Euclidean PDFs

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Introduction

• PDFs are defined on the light cone

• OPE: moments of PDFs are local matrix elements computable in LQCD

  • Problem: Power divergent mixing due to breaking of O(4) down to H(4) due to the lattice regulator

• Can we compute them in Euclidean LQCD calculations?

• Yes if we can overcome several difficulties
PDFs

Definition

\[ f_{j/N}^{(0)}(\xi) = \int_{-\infty}^{\infty} \frac{d\omega^-}{4\pi} e^{-i\xi P^+ \omega^-} \left\langle P \left| T \bar{\psi}_j(0, \omega^-, 0) W(\omega^-, 0) \gamma^+ \frac{\lambda^a}{2} \psi_j(0) \right| P \right\rangle_C \]

\[ W(\omega^-, 0) = \mathcal{P} \exp \left[ -ig_0 \int_0^{\omega^-} dy^- A^+_\alpha(0, y^-, 0) T_\alpha \right] \]

\[ \langle P' | P \rangle = (2\pi)^3 2P^+ \delta (P^+ - P'^+) \delta^{(2)} (P_T - P'_T) \]

Melin moments

\[ a_0^{(n)} = \int_0^1 d\xi \xi^{n-1} \left[ f_{j/N}^{(0)}(\xi) + (-1)^n f_{\bar{j}/N}^{(0)}(\xi) \right] = \int_{-1}^1 d\xi \xi^{n-1} f_{j/N}(\xi), \]

\[ f_{j/N}^{(0)}(-\xi) = -f_{j/N}^{(0)}(\xi), \]
Twist-2 operators

$$\langle P | \mathcal{O}_0^{\{\mu_1 \cdots \mu_n\}} | P \rangle = 2a_0^{(n)} (P^{\mu_1} \cdots P^{\mu_n} - \text{traces})$$

$$\mathcal{O}_0^{\{\mu_1 \cdots \mu_n\}} = i^{n-1} \bar{\psi}_j(0) \gamma^{\{\mu_1} D^{\mu_2} \cdots D^{\mu_n\}} \frac{\lambda^a}{2} \psi_j(0) - \text{traces}.$$
Ji’s suggestion

light-cone frame  Proton with a large boost
Ji’s suggestion

time local matrix element: computable on the lattice

\[
    h^{(0)}(zP_z, \Lambda_{QCD}/P_z, M_N/P_z) = \frac{1}{2P_z} \left< P_z \left| \overline{\psi}_j(z) W(0, z) \gamma_z \frac{\lambda^a}{2} \psi_j(0) \right| P_z \right>^C
\]

quasi-pdf:

\[
    q^{(0)}_{j/N}(\xi, \Lambda_{QCD}/P_z, M_N/P_z) = \int_{-\infty}^{\infty} \frac{dz}{2\pi} e^{-i\xi zP_z} P_z h^{(0)}(zP_z, \Lambda_{QCD}/P_z, M_N/P_z),
\]

These are bare quantities that are finite only if I have some a regulator (lattice?). In principle approach the light-cone PDFs if Pz goes to infinity.
Problems

• Large momentum:
  • Cut-off effects (technical)
  • Noise (technical)
  • Renormalization (conceptual)
moments of bare QPDFs

\[
h^{(0)}(zP_z, \Lambda_{QCD}/P_z, M_N/P_z) \bigg|_{A_z=0} = \frac{1}{2P_z} \left< P_z \left| \bar{\psi}_j(z) \gamma_z \frac{\lambda^a}{2} \psi_j(0) \right| P_z \right>_C.
\]

\[
\int_{-\infty}^{\infty} d\xi \, q^{(0)}_{j/N}(\xi, \Lambda_{QCD}/P_z, M_N/P_z) \bigg|_{A_z=0} = h^{(0)}(0, \Lambda_{QCD}/P_z, M_N/P_z) \bigg|_{A_z=0}.
\]

\[
\left( -i \frac{\partial}{P_z \partial z} \right)^{n-1} h^{(0)}(zP_z, \Lambda_{QCD}/P_z, M_N/P_z) = \int_{-\infty}^{\infty} d\xi \, \xi^{n-1} e^{i\xi z P_z} q^{(0)}_{j/N}(\xi, \Lambda_{QCD}/P_z, M_N/P_z).
\]
\[ b_n^{(0)} (\Lambda_{QCD}/P_z, M_N/P_z) = \int_{-\infty}^{\infty} d\xi \, \xi^{n-1} q_j^{(0)} (\xi, \Lambda_{QCD}/P_z, M_N/P_z) \]

\[ b_n^{(0)} (\Lambda_{QCD}/P_z, M_N/P_z) \Big|_{A_z=0} = \frac{1}{2P_z^n} \left\langle P_z \left| \bar{\psi}_j(z) \gamma_z \left( -i \frac{\xi}{D_z} \right)^{n-1} \frac{\lambda^a}{2} \psi_j(0) \right| P_z \right\rangle_C \]

removing the gauge fixing:

\[ b_n^{(0)} (\Lambda_{QCD}/P_z, M_N/P_z) = \frac{1}{2P_z^n} \left\langle P_z \left| \bar{\psi}_j(z) \gamma_z (-i \sqrt{D_z})^{(n-1)} \frac{\lambda^a}{2} \psi_j(0) \right| P_z \right\rangle_C \]

Operators are not traceless… corrections

\[ b_n^{(0)} = a_n^{(0)} \left( 1 + O(M_N^2/P_z^2) + O(\Lambda_{QCD}^2/P_z^2) \right) \]
Mass corrections

\[ K_n \left( \frac{M_N^2}{4P_z^2} \right) = \sum_{j=0}^{n/2} \binom{n-j}{j} \left( \frac{M_N^2}{4P_z^2} \right)^j. \]

\[ \frac{b_n^{(0)}}{K_n \left( \frac{M_N^2}{P_z^2} \right)} = a_n^{(0)} \left( 1 + O(\Lambda_{QCD}^2/P_z^2) \right) \]

Can be done exactly on the PDF

Chen et. al arXiv:1603.06664v1 [hep-ph]

rely on large momentum to remove higher twist effects
Renormalization

\[ f_{j/N}(x, \mu) = \int_{-\infty}^{\infty} \frac{d\xi}{\xi} Z \left( \frac{x}{\xi}, \frac{\mu}{P_z}, \frac{\Lambda}{P_z} \right) q_{j/N}^{(0)} \left( \xi, \frac{\Lambda}{P_z} \right) \]

Three scales: Momentum, cut-off, renormalization scale
Need to determine $Z$
$Z$ is short distance quantity independent of external states
Perturbative computations exist (Ji, Qiu, …)
Can we compute $Z$ non-perturbatively?

Alternatively (Qiu et. al.):
1. Define a renormalized quasi-PDF (at finite $P_z$)
2. Match the renormalized quasi-PDF to PDFs
results

Lattice result for $h$

Chen et. al arXiv:1603.06664v1 [hep-ph]
results

\[ P^z = \frac{2\pi}{L} n = n \times 0.43 \, \text{GeV} \ \ \ n = 1, 2, 3. \]

Chen et. al arXiv:1603.06664v1 [hep-ph]
renormalized and corrected PDF

Chen et. al arXiv:1603.06664v1 [hep-ph]
References

Parton Physics on Euclidean Lattice
Xiangdong Ji
Comments: 8 pages, 1 figure
Subjects: High Energy Physics – Phenomenology (hep-ph); High Energy Physics – Experiment (hep-ex); High Energy Physics – Lattice (hep-lat); Nuclear Experiment (nucl-ex); Nuclear Theory (nucl-th)

2. arXiv:1412.2688 [pdf, ps, other]
QCD Factorization and PDFs from Lattice QCD Calculation
Yan-Qing Ma, Jian-Wei Qiu
Comments: 8 pages, 2 figures, accepted contribution to the proceedings of "The QCD Evolution 2014 workshop", May 12–16, 2014, Santa Fe, NM
Subjects: High Energy Physics – Phenomenology (hep-ph); High Energy Physics – Lattice (hep-lat); Nuclear Theory (nucl-th)

A Lattice Calculation of Parton Distributions
Constantia Alexandrou, Krzysztof Cichy, Vincent Drach, Elena Garcia-Ramos, Kyriakos Hadjiyiannakou, Karl Jansen, Fernanda Steffens, Christian Wiese
Comments: Minor changes in the text. Version published in PRD. 19 pages, 6 figures

Nucleon Helicity and Transversity Parton Distributions from Lattice QCD
Jiunn-Wei Chen, Saul D. Cohen, Xiangdong Ji, Huey-Wen Lin, Jian-Hui Zhang
Comments: 21 pages, 6 figures

5. arXiv:1609.02018 [pdf, other]
Practical quasi parton distribution functions
Tomomi Ishikawa, Yan-Qing Ma, Jian-Wei Qiu, Shinsuke Yoshida
Comments: 28 pages, 7 figures
Subjects: High Energy Physics – Lattice (hep-lat); High Energy Physics – Phenomenology (hep-ph); Nuclear Theory (nucl-th)

Improved quasi parton distribution through Wilson line renormalization
Jiunn-Wei Chen, Xiangdong Ji, Jian-Hui Zhang
Comments: 9 pages, 4 figures