^i_\alpha(x)\bar{u}^j_\beta(y)\rangle$$
$$D^{ij}_{\alpha\beta}(x,y) = \langle d^i_\alpha(x)\bar{d}^j_\beta(y)\rangle,$$

Then U-contribution to three-point function given by

$$\Gamma^U_{\pi^+\mu\pi^+} = e_u \sum_{\vec{x},\vec{y}} e^{-i\vec{p}\cdot\vec{x}-i\vec{q}\cdot\vec{y}} \mathrm{Tr}\left\{\gamma_5 U(x,y)\gamma_\mu U(y,0)\gamma_5 D(0,x)\right\}$$

Quark propagator: $G^{ij}_{\alpha\beta}(x,y) = \langle q^i_\alpha(x)\bar{q}^j_\beta(y)\rangle$ satisfies

$$M^{ik}_{\alpha\gamma}(x,z)G^{kj}_{\gamma\beta}(z,y) = \delta_{ij}\delta_{\alpha\beta}\delta_{xy}; \quad G(y,x) = \gamma_5 G(x,y)^\dagger\gamma_5$$

Introduce Sequential Quark Propagator $H^u(y,0;t_f,\vec{p}) = \sum_{\vec{x}} e^{i\vec{p}\cdot\vec{x}} U(y,x)\gamma_5 D(x,0)\gamma_5$

Satisfies: $M(z,y)H^u(y,0;t_f,\vec{p}) = \delta_{t_z,t_f} e^{i\vec{p}\cdot\vec{z}}\gamma_5 D(z,0)\gamma_5$

Finally: $\Gamma^U_{\pi^+\mu\pi^+} = e_u \sum_{\vec{y}} e^{-i\vec{q}\cdot\vec{y}} \mathrm{Tr}\left\{H^u(y,0;t_f,\vec{p})^\dagger \gamma_5\gamma_\mu U(y,0)\gamma_5\right\}$

Anatomy of a Matrix Element Calculation - II

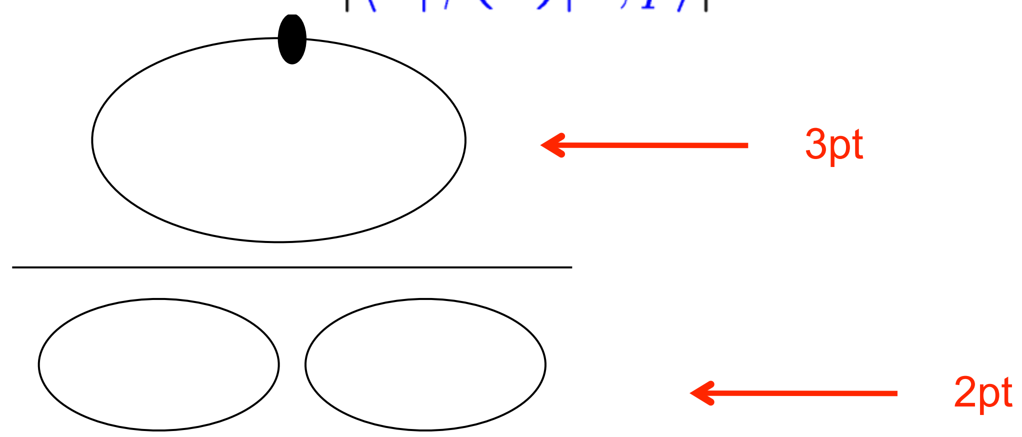
$$\Gamma_{\pi^+\mu\pi^+}(t_f,t;\vec{p},\vec{q}) = \sum_{\vec{x},\vec{y}} \langle 0|\phi(\vec{x},t_f)V_\mu(\vec{y},t)\phi^\dagger(\vec{0},0)|0\rangle e^{-i\vec{p}\cdot\vec{x}} e^{-i\vec{q}\cdot\vec{y}},$$

Resolution of unity – insert states

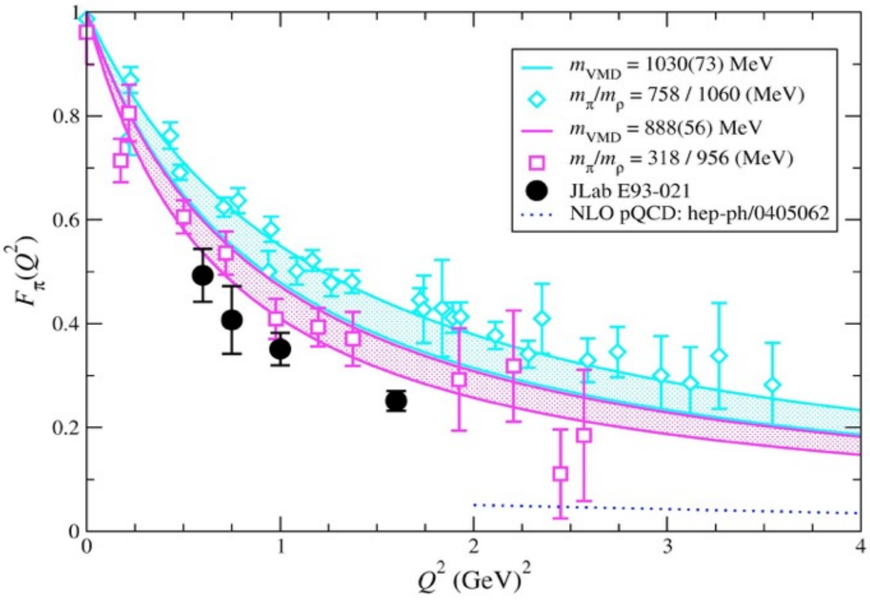
$$\langle 0 \mid \phi(0) \mid \pi, \vec{p}+\vec{q}\rangle\langle \pi, \vec{p}+\vec{q} \mid V_\mu(0) \mid \pi, \vec{p}\rangle\langle \pi, \vec{p} \mid \phi^\dagger \mid 0\rangle e^{-E(\vec{p}(t-t_i)} e^{-E(\vec{p}+\vec{q})(t_f-t)}$$

$$\Gamma_{\pi^+\pi^+}(t,0;\vec{p}) = \sum_{\vec{x}} \langle 0 \mid \phi(\vec{x},t_f)\phi^\dagger(0) \mid 0\rangle e^{-i\vec{p}\cdot\vec{x}}$$

$$\propto |\langle 0|\phi(0)|\pi,\vec{p}\rangle|^2 e^{-E(\vec{p})t}$$



Pion Form Factor

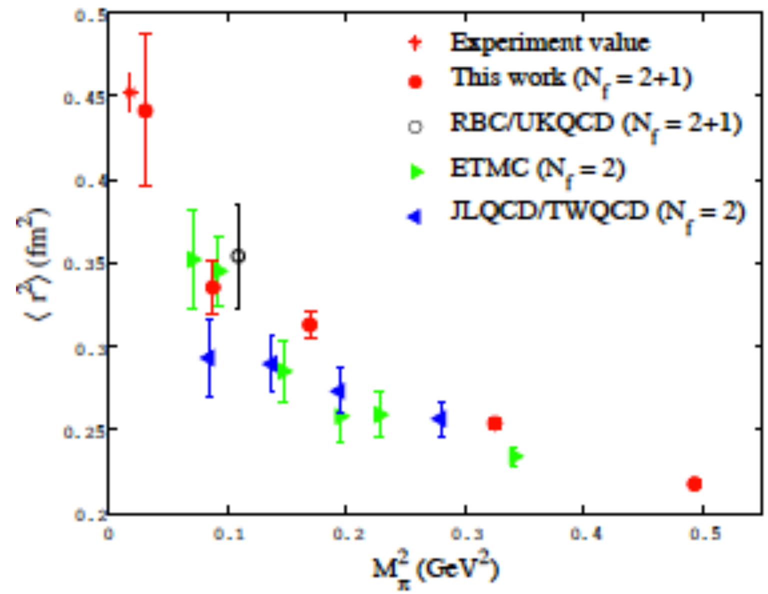


LHPC, Bonnet et al,
Phys.Rev. D72 (2005) 054506

$$F(Q^2) = \frac{1}{1 + Q^2/{M_{\mathrm{VMD}}}^2}$$

Charge radius Nguyen et al, 1102.3652

$$\langle r^2 \rangle = 6 \left. \frac{dF(q^2)}{dq^2} \right|_{q^2=0}$$



Nucleon EM Form Factors

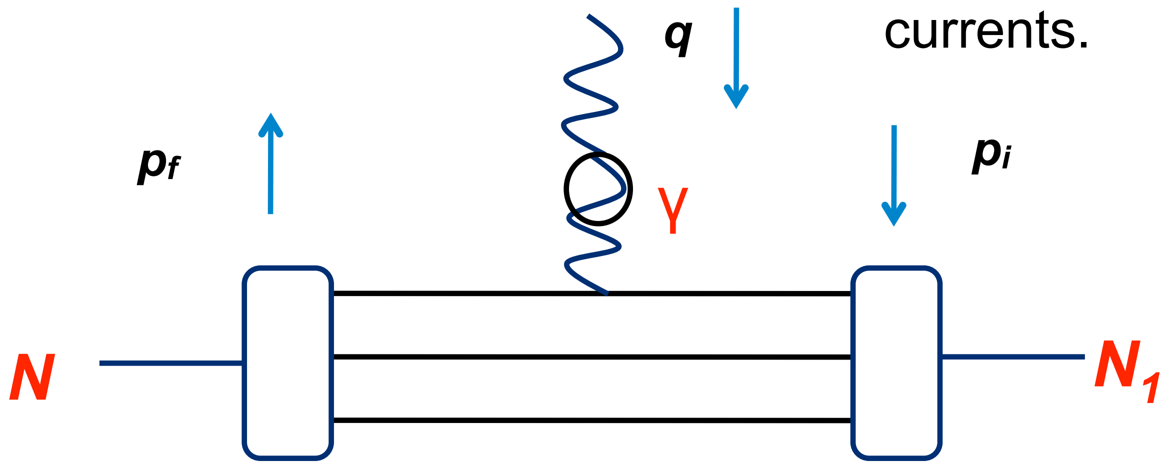
Two form factors

$$\langle p_f \mid V_\mu \mid p_i \rangle = \bar{u}(p_f)\left[\gamma_\mu \underbrace{F_1(q^2)}_{\text{Dirac}} + i q_\nu \frac{\sigma_{\mu\nu}}{2m_N} \underbrace{F_2(q^2)}_{\text{Pauli}}\right] u(p_i)$$

Related to familiar Sach's electromagnetic form factors through

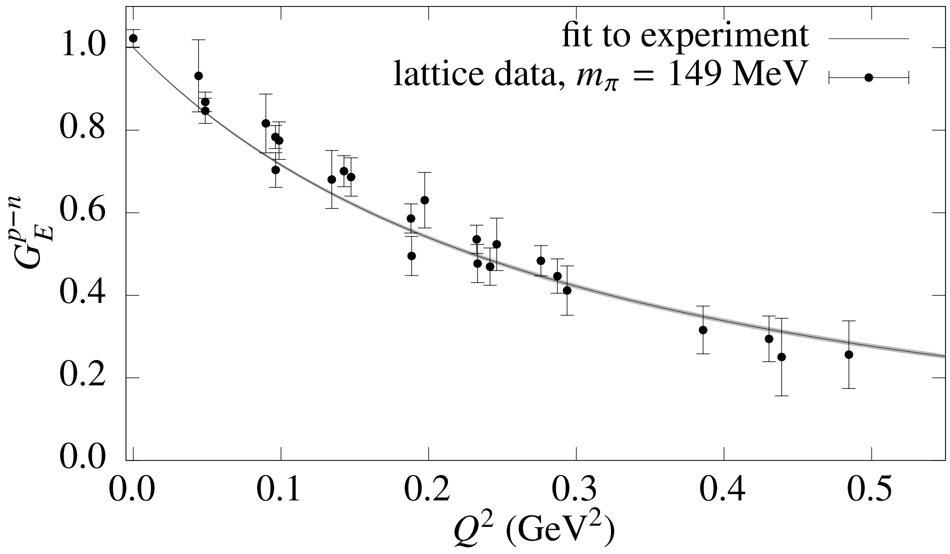
$$\begin{aligned} G_E(Q^2) &= F_1(Q^2) - \frac{Q^2}{(2m_N)^2} F_2(Q^2) \\ G_M(Q^2) &= F_1(Q^2) + F_2(Q^2) \end{aligned}$$

Isovector: difference between p and n or difference between u and d currents.



Electromagnetic Form Factors

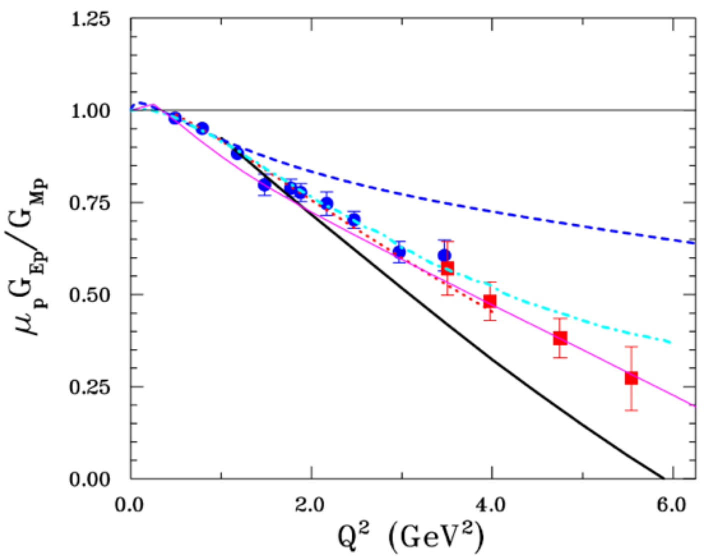
Wilson-clover lattices from BMW



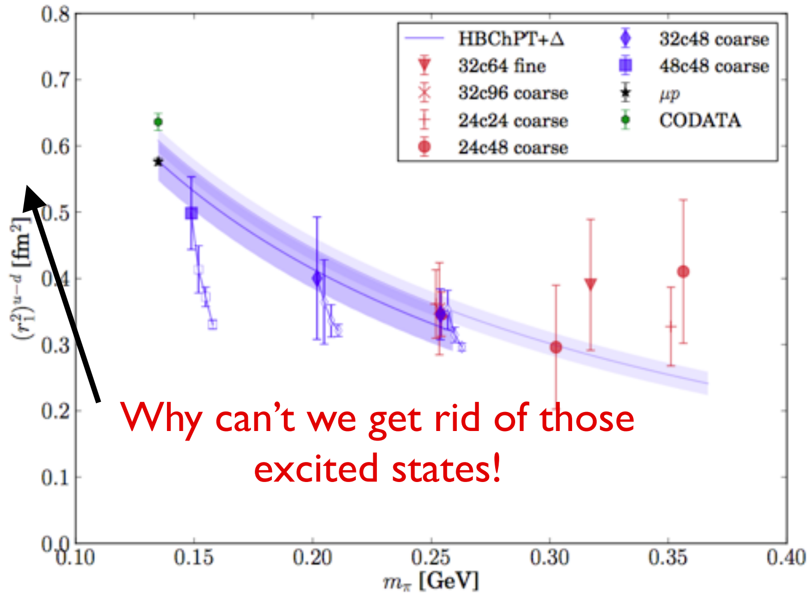
Green et al (LHPC), Phys. Rev. D 90, 074507 (2014)

Hadron structure at nearly-physical quark masses

Large Q² behavior: Hall C at JLab to 15 GeV²



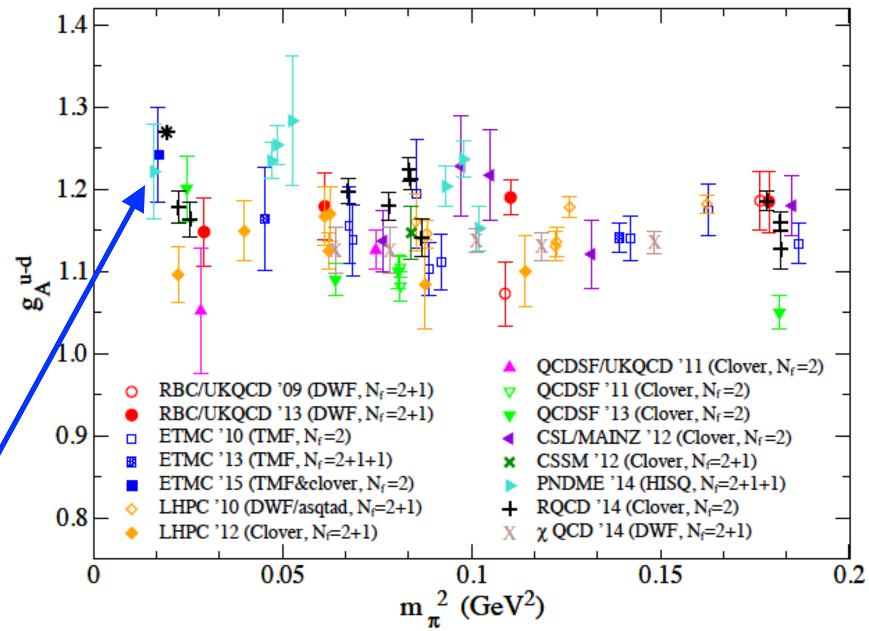
Hadron Structure



Green et al, arXiv:1404.40

M Constantinou, arXiv:1511.00214

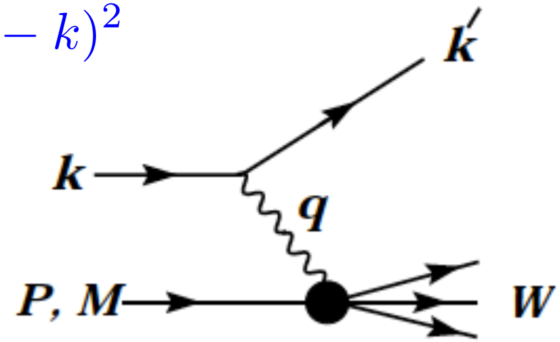
- Governs beta-decay rate
- Important for proton-proton fusion rate in solar models
- Benchmark for lattice QCD calculations of hadron structure



Luxury of large statistical errors! $m_\pi L < 4$

Structure Functions - I

$$Q^2 = -q^2 = (k' - k)^2$$
$$\nu = q \cdot P/M$$
$$x = \frac{Q^2}{2M\nu}$$



Bjorken limit:

$Q^2 \longrightarrow \infty, \nu \longrightarrow \infty, x$ fixed

The structure functions are defined in terms of the hadronic tensor:

$$W_{\mu\nu} = \frac{1}{4\pi} \int dz e^{iq\cdot z} \langle N(p,S) \mid J_\mu(z) J_\mu(0) \mid N(p,S) \rangle$$

Yields two unpolarized structure functions $F_1(x,Q^2)$ and $F_2(x,Q^2)$, and two polarized structure functions $g_1(x,Q^2)$ and $g_2(x,Q^2)$

Leading twist structure functions: product of currents at light-like $z^2 \rightarrow 0$

In Euclidean lattice QCD, use OPE to write in terms of local operators whose matrix elements we can compute in Euclidean space

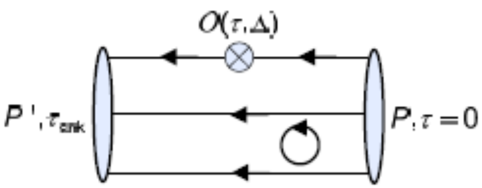
Structure Functions - II

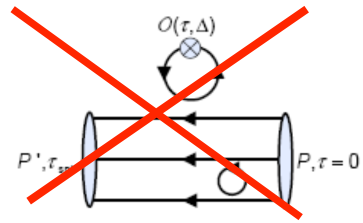
Operators

polarized

Capitani, this school

$$O_q^{\{\mu_1\mu_2\ldots\mu_n\}} = \bar{\psi}_q\gamma_5\gamma^{\{\mu_1}iD^{\mu_2}\ldots D^{\mu_n\}}\psi_q$$





Matrix elements related to moments of structure functions

Wilson coeffs

Operator renormalization

$$\int_0^1 dx\, x^{n-1} F_2(x,Q^2) = \sum_{q=u,d} C_n(\mu^2/Q^2), g(\mu))\langle x^n\rangle(\mu)$$

where

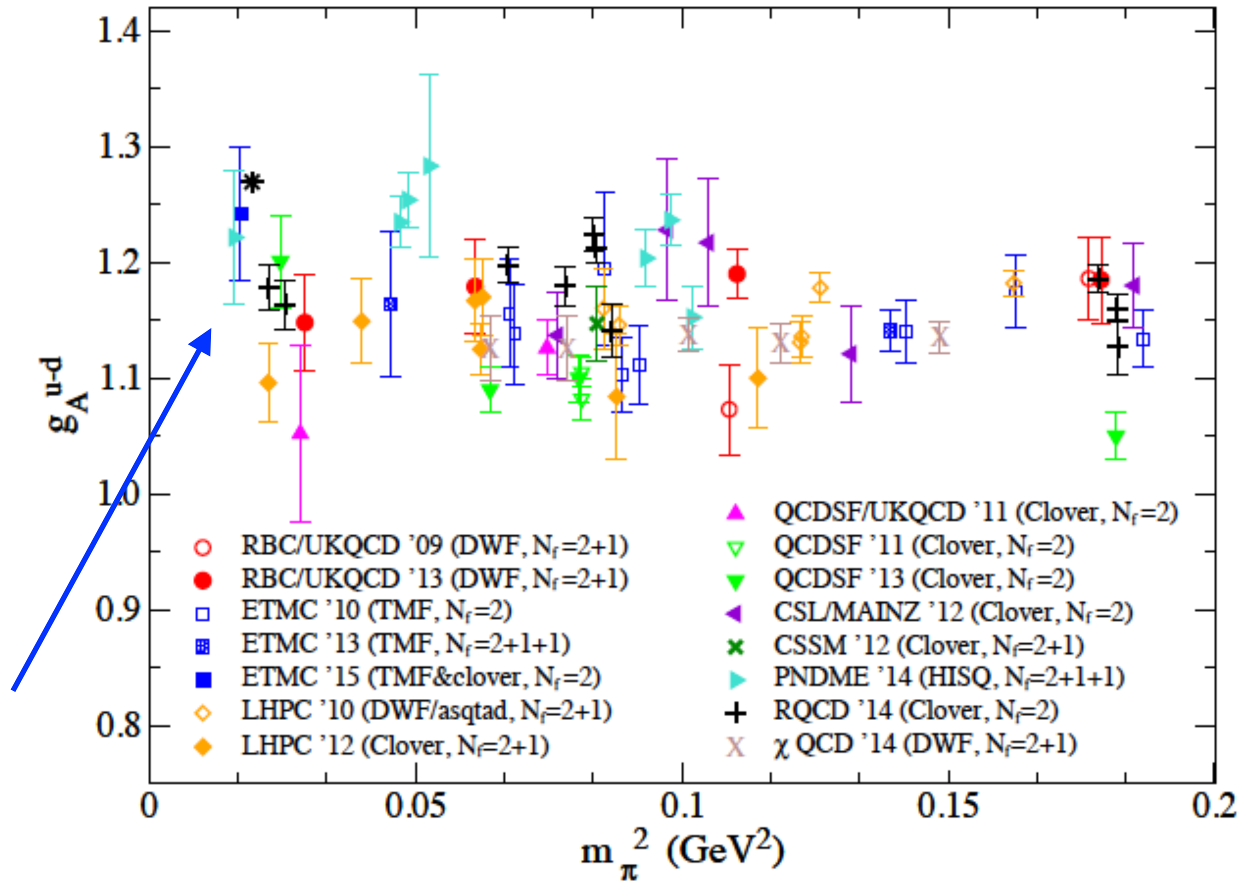
$$\langle N(p) \mid O_q^{\mu_1\ldots\mu_{n+1}} \mid N(p)\rangle = \langle x^n\rangle(\mu)[p_{\mu_1}\ldots p_{\mu_{n+1}}]$$

$$\mathcal{O}^{\rm cont} = Z\mathcal{O}^{\rm latt}$$

Perturbation theory

Non-perturbatively

Axial-vector Charge



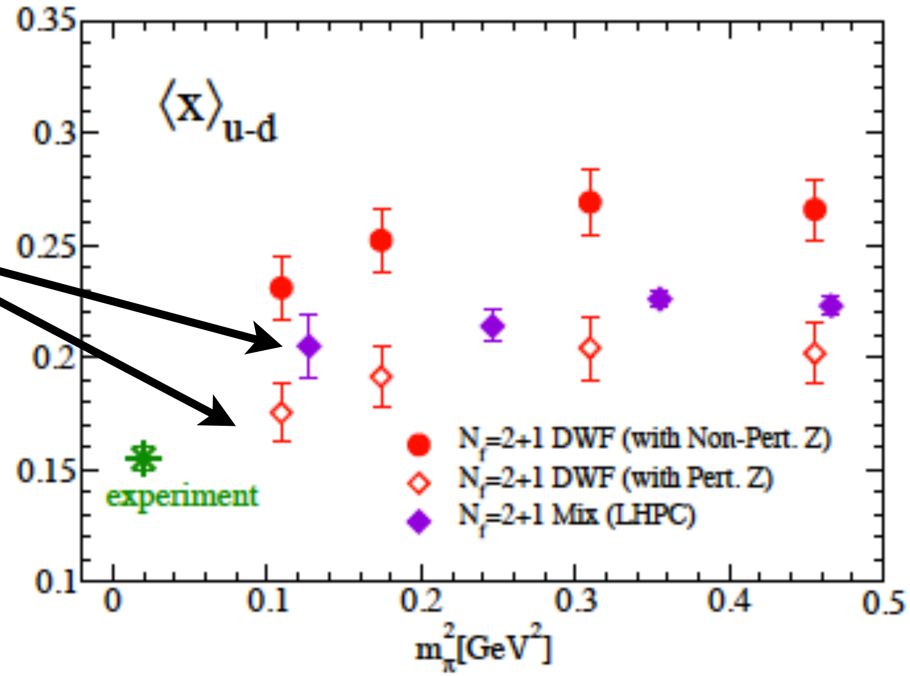
Luxury of large statistical errors! ***$m_\pi L < 4$***

M Constantinou, arXiv:1511.00214

Quark Momentum Fraction

RBC/UKQCD 2010: DWF

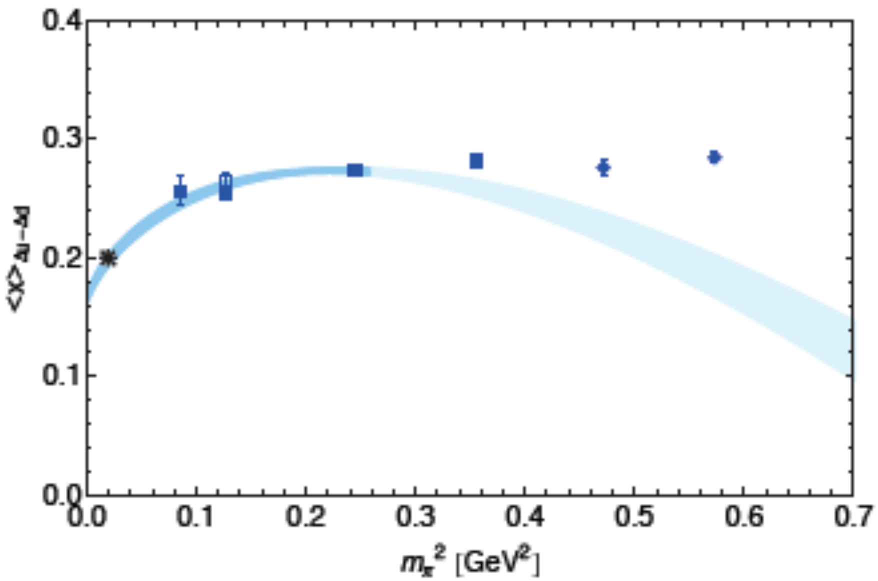
Similar renormalization prescription



- Need to go to approach physical light-quark masses: chiral behavior

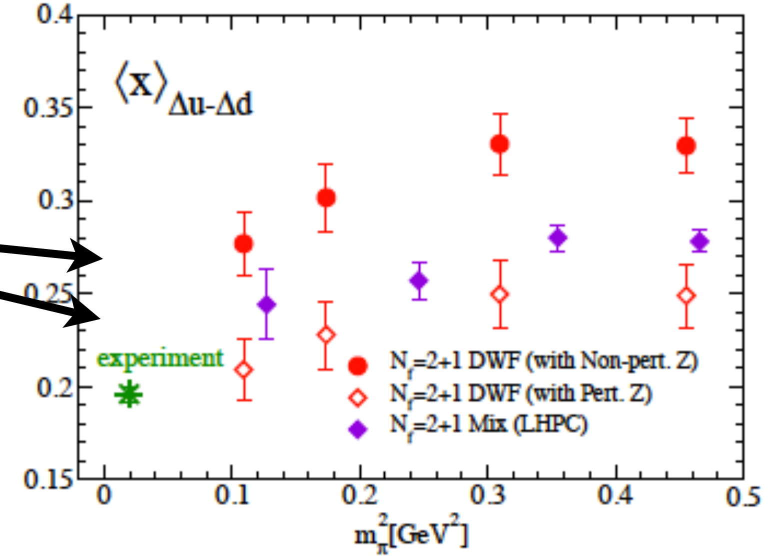
Quark Momentum Helicities

LHPC, 2010: DWF valence, Asqtad sea



HBChPT

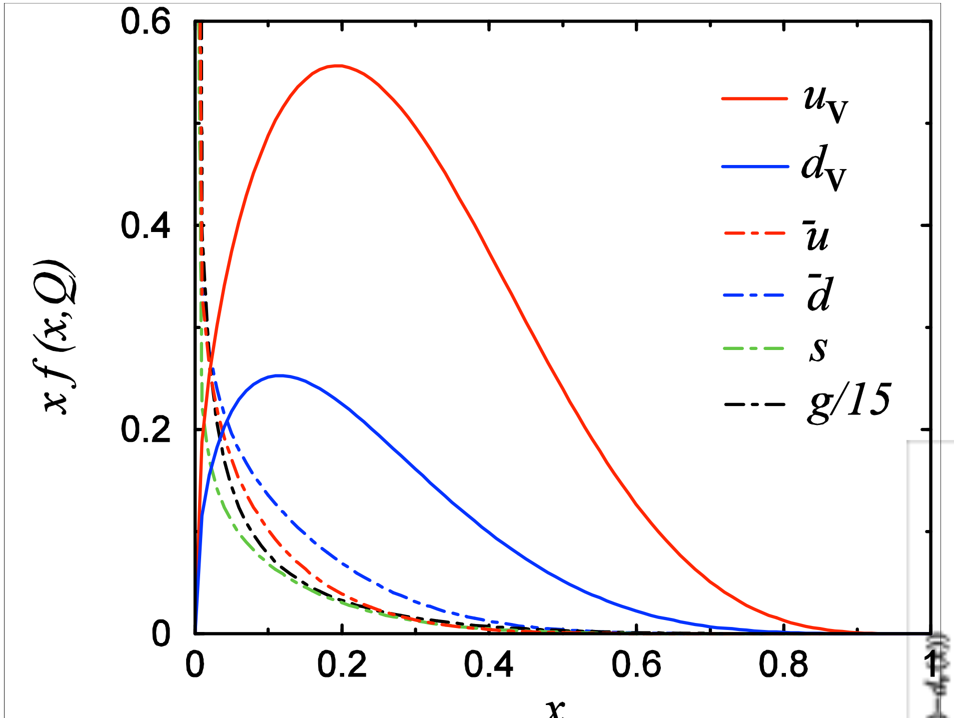
RBC/UKQCD 2010: DWF



Similar renormalization prescription

- Need to go to approach physical light-quark masses: chiral behavior

Moments of Parton Distributions

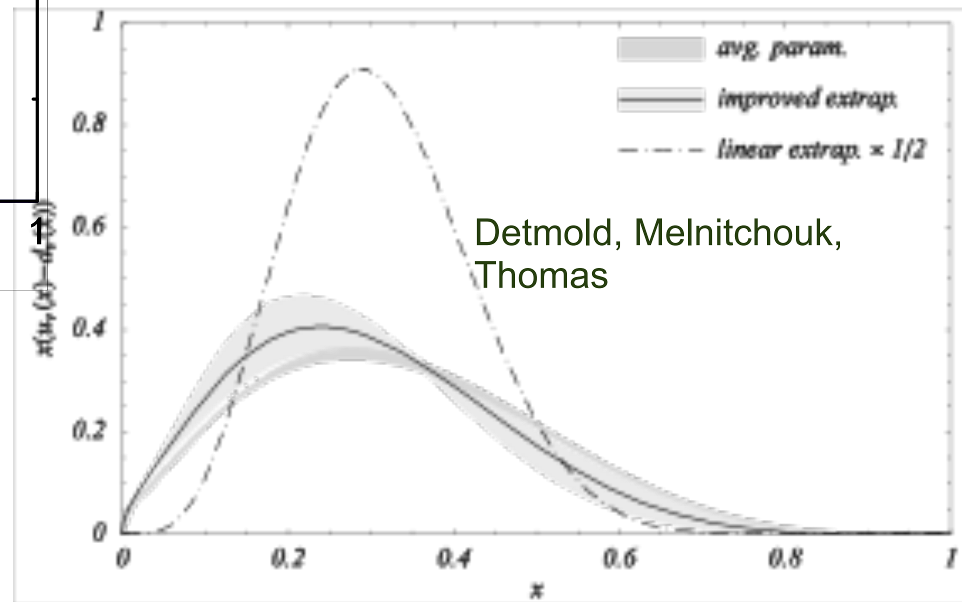


We are computing moments

$$O_q^{\{\mu_1\mu_2\ldots\mu_n\}} = \bar{\psi}_q\gamma_5\gamma^{\{\mu_1}iD^{\mu_2}\ldots D^{\mu_n\}}\psi_q$$

Do not have full Lorentz symmetry

n >= 5: operator mixing



Need to assume parametrization

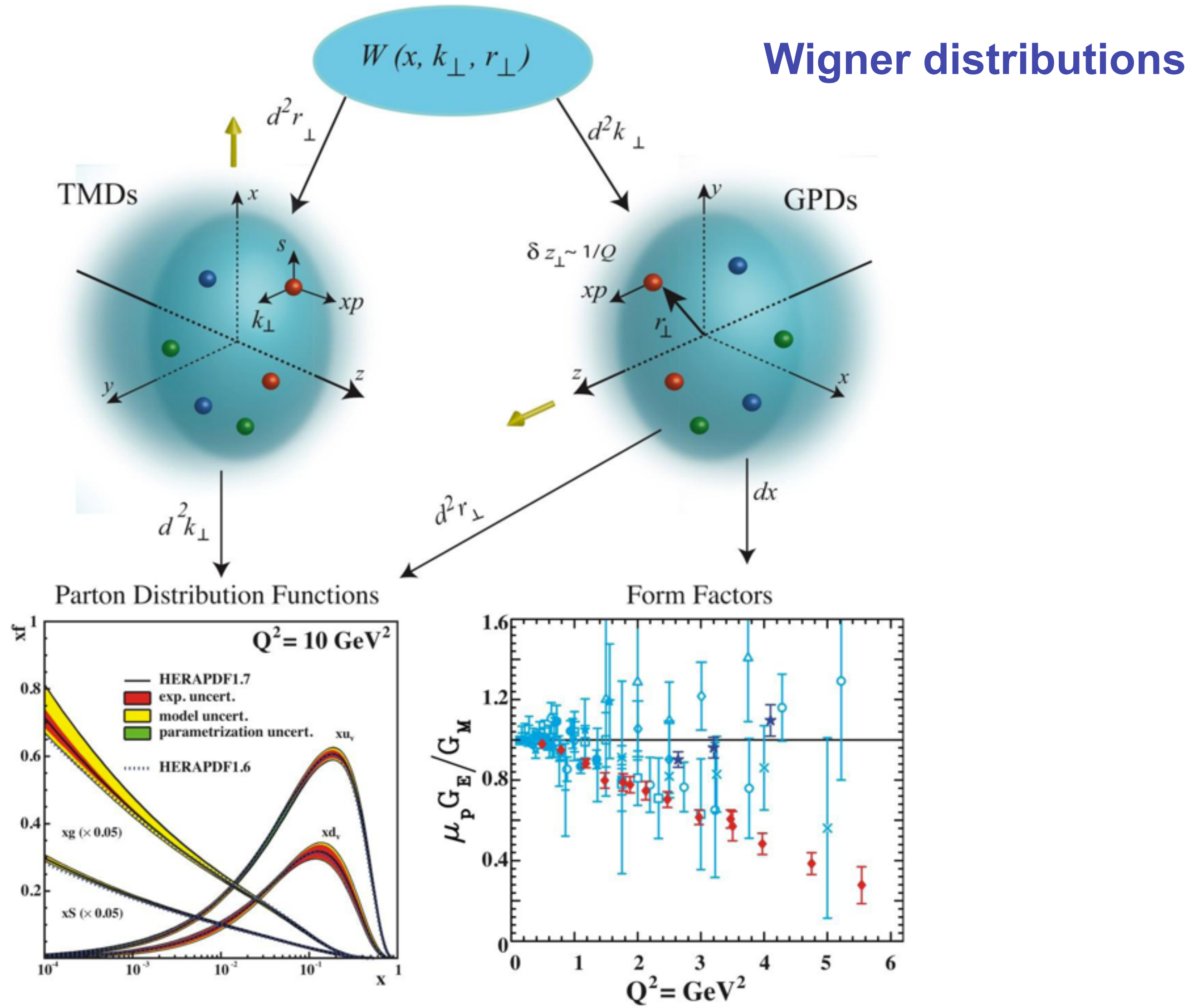
$$x(u_v(x) - d_v(x)) = a\,x^b(1-x)^c(1+\varepsilon\sqrt{x}+\gamma\ x)$$

3D Imaging of Nucleon

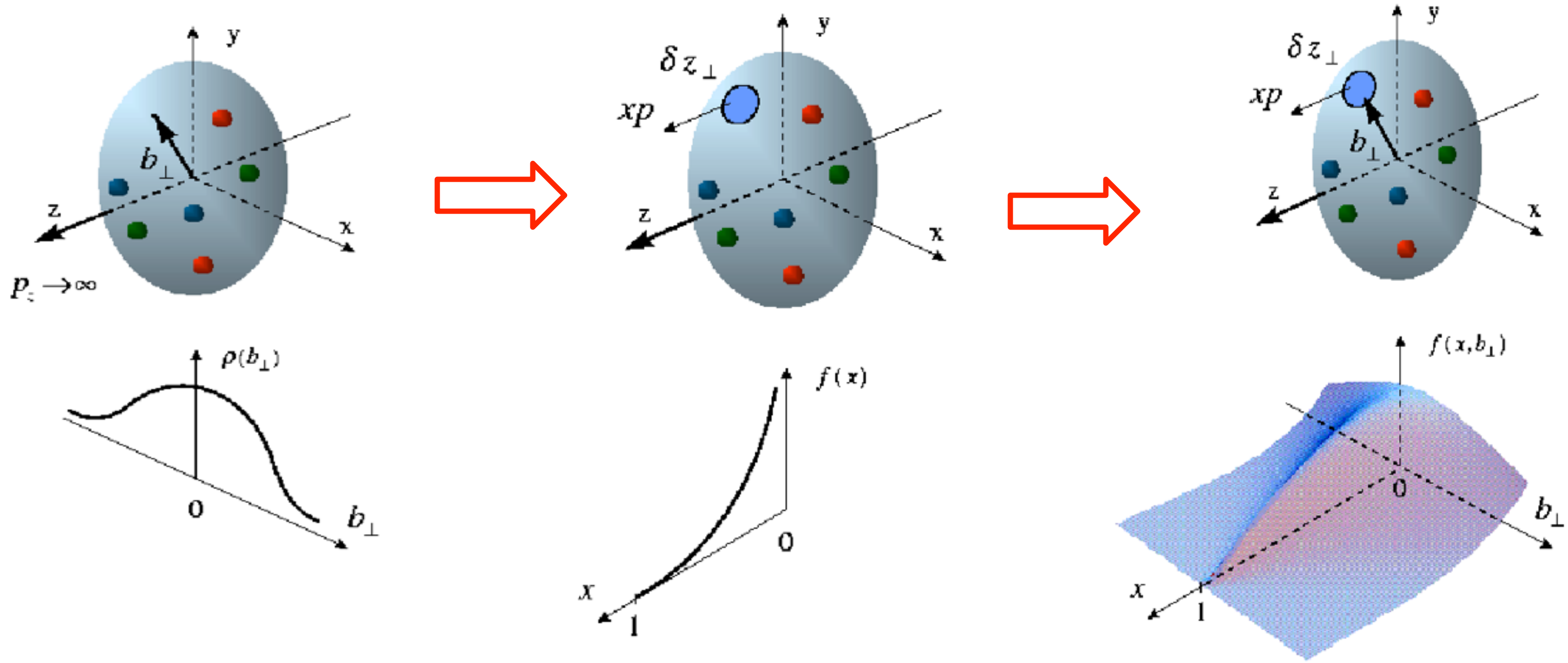
5D

3D

1D

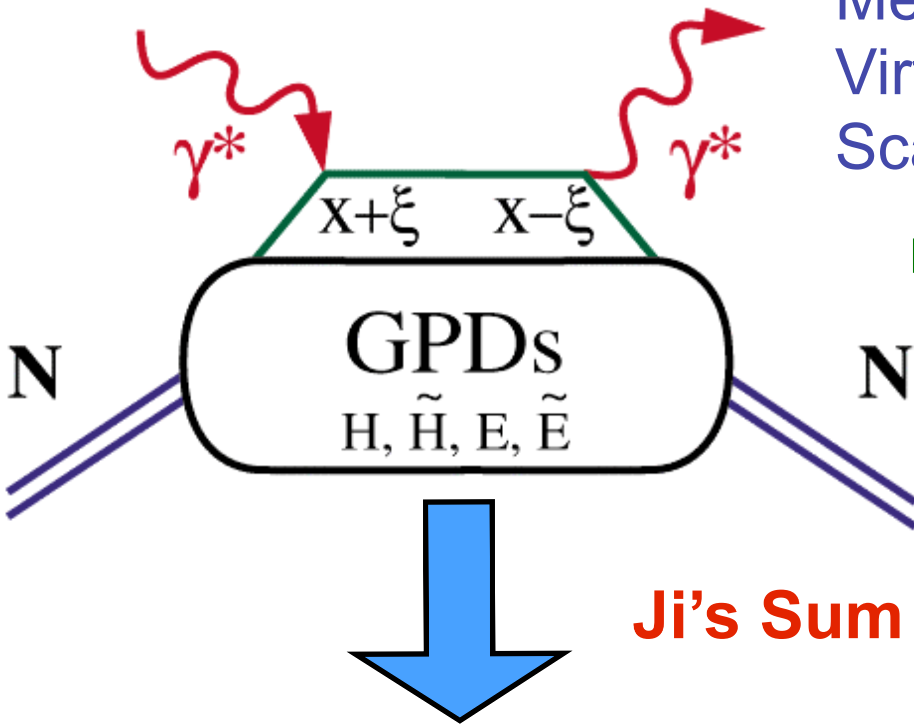


Different Regimes in Different Experiments



Form Factors transverse quark distribution in Coordinate space	**Structure Functions** longitudinal quark distribution in momentum space	**GPDs** Fully-correlated quark distribution in both coordinate and momentum space

Generalized Parton Distributions (GPDs)



Measured in Deeply Virtual Compton Scattering

D. Muller *et al* (1994), X. Ji & A. Radyushkin (1996)

Ji's Sum rule

$$J^q = \frac{1}{2} - J^G = \frac{1}{2}\int_{-1}^{1} x\,dx\,[H^q(x,\xi,0) + E^q(x,\xi,0)]$$

ξ is *skewness*

Moments of GPD's

- **Matrix elements of light-cone correlation functions**

$$\mathcal{O}(x) = \int \frac{d\lambda}{4\pi} e^{i\lambda x} \bar{\psi}\left(-\frac{\lambda}{2}n\right) n P e^{-ig\int_{\lambda/2}^{\lambda/2} d\alpha\, n\cdot A(\alpha n)} \psi\left(\frac{\lambda}{2}n\right)$$

- **Expand *O(x)* around light-cone**

$$O_q^{\{\mu_1\mu_2\cdots\mu_n\}} = \bar{\psi}_q \gamma^{\{\mu_1} iD^{\mu_2} \ldots D^{\mu_n\}} \psi_q$$

- ***Off-forward* matrix element**

$$\langle P'|O_q^{\{\mu_1\cdots\mu_n\}}|P\rangle \simeq \int dx\, x^{n-1}[H(x,\xi,t), E(x,\xi,t)]$$

$$\longrightarrow A_{ni}(t), B_{ni}(t), C_n(t), \tilde{A}_{ni}(t), \tilde{B}_{ni}(t), \tilde{C}_n(t)$$

↑

Co-efficient of ξ^i

LHPC, QCDSF, 2003

GPDs and Orbital Angular Momentum

- **Form factors of energy momentum tensor - *quark and gluon angular momentum***

"$\bar{q}\gamma_\mu D_\nu q$"

$$\frac{1}{2} = \sum_q J^q + J^g$$

X.D. Ji, PRL 78, 610 (1997)

$$= \frac{1}{2}\left\{\sum_q \left(A^q_{20}(t=0) + B^q_{20}(t=0)\right) + A^g_{20}(t=0) + B^g_{20}(t=0)\right\}$$

$$\sum_q \left(\frac{1}{2}\Delta\Sigma^q + L^q\right)$$

gluon operators - see later

Decomposition

- **Gauge-invariant**
- **Renormalization-scale dependent**
- **Handle on Quark orbital angular momentum**

Mathur et al., *Phys.Rev. D62 (2000) 114504*

Origin of Nucleon Spin

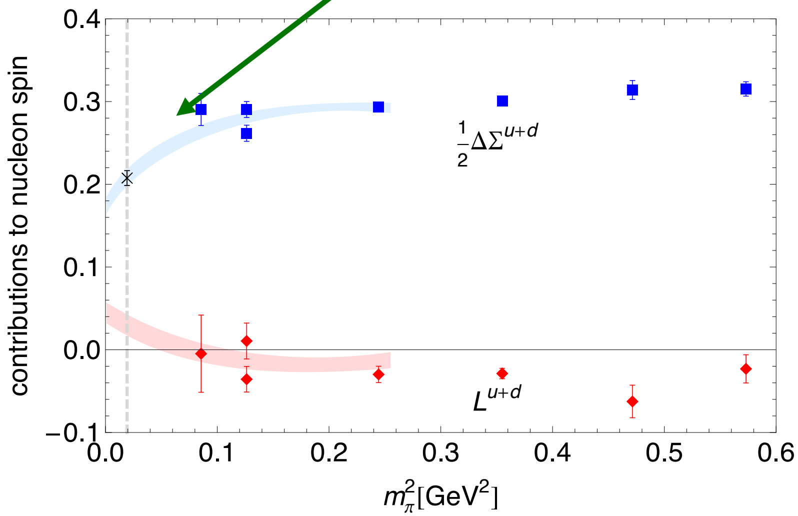
$$\begin{aligned} J^q &= 1/2\left(A^q_{20}(t=0)+B^q_{20}(t-0)\right) \\ \Delta\Sigma^q/2 &= \tilde{A}^q_{10}(t=0)/2 \\ \frac{1}{2} &= \frac{1}{2}\Delta\Sigma^{u+d}+L^{u+d}+J^g \end{aligned}$$

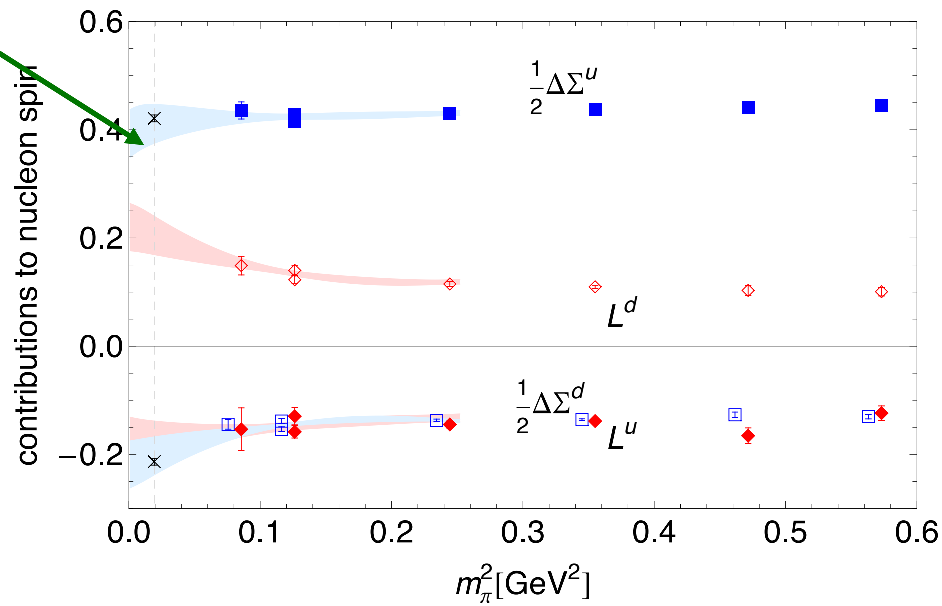
LHPC, Haegler et al., Phys. Rev. D 77, 094502 (2008); arXiv.1001.3620

Total orbital angular momentum carried by quarks small

Orbital angular momentum carried by quark flavors substantial

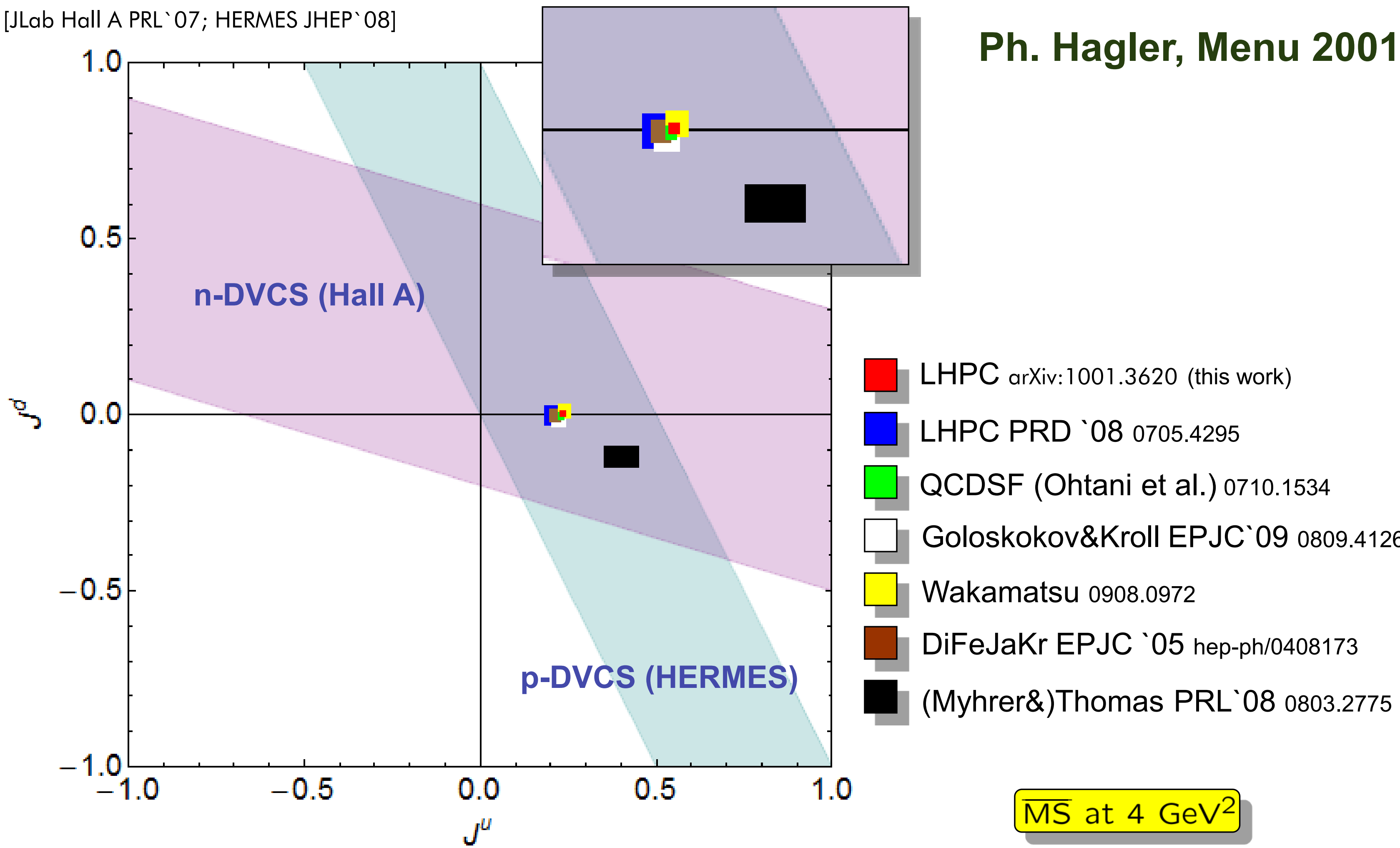
HERMES, PRD75 (2007)



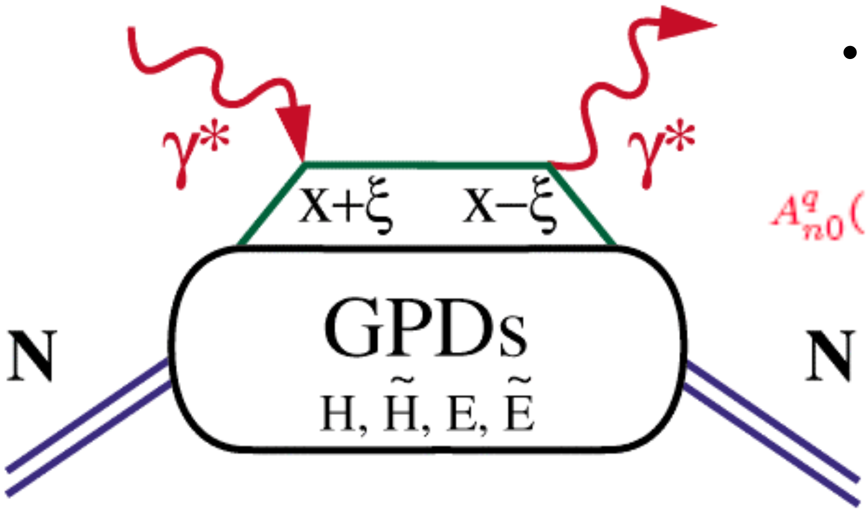


Origin of Nucleon Spin - II

Ph. Hagler, Menu 20010

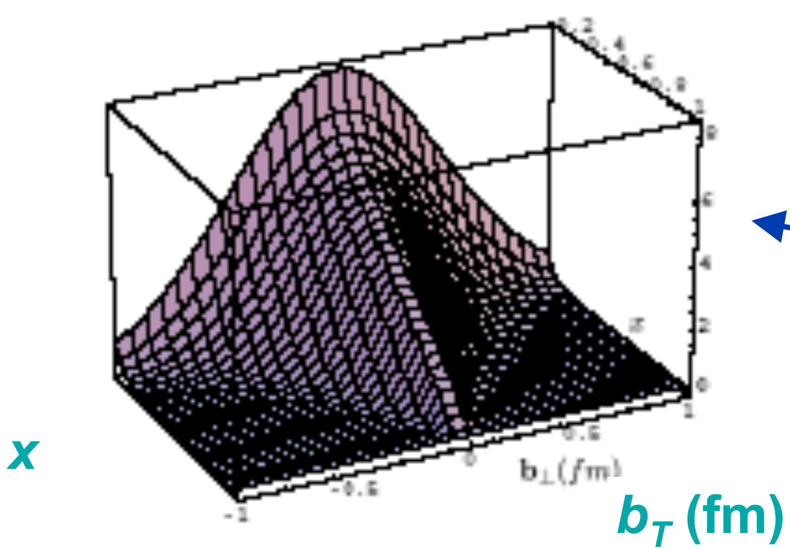


Transverse Distribution - I



- t-dependence ↔ impact parameter

$$A^q_{n0}(-\vec{\Delta}^2_\perp) = \int d^2b_\perp \, e^{i\vec{\Delta}_\perp\cdot\vec{b}_\perp} \int_{-1}^{1} dx \, x^{n-1} q(x,\vec{b}_\perp)$$



Compare to phenomenological models

Decrease slope : decreasing transverse size as x ! 1

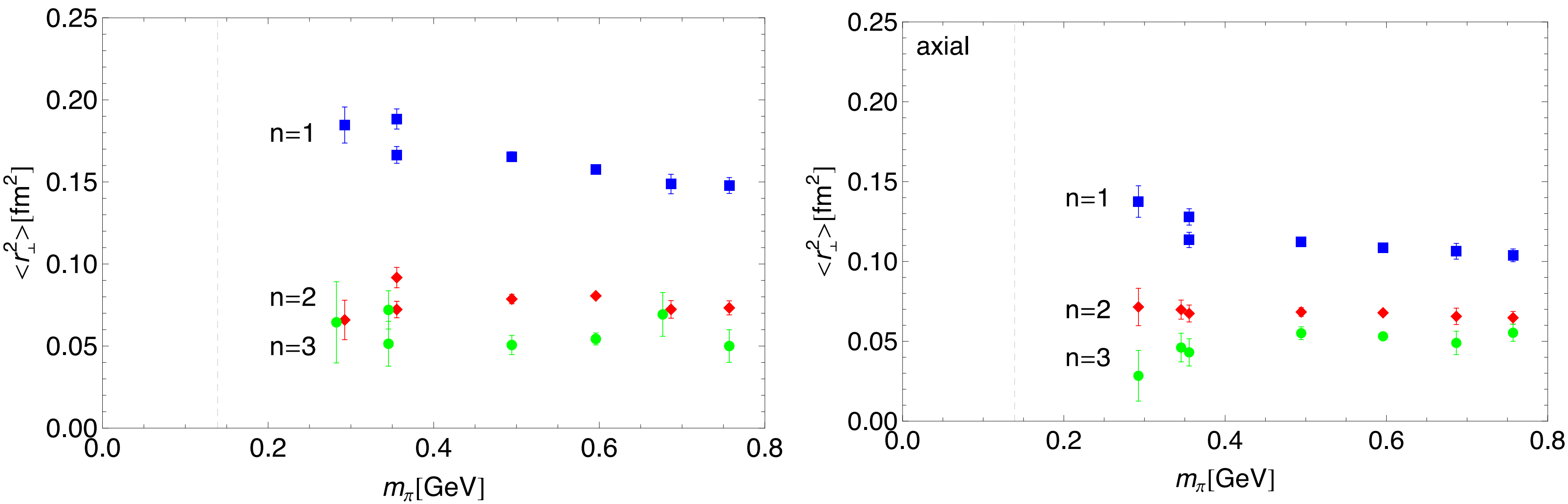
Burkardt

Transverse Distribution - II

Lattice results consistent with narrowing of transverse size with increasing x

LHPC, Haegler et al., Phys. Rev. D 77, 094502 (2008)

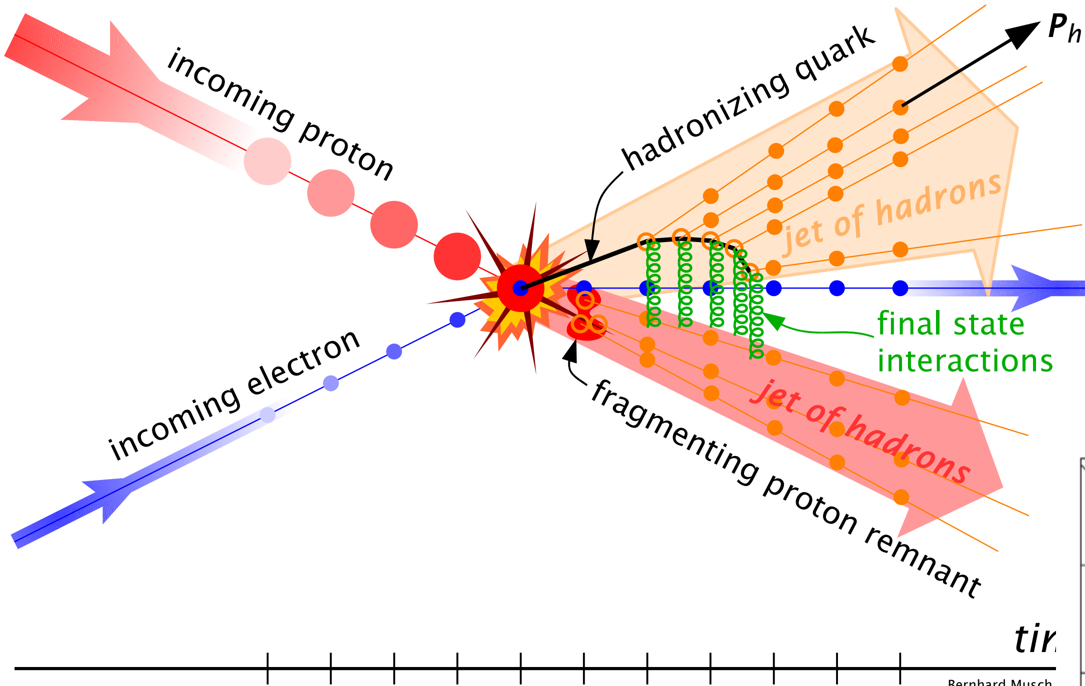
Flattening of GFFs with increasing n



Transverse momentum distributions (TMDs)

from experiment, e.g., SIDIS (semi-inclusive deep inelastic scattering)

HERMES, COMPASS, JLab 6 GeV, JLab 12 GeV , … , EIC



Cf: measured in Drell-Yan, eg at RHIC-spin

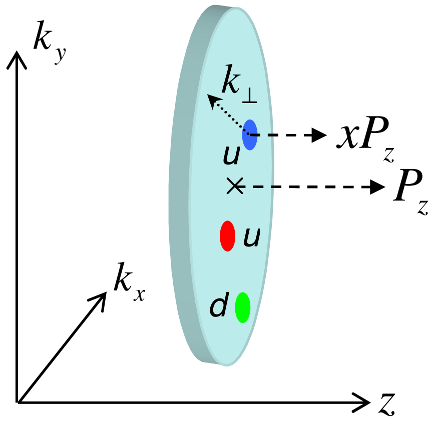
N \ q	U	L	T
U	f_1		$h_1^\perp$
L		g_1	$h_{1L}^\perp$
T	$f_{1T}^\perp$	g_{1T}	h_1 $h_{1T}^\perp$

Boer-Mulders

Sivers ← time-reversal odd

final state interactions!
explain large asymmetries otherwise forbidden!
signature of QCD!

Transverse-Momentum Distributions



B. Musch, PhD Thesis; Haegler, Musch, Negele, Schafer arXiv: 0908.1283

Introduce Momentum-space correlators

$$\Phi_\Gamma = \int d(n\cdot k)\int \frac{d^4 l}{2(2\pi)^4} e^{-ik\cdot l}\tilde{\Phi}_\Gamma(l;P,S)$$

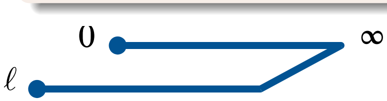
$$= \int d(n\cdot k)\int \frac{d^4 l}{2(2\pi)^4} e^{-ik\cdot l}\langle P,S \mid \bar{q}(l)\Gamma\mathcal{U}q(0) \mid P,S\rangle$$

continuum

$$\mathcal{U} \equiv \mathcal{P}\exp\left(-ig\int_0^\ell d\xi^\mu A_\mu(\xi)\right)$$

along path from 0 to ℓ

Choice of path - retain gauge invariance



Real world!: path runs to infinity

Lattice: equal time slice

Flavor-Singlet Hadron Structure

Flavor-singlet Quantities

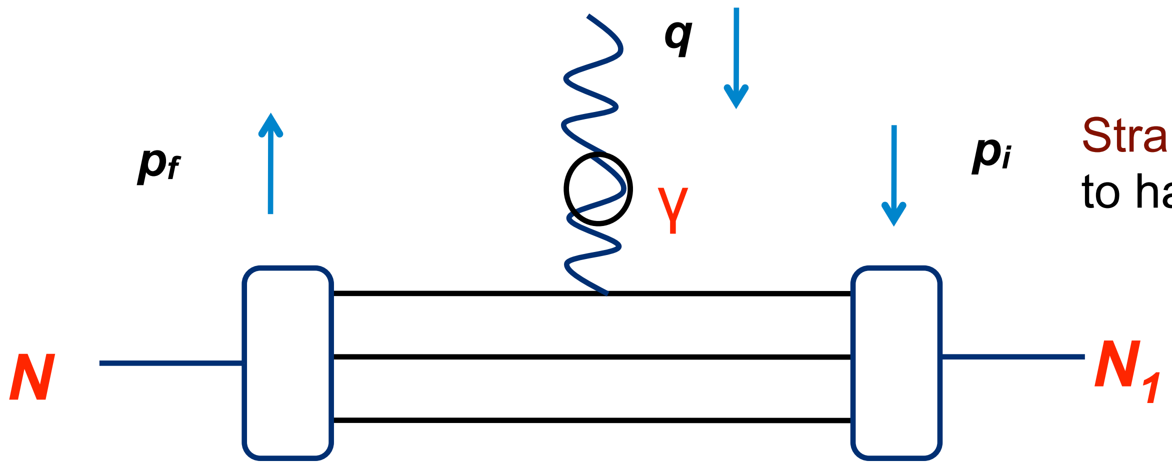
$$\langle p_f \mid V_\mu \mid p_i \rangle = \bar{u}(p_f) \left[\gamma_\mu F_1(q^2) + i q_\nu \frac{\sigma_{\mu\nu}}{2m_N} F_2(q^2) \right] u(p_i)$$

$$G_E(Q^2) = F_1(Q^2) - \frac{Q^2}{(2m_N)^2} F_2(Q^2)$$

$$G_M(Q^2) = F_1(Q^2) + F_2(Q^2)$$

Isoscalar: p and n separately, or u and d separated contribution.

$$V_\mu = \frac{2}{3}\bar{u}\gamma_\mu u - \frac{1}{3}\bar{d}\gamma_\mu d - \frac{1}{3}\bar{s}\gamma_\mu s$$



Strange-quark contribution to hadron structure

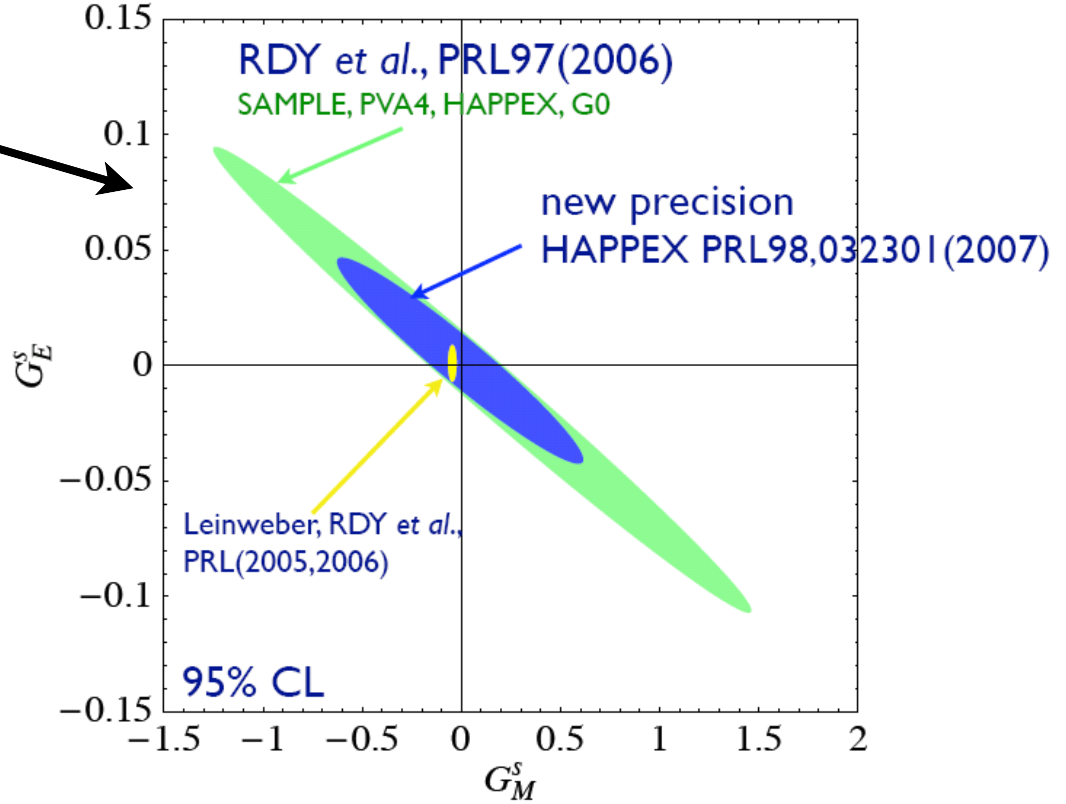
Flavor-singlet: Disconnected Contributions

Parity-violating electron scattering

$$G_{E/M}^{\gamma,p} = \frac{2}{3}G_{E,M}^{u} - \frac{1}{3}G_{E,M}^{d} - \frac{1}{3}G_{E,M}^{s}$$

$$G_{E/M}^{Z,p} = (1-\frac{8}{3}\sin^2\theta_W)G_{E,M}^{u} - (1-\frac{4}{3}\sin^2\theta_W)G_{E,M}^{d} - (1-\frac{4}{3}\sin^2\theta_W)G_{E,M}^{s}$$

Expected to be small



Spin carried by s-quark

$\Delta s = -0.085(13)(8)(9)$

HERMES: dominated by small x

Disconnected contributions

Three-point correlator looks like

$$\Gamma^{\rm disc}_{N\mu N}(t_f,t,0;\vec{p},\vec{q}) = \sum_{\vec{x},\vec{y}} \langle 0 \mid N(\vec{x},t_f)\bar{s}(\vec{y},t)\Gamma s(\vec{y},t)\bar{N}(\vec{0},0) \mid 0\rangle e^{-i\vec{p}\cdot\vec{x}} e^{-i\vec{q}\cdot\vec{y}}$$

$$= \sum_{\vec{x}} \langle 0 \mid N(\vec{x},t_f) \left(\sum_{\vec{y}} \bar{s}(\vec{y},t)\Gamma s(\vec{y},t) e^{-i\vec{q}\cdot\vec{y}} \right) \bar{N}(\vec{0},0) \mid 0\rangle e^{-i\vec{p}\cdot\vec{x}}$$

Need efficient means of evaluating $\sum_{\vec{y}} {\rm Tr}[M^{-1}(\vec{y},t;\vec{y},t)\Gamma]$

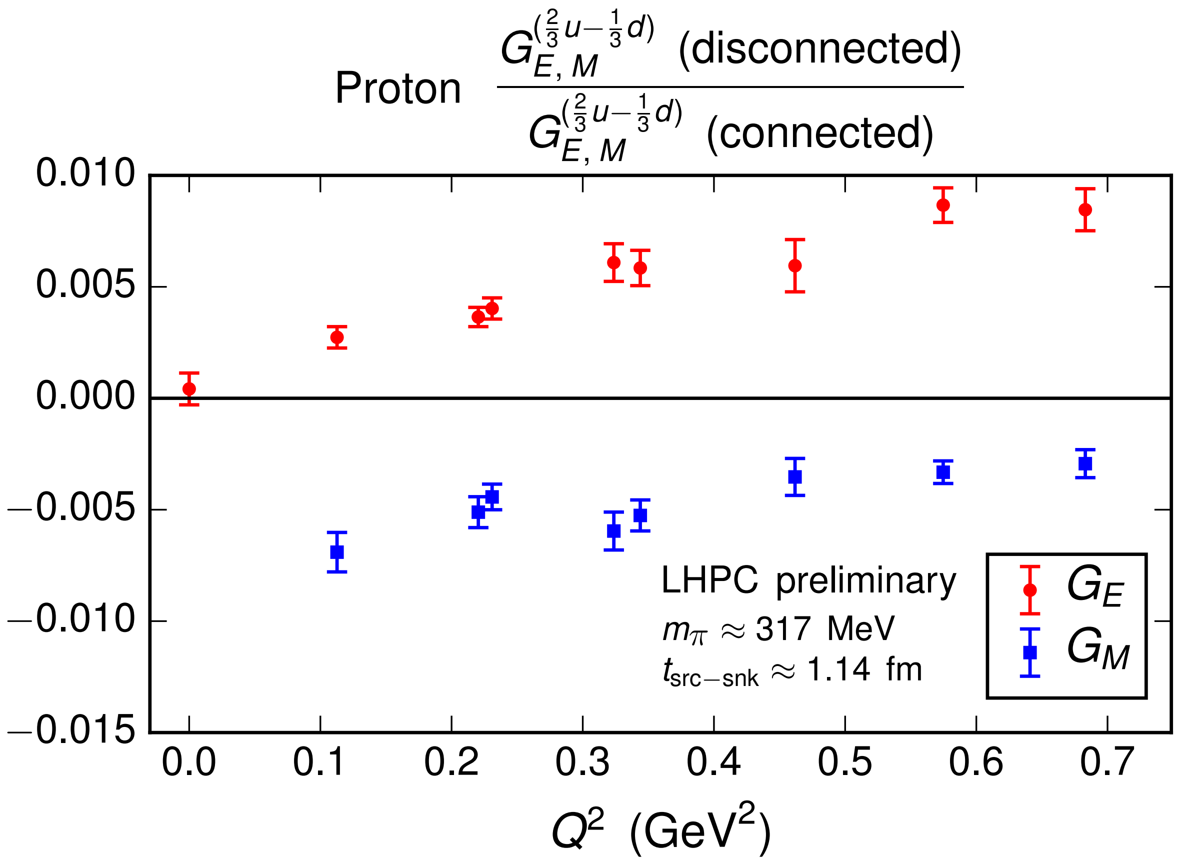
Straightforward way: introduce noise vectors such that $<\eta_i>=0; \quad <\eta_i\eta_j>=\delta_{ij}$

Solve $MX=\eta$: then $<M^{-1}_{ij}>=<\eta_j X_i>$

Error both from Gauge Noise and from Stochastic noise

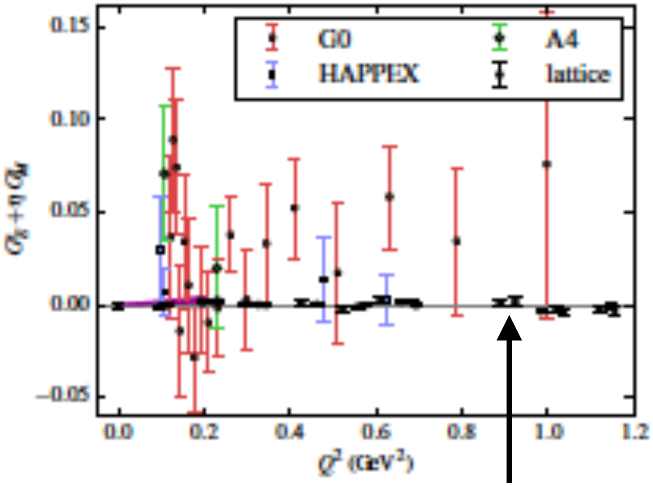
Noise-reduction methods
- Partitioning (“dilution”) - sources have support on, say, 8 timeslices
- Hopping parameter expansion
- Different stochastic sources

Flavor-separated Structure

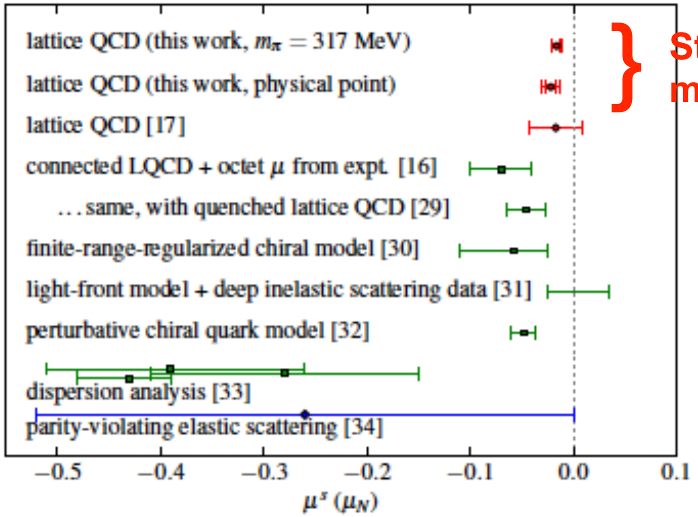


Isotropic Clover Gauge Generation for Hadron Structure at ORNL and at BlueWaters

Sea Quark Contributions

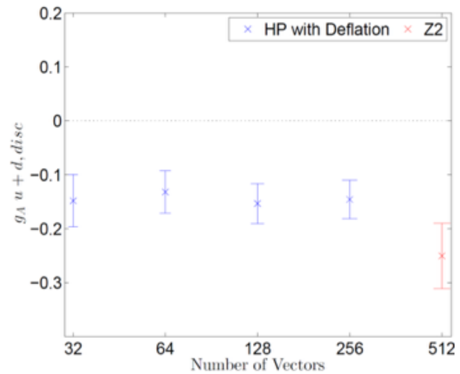


Combination *measured* in expt



} Strange magnetic moment of nucleon

J. Green, K. Orginos *et al.*, Phys. Rev. D 92, 031501 (2015)



Using *Hierarchical Probing* - A. Stathopoulos, J. Laeuchli, K. Orginos (2013)

A. Gambhir*, K. Orginos, A. Stathopoulos, arXiv:1603.05988. ****William and Mary student with SCGSR fellowship at JLab***

Synergy with computer scientists - precision calculation of sea quark contributions now possible

Mixing…

Quark and gluons mix under renormalization

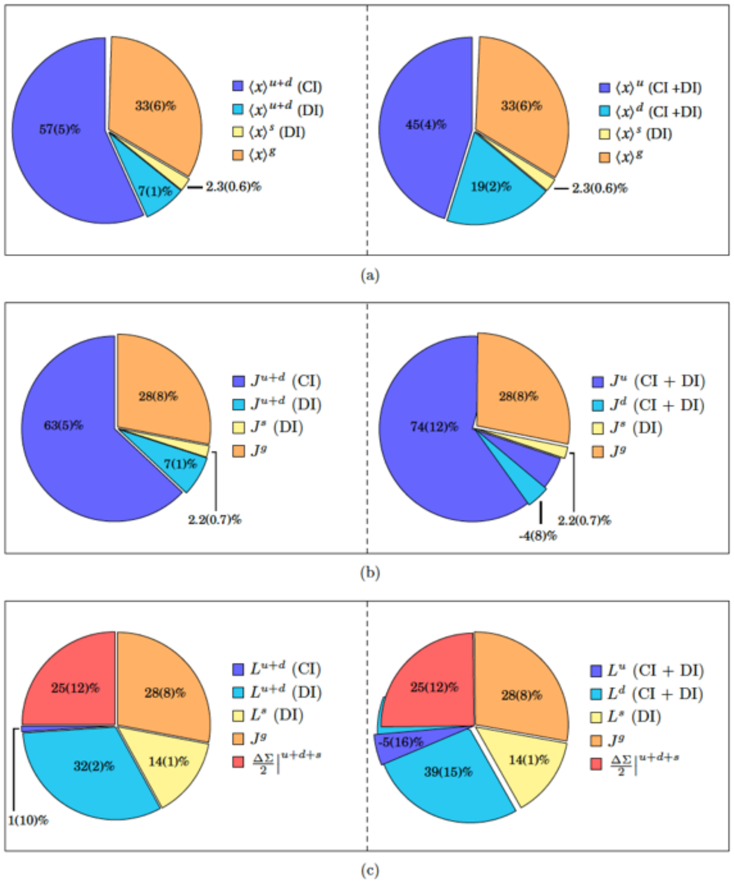
$$\frac{\partial}{\partial \ln \mu^2} \begin{pmatrix} q^S \\ g \end{pmatrix} = \frac{\alpha_s(\mu^2)}{2\pi} \begin{pmatrix} P_{qq} & 2n_f P_{qg} \\ P_{gq} & P_{gg} \end{pmatrix} \otimes \begin{pmatrix} q^S \\ g \end{pmatrix}$$

The local operators mix as follows:

$$O^{qS}_{\mu_1 \cdots \mu_N} = \frac{1}{2^N} \overline{\psi}\, \gamma_{[\mu_1} \overleftrightarrow{D}_{\mu_2} \cdots \overleftrightarrow{D}_{\mu_N]} (1 \pm \gamma_5) \psi$$

$$O^{gS}_{\mu_1 \cdots \mu_N} = \sum_\rho \mathrm{Tr} \left[F_{[\mu_1 \rho} \overleftrightarrow{D}_{\mu_2} \cdots \overleftrightarrow{D}_{\mu_{N-1}} F_{\rho \mu_N]} \right]$$

Flavor-separated and Gluon Contributions



Complete calculation of flavor-separated and gluonic contributions to nucleon spin

Deka et al, arXiv:1312.4816

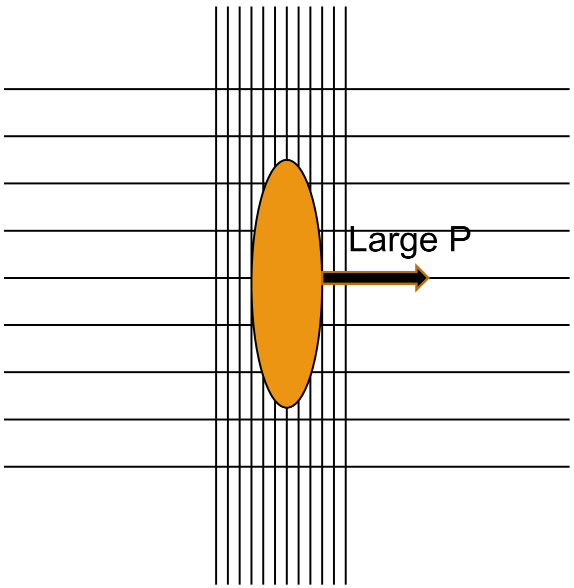
$$T_{\mu\nu} = \frac{1}{4}\bar{\psi}\gamma_{(\mu}D_{\nu)}\psi + G_{\mu\alpha}G_{\nu\alpha} - \frac{1}{4}\delta_{\mu\nu}G^2; \langle P \mid T_{\mu\nu} \mid P\rangle = P_\mu P_\nu / M$$

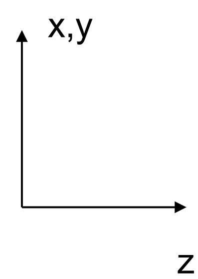
Parton Distributions - II

Formulation of LQCD in Euclidean space precludes direct calculation of light-cone correlation functions

→ LQCD computes Moments of parton distributions

New ideas: calculations of QUASI-distributions in *infinite-momentum frame*





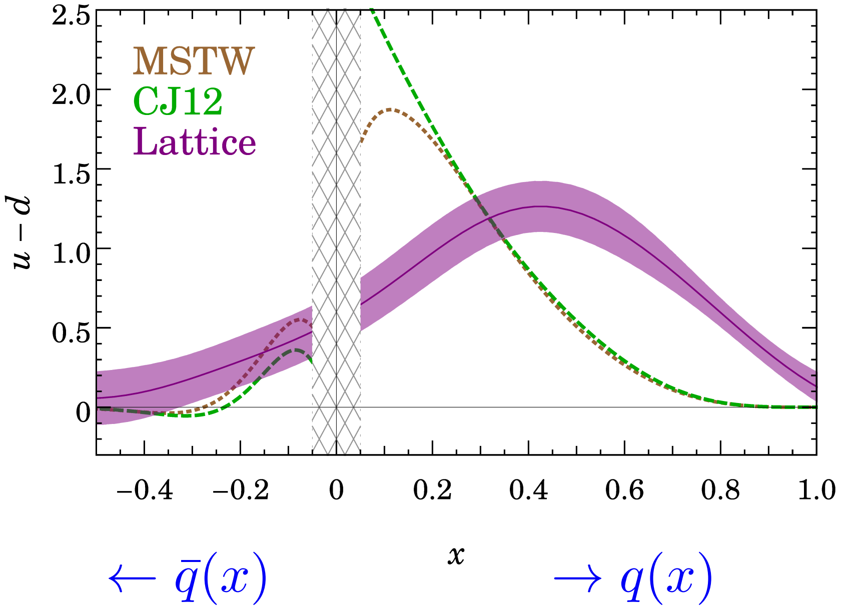
X. Ji, Phys. Rev. Lett. 110, 262002 (2013).
X. Ji, J. Zhang, and Y. Zhao, Phys. Rev. Lett. 111, 112002 (2013).
J. W. Qiu and Y. Q. Ma, arXiv:1404.686.

$$\tilde{q}(x,\mu,P_z) = \int \frac{dz}{4\pi} e^{-izk} \times \left\langle \vec{P} \middle| \bar{\psi}(z)\gamma_z e^{ig\int_0^z A_z(z')dz'} \psi(0) \middle| \vec{P} \right\rangle$$

“Equal time” correlator

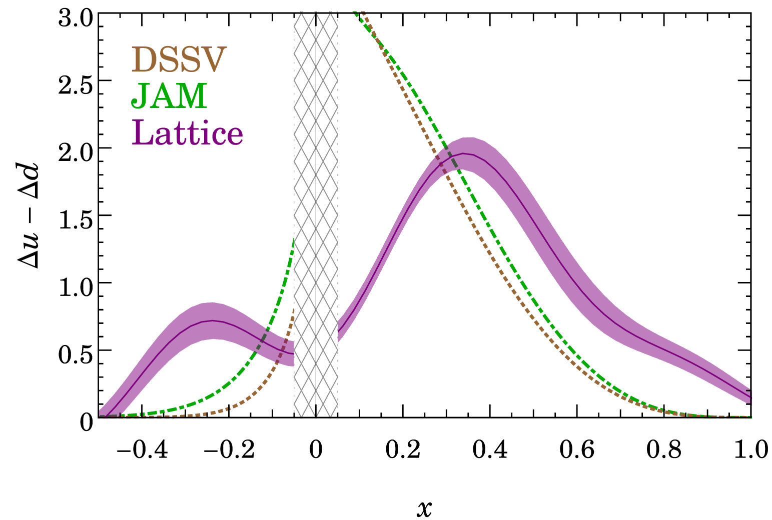
…Flavor Structure

$$\tilde{q}(x,\mu,P_z) = \int \frac{dy}{|y|} Z\left(\frac{x}{y}, \frac{\mu}{P_z}\right) q(y,\mu) + \mathcal{O}\left(\frac{\Lambda^2_{\rm QCD}}{P_z^2}, \frac{M_N^2}{P_z^2}\right) + \ldots$$



H.W. Lin et al, arXiv:1402.1462

First lattice calculations of Quasi Distributions



$\text{smallest } x \simeq 1/a$

12 GeV; Future EIC

$\text{Violation of Gottfried sum rule } \bar{d}(x) > \bar{u}(x)$

Summary

- **Lattice Calculations of the simplest quantities are now appearing at physical values of the quark masses**
- ***High-precision calculations of local matrix elements - relevant for searches for new physics in, e.g. UCN.***
  - **To directly explore x distributions, there are now a slew of new ideas… Ji et al, Qiu et al.**
- **Major effort underway in US in generating lattices designed for hadron structure calculations.**