Euclidean PDFs

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Introduction

- PDFs are defined on the light cone
- OPE: moments of PDFs are local matrix elements computable in LQCD
  - Problem: Power divergent mixing due to breaking of O(4) down to H(4) due to the lattice regulator
- Can we compute them in Euclidean LQCD calculations?
- Yes if we can overcome several difficulties

PDFs

Definition

$$f_{j/N}^{(0)}(\xi) = \int_{-\infty}^{\infty} \frac{\mathrm{d}\omega^-}{4\pi} e^{-i\xi P^+ \omega^-} \left\langle P \left| T\, \overline{\psi}_j(0,\omega^-,\mathbf{0}_{\mathrm{T}}) W(\omega^-,0)\gamma^+ \frac{\lambda^a}{2} \psi_j(0) \right| P \right\rangle_{\mathrm{C}} .$$

$$W(\omega^-,0) = \mathcal{P}\exp\left[-ig_0 \int_0^{\omega^-} \mathrm{d}y^- A_\alpha^+(0,y^-,\mathbf{0}_{\mathrm{T}}) T_\alpha\right]$$

$$\langle P'|P\rangle = (2\pi)^3 2P^+ \delta\left(P^+ - P'^{\,+}\right) \delta^{(2)}\left(\mathbf{P}_{\mathrm{T}} - \mathbf{P}'_{\mathrm{T}}\right)$$

Melin moments

$$a_0^{(n)} = \int_0^1 \mathrm{d}\xi\, \xi^{n-1} \left[f_{j/N}^{(0)}(\xi) + (-1)^n f_{\bar{j}/N}^{(0)}(\xi)\right] = \int_{-1}^1 \mathrm{d}\xi\, \xi^{n-1} f_{j/N}(\xi),$$

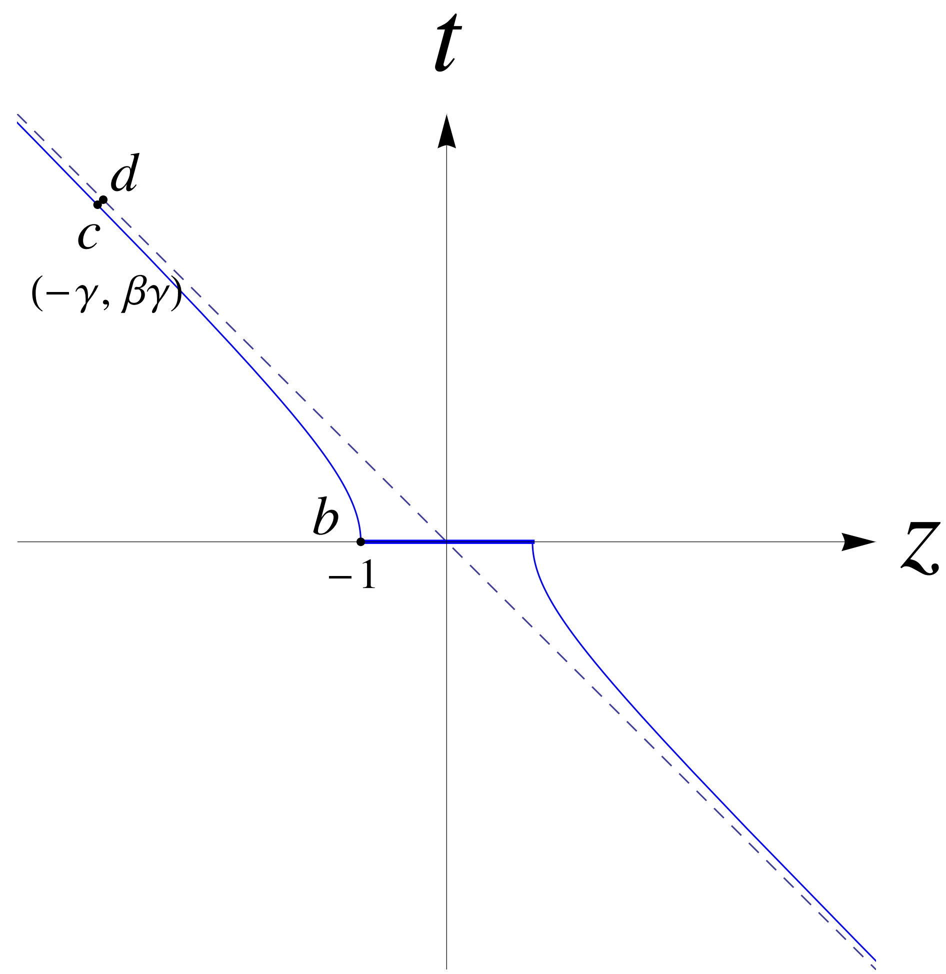
$$f_{j/N}^{(0)}(-\xi) = -f_{\bar{j}/N}^{(0)}(\xi),$$

Twist-2 operators

$$\left\langle P|\mathcal{O}_0^{\{\mu_1\ldots\mu_n\}}|P\right\rangle = 2a_0^{(n)}\left(P^{\mu_1}\cdots P^{\mu_n} - \mathrm{traces}\right) \text{ -}$$

$$\mathcal{O}_0^{\{\mu_1\cdots\mu_n\}} = i^{n-1}\overline{\psi}_j(0)\gamma^{\{\mu_1}D^{\mu_2}\cdots D^{\mu_n\}}\frac{\lambda^a}{2}\psi_j(0) - \mathrm{traces}\,.$$

Ji’s suggestion



light-cone frame ⟷ Proton with a large boost

Ji's suggestion

time local matrix element: computable on the lattice

$$h^{(0)}(zP_z, \Lambda_{\mathrm{QCD}}/P_z, M_{\mathrm{N}}/P_z) = \frac{1}{2P_z}\left\langle P_z \left| \overline{\psi}_j(z) W(0,z) \gamma_z \frac{\lambda^a}{2} \psi_j(0) \right| P_z \right\rangle_{\mathrm{C}}$$

quasi-pdf:

$$q^{(0)}_{j/N}(\xi, \Lambda_{\mathrm{QCD}}/P_z, M_{\mathrm{N}}/P_z) = \int_{-\infty}^{\infty} \frac{\mathrm{d}z}{2\pi} e^{-i\xi z P_z} P_z\, h^{(0)}(zP_z, \Lambda_{\mathrm{QCD}}/P_z, M_{\mathrm{N}}/P_z),$$

These are bare quantities that are finite only if I have some a regulator (lattice ?). In principle approach the light-cone PDFs if Pz goes to infinity.

Problems

- Large momentum:
  - Cut-off effects (technical)
  - Noise (technical)
  - Renormalization (conceptual)

moments of bare QPDFs

$$h^{(0)}(zP_z, \Lambda_{\mathrm{QCD}}/P_z, M_{\mathrm{N}}/P_z)\Big|_{A_z=0} = \frac{1}{2P_z}\left\langle P_z \left| \overline{\psi}_j(z)\gamma_z \frac{\lambda^a}{2}\psi_j(0) \right| P_z \right\rangle_{\mathrm{C}}.$$

$$\int_{-\infty}^{\infty} \mathrm{d}\xi \, q^{(0)}_{j/N}\left(\xi, \Lambda_{\mathrm{QCD}}/P_z, M_{\mathrm{N}}/P_z\right)\Big|_{A_z=0} = h^{(0)}(0, \Lambda_{\mathrm{QCD}}/P_z, M_{\mathrm{N}}/P_z)\Big|_{A_z=0}.$$

$$\left(\frac{-i}{P_z}\frac{\partial}{\partial z}\right)^{n-1} h^{(0)}(zP_z, \Lambda_{\mathrm{QCD}}/P_z, M_{\mathrm{N}}/P_z) = \int_{-\infty}^{\infty} \mathrm{d}\xi \, \xi^{n-1} e^{i\xi z P_z} q^{(0)}_{j/N}\left(\xi, \Lambda_{\mathrm{QCD}}/P_z, M_{\mathrm{N}}/P_z\right).$$

$$b_n^{(0)}\left(\Lambda_{\mathrm{QCD}}/P_z, M_{\mathrm{N}}/P_z\right) = \int_{-\infty}^{\infty} \mathrm{d}\xi\, \xi^{n-1} q_{j/N}^{(0)}\left(\xi, \Lambda_{\mathrm{QCD}}/P_z, M_{\mathrm{N}}/P_z\right)$$

$$b_n^{(0)}\left(\Lambda_{\mathrm{QCD}}/P_z, M_{\mathrm{N}}/P_z\right)\Big|_{A_z=0} = \frac{1}{2P_z^n}\left\langle P_z \left| \left[\overline{\psi}_j(z)\gamma_z \left(-i\overleftarrow{\partial}_z^{\,n-1}\right)\frac{\lambda^a}{2}\psi_j(0)\right]_{z=0}\right| P_z \right\rangle_{\mathrm{C}}$$

removing the gauge fixing:

$$b_n^{(0)}\left(\Lambda_{\mathrm{QCD}}/P_z, M_{\mathrm{N}}/P_z\right) = \frac{1}{2P_z^n}\left\langle P_z \left| \overline{\psi}_j \gamma_z (-i\overleftarrow{D}_z)^{(n-1)} \frac{\lambda^a}{2}\psi_j(0)\right| P_z \right\rangle_{\mathrm{C}}$$

Operators are not traceless… corrections

$$b_n^{(0)} = a_n^{(0)}\left(1 + O(M_N^2/P_z^2) + O(\Lambda_{QCD}^2/P_z^2)\right)$$

Mass corrections

$$K_n\left(\frac{M_{\rm N}^2}{4P_z^2}\right) = \sum_{j=0}^{n/2} \left(\begin{array}{c} n-j \\ j \end{array} \right) \left(\frac{M_{\rm N}^2}{4P_z^2}\right)^j .$$

$$\frac{b_n^{(0)}}{K_n(\frac{M_N^2}{P_z^2})} = a_n^{(0)} \left(1 + O(\Lambda_{QCD}^2/P_z^2)\right)$$

Can be done exactly on the PDF

Chen et. al arXiv:1603.06664v1 [hep-ph]

rely on large momentum to remove higher twist effects

Renormalization

$$f_{j/N}(x,\mu) = \int_{-\infty}^{\infty} \frac{d\xi}{\xi} Z\left(x/\xi, \mu/P_z, \Lambda/P_z\right) q^{(0)}_{j/N}\left(\xi, \Lambda/P_z\right)$$

Three scales: Momentum, cut-off, renormalization scale

Need to determine Z

Z is short distance quantity independent of external states

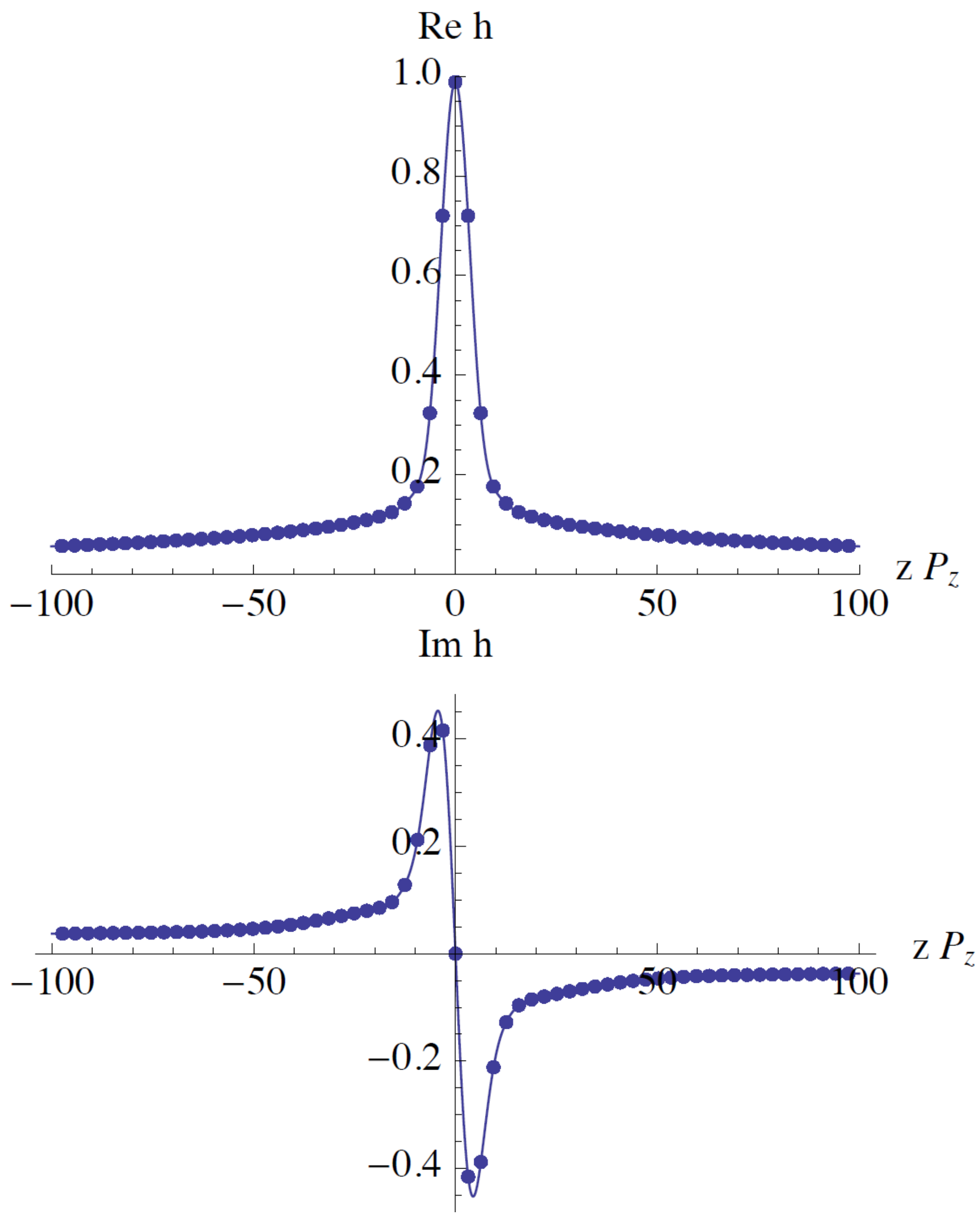
Perturbative computations exist (Ji, Qiu, …

Can we compute Z non-perturbatively?

Alternatively (Qiu et. al.):

1. Define a renormalized quasi-PDF (at finite Pz)
2. Match the renormalized quasi-PDF to PDFs

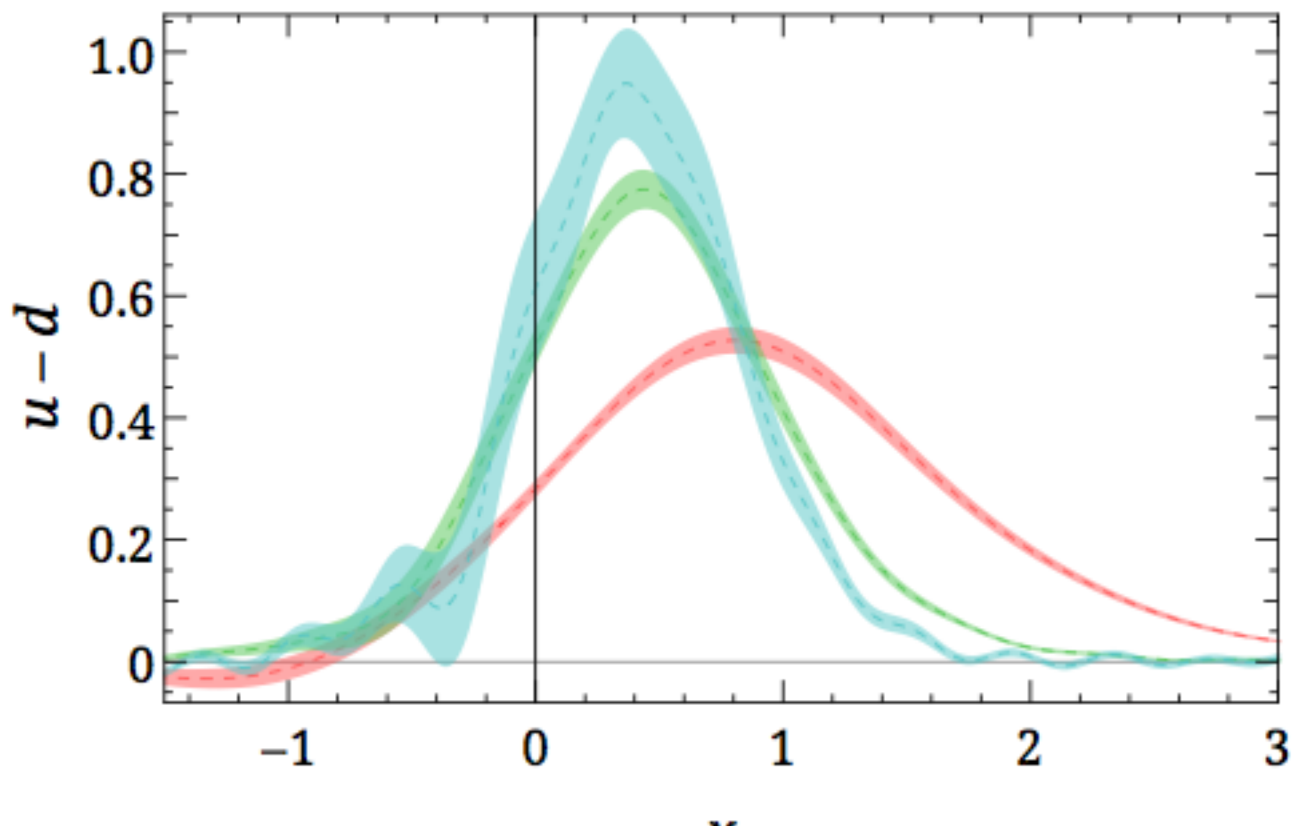
results



Lattice result for h

Chen et. al arXiv:1603.06664v1 [hep-ph]

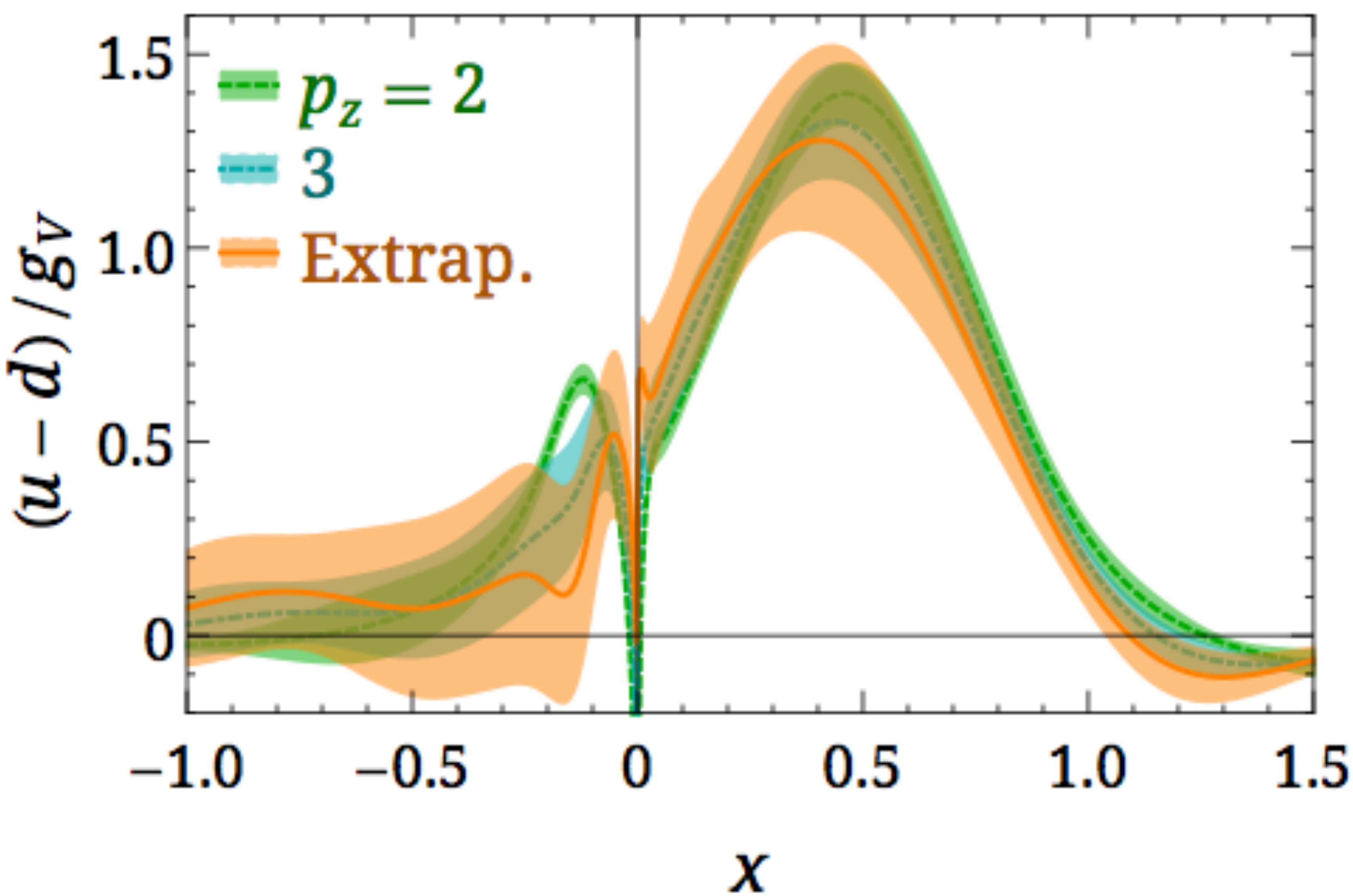
results



$$P^z = \frac{2\pi}{L} n = n \times 0.43\ GeV$$ n = 1, 2, 3.

Chen et. al arXiv:1603.06664v1 [hep-ph]

renormalized and corrected PDF



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