Resonance-DIS transition and low $Q^2$ phenomena

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+ F. Close (Oxford), O. Lalakulich & E. Paschos (Dortmund)
Outline

1. Introduction: resonance-DIS transition & Bloom-Gilman duality
2. Duality in QCD: moments & higher twists
3. Local duality in dynamical quark models
4. Phenomenological models
5. DIS at low $Q^2$
6. Summary
I.
Introduction
Resonance-DIS transition characterized by Bloom-Gilman duality

Jefferson Lab (Hall C)


Average over (strongly $Q^2$ dependent) resonances
$\approx Q^2$ independent scaling function
Bloom-Gilman duality

Duality in $F_2$ and $F_L$ structure functions (from longitudinal-transverse separation)
Integrated strength

~10% agreement for $Q^2 > 1$ GeV$^2$

Niculescu et al,
Moments

data from longitudinal-transverse separation!

Jefferson Lab (Hall C)
Nuclear structure functions

for larger nuclei, Fermi motion does resonance averaging automatically!
Neutron ($^3\text{He}$) $g_1$ structure function

Liyanage et al. (JLab Hall A)
2. Duality in QCD
Duality and the OPE

Operator product expansion

expand moments of structure functions
in powers of \(1/Q^2\)

\[ M_n(Q^2) = \int_0^1 dx \ x^{n-2} F_2(x, Q^2) \]

\[ = A_n^{(2)} + \frac{A_n^{(4)}}{Q^2} + \frac{A_n^{(6)}}{Q^4} + \cdots \]

matrix elements of operators
with specific “twist” \(\tau\)

\[ \tau = \text{dimension} - \text{spin} \]
Higher twists

\[ \tau = 2 \]

single quark scattering

\[ \tau > 2 \]

nonperturbative \( qq \) and \( qg \) correlations

(\( \rightarrow \) confinement)
Duality and the OPE

Operator product expansion

expand moments of structure functions in powers of $1/Q^2$

$$M_n(Q^2) = \int_0^1 dx \ x^{n-2} \ F_2(x, Q^2) = A_n^{(2)} + \frac{A_n^{(4)}}{Q^2} + \frac{A_n^{(6)}}{Q^4} + \cdots$$

If moment $\approx$ independent of $Q^2$

higher twist terms $A_n^{(\tau > 2)}$ small
Duality and the OPE

Operator product expansion

expand moments of structure functions in powers of $1/Q^2$

\[ M_n(Q^2) = \int_0^1 dx \, x^{n-2} \, F_2(x, Q^2) \]

\[ = A_n^{(2)} + \frac{A_n^{(4)}}{Q^2} + \frac{A_n^{(6)}}{Q^4} + \cdots \]

Duality ⇐⇒ suppression of higher twists

\[ de \ Rujula, \ Georgi, \ Politzer, \ Ann. \ Phys. \ 103 (1975) 315 \]
Moment of neutron $g_1$ structure function

- $\Gamma_1(Q^2) = \int_0^1 dx \ g_1(x, Q^2)$
  
  \[ = \Gamma_1^{(\tau=2)}(Q^2) + \Delta \Gamma_1(Q^2) \]


higher twist small down to $Q^2 \sim 1$ GeV$^2$
Moment of proton $g_1$ structure function


higher twist small down to $Q^2 \sim 2 \text{ GeV}^2$
Total higher twist small at $Q^2 \sim 1 - 2 \text{ GeV}^2$

- nonperturbative interactions between quarks and gluons not dominant at these scales

- suggests *strong cancellations* between resonances, resulting in dominance of *leading twist*

- OPE does not tell us *why* higher twists are small!

- need dynamical models to understand *how* cancellations between *coherent* resonances produce *incoherent* scaling function
3. Local duality in dynamical quark models
Coherence vs. incoherence

Exclusive form factors

\[ d\sigma \sim \left( \sum_i e_i \right)^2 \]

Inclusive structure functions

\[ d\sigma \sim \sum_i e_i^2 \]

how can the *square of a sum* become the *sum of squares*?
Pedagogical model

Two quarks bound in a harmonic oscillator potential
⇒ exactly solvable spectrum

Structure function given by sum of squares of transition form factors

$$F(\nu, q^2) \sim \sum_n |G_{0,n}(q^2)|^2 \delta(E_n - E_0 - \nu)$$

Charge operator \( \sum_i e_i \exp(iq \cdot r_i) \) excites

*even* partial waves with strength \( \propto (e_1 + e_2)^2 \)

*odd* partial waves with strength \( \propto (e_1 - e_2)^2 \)
Pedagogical model

Resulting structure function

\[ F(\nu, q^2) \sim \sum_n \left\{ (e_1 + e_2)^2 G^2_{0,2n} + (e_1 - e_2)^2 G^2_{0,2n+1} \right\} \]

If states degenerate, cross terms \((\sim e_1 e_2)\) cancel when averaged over nearby even and odd parity states

Minimum condition for duality:

\[ \text{at least one complete set of even and odd parity resonances must be summed over} \]

Even and odd parity states generalize to $56^+ (L=0)$ and $70^- (L=1)$ multiplets of spin-flavor SU(6).

Scaling occurs if contributions from $56^+$ and $70^-$ have equal overall strengths.

Simplified case: magnetic coupling of $\gamma^*$ to quark.

Expect dominance over electric at large $Q^2$. 

Quark model
Quark model

Even and odd parity states generalize to $56^+ (L=0)$ and $70^- (L=1)$ multiplets of spin-flavor SU(6)

Scaling occurs if contributions from $56^+$ and $70^-$ have equal overall strengths

<table>
<thead>
<tr>
<th>representation</th>
<th>$^28[56^+]$</th>
<th>$^410[56^+]$</th>
<th>$^28[70^-]$</th>
<th>$^48[70^-]$</th>
<th>$^210[70^-]$</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>$F_1^p$</td>
<td>$9\rho^2$</td>
<td>$8\lambda^2$</td>
<td>$9\rho^2$</td>
<td>$0$</td>
<td>$\lambda^2$</td>
<td>$18\rho^2 + 9\lambda^2$</td>
</tr>
<tr>
<td>$F_1^n$</td>
<td>$(3\rho + \lambda)^2/4$</td>
<td>$8\lambda^2$</td>
<td>$(3\rho - \lambda)^2/4$</td>
<td>$4\lambda^2$</td>
<td>$\lambda^2$</td>
<td>$(9\rho^2 + 27\lambda^2)/2$</td>
</tr>
<tr>
<td>$g_1^p$</td>
<td>$9\rho^2$</td>
<td>$-4\lambda^2$</td>
<td>$9\rho^2$</td>
<td>$0$</td>
<td>$\lambda^2$</td>
<td>$18\rho^2 - 3\lambda^2$</td>
</tr>
<tr>
<td>$g_1^n$</td>
<td>$(3\rho + \lambda)^2/4$</td>
<td>$-4\lambda^2$</td>
<td>$(3\rho - \lambda)^2/4$</td>
<td>$-2\lambda^2$</td>
<td>$\lambda^2$</td>
<td>$(9\rho^2 - 9\lambda^2)/2$</td>
</tr>
</tbody>
</table>

$\lambda (\rho) = (\text{anti}) \text{ symmetric component of ground state wfn.}$

$|N\rangle = \lambda \left| \varphi \otimes \chi \right\rangle_{\text{sym}} + \rho \left| \varphi \otimes \chi \right\rangle_{\text{antisym}}$
Quark model

Even and odd parity states generalize to $56^+ (L=0)$ and $70^- (L=1)$ multiplets of spin-flavor SU(6)

scaling occurs if contributions from $56^+$ and $70^-$ have equal overall strengths

Similarly for neutrinos ...

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</tr>
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<tr>
<td>$F_1^{vp}$</td>
<td>0</td>
<td>24$\lambda^2$</td>
<td>0</td>
<td>0</td>
<td>3$\lambda^2$</td>
<td>27$\lambda^2$</td>
</tr>
<tr>
<td>$F_1^{vn}$</td>
<td>($9\rho + \lambda)^2/4$</td>
<td>8$\lambda^2$</td>
<td>($9\rho - \lambda)^2/4$</td>
<td>4$\lambda^2$</td>
<td>$\lambda^2$</td>
<td>($81\rho^2 + 27\lambda^2)/2$</td>
</tr>
<tr>
<td>$g_1^{vp}$</td>
<td>0</td>
<td>$-12\lambda^2$</td>
<td>0</td>
<td>0</td>
<td>3$\lambda^2$</td>
<td>$-9\lambda^2$</td>
</tr>
<tr>
<td>$g_1^{vn}$</td>
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<td>$-2\lambda^2$</td>
<td>$\lambda^2$</td>
<td>($81\rho^2 - 9\lambda^2)/2$</td>
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$\lambda(\rho) = \text{(anti) symmetric component of ground state wfn.}$

Quark model

SU(6) limit $\rightarrow \lambda = \rho$

<table>
<thead>
<tr>
<th>$SU(6)$</th>
<th>$[56, 0^+]^28$</th>
<th>$[56, 0^+]^{410}$</th>
<th>$[70, 1^-]^28$</th>
<th>$[70, 1^-]^{48}$</th>
<th>$[70, 1^-]^{210}$</th>
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</tr>
</thead>
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<tr>
<td>$F_1^p$</td>
<td>9</td>
<td>8</td>
<td>9</td>
<td>0</td>
<td>1</td>
<td>27</td>
</tr>
<tr>
<td>$F_1^n$</td>
<td>4</td>
<td>8</td>
<td>1</td>
<td>4</td>
<td>1</td>
<td>18</td>
</tr>
<tr>
<td>$g_1^p$</td>
<td>9</td>
<td>-4</td>
<td>9</td>
<td>0</td>
<td>1</td>
<td>15</td>
</tr>
<tr>
<td>$g_1^n$</td>
<td>4</td>
<td>-4</td>
<td>1</td>
<td>-2</td>
<td>1</td>
<td>0</td>
</tr>
</tbody>
</table>

Summing over all resonances in $56^+$ and $70^-$ multiplets

$R^{np} = \frac{F_1^n}{F_1^p} = \frac{2}{3}$ \quad $A_1^p = \frac{g_1^p}{F_1^p} = \frac{5}{9}$ \quad $A_1^n = \frac{g_1^n}{F_1^n} = 0$

as in quark-parton model!
Quark model

SU(6) limit $\rightarrow \lambda = \rho$

<table>
<thead>
<tr>
<th>SU(6) : $[56, 0^+]^2$</th>
<th>$[56, 0^+]^{410}$</th>
<th>$[70, 1^-]^2$</th>
<th>$[70, 1^-]^{48}$</th>
<th>$[70, 1^-]^{210}$</th>
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<tr>
<td>$F_1^p$</td>
<td>9</td>
<td>8</td>
<td>9</td>
<td>0</td>
<td>1</td>
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<td>$F_1^n$</td>
<td>4</td>
<td>8</td>
<td>1</td>
<td>4</td>
<td>1</td>
</tr>
<tr>
<td>$g_1^p$</td>
<td>9</td>
<td>-4</td>
<td>9</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>$g_1^n$</td>
<td>4</td>
<td>-4</td>
<td>1</td>
<td>-2</td>
<td>1</td>
</tr>
</tbody>
</table>

$\rightarrow$ expect duality to appear earlier for $F_1^p$ than $F_1^n$

$\rightarrow$ earlier onset for $g_1^n$ than $g_1^p$

$\rightarrow$ cancellations within multiplets for $g_1^n$
Quark model

Similarly for neutrinos ...

SU(6) limit \((\lambda = \rho)\)

<table>
<thead>
<tr>
<th>SU(6)</th>
<th>([56, 0^+]) ^28</th>
<th>([56, 0^+]) ^410</th>
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<th>([70, 1^-]) ^48</th>
<th>([70, 1^-]) ^210</th>
<th>total</th>
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<tbody>
<tr>
<td>(F_{1}^{\nu p})</td>
<td>0</td>
<td>24</td>
<td>0</td>
<td>0</td>
<td>3</td>
<td>27</td>
</tr>
<tr>
<td>(F_{1}^{\nu n})</td>
<td>25</td>
<td>8</td>
<td>16</td>
<td>4</td>
<td>1</td>
<td>54</td>
</tr>
<tr>
<td>(g_{1}^{\nu p})</td>
<td>0</td>
<td>-12</td>
<td>0</td>
<td>0</td>
<td>3</td>
<td>-9</td>
</tr>
<tr>
<td>(g_{1}^{\nu n})</td>
<td>25</td>
<td>-4</td>
<td>16</td>
<td>-2</td>
<td>1</td>
<td>36</td>
</tr>
</tbody>
</table>

Summing over all resonances in \([56^+]\) and \([70^-]\) multiplets

\[ R^\nu = \frac{F_{1}^{\nu p}}{F_{1}^{\nu n}} = \frac{1}{2} \left( \frac{d}{u} \right) \]

\[ A_{1}^{\nu p} = -\frac{1}{3} \left( \frac{\Delta d}{d} \right) \]

\[ A_{1}^{\nu n} = \frac{2}{3} \left( \frac{\Delta u}{u} \right) \]

as in parton model!
Quark model

SU(6) may be \( \approx \) valid at \( x \sim 1/3 \)

**But** significant deviations at large \( x \)

\[ \text{which combinations of resonances reproduce behavior of structure functions at large } x? \]

<table>
<thead>
<tr>
<th>Model</th>
<th>SU(6)</th>
<th>No ( ^410 )</th>
<th>No ( ^210, ^410 )</th>
<th>No ( S_{3/2} )</th>
<th>No ( \sigma_{3/2} )</th>
<th>No ( \psi_\lambda )</th>
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<tr>
<td>( R^{np} )</td>
<td>2/3</td>
<td>10/19</td>
<td>1/2</td>
<td>6/19</td>
<td>3/7</td>
<td>1/4</td>
</tr>
<tr>
<td>( A_1^p )</td>
<td>5/9</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>( A_1^n )</td>
<td>0</td>
<td>2/5</td>
<td>1/3</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
</tbody>
</table>
Quark model

SU(6) may be \( \approx \) valid at \( x \sim 1/3 \)

**But** significant deviations at large \( x \)

which combinations of resonances reproduce behavior of structure functions at large \( x \)?

<table>
<thead>
<tr>
<th>Model</th>
<th>SU(6)</th>
<th>No ( ^4\text{10} )</th>
<th>No ( ^2\text{10}, ^4\text{10} )</th>
<th>No ( S_{3/2} )</th>
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<td>1/4</td>
</tr>
<tr>
<td>( A^p_1 )</td>
<td>5/9</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>( A^n_1 )</td>
<td>0</td>
<td>2/5</td>
<td>1/3</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
</tbody>
</table>

<ins>gives \( \Delta u/u > 1 \) inconsistent with duality</ins>
Quark model

SU(6) may be \( \approx \) valid at \( x \sim 1/3 \)

But significant deviations at large \( x \)

which combinations of resonances reproduce behavior of structure functions at large \( x \)?

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<tr>
<th>Model</th>
<th>SU(6)</th>
<th>No ( ^4!10 )</th>
<th>No ( ^2!10, \ ^4!10 )</th>
<th>No ( S_{3/2} )</th>
<th>No ( \sigma_{3/2} )</th>
<th>No ( \psi_{\lambda} )</th>
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<tr>
<td>( A^p_1 )</td>
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<td>1</td>
<td>1</td>
</tr>
<tr>
<td>( A^n_1 )</td>
<td>0</td>
<td>2/5</td>
<td>1/3</td>
<td>1</td>
<td>1</td>
<td>1</td>
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\( ^4\!10 \) \([56^+]\) and \( ^4\!8 \) \([70^-]\) suppressed
**Quark model**

SU(6) may be \( \approx \) valid at \( x \sim 1/3 \)

*But* significant deviations at large \( x \)

which combinations of resonances reproduce behavior of structure functions at large \( x \)?

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<th>SU(6)</th>
<th>No ( ^4\mathbf{10} )</th>
<th>No ( ^2\mathbf{10} , ^4\mathbf{10} )</th>
<th>No ( S_{3/2} )</th>
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<td>1/4</td>
</tr>
<tr>
<td>( A_1^p )</td>
<td>5/9</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>( A_1^n )</td>
<td>0</td>
<td>2/5</td>
<td>1/3</td>
<td>1</td>
<td>1</td>
<td>1</td>
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Quark model

SU(6) may be \( \approx \) valid at \( x \sim 1/3 \)

**But** significant deviations at large \( x \)

which combinations of resonances reproduce behavior of structure functions at large \( x \)?

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<th>Model</th>
<th>SU(6)</th>
<th>No ( ^4\mathbf{10} )</th>
<th>No ( ^2\mathbf{10}, , ^4\mathbf{10} )</th>
<th>No ( S_{3/2} )</th>
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<td>( 5/9 )</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>( A_n )</td>
<td>0</td>
<td>2/5</td>
<td>1/3</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
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* e.g. through \( \vec{S}_i \cdot \vec{S}_j \) interaction between quarks

suppression of symmetric part of spin-flavor wfn.
### Quark model

**SU(6) may be \(\approx\) valid at \(x \sim 1/3\)**

**But** significant deviations at large \(x\)

which combinations of resonances reproduce behavior of structure functions at large \(x\)?

<table>
<thead>
<tr>
<th>Model</th>
<th>SU(6)</th>
<th>No (^410)</th>
<th>No (^210, ^410)</th>
<th>No (S_{3/2})</th>
<th>No (\sigma_{3/2})</th>
<th>No (\psi_\lambda)</th>
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<tbody>
<tr>
<td>(R^v)</td>
<td>1/2</td>
<td>3/46</td>
<td>0</td>
<td>1/14</td>
<td>1/5</td>
<td>0</td>
</tr>
<tr>
<td>(A_1^{vp})</td>
<td>–1/3</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>–1/3</td>
<td></td>
</tr>
<tr>
<td>(A_1^{vn})</td>
<td>2/3</td>
<td>20/23</td>
<td>13/15</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
</tbody>
</table>

\[\text{gives } d/u, \Delta u/u, \Delta d/d \text{ inconsistent with } e \text{ scattering}\]
Quark model

SU(6) may be $\approx$ valid at $x \sim 1/3$

**But** significant deviations at large $x$

which combinations of resonances reproduce behavior of structure functions at large $x$?

<table>
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<th>No $^2_{10}$, $^4_{10}$</th>
<th>No $S_{3/2}$</th>
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<tr>
<td>$R^\nu$</td>
<td>1/2</td>
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<td>1/5</td>
<td>0</td>
</tr>
<tr>
<td>$A_1^{vp}$</td>
<td>-1/3</td>
<td>1</td>
<td></td>
<td>1</td>
<td></td>
<td>-1/3</td>
</tr>
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<td>1</td>
<td>1</td>
<td>1</td>
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consistent with duality in $e$ scattering
Fit to

\[ \left\{ \begin{array}{l}
\text{SU}(6) \text{ symmetry at } x \sim 1/3 \\
\text{SU}(6) \text{ breaking at } x \sim 1
\end{array} \right\}

\text{uncertainty in } F_2^n \text{ due to nuclear effects in deuteron}

\[ R^\nu (= d/u) \]
Polarization asymmetry $A_1^p$

![Graph showing polarization asymmetry $A_1^p$ vs. $x$.](image-url)
Polarization asymmetry $A_{1}^{n}$

![Graph showing polarization asymmetry $A_{1}^{n}$]
\[ \frac{\Delta d}{d} = \frac{4}{15} A_1^n \left( 4 + \frac{u}{d} \right) - \frac{1}{15} A_1^p \left( 1 + 4 \frac{u}{d} \right) \quad (= A_1^{\nu p}) \]

\[ \frac{u}{d} = \frac{4 - R^{np}}{4R^{np} - 1} \quad (= 1/R^{\nu}) \]


no sign of pQCD behavior
\[ \text{\( \lambda \) suppression model} \rightarrow \text{identical production rates in } 56^+ \text{ and } 70^- \text{ channels} \]

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<th>( ^28[56^+] )</th>
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<td>( F_1^p )</td>
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<td>18\rho^2 + 9\lambda^2</td>
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<td>0</td>
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for both \( e \) and \( \nu \) scattering

\[\rightarrow\] important test for future \( \nu \) experiments
4. Phenomenological models
Rein, Sehgal (1981): early model of $\pi$ production in $\nu$ scattering

based on relativistic HO model of Feynman, Kislinger & Ravndal (1971)


extended by Bodek, Yang to include DIS region

Bodek, Yang, hep-ph/0411202

Matsui, Sato, Lee (2005): CC and NC $\pi$ production in $\Delta$ region


Parameterize $\nu NN^*$ vertex function with phenomenological form factors

Phenomenological model

Neutrino form factors fitted to neutrino cross section data from BNL, ANL, BEBC, FNAL  (more to come with MINERvA)

Electromagnetic structure functions

Construct structure function from phenomenological \( N \rightarrow N^* \) transition form factors

\[
\delta(W^2 - M_R^2) \rightarrow \frac{M_R \Gamma_R}{\pi} \left( \frac{1}{(W^2 - M_R^2)^2 + M_R^2 \Gamma_R^2} \right)
\]

Lalakulich, WM, Paschos (2005)
Neutrino structure functions

\[ \frac{F_2^{\nu p} + F_2^{\nu n}}{2} \]

\[ \frac{x F_3^{\nu p} + x F_3^{\nu n}}{2} \]

\( Q^2 \) are 0.2, 0.5, 1.0 and 2.0

**Important to understand systematics of duality in \( \nu \) scattering cf. \( e \) scattering**

Lalakulich, WM, Paschos (2005)
Integrated structure functions

\[(F_2^{\gamma p} + F_2^{\gamma n})/2\]

\[(x F_3^{\gamma p} + x F_3^{\gamma n})/2\]

\[\text{Preliminary}\]

Integrated from \(W=1.1\) GeV to 1.6 GeV

\[P_{33}(1232) + D_{13}(1520)\]

Importance of background contribution.
5.

DIS at low $Q^2$
as $Q^2$ decreases, pQCD description (twist expansion) breaks down

near real photon point expand in $Q^2$ rather than $1/Q^2$

intriguing indications of duality even at $Q^2 = 0$

$\sigma_{\gamma p} = X (2M\nu)^{\alpha_p-1} + Y (2M\nu)^{\alpha_R-1}$

Donnachie, Landshoff (1992)
low $Q^2$ behavior constrained by (electromagnetic) gauge invariance

$$F_2(x, Q^2) \to Q^2$$
$$F_L(x, Q^2) \to Q^4$$ as $Q^2 \to 0$

since axial current only partially conserved

$$F_2^\gamma(x, Q^2) \to f_\pi^2 \sigma^{\pi N} \text{ as } Q^2 \to 0$$

model for $F_2^\gamma$ at low $Q^2$

$$F_2^\gamma = Q^2 \left( \frac{f_\rho}{1 + Q^2/m_\rho^2} \right)^2 \sigma^{\rho N} + f_\pi^2 \left( \frac{1}{1 + Q^2/m_{A_1}^2} \right)^2 \sigma^{\pi N}$$

VMD \hspace{1cm} PCAC
gauge invariance or dynamics?

$F_2$ valence-like at low $Q^2$ ?

$\rightarrow$ cf. $xF_3$

gauge invariance or dynamics?

\[ F_2 \sim Q^{0.5} \]


need lower \( Q^2 \) before behavior driven by gauge inv.
Target mass corrections

- Kinematical $1/Q^2$ corrections (twist-2) associated with finite value of $M/Q$
- Important at large $x^2M^2/Q^2$

Christy et al. (2005)
Target mass corrections

- TMCs for weak structure functions calculated by Kretzer & Reno (2004)

- difficulty with (well-known) threshold problem

\[
F_{T}^{\text{TMC}}(x, Q^2) = \frac{x^2}{\xi^2 \gamma} F_{T}^{\text{LT}}(\xi, Q^2) + \frac{2x^3 M^2}{Q^2 \gamma^2} \int_{\xi}^{1} \frac{dz}{z^2} F_{2}^{\text{LT}}(z, Q^2),
\]

\[
F_{2}^{\text{TMC}}(x, Q^2) = \frac{x^2}{\xi^2 \gamma^3} F_{2}^{\text{LT}}(\xi, Q^2) + \frac{6x^3 M^2}{Q^2 \gamma^4} \int_{\xi}^{1} \frac{dz}{z^2} F_{2}^{\text{LT}}(z, Q^2), + O(1/Q^4)
\]

\[
x F_{3}^{\text{TMC}}(x, Q^2) = \frac{x^2}{\xi^2 \gamma^2} \xi F_{3}^{\text{LT}}(\xi, Q^2) + \frac{2x^3 M^2}{Q^2 \gamma^3} \int_{\xi}^{1} \frac{dz}{z^2} z F_{3}^{\text{LT}}(z, Q^2).
\]

\[
\gamma = (1 + 4x^2 M^2 / Q^2)^{1/2}, \quad \xi = 2x/(1 + \gamma)
\]

since \( \xi(x = 1) < 1 \) \( \Rightarrow \) \( F_{i}^{\text{LT}}(\xi, Q^2) > 0 \)

\( \Rightarrow \) \( F_{i}^{\text{TMC}}(x \to 1, Q^2) \neq 0 \)
Target mass corrections

one solution (Kulagin/Petti) - expand in \(1/Q^2\)

\[
F_T^{\text{TMC}}(x, Q^2) = F_T^{\text{LT}}(x, Q^2) + \frac{x^3 M^2}{Q^2} \left( 2 \int_x^1 \frac{dz}{z^2} F_2^{\text{LT}}(z, Q^2) - \frac{\partial}{\partial x} F_T^{\text{LT}}(x, Q^2) \right),
\]

\[
F_2^{\text{TMC}}(x, Q^2) = \left( 1 - \frac{4x^2 M^2}{Q^2} \right) F_2^{\text{LT}}(x, Q^2) + \frac{x^3 M^2}{Q^2} \left( 6 \int_x^1 \frac{dz}{z^2} F_2^{\text{LT}}(z, Q^2) - \frac{\partial}{\partial x} F_2^{\text{LT}}(x, Q^2) \right),
\]

\[
x F_3^{\text{TMC}}(x, Q^2) = \left( 1 - \frac{2x^2 M^2}{Q^2} \right) x F_3^{\text{LT}}(x, Q^2) + \frac{x^3 M^2}{Q^2} \left( 2 \int_x^1 \frac{dz}{z^2} z F_3^{\text{LT}}(z, Q^2) - \frac{\partial}{\partial x} x F_3^{\text{LT}}(x, Q^2) \right).
\]

has correct threshold behavior

alternatively, work with \(\xi_0 = \xi(x = 1)\) dependent PDFs

\( \text{Kulagin, Petti} \)

\( \text{hep-ph/0412425} \)

\( \text{Steffens, WM 2005} \)
Phenomenological higher twists

usually parameterized as

\[ F_2(x, Q^2) = F_2^{LT}(x, Q^2) \left( 1 + \frac{C(x)}{Q^2} \right) \]
Phenomenological higher twists

- more recent JLab data analysis

\[ F_2(x, Q^2) = F_2^{LT}(x, Q^2) \left( 1 + \frac{C_{HT}(x)}{Q^2} + \Delta H(x, Q^2) \right) \]

+ TMC
+ large-\(x\) resummation

large-\(x\) resummation reduces \(C_{HT}\)
lower-\(W\) data require negative \(1/Q^4\) term

Liuti, Ent, Keppel, Niculescu,
Phenomenological higher twists

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Liuti, Ent, Keppel, Niculescu,
extrapolation to low $Q^2$

(Alekhin, Kulagin, Petti 2005)  
(→ talk of R. Petti)

- The leading-twist terms with the NNLO QCD evolution up to $Q^2 = 1$ GeV$^2$ (dominate structure functions for $Q^2 \gtrsim 10$ GeV$^2$).

- Phenomenological higher-twist terms are parameterized as additive corrections $H^{(t)}(x)/Q^{t-2}$. No $Q$–dependence of $H^{(t)}$ is assumed. The $t = 4$ terms are important for $Q^2 \lesssim 10$ GeV$^2$ and the $t = 6$ terms – at $Q^2 \lesssim 3$ GeV$^2$.

- The QCD structure functions are interpolated between $Q^2 = 1$ GeV$^2$ and $Q^2 = 0$ using cubic spline at fixed $x$ and the constraints due to current conservation $F_2 \sim Q^2$ and $F_L \sim Q^4$ as $Q^2 \to 0$. 

• Phenomenological higher twists
Phenomenological higher twists

- extrapolation to low $Q^2$
  (Alekhin, Kulagin, Petti 2005)

Comparison with JLAB data beyond resonance region
($W = 1.9 \div 2$ GeV)
 Phenomenological higher twists

**extrapolation to low $Q^2$**

(Alekhin, Kulagin, Petti 2005)

**NB:** $R^\gamma \nrightarrow 0$ as $Q^2 \rightarrow 0$

large twist 6! convergence?
Summary

- Remarkable confirmation of quark-hadron duality in structure functions
  - higher twists “small” down to low $Q^2$ ($\sim 1$ GeV$^2$)
  - provides quantitative handle on resonance-DIS transition

- Quark models provide clues to origin of resonance cancellations
  - study systematics of local duality in $\nu$ vs. $e$ scattering
  - detailed phenomenological study underway

- Intriguing low-$Q^2$ behavior
  - phenomenological extraction of higher twists
  - $Q^2 \to 0$ constraints for e.m. but different for $\nu$
  - TMCs not completely understood for large $x^2M^2/Q^2$
THE END